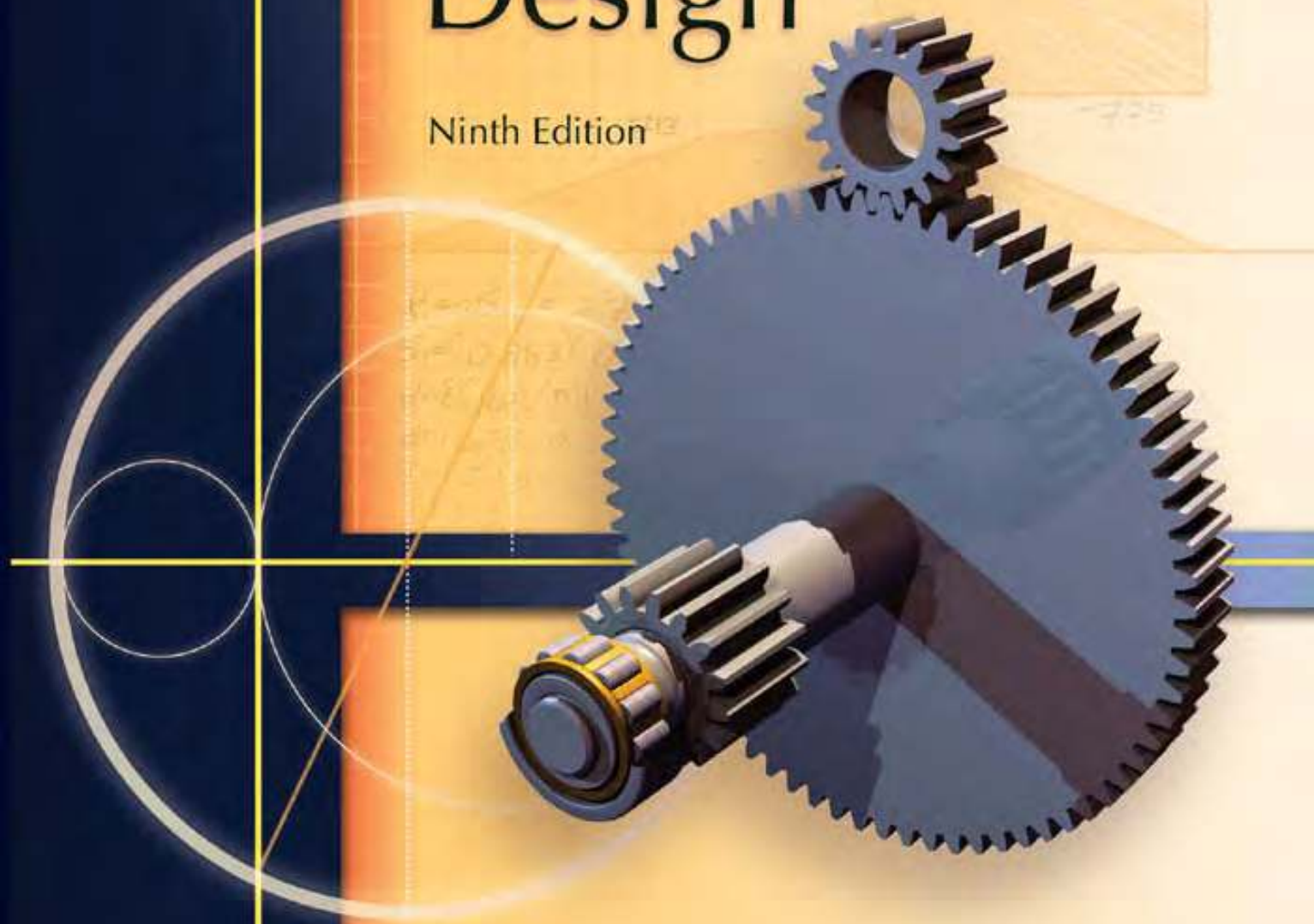
INSTRUCTOR'S SOLUTIONS MANUAL

Shigley's

Mechanical Engineering Design

Ninth Edition



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Chapter 1

Problems **1-1** through **1-6** are for student research. No standard solutions are provided.

1-7 From Fig. 1-2, cost of grinding to ± 0.0005 in is 270%. Cost of turning to ± 0.003 in is 60%.

Relative cost of grinding vs. turning = 270/60 = 4.5 times *Ans.*

1-8 $C_A = C_B$,

$$10 + 0.8\,P = 60 + 0.8\,P - 0.005\,P^2$$

$$P^2 = 50/0.005 \quad \Rightarrow \quad P = 100 \text{ parts} \quad \textit{Ans.}$$

1-9 Max. load = 1.10 P
Min. area = $(0.95)^2 A$
Min. strength = 0.85 S
To offset the absolute uncertainties, the design factor, from Eq. (1-1) should be

$$n_d = \frac{1.10}{0.85(0.95)^2} = 1.43 \qquad \textit{Ans.}$$

1-10 (**a**) $X_1 + X_2$:

$$x_1 + x_2 = X_1 + e_1 + X_2 + e_2$$

$$\text{error } = e = (x_1 + x_2) - (X_1 + X_2)$$

$$= e_1 + e_2 \qquad \textit{Ans.}$$

(**b**) $X_1 - X_2$:

$$x_1 - x_2 = X_1 + e_1 - (X_2 + e_2)$$

$$e = (x_1 - x_2) - (X_1 - X_2) = e_1 - e_2 \qquad \textit{Ans.}$$

(**c**) $X_1\,X_2$:

$$x_1 x_2 = (X_1 + e_1)(X_2 + e_2)$$

$$e = x_1 x_2 - X_1 X_2 = X_1 e_2 + X_2 e_1 + e_1 e_2$$

$$\doteq X_1 e_2 + X_2 e_1 = X_1 X_2 \left(\frac{e_1}{X_1} + \frac{e_2}{X_2} \right) \qquad \textit{Ans.}$$

(**d**) X_1/X_2:

$$\frac{x_1}{x_2} = \frac{X_1 + e_1}{X_2 + e_2} = \frac{X_1}{X_2}\left(\frac{1 + e_1/X_1}{1 + e_2/X_2}\right)$$

$$\left(1 + \frac{e_2}{X_2}\right)^{-1} \doteq 1 - \frac{e_2}{X_2} \quad \text{then} \quad \left(\frac{1 + e_1/X_1}{1 + e_2/X_2}\right) \doteq \left(1 + \frac{e_1}{X_1}\right)\left(1 - \frac{e_2}{X_2}\right) \doteq 1 + \frac{e_1}{X_1} - \frac{e_2}{X_2}$$

Thus, $e = \dfrac{x_1}{x_2} - \dfrac{X_1}{X_2} \doteq \dfrac{X_1}{X_2}\left(\dfrac{e_1}{X_1} - \dfrac{e_2}{X_2}\right)$ *Ans.*

1-11 (**a**) $x_1 = \sqrt{7} = 2.645\ 751\ 311\ 1$

$X_1 = 2.64$ (3 correct digits)

$x_2 = \sqrt{8} = 2.828\ 427\ 124\ 7$

$X_2 = 2.82$ (3 correct digits)

$x_1 + x_2 = 5.474\ 178\ 435\ 8$

$e_1 = x_1 - X_1 = 0.005\ 751\ 311\ 1$

$e_2 = x_2 - X_2 = 0.008\ 427\ 124\ 7$

$e = e_1 + e_2 = 0.014\ 178\ 435\ 8$

Sum $= x_1 + x_2 = X_1 + X_2 + e$

$= 2.64 + 2.82 + 0.014\ 178\ 435\ 8 = 5.474\ 178\ 435\ 8$ Checks

(**b**) $X_1 = 2.65,\ X_2 = 2.83$ (3 digit significant numbers)

$e_1 = x_1 - X_1 = -0.004\ 248\ 688\ 9$

$e_2 = x_2 - X_2 = -0.001\ 572\ 875\ 3$

$e = e_1 + e_2 = -0.005\ 821\ 564\ 2$

Sum $= x_1 + x_2 = X_1 + X_2 + e$

$= 2.65 + 2.83 - 0.001\ 572\ 875\ 3 = 5.474\ 178\ 435\ 8$ Checks

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1-12 $\sigma = \dfrac{S}{n_d} \quad \Rightarrow \quad \dfrac{16(1000)}{\pi d^3} = \dfrac{25(10^3)}{2.5} \quad \Rightarrow \quad d = 0.799 \text{ in}$ *Ans.*

Table A-17: $d = \frac{7}{8}$ in *Ans.*

Factor of safety: $n = \dfrac{S}{\sigma} = \dfrac{25(10^3)}{\dfrac{16(1000)}{\pi\left(\frac{7}{8}\right)^3}} = 3.29$ *Ans.*

1-13 Eq. (1-5): $R = \sum_{i=1}^{n} R_i = 0.98(0.96)0.94 = 0.88$

Overall reliability = 88 percent *Ans.*

1-14 $a = 1.500 \pm 0.001$ in
$b = 2.000 \pm 0.003$ in
$c = 3.000 \pm 0.004$ in
$d = 6.520 \pm 0.010$ in

(**a**) $\bar{w} = \bar{d} - \bar{a} - \bar{b} - \bar{c} = 6.520 - 1.5 - 2 - 3 = 0.020$ in

$t_w = \sum t_{\text{all}} = 0.001 + 0.003 + 0.004 + 0.010 = 0.018$

$w = 0.020 \pm 0.018$ in *Ans.*

(**b**) From part (a), $w_{\min} = 0.002$ in. Thus, must add 0.008 in to $\bar{d}$. Therefore,

$\bar{d} = 6.520 + 0.008 = 6.528$ in *Ans.*

1-15 $V = xyz$, and $x = a \pm \Delta a$, $y = b \pm \Delta b$, $z = c \pm \Delta c$,

$$\bar{V} = abc$$

$$V = (a \pm \Delta a)(b \pm \Delta b)(c \pm \Delta c)$$
$$= abc \pm bc\Delta a \pm ac\Delta b \pm ab\Delta c \pm a\Delta b\Delta c \pm b\Delta c\Delta a \pm c\Delta a\Delta b \pm \Delta a\Delta b\Delta c$$

The higher order terms in Δ are negligible. Thus,

$$\Delta V \doteq bc\Delta a + ac\Delta b + ab\Delta c$$

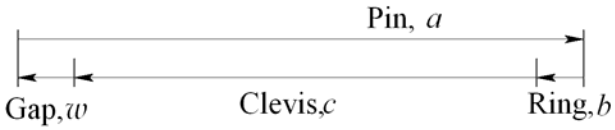
and, $$\frac{\Delta V}{\bar{V}} \doteq \frac{bc\Delta a + ac\Delta b + ab\Delta c}{abc} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} = \frac{\Delta a}{\bar{a}} + \frac{\Delta b}{\bar{b}} + \frac{\Delta c}{\bar{c}}$$ *Ans.*

For the numerical values given, $\bar{V} = 1.500(1.875)3.000 = 8.4375$ in^3

$$\frac{\Delta V}{\bar{V}} = \frac{0.002}{1.500} + \frac{0.003}{1.875} + \frac{0.004}{3.000} = 0.00427 \quad \Rightarrow \quad \Delta V = 0.00427(8.4375) = 0.036 \text{ in}^3$$

$V = 8.438 \pm 0.036$ in^3 *Ans.*

1-16



$w_{max} = 0.05$ in, $w_{min} = 0.004$ in

$$\bar{w} = \frac{0.05 + 0.004}{2} = 0.027 \text{ in}$$

Thus, $\Delta w = 0.05 - 0.027 = 0.023$ in, and then, $w = 0.027 \pm 0.023$ in.

$$\bar{w} = \bar{a} - \bar{b} - \bar{c}$$

$$0.027 = \bar{a} - 0.042 - 1.5$$

$$\bar{a} = 1.569 \text{ in}$$

$$t_w = \sum t_{all} \quad \Rightarrow \quad 0.023 = t_a + 0.002 + 0.005 \quad \Rightarrow \quad t_a = 0.016 \text{ in}$$

Thus, $a = 1.569 \pm 0.016$ in *Ans.*

1-17

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 3.734 + 2(0.139) = 4.012 \text{ in}$$

$$t_{D_o} = \sum t_{all} = 0.028 + 2(0.004) = 0.036 \text{ in}$$

$D_o = 4.012 \pm 0.036$ in *Ans.*

1-18 From O-Rings, Inc. (oringsusa.com), $D_i = 9.19 \pm 0.13$ mm, $d = 2.62 \pm 0.08$ mm

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 9.19 + 2(2.62) = 14.43 \text{ mm}$$

$$t_{D_o} = \sum t_{all} = 0.13 + 2(0.08) = 0.29 \text{ mm}$$

$D_o = 14.43 \pm 0.29$ mm *Ans.*

1-19 From O-Rings, Inc. (oringsusa.com), $D_i = 34.52 \pm 0.30$ mm, $d = 3.53 \pm 0.10$ mm

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 34.52 + 2(3.53) = 41.58 \text{ mm}$$

$$t_{D_o} = \sum t_{all} = 0.30 + 2(0.10) = 0.50 \text{ mm}$$

$D_o = 41.58 \pm 0.50$ mm *Ans.*

1-20 From O-Rings, Inc. (oringsusa.com), $D_i = 5.237 \pm 0.035$ in, $d = 0.103 \pm 0.003$ in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 5.237 + 2(0.103) = 5.443 \quad \text{in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.035 + 2(0.003) = 0.041 \quad \text{in}$$

$D_o = 5.443 \pm 0.041$ in *Ans.*

1-21 From O-Rings, Inc. (oringsusa.com), $D_i = 1.100 \pm 0.012$ in, $d = 0.210 \pm 0.005$ in

$$\bar{D}_o = \bar{D}_i + 2\bar{d} = 1.100 + 2(0.210) = 1.520 \quad \text{in}$$

$$t_{D_o} = \sum t_{\text{all}} = 0.012 + 2(0.005) = 0.022 \quad \text{in}$$

$D_o = 1.520 \pm 0.022$ in *Ans.*

1-22 From Table A-2,

(**a**) $\sigma = 150/6.89 = 21.8$ kpsi *Ans.*

(**b**) $F = 2/4.45 = 0.449$ kip $= 449$ lbf *Ans.*

(**c**) $M = 150/0.113 = 1330$ lbf · in $= 1.33$ kip · in *Ans.*

(**d**) $A = 1500/25.4^2 = 2.33$ in^2 *Ans.*

(**e**) $I = 750/2.54^4 = 18.0$ in^4 *Ans.*

(**f**) $E = 145/6.89 = 21.0$ Mpsi *Ans.*

(**g**) $v = 75/1.61 = 46.6$ mi/h *Ans.*

(**h**) $V = 1000/946 = 1.06$ qt *Ans.*

1-23 From Table A-2,

(**a**) $l = 5(0.305) = 1.53$ m *Ans.*

(**b**) $\sigma = 90(6.89) = 620$ MPa *Ans.*

(**c**) $p = 25(6.89) = 172$ kPa *Ans.*

(**d**) $Z = 12(16.4) = 197\ \text{cm}^3$ *Ans.*

(**e**) $w = 0.208(175) = 36.4$ N/m *Ans.*

(**f**) $\delta = 0.001\ 89(25.4) = 0.0480$ mm *Ans.*

(**g**) $v = 1200(0.0051) = 6.12$ m/s *Ans.*

(**h**) $\epsilon = 0.002\ 15(1) = 0.002\ 15$ mm/mm *Ans.*

(**i**) $V = 1830(25.4^3) = 30.0\ (10^6)\ \text{mm}^3$ *Ans.*

1-24

(**a**) $\sigma = M/Z = 1770/0.934 = 1895$ psi $= 1.90$ kpsi *Ans.*

(**b**) $\sigma = F/A = 9440/23.8 = 397$ psi *Ans.*

(**c**) $y = Fl^3/3EI = 270(31.5)^3/[3(30)10^6(0.154)] = 0.609$ in *Ans.*

(**d**) $\theta = Tl/GJ = 9740(9.85)/[11.3(10^6)(\pi/32)1.00^4] = 8.648(10^{-2})$ rad $= 4.95°$ *Ans.*

1-25

(**a**) $\sigma = F/wt = 1000/[25(5)] = 8$ MPa *Ans.*

(**b**) $I = bh^3/12 = 10(25)^3/12 = 13.0(10^3)\ \text{mm}^4$ *Ans.*

(**c**) $I = \pi d^4/64 = \pi(25.4)^4/64 = 20.4(10^3)\ \text{mm}^4$ *Ans.*

(**d**) $\tau = 16T/\pi d^3 = 16(25)10^3/[\pi(12.7)^3] = 62.2$ MPa *Ans.*

1-26

(**a**) $\tau = F/A = 2\ 700/[\pi(0.750)^2/4] = 6110$ psi $= 6.11$ kpsi *Ans.*

(**b**) $\sigma = 32Fa/\pi d^3 = 32(180)31.5/[\pi(1.25)^3] = 29\ 570$ psi $= 29.6$ kpsi *Ans.*

(**c**) $Z = \pi(d_o^4 - d_i^4)/(32\ d_o) = \pi(1.50^4 - 1.00^4)/[32(1.50)] = 0.266\ \text{in}^3$ *Ans.*

(**d**) $k = (d^4G)/(8D^3N) = 0.0625^4(11.3)10^6/[8(0.760)^3\ 32] = 1.53$ lbf/in *Ans.*

Chapter 2

2-1 From Tables A-20, A-21, A-22, and A-24c,

(**a**) UNS G10200 HR: S_{ut} = 380 (55) MPa (kpsi), S_{yt} = 210 (30) Mpa (kpsi) *Ans.*

(**b**) SAE 1050 CD: S_{ut} = 690 (100) MPa (kpsi), S_{yt} = 580 (84) Mpa (kpsi) *Ans.*

(**c**) AISI 1141 Q&T at 540°C (1000°F): S_{ut} = 896 (130) MPa (kpsi), S_{yt} = 765 (111) Mpa (kpsi) *Ans.*

(**d**) 2024-T4: S_{ut} = 446 (64.8) MPa (kpsi), S_{yt} = 296 (43.0) Mpa (kpsi) *Ans.*

(**e**) Ti-6Al-4V annealed: S_{ut} = 900 (130) MPa (kpsi), S_{yt} = 830 (120) Mpa (kpsi) *Ans.*

2-2 (**a**) Maximize yield strength: Q&T at 425°C (800°F) *Ans.*

(**b**)Maximize elongation: Q&T at 650°C (1200°F) *Ans.*

2-3 Conversion of kN/m^3 to kg/ m^3 multiply by $1(10^3) / 9.81 = 102$

AISI 1018 CD steel: Tables A-20 and A-5

$$\frac{S_y}{\rho} = \frac{370\left(10^3\right)}{76.5(102)} = 47.4 \ \text{kN} \cdot \text{m/kg} \qquad \textit{Ans.}$$

2011-T6 aluminum: Tables A-22 and A-5

$$\frac{S_y}{\rho} = \frac{169\left(10^3\right)}{26.6(102)} = 62.3 \ \text{kN} \cdot \text{m/kg} \qquad \textit{Ans.}$$

Ti-6Al-4V titanium: Tables A-24c and A-5

$$\frac{S_y}{\rho} = \frac{830\left(10^3\right)}{43.4(102)} = 187 \ \text{kN} \cdot \text{m/kg} \qquad \textit{Ans.}$$

ASTM No. 40 cast iron: Tables A-24a and A-5.Does not have a yield strength. Using the ultimate strength in tension

$$\frac{S_{ut}}{\rho} = \frac{42.5(6.89)\left(10^3\right)}{70.6(102)} = 40.7 \ \text{kN} \cdot \text{m/kg} \qquad \textit{Ans}$$

2-4

AISI 1018 CD steel: Table A-5

$$\frac{E}{\gamma} = \frac{30.0\left(10^6\right)}{0.282} = 106\left(10^6\right) \ \text{in} \qquad \textit{Ans.}$$

2011-T6 aluminum: Table A-5

$$\frac{E}{\gamma} = \frac{10.4\left(10^6\right)}{0.098} = 106\left(10^6\right) \ \text{in} \qquad \textit{Ans.}$$

Ti-6Al-6V titanium: Table A-5

$$\frac{E}{\gamma} = \frac{16.5\left(10^6\right)}{0.160} = 103\left(10^6\right) \quad \text{in} \qquad \textit{Ans.}$$

No. 40 cast iron: Table A-5

$$\frac{E}{\gamma} = \frac{14.5\left(10^6\right)}{0.260} = 55.8\left(10^6\right) \quad \text{in} \qquad \textit{Ans.}$$

2-5

$$2G(1+\nu) = E \quad \Rightarrow \quad \nu = \frac{E-2G}{2G}$$

From Table A-5

$$\text{Steel:} \quad \nu = \frac{30.0-2\left(11.5\right)}{2\left(11.5\right)} = 0.304 \qquad \textit{Ans.}$$

$$\text{Aluminum:} \quad \nu = \frac{10.4-2\left(3.90\right)}{2\left(3.90\right)} = 0.333 \qquad \textit{Ans.}$$

$$\text{Beryllium copper:} \quad \nu = \frac{18.0-2\left(7.0\right)}{2\left(7.0\right)} = 0.286 \qquad \textit{Ans.}$$

$$\text{Gray cast iron:} \quad \nu = \frac{14.5-2\left(6.0\right)}{2\left(6.0\right)} = 0.208 \qquad \textit{Ans.}$$

2-6 (**a**) $A_0 = \pi(0.503)^2/4, \ \sigma = P_i / A_0$

For data in elastic range, $\epsilon = \Delta l / l_0 = \Delta l / 2$

For data in plastic range, $\epsilon = \dfrac{\Delta l}{l_0} = \dfrac{l-l_0}{l_0} = \dfrac{l}{l_0} - 1 = \dfrac{A_0}{A} - 1$

On the next two pages, the data and plots are presented. Figure (*a*) shows the linear part of the curve from data points 1-7. Figure (*b*) shows data points 1-12. Figure (*c*) shows the complete range. **Note**: The exact value of A_0 is used without rounding off.

(**b**) From Fig. (*a*) the slope of the line from a linear regression is E = 30.5 Mpsi *Ans.*

From Fig. (*b*) the equation for the dotted offset line is found to be

$$\sigma = 30.5(10^6)\epsilon - 61\ 000 \qquad (1)$$

The equation for the line between data points 8 and 9 is

$$\sigma = 7.60(10^5)\epsilon + 42\ 900 \qquad (2)$$

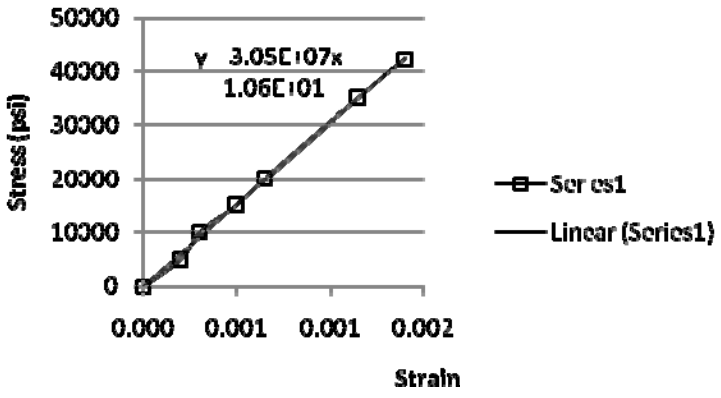
Solving Eqs. (1) and (2) simultaneously yields σ = 45.6 kpsi which is the 0.2 percent offset yield strength. Thus, S_y = 45.6 kpsi *Ans.*

The ultimate strength from Figure (*c*) is S_u = 85.6 kpsi *Ans.*

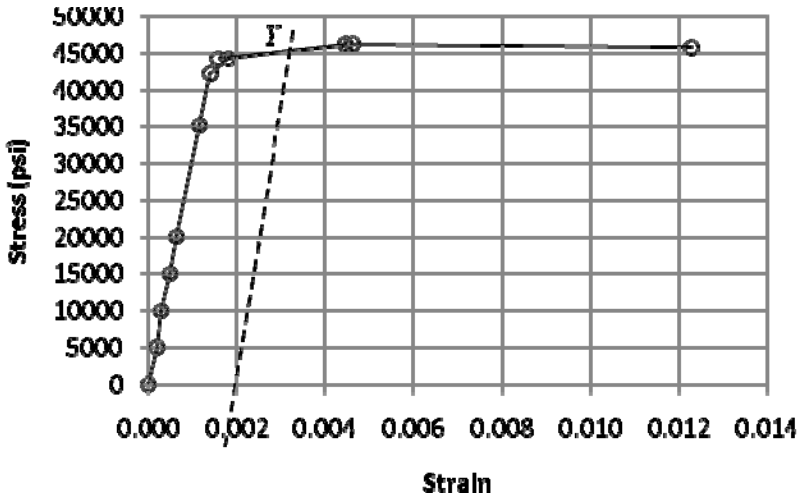
The reduction in area is given by Eq. (2-12) is

$$R = \frac{A_0 - A_f}{A_0}(100) = \frac{0.1987 - 0.1077}{0.1987}(100) = 45.8\ \%$$ *Ans.*

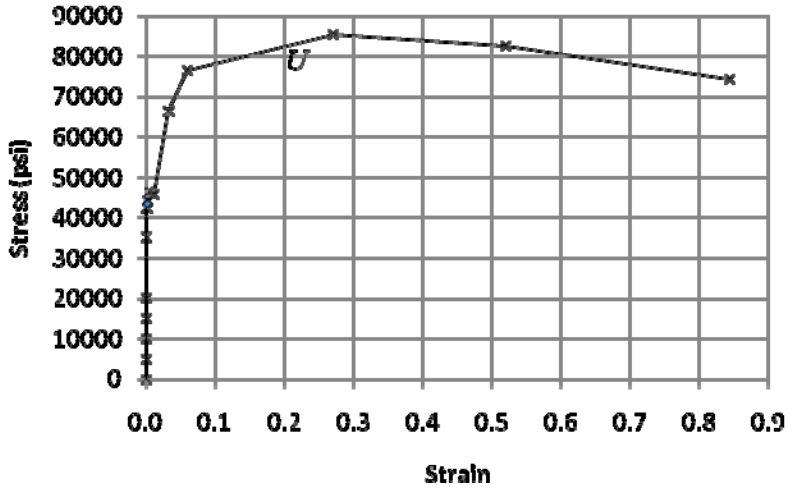
Data Point	P_i	Δl, A_i	ϵ	σ
1	0	0	0	0
2	1000	0.0004	0.00020	5032
3	2000	0.0006	0.00030	10065
4	3000	0.001	0.00050	15097
5	4000	0.0013	0.00065	20130
6	7000	0.0023	0.00115	35227
7	8400	0.0028	0.00140	42272
8	8800	0.0036	0.00180	44285
9	9200	0.0089	0.00445	46298
10	8800	0.1984	0.00158	44285
11	9200	0.1978	0.00461	46298
12	9100	0.1963	0.01229	45795
13	13200	0.1924	0.03281	66428
14	15200	0.1875	0.05980	76492
15	17000	0.1563	0.27136	85551
16	16400	0.1307	0.52037	82531
17	14800	0.1077	0.84506	74479



(*a*) Linear range



(*b*) Offset yield



(*c*) Complete range

(**c**) The material is ductile since there is a large amount of deformation beyond yield.

(**d**) The closest material to the values of S_y, S_{ut}, and R is SAE 1045 HR with S_y = 45 kpsi, S_{ut} = 82 kpsi, and R = 40 %. *Ans.*

2-7 To plot σ_{true} vs. ε, the following equations are applied to the data.

$$\sigma_{\text{true}} = \frac{P}{A}$$

Eq. (2-4)

$$\varepsilon = \ln \frac{l}{l_0} \quad \text{for } 0 \le \Delta l \le 0.0028 \text{ in}$$

$$\varepsilon = \ln \frac{A_0}{A} \quad \text{for } \Delta l > 0.0028 \text{ in}$$

where $A_0 = \dfrac{\pi(0.503)^2}{4} = 0.1987 \text{ in}^2$

The results are summarized in the table below and plotted on the next page. The last 5 points of data are used to plot log σ vs log ε

The curve fit gives $m = 0.2306$

$\log \sigma_0 = 5.1852 \Rightarrow \sigma_0 = 153.2$ kpsi *Ans.*

For 20% cold work, Eq. (2-14) and Eq. (2-17) give,

$$A = A_0 (1 - W) = 0.1987 (1 - 0.2) = 0.1590 \text{ in}^2$$

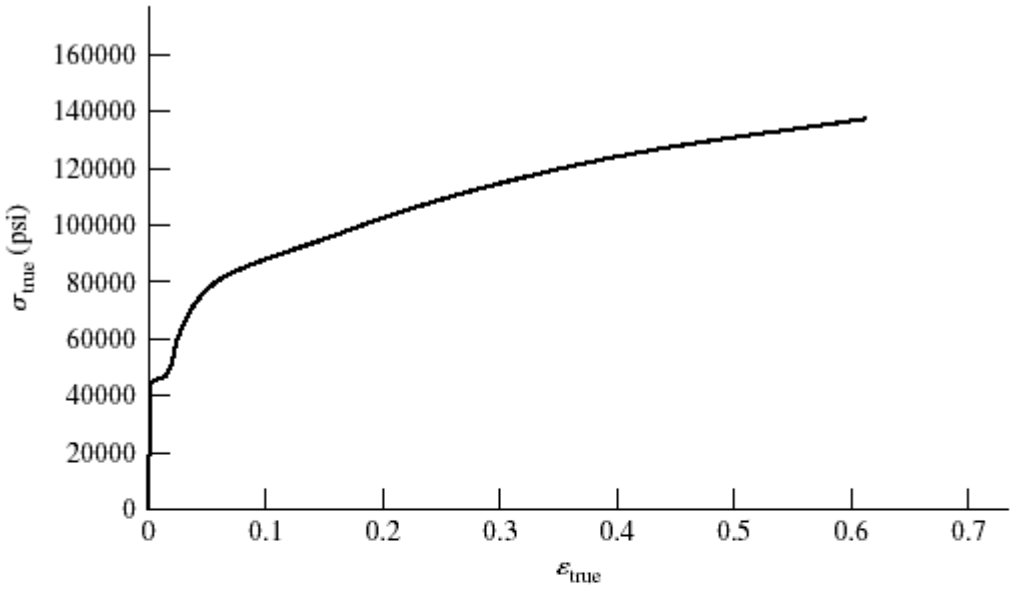
$$\varepsilon = \ln \frac{A_0}{A} = \ln \frac{0.1987}{0.1590} = 0.2231$$

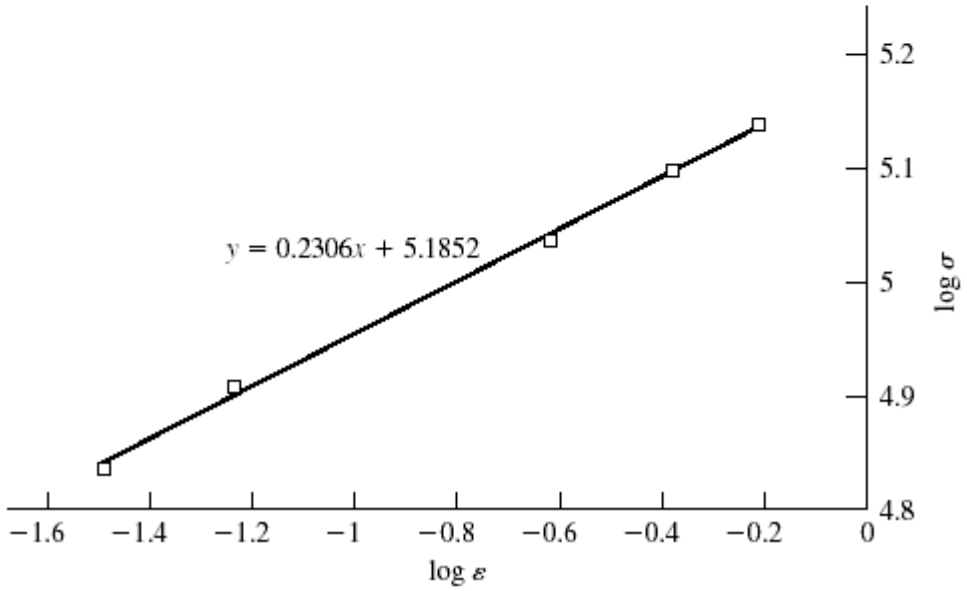
Eq. (2-18): $S_{y'} = \sigma_0 \varepsilon^m = 153.2(0.2231)^{0.2306} = 108.4$ kpsi *Ans.*

Eq. (2-19), with $S_u = 85.6$ from Prob. 2-6,

$$S'_u = \frac{S_u}{1-W} = \frac{85.6}{1-0.2} = 107 \text{ kpsi} \quad \textit{Ans.}$$

P	ΔL	A	ε	σ_{true}	$\log \varepsilon$	$\log \sigma_{\text{true}}$
0	0	0.198 713	0	0		
1000	0.0004	0.198 713	0.000 2	5032.388	-3.699 01	3.701 774
2000	0.0006	0.198 713	0.000 3	10 064.78	-3.522 94	4.002 804
3000	0.001	0.198 713	0.000 5	15 097.17	-3.301 14	4.178 895
4000	0.0013	0.198 713	0.000 65	20 129.55	-3.187 23	4.303 834
7000	0.0023	0.198 713	0.001 149	35 226.72	-2.939 55	4.546 872
8400	0.0028	0.198 713	0.001 399	42 272.06	-2.854 18	4.626 053
8800	0.0036	0.198 4	0.001 575	44 354.84	-2.802 61	4.646 941
9200	0.0089	0.197 8	0.004 604	46 511.63	-2.336 85	4.667 562
9100		0.196 3	0.012 216	46 357.62	-1.913 05	4.666 121
13200		0.192 4	0.032 284	68 607.07	-1.491 01	4.836 369
15200		0.187 5	0.058 082	81 066.67	-1.235 96	4.908 842
17000		0.156 3	0.240 083	108 765.20	-0.619 64	5.036 49
16400		0.130 7	0.418 956	125 478.20	-0.377 83	5.098 568
14800		0.107 7	0.612 511	137 418.80	-0.212 89	5.138 046





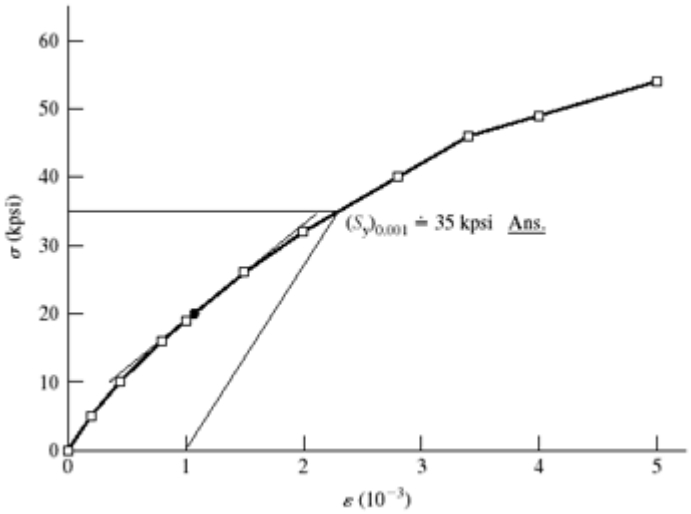
2-8 Tangent modulus at σ = 0 is

$$E = \frac{\Delta\sigma}{\Delta\epsilon} \doteq \frac{5000 - 0}{0.2\left(10^{-3}\right) - 0} = 25\left(10^{6}\right) \text{ psi} \qquad \textit{Ans.}$$

At σ = 20 kpsi

$$E_{20} \doteq \frac{(26-19)(10^3)}{(1.5-1)(10^{-3})} = 14.0(10^6) \text{ psi} \qquad \textit{Ans.}$$

ϵ (10^{-3})	σ (kpsi)
0	0
0.20	5
0.44	10
0.80	16
1.0	19
1.5	26
2.0	32
2.8	40
3.4	46
4.0	49
5.0	54



2-9 $W = 0.20$,

(**a**) Before cold working: Annealed AISI 1018 steel. Table A-22, S_y = 32 kpsi, S_u = 49.5 kpsi, σ_0 = 90.0 kpsi, m = 0.25, ϵ_f = 1.05

After cold working: Eq. (2-16), $\epsilon_u = m = 0.25$

Eq. (2-14), $$\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$$

Eq. (2-17), $$\varepsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 < \varepsilon_u$$

Eq. (2-18), $$S_y' = \sigma_0 \varepsilon_i^m = 90(0.223)^{0.25} = 61.8 \text{ kpsi} \quad \textit{Ans.} \quad 93\% \text{ increase} \quad \textit{Ans.}$$

Eq. (2-19), $$S_u' = \frac{S_u}{1-W} = \frac{49.5}{1-0.20} = 61.9 \text{ kpsi} \quad \textit{Ans.} \quad 25\% \text{ increase} \quad \textit{Ans.}$$

(**b**) Before: $$\frac{S_u}{S_y} = \frac{49.5}{32} = 1.55 \qquad \text{After:} \quad \frac{S_u'}{S_y'} = \frac{61.9}{61.8} = 1.00 \quad \textit{Ans.}$$

Lost most of its ductility

2-10 $W = 0.20$,

(**a**) Before cold working: AISI 1212 HR steel. Table A-22, S_y = 28 kpsi, S_u = 61.5 kpsi, σ_0 = 110 kpsi, m = 0.24, ϵ_f = 0.85

After cold working: Eq. (2-16), $\epsilon_u = m = 0.24$

Eq. (2-14), $\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$

Eq. (2-17), $\varepsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 < \varepsilon_u$

Eq. (2-18), $S_y' = \sigma_0 \varepsilon_i^m = 110\left(0.223\right)^{0.24} = 76.7$ kpsi *Ans.* 174% increase *Ans.*

Eq. (2-19), $S_u' = \frac{S_u}{1-W} = \frac{61.5}{1-0.20} = 76.9$ kpsi *Ans.* 25% increase *Ans.*

(**b**) Before: $\frac{S_u}{S_y} = \frac{61.5}{28} = 2.20$ After: $\frac{S_u'}{S_y'} = \frac{76.9}{76.7} = 1.00$ *Ans.*

Lost most of its ductility

2-11 $W = 0.20$,

(**a**) Before cold working: 2024-T4 aluminum alloy. Table A-22, S_y = 43.0 kpsi, S_u = 64.8 kpsi, σ_0 = 100 kpsi, m = 0.15, ϵ_f = 0.18

After cold working: Eq. (2-16), $\epsilon_u = m = 0.15$

Eq. (2-14), $\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$

Eq. (2-17), $\varepsilon_i = \ln \frac{A_0}{A_i} = \ln 1.25 = 0.223 > \varepsilon_f$ Material fractures. *Ans.*

2-12 For H_B = 275, Eq. (2-21), S_u = 3.4(275) = 935 MPa *Ans.*

2-13 Gray cast iron, H_B = 200.

Eq. (2-22), S_u = 0.23(200) – 12.5 = 33.5 kpsi *Ans.*

From Table A-24, this is probably ASTM No. 30 Gray cast iron *Ans.*

2-14 Eq. (2-21), $0.5H_B = 100 \Rightarrow H_B = 200$ *Ans.*

2-15 For the data given, converting H_B to S_u using Eq. (2-21)

H_B	S_u (kpsi)	S_u^2 (kpsi)
230	115	13225
232	116	13456
232	116	13456
234	117	13689
235	117.5	13806.25
235	117.5	13806.25
235	117.5	13806.25
236	118	13924
236	118	13924
239	119.5	14280.25
$\Sigma S_u =$	1172	$\Sigma S_u^2 =$ 137373

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{1172}{10} = 117.2 \doteq 117 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{137373 - 10(117.2)^2}{9}} = 1.27 \text{ kpsi} \qquad \textit{Ans.}$$

2-16 For the data given, converting H_B to S_u using Eq. (2-22)

H_B	S_u (kpsi)	S_u^2 (kpsi)
230	40.4	1632.16
232	40.86	1669.54
232	40.86	1669.54
234	41.32	1707.342
235	41.55	1726.403
235	41.55	1726.403
235	41.55	1726.403
236	41.78	1745.568
236	41.78	1745.568
239	42.47	1803.701
$\Sigma S_u =$	414.12	$\Sigma S_u^2 =$ 17152.63

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{414.12}{10} = 41.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{17152.63 - 10(41.4)^2}{9}} = 1.20 \qquad \textit{Ans.}$$

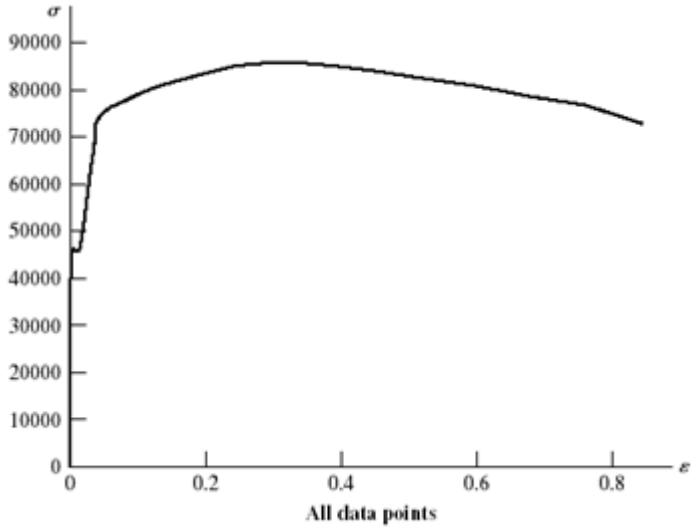
2-17 (**a**) $$u_R \doteq \frac{45.5^2}{2(30)} = 34.5 \text{ in} \cdot \text{lbf} / \text{in}^3 \quad \textit{Ans.}$$

(**b**)

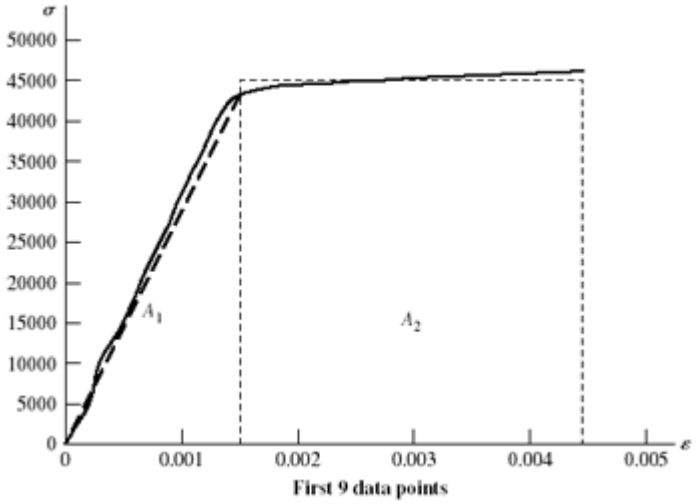
P	ΔL	A	$A_0 / A - 1$	ϵ	$\sigma = P/A_0$
0	0			0	0
1000	0.0004			0.0002	5 032.39
2000	0.0006			0.0003	10 064.78
3000	0.0010			0.0005	15 097.17
4000	0.0013			0.000 65	20 129.55
7000	0.0023			0.001 15	35 226.72
8400	0.0028			0.0014	42 272.06
8800	0.0036			0.0018	44 285.02
9200	0.0089			0.004 45	46 297.97
9100		0.1963	0.012 291	0.012 291	45 794.73
13200		0.1924	0.032 811	0.032 811	66 427.53
15200		0.1875	0.059 802	0.059 802	76 492.30
17000		0.1563	0.271 355	0.271 355	85 550.60
16400		0.1307	0.520 373	0.520 373	82 531.17
14800		0.1077	0.845 059	0.845 059	74 479.35

From the figures on the next page,

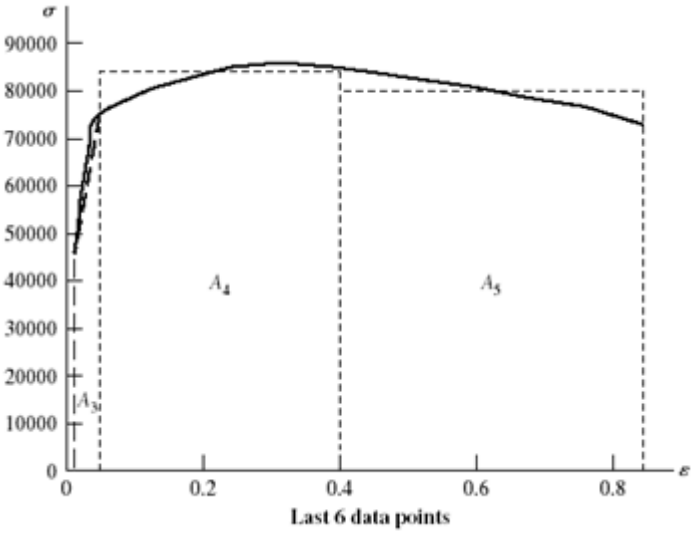
$$\begin{aligned} u_T &\doteq \sum_{i=1}^{5} A_i = \frac{1}{2}(43\ 000)(0.001\ 5) + 45\ 000(0.004\ 45 - 0.001\ 5) \\ &\quad + \frac{1}{2}(45\ 000 + 76\ 500) + (0.059\ 8 - 0.004\ 45) \\ &\quad + 81\ 000(0.4 - 0.059\ 8) + 80\ 000(0.845 - 0.4) \\ &\doteq 66.7\left(10^3\right) \text{in} \cdot \text{lbf/in}^3 \quad \textit{Ans.} \end{aligned}$$



All data points



First 9 data points



Last 6 data points

2-18, 2-19 These problems are for student research. No standard solutions are provided.

2-20 Appropriate tables: Young's modulus and Density (Table A-5)1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:

For diameter, $\sigma = \dfrac{F}{A} = \dfrac{F}{(\pi/4)d^2} = S_y \qquad \Rightarrow \qquad d = \sqrt{\dfrac{4F}{\pi S_y}}$

Weight/length = ρA, Cost/length = \$/in = (\$/lbf) Weight/length,
Deflection/length = $\delta/L = F/(AE)$

With F = 100 kips = 100(10^3) lbf,

Material	Young's Modulus	Density	Yield Strength	Cost/lbf	Diameter	Weight/ length	Cost/ length	Deflection/ length
units	Mpsi	lbf/in^3	kpsi	\$/lbf	in	lbf/in	\$/in	in/in
1020 HR	30	0.282	30	\$0.27	2.060	0.9400	\$0.25	1.000E-03
1020 CD	30	0.282	57	\$0.30	1.495	0.4947	\$0.15	1.900E-03
1040	30	0.282	80	\$0.35	1.262	0.3525	\$0.12	2.667E-03
4140	30	0.282	165	\$0.80	0.878	0.1709	\$0.14	5.500E-03
Al	10.4	0.098	50	\$1.10	1.596	0.1960	\$0.22	4.808E-03
Ti	16.5	0.16	120	\$7.00	1.030	0.1333	\$0.93	7.273E-03

The selected materials with minimum values are shaded in the table above. *Ans.*

2-21 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three would favor steel, cast iron, or maybe a less common ferrous material. The expectation would likely be hot-rolled steel. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 7.95 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{7.95 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.281 \text{ lbf/in}^3 \doteq 0.28 \text{ lbf/in}^3$$

which agrees well with the unit weight of 0.282 lbf/in^3 reported in Table A-5 for carbon steel. Nickel steel and stainless steel have similar unit weights, but surface finish and darker coloring do not favor their selection. To select a likely specification from Table

A-20, perform a Brinell hardness test, then use Eq. (2-21) to estimate an ultimate strength of $S_u = 0.5H_B = 0.5(200) = 100$ kpsi. Assuming the material is hot-rolled due to the rough surface finish, appropriate choices from Table A-20 would be one of the higher carbon steels, such as hot-rolled AISI 1050, 1060, or 1080. *Ans.*

2-22 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous material like aluminum. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 2.90 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{2.9 \text{ lbf}}{[\pi(1 \text{ in})^2 / 4](36 \text{ in})} = 0.103 \text{ lbf/in}^3 \doteq 0.10 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.098 lbf/in^3 reported in Table A-5 for aluminum. No other materials come close to this unit weight, so the material is likely aluminum. *Ans.*

2-23 First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous copper-based material such as copper, brass, or bronze. To further distinguish the material, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 9 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{9.0 \text{ lbf}}{[\pi(1 \text{ in})^2 / 4](36 \text{ in})} = 0.318 \text{ lbf/in}^3 \doteq 0.32 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.322 lbf/in^3 reported in Table A-5 for copper. Brass is not far off (0.309 lbf/in^3), so the deflection test could be used to gain additional insight. From the measured deflection and utilizing the deflection equation for an end-loaded cantilever beam from Table A-9, Young's modulus is determined to be

$$E = \frac{Fl^3}{3Iy} = \frac{100\left(24\right)^3}{3\left(\pi(1)^4/64\right)(17/32)} = 17.7 \text{ Mpsi}$$

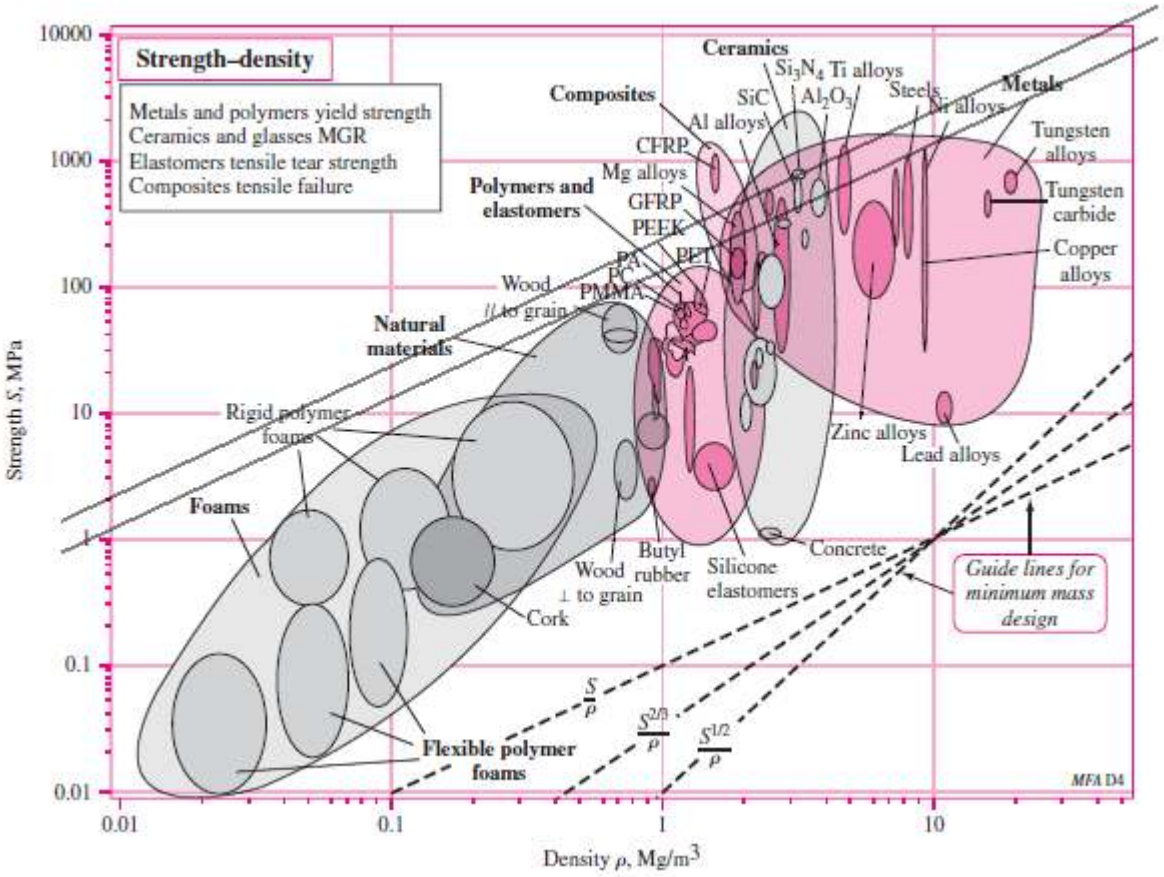
which agrees better with the modulus for copper (17.2 Mpsi) than with brass (15.4 Mpsi). The conclusion is that the material is likely copper. *Ans.*

2-24 and 2-25 These problems are for student research. No standard solutions are provided.

2-26 For strength, $\sigma = F/A = S \quad \Rightarrow \quad A = F/S$
For mass, $m = Al\rho = (F/S)\, l\rho$

Thus, $\quad f_3(M) = \rho/S$, and maximize $S/\rho \quad (\beta = 1)$

In Fig. (2-19), draw lines parallel to S/ρ

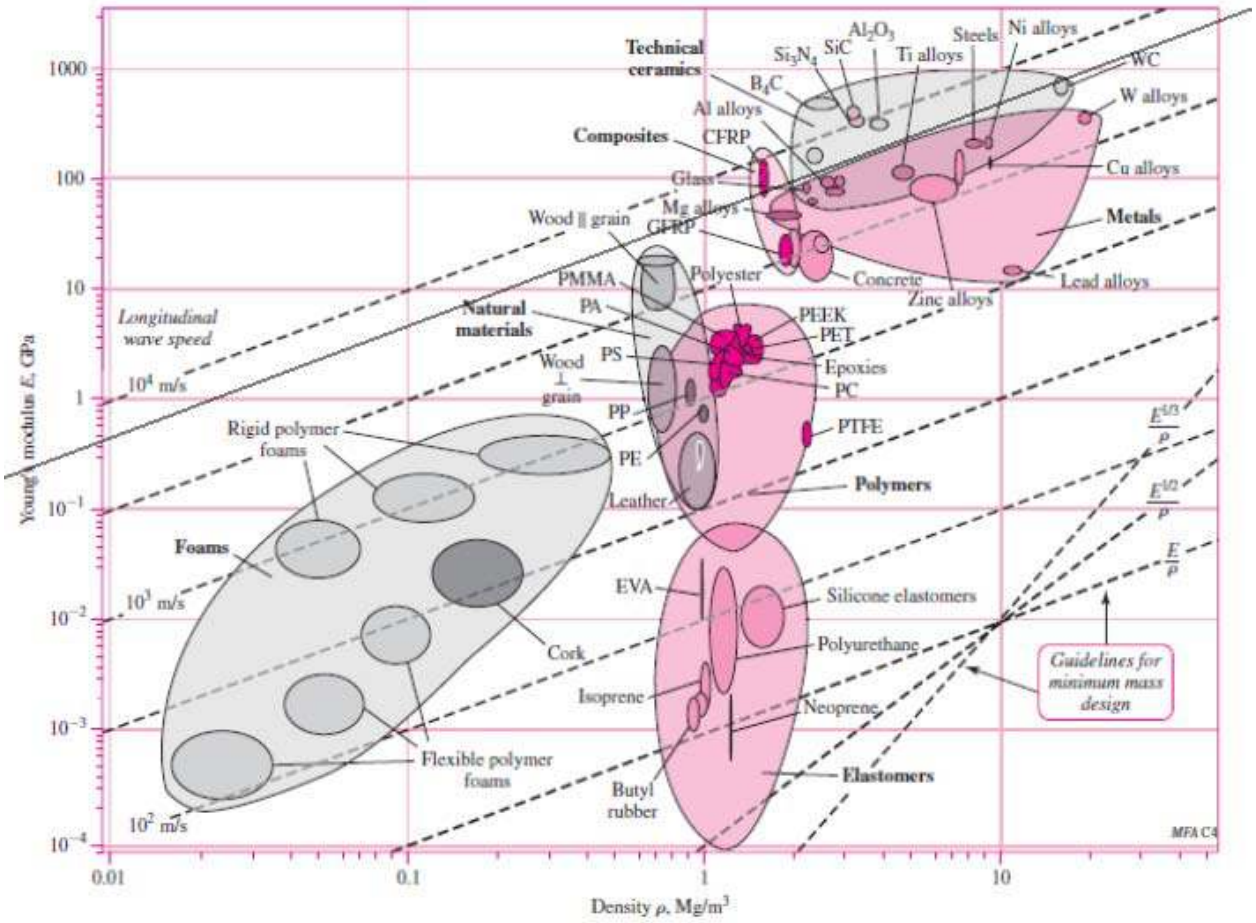


From the list of materials given, both **aluminum alloy** and **high carbon heat treated steel** are good candidates, having greater potential than tungsten carbide or polycarbonate. The higher strength aluminum alloys have a slightly greater potential. Other factors, such as cost or availability, may dictate which to choose. *Ans*.

2-27 For stiffness, $k = AE/l \quad \Rightarrow \quad A = kl/E$
For mass, $m = Al\rho = (kl/E)\, l\rho = kl^2\, \rho/E$

Thus, $\quad f_3(M) = \rho/E$, and maximize $E/\rho \quad (\beta = 1)$

In Fig. (2-16), draw lines parallel to E/ρ



From the list of materials given, **tungsten carbide** (WC) is best, closely followed by aluminum alloys, and then followed by high carbon heat-treated steel. They are close enough that other factors, like cost or availability, would likely dictate the best choice. Polycarbonate polymer is clearly not a good choice compared to the other candidate materials. *Ans.*

2-28 For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

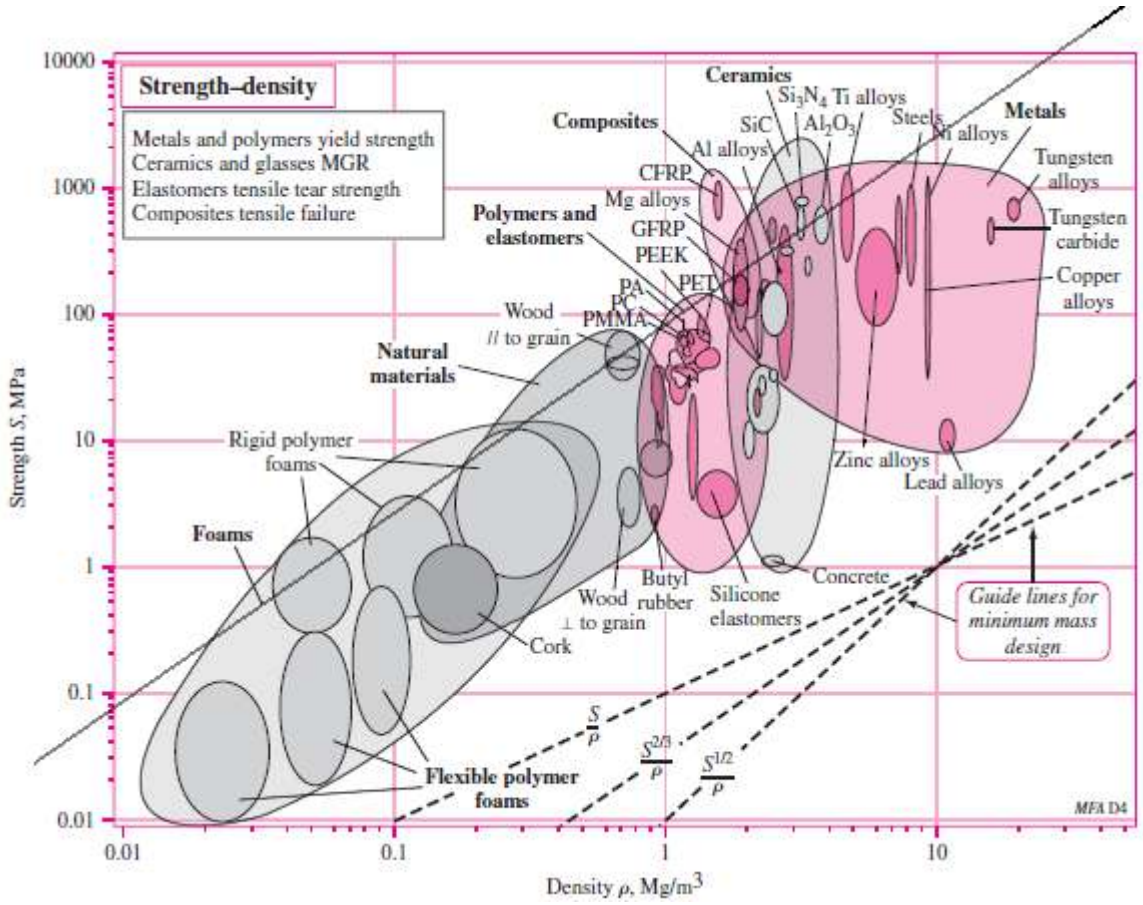
where Fl is the bending moment and Z is the section modulus [see Eq. (3-26*b*), p. 90]. The section modulus is strictly a function of the dimensions of the cross section and has the units in^3 (ips) or m^3 (SI). Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where C is a number. For example, for a circular cross section, $C = \left(4\sqrt{\pi}\right)^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass, $$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3}\left(\frac{\rho}{S^{2/3}}\right)$$

Thus, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$ $(\beta = 2/3)$

In Fig. (2-19), draw lines parallel to $S^{2/3}/\rho$



From the list of materials given, a higher strength **aluminum alloy** has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. *.Ans*.

2-29 Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For example, consider a rectangular section of height h and width b, where for a given scaled shape, $h = cb$, where c is a

constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$ (a constant).

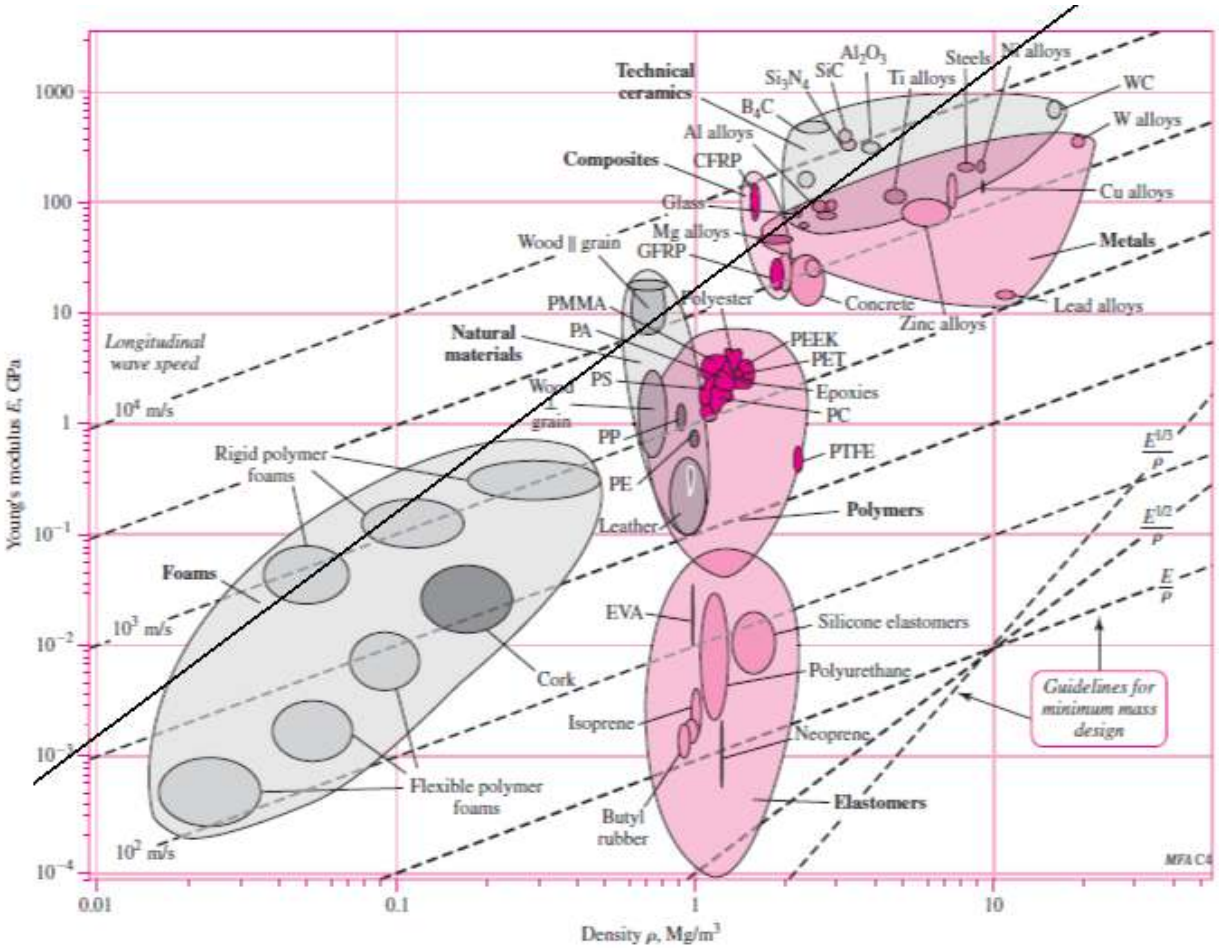
Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE}\right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C}\right)^{1/2} l^{5/2}\left(\frac{\rho}{E^{1/2}}\right)$$

Thus, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. From Fig. (2-16)



From the list of materials given, **aluminum alloys** are clearly the best followed by steels and tungsten carbide. Polycarbonate polymer is not a good choice compared to the other candidate materials. *Ans*.

2-30 For stiffness, $k = AE/l \;\Rightarrow\; A = kl/E$

For mass, $m = Al\rho = (kl/E)\, l\rho = kl^2\,\rho/E$

So, $f_3(M) = \rho/E$, and maximize E/ρ. Thus, $\beta = 1$. *Ans.*

2-31 For strength, $\sigma = F/A = S \Rightarrow A = F/S$
For mass, $m = Al\rho = (F/S)\, l\rho$

So, $f_3(M) = \rho/S$, and maximize S/ρ. Thus, $\beta = 1$. *Ans.*

2-32 Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where C is a constant. For example, consider a rectangular section of height h and width b, where for a given scaled shape, $h = cb$, where c is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$.

Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE}\right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C}\right)^{1/2} l^{5/2}\left(\frac{\rho}{E^{1/2}}\right)$$

So, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. Thus, $\beta = 1/2$. *Ans.*

2-33 For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

where Fl is the bending moment and Z is the section modulus [see Eq. (3-26*b*), p. 90]. The section modulus is strictly a function of the dimensions of the cross section and has the units in^3 (ips) or m^3 (SI). Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where C is a number. For example, for a circular cross section, $C = \left(4\sqrt{\pi}\right)^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \qquad \Rightarrow \qquad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass,
$$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3}\left(\frac{\rho}{S^{2/3}}\right)$$

So, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$. Thus, $\beta = 2/3$. *Ans.*

2-34 <u>For stiffness</u>, $k = AE/l$, or, $A = kl/E$.

Thus, $m = \rho Al = \rho (kl/E)l = kl^2 \rho/E$. Then, $M = E/\rho$ and $\beta = 1$.

From Fig. 2-16, lines parallel to E/ρ for ductile materials include steel, titanium, molybdenum, aluminum alloys, and composites.

For strength, $S = F/A$, or, $A = F/S$.

Thus, $m = \rho Al = \rho F/Sl = Fl \rho/S$. Then, $M = S/\rho$ and $\beta = 1$.

From Fig. 2-19, lines parallel to S/ρ give for ductile materials, steel, aluminum alloys, nickel alloys, titanium, and composites.

Common to both stiffness and strength are steel, titanium, aluminum alloys, and composites. *Ans.*

Chapter 3

3-1

$\Sigma M_o = 0$

$18R_B - 6(100) = 0$

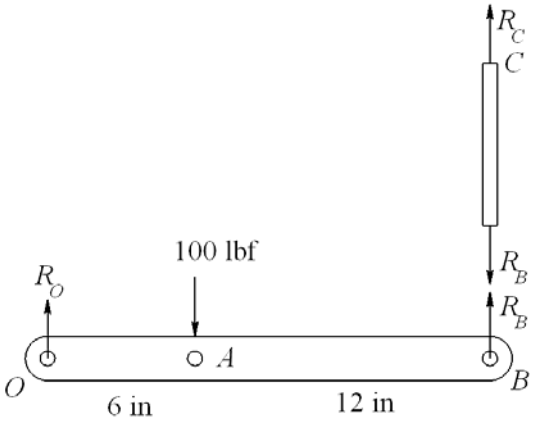
$R_B = 33.3$ lbf *Ans.*

$\Sigma F_y = 0$

$R_o + R_B - 100 = 0$

$R_o = 66.7$ lbf *Ans.*

$R_C = R_B = 33.3$ lbf *Ans.*



3-2

Body *AB*:

$\Sigma F_x = 0 \qquad R_{Ax} = R_{Bx}$

$\Sigma F_y = 0 \qquad R_{Ay} = R_{By}$

$\Sigma M_B = 0 \qquad R_{Ay}(10) - R_{Ax}(10) = 0$

$R_{Ax} = R_{Ay}$

Body *OAC*:

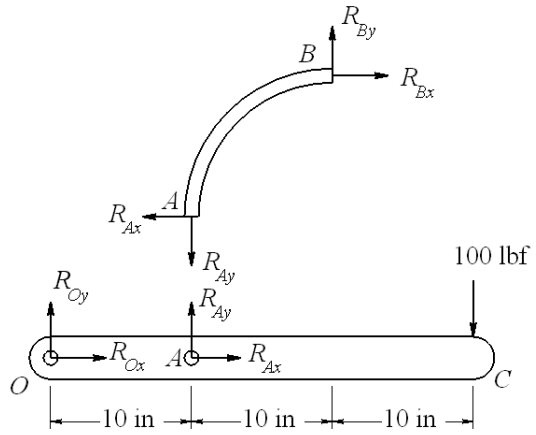
$\Sigma M_O = 0 \qquad R_{Ay}(10) - 100(30) = 0$

$R_{Ay} = 300$ lbf *Ans.*

$\Sigma F_x = 0 \qquad R_{Ox} = -R_{Ax} = -300$ lbf *Ans.*

$\Sigma F_y = 0 \qquad R_{Oy} + R_{Ay} - 100 = 0$

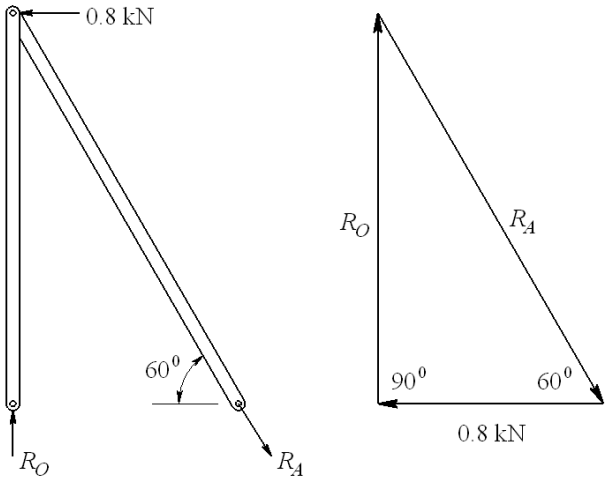
$R_{Oy} = -200$ lbf *Ans.*



3-3

$$R_O = \frac{0.8}{\tan 30^\circ} = 1.39 \text{ kN} \quad Ans.$$

$$R_A = \frac{0.8}{\sin 30^\circ} = 1.6 \text{ kN} \quad Ans.$$



3-4

Step 1: Find R_A & R_E

$$h = \frac{4.5}{\tan 30^\circ} = 7.794 \text{ m}$$

$$\Sigma M_A = 0$$

$$9R_E - 7.794(400\cos 30^\circ) - 4.5(400\sin 30^\circ) = 0$$

$$R_E = 400 \text{ N} \quad Ans.$$

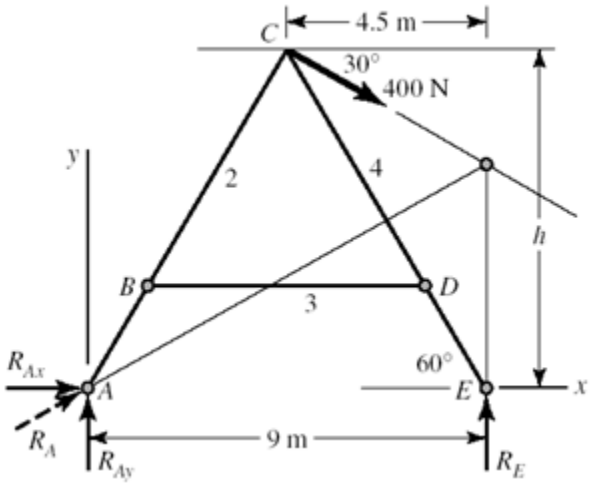
$$\sum F_x = 0 \quad R_{Ax} + 400\cos 30^\circ = 0$$

$$R_{Ax} = -346.4 \text{ N}$$

$$\sum F_y = 0 \quad R_{Ay} + 400 - 400\sin 30^\circ = 0$$

$$R_{Ay} = -200 \text{ N}$$

$$R_A = \sqrt{346.4^2 + 200^2} = 400 \text{ N} \quad Ans.$$



Step 2: Find components of R_C on link 4 and R_D

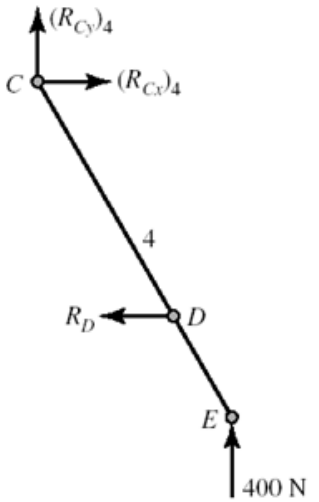
$$\sum M_C = 0$$

$$400(4.5) - (7.794 - 1.9)R_D = 0$$

$$R_D = 305.4 \text{ N} \quad Ans.$$

$$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4 \text{ N}$$

$$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400 \text{ N}$$



Step 3: Find components of R_C on link 2

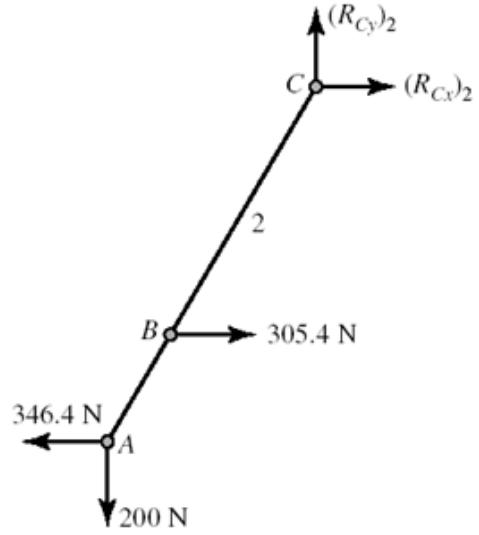
$$\sum F_x = 0$$

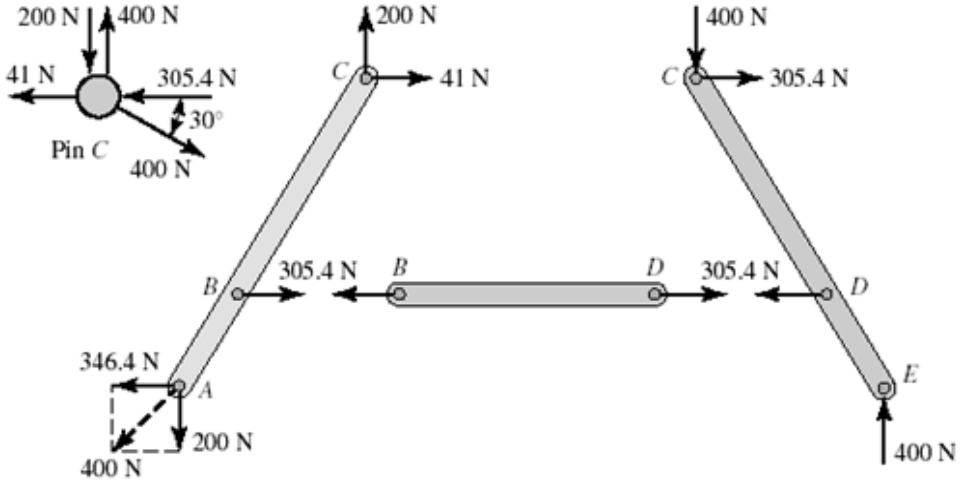
$$(R_{Cx})_2 + 305.4 - 346.4 = 0$$

$$(R_{Cx})_2 = 41 \text{ N}$$

$$\sum F_y = 0$$

$$(R_{Cy})_2 = 200 \text{ N}$$





3-5

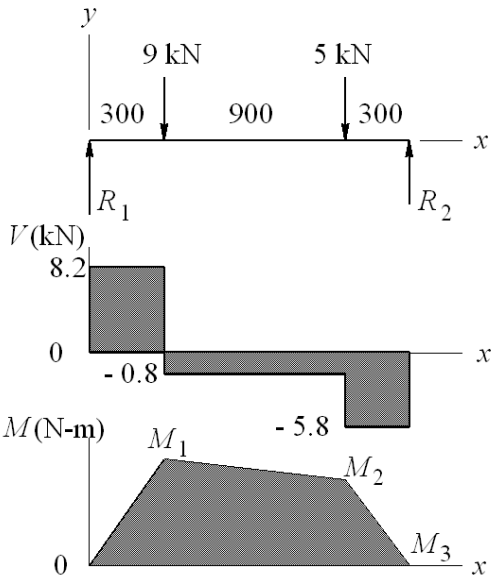
$\Sigma M_C = 0$

$-1500R_1 + 300(5) + 1200(9) = 0$

$R_1 = 8.2 \text{ kN}$ *Ans.*

$\Sigma F_y = 0$

$8.2 - 9 - 5 + R_2 = 0 \;\; R_2 = 5.8 \text{ kN}$ *Ans.*



$M_1 = 8.2(300) = 2460 \text{ N} \cdot \text{m}$ *Ans.*

$M_2 = 2460 - 0.8(900) = 1740 \text{ N} \cdot \text{m}$ *Ans.*

$M_3 = 1740 - 5.8(300) = 0$ checks!

3-6

$\Sigma F_y = 0$

$R_0 = 500 + 40(6) = 740 \text{ lbf}$ *Ans.*

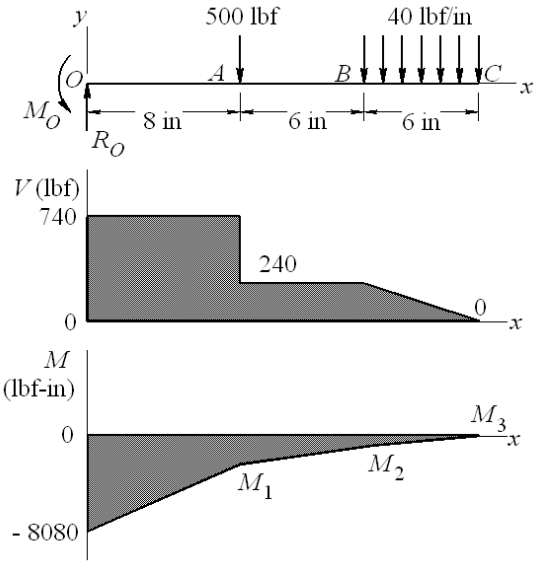
$\Sigma M_0 = 0$

$M_0 = 500(8) + 40(6)(17) = 8080 \text{ lbf} \cdot \text{in}$ *Ans.*

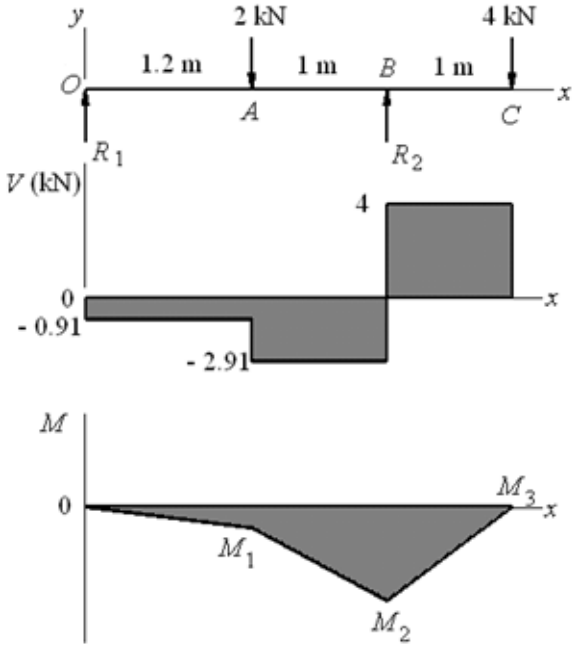
$M_1 = -8080 + 740(8) = -2160 \text{ lbf} \cdot \text{in}$ *Ans.*

$M_2 = -2160 + 240(6) = -720 \text{ lbf} \cdot \text{in}$ *Ans.*

$M_3 = -720 + \frac{1}{2}(240)(6) = 0$ checks!



3-7



$\Sigma M_B = 0$

$-2.2R_1 + 1(2) - 1(4) = 0$

$R_1 = -0.91$ kN *Ans.*

$\Sigma F_y = 0$

$-0.91 - 2 + R_2 - 4 = 0$

$R_2 = 6.91$ kN *Ans.*

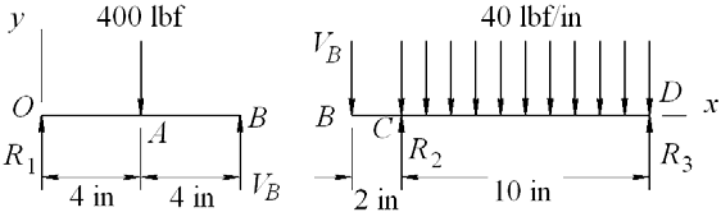
$M_1 = -0.91(1.2) = -1.09$ kN · m *Ans.*

$M_2 = -1.09 - 2.91(1) = -4$ kN · m *Ans.*

$M_3 = -4 + 4(1) = 0$ checks!

3-8

Break at the hinge at *B*



Beam *OB*:

From symmetry,

$R_1 = V_B = 200$ lbf *Ans.*

Beam *BD*:

$\Sigma M_D = 0$

$200(12) - R_2(10) + 40(10)(5) = 0$

$R_2 = 440$ lbf *Ans.*

$\Sigma F_y = 0$

$-200 + 440 - 40(10) + R_3 = 0$

$R_3 = 160$ lbf *Ans.*

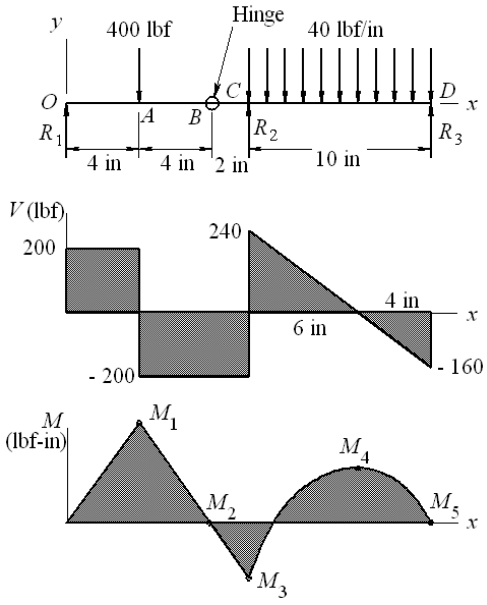
$M_1 = 200(4) = 800 \text{ lbf} \cdot \text{in}$ *Ans.*

$M_2 = 800 - 200(4) = 0$ checks at hinge

$M_3 = 800 - 200(6) = -400 \text{ lbf} \cdot \text{in}$ *Ans.*

$M_4 = -400 + \frac{1}{2}(240)(6) = 320 \text{ lbf} \cdot \text{in}$ *Ans.*

$M_5 = 320 - \frac{1}{2}(160)(4) = 0$ checks!



3-9

$$q = R_1\langle x\rangle^{-1} - 9\langle x-300\rangle^{-1} - 5\langle x-1200\rangle^{-1} + R_2\langle x-1500\rangle^{-1}$$

$$V = R_1 - 9\langle x-300\rangle^{0} - 5\langle x-1200\rangle^{0} + R_2\langle x-1500\rangle^{0} \qquad (1)$$

$$M = R_1 x - 9\langle x-300\rangle^{1} - 5\langle x-1200\rangle^{1} + R_2\langle x-1500\rangle^{1} \qquad (2)$$

At $x = 1500^+$ $V = M = 0$. Applying Eqs. (1) and (2),

$R_1 - 9 - 5 + R_2 = 0 \quad \Rightarrow \quad R_1 + R_2 = 14$

$1500R_1 - 9(1500-300) - 5(1500-1200) = 0 \quad \Rightarrow \quad R_1 = 8.2 \text{ kN}$ *Ans.*

$R_2 = 14 - 8.2 = 5.8 \text{ kN}$ *Ans.*

$0 \le x \le 300$: $V = 8.2 \text{ kN}$, $M = 8.2x \text{ N} \cdot \text{m}$

$300 \le x \le 1200$: $V = 8.2 - 9 = -0.8 \text{ kN}$

$M = 8.2x - 9(x-300) = -0.8x + 2700 \text{ N} \cdot \text{m}$

$1200 \le x \le 1500$: $V = 8.2 - 9 - 5 = -5.8 \text{ kN}$

$M = 8.2x - 9(x-300) - 5(x-1200) = -5.8x + 8700 \text{ N} \cdot \text{m}$

Plots of V and M are the same as in Prob. 3-5.

3-10

$$q = R_0\langle x\rangle^{-1} - M_0\langle x\rangle^{-2} - 500\langle x-8\rangle^{-1} - 40\langle x-14\rangle^{0} + 40\langle x-20\rangle^{0}$$

$$V = R_0 - M_0\langle x\rangle^{-1} - 500\langle x-8\rangle^{0} - 40\langle x-14\rangle^{1} + 40\langle x-20\rangle^{1} \qquad (1)$$

$$M = R_0 x - M_0 - 500\langle x-8\rangle^{1} - 20\langle x-14\rangle^{2} + 20\langle x-20\rangle^{2} \qquad (2)$$

at $x = 20^{+}$ in, $V = M = 0$, Eqs. (1) and (2) give

$$R_0 - 500 - 40(20-14) = 0 \quad \Rightarrow \quad R_0 = 740 \text{ lbf} \qquad \textit{Ans.}$$

$$R_0(20) - M_0 - 500(20-8) - 20(20-14)^2 = 0 \quad \Rightarrow \quad M_0 = 8080 \text{ lbf}\cdot\text{in} \qquad \textit{Ans.}$$

$0 \le x \le 8$: $V = 740$ lbf, $M = 740x - 8080$ lbf·in

$8 \le x \le 14$: $V = 740 - 500 = 240$ lbf

$M = 740x - 8080 - 500(x-8) = 240x - 4080$ lbf·in

$14 \le x \le 20$: $V = 740 - 500 - 40(x-14) = -40x + 800$ lbf

$M = 740x - 8080 - 500(x-8) - 20(x-14)^2 = -20x^2 + 800x - 8000$ lbf·in

Plots of V and M are the same as in Prob. 3-6.

3-11

$$q = R_1\langle x\rangle^{-1} - 2\langle x-1.2\rangle^{-1} + R_2\langle x-2.2\rangle^{-1} - 4\langle x-3.2\rangle^{-1}$$

$$V = R_1 - 2\langle x-1.2\rangle^{0} + R_2\langle x-2.2\rangle^{0} - 4\langle x-3.2\rangle^{0} \qquad (1)$$

$$M = R_1 x - 2\langle x-1.2\rangle^{1} + R_2\langle x-2.2\rangle^{1} - 4\langle x-3.2\rangle^{1} \qquad (2)$$

at $x = 3.2^{+}$, $V = M = 0$. Applying Eqs. (1) and (2),

$$R_1 - 2 + R_2 - 4 = 0 \quad \Rightarrow \quad R_1 + R_2 = 6 \qquad (3)$$

$$3.2R_1 - 2(2) + R_2(1) = 0 \quad \Rightarrow \quad 3.2R_1 + R_2 = 4 \qquad (4)$$

Solving Eqs. (3) and (4) simultaneously,
R_1 = -0.91 kN, R_2 = 6.91 kN *Ans.*

$0 \le x \le 1.2$: $V = -0.91$ kN, $M = -0.91x$ kN·m

$1.2 \le x \le 2.2$: $V = -0.91 - 2 = -2.91$ kN

$M = -0.91x - 2(x-1.2) = -2.91x + 2.4$ kN·m

$2.2 \le x \le 3.2$: $V = -0.91 - 2 + 6.91 = 4$ kN

$M = -0.91x - 2(x-1.2) + 6.91(x-2.2) = 4x - 12.8$ kN·m

Plots of V and M are the same as in Prob. 3-7.

3-12

$$q = R_1\langle x\rangle^{-1} - 400\langle x-4\rangle^{-1} + R_2\langle x-10\rangle^{-1} - 40\langle x-10\rangle^{0} + 40\langle x-20\rangle^{0} + R_3\langle x-20\rangle^{-1}$$

$$V = R_1 - 400\langle x-4\rangle^{0} + R_2\langle x-10\rangle^{0} - 40\langle x-10\rangle^{1} + 40\langle x-20\rangle^{1} + R_3\langle x-20\rangle^{0} \qquad (1)$$

$$M = R_1 x - 400\langle x-4\rangle^{1} + R_2\langle x-10\rangle^{1} - 20\langle x-10\rangle^{2} + 20\langle x-20\rangle^{2} + R_3\langle x-20\rangle^{1} \qquad (2)$$

$M = 0$ at $x = 8$ in $\quad \therefore 8R_1 - 400(8-4) = 0 \quad \Rightarrow \quad R_1 = 200$ lbf *Ans.*

at $x = 20^+$, $V = M = 0$. Applying Eqs. (1) and (2),

$$200 - 400 + R_2 - 40(10) + R_3 = 0 \quad \Rightarrow \quad R_2 + R_3 = 600$$

$$200(20) - 400(16) + R_2(10) - 20(10)^2 = 0 \quad \Rightarrow \quad R_2 = 440 \text{ lbf} \quad \textit{Ans.}$$

$$R_3 = 600 - 440 = 160 \text{ lbf} \quad \textit{Ans.}$$

$0 \le x \le 4$: $\quad V = 200$ lbf, $\quad M = 200x$ lbf·in

$4 \le x \le 10$: $\quad V = 200 - 400 = -200$ lbf,

$$M = 200x - 400(x-4) = -200x + 1600 \text{ lbf}\cdot\text{in}$$

$10 \le x \le 20$: $\quad V = 200 - 400 + 440 - 40(x-10) = 640 - 40x$ lbf

$$M = 200x - 400(x-4) + 440(x-10) - 20\langle x-10\rangle^2 = -20x^2 + 640x - 4800 \text{ lbf}\cdot\text{in}$$

Plots of V and M are the same as in Prob. 3-8.

3-13 Solution depends upon the beam selected.

3-14

(a) Moment at center,

$$x_c = \frac{(l-2a)}{2}$$

$$M_c = \frac{w}{2}\left[\frac{l}{2}(l-2a) - \left(\frac{l}{2}\right)^2\right] = \frac{wl}{2}\left(\frac{l}{4} - a\right)$$

At reaction, $|M_r| = wa^2/2$

$a = 2.25$, $l = 10$ in, $w = 100$ lbf/in

$$M_c = \frac{100(10)}{2}\left(\frac{10}{4} - 2.25\right) = 125 \text{ lbf}\cdot\text{in}$$

$$|M_r| = \frac{100\left(2.25^2\right)}{2} = 253 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(b) Optimal occurs when $M_c = |M_r|$

$$\frac{wl}{2}\left(\frac{l}{4}-a\right)=\frac{wa^2}{2}\Rightarrow a^2+al-0.25l^2=0$$

Taking the positive root

$$a=\frac{1}{2}\left[-l+\sqrt{l^2+4\left(0.25l^2\right)}\right]=\frac{l}{2}\left(\sqrt{2}-1\right)=0.207\ l \qquad \textit{Ans.}$$

for $l = 10$ in, $w = 100$ lbf, $a = 0.207(10) = 2.07$ in

$$M_{\min}=\left(100/2\right)2.07^2=214\ \text{lbf}\cdot\text{in}$$

3-15

(a)

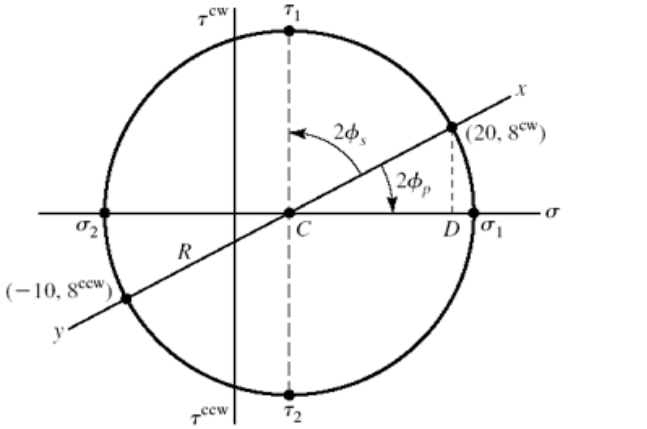
$$C=\frac{20-10}{2}=5\ \text{kpsi}$$

$$CD=\frac{20+10}{2}=15\ \text{kpsi}$$

$$R=\sqrt{15^2+8^2}=17\ \text{kpsi}$$

$$\sigma_1=5+17=22\ \text{kpsi}$$

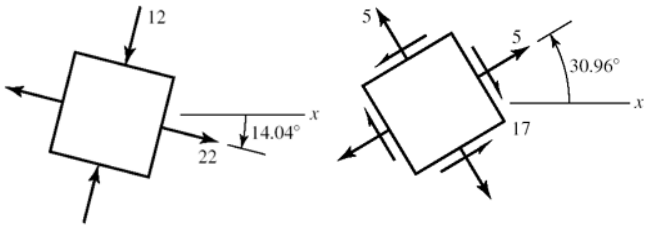
$$\sigma_2=5-17=-12\ \text{kpsi}$$



$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{8}{15}\right)=14.04^\circ\ \text{cw}$$

$$\tau_1=R=17\ \text{kpsi}$$

$$\phi_s=45^\circ-14.04^\circ=30.96^\circ\ \text{ccw}$$



(b)

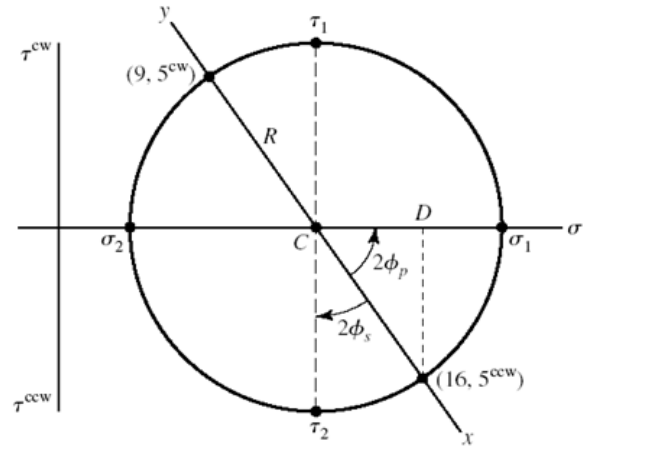
$$C=\frac{9+16}{2}=12.5\ \text{kpsi}$$

$$CD=\frac{16-9}{2}=3.5\ \text{kpsi}$$

$$R=\sqrt{5^2+3.5^2}=6.10\ \text{kpsi}$$

$$\sigma_1=12.5+6.1=18.6\ \text{kpsi}$$

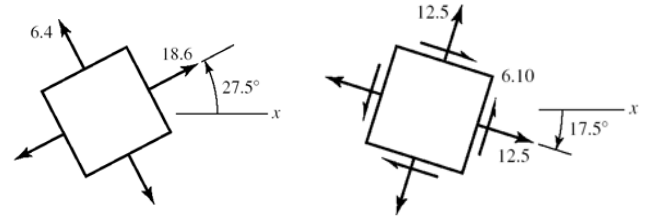
$$\sigma_2=12.5-6.1=6.4\ \text{kpsi}$$



$$\phi_p=\frac{1}{2}\tan^{-1}\left(\frac{5}{3.5}\right)=27.5^\circ\ \text{ccw}$$

$$\tau_1=R=6.10\ \text{kpsi}$$

$$\phi_s=45^\circ-27.5^\circ=17.5^\circ\ \text{cw}$$



(c)

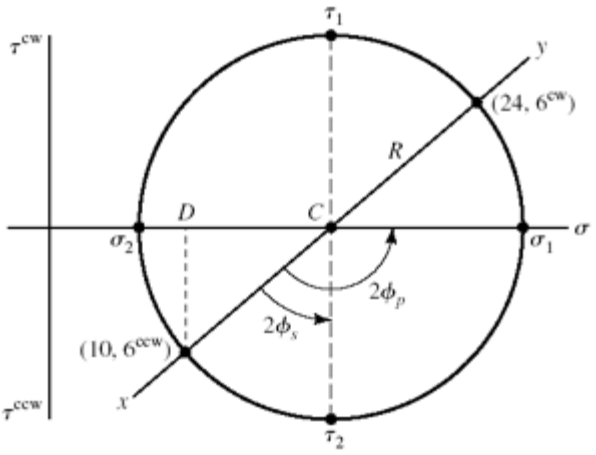
$$C = \frac{24+10}{2} = 17 \text{ kpsi}$$

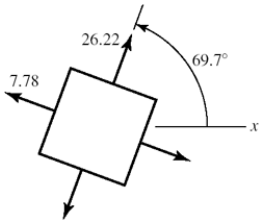
$$CD = \frac{24-10}{2} = 7 \text{ kpsi}$$

$$R = \sqrt{7^2 + 6^2} = 9.22 \text{ kpsi}$$

$$\sigma_1 = 17 + 9.22 = 26.22 \text{ kpsi}$$

$$\sigma_2 = 17 - 9.22 = 7.78 \text{ kpsi}$$

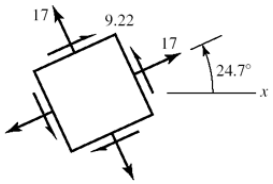




$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7}{6}\right)\right] = 69.7^\circ \text{ ccw}$$

$$\tau_1 = R = 9.22 \text{ kpsi}$$

$$\phi_s = 69.7^\circ - 45^\circ = 24.7^\circ \text{ ccw}$$



(d)

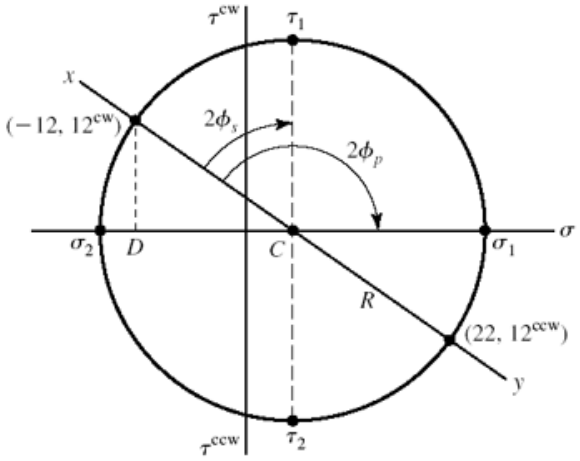
$$C = \frac{-12+22}{2} = 5 \text{ kpsi}$$

$$CD = \frac{12+22}{2} = 17 \text{ kpsi}$$

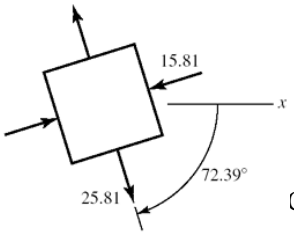
$$R = \sqrt{17^2 + 12^2} = 20.81 \text{ kpsi}$$

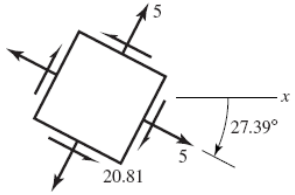
$$\sigma_1 = 5 + 20.81 = 25.81 \text{ kpsi}$$

$$\sigma_2 = 5 - 20.81 = -15.81 \text{ kpsi}$$



$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{17}{12}\right)\right] = 72.39^\circ \text{ cw}$$





Ch

$\tau_1 = R = 20.81$ kpsi

$\phi_s = 72.39 - 45 = 27.39^\circ$ cw

3-16

(a)

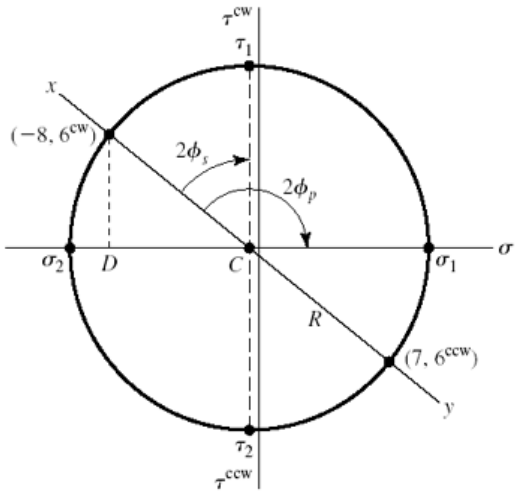
$$C = \frac{-8+7}{2} = -0.5 \text{ MPa}$$

$$CD = \frac{8+7}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 6^2} = 9.60 \text{ MPa}$$

$$\sigma_1 = 9.60 - 0.5 = 9.10 \text{ MPa}$$

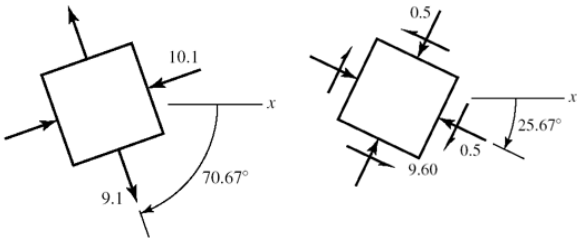
$$\sigma_2 = -0.5 - 9.6 = -10.1 \text{ Mpa}$$



$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7.5}{6}\right)\right] = 70.67^\circ \text{ cw}$$

$$\tau_1 = R = 9.60 \text{ MPa}$$

$$\phi_s = 70.67^\circ - 45^\circ = 25.67^\circ \text{ cw}$$



(b)

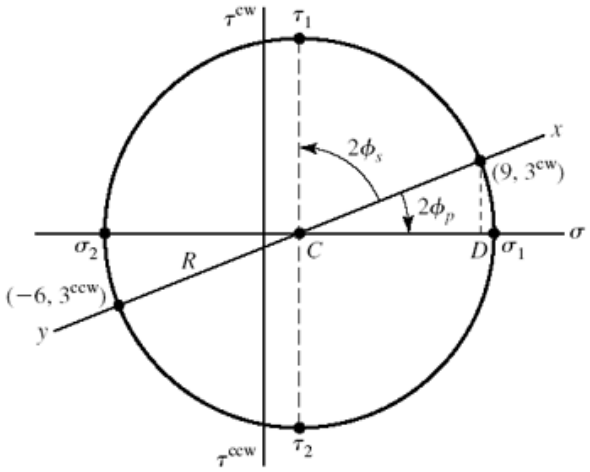
$$C = \frac{9-6}{2} = 1.5 \text{ MPa}$$

$$CD = \frac{9+6}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078 \text{ MPa}$$

$$\sigma_1 = 1.5 + 8.078 = 9.58 \text{ MPa}$$

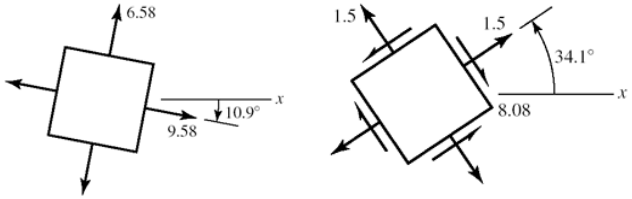
$$\sigma_2 = 1.5 - 8.078 = -6.58 \text{ MPa}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{3}{7.5}\right) = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078 \text{ MPa}$$

$$\phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



(c)

$$C = \frac{12-4}{2} = 4 \text{ MPa}$$

$$CD = \frac{12+4}{2} = 8 \text{ MPa}$$

$$R = \sqrt{8^2 + 7^2} = 10.63 \text{ MPa}$$

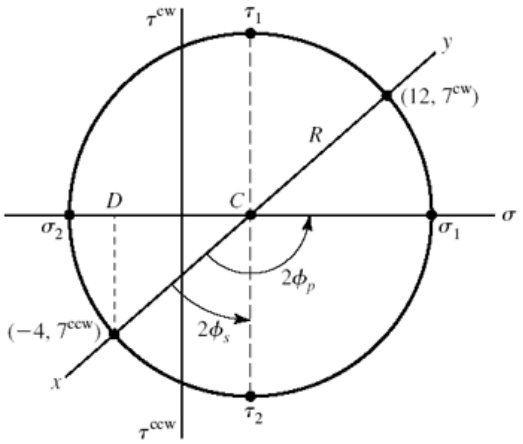
$$\sigma_1 = 4 + 10.63 = 14.63 \text{ MPa}$$

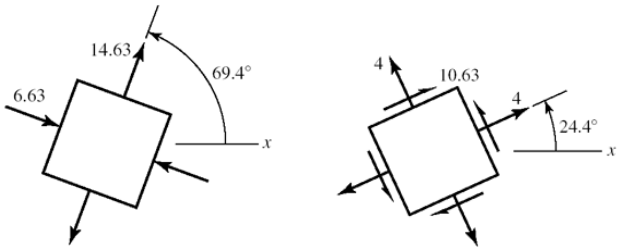
$$\sigma_2 = 4 - 10.63 = -6.63 \text{ MPa}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{8}{7}\right)\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63 \text{ MPa}$$

$$\phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$





(d)

$$C = \frac{6-5}{2} = 0.5 \text{ MPa}$$

$$CD = \frac{6+5}{2} = 5.5 \text{ MPa}$$

$$R = \sqrt{5.5^2 + 8^2} = 9.71 \text{ MPa}$$

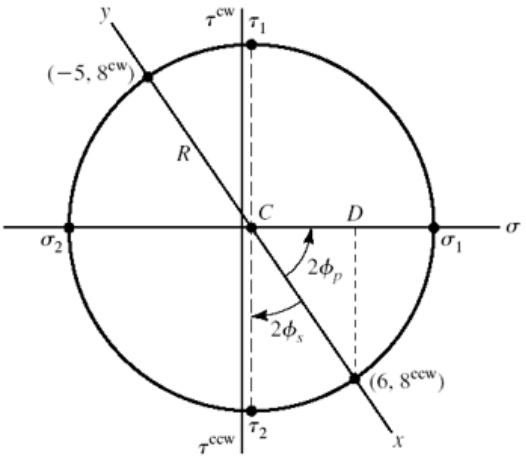
$$\sigma_1 = 0.5 + 9.71 = 10.21 \text{ MPa}$$

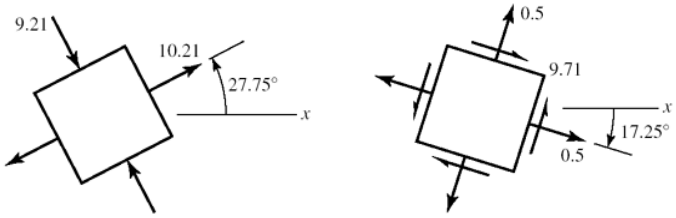
$$\sigma_2 = 0.5 - 9.71 = -9.21 \text{ MPa}$$

$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{8}{5.5}\right) = 27.75^\circ \text{ ccw}$$

$$\tau_1 = R = 9.71 \text{ MPa}$$

$$\phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$





3-17

(a)

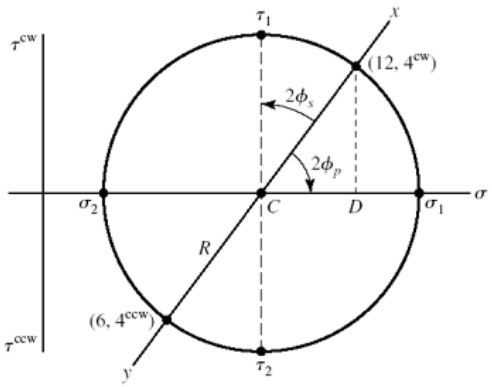
$$C = \frac{12+6}{2} = 9 \text{ kpsi}$$

$$CD = \frac{12-6}{2} = 3 \text{ kpsi}$$

$$R = \sqrt{3^2+4^2} = 5 \text{ kpsi}$$

$$\sigma_1 = 5+9 = 14 \text{ kpsi}$$

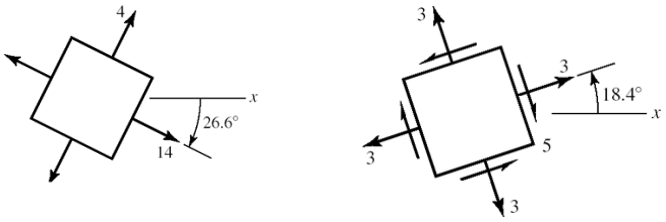
$$\sigma_2 = 9-5 = 4 \text{ kpsi}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = 26.6^\circ \text{ ccw}$$

$$\tau_1 = R = 5 \text{ kpsi}$$

$$\phi_s = 45^\circ - 26.6^\circ = 18.4^\circ \text{ ccw}$$



(b)

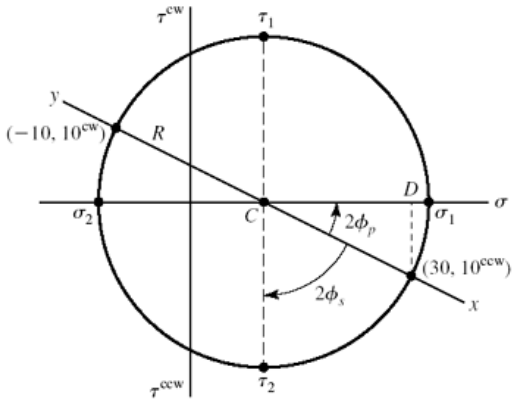
$$C = \frac{30-10}{2} = 10 \text{ kpsi}$$

$$CD = \frac{30+10}{2} = 20 \text{ kpsi}$$

$$R = \sqrt{20^2+10^2} = 22.36 \text{ kpsi}$$

$$\sigma_1 = 10+22.36 = 32.36 \text{ kpsi}$$

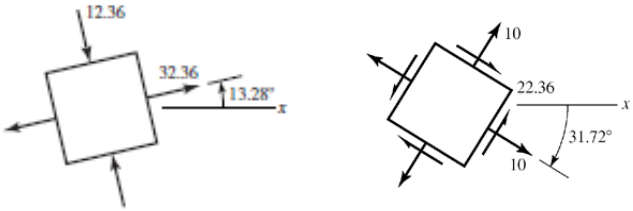
$$\sigma_2 = 10-22.36 = -12.36 \text{ kpsi}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{10}{20}\right) = 13.28^\circ \text{ ccw}$$

$$\tau_1 = R = 22.36 \text{ kpsi}$$

$$\phi_s = 45^\circ - 13.28^\circ = 31.72^\circ \text{ cw}$$



(c)

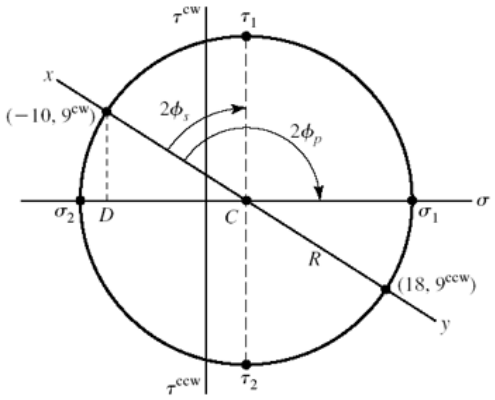
$$C = \frac{-10+18}{2} = 4 \text{ kpsi}$$

$$CD = \frac{10+18}{2} = 14 \text{ kpsi}$$

$$R = \sqrt{14^2 + 9^2} = 16.64 \text{ kpsi}$$

$$\sigma_1 = 4 + 16.64 = 20.64 \text{ kpsi}$$

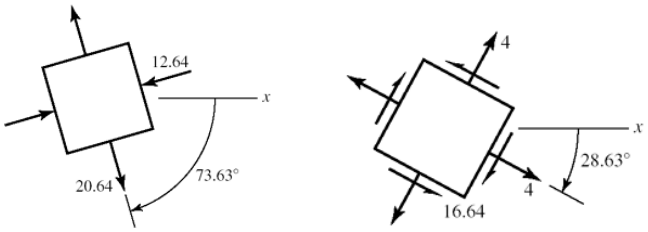
$$\sigma_2 = 4 - 16.64 = -12.64 \text{ kpsi}$$



$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{14}{9}\right)\right] = 73.63^\circ \text{ cw}$$

$$\tau_1 = R = 16.64 \text{ kpsi}$$

$$\phi_s = 73.63 - 45 = 28.63^\circ \text{ cw}$$



(d)

$$C = \frac{9+19}{2} = 14 \text{ kpsi}$$

$$CD = \frac{19-9}{2} = 5 \text{ kpsi}$$

$$R = \sqrt{5^2 + 8^2} = 9.434 \text{ kpsi}$$

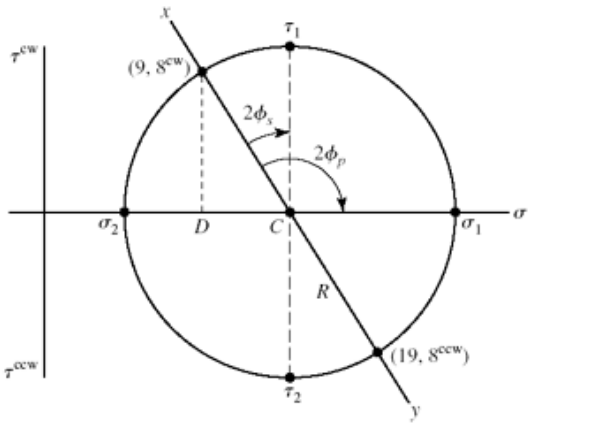
$$\sigma_1 = 14 + 9.43 = 23.43 \text{ kpsi}$$

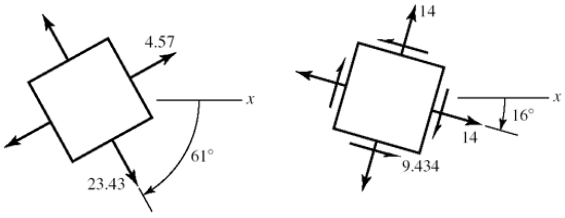
$$\sigma_2 = 14 - 9.43 = 4.57 \text{ kpsi}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{5}{8}\right)\right] = 61.0^\circ \text{ cw}$$

$$\tau_1 = R = 9.34 \text{ kpsi}$$

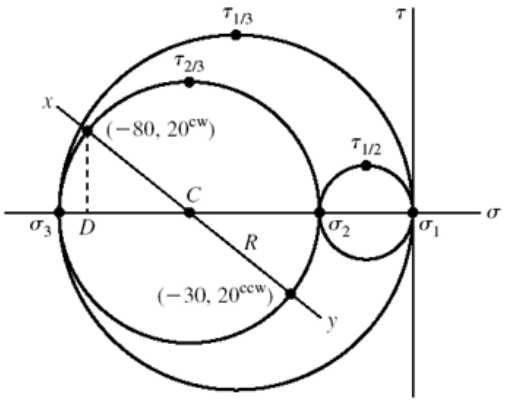
$$\phi_s = 61^\circ - 45^\circ = 16^\circ \text{ cw}$$





3-18

(a)



$$C = \frac{-80-30}{2} = -55 \text{ MPa}$$

$$CD = \frac{80-30}{2} = 25 \text{ MPa}$$

$$R = \sqrt{25^2 + 20^2} = 32.02 \text{ MPa}$$

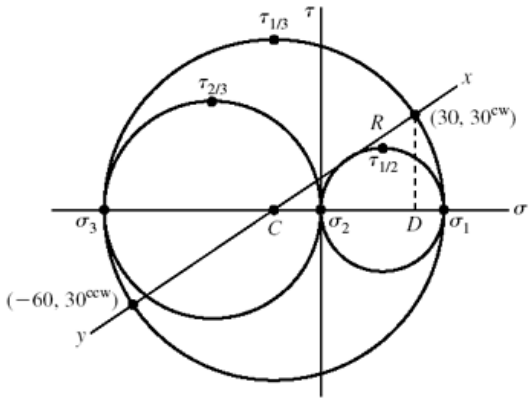
$$\sigma_1 = 0 \text{ MPa}$$

$$\sigma_2 = -55 + 32.02 = -22.98 = -23.0 \text{ MPa}$$

$$\sigma_3 = -55 - 32.0 = -87.0 \text{ MPa}$$

$$\tau_{1/2} = \frac{23}{2} = 11.5 \text{ MPa}, \qquad \tau_{2/3} = 32.0 \text{ MPa}, \qquad \tau_{1/3} = \frac{87}{2} = 43.5 \text{ MPa}$$

(b)



$$C = \frac{30-60}{2} = -15 \text{ MPa}$$

$$CD = \frac{60+30}{2} = 45 \text{ MPa}$$

$$R = \sqrt{45^2 + 30^2} = 54.1 \text{ MPa}$$

$$\sigma_1 = -15 + 54.1 = 39.1 \text{ MPa}$$

$$\sigma_2 = 0 \text{ MPa}$$

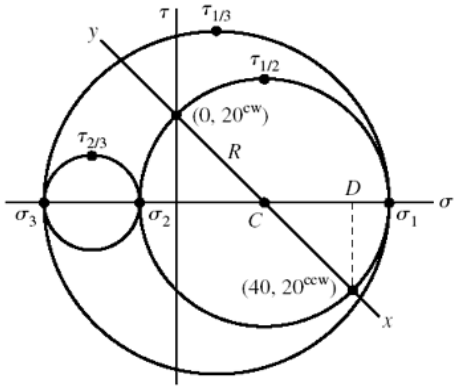
$$\sigma_3 = -15 - 54.1 = -69.1 \text{ MPa}$$

$$\tau_{1/3} = \frac{39.1 + 69.1}{2} = 54.1 \text{ MPa}$$

$$\tau_{1/2} = \frac{39.1}{2} = 19.6 \text{ MPa}$$

$$\tau_{2/3} = \frac{69.1}{2} = 34.6 \text{ MPa}$$

(c)



$$C = \frac{40+0}{2} = 20 \text{ MPa}$$

$$CD = \frac{40-0}{2} = 20 \text{ MPa}$$

$$R = \sqrt{20^2 + 20^2} = 28.3 \text{ MPa}$$

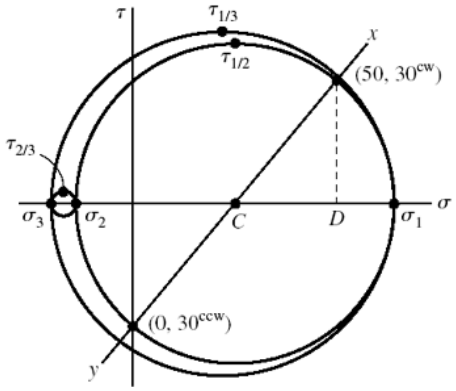
$$\sigma_1 = 20 + 28.3 = 48.3 \text{ MPa}$$

$$\sigma_2 = 20 - 28.3 = -8.3 \text{ MPa}$$

$$\sigma_3 = \sigma_z = -30 \text{ MPa}$$

$$\tau_{1/3} = \frac{48.3+30}{2} = 39.1 \text{ MPa}, \qquad \tau_{1/2} = 28.3 \text{ MPa}, \qquad \tau_{2/3} = \frac{30-8.3}{2} = 10.9 \text{ MPa}$$

(d)



$$C = \frac{50}{2} = 25 \text{ MPa}$$

$$CD = \frac{50}{2} = 25 \text{ MPa}$$

$$R = \sqrt{25^2 + 30^2} = 39.1 \text{ MPa}$$

$$\sigma_1 = 25 + 39.1 = 64.1 \text{ MPa}$$

$$\sigma_2 = 25 - 39.1 = -14.1 \text{ MPa}$$

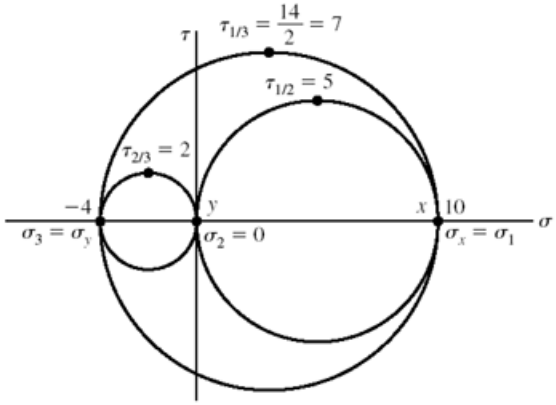
$$\sigma_3 = \sigma_z = -20 \text{ MPa}$$

$$\tau_{1/3} = \frac{64.1+20}{2} = 42.1 \text{ MPa}, \qquad \tau_{1/2} = 39.1 \text{ MPa}, \qquad \tau_{2/3} = \frac{20-14.1}{2} = 2.95 \text{ MPa}$$

3-19

(a)

Since there are no shear stresses on the stress element, the stress element already represents principal stresses.



$$\sigma_1 = \sigma_x = 10 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}$$

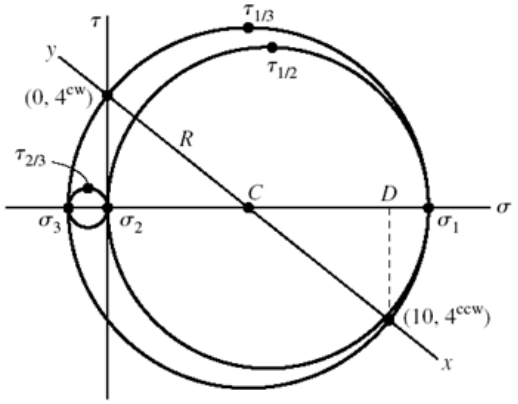
$$\sigma_3 = \sigma_y = -4 \text{ kpsi}$$

$$\tau_{1/3} = \frac{10-(-4)}{2} = 7 \text{ kpsi}$$

$$\tau_{1/2} = \frac{10}{2} = 5 \text{ kpsi}$$

$$\tau_{2/3} = \frac{0-(-4)}{2} = 2 \text{ kpsi}$$

(b)



$$C = \frac{0+10}{2} = 5 \text{ kpsi}$$

$$CD = \frac{10-0}{2} = 5 \text{ kpsi}$$

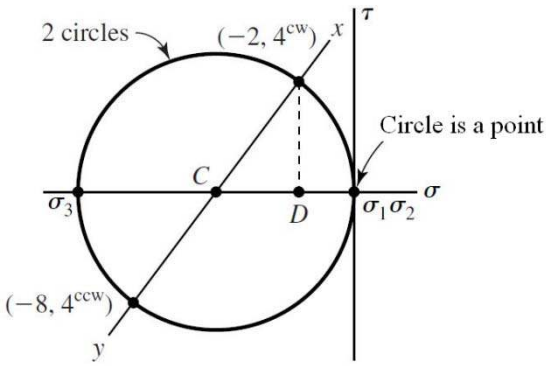
$$R = \sqrt{5^2 + 4^2} = 6.40 \text{ kpsi}$$

$$\sigma_1 = 5 + 6.40 = 11.40 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}, \quad \sigma_3 = 5 - 6.40 = -1.40 \text{ kpsi}$$

$$\tau_{1/3} = R = 6.40 \text{ kpsi}, \qquad \tau_{1/2} = \frac{11.40}{2} = 5.70 \text{ kpsi}, \qquad \tau_3 = \frac{1.40}{2} = 0.70 \text{ kpsi}$$

(c)



$$C = \frac{-2-8}{2} = -5 \text{ kpsi}$$

$$CD = \frac{8-2}{2} = 3 \text{ kpsi}$$

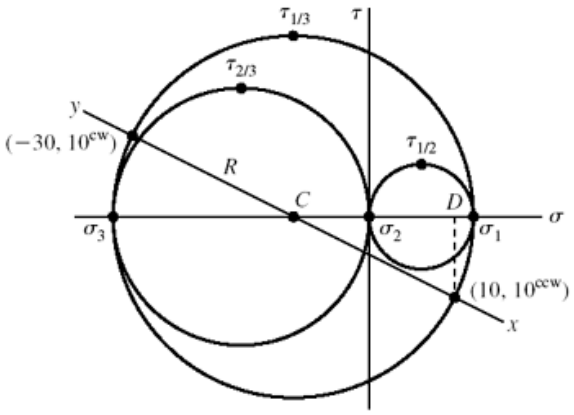
$$R = \sqrt{3^2 + 4^2} = 5 \text{ kpsi}$$

$$\sigma_1 = -5 + 5 = 0 \text{ kpsi}, \quad \sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -5 - 5 = -10 \text{ kpsi}$$

$$\tau_{1/3} = \frac{10}{2} = 5 \text{ kpsi}, \qquad \tau_{1/2} = 0 \text{ kpsi}, \qquad \tau_{2/3} = 5 \text{ kpsi}$$

(d)



$$C = \frac{10-30}{2} = -10 \text{ kpsi}$$

$$CD = \frac{10+30}{2} = 20 \text{ kpsi}$$

$$R = \sqrt{20^2 + 10^2} = 22.36 \text{ kpsi}$$

$$\sigma_1 = -10 + 22.36 = 12.36 \text{ kpsi}$$

$$\sigma_2 = 0 \text{ kpsi}$$

$$\sigma_3 = -10 - 22.36 = -32.36 \text{ kpsi}$$

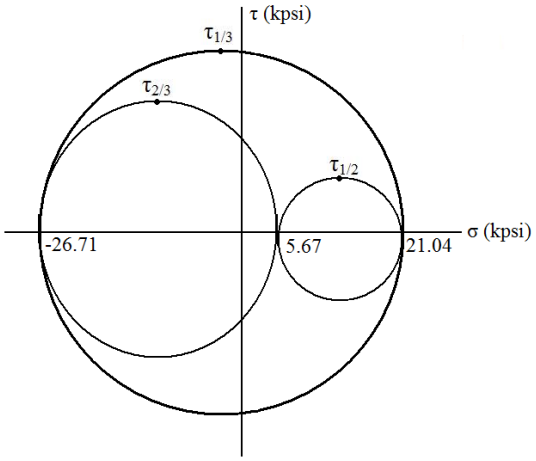
$$\tau_{1/3} = 22.36 \text{ kpsi}, \qquad \tau_{1/2} = \frac{12.36}{2} = 6.18 \text{ kpsi}, \qquad \tau_{2/3} = \frac{32.36}{2} = 16.18 \text{ kpsi}$$

3-20 From Eq. (3-15),

$$\sigma^3-(-6+18-12)\sigma^2+\left[-6(18)+(-6)(-12)+18(-12)-9^2-6^2-(-15)^2\right]\sigma$$
$$-\left[-6(18)(-12)+2(9)(6)(-15)-(-6)(6)^2-18(-15)^2-(-12)(9)^2\right]=0$$
$$\sigma^3-594\sigma+3186=0$$

Roots are: 21.04, 5.67, –26.71 kpsi *Ans.*

$$\tau_{1/2}=\frac{21.04-5.67}{2}=7.69 \text{ kpsi}$$
$$\tau_{2/3}=\frac{5.67+26.71}{2}=16.19 \text{ kpsi}$$
$$\tau_{max}=\tau_{1/3}=\frac{21.04+26.71}{2}=23.88 \text{ kpsi} \quad \textit{Ans.}$$



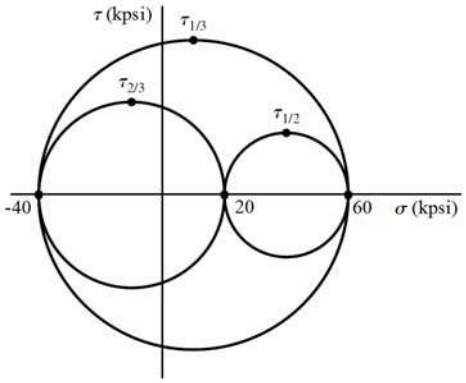
3-21

From Eq. (3-15)

$$\sigma^3-(20+0+20)\sigma^2+\left[20(0)+20(20)+0(20)-40^2-\left(-20\sqrt{2}\right)^2-0^2\right]\sigma$$
$$-\left[20(0)(20)+2(40)\left(-20\sqrt{2}\right)(0)-20\left(-20\sqrt{2}\right)^2-0(0)^2-20(40)^2\right]=0$$
$$\sigma^3-40\sigma^2-2\,000\sigma+48\,000=0$$

Roots are: 60, 20, –40 kpsi *Ans.*

$$\tau_{1/2}=\frac{60-20}{2}=20 \text{ kpsi}$$
$$\tau_{2/3}=\frac{20+40}{2}=30 \text{ kpsi}$$
$$\tau_{max}=\tau_{1/3}=\frac{60+40}{2}=50 \text{ kpsi} \quad \textit{Ans.}$$

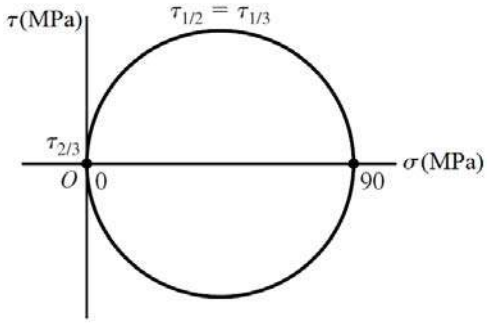


3-22

From Eq. (3-15)

$$\sigma^3-(10+40+40)\sigma^2+\left[10(40)+10(40)+40(40)-20^2-\left(-40\right)^2-\left(-20\right)^2\right]\sigma$$
$$-\left[10(40)(40)+2(20)(-40)(-20)-10(-40)^2-40(-20)^2-40(20)^2\right]=0$$
$$\sigma^3-90\sigma^2=0$$

Roots are: 90, 0, 0 MPa *Ans.*



$\tau_{2/3}=0$

$\tau_{1/2}=\tau_{1/3}=\tau_{\max}=\dfrac{90}{2}=45$ MPa *Ans.*

3-23

$$\sigma=\frac{F}{A}=\frac{15000}{(\pi/4)\left(0.75^2\right)}=33\ 950\text{ psi}=34.0\text{ kpsi}\quad \textit{Ans.}$$

$$\delta=\frac{FL}{AE}=\sigma\frac{L}{E}=33\ 950\frac{60}{30\left(10^6\right)}=0.0679\text{ in}\quad \textit{Ans.}$$

$$\epsilon_1=\frac{\delta}{L}=\frac{0.0679}{60}=1130\left(10^{-6}\right)=1130\mu\quad \textit{Ans.}$$

From Table A-5, $\nu=0.292$

$$\epsilon_2=-\nu\epsilon_1=-0.292(1130)=-330\mu\quad \textit{Ans.}$$

$$\Delta d=\epsilon_2 d=-330\left(10^{-6}\right)(0.75)=-248\left(10^{-6}\right)\text{ in}\quad \textit{Ans.}$$

3-24

$$\sigma=\frac{F}{A}=\frac{3000}{(\pi/4)\left(0.75^2\right)}=6790\text{ psi}=6.79\text{ kpsi}\quad \textit{Ans.}$$

$$\delta=\frac{FL}{AE}=\sigma\frac{L}{E}=6790\frac{60}{10.4\left(10^6\right)}=0.0392\text{ in}\quad \textit{Ans.}$$

$$\epsilon_1=\frac{\delta}{L}=\frac{0.0392}{60}=653\left(10^{-6}\right)=653\mu\quad \textit{Ans.}$$

From Table A-5, $\nu=0.333$

$$\epsilon_2=-\nu\epsilon_1=-0.333(653)=-217\mu\quad \textit{Ans.}$$

$$\Delta d=\epsilon_2 d=-217\left(10^{-6}\right)(0.75)=-163\left(10^{-6}\right)\text{ in}\quad \textit{Ans.}$$

3-25

$$\epsilon_2 = \frac{\Delta d}{d} = \frac{-0.0001d}{d} = -0.0001$$

From Table A-5, $v = 0.326$, $E = 119$ GPa

$$\epsilon_1 = \frac{-\epsilon_2}{v} = \frac{-0.0001}{0.326} = 306.7\left(10^{-6}\right)$$

$\delta = \dfrac{FL}{AE}$ and $\sigma = \dfrac{F}{A}$, so

$$\sigma = \frac{\delta E}{L} = \epsilon_1 E = 306.7\left(10^{-6}\right)(119)\left(10^{9}\right) = 36.5 \text{ MPa}$$

$$F = \sigma A = 36.5\left(10^{6}\right)\frac{\pi\left(0.03\right)^2}{4} = 25\ 800 \text{ N} = 25.8 \text{ kN} \qquad \textit{Ans.}$$

$S_y = 70$ MPa $> \sigma$, so elastic deformation assumption is valid.

3-26

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 20\ 000\frac{8(12)}{10.4\left(10^{6}\right)} = 0.185 \text{ in} \qquad \textit{Ans.}$$

3-27

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 140\left(10^{6}\right)\frac{3}{71.7\left(10^{9}\right)} = 0.00586 \text{ m} = 5.86 \text{ mm} \qquad \textit{Ans.}$$

3-28

$$\delta = \frac{FL}{AE} = \sigma\frac{L}{E} = 15\ 000\frac{10(12)}{10.4\left(10^{6}\right)} = 0.173 \text{ in} \qquad \textit{Ans.}$$

3-29

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\epsilon_x & -v \\ E\epsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -v \\ -v & 1 \end{vmatrix}} = \frac{E\epsilon_x + vE\epsilon_y}{1-v^2} = \frac{E\left(\epsilon_x + v\epsilon_y\right)}{1-v^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\epsilon_y + \nu\epsilon_x\right)}{1-\nu^2}$$

From Table A-5, E = 207 GPa and ν = 0.292. Thus,

$$\sigma_x = \frac{E\left(\epsilon_x + \nu\epsilon_y\right)}{1-\nu^2} = \frac{207\left(10^9\right)\left[0.0019 + 0.292\left(-0.000\ 72\right)\right]}{1-0.292^2}\left(10^{-6}\right) = 382 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_y = \frac{207\left(10^9\right)\left[-0.000\ 72 + 0.292\left(0.0019\right)\right]}{1-0.292^2}\left(10^{-6}\right) = -37.4 \text{ MPa} \quad \textit{Ans.}$$

3-30

With $\sigma_z = 0$, solve the first two equations of Eq. (3-19) simulatenously. Place E on the left-hand side of both equations, and using Cramer's rule,

$$\sigma_x = \frac{\begin{vmatrix} E\epsilon_x & -\nu \\ E\epsilon_y & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\nu \\ -\nu & 1 \end{vmatrix}} = \frac{E\epsilon_x + \nu E\epsilon_y}{1-\nu^2} = \frac{E\left(\epsilon_x + \nu\epsilon_y\right)}{1-\nu^2}$$

Likewise,

$$\sigma_y = \frac{E\left(\epsilon_y + \nu\epsilon_x\right)}{1-\nu^2}$$

From Table A-5, E = 71.7 GPa and ν = 0.333. Thus,

$$\sigma_x = \frac{E\left(\epsilon_x + \nu\epsilon_y\right)}{1-\nu^2} = \frac{71.7\left(10^9\right)\left[0.0019 + 0.333\left(-0.000\ 72\right)\right]}{1-0.333^2}\left(10^{-6}\right) = 134 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_y = \frac{71.7\left(10^9\right)\left[-0.000\ 72 + 0.333\left(0.0019\right)\right]}{1-0.333^2}\left(10^{-6}\right) = -7.04 \text{ MPa} \quad \textit{Ans.}$$

3-31

(a) $R_1 = \dfrac{c}{l}F \qquad M_{\max} = R_1 a = \dfrac{ac}{l}F$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{ac}{l}F \Rightarrow F = \frac{\sigma bh^2 l}{6ac} \quad \textit{Ans.}$$

(b) $$\frac{F_m}{F} = \frac{\left(\sigma_m/\sigma\right)\left(b_m/b\right)\left(h_m/h\right)^2\left(l_m/l_1\right)}{\left(a_m/a\right)\left(c_m/c\right)} = \frac{1(s)(s)^2(s)}{(s)(s)} = s^2 \quad \textit{Ans.}$$

For equal stress, the model load varies by the square of the scale factor.

3-32

(a) $R_1 = \dfrac{wl}{2}, \quad M_{\max}\big|_{x=l/2} = \dfrac{w}{2}\dfrac{l}{2}\left(l - \dfrac{l}{2}\right) = \dfrac{wl^2}{8}$

$$\sigma = \frac{6M}{bh^2} = \frac{6}{bh^2}\frac{wl^2}{8} = \frac{3Wl}{4bh^2} \quad \Rightarrow \quad W = \frac{4}{3}\frac{\sigma bh^2}{l} \qquad Ans.$$

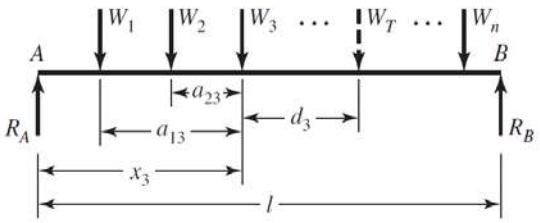
(b) $$\frac{W_m}{W} = \frac{(\sigma_m/\sigma)(b_m/b)(h_m/h)^2}{l_m/l} = \frac{1(s)(s)^2}{s} = s^2 \qquad Ans.$$

$$\frac{w_m l_m}{wl} = s^2 \quad \Rightarrow \quad \frac{w_m}{w} = \frac{s^2}{s} = s \qquad Ans.$$

For equal stress, the model load w varies linearly with the scale factor.

3-33

(a) Can solve by iteration *or* derive equations for the general case. Find maximum moment under wheel W_3.



$W_T = \Sigma W$ at centroid of W's

$$R_A = \frac{l - x_3 - d_3}{l} W_T$$

Under wheel 3,

$$M_3 = R_A x_3 - W_1 a_{13} - W_2 a_{23} = \frac{(l - x_3 - d_3)}{l} W_T x_3 - W_1 a_{13} - W_2 a_{23}$$

For maximum, $\dfrac{dM_3}{dx_3} = 0 = (l - d_3 - 2x_3)\dfrac{W_T}{l} \quad \Rightarrow \quad x_3 = \dfrac{l - d_3}{2}$

Substitute into $M \Rightarrow M_3 = \dfrac{(l - d_3)^2}{4l} W_T - W_1 a_{13} - W_2 a_{23}$

This means the midpoint of d_3 intersects the midpoint of the beam.

For wheel *i*, $\quad x_i = \dfrac{l - d_i}{2}, \quad M_i = \dfrac{(l - d_i)^2}{4l} W_T - \sum_{j=1}^{i-1} W_j a_{ji}$

Note for wheel 1: $\Sigma W_j a_{ji} = 0$

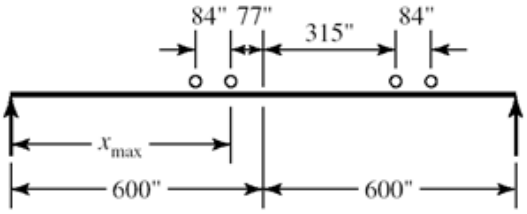
$$W_T = 104.4, \quad W_1 = W_2 = W_3 = W_4 = \frac{104.4}{4} = 26.1 \text{ kips}$$

Wheel 1: $d_1 = \dfrac{476}{2} = 238$ in, $\quad M_1 = \dfrac{(1200 - 238)^2}{4(1200)}(104.4) = 20\,128$ kip·in

Wheel 2: $d_2 = 238 - 84 = 154$ in

$$M_2 = \frac{(1200-154)^2}{4(1200)}(104.4) - 26.1(84) = 21\,605 \text{ kip}\cdot\text{in} = M_{\max} \qquad Ans.$$

Check if all of the wheels are on the rail.



(b) $x_{\max} = 600 - 77 = 523$ in *Ans.*

(c) See above sketch.

(d) Inner axles

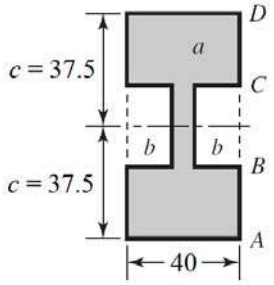
3-34

(a) Let a = total area of entire envelope
Let b = area of side notch

$$A = a - 2b = 40(3)(25) - 25(34) = 2150 \text{ mm}^2$$

$$I = I_a - 2I_b = \frac{1}{12}(40)(75)^3 - \frac{1}{12}(34)(25)^3$$

$$I = 1.36(10^6) \text{ mm}^4 \qquad Ans.$$



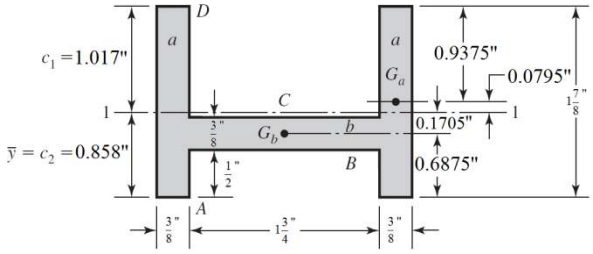
Dimensions in mm.

(b)

$$A_a = 0.375(1.875) = 0.703\,125 \text{ in}^2$$

$$A_b = 0.375(1.75) = 0.656\,25 \text{ in}^2$$

$$A = 2(0.703125) + 0.656\,25 = 2.0625 \text{ in}^2$$



$$\bar{y} = \frac{2(0.703\,125)(0.9375) + 0.656\,25(0.6875)}{2.0625} = 0.858 \text{ in} \qquad Ans.$$

$$I_a = \frac{0.375(1.875)^3}{12} = 0.206 \text{ in}^4$$

$$I_b = \frac{1.75(0.375)^3}{12} = 0.007\,69 \text{ in}^4$$

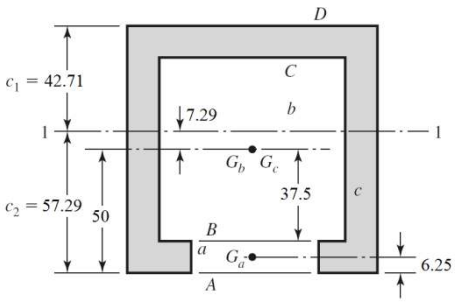
$$I_1 = 2\left[0.206 + 0.703\,125(0.0795)^2\right] + \left[0.00769 + 0.656\,25(0.1705)^2\right] = 0.448 \text{ in}^4 \qquad Ans.$$

(c)

Use two negative areas.

$$A_a = 625 \text{ mm}^2,\ A_b = 5625 \text{ mm}^2,\ A_c = 10\,000 \text{ mm}^2$$

$$A = 10\,000 - 5625 - 625 = 3750 \text{ mm}^2;$$



$\overline{y}_a = 6.25$ mm, $\overline{y}_b = 50$ mm, $\overline{y}_c = 50$ mm

$$\overline{y} = \frac{10\,000(50) - 5625(50) - 625(6.25)}{3750} = 57.29 \text{ mm} \quad Ans.$$

$$c_1 = 100 - 57.29 = 42.71 \text{ mm} \quad Ans.$$

$$I_a = \frac{50(12.5)^3}{12} = 8138 \text{ mm}^4$$

$$I_b = \frac{75(75)^3}{12} = 2.637\left(10^6\right) \text{ mm}^4$$

$$I_c = \frac{100(100)^3}{12} = 8.333\left(10^6\right) \text{in}^4$$

$$I_1 = \left[8.333\left(10^6\right) + 10\,000(7.29)^2\right] - \left[2.637\left(10^6\right) + 5625\left(7.29\right)^2\right] - \left[8138 + 625\left(57.29 - 6.25\right)^2\right]$$

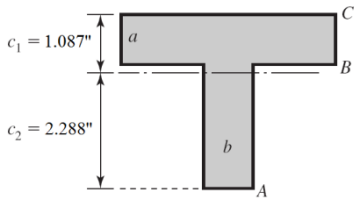
$$I_1 = 4.29\left(10^6\right) \text{ in}^4 \quad Ans.$$

(d)

$$A_a = 4\left(0.875\right) = 3.5 \text{ in}^2$$

$$A_b = 2.5\left(0.875\right) = 2.1875 \text{ in}^2$$

$$A = A_a + A_b = 5.6875 \text{ in}^2$$

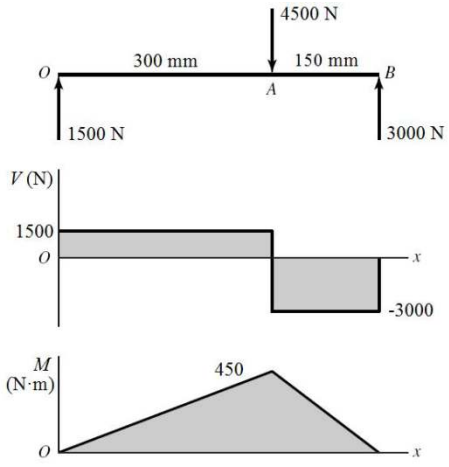


$$\overline{y} = \frac{2.9375\left(3.5\right) + 1.25(2.1875)}{5.6875} = 2.288 \text{ in} \quad Ans.$$

$$I = \frac{1}{12}(4)\left(0.875\right)^3 + 3.5\left(2.9375 - 2.288\right)^2 + \frac{1}{12}\left(0.875\right)\left(2.5\right)^3 + 2.1875\left(2.288 - 1.25\right)^2$$

$$I = 5.20 \text{ in}^4 \quad Ans.$$

3-35



$$I = \frac{1}{12}(20)(40)^3 = 1.067\left(10^5\right) \text{ mm}^4$$

$$A = 20(40) = 800 \text{ mm}^2$$

$M_{\max}$ is at A. At the bottom of the section,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{450\ 000(20)}{1.067\left(10^5\right)} = 84.3 \text{ MPa} \quad Ans.$$

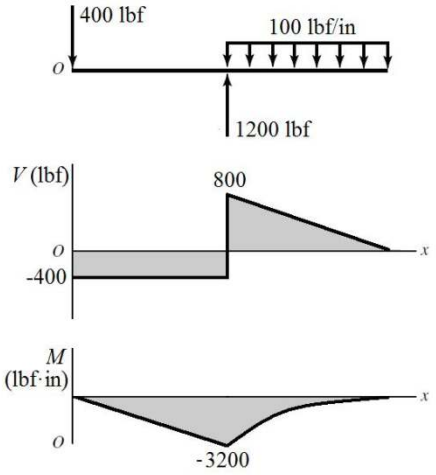
Due to V, $\tau_{\max}$ is between A and B at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{3000}{800}\right) = 5.63 \text{ MPa} \quad Ans.$$

3-36

$$I = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$A = 1(2) = 2 \text{ in}^2$$



$$\Sigma M_o = 0$$

$$8R_A - 100(8)(12) = 0$$

$$R_A = 1200 \text{ lbf}$$

$$R_o = 1200 - 100(8) = 400 \text{ lbf}$$

$M_{\max}$ is at A. At the top of the beam,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{3200(0.5)}{0.6667} = 2400 \text{ psi} \quad \textit{Ans.}$$

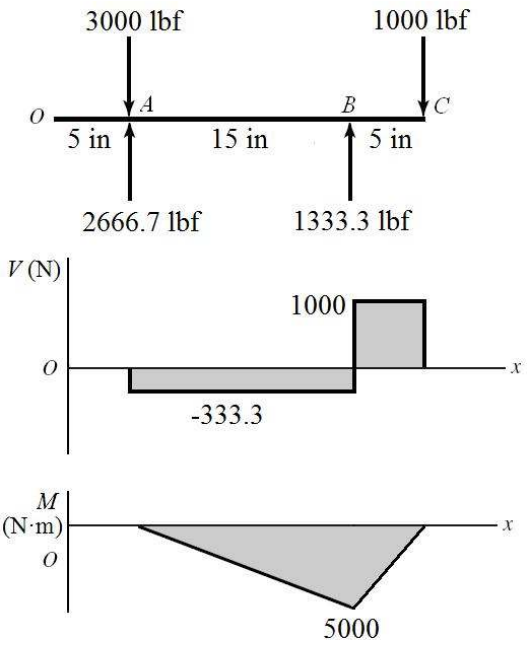
Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{800}{2}\right) = 600 \text{ psi} \quad \textit{Ans.}$$

3-37

$$I = \frac{1}{12}(0.75)(2)^3 = 0.5 \text{ in}^4$$

$$A = (0.75)(2) = 1.5 \text{ in}^2$$



$$\Sigma M_A = 0$$

$$15R_B - 1000(20) = 0$$

$$R_B = 1333.3 \text{ lbf}$$

$$R_A = 3000 - 1333.3 + 1000 = 2666.7 \text{ lbf}$$

$M_{\max}$ is at B. At the top of the beam,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{5000(1)}{0.5} = 10\,000 \text{ psi} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is between B and C at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{1000}{1.5}\right) = 1000 \text{ psi} \quad \textit{Ans.}$$

3-38

$$I = \frac{\pi d^4}{64} = \frac{\pi(50)^4}{64} = 306.796\left(10^3\right) \text{ mm}^4$$

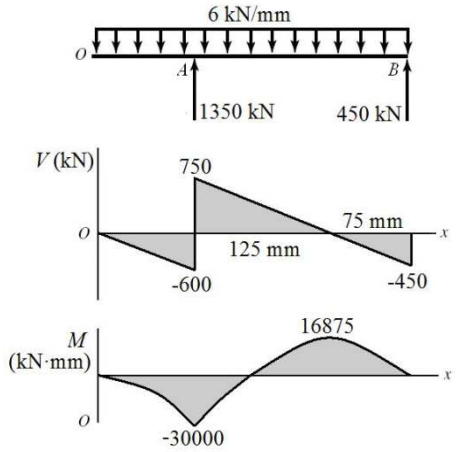
$$A = \frac{\pi d^2}{4} = \frac{\pi(50)^2}{4} = 1963 \text{ mm}^2$$

$$\Sigma M_B = 0$$

$$6(300)(150) - 200R_A = 0$$

$$R_A = 1350 \text{ kN}$$

$$R_B = 6(300) - 1350 = 450 \text{ kN}$$



$M_{\max}$ is at A. At the top,

$$\sigma_{\max} = \frac{Mc}{I} =$$

Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{750}{1963}\right) = 0.509 \text{ kN/mm}^2 = 509 \text{ MPa} \quad \textit{Ans.}$$

3-39

$$M_{\max} = \frac{wl^2}{8} \Rightarrow \sigma_{\max} = \frac{wl^2 c}{8I} \Rightarrow \quad w = \frac{8\sigma_{\max} I}{cl^2}$$

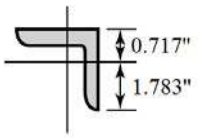
(a) $l = 48$ in; Table A-8, $I = 0.537$ in^4

$$w = \frac{8(12)\left(10^3\right)(0.537)}{1\left(48^2\right)} = 22.38 \text{ lbf/in} \quad \textit{Ans.}$$

(b) $l = 60$ in, $I \doteq (1/12)(2)\left(3^3\right) - (1/12)(1.625)\left(2.625^3\right) = 2.051$ in^4

$$w = \frac{8(12)\left(10^3\right)(2.051)}{(1.5)\left(60^2\right)} = 36.5 \text{ lbf/in} \quad \textit{Ans.}$$

(c) $l = 60$ in; Table A-6, $I = 2(0.703) = 1.406$ in^4



$y = 0.717$ in, $c_{\max} = 1.783$ in

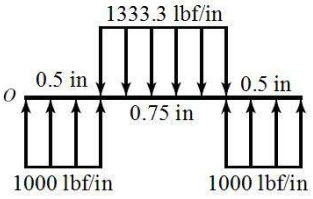
$$w = \frac{8(12)\left(10^3\right)(1.406)}{1.783\left(60^2\right)} = 21.0 \text{ lbf/in} \quad \textit{Ans.}$$

(d) $l = 60$ in, Table A-7, $I = 2.07$ in^4

$$w = \frac{8(12)\left(10^3\right)(2.07)}{1.5\left(60^2\right)} = 36.8 \text{ lbf/in} \quad \textit{Ans.}$$

3-40

$$I = \frac{\pi}{64}\left(0.5^4\right) = 3.068\left(10^{-3}\right)\ \text{in}^4,\quad A = \frac{\pi}{4}\left(0.5^2\right) = 0.1963\ \text{in}^2$$



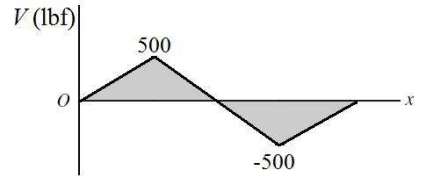
Model

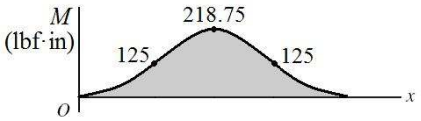
$$M = \frac{500(0.5)}{2} + \frac{500(0.75/2)}{2} = 218.75\ \text{lbf}\cdot\text{in}$$

(*c*)
$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068\left(10^{-3}\right)}$$

$$\sigma = 17\ 825\ \text{psi} = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} = 3.4\ \text{kpsi} \quad \textit{Ans.}$$





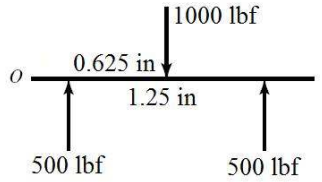
Model (*d*)

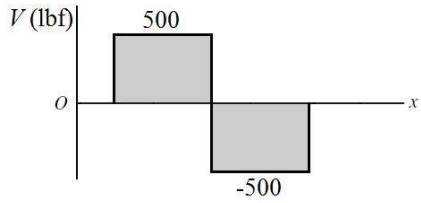
$$M = 500(0.625) = 312.5\ \text{lbf}\cdot\text{in}$$

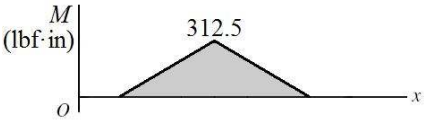
$$\sigma = \frac{Mc}{I} = \frac{312.5(0.25)}{3.068\left(10^{-3}\right)}$$

$$\sigma = 25\ 464\ \text{psi} = 25.5\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} = 3.4\ \text{kpsi} \quad \textit{Ans.}$$







Model

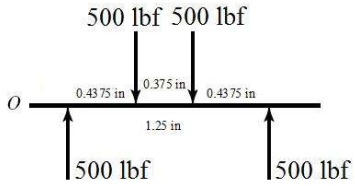
$$M = 500(0.4375) = 218.75\ \text{lbf}\cdot\text{in}$$

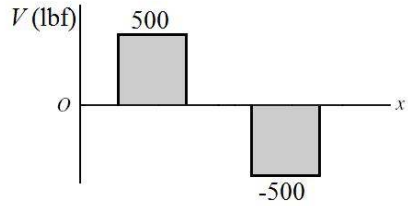
$$\sigma = \frac{Mc}{I} = \frac{218.75(0.25)}{3.068\left(10^{-3}\right)}$$

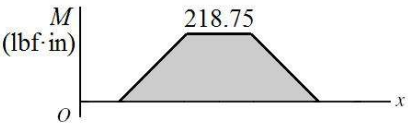
(*e*)

$$\sigma = 17\ 825\ \text{psi} = 17.8\ \text{kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\frac{500}{0.1963} = 3400\ \text{psi} = 3.4\ \text{kpsi} \quad \textit{Ans.}$$







3-41

$$I = \frac{\pi}{64}\left(12^4\right) = 1018 \text{ mm}^4,\ A = \frac{\pi}{4}\left(12^2\right) = 113.1 \text{ mm}^2$$

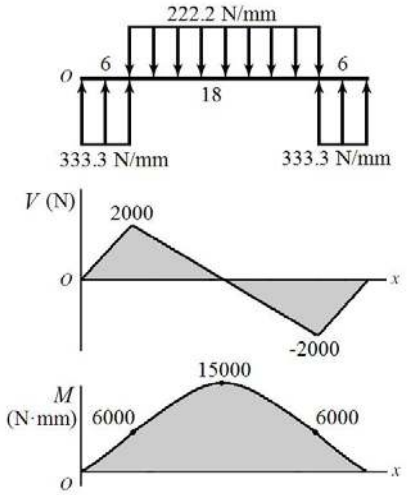
Model (*c*)

$$M = \frac{2000(6)}{2} + \frac{2000(9)}{2} = 15\,000 \text{ N}\cdot\text{mm}$$

$$\sigma = \frac{Mc}{I} = \frac{15\,000(6)}{1018}$$

$$\sigma = 88.4 \text{ N/mm}^2 = 88.4 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{2000}{113.1}\right) = 23.6 \text{ N/mm}^2 = 23.6 \text{ MPa} \quad \textit{Ans.}$$



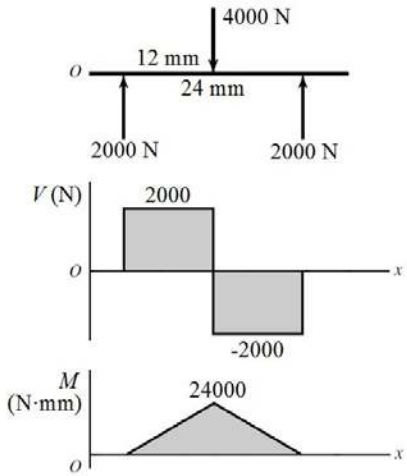
Model (*d*)

$$M = 2000(12) = 24\,000 \text{ N}\cdot\text{mm}$$

$$\sigma = \frac{Mc}{I} = \frac{24\,000(6)}{1018}$$

$$\sigma = 141.5 \text{ N/mm}^2 = 141.5 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{2000}{113.1}\right) = 23.6 \text{ N/mm}^2 = 23.6 \text{ MPa} \quad \textit{Ans.}$$



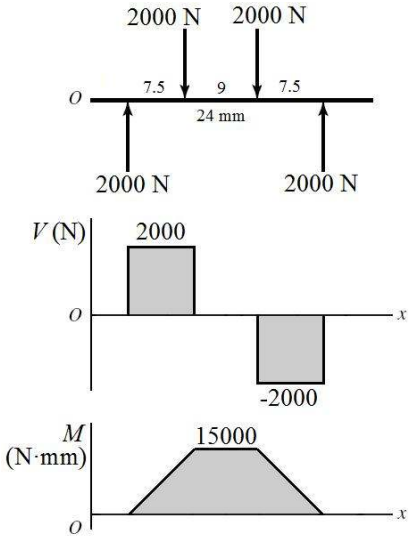
Model (e)

$$M = 2000(7.5) = 15000 \text{ N}\cdot\text{mm}$$

$$\sigma = \frac{Mc}{I} = \frac{15000(6)}{1018}$$

$$\sigma = 88.4 \text{ N/mm}^2 = 88.4 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{4}{3}\frac{V}{A} = \frac{4}{3}\left(\frac{2000}{113.1}\right) = 23.6 \text{ N/mm}^2 = 23.6 \text{ MPa} \quad \textit{Ans.}$$



3-42 **(a)** $\sigma = \dfrac{Mc}{I} = \dfrac{M\left(d/2\right)}{\pi d^4/64} = \dfrac{32M}{\pi d^3}$

$$d = \sqrt[3]{\frac{32M}{\pi\sigma}} = \sqrt[3]{\frac{32(218.75)}{\pi(30\ 000)}} = 0.420 \text{ in} \quad \textit{Ans.}$$

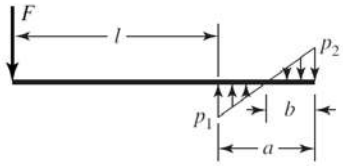
(b) $\tau = \dfrac{V}{A} = \dfrac{V}{\pi d^2/4}$

$$d = \sqrt{\frac{4V}{\pi\tau}} = \sqrt{\frac{4(500)}{\pi(15\,000)}} = 0.206 \text{ in} \quad \textit{Ans.}$$

(c) $\tau = \dfrac{4}{3}\dfrac{V}{A} = \dfrac{4}{3}\dfrac{V}{\left(\pi d^2/4\right)}$

$$d = \sqrt{\frac{4}{3}\frac{4V}{\pi\tau}} = \sqrt{\frac{4}{3}\frac{4(500)}{\pi(15\,000)}} = 0.238 \text{ in} \quad \textit{Ans.}$$

3-43



$$q = -F\langle x\rangle^{-1} + p_1\langle x-l\rangle^{0} - \frac{p_1+p_2}{a}\langle x-l\rangle^{1} + \text{terms for } x > l+a$$

$$V = -F + p_1\langle x-l\rangle^{1} - \frac{p_1+p_2}{2a}\langle x-l\rangle^{2} + \text{ terms for } x > l+a$$

$$M = -Fx + \frac{p_1}{2}\langle x-l\rangle^{2} - \frac{p_1+p_2}{6a}\langle x-l\rangle^{3} + \text{ terms for } x > l+a$$

At $x = (l+a)^+$, $V = M = 0$, terms for $x > l + a = 0$

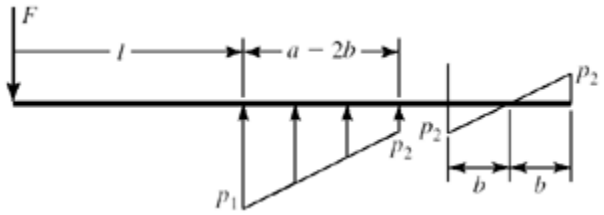
$$-F + p_1 a - \frac{p_1+p_2}{2a}a^2 = 0 \quad \Rightarrow \quad p_1 - p_2 = \frac{2F}{a} \qquad (1)$$

$$-F(l+a) + \frac{p_1 a^2}{2} - \frac{p_1+p_2}{6a}a^3 = 0 \quad \Rightarrow \quad 2p_1 - p_2 = \frac{6F(l+a)}{a^2} \qquad (2)$$

From (1) and (2) $\qquad p_1 = \dfrac{2F}{a^2}(3l+2a), \qquad p_2 = \dfrac{2F}{a^2}(3l+a) \qquad (3)$

From similar triangles $\qquad \dfrac{b}{p_2} = \dfrac{a}{p_1+p_2} \quad \Rightarrow \quad b = \dfrac{ap_2}{p_1+p_2} \qquad (4)$

$M_{\max}$ occurs where $V = 0$



$$x_{\max} = l + a - 2b$$

$$M_{\max} = -F(l + a - 2b) + \frac{p_1}{2}(a - 2b)^2 - \frac{p_1 + p_2}{6a}(a - 2b)^3$$

$$= -Fl - F(a - 2b) + \frac{p_1}{2}(a - 2b)^2 - \frac{p_1 + p_2}{6a}(a - 2b)^3$$

Normally $M_{\max} = -Fl$

The fractional increase in the magnitude is

$$\Delta = \frac{F(a - 2b) - (p_1/2)(a - 2b)^2 + \left[(p_1 + p_2)/6a\right](a - 2b)^3}{Fl} \qquad (5)$$

For example, consider $F = 1500$ lbf, $a = 1.2$ in, $l = 1.5$ in

(3) $$p_1 = \frac{2(1500)}{1.2^2}\left[3(1.5) + 2(1.2)\right] = 14\ 375 \text{ lbf/in}$$

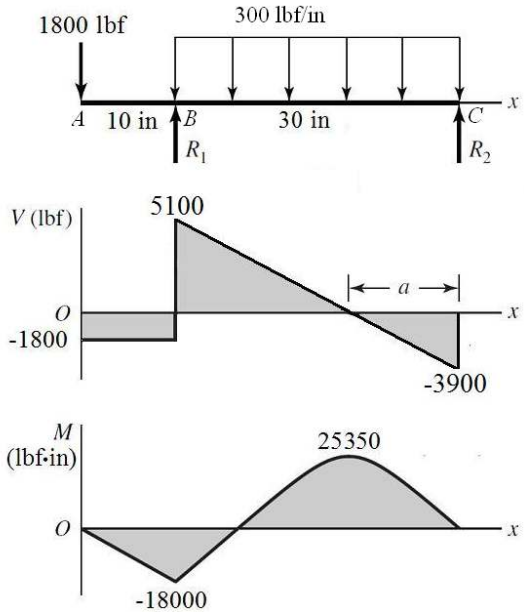
$$p_2 = \frac{2(1500)}{1.2^2}\left[3(1.5) + 1.2\right] = 11\ 875 \text{ lbf/in}$$

(4) $b = 1.2(11\ 875)/(14\ 375 + 11\ 875) = 0.5429$ in

Substituting into (5) yields

$\Delta = 0.036\ 89$ or 3.7% higher than $-Fl$

3-44



$$R_1 = \frac{300(30)}{2} + \frac{40}{30}1800 = 6900 \text{ lbf}$$

$$R_2 = \frac{300(30)}{2} - \frac{10}{30}1800 = 3900 \text{ lbf}$$

$$a = \frac{3900}{300} = 13 \text{ in}$$

$M_B = -1800(10) = -18\,000$ lbf·in

$M_{x=27\text{ in}} = (1/2)3900(13) = 25\,350$ lbf·in

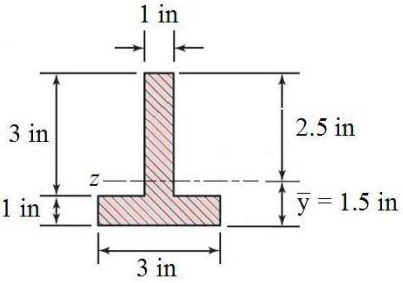
$$\bar{y} = \frac{0.5(3) + 2.5(3)}{6} = 1.5 \text{ in}$$

$$I_1 = \frac{1}{12}(3)(1^3) = 0.25 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(3^3) = 2.25 \text{ in}^4$$

Applying the parallel-axis theorem,

$$I_z = \left[0.25 + 3(1.5 - 0.5)^2\right] + \left[2.25 + 3(2.5 - 1.5)^2\right] = 8.5 \text{ in}^4$$



(a)

At $x = 10$ in, $y = -1.5$ in, $\sigma_x = -\dfrac{-18000(-1.5)}{8.5} = -3176$ psi

At $x = 10$ in, $y = 2.5$ in, $\sigma_x = -\dfrac{-18000(2.5)}{8.5} = 5294$ psi

At $x = 27$ in, $y = -1.5$ in, $\sigma_x = -\dfrac{25350(-1.5)}{8.5} = 4474$ psi

At $x = 27$ in, $y = 2.5$ in, $\sigma_x = -\dfrac{25350(2.5)}{8.5} = -7456$ psi

Max tension = 5294 psi *Ans.*

Max compression = −7456 psi *Ans.*

(b) The maximum shear stress due to V is at B, at the neutral axis.

$V_{\max} = 5100$ lbf

$Q = \bar{y}'A' = 1.25(2.5)(1) = 3.125 \text{ in}^3$

$$\left(\tau_{\max}\right)_V = \frac{VQ}{Ib} = \frac{5100(3.125)}{8.5(1)} = 1875 \text{ psi} \qquad \textit{Ans.}$$

(c) There are three potentially critical locations for the maximum shear stress, all at x = 27 in: (i) at the top where the bending stress is maximum, (ii) at the neutral axis where

the transverse shear is maximum, or (iii) in the web just above the flange where bending stress and shear stress are in their largest combination.

For (i):

The maximum bending stress was previously found to be -7456 psi, and the shear stress is zero. From Mohr's circle,

$$\tau_{\max} = \frac{|\sigma_{\max}|}{2} = \frac{7456}{2} = 3728 \text{ psi}$$

For (ii):

The bending stress is zero, and the transverse shear stress was found previously to be 1875 psi. Thus, $\tau_{\max}$ = 1875 psi.

For (iii):

The bending stress at $y = -0.5$ in is

$$\sigma_x = -\frac{-18000(-0.5)}{8.5} = -1059 \text{ psi}$$

The transverse shear stress is

$$Q = \bar{y}'A' = (1)(3)(1) = 3.0 \text{ in}^3$$

$$\tau = \frac{VQ}{Ib} = \frac{5100(3.0)}{8.5(1)} = 1800 \text{ psi}$$

From Mohr's circle,

$$\tau_{\max} = \sqrt{\left(\frac{-1059}{2}\right)^2 + 1800^2} = 1876 \text{ psi}$$

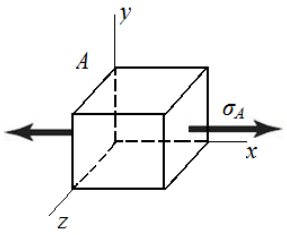
The critical location is at x = 27 in, at the top surface, where $\tau_{\max}$ = 3728 psi. *Ans.*

3-45 **(a)** L = 10 in. Element A:

$$\sigma_A = -\frac{My}{I} = -\frac{-(1000)(10)(0.5)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 101.9 \text{ kpsi}$$

$$\tau_A = \frac{VQ}{Ib}, \quad Q = 0 \quad \Rightarrow \quad \tau_A = 0$$

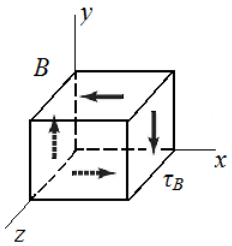
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \sqrt{\left(\frac{101.9}{2}\right)^2 + (0)^2} = 50.9 \text{ kpsi} \quad \textit{Ans.}$$



Element B:

$$\sigma_B = -\frac{My}{I}, \quad y = 0 \quad \Rightarrow \quad \sigma_B = 0$$

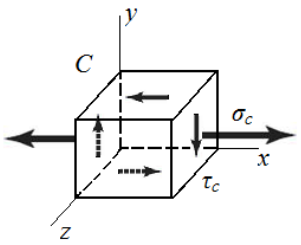
$$Q = \bar{y}'A' = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{4r^3}{6} = \frac{4(0.5)^3}{6} = 1/12 \text{ in}^3$$



$$\tau_B = \frac{VQ}{Ib} = \frac{(1000)(1/12)}{(\pi/64)(1)^4(1)}\left(10^{-3}\right) = 1.698 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2 + 1.698^2} = 1.698 \text{ kpsi} \quad \textit{Ans.}$$

Element C:

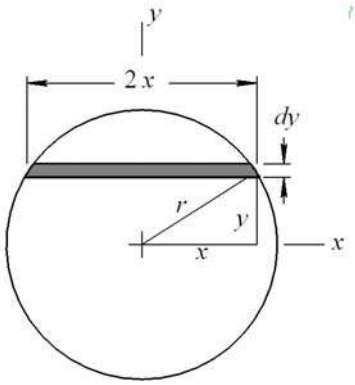


$$\sigma_C = -\frac{My}{I} = -\frac{-(1000)(10)(0.25)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 50.93 \text{ kpsi}$$

$$Q = \int_{y_1}^{r} y\,dA = \int_{y_1}^{r} y(2x)dy = \int_{y_1}^{r} y\left(2\sqrt{r^2 - y^2}\right)dy$$

$$= -\frac{2}{3}\left(r^2 - y^2\right)^{3/2}\Big|_{y_1}^{r} = -\frac{2}{3}\left[\left(r^2 - r^2\right)^{3/2} - \left(r^2 - y_1^2\right)^{3/2}\right]$$

$$= \frac{2}{3}\left(r^2 - y_1^2\right)^{3/2}$$



For C, $y_1 = r/2 = 0.25$ in

$$Q = \frac{2}{3}\left(0.5^2 - 0.25^2\right)^{3/2} = 0.05413 \text{ in}^3$$

$$b = 2x = 2\sqrt{r^2 - y_1^2} = 2\sqrt{0.5^2 - 0.25^2} = 0.866 \text{ in}$$

$$\tau_C = \frac{VQ}{Ib} = \frac{(1000)(0.05413)}{(\pi/64)(1)^4(0.866)}\left(10^{-3}\right) = 1.273 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2 + (1.273)^2} = 25.50 \text{ kpsi} \quad \textit{Ans.}$$

(b) Neglecting transverse shear stress:

Element A: Since the transverse shear stress at point A is zero, there is no change.

$\tau_{\max} = 50.9$ kpsi *Ans.*

% error = 0% *Ans.*

Element B: Since the only stress at point B is transverse shear stress, neglecting the transverse shear stress ignores the entire stress.

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2} = 0 \text{ psi} \quad \textit{Ans.}$$

$$\% \text{ error} = \left(\frac{1.698 - 0}{1.698}\right) * (100) = 100\% \quad \textit{Ans.}$$

Element *C*:

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2} = 25.47 \text{ kpsi} \quad Ans.$$

$$\% \text{ error} = \left(\frac{25.50 - 25.47}{25.50}\right) * (100) = 0.12\% \quad Ans.$$

(c) Repeating the process with different beam lengths produces the results in the table.

	Bending stress, σ (kpsi)	Transverse shear stress, τ (kpsi)	Max shear stress, $\tau_{\max}$ (kpsi)	Max shear stress, neglecting τ, $\tau_{\max}$ (kpsi)	% error
***L* = 10 in**					
A	102	0	50.9	50.9	0
B	0	1.70	1.70	0	100
C	50.9	1.27	25.50	25.47	0.12
***L* = 4 in**					
A	40.7	0	20.4	20.4	0
B	0	1.70	1.70	0	100
C	20.4	1.27	10.26	10.19	0.77
***L* = 1 in**					
A	10.2	0	5.09	5.09	0
B	0	1.70	1.70	0	100
C	5.09	1.27	2.85	2.55	10.6
***L* = 0.1in**					
A	1.02	0	0.509	0.509	0
B	0	1.70	1.70	0	100
C	0.509	1.27	1.30	0.255	80.4

Discussion:
The transverse shear stress is only significant in determining the critical stress element as the length of the cantilever beam becomes smaller. As this length decreases, bending stress reduces greatly and transverse shear stress stays the same. This causes the critical element location to go from being at point *A*, on the surface, to point *B*, in the center. The maximum shear stress is on the outer surface at point *A* for all cases except $L = 0.1$ in, where it is at point *B* at the center. When the critical stress element is at point *A,* there is no error from neglecting transverse shear stress, since it is zero at that location. Neglecting the transverse shear stress has extreme significance at the stress element at the center at point *B*, but that location is probably only of practical significance for very short beam lengths.

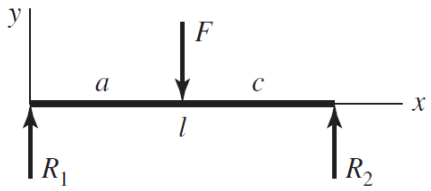
3-46

$$R_1 = \frac{c}{l}F$$

$$M = \frac{c}{l}Fx \quad 0 \le x \le a$$

$$\sigma = \frac{6M}{bh^2} = \frac{6(c/l)Fx}{bh^2}$$

$$h = \sqrt{\frac{6Fcx}{lb\sigma_{\max}}} \quad 0 \le x \le a \quad \textit{Ans.}$$



3-47

From Problem 3-46, $R_1 = \frac{c}{l}F = V,\ 0 \le x \le a$

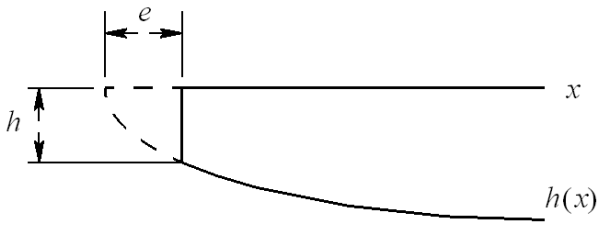
$$\tau_{\max} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{(c/l)F}{bh} \quad \Rightarrow \quad h = \frac{3}{2}\frac{Fc}{lb\tau_{\max}} \quad \textit{Ans.}$$

From Problem 3-46, $h(x) = \sqrt{\frac{6Fcx}{lb\sigma_{\max}}}$.

Sub in $x = e$ and equate to h above.

$$\frac{3}{2}\frac{Fc}{lb\tau_{\max}} = \sqrt{\frac{6Fce}{lb\sigma_{\max}}}$$

$$e = \frac{3}{8}\frac{Fc\sigma_{\max}}{lb\tau_{\max}^2} \quad \textit{Ans.}$$



3-48 **(a)**

<u>*x-z*</u> plane

$$\Sigma M_O = 0 = 1.5(0.5) + 2(1.5)\sin(30^\circ)(2.25) - R_{2z}(3)$$

$$R_{2z} = 1.375 \text{ kN} \quad \textit{Ans.}$$

$$\Sigma F_z = 0 = R_{1z} - 1.5 - 2(1.5)\sin(30^\circ) + 1.375$$

$$R_{1z} = 1.625 \text{ kN} \quad \textit{Ans.}$$

x-y plane

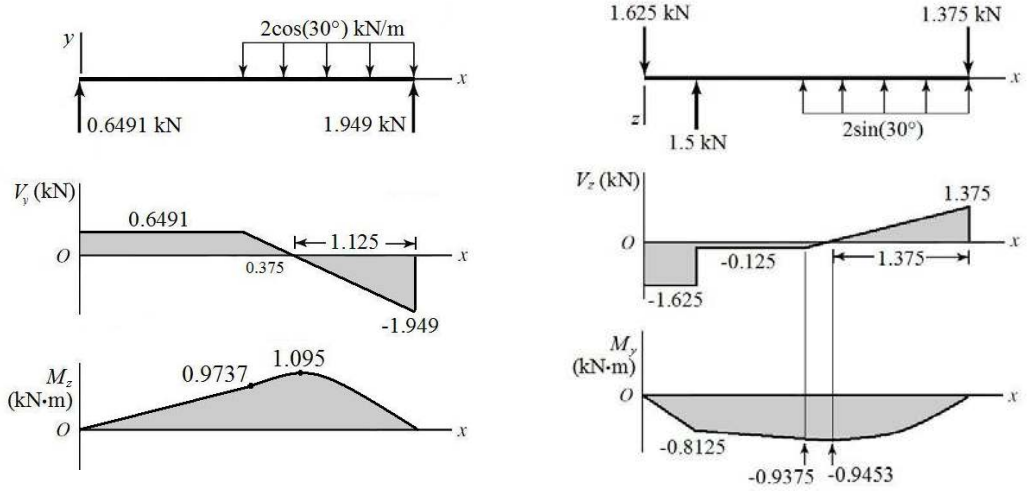
$$\Sigma M_O = 0 = -2(1.5)\cos(30^\circ)(2.25) + R_{2y}(3)$$

$$R_{2y} = 1.949 \text{ kN} \quad \textit{Ans.}$$

$$\Sigma F_y = 0 = R_{1y} - 2(1.5)\cos(30^\circ) + 1.949$$

$$R_{1y} = 0.6491 \text{ kN} \quad \textit{Ans.}$$

(b)



(c) The transverse shear and bending moments for most points of interest can readily be taken straight from the diagrams. For $1.5 < x < 3$, the bending moment equations are parabolic, and are obtained by integrating the linear expressions for shear. For convenience, use a coordinate shift of $x' = x - 1.5$. Then, for $0 < x' < 1.5$,

$$V_z = x' - 0.125$$

$$M_y = \int V_z dx' = \frac{(x')^2}{2} - 0.125x' + C$$

$$\text{At } x' = 0, M_y = C = -0.9375 \;\Rightarrow\; M_y = 0.5(x')^2 - 0.125x' + 0.9375$$

$$V_y = -\frac{1.949}{1.125}x' + 0.6491 = -1.732x' + 0.6491$$

$$M_z = \frac{-1.732}{2}(x')^2 + 0.6491x' + C$$

$$\text{At } x' = 0, M_z = C = 0.9737 \;\Rightarrow\; M_z = -0.8662(x')^2 - 0.125x' - 0.9375$$

By programming these bending moment equations, we can find M_y, M_z, and their vector combination at any point along the beam. The maximum combined bending moment is found to be at $x = 1.79$ m, where $M = 1.433$ kN·m. The table below shows values at key locations on the shear and bending moment diagrams.

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–1.625	0.6491	1.750	0	0	0
0.5^-	–1.625	0.6491	1.750	–0.8125	0.3246	0.8749
1.5	–0.1250	0.6491	0.6610	0.9375	0.9737	1.352
1.625	0	0.4327	0.4327	–0.9453	1.041	1.406
1.875	0.2500	0	0.2500	–0.9141	1.095	1.427
3^-	1.375	–1.949	2.385	0	0	0

(d) The bending stress is obtained from Eq. (3-27),

$$\sigma_x = \frac{-M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

The maximum tensile bending stress will be at point *A* in the cross section of Prob. 3-34 (*a*), where distances from the neutral axes for both bending moments will be maximum. At *A*, for M_z, $y_A = -37.5$ mm, and for M_y, $z_A = -20$ mm.

$$I_z = \frac{40(75)^3}{12} - \frac{34(25)^3}{12} = 1.36(10^6)\ \text{mm}^4 = 1.36(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{25(40)^3}{12}\right] + \frac{25(6)^3}{12} = 2.67(10^5)\ \text{mm}^4 = 2.67(10^{-7})\ \text{m}^4$$

It is apparent the maximum bending moment, and thus the maximum stress, will be in the parabolic section of the bending moment diagrams. Programming Eq. (3-27) with the bending moment equations previously derived, the maximum tensile bending stress is found at $x = 1.77$ m, where $M_y = -0.9408$ kN·m, $M_z = 1.075$ kN·m, and $\sigma_x = 100.1$ MPa. *Ans.*

3-49

(a) *x-z* plane

$$\Sigma M_O = 0 = \frac{3}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oy}$$

$$M_{Oy} = 1842.6\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_z = 0 = R_{Oz} - \frac{3}{5}(1000) + \frac{600}{\sqrt{2}}$$

$$R_{Oz} = 175.7\ \text{lbf} \quad \textit{Ans.}$$

x-y plane

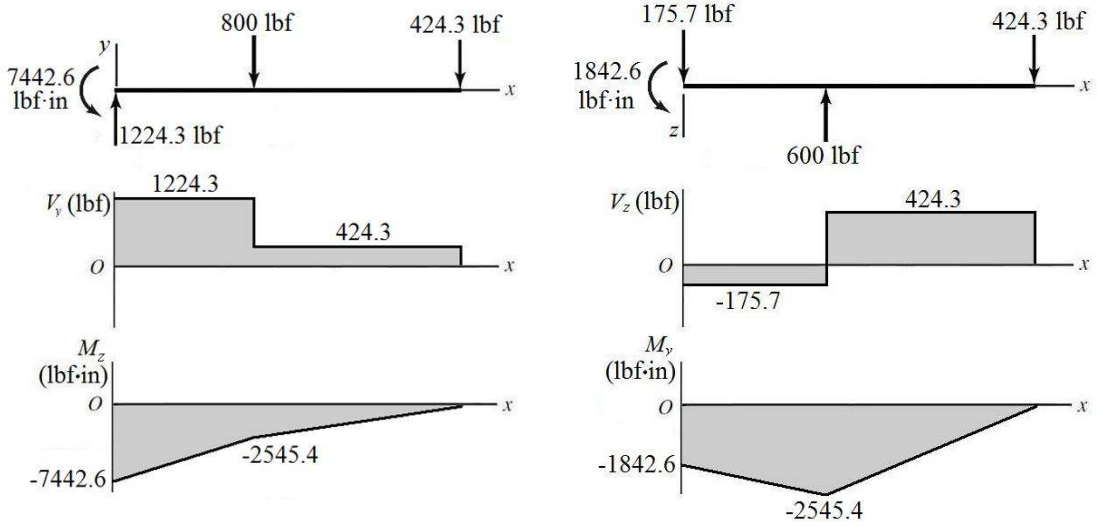
$$\Sigma M_O = 0 = -\frac{4}{5}(1000)(4) - \frac{600}{\sqrt{2}}(10) + M_{Oz}$$

$$M_{Oz} = 7442.5\ \text{lbf}\cdot\text{in} \quad \textit{Ans.}$$

$$\Sigma F_y = 0 = R_{Oy} - \frac{4}{5}(1000) - \frac{600}{\sqrt{2}}$$

$$R_{Oy} = 1224.3\ \text{lbf} \quad \textit{Ans.}$$

(b)



(c)

$$V(x) = \left[V_y(x)^2 + V_z(x)^2 \right]^{1/2}$$

$$M(x) = \left[M_y(x)^2 + M_z(x)^2 \right]^{1/2}$$

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–175.7	1224.3	1237	–1842.6	–7442.6	7667
4^-	–175.7	1224.3	1237	–2545.4	–2545.4	3600
10^-	424.3	424.3	600	0	0	0

(d) The maximum tensile bending stress will be at the outer corner of the cross section in the positive y, negative z quadrant, where $y = 1.5$ in and $z = -1$ in.

$$I_z = \frac{2(3)^3}{12} - \frac{(1.625)(2.625)^3}{12} = 2.051 \text{ in}^4$$

$$I_y = \frac{3(2)^3}{12} - \frac{(2.625)(1.625)^3}{12} = 1.601 \text{ in}^4$$

At $x = 0$, using Eq. (3-27),

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

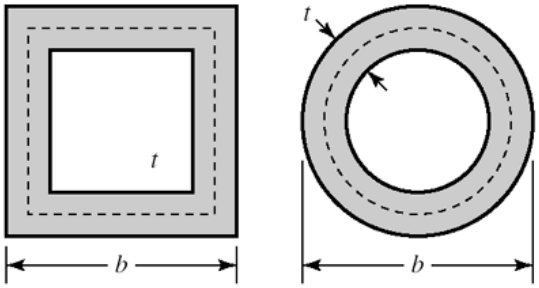
$$\sigma_x = -\frac{(-7442.6)(1.5)}{2.051} + \frac{(-1842.6)(-1)}{1.601} = 6594 \text{ psi}$$

Check at $x = 4$ in,

$$\sigma_x = -\frac{(-2545.4)(1.5)}{2.051} + \frac{(-2545.4)(-1)}{1.601} = 2706 \text{ psi}$$

The critical location is at $x = 0$, where $\sigma_x = 6594$ psi. *Ans.*

3-50 The area within the wall median line, A_m, is



Square: $A_m = (b-t)^2$. From Eq. (3-45)

$$T_{\text{sq}} = 2A_m t\tau_{\text{all}} = 2(b-t)^2 t\tau_{\text{all}}$$

Round: $A_m = \pi(b-t)^2/4$

$$T_{\text{rd}} = 2\pi(b-t)^2 t\tau_{\text{all}}/4$$

Ratio of Torques

$$\frac{T_{\text{sq}}}{T_{\text{rd}}} = \frac{2(b-t)^2 t\tau_{\text{all}}}{\pi(b-t)^2 t\tau_{\text{all}}/2} = \frac{4}{\pi} = 1.27$$

Twist per unit length from Eq. (3-46) is

$$\theta_1 = \frac{TL_m}{4GA_m^2 t} = \frac{2A_m t\tau_{\text{all}} L_m}{4GA_m^2 t} = \frac{\tau_{\text{all}}}{2G}\frac{L_m}{A_m} = C\frac{L_m}{A_m}$$

Square:

$$\theta_{\text{sq}} = C\frac{4(b-t)}{(b-t)^2}$$

Round:

$$\theta_{\text{rd}} = C\frac{\pi(b-t)}{\pi(b-t)^2/4} = C\frac{4(b-t)}{(b-t)^2}$$

Ratio equals 1. Twists are the same.

3-51

(a) The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 = 0.8789$ in^2 and the length of the section median line is $L_m = 4(1 - 0.0625) = 3.75$ in. From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8789)(0.0625)(12\ 000) = 1318\ \text{lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1318)(3.75)(36)}{4(11.5)(10^6)(0.8789)^2(0.0625)} = 0.0801\ \text{rad} = 4.59^\circ \quad \textit{Ans.}$$

(b) The radius at the median line is $r_m = 0.125 + (0.5)(0.0625) = 0.15625$ in. The area enclosed by the section median line is $A_m = (1 - 0.0625)^2 - 4(0.15625)^2 + 4(\pi/4)(0.15625)^2 = 0.8579$ in^2. The length of the section median line is $L_m = 4[1 - 0.0625 - 2(0.15625)] + 2\pi(0.15625) = 3.482$ in.

From Eq. (3-45),

$$T = 2A_m t\tau = 2(0.8579)(0.0625)(12\ 000) = 1287\ \text{lbf}\cdot\text{in} \qquad \textit{Ans.}$$

From Eq. (3-46),

$$\theta = \theta_1 l = \frac{TL_m l}{4GA_m^2 t} = \frac{(1287)(3.482)(36)}{4(11.5)(10^6)(0.8579)^2(0.0625)} = 0.0762\ \text{rad} = 4.37^\circ \quad \textit{Ans.}$$

3-52

$$\theta_1 = \frac{3T_i}{GL_i c_i^3} \qquad \Rightarrow \qquad T_i = \frac{\theta_1 GL_i c_i^3}{3}$$

$$T = T_1 + T_2 + T_3 = \frac{\theta_1 G}{3}\sum_{i=1}^{3} L_i c_i^3 \qquad \textit{Ans.}$$

From Eq. (3-47), $\tau = G\theta_1 c$

G and θ_1 are constant, therefore the largest shear stress occurs when c is a maximum.

$$\tau_{\max} = G\theta_1 c_{\max} \qquad \textit{Ans.}$$

3-53

(b) Solve part (b) first since the twist is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12(6.89) = 82.7\ \text{MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7(10^6)}{79.3(10^9)(0.003)} = 0.348\ \text{rad/m} \qquad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 GL_1 c_1^3}{3} = \frac{0.348(79.3)(10^9)(0.020)(0.002^3)}{3} = 1.47\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 GL_2 c_2^3}{3} = \frac{0.348(79.3)(10^9)(0.030)(0.003^3)}{3} = 7.45\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 GL_3 c_3^3}{3} = \frac{0.348(79.3)(10^9)(0)(0^3)}{3} = 0 \qquad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 0 = 8.92\ \text{N}\cdot\text{m} \qquad \textit{Ans.}$$

3-54

(b) Solve part (b) first since the twist is needed for part (a).

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{12\,000}{11.5\left(10^6\right)(0.125)} = 8.35\left(10^{-3}\right) \text{ rad/in} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.75)\left(0.0625^3\right)}{3} = 5.86 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(1)\left(0.125^3\right)}{3} = 62.52 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{(8.35)\left(10^{-3}\right)(11.5)\left(10^6\right)(0.625)\left(0.0625^3\right)}{3} = 4.88 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 5.86 + 62.52 + 4.88 = 73.3 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

3-55

(b) Solve part (b) first since the twist is needed for part (a).

$$\tau_{\max} = \tau_{\text{allow}} = 12(6.89) = 82.7 \text{ MPa}$$

$$\theta_1 = \frac{\tau_{\max}}{Gc_{\max}} = \frac{82.7\left(10^6\right)}{79.3\left(10^9\right)(0.003)} = 0.348 \text{ rad/m} \quad \textit{Ans.}$$

(a)

$$T_1 = \frac{\theta_1 G L_1 c_1^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.020)(0.002^3)}{3} = 1.47 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$T_2 = \frac{\theta_2 G L_2 c_2^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.030)(0.003^3)}{3} = 7.45 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$T_3 = \frac{\theta_3 G L_3 c_3^3}{3} = \frac{0.348(79.3)\left(10^9\right)(0.025)(0.002^3)}{3} = 1.84 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$T = T_1 + T_2 + T_3 = 1.47 + 7.45 + 1.84 = 10.8 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

3-56

(a) From Eq. (3-40), with two 2-mm strips,

$$T = \frac{\tau_{\max} b c^2}{3 + 1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.002^2\right)}{3 + 1.8/\left(0.030/0.002\right)} = 3.08 \text{ N} \cdot \text{m}$$

$$T_{\max} = 2(3.08) = 6.16 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

From the table on p. 102, with $b/c = 30/2 = 15$, $\alpha = \beta$ and has a value between 0.313 and 0.333.

From Eq. (3-40),

$$\alpha \doteq \frac{1}{3 + 1.8/(30/2)} = 0.321$$

From Eq. (3-41),

$$\theta = \frac{Tl}{\beta b c^3 G} = \frac{3.08(0.3)}{0.321(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \quad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{6.16}{0.151} = 40.8 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

From Eq. (3-40), with a single 4-mm strip,

$$T_{\max} = \frac{\tau_{\max} b c^2}{3 + 1.8/(b/c)} = \frac{(80)\left(10^6\right)(0.030)\left(0.004^2\right)}{3 + 1.8/(0.030/0.004)} = 11.9 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

Interpolating from the table on p. 102, with $b/c = 30/4 = 7.5$,

$$\beta = \frac{7.5 - 6}{8 - 6}(0.307 - 0.299) + 0.299 = 0.305$$

From Eq. (3-41)

$$\theta = \frac{Tl}{\beta b c^3 G} = \frac{11.9(0.3)}{0.305(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0769 \text{ rad} \quad \textit{Ans.}$$

$$k_t = \frac{T}{\theta} = \frac{11.9}{0.0769} = 155 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

(b) From Eq. (3-47), with two 2-mm strips,

$$T = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.002^2\right)(80)\left(10^6\right)}{3} = 3.20 \text{ N} \cdot \text{m}$$

$$T_{\max} = 2(3.20) = 6.40 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3 G} = \frac{3(3.20)(0.3)}{(0.030)\left(0.002^3\right)(79.3)\left(10^9\right)} = 0.151 \text{ rad} \quad \textit{Ans.}$$

$$k_t = T/\theta = 6.40/0.151 = 42.4 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

From Eq. (3-47), with a single 4-mm strip,

$$T_{\max} = \frac{Lc^2\tau}{3} = \frac{(0.030)\left(0.004^2\right)(80)\left(10^6\right)}{3} = 12.8 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

$$\theta = \frac{3Tl}{Lc^3 G} = \frac{3(12.8)(0.3)}{(0.030)\left(0.004^3\right)(79.3)\left(10^9\right)} = 0.0757 \text{ rad} \quad \textit{Ans.}$$

$$k_t = T/\theta = 12.8/0.0757 = 169 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

The results for the spring constants when using Eq. (3-47) are slightly larger than when using Eq. (3-40) and Eq. (3-41) because the strips are not infinitesimally thin (i.e. b/c does not equal infinity). The spring constants when considering one solid strip are significantly larger (almost four times larger) than when considering two thin strips because two thin strips would be able to slip along the center plane.

3-57

(a) Obtain the torque from the given power and speed using Eq. (3-44).

$$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{2500} = 152.8 \text{ N} \cdot \text{m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi \tau_{\max}}\right)^{1/3} = \left[\frac{16(152.8)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0223 \text{ m} = 22.3 \text{ mm} \quad \textit{Ans.}$$

(b) $$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{250} = 1528 \text{ N} \cdot \text{m}$$

$$d = \left[\frac{16(1528)}{\pi(70)\left(10^6\right)}\right]^{1/3} = 0.0481 \text{ m} = 48.1 \text{ mm} \quad \textit{Ans.}$$

3-58

(a) Obtain the torque from the given power and speed using Eq. (3-42).

$$T = \frac{63\,025H}{n} = \frac{63\,025(50)}{2500} = 1261 \text{ lbf} \cdot \text{in}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi \tau_{\max}}\right)^{1/3} = \left[\frac{16(1261)}{\pi(20\,000)}\right]^{1/3} = 0.685 \text{ in} \quad \textit{Ans.}$$

(b) $$T = \frac{63\,025H}{n} = \frac{63\,025(50)}{250} = 12\,610 \text{ lbf} \cdot \text{in}$$

$$d = \left[\frac{16(12\,610)}{\pi(20\,000)}\right]^{1/3} = 1.48 \text{ in} \quad \textit{Ans.}$$

3-59

$$\tau_{\max} = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\tau_{\max}\pi d^3}{16} = \frac{(50)\left(10^6\right)\pi\left(0.03^3\right)}{16} = 265 \text{ N}\cdot\text{m}$$

Eq. (3-44), $H = \frac{Tn}{9.55} = \frac{265(2000)}{9.55} = 55.5\left(10^3\right) \text{ W} = 55.5 \text{ kW}$ *Ans.*

3-60

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3 = \frac{\pi}{16}(110)\left(10^6\right)\left(0.020^3\right) = 173 \text{ N}\cdot\text{m}$$

$$\theta = \frac{Tl}{JG} \quad \Rightarrow \quad l = \frac{\pi d^4 G\theta}{32T} = \frac{\pi\left(0.020^4\right)(79.3)\left(10^9\right)\left(15\frac{\pi}{180}\right)}{32(173)}$$

$l = 1.89 \text{ m}$ *Ans.*

3-61

$$\tau = \frac{16T}{\pi d^3} \quad \Rightarrow \quad T = \frac{\pi}{16}\tau d^3 = \frac{\pi}{16}(30\,000)\left(0.75^3\right) = 2485 \text{ lbf}\cdot\text{in}$$

$$\theta = \frac{Tl}{JG} = \frac{32Tl}{\pi d^4 G} = \frac{32(2485)(24)}{\pi\left(0.75^4\right)(11.5)\left(10^6\right)} = 0.167 \text{ rad} = 9.57^\circ \quad \textit{Ans.}$$

3-62

(a) $T_{\text{solid}} = \frac{J\tau_{\max}}{r} = \frac{\pi d_o^4 \tau_{\max}}{16 d_o}$ $\qquad T_{\text{hollow}} = \frac{J\tau_{\max}}{r} = \frac{\pi(d_o^4 - d_i^4)\tau_{\max}}{16 d_o}$

$$\%\Delta T = \frac{T_{\text{solid}} - T_{\text{hollow}}}{T_{\text{solid}}}(100\%) = \frac{d_i^4}{d_o^4}(100\%) = \frac{\left(36^4\right)}{\left(40^4\right)}(100\%) = 65.6\% \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd_o^2, \qquad W_{\text{hollow}} = k\left(d_o{}^2 - d_i{}^2\right)$

$$\%\Delta W = \frac{W_{\text{solid}} - W_{\text{hollow}}}{W_{\text{solid}}}(100\%) = \frac{d_i^2}{d_o^2}(100\%) = \frac{\left(36^2\right)}{\left(40^2\right)}(100\%) = 81.0\% \quad \textit{Ans.}$$

3-63

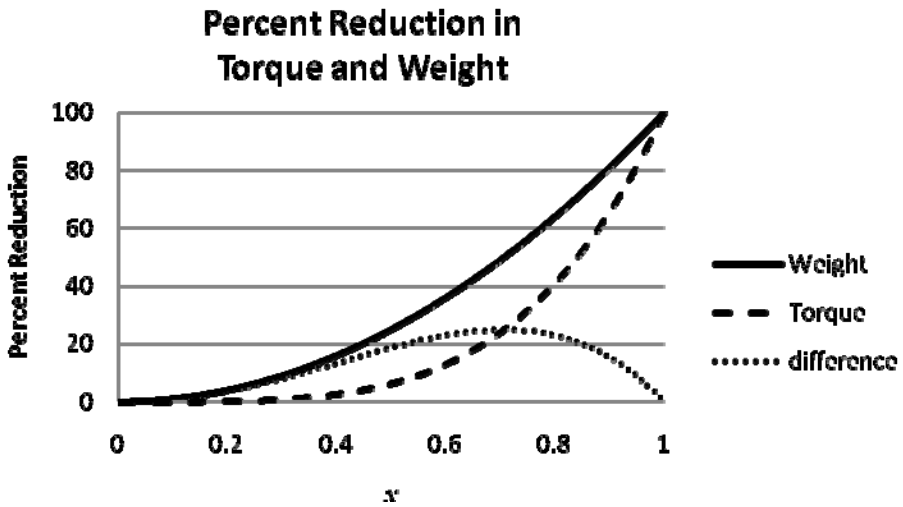
(a) $T_{\text{solid}} = \frac{J\tau_{\max}}{r} = \frac{\pi d^4 \tau_{\max}}{16 d}$ $\qquad T_{\text{hollow}} = \frac{J\tau_{\max}}{r} = \frac{\pi\left[d^4 - (xd)^4\right]\tau_{\max}}{16 d}$

$$\%\Delta T = \frac{T_{\text{solid}} - T_{\text{hollow}}}{T_{\text{solid}}}(100\%) = \frac{(xd)^4}{d^4}(100\%) = x^4(100\%) \quad \textit{Ans.}$$

(b) $W_{\text{solid}} = kd^2 \qquad W_{\text{hollow}} = k\left(d^2 - (xd)^2\right)$

$$\%\Delta W = \frac{W_{\text{solid}} - W_{\text{hollow}}}{W_{\text{solid}}}(100\%) = \frac{(xd)^2}{d^2}(100\%) = x^2(100\%) \qquad \textit{Ans.}$$

Plot $\%\Delta T$ and $\%\Delta W$ versus x.



The value of greatest difference in percent reduction of weight and torque is 25% and occurs at $x = \sqrt{2}/2$.

3-64

(a) $\tau = \dfrac{Tc}{J} \quad \Rightarrow \quad 120\left(10^6\right) = \dfrac{4200(d/2)}{(\pi/32)\left[d^4 - (0.70d)^4\right]} = \dfrac{2.8149\left(10^4\right)}{d^3}$

$$d = \left(\frac{2.8149\left(10^4\right)}{120(10^6)}\right)^{1/3} = 6.17\left(10^{-2}\right) \text{ m} = 61.7 \text{ mm}$$

From Table A-17, the next preferred size is $d = 80$ mm. *Ans.*
$d_i = 0.7d = 56$ mm. The next preferred size smaller is $d_i = 50$ mm *Ans.*

(b)

$$\tau = \frac{Tc}{J} = \frac{4200(d_i/2)}{(\pi/32)\left[d^4 - (d_i)^4\right]} = \frac{4200(0.050/2)}{(\pi/32)\left[(0.080)^4 - (0.050)^4\right]} = 30.8 \text{ MPa} \qquad \textit{Ans.}$$

3-65

$$T = 9.55\frac{H}{n} = 9.55\frac{(1500)}{10} = 1433 \text{ N}\cdot\text{m}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(1433)}{\pi(80)(10^6)}\right]^{1/3} = 0.045 \text{ m} = 45 \text{ mm}$$

From Table A-17, select 50 mm. *Ans.*

(a) $\tau_{\text{start}} = \dfrac{16(2)(1433)}{\pi(0.050^3)} = 117(10^6) \text{ Pa} = 117 \text{ MPa}$ *Ans.*

(b) Design activity

3-66

$$T = \frac{63\ 025H}{n} = \frac{63\ 025(1)}{8} = 7880 \text{ lbf}\cdot\text{in}$$

$$\tau = \frac{16T}{\pi d_C^3} \quad \Rightarrow \quad d_C = \left(\frac{16T}{\pi\tau}\right)^{1/3} = \left[\frac{16(7880)}{\pi(15\ 000)}\right]^{1/3} = 1.39 \text{ in}$$

From Table A-17, select 1.40 in. *Ans.*

3-67 For a square cross section with side length b, and a circular section with diameter d,

$$A_{\text{square}} = A_{\text{circular}} \quad \Rightarrow \quad b^2 = \frac{\pi}{4}d^2 \quad \Rightarrow \quad b = \frac{\sqrt{\pi}}{2}d$$

From Eq. (3-40) with $b = c$,

$$(\tau_{\max})_{\text{square}} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) = \frac{T}{b^3}\left(3 + \frac{1.8}{1}\right) = \frac{T}{d^3}\left(\frac{2}{\sqrt{\pi}}\right)^3 (4.8) = 6.896\frac{T}{d^3}$$

For the circular cross section,

$$(\tau_{\max})_{\text{circular}} = \frac{16T}{\pi d^3} = 5.093\frac{T}{d^3}$$

$$\frac{(\tau_{\max})_{\text{square}}}{(\tau_{\max})_{\text{circular}}} = \frac{6.896\dfrac{T}{d^3}}{5.093\dfrac{T}{d^3}} = 1.354$$

The shear stress in the square cross section is 35.4% greater. *Ans.*

(b) For the square cross section, from the table on p. 102, $\beta = 0.141$. From Eq. (3-41),

$$\theta_{\text{square}} = \frac{Tl}{\beta bc^3 G} = \frac{Tl}{\beta b^4 G} = \frac{Tl}{0.141\left(\frac{\sqrt{\pi}}{2}d\right)^4 G} = 11.50\frac{Tl}{d^4 G}$$

For the circular cross section,

$$\theta_{rd} = \frac{Tl}{GJ} = \frac{Tl}{G\left(\pi d^4/32\right)} = 10.19\frac{Tl}{d^4 G}$$

$$\frac{\theta_{sq}}{\theta_{rd}} = \frac{11.50\dfrac{Tl}{d^4 G}}{10.19\dfrac{Tl}{d^4 G}} = 1.129$$

The angle of twist in the square cross section is 12.9% greater. *Ans.*

3-68 **(a)**

$$T_1 = 0.15T_2$$

$$\sum T = 0 = (500-75)(4) - \left(T_2 - T_1\right)(5) = 1700 - \left(T_2 - 0.15T_2\right)(5)$$

$$1700 - 4.25T_2 = 0 \qquad \Rightarrow \qquad T_2 = 400 \text{ lbf} \qquad \textit{Ans.}$$

$$T_1 = 0.15(400) = 60 \text{ lbf} \qquad \textit{Ans.}$$

(b)

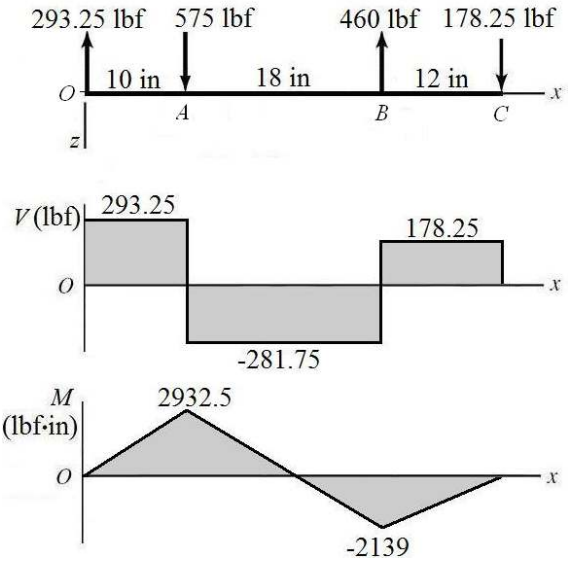
$$\sum M_O = 0 = -575(10) + 460(28) - R_C(40)$$

$$R_C = 178.25 \doteq 178 \text{ lbf} \qquad \textit{Ans.}$$

$$\sum F = 0 = R_O + 575 - 460 + 178.25$$

$$R_O = -293.25 \text{ lbf} \qquad \textit{Ans.}$$

(c)



(d) The maximum bending moment is at $x = 10$ in, and is $M = 2932.5$ lbf·in. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (500 - 75)(4) = 1700$ lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(2932.5)}{\pi(1.25)^3} = 15\,294 \text{ psi} = 15.3 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1700)}{\pi(1.25)^3} = 4433 \text{ psi} = 4.43 \text{ kpsi} \quad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{15.3}{2} \pm \sqrt{\left(\frac{15.3}{2}\right)^2 + (4.43)^2}$$

$$\sigma_1 = 16.5 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -1.19 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{15.3}{2}\right)^2 + (4.43)^2} = 8.84 \text{ kpsi} \quad \textit{Ans.}$$

3-69 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (1800 - 270)(200) + (T_2 - T_1)(125) = 306(10^3) + 125(0.15T_1 - T_1)$$

$$306(10^3) - 106.25T_1 = 0 \quad \Rightarrow \quad T_1 = 2880 \text{ N} \quad \textit{Ans.}$$

$$T_2 = 0.15(2880) = 432 \text{ N} \quad \textit{Ans.}$$

(b)

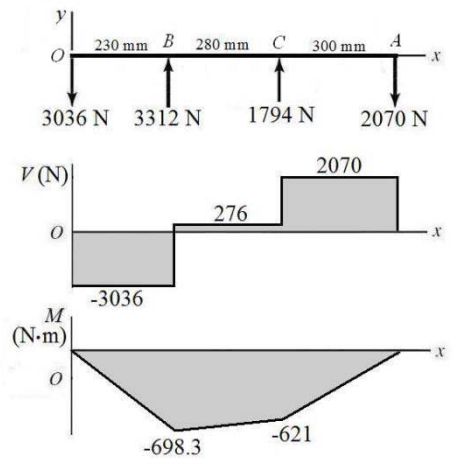
$$\sum M_O = 0 = 3312(230) + R_C(510) - 2070(810)$$

$$R_C = 1794 \text{ N} \quad \textit{Ans.}$$

$$\sum F_y = 0 = R_O + 3312 + 1794 - 2070$$

$$R_O = -3036 \text{ N} \quad \textit{Ans.}$$

(c)



(d) The maximum bending moment is at $x = 230$ mm, and is $M = -698.3$ N·m. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (1800 - 270)(0.200) = 306$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(698.3)}{\pi(0.030)^3} = 263(10^3) \text{ Pa} = 263 \text{ MPa} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(306)}{\pi(0.030)^3} = 57.7(10^6) \text{ Pa} = 57.7\text{MPa} \quad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{263}{2} \pm \sqrt{\left(\frac{263}{2}\right)^2 + (57.7)^2}$$

$$\sigma_1 = 275 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_2 = -12.1 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{263}{2}\right)^2 + (57.7)^2} = 144 \text{ MPa} \quad \textit{Ans.}$$

3-70

(a)

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (300 - 50)(4) + (T_2 - T_1)(3) = 1000 + (0.15T_1 - T_1)(3)$$

$$1000 - 2.55T_1 = 0 \quad \Rightarrow \quad T_1 = 392.16 \text{ lbf} \quad \textit{Ans.}$$

$$T_2 = 0.15(392.16) = 58.82 \text{ lbf} \quad \textit{Ans.}$$

(b)

$$\sum M_{Oy} = 0 = -450.98(16) - R_{Cz}(22)$$

$$R_{Cz} = -327.99 \text{ lbf} \quad Ans.$$

$$\sum F_z = 0 = R_{Oz} + 450.98 - 327.99$$

$$R_{Oz} = -122.99 \text{ lbf} \quad Ans.$$

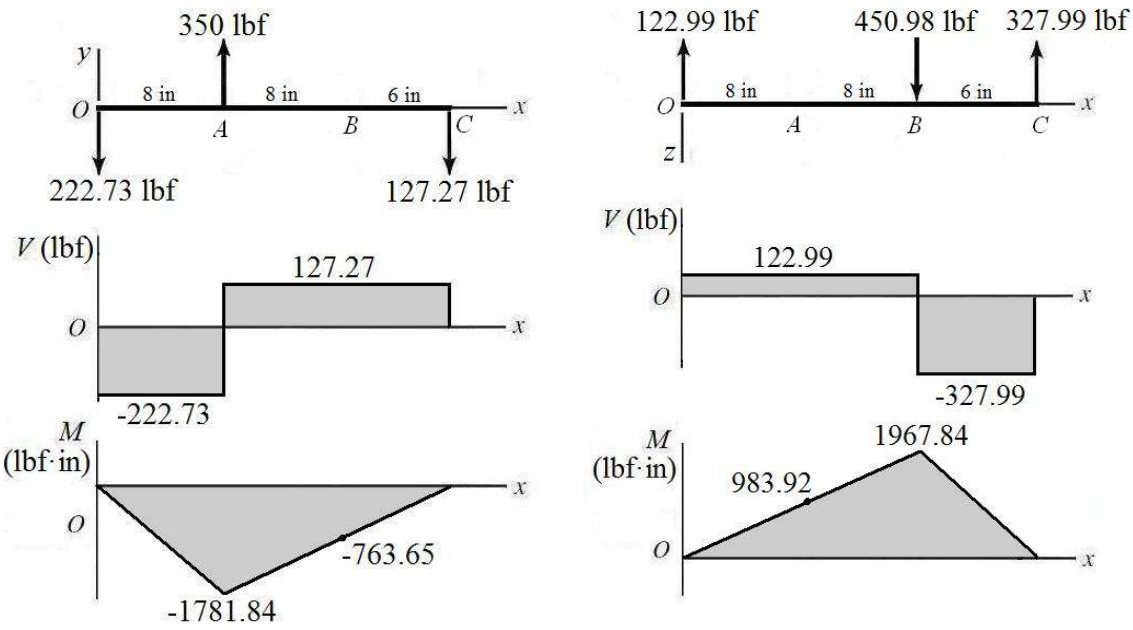
$$\sum M_{Oz} = 0 = 350(8) + R_{Cy}(22)$$

$$R_{Cy} = -127.27 \text{ lbf} \quad Ans.$$

$$\sum F_y = 0 = R_{Oy} + 350 - 127.27$$

$$R_{Oy} = -222.73 \text{ lbf} \quad Ans.$$

(c)



(d) Combine the bending moments from both planes at *A* and *B* to find the critical location.

$$M_A = \sqrt{(983.92)^2 + (-1781.84)^2} = 2035 \text{ lbf} \cdot \text{in}$$

$$M_B = \sqrt{(1967.84)^2 + (-763.65)^2} = 2111 \text{ lbf} \cdot \text{in}$$

The critical location is at *B*. The torque transmitted through the shaft from *A* to *B* is $T = (300 - 50)(4) = 1000$ lbf·in. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(2111)}{\pi(1)^3} = 21\,502 \text{ psi} = 21.5 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi} \quad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{21.5}{2} \pm \sqrt{\left(\frac{21.5}{2}\right)^2 + (5.09)^2}$$

$$\sigma_1 = 22.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -1.14 \text{ kpsi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{21.5}{2}\right)^2 + (5.09)^2} = 11.9 \text{ kpsi} \quad \textit{Ans.}$$

3-71 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (300-45)(125) + (T_2 - T_1)(150) = 31\,875 + (0.15T_1 - T_1)(150)$$

$$31\,875 - 127.5T_1 = 0 \qquad \Rightarrow \qquad T_1 = 250 \text{ N}\cdot\text{mm} \quad \textit{Ans.}$$

$$T_2 = 0.15(250) = 37.5 \text{ N}\cdot\text{mm} \quad \textit{Ans.}$$

(b)

$$\sum M_{Oy} = 0 = 345\sin 45^\circ(300) - 287.5(700) - R_{Cz}(850)$$

$$R_{Cz} = -150.7 \text{ N} \qquad \textit{Ans.}$$

$$\sum F_z = 0 = R_{Oz} - 345\cos 45^\circ + 287.5 - 150.7$$

$$R_{Oz} = 107.2 \text{ N} \qquad \textit{Ans.}$$

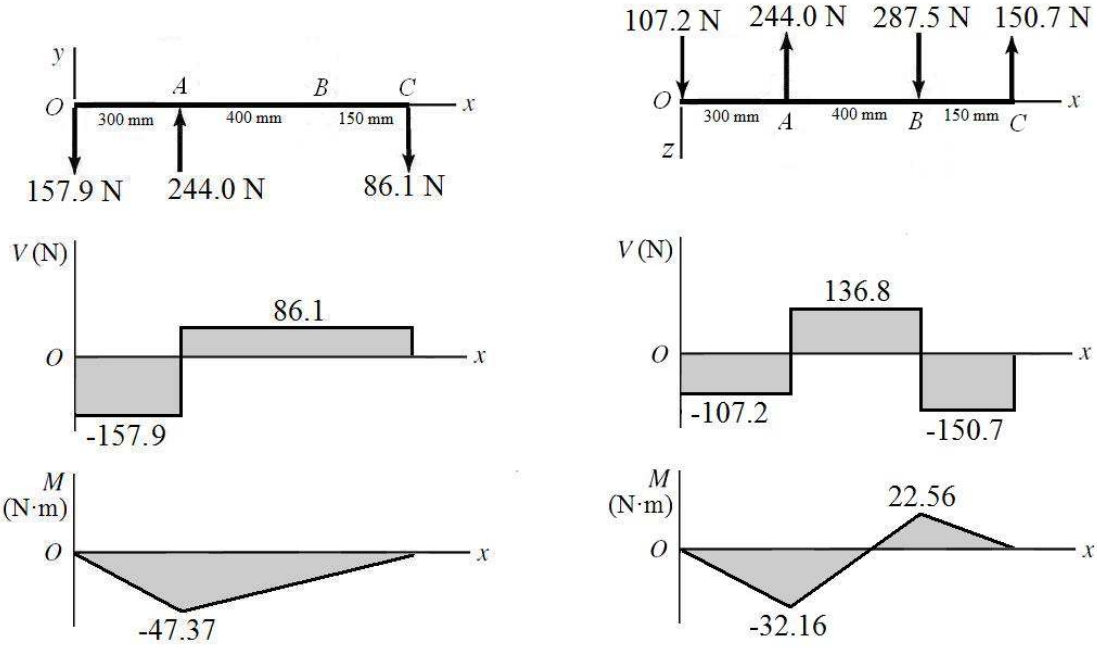
$$\sum M_{Oz} = 0 = 345\sin 45^\circ(300) + R_{Cy}(850)$$

$$R_{Cy} = -86.10 \text{ N} \qquad \textit{Ans.}$$

$$\sum F_y = 0 = R_{Oy} + 345\cos 45^\circ - 86.10$$

$$R_{Oy} = -157.9 \text{ N} \qquad \textit{Ans.}$$

(c)



n

ding moment diagrams, it is clear that the critical location is at *A* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M = \sqrt{(-47.37)^2 + (-32.16)^2} = 57.26 \text{ N}\cdot\text{m}$$

The torque transmitted through the shaft from A to B is $T = (300 - 45)(0.125) = 31.88$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(57.26)}{\pi(0.020)^3} = 72.9\left(10^6\right) \text{ Pa} = 72.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(31.88)}{\pi(0.020)^3} = 20.3\left(10^6\right) \text{ Pa} = 20.3 \text{ MPa} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{72.9}{2} \pm \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2}$$

$$\sigma_1 = 78.2 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_2 = -5.27 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2} = 41.7 \text{ MPa} \qquad \textit{Ans.}$$

3-72

(a)

$$\sum T = 0 = -300(\cos 20^\circ)(10) + F_B(\cos 20^\circ)(4)$$

$$F_B = 750 \text{ lbf} \quad \textit{Ans.}$$

(b)

$$\sum M_{Oz} = 0 = 300(\cos 20^\circ)(16) - 750(\sin 20^\circ)(39) + R_{Cy}(30)$$

$$R_{Cy} = 183 \text{ lbf} \quad \textit{Ans.}$$

$$\sum F_y = 0 = R_{Oy} + 300(\cos 20^\circ) + 183 - 750(\sin 20^\circ)$$

$$R_{Oy} = -208 \text{ lbf} \quad \textit{Ans.}$$

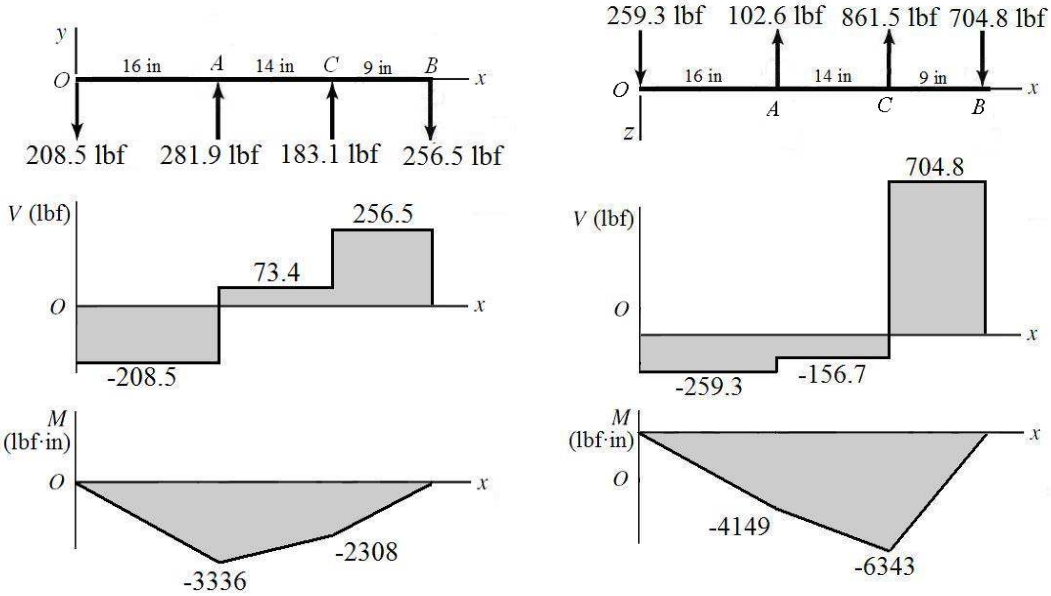
$$\sum M_{Oy} = 0 = 300(\sin 20^\circ)(16) - R_{Cz}(30) - 750(\cos 20^\circ)(39)$$

$$R_{Cz} = -861 \text{ lbf} \quad \textit{Ans.}$$

$$\sum F_z = 0 = R_{Oz} - 300(\sin 20^\circ) - 861 + 750(\cos 20^\circ)$$

$$R_{Oz} = 259 \text{ lbf} \quad \textit{Ans.}$$

(c)



(d) Combine the bending moments from both planes at *A* and C to find the critical location.

$$M_A = \sqrt{(-3336)^2 + (-4149)^2} = 5324 \text{ lbf} \cdot \text{in}$$

$$M_C = \sqrt{(-2308)^2 + (-6343)^2} = 6750 \text{ lbf} \cdot \text{in}$$

The critical location is at *C*. The torque transmitted through the shaft from *A* to *B* is $T = 300\cos(20^\circ)(10) = 2819 \text{ lbf} \cdot \text{in}$. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(6750)}{\pi(1.25)^3} = 35\,203 \text{ psi} = 35.2 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(2819)}{\pi(1.25)^3} = 7351 \text{ psi} = 7.35 \text{ kpsi} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{35.2}{2} \pm \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2}$$

$$\sigma_1 = 36.7 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_2 = -1.47 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{35.2}{2}\right)^2 + (7.35)^2} = 19.1 \text{ kpsi} \qquad \textit{Ans.}$$

3-73

(a)

$$\sum T = 0 = -11\,000(\cos 20^\circ)(300) + F_B(\cos 25^\circ)(150)$$

$$F_B = 22\,810 \text{ N} \qquad Ans.$$

(b)

$$\sum M_{Oz} = 0 = -11\,000(\sin 20^\circ)(400) - 22\,810(\sin 25^\circ)(750) + R_{Cy}(1050)$$

$$R_{Cy} = 8319 \text{ N} \qquad Ans.$$

$$\sum F_y = 0 = R_{Oy} - 11000(\sin 20^\circ) - 22\,810\sin(25^\circ) + 8319$$

$$R_{Oy} = 5083 \text{ N} \qquad Ans.$$

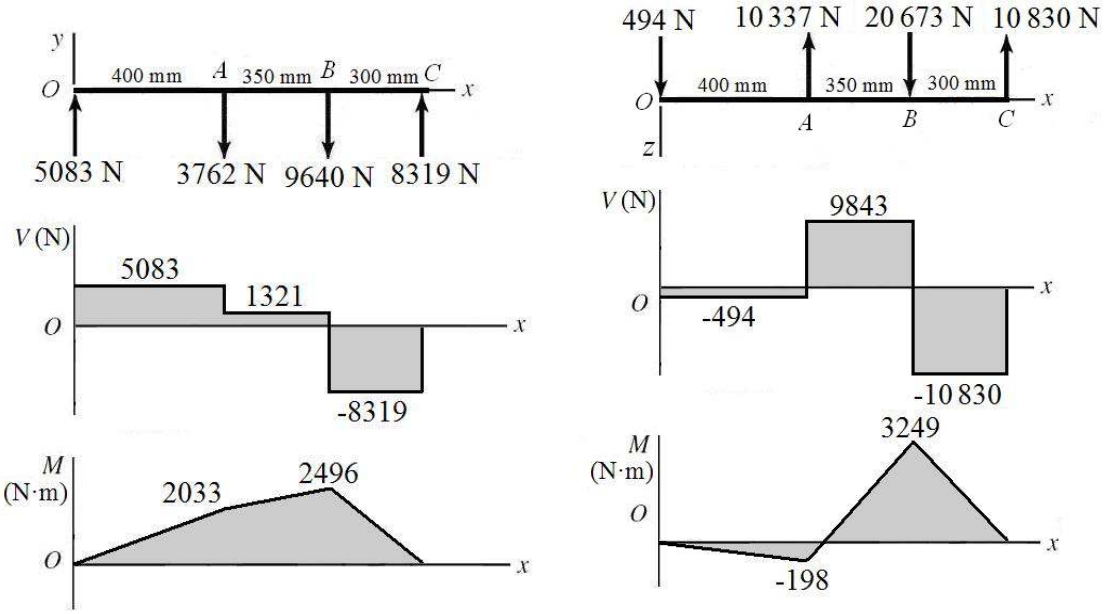
$$\sum M_{Oy} = 0 = 11\,000(\cos 20^\circ)(400) - 22\,810(\cos 25^\circ)(750) - R_{Cz}(1050)$$

$$R_{Cz} = -10\,830 \text{ N} \quad Ans.$$

$$\sum F_z = 0 = R_{Oz} - 11\,000(\cos 20^\circ) + 22\,810(\cos 25^\circ) - 10\,830$$

$$R_{Oz} = 494 \text{ N} \quad Ans.$$

(c)



(d) From the bending moment diagrams, it is clear that the critical location is at *B* where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M = \sqrt{(2496)^2 + (3249)^2} = 4097 \text{ N} \cdot \text{m}$$

The torque transmitted through the shaft from *A* to *B* is

$$T = 11\,000\cos(20^\circ)(0.3) = 3101 \text{ N} \cdot \text{m}.$$

For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(4097)}{\pi(0.050)^3} = 333.9\left(10^6\right) \text{ Pa} = 333.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(3101)}{\pi(0.050)^3} = 126.3\left(10^6\right) \text{ Pa} = 126.3 \text{ MPa} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{333.9}{2} \pm \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2}$$

$\sigma_1 = 376$ MPa *Ans.*

$\sigma_2 = -42.4$ MPa *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{333.9}{2}\right)^2 + (126.3)^2} = 209 \text{ MPa} \qquad \textit{Ans.}$$

3-74

(a)

$$\left(\Sigma M_D\right)_z = 6.13C_x - 3.8(92.8) - 3.88(362.8) = 0$$

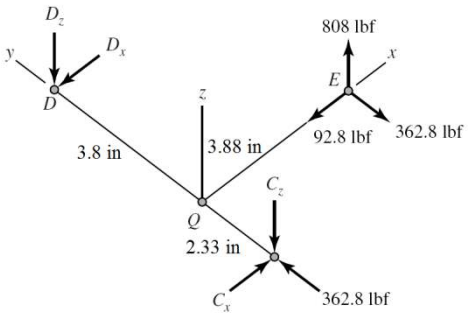
$C_x = 287.2$ lbf *Ans.*

$$\left(\Sigma M_C\right)_z = 6.13D_x + 2.33(92.8) - 3.88(362.8) = 0$$

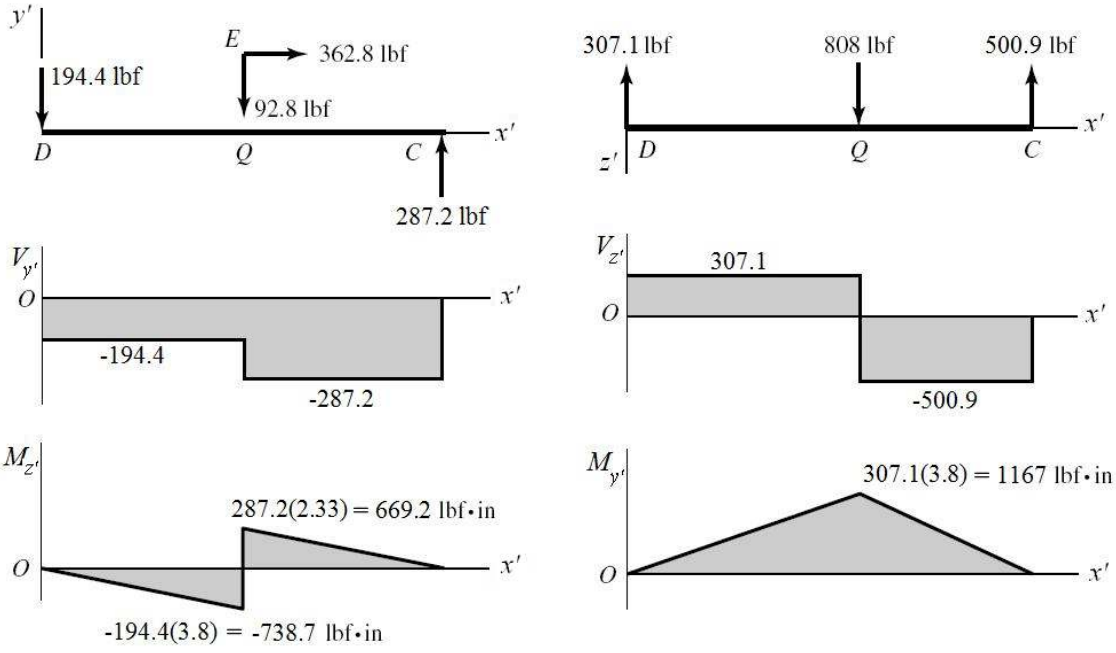
$D_x = 194.4$ lbf *Ans.*

$$\left(\Sigma M_D\right)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(808) = 500.9 \text{ lbf} \qquad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(808) = 307.1 \text{ lbf} \qquad \textit{Ans.}$$



(b) For *DQC*, let *x′, y′, z′* correspond to the original – *y, x, z* axes.



(c) The critical stress element is just to the right of Q, where the bending moment in both planes is maximum, and where the torsional and axial loads exist.

$$T = 808(3.88) = 3135 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{669.2^2 + 1167^2} = 1345 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(3135)}{\pi\left(1.13^3\right)} = 11\,070 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(1345)}{\pi\left(1.13^3\right)} = \pm 9495 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{362.8}{(\pi/4)\left(1.13^2\right)} = -362 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -9495 - 362 = -9857 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2} = 12\,118 \text{ psi} = 12.1 \text{ kpsi} \quad \textit{Ans.}$$

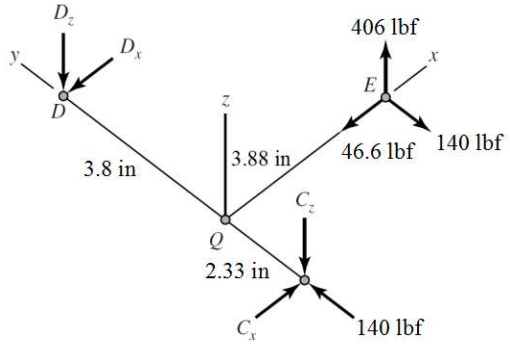
$$\sigma_1, \sigma_2 = \frac{-9857}{2} \pm \sqrt{\left(\frac{-9857}{2}\right)^2 + 11\,070^2}$$

$$\sigma_1 = 7189 \text{ psi} = 7.19 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -17\,046 \text{ psi} = -17.0 \text{ kpsi} \quad \textit{Ans.}$$

3-75

(a)



$$\left(\Sigma M_D\right)_z = 0$$

$$6.13C_x - 3.8(46.6) - 3.88(140) = 0$$

$$C_x = 117.5 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_z = 0$$

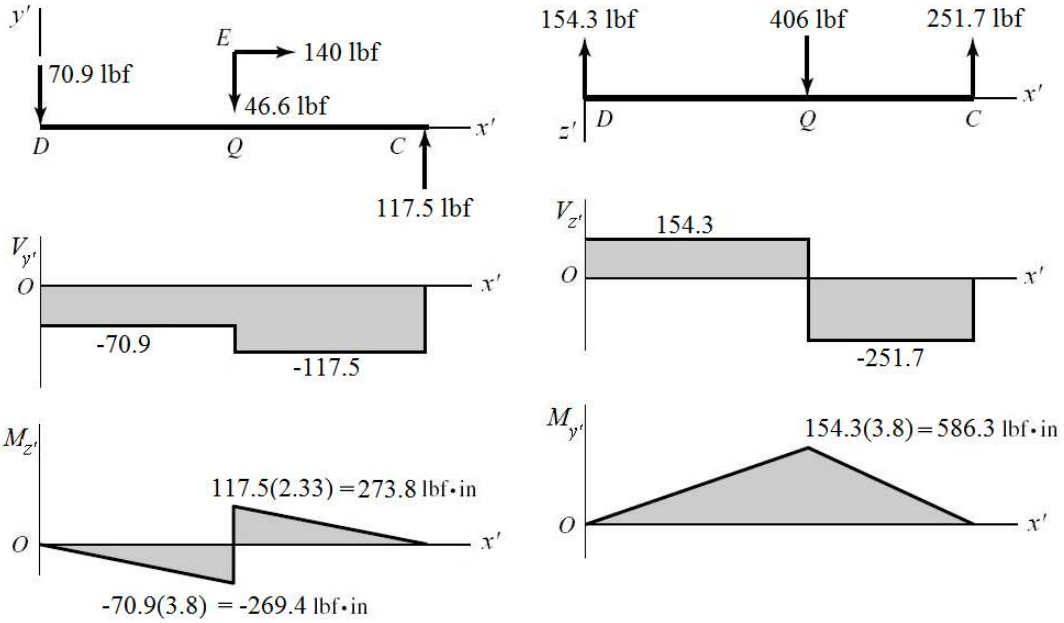
$$-6.13D_x - 2.33(46.6) + 3.88(140) = 0$$

$$D_x = 70.9 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_D\right)_x = 0 \quad \Rightarrow \quad C_z = \frac{3.8}{6.13}(406) = 251.7 \text{ lbf} \quad \textit{Ans.}$$

$$\left(\Sigma M_C\right)_x = 0 \quad \Rightarrow \quad D_z = \frac{2.33}{6.13}(406) = 154.3 \text{ lbf} \quad \textit{Ans.}$$

(b) For *DQC*, let *x′, y′, z′* correspond to the original – *y, x, z* axes.



(c) The critical stress element is just to the right of *Q*, where the bending moment in both planes is maximum, and where the torsional and axial loads exist.

$$T = 406(3.88) = 1575 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{273.8^2 + 586.3^2} = 647.1 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16T}{\pi d^3} = \frac{16(1575)}{\pi\left(1^3\right)} = 8021 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(647.1)}{\pi\left(1^3\right)} = \pm 6591 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{140}{(\pi/4)\left(1^2\right)} = -178.3 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress element will be where the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -6591 - 178.3 = -6769 \text{ psi}$$

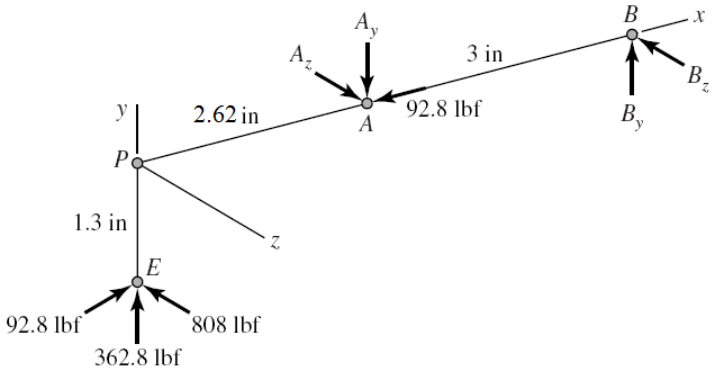
$$\tau_{\max} = \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2} = 8706 \text{ psi} = 8.71 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_1, \sigma_2 = \frac{-6769}{2} \pm \sqrt{\left(\frac{-6769}{2}\right)^2 + 8021^2}$$

$\sigma_1 = 5321 \text{ psi} = 5.32 \text{ kpsi}$ *Ans.*

$\sigma_2 = -12\,090 \text{ psi} = -12.1 \text{ kpsi}$ *Ans.*

3-76



$$(\Sigma M_B)_z = -5.62(362.8) + 1.3(92.8) + 3A_y = 0$$

$$A_y = 639.4 \text{ lbf} \qquad \textit{Ans.}$$

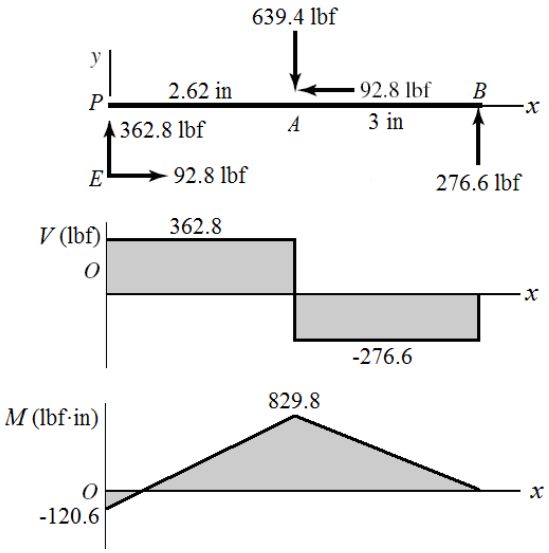
$$(\Sigma M_A)_z = -2.62(362.8) + 1.3(92.8) + 3B_y = 0$$

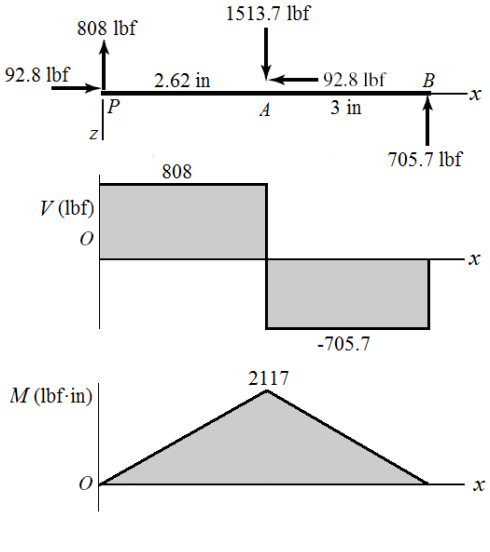
$$B_y = 276.6 \text{ lbf} \qquad \textit{Ans.}$$

$$(\Sigma M_B)_y = 0 \quad \Rightarrow \quad A_z = \frac{5.62}{3}(808) = 1513.7 \text{ lbf} \qquad \textit{Ans.}$$

$$(\Sigma M_A)_y = 0 \quad \Rightarrow \quad B_z = \frac{2.62}{3}(808) = 705.7 \text{ lbf} \qquad \textit{Ans.}$$

(b)





(c) The critical stress element is just to the left of A, where the bending moment in both planes is maximum, and where the torsional and axial loads exist.

$$T = 808(1.3) = 1050 \text{ lbf} \cdot \text{in}$$

$$\tau = \frac{16(1050)}{\pi\left(0.88^3\right)} = 7847 \text{ psi} \quad \textit{Ans.}$$

$$M = \sqrt{(829.8)^2 + (2117)^2} = 2274 \text{ lbf} \cdot \text{in}$$

$$\sigma_b = \pm\frac{32M}{\pi d^3} = \pm\frac{32(2274)}{\pi\left(0.88^3\right)} = \pm 33\,990 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_a = -\frac{F}{A} = -\frac{92.8}{(\pi/4)\left(0.88^2\right)} = -153 \text{ psi} \quad \textit{Ans.}$$

(d) The critical stress will occur when the bending stress and axial stress are both in compression.

$$\sigma_{\max} = -33\,990 - 153 = -34\,143 \text{ psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{-34\,143}{2}\right)^2 + 7847^2} = 18\,789 \text{ psi} = 18.8 \text{ kpsi} \quad \textit{Ans.}$$

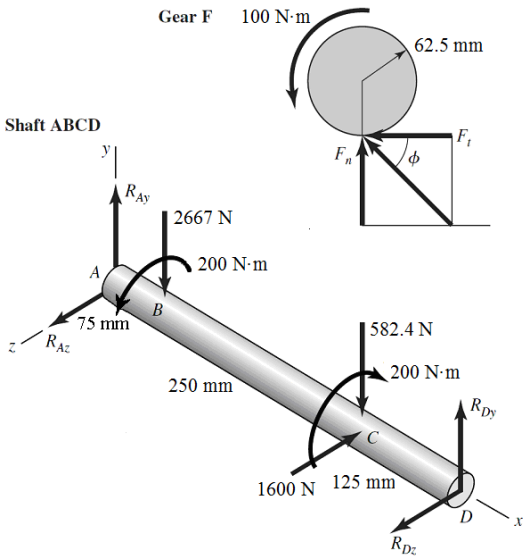
$$\sigma_1, \sigma_2 = \frac{-34143}{2} \pm \sqrt{\left(\frac{-34143}{2}\right)^2 + 7847^2}$$

$$\sigma_1 = 1717 \text{ psi} = 1.72 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_2 = -35\,860 \text{ psi} = -35.9 \text{ kpsi} \quad \textit{Ans.}$$

3-77

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600 \tan 20 = 582.4 \text{ N}$$

$$T_C = F_t\left(b/2\right) = 1600\left(0.250/2\right) = 200 \text{ N} \cdot \text{m}$$

$$P = \frac{T_C}{\left(a/2\right)} = \frac{200}{\left(0.150/2\right)} = 2667 \text{ N}$$

$$\sum\left(M_A\right)_z = 0$$

$$450R_{Dy} - 582.4(325) - 2667(75) = 0$$

$$R_{Dy} = 865.1 \text{ N}$$

$$\sum\left(M_A\right)_y = 0 = -450R_{Dz} + 1600(325) \quad \Rightarrow \quad R_{Dz} = 1156 \text{ N}$$

$$\sum F_y = 0 = R_{Ay} + 865.1 - 582.4 - 2667 \quad \Rightarrow \quad R_{Ay} = 2384 \text{ N}$$

$$\sum F_z = 0 = R_{Az} + 1156 - 1600 \quad \Rightarrow \quad R_{Az} = 444 \text{ N}$$

AB The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch the shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{2384^2 + 444^2} = 181.9 \text{ N}\cdot\text{m}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{865.1^2 + 1156^2} = 180.5 \text{ N}\cdot\text{m}$$

The stresses at *B* and *C* are almost identical, but the maximum stresses occur at *B*. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(181.9)}{\pi\left(0.030^3\right)} = 68.6\left(10^6\right) \text{ Pa} = 68.6 \text{ MPa}$$

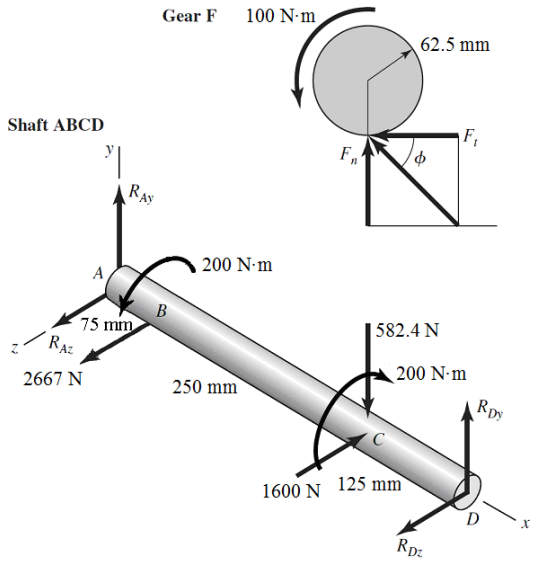
$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi\left(0.030^3\right)} = 37.7\left(10^6\right) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{68.6}{2} + \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 85.3 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{68.6}{2}\right)^2 + 37.7^2} = 51.0 \text{ MPa} \quad \textit{Ans.}$$

3-78

$$F_t = \frac{T}{c/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$



$$F_n = 1600\tan 20 = 582.4 \text{ N}$$

$$T_C = F_t\left(b/2\right) = 1600\left(0.250/2\right) = 200 \text{ N}\cdot\text{m}$$

$$P = \frac{T_C}{\left(a/2\right)} = \frac{200}{\left(0.150/2\right)} = 2667 \text{ N}$$

$$\sum\left(M_A\right)_z = 0 = 450R_{Dy} - 582.4(325) \quad \Rightarrow \quad R_{Dy} = 420.6 \text{ N}$$

$$\sum\left(M_A\right)_y = 0 = -450R_{Dz} + 1600(325) - 2667(75) \quad \Rightarrow \quad R_{Dz} = 711.1 \text{ N}$$

$$\sum F_y = 0 = R_{Ay} + 420.6 - 582.4 \quad \Rightarrow \quad R_{Ay} = 161.8 \text{ N}$$

$$\sum F_z = 0 = R_{Az} + 711.1 - 1600 + 2667 \quad \Rightarrow \quad R_{Az} = -1778 \text{ N}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 0.075\sqrt{161.8^2 + (-1778)^2} = 133.9 \text{ N}\cdot\text{m}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 0.125\sqrt{420.6^2 + 711.1^2} = 103.3 \text{ N}\cdot\text{m}$$

The maximum stresses occur at *B*. *Ans.*

$$\sigma_B = \frac{32M_B}{\pi d^3} = \frac{32(133.9)}{\pi\left(0.030^3\right)} = 50.5\left(10^6\right) \text{ Pa} = 50.5 \text{ MPa}$$

$$\tau_B = \frac{16T_B}{\pi d^3} = \frac{16(200)}{\pi\left(0.030^3\right)} = 37.7\left(10^6\right) \text{ Pa} = 37.7 \text{ MPa}$$

$$\sigma_{\max} = \frac{\sigma_B}{2} + \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \frac{50.5}{2} + \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 70.6 \text{ MPa} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_B}{2}\right)^2 + \tau_B^2} = \sqrt{\left(\frac{50.5}{2}\right)^2 + 37.7^2} = 45.4 \text{ MPa} \quad \textit{Ans.}$$

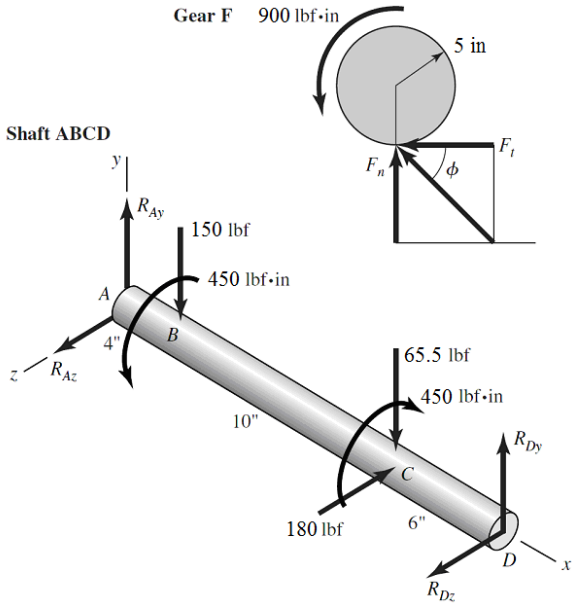
3-79

$$F_t = \frac{T}{c/2} = \frac{900}{10/2} = 180 \text{ lbf}$$

$$F_n = 180\tan 20 = 65.5 \text{ lbf}$$

$$T_C = F_t(b/2) = 180(5/2) = 450 \text{ lbf}\cdot\text{in}$$

$$P = \frac{T_C}{(a/2)} = \frac{450}{(6/2)} = 150 \text{ lbf}$$



$$\sum (M_A)_z = 0 = 20R_{Dy} - 65.5(14) - 150(4) \quad \Rightarrow \quad R_{Dy} = 75.9 \text{ lbf}$$

$$\sum (M_A)_y = 0 = -20R_{Dz} + 180(14) \quad \Rightarrow \quad R_{Dz} = 126 \text{ lbf}$$

$$\sum F_y = 0 = R_{Ay} + 75.9 - 65.5 - 150 \quad \Rightarrow \quad R_{Ay} = 140 \text{ lbf}$$

$$\sum F_z = 0 = R_{Az} + 126 - 180 \quad \Rightarrow \quad R_{Az} = 54.0 \text{ lbf}$$

The maximum bending moment will either be at *B* or *C*. If this is not obvious, sketch shear and bending moment diagrams. We will directly obtain the combined moments from each plane.

$$M_B = \overline{AB}\sqrt{R_{A_y}^2 + R_{A_z}^2} = 4\sqrt{140^2 + 54^2} = 600 \text{ lbf} \cdot \text{in}$$

$$M_C = \overline{CD}\sqrt{R_{D_y}^2 + R_{D_z}^2} = 6\sqrt{75.9^2 + 126^2} = 883 \text{ lbf} \cdot \text{in}$$

The maximum stresses occur at *C*. *Ans.*

$$\sigma_C = \frac{32M_C}{\pi d^3} = \frac{32(883)}{\pi\left(1.375^3\right)} = 3460 \text{ psi}$$

$$\tau_C = \frac{16T_C}{\pi d^3} = \frac{16(450)}{\pi\left(1.375^3\right)} = 882 \text{ psi}$$

$$\sigma_{\max} = \frac{\sigma_C}{2} + \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \frac{3460}{2} + \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 3670 \text{ psi} \quad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_C}{2}\right)^2 + \tau_C^2} = \sqrt{\left(\frac{3460}{2}\right)^2 + 882^2} = 1940 \text{ psi} \quad \textit{Ans.}$$

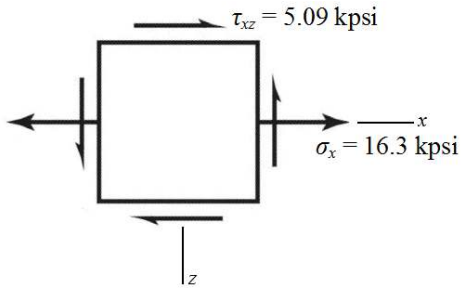
3-80

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moment at the wall. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at the wall, at either the top (compression) or the bottom (tension) on the *y* axis. We will select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M\left(d/2\right)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(8)(200)}{\pi\left(1\right)^3} = 16\,297 \text{ psi} = 16.3 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T\left(d/2\right)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi\left(1\right)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{16.3}{2} \pm \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 17.8$ kpsi *Ans.*

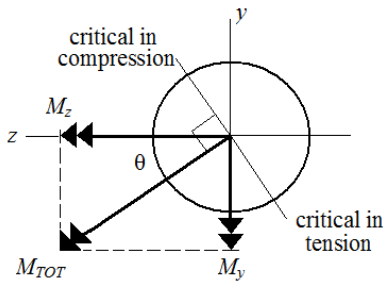
$\sigma_2 = -1.46$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{16.3}{2}\right)^2 + (5.09)^2} = 9.61 \text{ kpsi} \quad \textit{Ans.}$$

3-81

(a) Rod *AB* experiences constant torsion throughout its length, and maximum bending moments at the wall in both planes of bending. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

$$M_y = -(100)(8) = -800 \text{ lbf·in}$$
$$M_z = (175)(8) = 1400 \text{ lbf·in}$$
$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2} = \sqrt{(-800)^2 + 1400^2} = 1612 \text{ lbf} \cdot \text{in}$$
$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{800}{1400}\right) = 29.7°$$

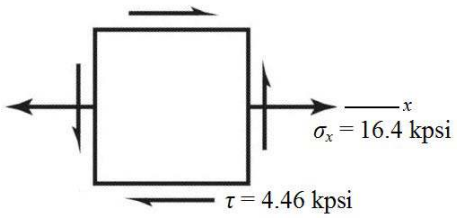


The combined bending moment vector is at an angle of 29.7º CCW from the *z* axis. The critical bending stress location, and thus the critical stress element, will be ±90º from this vector, as shown. There are two equally critical stress elements, one in tension (119.7º CCW from the *z* axis) and the other in compression (60.3º CW from the *z* axis). We'll continue the analysis with the element in tension.

(b) Transverse shear is zero at the critical stress elements on the outer surfaces.

$$\sigma_x = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(1612)}{\pi(1)^3} = 16\,420 \text{ psi} = 16.4 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(5)(175)}{\pi(1)^3} = 4456 \text{ psi} = 4.46 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{16.4}{2} \pm \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2}$$

$\sigma_1 = 17.5$ kpsi *Ans.*

$\sigma_2 = -1.13$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{16.4}{2}\right)^2 + (4.46)^2} = 9.33 \text{ kpsi} \quad \textit{Ans.}$$

3-82

(a) Rod *AB* experiences constant torsion and constant axial tension throughout its length, and maximum bending moments at the wall from both planes of bending. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

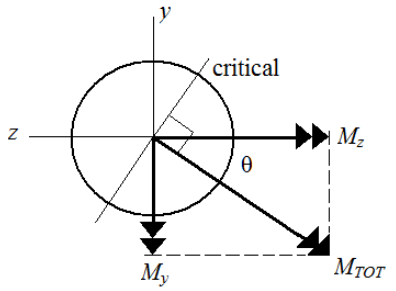
$$M_y = -(100)(8) - (75)(5) = -1175 \text{ lbf·in}$$

$$M_z = (-200)(8) = -1600 \text{ lbf·in}$$

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(-1175)^2 + (-1600)^2} = 1985 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{1175}{1600}\right) = 36.3°$$



The combined bending moment vector is at an angle of 36.3º CW from the negative *z* axis. The critical bending stress location will be ±90º from this vector, as shown. Since there is an axial stress in tension, the critical stress element will be where the bending is also in tension. The critical stress element is therefore on the outer surface at the wall, at an angle of 36.3º CW from the *y* axis.

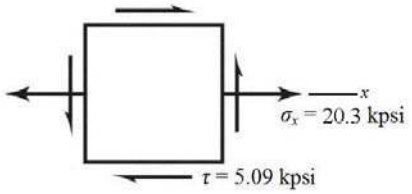
(b) Transverse shear is zero at the critical stress element on the outer surface.

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}} c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32 M_{\text{tot}}}{\pi d^3} = \frac{32(1985)}{\pi(1)^3} = 20\,220 \text{ psi} = 20.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{75}{\pi(1)^2/4} = 95.5 \text{ psi} = 0.1 \text{ kpsi}\ \text{, which is essentially negligible}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 20\,220 + 95.5 = 20\,316 \text{ psi} = 20.3 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{20.3}{2} \pm \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 21.5$ kpsi *Ans.*

$\sigma_2 = -1.20$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2} = 11.4 \text{ kpsi} \quad \textit{Ans.}$$

3-83

$T = (2)(200) = 400$ lbf·in

The maximum shear stress due to torsion occurs in the middle of the longest side of the rectangular cross section. From the table on p. 102, with $b/c = 1.5/0.25 = 6$, $\alpha = 0.299$. From Eq. (3-40),

$$\tau_{\max} = \frac{T}{\alpha b c^2} = \frac{400}{(0.299)(1.5)(0.25)^2} = 14\,270 \text{ psi} = 14.3 \text{ kpsi} \quad \textit{Ans.}$$

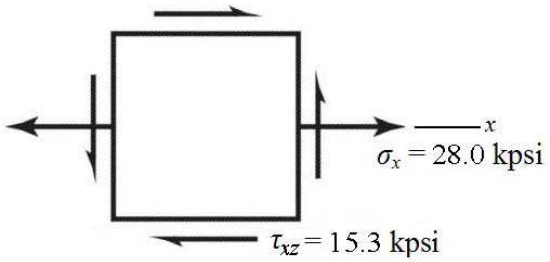
3-84

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$$\sigma_x = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(11)(250)}{\pi(1)^3} = 28\,011 \text{ psi} = 28.0 \text{ kpsi}$$

$$\tau_{xz} = \frac{Tr}{J} = \frac{T(d/2)}{\pi d^4/32} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{28.0}{2} \pm \sqrt{\left(\frac{28.0}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 34.7$ kpsi *Ans.*

$\sigma_2 = -6.7$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{28.0}{2}\right)^2 + (15.3)^2} = 20.7 \text{ kpsi} \quad \textit{Ans.}$$

3-85

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

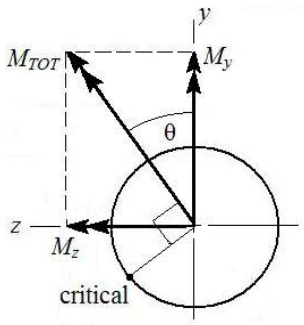
$M_y = (300)(12) = 3600$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(3600)^2 + (2750)^2} = 4530 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{3600}\right) = 37.4^\circ$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

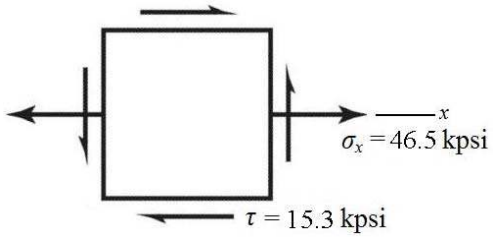
(b)

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}} c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32 M_{\text{tot}}}{\pi d^3} = \frac{32(4530)}{\pi(1)^3} = 46\,142 \text{ psi} = 46.1 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{300}{\pi(1)^2/4} = 382 \text{ psi} = 0.382 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 46\,142 + 382 = 46\,524 \text{ psi} = 46.5 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{46.5}{2} \pm \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 51.1$ kpsi *Ans.*

$\sigma_2 = -4.58$ kpsi *Ans.*

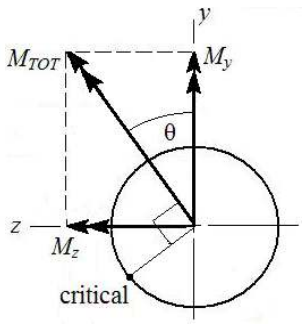
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{46.5}{2}\right)^2 + (15.3)^2} = 27.8 \text{ kpsi} \qquad \textit{Ans.}$$

3-86

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

$M_y = (300)(12) - (-100)(11) = 4700$ lbf·in
$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$
$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf} \cdot \text{in}$$
$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^\circ$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

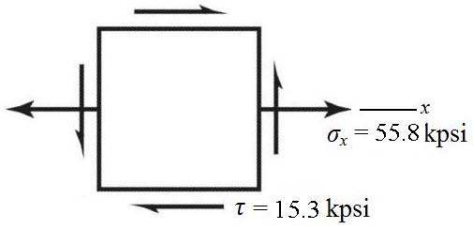
(b)

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}} c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32 M_{\text{tot}}}{\pi d^3} = \frac{32(5445)}{\pi (1)^3} = 55\,462 \text{ psi} = 55.5 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{300}{\pi(1)^2/4} = 382 \text{ psi} = 0.382 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 55\,462 + 382 = 55\,844 \text{ psi} = 55.8 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(12)(250)}{\pi(1)^3} = 15\,279 \text{ psi} = 15.3 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{55.8}{2} \pm \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2}$$

$\sigma_1 = 59.7$ kpsi *Ans.*

$\sigma_2 = -3.92$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{55.8}{2}\right)^2 + (15.3)^2} = 31.8 \text{ kpsi} \quad \textit{Ans.}$$

3-87

(a) The cross section at *A* will experience bending, torsion, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be at either the top (compression) or the bottom (tension) on the *y* axis. We'll select the bottom element for this analysis.

(b) Transverse shear is zero at the critical stress elements on the top and bottom surfaces.

$r/d = 0.125/1 = 0.125$

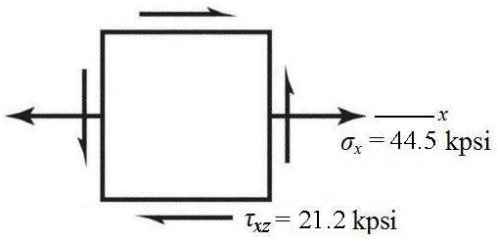
$D/d = 1.5/1 = 1.5$

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_x = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(11)(250)}{\pi(1)^3} = 44\,538 \text{ psi} = 44.5 \text{ kpsi}$$

$$\tau_{xz} = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \frac{44.5}{2} \pm \sqrt{\left(\frac{44.5}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 53.0$ kpsi *Ans.*

$\sigma_2 = -8.48$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = \sqrt{\left(\frac{44.5}{2}\right)^2 + (21.2)^2} = 30.7 \text{ kpsi} \quad \textit{Ans.}$$

3-88

(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface, where the stress concentration will also be applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

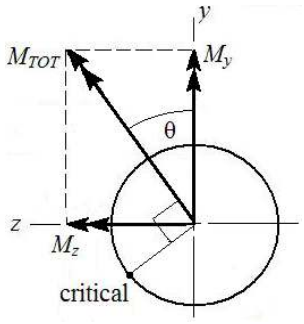
$M_y = (300)(12) = 3600$ lbf·in

$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(3600)^2 + (2750)^2} = 4530 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{3600}\right) = 37.4°$$



The combined bending moment vector is at an angle of 37.4º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 37.4º CCW from the *z* axis.

(b)

$r / d = 0.125 / 1 = 0.125$

$D / d = 1.5 / 1 = 1.5$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

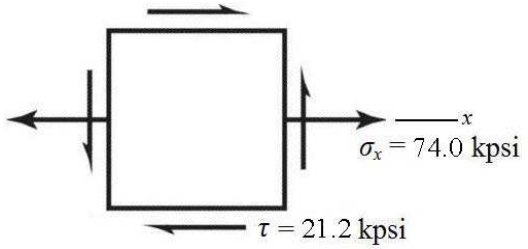
$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(4530)}{\pi(1)^3} = 73\,366 \text{ psi} = 73.4 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}} \frac{F_x}{A} = (1.75)\frac{300}{\pi(1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 73\,366 + 668 = 74\,034 \text{ psi} = 74.0 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{74.0}{2} \pm \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 79.6$ kpsi *Ans.*

$\sigma_2 = -5.64$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{74.0}{2}\right)^2 + (21.2)^2} = 42.6 \text{ kpsi} \quad \textit{Ans.}$$

3-89

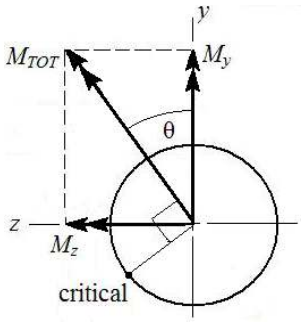
(a) The cross section at *A* will experience bending, torsion, axial, and transverse shear. Both torsional shear stress and bending stress will be maximum on the outer surface, where the stress concentration is also applicable. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface, with its critical location determined by the plane of the combined bending moments.

$M_y = (300)(12) - (-100)(11) = 4700$ lbf·in
$M_z = (250)(11) = 2750$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2}$$

$$= \sqrt{(4700)^2 + (2750)^2} = 5445 \text{ lbf} \cdot \text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_z}{M_y}\right|\right) = \tan^{-1}\left(\frac{2750}{4700}\right) = 30.3^\circ$$



The combined bending moment vector is at an angle of 30.3º CCW from the *y* axis. The critical bending stress location will be 90º CCW from this vector, where the tensile bending stress is additive with the tensile axial stress. The critical stress element is therefore on the outer surface, at an angle of 30.3º CCW from the *z* axis.

(b)

$r / d = 0.125 / 1 = 0.125$

$D / d = 1.5 / 1 = 1.5$

$K_{t,axial} = 1.75$ Fig. A-15-7

$K_{t,\text{torsion}} = 1.39$ Fig. A-15-8

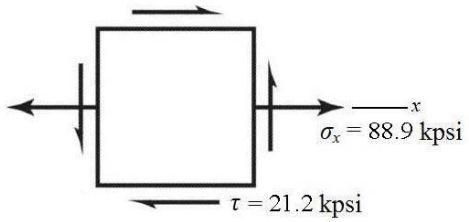
$K_{t,\text{bend}} = 1.59$ Fig. A-15-9

$$\sigma_{x,\text{bend}} = K_{t,\text{bend}} \frac{Mc}{I} = K_{t,\text{bend}} \frac{32M}{\pi d^3} = (1.59)\frac{32(5445)}{\pi(1)^3} = 88\,185 \text{ psi} = 88.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = K_{t,\text{axial}} \frac{F_x}{A} = (1.75)\frac{300}{\pi(1)^2/4} = 668 \text{ psi} = 0.668 \text{ kpsi}$$

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 88\,185 + 668 = 88\,853 \text{ psi} = 88.9 \text{ kpsi}$$

$$\tau = K_{t,\text{torsion}} \frac{Tr}{J} = K_{t,\text{torsion}} \frac{16T}{\pi d^3} = (1.39)\frac{16(12)(250)}{\pi(1)^3} = 21\,238 \text{ psi} = 21.2 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{88.9}{2} \pm \sqrt{\left(\frac{88.9}{2}\right)^2 + (21.2)^2}$$

$\sigma_1 = 93.7$ kpsi *Ans.*

$\sigma_2 = -4.80$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{88.9}{2}\right)^2 + (21.2)^2} = 49.2 \text{ kpsi} \quad \textit{Ans.}$$

3-90

(a) $M = F(p/4),\ c = p/4,\ I = bh^3/12,\ b = \pi d_r n_t,\ \ h = p/2$

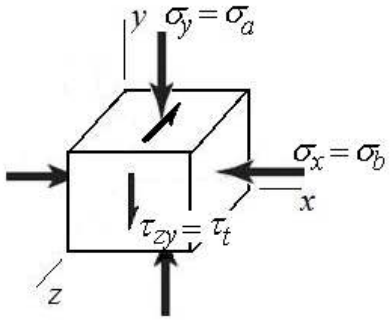
$$\sigma_b = \pm\frac{Mc}{I} = \pm\frac{\left[F\left(p/4\right)\right]\left(p/4\right)}{bh^3/12} = \pm\frac{Fp^2}{16\left(\pi d_r n_t\right)\left(p/2\right)^3/12}$$

$$\sigma_b = \pm\frac{6F}{\pi d_r n_t p} \qquad \textit{Ans.}$$

(b) $$\sigma_a = -\frac{F}{A} = -\frac{F}{\pi d_r^2/4} = -\frac{4F}{\pi d_r^2} \qquad \textit{Ans.}$$

$$\tau_t = \frac{Tr}{J} = \frac{T\left(d_r/2\right)}{\pi d_r^4/32} = \frac{16T}{\pi d_r^3} \qquad \textit{Ans.}$$

(c) The bending stress causes compression in the *x* direction. The axial stress causes compression in the *y* direction. The torsional stress shears across the *y* face in the negative *z* direction.



(d) Analyze the stress element from part (c) using the equations developed in parts (a) and (b).

$d_r = d - p = 1.5 - 0.25 = 1.25$ in

$$\sigma_x = \sigma_b = -\frac{6F}{\pi d_r n_t p} = -\frac{6\left(1500\right)}{\pi\left(1.25\right)\left(2\right)\left(0.25\right)} = -4584 \text{ psi} = -4.584 \text{ kpsi}$$

$$\sigma_y = \sigma_a = -\frac{4F}{\pi d_r^2} = -\frac{4\left(1500\right)}{\pi\left(1.25^2\right)} = -1222 \text{ psi} = -1.222 \text{ kpsi}$$

$$\tau_{yz} = -\tau_t = -\frac{16T}{\pi d_r^3} = -\frac{16\left(235\right)}{\pi\left(1.25^3\right)} = -612.8 \text{ psi} = -0.6128 \text{ kpsi}$$

Use Eq. (3-15) for the three-dimensional stress element.

$$\sigma^3 - \left(-4.584 - 1.222\right)\sigma^2 + \left[\left(-4.584\right)\left(-1.222\right) - \left(-0.6128\right)^2\right]\sigma - \left[-\left(-4.584\right)\left(-0.6128\right)^2\right] = 0$$

$$\sigma^3 + 5.806\sigma^2 + 5.226\sigma - 1.721 = 0$$

The roots are at 0.2543, – 4.584, and –1.476. Thus, the ordered principal stresses are

$\sigma_1 = 0.2543$ kpsi, $\sigma_2 = -1.476$ kpsi, and $\sigma_3 = -4.584$ kpsi. *Ans.*

From Eq. (3-16), the principal shear stresses are

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} = \frac{0.2543 - (-1.476)}{2} = 0.8652 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} = \frac{(-1.476) - (-4.584)}{2} = 1.554 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0.2543 - (-4.584)}{2} = 2.419 \text{ kpsi} \qquad \textit{Ans.}$$

3-91 As shown in Fig. 3-32, the maximum stresses occur at the inside fiber where $r = r_i$. Therefore, from Eq. (3-50)

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right)$$

$$= p_i \left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \qquad \textit{Ans.}$$

$$\sigma_{r,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i \qquad \textit{Ans.}$$

3-92 If $p_i = 0$, Eq. (3-49) becomes

$$\sigma_t = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= -\frac{p_o r_o^2}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

The maximum tangential stress occurs at $r = r_i$. So

$$\sigma_{t,\max} = -\frac{2 p_o r_o^2}{r_o^2 - r_i^2} \qquad \textit{Ans.}$$

For σ_r, we have

$$\sigma_r = \frac{-p_o r_o^2 - r_i^2 r_o^2 p_o / r^2}{r_o^2 - r_i^2}$$

$$= \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r^2} - 1\right)$$

So $\sigma_r = 0$ at $r = r_i$. Thus at $r = r_o$

$$\sigma_{r,\max} = \frac{p_o r_o^2}{r_o^2 - r_i^2}\left(\frac{r_i^2 - r_o^2}{r_o^2}\right) = -p_o \qquad \textit{Ans.}$$

3-93 The force due to the pressure on half of the sphere is resisted by the stress that is distributed around the center plane of the sphere. All planes are the same, so

$$(\sigma_t)_{\text{av}} = \sigma_1 = \sigma_2 = \frac{p(\pi/4)d_i^2}{\pi d_i t} = \frac{pd_i}{4t} \quad \textit{Ans.}$$

The radial stress on the inner surface of the shell is, $\sigma_3 = -p$ *Ans.*

3-94 $\sigma_t > \sigma_l > \sigma_r$
$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{r_o^2 - r_i^2}{r_o^2}\tau_{\max} = \frac{3^2 - 2.75^2}{3^2}(10\ 000) = 1597 \text{ psi} \quad \textit{Ans.}$$

3-95 $\sigma_t > \sigma_l > \sigma_r$
$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{(\tau_{\max} - p_i)}{\tau_{\max}}} = 100\sqrt{\frac{(25-4)10^6}{25(10^6)}} = 91.7 \text{ mm}$$

$$t = r_o - r_i = 100 - 91.7 = 8.3 \text{ mm} \quad \textit{Ans.}$$

3-96 $\sigma_t > \sigma_l > \sigma_r$
$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$= \frac{4^2(500)}{4^2 - 3.75^2} = 4129 \text{ psi} \quad \textit{Ans.}$$

3-97 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_o = \frac{r_o^2 - r_i^2}{r_i^2}\tau_{\max} = \frac{3^2 - 2.75^2}{2.75^2}(10\ 000) = 1900 \text{ psi} \quad \textit{Ans.}$$

3-98 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{\tau_{\max}}{(\tau_{\max} + p_o)}} = 100\sqrt{\frac{25(10^6)}{(25+4)10^6}} = 92.8 \text{ mm}$$

$$t = r_o - r_i = 100 - 92.8 = 7.2 \text{ mm} \quad \textit{Ans.}$$

3-99 From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

$$\sigma_r = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r^2}\right)$$

$\sigma_t > \sigma_l > \sigma_r$, and since σ_t and σ_r are negative,
$\tau_{\max} = (\sigma_r - \sigma_t)/2$ at $r = r_o$

$$\tau_{\max} = \frac{1}{2}\left[-\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 - \frac{r_i^2}{r_o^2}\right) + \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r_o^2}\right)\right] = \frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(\frac{r_i^2}{r_o^2}\right) = \frac{r_i^2 p_o}{r_o^2 - r_i^2}$$

$$= \frac{3.75^2(500)}{4^2 - 3.75^2} = 3629 \text{ psi} \quad \textit{Ans.}$$

3-100 From Table A-20, S_y=490 MPa
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{2r_o^2} = -\frac{[0.8(-490)]\left(25^2 - 19^2\right)}{2(25^2)} = 82.8 \text{ MPa} \quad \textit{Ans.}$$

3-101 From Table A-20, $S_y = 71$ kpsi
From Eq. (3-49) with $p_i = 0$,

$$\sigma_t = -\frac{r_o^2 p_o}{r_o^2 - r_i^2}\left(1 + \frac{r_i^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} \Rightarrow p_o = -\frac{\sigma_{t,\max}\left(r_o^2 - r_i^2\right)}{2r_o^2} = -\frac{[0.8(-71)]\left(1^2 - 0.75^2\right)}{2(1^2)} = 12.4 \text{ kpsi} \quad \textit{Ans.}$$

3-102 From Table A-20, S_y=490 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i\left(r_o^2 + r_i^2\right)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{[0.8(490)](25^2 - 19^2)}{(25^2 + 19^2)} = 105 \text{ MPa} \quad \textit{Ans.}$$

3-103 From Table A-20, S_y =71 MPa
From Eq. (3-50)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

Maximum will occur at $r = r_i$

$$\sigma_{t,\max} = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = \frac{p_i(r_o^2 + r_i^2)}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad p_i = \frac{\sigma_{t,\max}(r_o^2 - r_i^2)}{r_o^2 + r_i^2} = \frac{[0.8(71)](1^2 - 0.75^2)}{(1^2 + 0.75^2)} = 15.9 \text{ ksi} \quad \textit{Ans.}$$

3-104 The longitudinal stress will be due to the weight of the vessel above the maximum stress point. From Table A-5, the unit weight of steel is γ_s = 0.282 lbf/in^3. The area of the wall is

$$A_{\text{wall}} = (\pi/4)(360^2 - 358.5^2) = 846.\ 5 \text{ in}^2$$

The volume of the wall and dome are

$$V_{\text{wall}} = A_{\text{wall}}\, h = 846.5\ (720) = 609.5\ (10^3) \text{ in}^3$$

$$V_{\text{dome}} = (2\pi/3)(180^3 - 179.25^3) = 152.0\ (10^3) \text{ in}^3$$

The weight of the structure on the wall area at the tank bottom is

$$W = \gamma_s\, V_{\text{total}} = 0.282(609.5 + 152.0)\ (10^3) = 214.7(10^3) \text{ lbf}$$

$$\sigma_l = -\frac{W}{A_{\text{wall}}} = -\frac{214.7\left(10^3\right)}{846.5} = -254 \text{ psi}$$

The maximum pressure will occur at the bottom of the tank, $p_i = \gamma_{\text{water}}\, h$. From Eq. (3-50) with $r = r_i$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right)$$

$$= \left[62.4(55)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)\right]\left(\frac{180^2 + 179.25^2}{180^2 - 179.25^2}\right) = 5708 \doteq 5710 \text{ psi} \quad \textit{Ans.}$$

$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right) = -p_i = -62.4(55)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right) = -23.8 \text{ psi} \quad \textit{Ans.}$$

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

Since $\sigma_1 \geq \sigma_2 \geq \sigma_3$, $\sigma_1 = \sigma_t = 5708$ psi, $\sigma_2 = \sigma_r = -24$ psi and $\sigma_3 = \sigma_l = -254$ psi. From Eq. (3-16),

$$\tau_{1/3} = \frac{5708 + 254}{2} = 2981 \doteq 2980 \text{ psi}$$

$$\tau_{1/2} = \frac{5708 + 24}{2} = 2866 \doteq 2870 \text{ psi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{-24 + 254}{2} = 115 \text{ psi}$$

3-105 Stresses from additional pressure are,

Eq. (3-51),

$$(\sigma_l)_{50\text{psi}} = \frac{50\left(179.25^2\right)}{180^2 - 179.25^2} = 5963 \text{ psi}$$

$$(\sigma_r)_{50 \text{ psi}} = -50 \text{ psi}$$

Eq. (3-50)

$$(\sigma_t)_{50\text{psi}} = 50\frac{180^2 + 179.25^2}{180^2 - 179.25^2} = 11\,975 \text{ psi}$$

Adding these to the stresses found in Prob. 3-104 gives

$\sigma_t = 5708 + 11\,975 = 17683$ psi $= 17.7$ kpsi *Ans.*

$\sigma_r = -23.8 - 50 = -73.8$ psi *Ans.*

$\sigma_l = -254 + 5963 = 5709$ psi *Ans.*

Note: These stresses are very idealized as the floor of the tank will restrict the values calculated.

From Eq. (3-16)

$$\tau_{1/3} = \frac{17\,683 + 73.8}{2} = 8879 \text{ psi}$$

$$\tau_{1/2} = \frac{17\,683 - 5709}{2} = 5987 \text{ psi} \qquad \textit{Ans.}$$

$$\tau_{2/3} = \frac{5709 + 23.8}{2} = 2866 \text{ psi}$$

3-106 Since σ_t and σ_r are both positive and $\sigma_t > \sigma_r$

$$\tau_{\max} = (\sigma_t)_{\max} / 2$$

From Eq. (3-55), σ_t is maximum at $r = r_i = 0.3125$ in. The term

$$\rho\omega^2\left(\frac{3+\nu}{8}\right)=\frac{0.282}{386}\left[\frac{2\pi(5000)}{60}\right]^2\left(\frac{3+0.292}{8}\right)=82.42 \text{ lbf/in}$$

$$(\sigma_t)_{\max}=82.42\left[0.3125^2+2.75^2+\frac{\left(0.3125^2\right)\left(2.75^2\right)}{0.3125^2}-\frac{1+3(0.292)}{3+0.292}\left(0.3125^2\right)\right]$$

$$=1260 \text{ psi}$$

$$\tau_{\max}=\frac{1260}{2}=630 \text{ psi} \quad \textit{Ans.}$$

Radial stress:

$$\sigma_r=k\left(r_i^2+r_o^2-\frac{r_i^2r_o^2}{r^2}-r^2\right)$$

Maxima:

$$\frac{d\sigma_r}{dr}=k\left(2\frac{r_i^2r_o^2}{r^3}-2r\right)=0 \quad \Rightarrow \quad r=\sqrt{r_ir_o}=\sqrt{0.3125(2.75)}=0.927 \text{ in}$$

$$(\sigma_r)_{\max}=82.42\left[0.3125^2+2.75^2-\frac{0.3125^2\left(2.75^2\right)}{0.927^2}-0.927^2\right]$$

$$=490 \text{ psi} \quad \textit{Ans.}$$

3-107 $\omega=2\pi(2000)/60=209.4$ rad/s, $\rho=3320$ 20 kg/m^3, $\nu=0.24$, $r_i=0.01$ m, $r_o=0.125$ m

Using Eq. (3-55)

$$\sigma_t=3320(209.4)^2\left(\frac{3+0.24}{8}\right)\left[(0.01)^2+(0.125)^2+(0.125)^2-\frac{1+3(0.24)}{3+0.24}(0.01)^2\right](10)^{-6}$$

$$=1.85 \text{ MPa} \quad \textit{Ans.}$$

3-108 $\omega=2\pi(12\,000)/60=1256.6$ rad/s,

$$\rho=\frac{(5/16)}{386(1/16)(\pi/4)\left(5^2-0.75^2\right)}=6.749\left(10^{-4}\right) \text{ lbf}\cdot\text{s}^2/\text{ in}^4$$

The maximum shear stress occurs at bore where $\tau_{\max}=\sigma_t/2$. From Eq. (3-55)

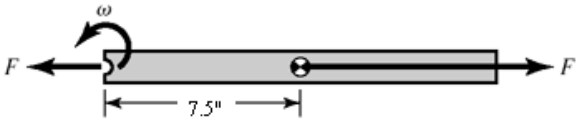
$$(\sigma_t)_{\max}=6.749(10^{-4})(1256.6)^2\left(\frac{3+0.20}{8}\right)\left[0.375^2+2.5^2+2.5^2-\frac{1+3(0.20)}{3+0.20}(0.375)^2\right]$$

$$=5360 \text{ psi}$$

τ_{max} = 5360 / 2 = 2680 psi *Ans*.

3-109 $\omega = 2\pi(3500)/60 = 366.5$ rad/s,
mass of blade = $m = \rho V = (0.282 / 386)\,[1.25(30)(0.125)] = 3.425(10^{-3})$ lbf·s^2/in

$F = (m/2)\,\omega^2 r$
$= [3.425(10^{-3})/2](\,366.5^2)(7.5)$
$= 1725$ lbf



$A_{nom} = (1.25 - 0.5)(1/8) = 0.093\,75$ in^2

$\sigma_{nom} = F/A_{nom} = 1725/0.093\,75 = 18\,400$ psi *Ans*.

Note: Stress concentration Fig. A-15-1 gives $K_t = 2.25$ which increases σ_{max} and fatigue.

3-110 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm
Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{max} = \frac{1}{2}[50.042 - 50.000] = 0.021 \text{ mm} \quad \textit{Ans}.$$

Minimum interference,

$$\delta_{min} = \frac{1}{2}[50.026 - 50.025] = 0.0005 \text{ mm} \quad \textit{Ans}.$$

From Eq. (1)

$p_{max} = 3.105(10^3)(0.021) = 65.2$ MPa *Ans*.

$p_{min} = 3.105(10^3)(0.0005) = 1.55$ MPa *Ans*.

3-111 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in
Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0016 - 2.0000] = 0.0008 \text{ in} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0010 - 2.0010] = 0 \quad \textit{Ans.}$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.0008) = 9\,000 \text{ psi} \quad \textit{Ans.}$$

$$p_{\min} = 1.125(10^7)(0) = 0 \quad \textit{Ans.}$$

3-112 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right]\left(10^{-9}\right) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.059 - 50.000] = 0.0295 \text{ mm} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.043 - 50.025] = 0.009 \text{ mm} \quad \textit{Ans.}$$

From Eq. (1)

$$p_{\max} = 3.105(10^3)(0.0295) = 91.6 \text{ MPa} \quad \textit{Ans.}$$

$$p_{\min} = 3.105(10^3)(0.009) = 27.9 \text{ MPa} \quad \textit{Ans.}$$

3-113 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0023 - 2.0000] = 0.00115 \text{ in} \quad \textit{Ans.}$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0017 - 2.0010] = 0.00035 \quad Ans.$$

From Eq. (1),

$$p_{\max} = 1.125(10^7)(0.00115) = 12\,940 \text{ psi} \quad Ans.$$

$$p_{\min} = 1.125(10^7)(0.00035) = 3\,938 \quad Ans.$$

3-114 $\nu = 0.292$, $E = 207$ GPa, $r_i = 0$, $R = 25$ mm, $r_o = 50$ mm

Eq. (3-57),

$$p = \frac{207(10^9)\delta}{2(0.025)^3}\left[\frac{(0.05^2 - 0.025^2)(0.025^2 - 0)}{(0.05^2 - 0)}\right](10^{-9}) = 3.105(10^3)\delta \qquad (1)$$

where p is in MPa and δ is in mm.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[50.086 - 50.000] = 0.043 \text{ mm} \quad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[50.070 - 50.025] = 0.0225 \text{ mm} \quad Ans.$$

From Eq. (1)

$$p_{\max} = 3.105(10^3)(0.043) = 134 \text{ MPa} \quad Ans.$$

$$p_{\min} = 3.105(10^3)(0.0225) = 69.9 \text{ MPa} \quad Ans.$$

3-115 $\nu = 0.292$, $E = 30$ Mpsi, $r_i = 0$, $R = 1$ in, $r_o = 2$ in

Eq. (3-57),

$$p = \frac{30(10^6)\delta}{2(1^3)}\left[\frac{(2^2 - 1^2)(1^2 - 0)}{(2^2 - 0)}\right] = 1.125(10^7)\delta \qquad (1)$$

where p is in psi and δ is in inches.

Maximum interference,

$$\delta_{\max} = \frac{1}{2}[2.0034 - 2.0000] = 0.0017 \text{ in} \quad Ans.$$

Minimum interference,

$$\delta_{\min} = \frac{1}{2}[2.0028 - 2.0010] = 0.0009 \quad Ans.$$

From Eq. (1),

$p_{\max} = 1.125(10^7)(0.0017) = 19\ 130$ psi *Ans.*

$p_{\min} = 1.125(10^7)(0.0009) = 10\ 130$ *Ans.*

3-116 From Table A-5, $E_i = E_o = 30$ Mpsi, $\nu_i = \nu_o = 0.292$. $r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001$ in *Ans.*

Eq. (3-57),

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right] = \frac{30\left(10^6\right)0.001}{2\left(1^3\right)}\left[\frac{\left(1.5^2 - 1^2\right)\left(1^2 - 0\right)}{\left(1.5^2 - 0\right)}\right]$$

$$= 8333 \text{ psi} \doteq 83.3 \text{ kpsi} \qquad \textit{Ans.}$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(8333)\frac{1^2 + 0^2}{1^2 - 0^2} = -8333 \text{ psi} \doteq -8.33 \text{ kpsi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (8333)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 21\ 670 \text{ psi} \doteq 21.7 \text{ kpsi} \quad \textit{Ans.}$$

3-117 From Table A-5, $E_i = 30$ Mpsi, $E_o = 14.5$ Mpsi, $\nu_i = 0.292$, $\nu_o = 0.211$.
$r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The radial interference is $\delta = \frac{1}{2}(2.002 - 2.000) = 0.001$ in *Ans.*

Eq. (3-56),

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{0.001}{1\left[\frac{1}{14.5\left(10^6\right)}\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} + 0.211\right) + \frac{1}{30\left(10^6\right)}\left(\frac{1^2 + 0^2}{1^2 - 0^2} - 0.292\right)\right]} = 4599 \text{ psi} \qquad \textit{Ans.}$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(4599)\frac{1^2 + 0^2}{1^2 - 0^2} = -4599 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (4599)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 11\,960 \text{ psi} \quad \textit{Ans.}$$

3-118 From Table A-5, $E_i = E_o = 30$ Mpsi, $\nu_i = \nu_o = 0.292$. $r_i = 0$, $R = 0.5$ in, $r_o = 1$ in
The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}(1.002 - 1.002) = 0.000 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\max} = \frac{1}{2}(1.003 - 1.001) = 0.001 \text{ in} \quad \textit{Ans.}$$

Since the minimum interference is zero, the minimum pressure and tangential stresses are zero. *Ans.*

The maximum pressure is obtained from Eq. (3-57).

$$p = \frac{E\delta}{2R^3}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{r_o^2 - r_i^2}\right]$$

$$p = \frac{30\left(10^6\right)0.001}{2\left(0.5^3\right)}\left[\frac{\left(1^2 - 0.5^2\right)\left(0.5^2 - 0\right)}{\left(1^2 - 0\right)}\right] = 22\,500 \text{ psi} \qquad \textit{Ans}$$

The maximum tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

$$(\sigma_t)_i\big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(22\,500)\frac{0.5^2 + 0^2}{0.5^2 - 0^2} = -22\,500 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2} = (22\,500)\frac{1^2 + 0.5^2}{1^2 - 0.5^2} = 37\,500 \text{ psi} \quad \textit{Ans.}$$

3-119 From Table A-5, $E_i = 10.4$ Mpsi, $E_o = 30$ Mpsi, $\nu_i = 0.333$, $\nu_o = 0.292$.
$r_i = 0$, $R = 1$ in, $r_o = 1.5$ in

The minimum and maximum radial interferences are

$$\delta_{\min} = \frac{1}{2}[2.003 - 2.002] = 0.0005 \text{ in} \quad \textit{Ans.}$$

$$\delta_{\max} = \frac{1}{2}[2.006 - 2.000] = 0.003 \text{ in} \quad \textit{Ans.}$$

Eq. (3-56),

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

$$p = \frac{\delta}{1\left[\frac{1}{30(10^6)}\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2} + 0.292\right) + \frac{1}{10.4(10^6)}\left(\frac{1^2 + 0^2}{1^2 - 0^2} - 0.333\right)\right]}$$

$$p = 6.229(10^6)\,\delta \text{ psi} \qquad \textit{Ans.}$$

$$p_{\min} = 6.229(10^6)\,\delta_{\min} = 6.229(10^6)(0.0005) = 3114.6 \text{ psi} = 3.11 \text{ kpsi} \qquad \textit{Ans.}$$

$$p_{\max} = 6.229(10^6)\,\delta_{\max} = 6.229(10^6)(0.003) = 18\,687 \text{ psi} = 18.7 \text{ kpsi} \qquad \textit{Ans.}$$

The tangential stresses at the interface for the inner and outer members are given by Eqs. (3-58) and (3-59), respectively.

Minimum interference:

$$(\sigma_t)_i\big|_{\min} = -p_{\min}\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(3.11)\frac{1^2 + 0^2}{1^2 - 0^2} = -3.11 \text{ kpsi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{\min} = p_{\min}\frac{r_o^2 + R^2}{r_o^2 - R^2} = (3.11)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 8.09 \text{ kpsi} \quad \textit{Ans.}$$

Maximum interference:

$$(\sigma_t)_i\big|_{\max} = -p_{\max}\frac{R^2 + r_i^2}{R^2 - r_i^2} = -(18.7)\frac{1^2 + 0^2}{1^2 - 0^2} = -18.7 \text{ kpsi} \quad \textit{Ans.}$$

$$(\sigma_t)_o\big|_{\max} = p_{\max}\frac{r_o^2 + R^2}{r_o^2 - R^2} = (18.7)\frac{1.5^2 + 1^2}{1.5^2 - 1^2} = 48.6 \text{ kpsi} \quad \textit{Ans.}$$

3-120 $d = 20$ mm, $r_i = 37.5$ mm, $r_o = 57.5$ mm

From Table 3-4, for $R = 10$ mm,

$$r_c = 37.5 + 10 = 47.5 \text{ mm}$$

$$r_n = \frac{10^2}{2\left(47.5 - \sqrt{47.5^2 - 10^2}\right)} = 46.96772 \text{ mm}$$

$$e = r_c - r_n = 47.5 - 46.96772 = 0.53228 \text{ mm}$$

$$c_i = r_n - r_i = 46.9677 - 37.5 = 9.4677 \text{ mm}$$

$$c_o = r_o - r_n = 57.5 - 46.9677 = 10.5323 \text{ mm}$$

$$A = \pi d^2 / 4 = \pi(20)^2 / 4 = 314.16 \text{ mm}^2$$

$$M = Fr_c = 4000(47.5) = 190\,000 \text{ N} \cdot \text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{4000}{314.16} + \frac{190\ 000(9.4677)}{314.16(0.53228)(37.5)} = 300 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{4000}{314.16} - \frac{190\ 000(10.5323)}{314.16(0.53228)(57.5)} = -195 \text{ MPa} \quad \textit{Ans.}$$

3-121 $d = 0.75$ in, $r_i = 1.25$ in, $r_o = 2.0$ in

From Table 3-4, for $R = 0.375$ in,

$$r_c = 1.25 + 0.375 = 1.625 \text{ in}$$

$$r_n = \frac{0.375^2}{2\left(1.625 - \sqrt{1.625^2 - 0.375^2}\right)} = 1.60307 \text{ in}$$

$$e = r_c - r_n = 1.625 - 1.60307 = 0.02193 \text{ in}$$

$$c_i = r_n - r_i = 1.60307 - 1.25 = 0.35307 \text{ in}$$

$$c_o = r_o - r_n = 2.0 - 1.60307 = 0.39693 \text{ in}$$

$$A = \pi d^2 / 4 = \pi(0.75)^2 / 4 = 0.44179 \text{ in}^2$$

$$M = Fr_c = 750(1.625) = 1218.8 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{750}{0.44179} + \frac{1218.8(0.35307)}{0.44179(0.02193)(1.25)} = 37\ 230 \text{ psi} = 37.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{750}{0.44179} - \frac{1218.8(0.39693)}{0.44179(0.02193)(2.0)} = -23\ 269 \text{ psi} = -23.3 \text{ kpsi} \quad \textit{Ans.}$$

3-122 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for $R = 3$ mm,

$$r_c = 10 + 3 = 13 \text{ mm}$$

$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456 \text{ mm}$$

$$e = r_c - r_n = 13 - 12.82456 = 0.17544 \text{ mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456 \text{ mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544 \text{ mm}$$

$$A = \pi d^2 / 4 = \pi(6)^2 / 4 = 28.2743 \text{ mm}^2$$

$$M = Fr_c = 300(13) = 3900 \text{ N} \cdot \text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{300}{28.2743} + \frac{3900(2.82456)}{28.2743(0.17544)(10)} = 233 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{300}{28.2743} - \frac{3900(3.17544)}{28.2743(0.17544)(16)} = -145 \text{ MPa} \quad \textit{Ans.}$$

3-123 $d = 6$ mm, $r_i = 10$ mm, $r_o = 16$ mm

From Table 3-4, for $R = 3$ mm,

$$r_c = 10 + 3 = 13 \text{ mm}$$

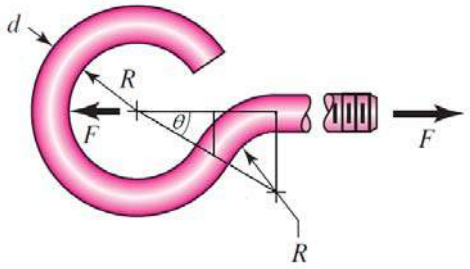
$$r_n = \frac{3^2}{2\left(13 - \sqrt{13^2 - 3^2}\right)} = 12.82456 \text{ mm}$$

$$e = r_c - r_n = 13 - 12.82456 = 0.17544 \text{ mm}$$

$$c_i = r_n - r_i = 12.82456 - 10 = 2.82456 \text{ mm}$$

$$c_o = r_o - r_n = 16 - 12.82456 = 3.17544 \text{ mm}$$

$$A = \pi d^2 / 4 = \pi(6)^2 / 4 = 28.2743 \text{ mm}^2$$



The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{10 + 6/2}{10 + 6 + 10}\right) = 30^\circ$$

$$M = F\left(R + d/2\right)\sin\theta = 300\left(10 + 6/2\right)\sin 30^\circ = 1950 \text{ N} \cdot \text{mm}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{300\sin 30^\circ}{28.2743} + \frac{1950(2.82456)}{28.2743(0.17544)(10)} = 116 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{300\sin 30^\circ}{28.2743} - \frac{1950(3.17544)}{28.2743(0.17544)(16)} = -72.7 \text{ MPa} \quad \textit{Ans.}$$

Note that the shear stress due to the shear force is zero at the surface.

3-124 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$

$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$

$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$

$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$

$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$

$$A = \pi d^2 / 4 = \pi(0.25)^2 / 4 = 0.049087 \text{ in}^2$$
$$M = Fr_c = 75(0.625) = 46.875 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

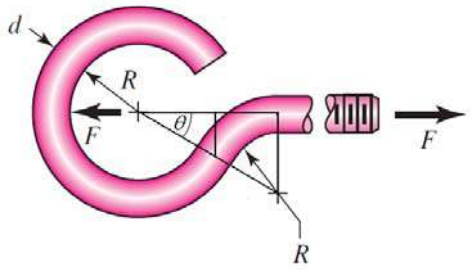
$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{75}{0.049087} + \frac{46.875(0.118686)}{0.049087(0.006314)(0.5)} = 37\,428 \text{ psi} = 37.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{75}{0.049087} - \frac{46.875(0.131314)}{0.049087(0.006314)(0.75)} = -24\,952 \text{ psi} = -25.0 \text{ kpsi} \quad \textit{Ans.}$$

3-125 $d = 0.25$ in, $r_i = 0.5$ in, $r_o = 0.75$ in

From Table 3-4, for $R = 0.125$ in,

$$r_c = 0.5 + 0.125 = 0.625 \text{ in}$$
$$r_n = \frac{0.125^2}{2\left(0.625 - \sqrt{0.625^2 - 0.125^2}\right)} = 0.618686 \text{ in}$$
$$e = r_c - r_n = 0.625 - 0.618686 = 0.006314 \text{ in}$$
$$c_i = r_n - r_i = 0.618686 - 0.5 = 0.118686 \text{ in}$$
$$c_o = r_o - r_n = 0.75 - 0.618686 = 0.131314 \text{ in}$$
$$A = \pi d^2 / 4 = \pi(0.25)^2 / 4 = 0.049087 \text{ in}^2$$



The angle θ of the line of radius centers is

$$\theta = \sin^{-1}\left(\frac{R + d/2}{R + d + R}\right) = \sin^{-1}\left(\frac{0.5 + 0.25/2}{0.5 + 0.25 + 0.5}\right) = 30^\circ$$
$$M = F\left(R + d/2\right)\sin\theta = 75\left(0.5 + 0.25/2\right)\sin 30^\circ = 23.44 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F\sin\theta}{A} + \frac{Mc_i}{Aer_i} = \frac{75\sin 30^\circ}{0.049087} + \frac{23.44(0.118686)}{0.049087(0.006314)(0.5)} = 18\,716 \text{ psi} = 18.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F\sin\theta}{A} - \frac{Mc_o}{Aer_o} = \frac{75\sin 30^\circ}{0.049087} - \frac{23.44(0.131314)}{0.049087(0.006314)(0.75)} = -12\,478 \text{ psi} = -12.5 \text{ kpsi} \quad \textit{Ans.}$$

Note that the shear stress due to the shear force is zero at the surface.

3-126

(a) $$\sigma = \pm\frac{Mc}{I} = \pm\frac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi} \quad \textit{Ans.}$$

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1094) = 0.1797 \text{ in}$$

$$r_n = \frac{0.1094}{\ln(0.2344/0.125)} = 0.174006 \text{ in}$$

$$e = r_c - r_n = 0.1797 - 0.174006 = 0.005694 \text{ in}$$

$$c_i = r_n - r_i = 0.174006 - 0.125 = 0.049006 \text{ in}$$

$$c_o = r_o - r_n = 0.2344 - 0.174006 = 0.060394 \text{ in}$$

$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-34. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.049006)}{0.08205(0.005694)(0.125)} = -10\ 070 \text{ psi} = -10.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.060394)}{0.08205(0.005694)(0.2344)} = 6618 \text{ psi} = 6.62 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-10.1}{-8.02} = 1.26$ *Ans.*

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{6.62}{8.02} = 0.825$ *Ans.*

3-127

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1406)]}{(0.75)(0.1406^3)/12} = \pm 4856 \text{ psi} = \pm 4.86 \text{ kpsi}$ *Ans.*

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1406 = 0.2656$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1406) = 0.1953 \text{ in}$$

$$r_n = \frac{0.1406}{\ln(0.2656/0.125)} = 0.186552 \text{ in}$$

$$e = r_c - r_n = 0.1953 - 0.186552 = 0.008748 \text{ in}$$

$$c_i = r_n - r_i = 0.186552 - 0.125 = 0.061552 \text{ in}$$

$$c_o = r_o - r_n = 0.2656 - 0.186552 = 0.079048 \text{ in}$$

$$A = bh = 0.75(0.1406) = 0.10545 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-34. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.061552)}{0.10545(0.008748)(0.125)} = -6406 \text{ psi} = -6.41 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.079048)}{0.10545(0.008748)(0.2656)} = 3872 \text{ psi} = 3.87 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-6.41}{-4.86} = 1.32 \quad \textit{Ans.}$

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{3.87}{4.86} = 0.80 \quad \textit{Ans.}$

3-128

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi} \quad \textit{Ans.}$

(b) $r_i = 0.25$ in, $r_o = r_i + h = 0.25 + 0.1094 = 0.3594$ in

From Table 3-4,

$r_c = 0.25 + (0.5)(0.1094) = 0.3047$ in

$r_n = \dfrac{0.1094}{\ln(0.3594/0.25)} = 0.301398$ in

$e = r_c - r_n = 0.3047 - 0.301398 = 0.003302$ in

$c_i = r_n - r_i = 0.301398 - 0.25 = 0.051398$ in

$c_o = r_o - r_n = 0.3594 - 0.301398 = 0.058002$ in

$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$

$M = -3(4) = -12 \text{ lbf} \cdot \text{in}$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-34. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.051398)}{0.08205(0.003302)(0.25)} = -9106 \text{ psi} = -9.11 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.058002)}{0.08205(0.003302)(0.3594)} = 7148 \text{ psi} = 7.15 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-9.11}{-8.02} = 1.14 \quad \textit{Ans.}$

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{7.15}{8.02} = 0.89 \quad \textit{Ans.}$

3-129 $r_i = 25$ mm, $r_o = r_i + h = 25 + 87 = 112$ mm, $r_c = 25 + 87/2 = 68.5$ mm

The radius of the neutral axis is found from Eq. (3-63), given below.

$$r_n = \frac{A}{\int (dA/r)}$$

For a rectangular area with constant width b, the denominator is

$$\int_{r_i}^{r_o} \left(\frac{b\,dr}{r} \right) = b \ln \frac{r_o}{r_i}$$

Applying this equation over each of the four rectangular areas,

$$\int \frac{dA}{r} = 9\left(\ln \frac{45}{25} \right) + 31\left(\ln \frac{54.5}{45} \right) + 31\left(\ln \frac{92}{82.5} \right) + 9\left(\ln \frac{112}{92} \right) = 16.3769$$

$$A = 2\left[20(9) + 31(9.5) \right] = 949 \text{ mm}^2$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{949}{16.3769} = 57.9475 \text{ mm}$$

$$e = r_c - r_n = 68.5 - 57.9475 = 10.5525 \text{ mm}$$

$$c_i = r_n - r_i = 57.9475 - 25 = 32.9475 \text{ mm}$$

$$c_o = r_o - r_n = 112 - 57.9475 = 54.0525 \text{ mm}$$

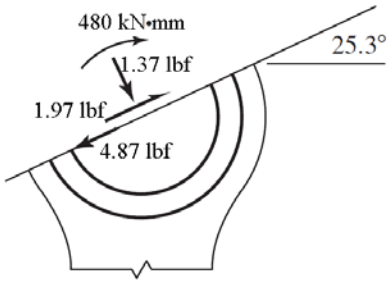
$$M = 150F_2 = 150(3.2) = 480 \text{ kN·mm}$$

We need to find the forces transmitted through the section in order to determine the axial stress. It is not immediately obvious which plane should be used for resolving the axial versus shear directions. It is convenient to use the plane containing the reaction force at the bushing, which assumes its contribution resolves entirely into shear force. To find the angle of this plane, find the resultant of F_1 and F_2.

$$F_x = F_{1x} + F_{2x} = 2.4\cos 60^\circ + 3.2\cos 0^\circ = 4.40 \text{ kN}$$

$$F_y = F_{1y} + F_{2y} = 2.4\sin 60^\circ + 3.2\sin 0^\circ = 2.08 \text{ kN}$$

$$F = \left(4.40^2 + 2.08^2 \right)^{1/2} = 4.87 \text{ kN}$$



This is the pin force on the lever which acts in a direction

$$\theta = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \frac{2.08}{4.40} = 25.3^\circ$$

On the surface 25.3° from the horizontal, find the internal forces in the tangential and normal directions. Resolving F_1 into components,

$$F_t = 2.4\cos\left(60^\circ - 25.3^\circ \right) = 1.97 \text{ kN}$$

$$F_n = 2.4\sin\left(60^\circ - 25.3^\circ \right) = 1.37 \text{ kN}$$

The transverse shear stress is zero at the inner and outer surfaces. Using Eq. (3-65) for the bending stress, and combining with the axial stress due to F_n,

$$\sigma_i = \frac{F_n}{A} + \frac{Mc_i}{Aer_i} = \frac{1370}{949} + \frac{\left[(3200)(150)\right](32.9475)}{949(10.5525)(25)} = 64.6 \text{ MPa} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F_n}{A} - \frac{Mc_o}{Aer_o} = \frac{1370}{949} - \frac{\left[(3200)(150)\right](54.0525)}{949(10.5525)(112)} = -21.7 \text{ MPa} \quad \textit{Ans.}$$

3-130 $r_i = 2$ in, $r_o = r_i + h = 2 + 4 = 6$ in, $r_c = 2 + 0.5(4) = 4$ in

$A = (6 - 2 - 0.75)(0.75) = 2.4375 \text{ in}^2$

Similar to Prob. 3-129,

$$\int \frac{dA}{r} = 0.75 \ln \frac{3.625}{2} + 0.75 \ln \frac{6}{4.375} = 0.682\,920 \text{ in}$$

$$r_n = \frac{A}{\int (dA/r)} = \frac{2.4375}{0.682\,920} = 3.56923 \text{ in}$$

$$e = r_c - r_n = 4 - 3.56923 = 0.43077 \text{ in}$$

$$c_i = r_n - r_i = 3.56923 - 2 = 1.56923 \text{ in}$$

$$c_o = r_o - r_n = 6 - 3.56923 = 2.43077 \text{ in}$$

$$M = Fr_c = 6000(4) = 24\,000 \text{ lbf} \cdot \text{in}$$

Using Eq. (3-65) for the bending stress, and combining with the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{6000}{2.4375} + \frac{24\,000(1.56923)}{2.4375(0.43077)(2)} = 20\,396 \text{ psi} = 20.4 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{6000}{2.4375} - \frac{24\,000(2.43077)}{2.4375(0.43077)(6)} = -6\,799 \text{ psi} = -6.80 \text{ kpsi} \quad \textit{Ans.}$$

3-131 $r_i = 12$ in, $r_o = r_i + h = 12 + 3 = 15$ in, $r_c = 12 + 3/2 = 13.5$ in

$$I = \frac{\pi}{4} a^3 b = \frac{\pi}{4}(1.5^3)(0.75) = 1.988 \text{ in}^4$$

$$A = \pi ab = \pi(1.5)(0.75) = 3.534$$

$$M = 20(3 + 1.5) = 90 \text{ kip} \cdot \text{in}$$

Since the radius is large compared to the cross section, assume Eq. 3-67 is applicable for the bending stress. Combining the bending stress and the axial stress,

$$\sigma_i = \frac{F}{A} + \frac{Mc_i r_c}{Ir_i} = \frac{20}{3.534} + \frac{90(1.5)(13.5)}{(1.988)(12)} = 82.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o r_c}{Ir_o} = \frac{20}{3.534} - \frac{90(1.5)(13.5)}{1.988(15)} = -55.5 \text{ kpsi} \quad \textit{Ans.}$$

3-132 $r_i = 1.25$ in, $r_o = r_i + h = 1.25 + 0.5 + 1 + 0.5 = 3.25$ in

$r_c = (r_i + r_o)/2 = (1.25 + 3.25)/2 = 2.25$ in *Ans.*

For outer rectangle, $\left(\int \frac{dA}{r}\right)_{\square} = b \ln \frac{r_o}{r_i}$

For circle, $\left[\frac{A}{\int (dA/r)}\right]_{\mathrm{O}} = \left[\frac{r^2}{2\left(r_c - \sqrt{r_c^2 - r^2}\right)}\right]_{\mathrm{O}}$, $A_{\mathrm{O}} = \pi r^2$

$$\therefore \left[\int \frac{dA}{r}\right]_{\mathrm{O}} = 2\pi(r_c - \sqrt{r_c^2 - r^2})$$

Combine the integrals subtracting the circle from the rectangle

$$\Sigma \int \frac{dA}{r} = 1.25 \ln \frac{3.25}{1.25} - 2\pi\left(2.25 - \sqrt{2.25^2 - 0.5^2}\right) = 0.840\,904 \text{ in}$$

$$A = 1.25(2) - \pi(0.5^2) = 1.714\,60 \text{ in}^2 \quad \textit{Ans.}$$

$$r_n = \frac{A}{\sum \int (dA/r)} = \frac{1.714\,60}{0.840\,904} = 2.0390 \text{ in} \quad \textit{Ans.}$$

$$e = r_c - r_n = 2.25 - 2.0390 = 0.2110 \text{ in} \quad \textit{Ans.}$$

$$c_i = r_n - r_i = 2.0390 - 1.25 = 0.7890 \text{ in}$$

$$c_o = r_o - r_n = 3.25 - 2.0390 = 1.2110 \text{ in}$$

$$M = 2000(4.5 + 1.25 + 0.5 + 0.5) = 13\,500 \text{ lbf} \cdot \text{in}$$

$$\sigma_i = \frac{F}{A} + \frac{Mc_i}{Aer_i} = \frac{2000}{1.7146} + \frac{13\,500(0.7890)}{1.7146(0.2110)(1.25)} = 20\,720 \text{ psi} = 20.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = \frac{F}{A} - \frac{Mc_o}{Aer_o} = \frac{2000}{1.7146} - \frac{13\,500(1.2110)}{1.7146(0.2110)(3.25)} = -12\,738 \text{ psi} = -12.7 \text{ kpsi} \quad \textit{Ans.}$$

3-133 From Eq. (3-68),

$$a = KF^{1/3} = F^{1/3}\left\{\left(\frac{3}{8}\right)\frac{2\left[\left(1-\nu^2\right)/E\right]}{2\left(1/d\right)}\right\}^{1/3}$$

Use $\nu = 0.292$, F in newtons, E in N/mm^2 and d in mm, then

$$K = \left\{\left(\frac{3}{8}\right)\frac{[(1-0.292^2)/207\,000]}{1/30}\right\}^{1/3} = 0.03685$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi(KF^{1/3})^2} = \frac{3F^{1/3}}{2\pi K^2} = \frac{3F^{1/3}}{2\pi(0.03685)^2} = 352F^{1/3} \text{ MPa}$$

From Eq. (3-71), the maximum principal stress occurs on the surface where $z = 0$, and is equal to $-p_{\max}$.

$$\sigma_{\max} = \sigma_z = -p_{\max} = -352F^{1/3} \text{ MPa} \qquad Ans.$$

From Fig. 3-37,

$$\tau_{\max} = 0.3p_{\max} = 106F^{1/3} \text{ MPa} \qquad Ans.$$

3-134 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(10)}{8}\right)\frac{\left(1-0.292^2\right)/(207\,000) + \left(1-0.333^2\right)/(71\,700)}{1/25 + 1/40}} = 0.0990 \text{ mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(10)}{2\pi\left(0.0990^2\right)} = 487.2 \text{ MPa}$$

From Fig. 3-37, the maximum shear stress occurs at a depth of $z = 0.48\,a$.

$$z = 0.48a = 0.48(0.0990) = 0.0475 \text{ mm} \qquad Ans.$$

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of $z/a = 0.48$.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1 - 0.48\tan^{-1}(1/0.48)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -101.3 \text{ MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0 \text{ MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-101.3) - (-396.0)}{2} = 147.4 \text{ MPa} \qquad Ans.$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-37 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For $\nu = 0.333$ for aluminum, the maximum shear stress occurs at a depth of $z = 0.492a$ with $\tau_{\max} = 0.3025\,p_{\max}$.

This gives $\tau_{max} = 0.3025\, p_{max} = (0.3025)(487.2) = 147.38$ MPa. Even though the depth assumption was a little off, it did not have significant effect on the the maximum shear stress.

3-135 From the solution to Prob. 3-134, $a = 0.0990$ mm and $p_{max} = 487.2$ MPa. Assuming applicability of Fig. 3-37, the maximum shear stress occurs at a depth of $z = 0.48\, a = 0.0475$ mm. *Ans.*

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of $z/a = 0.48$.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1 - 0.48\tan^{-1}(1/0.48)\right](1+0.292) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -92.09 \text{ MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0 \text{ MPa}$$

From Eq. (3-72),

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-92.09) - (-396.0)}{2} = 152.0 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-37 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.292 for steel, the maximum shear stress occurs at a depth of $z = 0.478a$ with $\tau_{max} = 0.3119\, p_{max}$.

3-136 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{2\left(1-\nu^2\right)/E}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(20)}{8}\right)\frac{2\left(1-0.292^2\right)/(207\,000)}{1/30 + 1/\infty}} = 0.1258 \text{ mm}$$

From Eq. (3-69),

$$p_{max} = \frac{3F}{2\pi a^2} = \frac{3(20)}{2\pi\left(0.1258^2\right)} = 603.4 \text{ MPa}$$

From Fig. 3-37, the maximum shear stress occurs at a depth of

$$z = 0.48a = 0.48(0.1258) = 0.0604 \text{ mm} \qquad \textit{Ans.}$$

Also from Fig. 3-37, the maximum shear stress is

$$\tau_{max} = 0.3\, p_{max} = 0.3(603.4) = 181 \text{ MPa} \qquad \textit{Ans.}$$

3-137 Aluminum **Plate-Ball interface:** From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)(10^6)\right] + \left(1-0.333^2\right)/\left[(10.4)(10^6)\right]}{1/1 + 1/\infty}} = 3.517\left(10^{-3}\right)F^{1/3} \text{ in}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left[3.517\left(10^{-3}\right)F^{1/3}\right]^2} = 3.860\left(10^4\right)F^{1/3} \text{ psi}$$

By examination of Eqs. (3-70), (3-71), and (3-72), it can be seen that the only difference in the maximum shear stress for the plate and the ball will be due to poisson's ratio in Eq. (3-70). The larger poisson's ratio will create the greater maximum shear stress, so the aluminum plate will be the critical element in this interface. Applying the equations for the aluminum plate,

$$\sigma_1 = -3.86\left(10^4\right)F^{1/3}\left\{\left[1-0.48\tan^{-1}\left(1/0.48\right)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -8025F^{1/3} \text{ psi}$$

$$\sigma_3 = \frac{-3.86\left(10^4\right)F^{1/3}}{1+0.48^2} = -3.137\left(10^4\right)F^{1/3} \text{ psi}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\left(-8025F^{1/3}\right) - \left(-3.137\left(10^4\right)F^{1/3}\right)}{2} = 1.167\left(10^4\right)F^{1/3} \text{ psi}$$

Comparing this stress to the allowable stress, and solving for F,

$$F = \left[\frac{20\,000}{1.167\left(10^4\right)}\right]^3 = 5.03 \text{ lbf}$$

Table-Ball interface: From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-0.292^2\right)/\left[(30)(10^6)\right] + \left(1-0.211^2\right)/\left[(14.5)(10^6)\right]}{1/1 + 1/\infty}} = 3.306\left(10^{-3}\right)F^{1/3} \text{ in}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3F}{2\pi\left[3.306\left(10^{-3}\right)F^{1/3}\right]^2} = 4.369\left(10^4\right)F^{1/3} \text{ psi}$$

The steel ball has a higher poisson's ratio than the cast iron table, so it will dominate.

$$\sigma_1 = -4.369\left(10^4\right)F^{1/3}\left\{\left[1-0.48\tan^{-1}\left(1/0.48\right)\right](1+0.292) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -8258F^{1/3} \text{ psi}$$

$$\sigma_3 = \frac{-4.369\left(10^4\right)F^{1/3}}{1+0.48^2} = -3.551\left(10^4\right)F^{1/3} \text{ psi}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\left(-8258F^{1/3}\right) - \left(-3.551\left(10^4\right)F^{1/3}\right)}{2} = 1.363\left(10^4\right)F^{1/3} \text{ psi}$$

Comparing this stress to the allowable stress, and solving for F,

$$F = \left[\frac{20\,000}{1.363\left(10^4\right)}\right]^3 = 3.16 \text{ lbf}$$

The steel ball is critical, with $F = 3.16$ lbf. *Ans.*

3-138 $\nu_1 = 0.333$, $E_1 = 10.4$ Mpsi, $l = 2$ in, $d_1 = 1.25$ in, $\nu_2 = 0.211$, $E_2 = 14.5$ Mpsi, $d_2 = -12$ in.

With $b = K_c F^{1/2}$

$$K_c = \left(\frac{2}{\pi(2)}\frac{\left(1-0.333^2\right)/\left[10.4\left(10^6\right)\right] + \left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/1.25 + 1/12}\right)^{1/2}$$

$$= 2.336\left(10^{-4}\right)$$

By examination of Eqs. (3-75), (3-76), and (3-77, it can be seen that the only difference in the maximum shear stress for the two materials will be due to poisson's ratio in Eq. (3-75). The larger poisson's ratio will create the greater maximum shear stress, so the aluminum roller will be the critical element in this interface. Instead of applying these equations, we will assume the poisson's ratio for aluminum of 0.333 is close enough to 0.3 to make Fig. 3-39 applicable.

$$\tau_{\max} = 0.3 p_{\max}$$

$$p_{\max} = \frac{4000}{0.3} = 13\,300 \text{ psi}$$

From Eq. (3-74), $p_{\max} = 2F / (\pi bl)$, so we have

$$p_{\max} = \frac{2F}{\pi l K_c F^{1/2}} = \frac{2F^{1/2}}{\pi l K_c}$$

So,

$$F = \left(\frac{\pi l K_c p_{\max}}{2} \right)^2$$

$$= \left(\frac{\pi (2)(2.336)\left(10^{-4}\right)(13\ 300)}{2} \right)^2$$

$$= 95.3 \text{ lbf} \quad \textit{Ans.}$$

3-139

$\nu = 0.292$, $E = 30$ Mpsi, $l = 0.75$ in, $d_1 = 2(0.47) = 0.94$ in, $d_2 = 2(0.62) = 1.24$ in.
Eq. (3-73):

$$b = \left(\frac{2(40)}{\pi(0.75)} \frac{2\left(1 - 0.292^2\right) \Big/ \left[30\left(10^6\right)\right]}{1/0.94 + 1/1.24} \right)^{1/2} = 1.052\left(10^{-3}\right) \text{ in}$$

Eq. (3-74):

$$p_{\max} = \frac{2F}{\pi b l} == \frac{2(40)}{\pi(1.052)\left(10^{-3}\right)(0.75)} = 32\ 275 \text{ psi} = 32.3 \text{ kpsi} \qquad \textit{Ans.}$$

From Fig. 3-39,

$$\tau_{\max} = 0.3 p_{\max} = 0.3(32\ 275) = 9682.5 \text{ psi} = 9.68 \text{ kpsi} \qquad \textit{Ans.}$$

3-140 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{(1 - \nu_1^2)/E_1 + (1 - \nu_2^2)/E_2}{(1/d_1) + (1/d_2)} \right)^{1/2}$$

$$= \left(\frac{2(600)}{\pi(2)} \frac{(1 - 0.292^2)/(30(10^6)) + (1 - 0.292^2)/(30(10^6))}{1/5 + 1/\infty} \right)^{1/2}$$

$$b = 0.007\ 631 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi b l} = \frac{2(600)}{\pi(0.007\ 631)(2)} = 25\ 028 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}}-\left|\frac{z}{b}\right|\right) = -2(0.292)(25\,028)\left(\sqrt{1+0.786^2}-0.786\right)$$

$$= -7102 \text{ psi} = -7.10 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}}-2\left|\frac{z}{b}\right|\right) = -25\,028\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}}-2(0.786)\right)$$

$$= -4\,646 \text{ psi} = -4.65 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\dfrac{z^2}{b^2}}} = \frac{-25\,028}{\sqrt{1+0.786^2}} = -19\,677 \text{ psi} = -19.7 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-4\,646-(-19\,677)}{2} = 7\,516 \text{ psi} = 7.52 \text{ kpsi} \qquad \textit{Ans.}$$

3-141 Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l}\frac{\left(1-\nu_1^2\right)/E_1+\left(1-\nu_2^2\right)/E_2}{1/d_1+1/d_2}\right)^{1/2}$$

$$= \left(\frac{2(2000)}{\pi(40)}\frac{\left(1-0.292^2\right)/\left[207\left(10^3\right)\right]+\left(1-0.211^2\right)/\left[100\left(10^3\right)\right]}{1/150+1/\infty}\right)^{1/2}$$

$$b = 0.2583 \text{ mm}$$

$$p_{\max} = \frac{2F}{\pi b l} = \frac{2(2000)}{\pi(0.2583)(40)} = 123.2 \text{ MPa}$$

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}}-\left|\frac{z}{b}\right|\right) = -2(0.292)(123.2)\left(\sqrt{1+0.786^2}-0.786\right)$$

$$= -35.0 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}}-2\left|\frac{z}{b}\right|\right) = -123.2\left(\frac{1+2\left(0.786^2\right)}{\sqrt{1+\left(0.786^2\right)}}-2(0.786)\right)$$

$$= -22.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\dfrac{z^2}{b^2}}} = \frac{-123.2}{\sqrt{1+0.786^2}} = -96.9 \text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-22.9 - (-96.9)}{2} = 37.0 \text{ MPa} \qquad \textit{Ans.}$$

3-142 **Note to the Instructor**: The first printing incorrectly had a width $w = 1.25$ mm instead of $w = 1.25$ in. The solution presented here reflects the correction which will be made in subsequent printings.

Use Eqs. (3-73) through (3-77).

$$b = \left(\frac{2F}{\pi l} \frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2} \right)^{1/2}$$

$$= \left(\frac{2(250)}{\pi(1.25)} \frac{\left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right] + \left(1-0.211^2\right)/\left[14.5\left(10^6\right)\right]}{1/3 + 1/\infty} \right)^{1/2}$$

$$b = 0.007\,095 \text{ in}$$

$$p_{\max} = \frac{2F}{\pi b l} = \frac{2(250)}{\pi(0.007\,095)(1.25)} = 17\,946 \text{ psi}$$

$$\sigma_x = -2\nu p_{\max} \left(\sqrt{1 + \frac{z^2}{b^2}} - \left| \frac{z}{b} \right| \right) = -2(0.211)(17\,946)\left(\sqrt{1 + 0.786^2} - 0.786 \right)$$

$$= -3\,680 \text{ psi} = -3.68 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_y = -p_{\max} \left(\frac{1 + 2\frac{z^2}{b^2}}{\sqrt{1 + \frac{z^2}{b^2}}} - 2\left| \frac{z}{b} \right| \right) = -17\,946 \left(\frac{1 + 2\left(0.786^2\right)}{\sqrt{1 + \left(0.786^2\right)}} - 2(0.786) \right)$$

$$= -3\,332 \text{ psi} = -3.33 \text{ kpsi} \qquad \textit{Ans.}$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1 + \frac{z^2}{b^2}}} = \frac{-17\,946}{\sqrt{1 + 0.786^2}} = -14\,109 \text{ psi} = -14.1 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{\max} = \frac{\sigma_y - \sigma_z}{2} = \frac{-3\,332 - (-14\,109)}{2} = 5\,389 \text{ psi} = 5.39 \text{ kpsi} \qquad \textit{Ans.}$$

Chapter 4

4-1 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For a cantilever, $k_l = F/\delta, \delta = F/k_l$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta$
Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l}$$

Solving for k

$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l}} = \frac{k_l k_T}{k_l l^2 + k_T} \qquad \textit{Ans.}$$

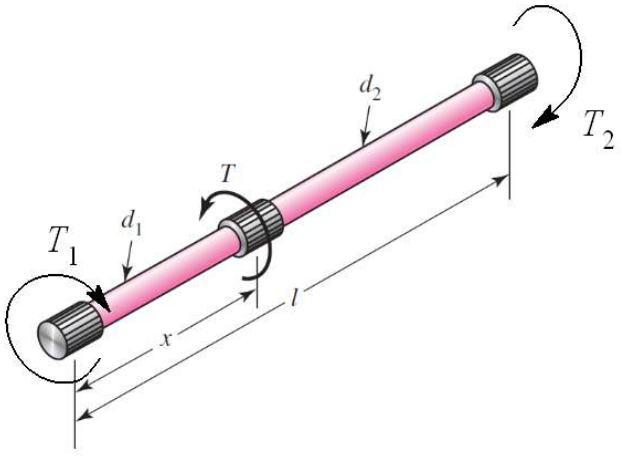
4-2 For a torsion bar, $k_T = T/\theta = Fl/\theta$, and so $\theta = Fl/k_T$. For each cantilever, $k_l = F/\delta_l$, $\delta_l = F/k_l$, and, $\delta_L = F/k_L$. For the assembly, $k = F/y$, or, $y = F/k = l\theta + \delta_l + \delta_L$.
Thus

$$y = \frac{F}{k} = \frac{Fl^2}{k_T} + \frac{F}{k_l} + \frac{F}{k_L}$$

Solving for k

$$k = \frac{1}{\dfrac{l^2}{k_T} + \dfrac{1}{k_l} + \dfrac{1}{k_L}} = \frac{k_L k_l k_T}{k_l k_L l^2 + k_T k_L + k_T k_l} \qquad \textit{Ans.}$$

4-3 **(a)** For a torsion bar, $k = T/\theta = GJ/l$.



Two springs in parallel, with $J = \pi d_i^{\,4}/32$, and $d_1 = d_1 = d$,

$$k = \frac{J_1 G}{x} + \frac{J_2 G}{l-x} = \frac{\pi}{32} G \left(\frac{d_1^4}{x} + \frac{d_2^4}{l-x} \right)$$

$$= \frac{\pi}{32} G d^4 \left(\frac{1}{x} + \frac{1}{l-x} \right) \qquad \textit{Ans.} \quad (1)$$

Deflection equation,

$$\theta = \frac{T_1 x}{JG} = \frac{T_2 (l-x)}{JG}$$

results in $$T_1 = \frac{T_2 (l-x)}{x} \qquad (2)$$

From statics, $T_1 + T_2 = T = 1500$. Substitute Eq. (2)

▲

$$T_2\left(\frac{l-x}{x}\right)+T_2=1500 \quad \Rightarrow \quad T_2=1500\frac{x}{l} \qquad \textit{Ans.} \quad (3)$$

Substitute into Eq. (2) resulting in $\quad T_1=1500\dfrac{l-x}{l} \qquad \textit{Ans.} \quad (4)$

(**b**) From Eq. (1), $k=\dfrac{\pi}{32}\left(0.5^4\right)11.5\left(10^6\right)\left(\dfrac{1}{5}+\dfrac{1}{10-5}\right)=28.2\left(10^3\right)$ lbf · in/rad *Ans.*

From Eq. (4), $T_1=1500\dfrac{10-5}{10}=750$ lbf · in *Ans.*

From Eq. (3), $T_2=1500\dfrac{5}{10}=750$ lbf · in *Ans.*

From either section, $\tau=\dfrac{16T_i}{\pi d_i^3}=\dfrac{16(1500)}{\pi\left(0.5^3\right)}=30.6\left(10^3\right)$ psi $=30.6$ kpsi *Ans.*

4-4 Deflection to be the same as Prob. 4-3 where T_1 = 750 lbf·in, $l_1 = l / 2 = 5$ in, and $d_1 = 0.5$ in

$$\theta_1=\theta_2=\theta$$

$$\frac{T_1(4)}{\frac{\pi}{32}d_1^4G}=\frac{T_2(6)}{\frac{\pi}{32}d_2^4G}=\frac{750(5)}{\frac{\pi}{32}\left(0.5^4\right)G} \quad \Rightarrow \quad \frac{4T_1}{d_1^4}=\frac{6T_2}{d_2^4}=60\left(10^3\right) \qquad (1)$$

Or, $\quad T_1=15\left(10^3\right)d_1^4 \qquad (2)$

$$T_2=10\left(10^3\right)d_2^4 \qquad (3)$$

Equal stress, $\tau_1=\tau_2 \;\Rightarrow\; \dfrac{16T_1}{\pi d_1^3}=\dfrac{16T_2}{\pi d_2^3} \;\Rightarrow\; \dfrac{T_1}{d_1^3}=\dfrac{T_2}{d_2^3} \qquad (4)$

Divide Eq. (4) by the first two equations of Eq.(1) results in

$$\frac{\frac{T_1}{d_1^3}}{\frac{4T_1}{d_1^4}}=\frac{\frac{T_2}{d_2^3}}{\frac{4T_2}{d_2^4}} \quad \Rightarrow \quad d_2=1.5d_1 \qquad (5)$$

Statics, $T_1 + T_2 = 1500$ (6)

Substitute in Eqs. (2) and (3), with Eq. (5) gives

$$15\left(10^3\right)d_1^4+10\left(10^3\right)\left(1.5d_1\right)^4=1500$$

Solving for d_1 and substituting it back into Eq. (5) gives

d_1 = 0.388 8 in, d_2 = 0.583 2 in *Ans.*

From Eqs. (2) and (3),

$T_1 = 15(10^3)(0.388\ 8)^4 = 343$ lbf·in *Ans.*
$T_2 = 10(10^3)(0.583\ 2)^4 = 1\ 157$ lbf·in *Ans.*

Deflection of T is $$\theta_1 = \frac{T_1 l_1}{J_1 G} = \frac{343(4)}{(\pi/32)(0.388\ 8^4)11.5(10^6)} = 0.053\ 18 \text{ rad}$$

Spring constant is $$k = \frac{T}{\theta_1} = \frac{1500}{0.053\ 18} = 28.2(10^3) \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

The stress in d_1 is $$\tau_1 = \frac{16T_1}{\pi d_1^3} = \frac{16(343)}{\pi(0.388\ 8)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \quad \textit{Ans.}$$

The stress in d_1 is $$\tau_2 = \frac{16T_2}{\pi d_2^3} = \frac{16(1\ 157)}{\pi(0.583\ 2)^3} = 29.7(10^3) \text{ psi} = 29.7 \text{ kpsi} \quad \textit{Ans.}$$

4-5 (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of x is $r = r_1 + x\tan\alpha$, and the area is $A = \pi(r_1 + x\tan\alpha)^2$. The deflection of the tapered portion is

$$\delta = \int_0^l \frac{F}{AE}dx = \frac{F}{\pi E}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^2} = -\frac{F}{\pi E}\left.\frac{1}{(r_1 + x\tan\alpha)\tan\alpha}\right|_0^l$$

$$= \frac{F}{\pi E}\left[\frac{1}{r_1\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)}\right] = \frac{F}{\pi E\tan\alpha}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{F}{\pi E\tan\alpha}\frac{r_2 - r_1}{r_1 r_2} = \frac{F}{\pi E\tan\alpha}\frac{l\tan\alpha}{r_1 r_2} = \frac{Fl}{\pi r_1 r_2 E}$$

$$= \frac{4Fl}{\pi d_1 d_2 E} \quad \textit{Ans.}$$

(**b**) For section 1,

$$\delta_1 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.5^2)(30)(10^6)} = 3.40(10^{-4}) \text{ in} \quad \textit{Ans.}$$

For the tapered section,

$$\delta = \frac{4}{\pi}\frac{Fl}{d_1 d_2 E} = \frac{4}{\pi}\frac{1000(2)}{(0.5)(0.75)(30)(10^6)} = 2.26(10^{-4}) \text{ in} \quad \textit{Ans.}$$

For section 2,

$$\delta_2 = \frac{Fl}{AE} = \frac{4Fl}{\pi d_1^2 E} = \frac{4(1000)(2)}{\pi(0.75^2)(30)(10^6)} = 1.51(10^{-4}) \text{ in} \qquad \textit{Ans.}$$

4-6 (**a**) Let the radii of the straight sections be $r_1 = d_1/2$ and $r_2 = d_2/2$. Let the angle of the taper be α where $\tan\alpha = (r_2 - r_1)/2$. Thus, the radius in the taper as a function of x is $r = r_1 + x\tan\alpha$, and the polar second area moment is $J = (\pi/2)(r_1 + x\tan\alpha)^4$. The angular deflection of the tapered portion is

$$\theta = \int_0^l \frac{T}{GJ}dx = \frac{2T}{\pi G}\int_0^l \frac{dx}{(r_1 + x\tan\alpha)^4} = -\frac{1}{3}\frac{2T}{\pi G}\left.\frac{1}{(r_1 + x\tan\alpha)^3\tan\alpha}\right|_0^l$$

$$= \frac{2}{3\pi}\frac{T}{G}\left[\frac{1}{r_1^3\tan\alpha} - \frac{1}{\tan\alpha(r_1 + l\tan\alpha)^3}\right] = \frac{2}{3\pi}\frac{T}{G\tan\alpha}\left(\frac{1}{r_1^3} - \frac{1}{r_2^3}\right)$$

$$= \frac{2}{3\pi}\frac{T}{G\tan\alpha}\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{T}{G}\left(\frac{l}{r_2 - r_1}\right)\frac{r_2^3 - r_1^3}{r_1^3 r_2^3} = \frac{2}{3\pi}\frac{Tl}{G}\frac{(r_1^2 + r_1 r_2 + r_2^2)}{r_1^3 r_2^3}$$

$$= \frac{32}{3\pi}\frac{Tl}{G}\frac{(d_1^2 + d_1 d_2 + d_2^2)}{d_1^3 d_2^3} \qquad \textit{Ans.}$$

(**b**) The deflections, in degrees, are
For section 1,

$$\theta_1 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_1^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.5^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 2.44 \text{ deg} \qquad \textit{Ans.}$$

For the tapered section,

$$\theta = \frac{32}{3\pi}\frac{Tl(d_1^2 + d_1 d_2 + d_2^2)}{G d_1^3 d_2^3}\left(\frac{180}{\pi}\right)$$

$$= \frac{32}{3\pi}\frac{(1500)(2)\left[0.5^2 + (0.5)(0.75) + 0.75^2\right]}{11.5(10^6)(0.5^3)(.75^3)}\left(\frac{180}{\pi}\right) = 1.14 \text{ deg} \qquad \textit{Ans.}$$

For section 2,

$$\theta_2 = \frac{Tl}{GJ}\left(\frac{180}{\pi}\right) = \frac{32Tl}{\pi d_2^4 G}\left(\frac{180}{\pi}\right) = \frac{32(1500)(2)}{\pi(0.75^4)11.5(10^6)}\left(\frac{180}{\pi}\right) = 0.481 \text{ deg} \qquad \textit{Ans.}$$

4-7 The area and the elastic modulus remain constant, however the force changes with respect to x. From Table A-5 the unit weight of steel is $\gamma = 0.282$ lbf/in^3, and the elastic modulus is $E = 30$ Mpsi. Starting from the top of the cable (i.e. $x = 0$, at the top).

$$F = \gamma(A)(l - x)$$

$$\delta_c = \int_o^l \frac{F dx}{AE} = \frac{w}{E}\int_0^l (l-x)dx = \frac{\gamma}{E}\left(lx - \frac{1}{2}x^2\right)\Big|_0^l = \frac{\gamma l^2}{2E} = \frac{0.282\left[500(12)\right]^2}{2(30)10^6} = 0.169 \text{ in}$$

From the weight at the bottom of the cable,

$$\delta_W = \frac{Wl}{AE} = \frac{4Wl}{\pi d^2 E} = \frac{4(5000)\left[500(12)\right]}{\pi(0.5^2)30(10^6)} = 5.093 \text{ in}$$

$$\delta = \delta_c + \delta_W = 0.169 + 5.093 = 5.262 \text{ in} \qquad \textit{Ans.}$$

The percentage of total elongation due to the cable's own weight

$$\frac{0.169}{5.262}(100) = 3.21\% \qquad \textit{Ans.}$$

4-8 $\Sigma F_y = 0 = R_1 - F \;\Rightarrow\; R_1 = F$
$\Sigma M_A = 0 = M_1 - Fa \;\Rightarrow\; M_1 = Fa$
$V_{AB} = F,\ M_{AB} = F(x - a),\ V_{BC} = M_{BC} = 0$

Section *AB*:

$$\theta_{AB} = \frac{1}{EI}\int F\left(x-a\right)dx = \frac{F}{EI}\left(\frac{x^2}{2} - ax\right) + C_1 \qquad (1)$$

$\theta_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_1 = 0$

$$y_{AB} = \frac{F}{EI}\int\left(\frac{x^2}{2} - ax\right)dx = \frac{F}{EI}\left(\frac{x^3}{6} - a\frac{x^2}{2}\right) + C_2 \qquad (2)$$

$y_{AB} = 0$ at $x = 0 \;\Rightarrow\; C_2 = 0$

$$\therefore \qquad y_{AB} = \frac{Fx^2}{6EI}\left(x - 3a\right) \qquad \textit{Ans.}$$

Section *BC*:

$$\theta_{BC} = \frac{1}{EI}\int\left(0\right)dx = 0 + C_3$$

From Eq. (1), at $x = a$ (with $C_1 = 0$), $\theta = \frac{F}{EI}\left(\frac{a^2}{2} - a(a)\right) = -\frac{Fa^2}{2EI} = C_3$. Thus,

$$\theta_{BC} = -\frac{Fa^2}{2EI}$$

$$y_{BC} = -\frac{Fa^2}{2EI}\int dx = -\frac{Fa^2}{2EI}x + C_4 \qquad (3)$$

From Eq. (2), at $x = a$ (with $C_2 = 0$), $y = \frac{F}{EI}\left(\frac{a^3}{6} - a\frac{a^2}{2}\right) = -\frac{Fa^3}{3EI}$. Thus, from Eq. (3)

$$-\frac{Fa^2}{2EI}a + C_4 = -\frac{Fa^3}{3EI} \quad \Rightarrow \quad C_4 = \frac{Fa^3}{6EI} \quad \text{Substitute into Eq. (3)}$$

$$\therefore \quad y_{BC} = -\frac{Fa^2}{2EI}x + \frac{Fa^3}{6EI} = \frac{Fa^2}{6EI}(a - 3x) \qquad \textit{Ans.}$$

The maximum deflection occurs at $x = l$,

$$y_{\max} = \frac{Fa^2}{6EI}(a - 3l) \qquad \textit{Ans.}$$

4-9 $\Sigma M_C = 0 = F\,(l/2) - R_1\,l \;\Rightarrow\; R_1 = F/2$

$\Sigma F_y = 0 = F/2 + R_2 - F \;\Rightarrow\; R_2 = F/2$

Break at $0 \le x \le l/2$:

$V_{AB} = R_1 = F/2, \qquad M_{AB} = R_1\,x = Fx/2$

Break at $l/2 \le x \le l$:

$V_{BC} = R_1 - F = -R_2 = -F/2, \qquad M_{BC} = R_1\,x - F\,(x - l/2) = F\,(l - x)/2$

Section *AB*:

$$\theta_{AB} = \frac{1}{EI}\int \frac{Fx}{2}dx = \frac{F}{EI}\frac{x^2}{4} + C_1$$

From symmetry, $\theta_{AB} = 0$ at $x = l/2 \;\Rightarrow\; \frac{F\left(\frac{l}{2}\right)^2}{4EI} + C_1 = 0 \quad \Rightarrow \quad C_1 = -\frac{Fl^2}{16EI}$. Thus,

$$\theta_{AB} = \frac{F}{EI}\frac{x^2}{4} - \frac{Fl^2}{16EI} = \frac{F}{16EI}\left(4x^2 - l^2\right) \qquad (1)$$

$$y_{AB} = \frac{F}{16EI}\int\left(4x^2 - l^2\right)dx = \frac{F}{16EI}\left(\frac{4x^3}{3} - l^2x\right) + C_2$$

$y_{AB} = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$, and,

$$\therefore \qquad y_{AB} = \frac{Fx}{48EI}\left(4x^2 - 3l^2\right) \qquad (2)$$

y_{BC} is not given, because with symmetry, Eq. (2) can be used in this region. The maximum deflection occurs at $x = l/2$,

$$\therefore \qquad y_{\max} = \frac{F\left(\frac{l}{2}\right)}{48EI}\left[4\left(\frac{l}{2}\right)^2 - 3l^2\right] = -\frac{Fl^3}{48EI} \qquad \textit{Ans.}$$

4-10 From Table A-6, for each angle, $I_{1\text{-}1} = 207\ \text{cm}^4$. Thus, $I = 2(207)\,(10^4) = 4.14(10^6)\ \text{mm}^4$

From Table A-9, use beam 2 with $F = 2500$ N, $a = 2000$ mm, and $l = 3000$ mm; and beam 3 with $w = 1$ N/mm and $l = 3000$ mm.

$$y_{\max} = \frac{Fa^2}{6EI}(a - 3l) - \frac{wl^4}{8EI}$$

$$= \frac{2500(2000)^2}{6(207)10^3(4.14)10^6}\left[2000 - 3(3000)\right] - \frac{(1)(3000)^4}{8(207)(10^3)(4.14)(10^6)}$$

$$= -25.4\ \text{mm} \qquad \textit{Ans.}$$

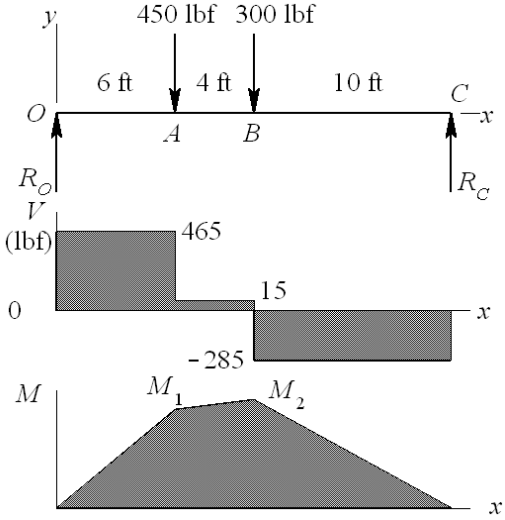
$$M_O = -Fa - (wl^2/2)$$

$$= -2500(2000) - [1(3000^2)/2] = -9.5(10^6)\ \text{N}\cdot\text{mm}$$

From Table A-6, from centroid to upper surface is $y = 29$ mm. From centroid to bottom surface is $y = 29.0 - 100 = -71$ mm. The maximum stress is compressive at the bottom of the beam at the wall. This stress is

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5(10^6)(-71)}{4.14(10^6)} = -163\ \text{MPa} \qquad \textit{Ans.}$$

4-11



$$R_O = \frac{14}{20}(450) + \frac{10}{20}(300) = 465 \text{ lbf}$$

$$R_C = \frac{6}{20}(450) + \frac{10}{20}(300) = 285 \text{ lbf}$$

$M_1 = 465(6)12 = 33.48(10^3)$ lbf·in

$M_2 = 33.48(10^3) + 15(4)12$

$= 34.20(10^3)$ lbf·in

$$\sigma_{\max} = \frac{M_{\max}}{Z} \quad \Rightarrow \quad 15 = \frac{34.2}{Z} \qquad Z = 2.28 \text{ in}^3$$

For deflections, use beams 5 and 6 of Table A-9

$$y\big|_{x=10\text{ft}} = \frac{F_1 a[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2 + a^2 - 2l\frac{l}{2}\right] - \frac{F_2 l^3}{48EI}$$

$$-0.5 = \frac{450(72)(120)}{6(30)(10^6)I(240)}\left(120^2 + 72^2 - 240^2\right) - \frac{300(240^3)}{48(30)(10^6)I}$$

$$I = 12.60 \text{ in}^4 \;\Rightarrow\; I/2 = 6.30 \text{ in}^4$$

Select two 5 in-6.7 lbf/ft channels from Table A-7, $I = 2(7.49) = 14.98$ in^4, $Z = 2(3.00) = 6.00$ in^3

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in}$$

$$\sigma_{\max} = \frac{34.2}{6.00} = 5.70 \text{ kpsi}$$

4-12

$$I = \frac{\pi}{64}(1.5^4) = 0.2485 \text{ in}^4$$

From Table A-9 by superposition of beams 6 and 7, at $x = a = 15$ in, with $b = 24$ in and $l = 39$ in

$$y = \frac{Fba}{6EIl}[a^2 + b^2 - l^2] + \frac{wa}{24EI}(2la^2 - a^3 - l^3)$$

$$y_A = \frac{340(24)15}{6(30)10^6(0.2485)39}\left[15^2 + 24^2 - 39^2\right]$$

$$+ \frac{(150/12)(15)}{24(30)10^6(0.2485)}\left[2(39)(15^2) - 15^3 - 39^3\right] = -0.0978 \text{ in} \quad \textit{Ans.}$$

At $x = l/2 = 19.5$ in

$$y=\frac{Fa[l-(l/2)]}{6EIl}\left[\left(\frac{l}{2}\right)^2+a^2-2l\frac{l}{2}\right]+\frac{w(l/2)}{24EI}\left[2l\left(\frac{l}{2}\right)^2-\left(\frac{l}{2}\right)^3-l^3\right]$$

$$y=\frac{340(15)(19.5)}{6(30)(10^6)(0.2485)(39)}\left[19.5^2+15^2-39^2\right]$$

$$+\frac{(150/12)(19.5)}{24(30)(10^6)(0.2485)}\left[2(39)(19.5^2)-19.5^3-39^3\right]=-0.1027 \text{ in} \qquad \textit{Ans.}$$

$$\%\text{ difference}=\frac{-0.1027+0.0978}{-0.0978}(100)=5.01\% \qquad \textit{Ans.}$$

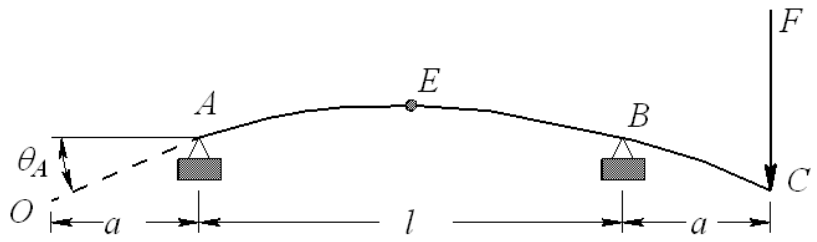
4-13 $I=\frac{1}{12}(6)(32^3)=16.384\left(10^3\right)\text{ mm}^4$

From Table A-9-10, beam 10

$$y_C=-\frac{Fa^2}{3EI}(l+a)$$

$$y_{AB}=\frac{Fax}{6EIl}\left(l^2-x^2\right)$$

$$\frac{dy_{AB}}{dx}=\frac{Fa}{6EIl}(l^2-3x^2)$$



At $x=0$, $\frac{dy_{AB}}{dx}=\theta_A$

$$\theta_A=\frac{Fal^2}{6EIl}=\frac{Fal}{6EI}$$

$$y_O=-\theta_A a=-\frac{Fa^2l}{6EI}$$

With both loads,

$$y_O=-\frac{Fa^2l}{6EI}-\frac{Fa^2}{3EI}(l+a)$$

$$=-\frac{Fa^2}{6EI}(3l+2a)=-\frac{400(300^2)}{6(207)10^3(16.384)10^3}\left[3(500)+2(300)\right]=-3.72\text{ mm} \qquad \textit{Ans.}$$

At midspan,

$$y_E=\frac{2Fa(l/2)}{6EIl}\left[l^2-\left(\frac{l}{2}\right)^2\right]=\frac{3}{24}\frac{Fal^2}{EI}=\frac{3}{24}\frac{400(300)(500^2)}{207\left(10^3\right)16.384\left(10^3\right)}=1.11\text{ mm} \qquad \textit{Ans.}$$

4-14 $I=\frac{\pi}{64}(2^4-1.5^4)=0.5369\text{ in}^4$

From Table A-5, $E = 10.4$ Mpsi
From Table A-9, beams 1 and 2, by superposition

$$y_B = -\frac{F_B l^3}{3EI} + \frac{F_A a^2}{6EI}(a - 3l) = \frac{-200\left[4(12)\right]^3}{3(10.4)10^6(0.5369)} + \frac{300\left[2(12)\right]^2}{6(10.4)10^6(0.5369)}\left[2(12) - 3(4)(12)\right]$$

$$y_B = -1.94 \text{ in} \quad \textit{Ans.}$$

4-15 From Table A-7, $I = 2(1.85) = 3.70$ in^4
From Table A-5, $E = 30.0$ Mpsi
From Table A-9, beams 1 and 3, by superposition

$$y_A = -\frac{Fl^3}{3EI} - \frac{(w + w_c)l^4}{8EI} = -\frac{150(60^3)}{3(30)10^6(3.70)} - \frac{\left[5 + 2(5/12)\right](60^4)}{8(30)10^6(3.70)} = -0.182 \text{ in} \quad \textit{Ans.}$$

4-16 $I = \dfrac{\pi}{64} d^4$

From Table A-5, $E = 207(10^3)$ MPa
From Table A-9, beams 5 and 9, with $F_C = F_A = F$, by superposition

$$y_B = -\frac{F_B l^3}{48EI} + \frac{Fa}{24EI}(4a^2 - 3l^2) \quad \Rightarrow \quad I = \frac{1}{48Ey_B}\left[-F_B l^3 + 2Fa(4a^2 - 3l^2)\right]$$

$$I = \frac{1}{48(207)10^3(-2)}\left\{-550(1000^3) + 2(375)(250)\left[4(250^2) - 3(1000^2)\right]\right\}$$

$$= 53.624\left(10^3\right) \text{ mm}^4$$

$$d = \sqrt[4]{\frac{64}{\pi} I} = \sqrt[4]{\frac{64}{\pi}(53.624)10^3} = 32.3 \text{ mm} \quad \textit{Ans.}$$

4-17 From Table A-9, beams 8 (region *BC* for this beam with $a = 0$) and 10 (with $a = a$), by superposition

$$y_{AB} = \frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right) + \frac{Fax}{6EIl}\left(l^2 - x^2\right)$$

$$= \frac{1}{6EIl}\left[M_A\left(x^3 - 3lx^2 + 2l^2x\right) + Fax\left(l^2 - x^2\right)\right] \quad \textit{Ans.}$$

$$y_{BC} = \left\{\frac{d}{dx}\left[\frac{M_A}{6EIl}\left(x^3 - 3lx^2 + 2l^2x\right)\right]\right\}_{x=l}(x - l) + \frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x - l)\right]$$

$$= -\frac{M_A l}{6EI}(x - l) + \frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x - l)\right]$$

$$= \frac{(x-l)}{6EI}\left\{-M_A l + F\left[(x-l)^2 - a(3x-l)\right]\right\} \quad \textit{Ans.}$$

4-18 **Note to the instructor:** Beams with discontinuous loading are better solved using singularity functions. This eliminates matching the slopes and displacements at the discontinuity as is done in this solution.

$$\sum M_C = 0 = R_1 l - wa\left(l - a + \frac{a}{2}\right) \quad \Rightarrow \quad R_1 = \frac{wa}{2l}(2l - a) \quad \textit{Ans.}$$

$$\sum F_y = 0 = \frac{wa}{2l}(2l - a) + R_2 - wa \quad \Rightarrow \quad R_2 = \frac{wa^2}{2l} \quad \textit{Ans.}$$

$$V_{AB} = R_1 - wx = \frac{wa}{2l}(2l - a) - wx = \frac{w}{2l}\left[2l(a - x) - a^2\right] \quad \textit{Ans.}$$

$$V_{BC} = -R_2 = -\frac{wa^2}{2l} \quad \textit{Ans.}$$

$$M_{AB} = \int V_{AB}\,dx = \frac{w}{2l}\left[2l\left(ax - \frac{x^2}{2}\right) - a^2 x\right] + C_1$$

$$M_{AB} = 0 \text{ at } x = 0 \ \therefore C_1 = 0 \quad \Rightarrow \quad M_{AB} = \frac{wx}{2l}\left[2al - a^2 - lx\right] \quad \textit{Ans.}$$

$$M_{BC} = \int V_{BC}\,dx = \int -\frac{wa^2}{2l}dx = -\frac{wa^2}{2l}x + C_2$$

$$M_{BC} = 0 \text{ at } x = l \ \therefore \ C_2 = \frac{wa^2}{2} \quad \Rightarrow \quad M_{BC} = \frac{wa^2}{2l}(l - x) \quad \textit{Ans.}$$

$$\theta_{AB} = \int \frac{M_{AB}}{EI}dx = \frac{1}{EI}\int \frac{wx}{2l}\left(2al - a^2 - lx\right)dx = \frac{1}{EI}\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]$$

$$y_{AB} = \int \theta_{AB}\,dx = \frac{1}{EI}\int\left[\frac{w}{2l}\left(alx^2 - \frac{1}{2}a^2x^2 - \frac{1}{3}lx^3\right) + C_3\right]dx$$

$$= \frac{1}{EI}\left[\frac{w}{2l}\left(\frac{1}{3}alx^3 - \frac{1}{6}a^2x^3 - \frac{1}{12}lx^4\right) + C_3 x + C_4\right]$$

$$y_{AB} = 0 \text{ at } x = 0 \ \therefore \ C_4 = 0$$

$$\theta_{BC} = \int \frac{M_{BC}}{EI}dx = \frac{1}{EI}\int \frac{wa^2}{2l}(l - x)\,dx = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(lx - \frac{1}{2}x^2\right) + C_5\right]$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = a \ \therefore$$

$$\frac{1}{EI}\left[\frac{w}{2l}\left(ala^2 - \frac{1}{2}a^4 - \frac{1}{3}la^3\right) + C_3\right] = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(la - \frac{1}{2}a^2\right) + C_5\right] \Rightarrow C_3 = \frac{wa^3}{6} + C_5 \qquad (1)$$

$$y_{BC} = \int \theta_{BC} dx = \frac{1}{EI}\int\left[\frac{wa^2}{2l}\left(lx - \frac{1}{2}x^2\right) + C_5\right]dx = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2 - \frac{1}{6}x^3\right) + C_5 x + C_6\right]$$

$$y_{BC} = 0 \text{ at } x = l \;\; \therefore \;\; C_6 = -\frac{wa^2 l^2}{6} - C_5 l$$

$$y_{BC} = \frac{1}{EI}\left[\frac{wa^2}{2l}\left(\frac{1}{2}lx^2 - \frac{1}{6}x^3 - \frac{1}{3}l^3\right) + C_5(x-l)\right]$$

$$y_{AB} = y_{BC} \text{ at } x = a \;\; \therefore$$

$$\frac{w}{2l}\left(\frac{1}{3}ala^3 - \frac{1}{6}a^5 - \frac{1}{12}la^4\right) + C_3 a = \frac{wa^2}{2l}\left(\frac{1}{2}la^2 - \frac{1}{6}a^3 - \frac{1}{3}l^3\right) + C_5(a-l)$$

$$C_3 a = \frac{wa^2}{24l}\left(3la^2 - 4l^3\right) + C_5(a-l) \qquad (2)$$

Substituting (1) into (2) yields $C_5 = \frac{wa^2}{24l}\left(-a^2 - 4l^2\right)$. Substituting this back into (2) gives

$C_3 = \frac{wa^2}{24l}\left(4al - a^2 - 4l^2\right)$. Thus,

$$y_{AB} = \frac{w}{24EIl}\left(4alx^3 - 2a^2x^3 - lx^4 + 4a^3lx - a^4x - 4a^2l^2x\right)$$

$$\Rightarrow \quad y_{AB} = \frac{wx}{24EIl}\left[2ax^2(2l-a) - lx^3 - a^2\left(2l-a\right)^2\right] \qquad \textit{Ans.}$$

$$y_{BC} = \frac{w}{24EIl}\left(6a^2lx^2 - 2a^2x^3 - a^4x - 4a^2l^2x + a^4l\right) \qquad \textit{Ans.}$$

This result is sufficient for y_{BC}. However, this can be shown to be equivalent to

$$y_{BC} = \frac{w}{24EIl}\left(4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x\right) + \frac{w}{24EI}(x-a)^4$$

$$y_{BC} = y_{AB} + \frac{w}{24EI}(x-a)^4 \qquad \textit{Ans.}$$

by expanding this or by solving the problem using singularity functions.

4-19 The beam can be broken up into a uniform load w downward from points A to C and a uniform load upward from points A to B.

$$y_{AB} = \frac{wx}{24EIl}\left[2bx^2(2l-b) - lx^3 - b^2\left(2l-b\right)^2\right] - \frac{wx}{24EIl}\left[2ax^2(2l-a) - lx^3 - a^2\left(2l-a\right)^2\right]$$

$$= \frac{wx}{24EIl}\left[2bx^2(2l-b) - b^2\left(2l-b\right)^2 - 2ax^2(2l-a) + a^2\left(2l-a\right)^2\right] \qquad \textit{Ans.}$$

$$y_{BC} = \frac{w}{24EIl}\left[2bx^3(2l-b) - lx^4 - b^2x\left(2l-b\right)^2 - \left(4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x\right) - l(x-a)^4\right] \qquad \textit{Ans.}$$

$$y_{CD} = \frac{w}{24EIl}\left[4blx^3 - 2b^2x^3 - lx^4 - 4b^2l^2x + 4b^3lx - b^4x + l(x-b)^4\right]$$

$$-\frac{w}{24EIl}\left[4alx^3 - 2a^2x^3 - lx^4 - 4a^2l^2x + 4a^3lx - a^4x + l(x-a)^4\right]$$

$$= \frac{w}{24EI}\left[(x-b)^4 - (x-a)^4\right] + y_{AB} \qquad \textit{Ans.}$$

4-20 **Note to the instructor:** See the note in the solution for Problem 4-18.

$$\sum F_y = 0 = R_B - \frac{wa^2}{2l} - wa \quad \Rightarrow \quad R_B = \frac{wa}{2l}(2l + a) \qquad \textit{Ans.}$$

For region *BC*, isolate right-hand element of length ($l + a - x$)

$$V_{AB} = -R_A = -\frac{wa^2}{2l}, \qquad V_{BC} = w(l + a - x) \qquad \textit{Ans.}$$

$$M_{AB} = -R_A x = -\frac{wa^2}{2l}x, \qquad M_{BC} = -\frac{w}{2}(l + a - x)^2 \qquad \textit{Ans.}$$

$$EI\theta_{AB} = \int M_{AB}dx = -\frac{wa^2}{4l}x^2 + C_1$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x + C_2$$

$$y_{AB} = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0 \quad \therefore \quad EIy_{AB} = -\frac{wa^2}{12l}x^3 + C_1x$$

$$y_{AB} = 0 \text{ at } x = l \quad \Rightarrow \quad C_1 = \frac{wa^2l}{12} \quad \therefore$$

$$EIy_{AB} = -\frac{wa^2}{12l}x^3 + \frac{wa^2l}{12}x = \frac{wa^2x}{12l}(l^2 - x^2) \quad \Rightarrow \quad y_{AB} = \frac{wa^2x}{12EIl}(l^2 - x^2) \quad \textit{Ans.}$$

$$EI\theta_{BC} = \int M_{BC}dx = -\frac{w}{6}(l + a - x)^3 + C_3$$

$$EIy_{BC} = -\frac{w}{24}(l + a - x)^4 + C_3x + C_4$$

$$y_{BC} = 0 \text{ at } x = l \quad \Rightarrow \quad -\frac{wa^4}{24} + C_3l + C_4 = 0 \quad \Rightarrow \quad C_4 = \frac{wa^4}{24} - C_3l \qquad (1)$$

$$\theta_{AB} = \theta_{BC} \text{ at } x = l \quad \Rightarrow \quad -\frac{wa^2l}{4} + \frac{wa^2l}{12} = \frac{wa^3}{6} + C_3 \quad \Rightarrow \quad C_3 = -\frac{wa^2}{6}(l + a)$$

Substitute C_3 into Eq. (1) gives $C_4 = \frac{wa^2}{24}\left[a^2 + 4l(l + a)\right]$. Substitute back into y_{BC}

$$y_{BC} = \frac{1}{EI}\left[-\frac{w}{24}(l + a - x)^4 - \frac{wa^2}{6}x(l + a) + \frac{wa^4}{24} + \frac{wa^2l}{6}(l + a)\right]$$

$$= -\frac{w}{24EI}\left[(l + a - x)^4 - 4a^2(l - x)(l + a) - a^4\right] \qquad \textit{Ans.}$$

4-21 Table A-9, beam 7,

$$R_1 = R_2 = \frac{wl}{2} = \frac{100(10)}{2} = 500 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wx}{24EI}\left(2lx^2 - x^3 - l^3\right) = \frac{100x}{24(30)10^6(0.05)}\left[2(10)x^2 - x^3 - 10^3\right]$$

$$= 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right)$$

Slope: $\theta_{AB} = \dfrac{d\,y_{AB}}{d\,x} = \dfrac{w}{24EI}\left(6lx^2 - 4x^3 - l^3\right)$

At $x = l$, $\theta_{AB}\big|_{x=l} = \dfrac{w}{24EI}\left(6l\,l^2 - 4l^3 - l^3\right) = \dfrac{wl^3}{24EI}$

$$y_{BC} = \theta_{AB}\big|_{x=l}(x-l) = \frac{wl^3}{24EI}(x-l) = \frac{100\left(10^3\right)}{24(30)10^6(0.05)}(x-10) = 2.7778\left(10^{-3}\right)(x-10)$$

From Prob. 4-20,

$$R_A = \frac{wa^2}{2l} = \frac{100\left(4^2\right)}{2(10)} = 80 \text{ lbf } \downarrow \qquad R_B = \frac{wa}{2l}(2l+a) = \frac{100(4)}{2(10)}\left[2(10)+4\right] = 480 \text{ lbf } \uparrow$$

$$y_{AB} = \frac{wa^2x}{12EIl}\left(l^2 - x^2\right) = \frac{100\left(4^2\right)x}{12(30)10^6(0.05)}\left(10^2 - x^2\right) = 8.8889\left(10^{-6}\right)x\left(100 - x^2\right)$$

$$y_{BC} = -\frac{w}{24EI}\left[(l+a-x)^4 - 4a^2(l-x)(l+a) - a^4\right]$$

$$= -\frac{100}{24(30)10^6(0.05)}\left[(10+4-x)^4 - 4\left(4^2\right)(10-x)(10+4) - 4^4\right]$$

$$= -2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right]$$

Superposition,

$R_A = 500 - 80 = 420$ lbf $\uparrow$ $\qquad R_B = 500 + 480 = 980$ lbf $\uparrow$ *Ans.*

$$y_{AB} = 2.7778\left(10^{-6}\right)x\left(20x^2 - x^3 - 1000\right) + 8.8889\left(10^{-6}\right)x\left(100 - x^2\right) \quad \textit{Ans.}$$

$$y_{BC} = 2.7778\left(10^{-3}\right)(x-10) - 2.7778\left(10^{-6}\right)\left[(14-x)^4 + 896x - 9216\right] \quad \textit{Ans.}$$

The deflection equations can be simplified further. However, they are sufficient for plotting.

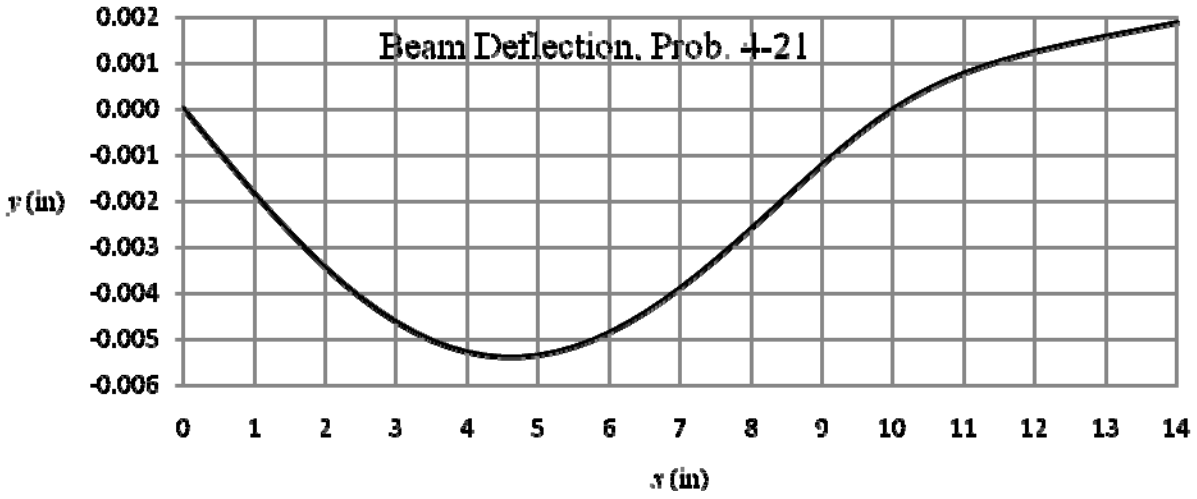
Using a spreadsheet,

x	0	0.5	1	1.5	2	2.5	3	3.5
y	0.000000	-0.000939	-0.001845	-0.002690	-0.003449	-0.004102	-0.004632	-0.005027

x	4	4.5	5	5.5	6	6.5	7	7.5
y	-0.005280	-0.005387	-0.005347	-0.005167	-0.004853	-0.004421	-0.003885	-0.003268

x	8	8.5	9	9.5	10	10.5	11	11.5
y	-0.002596	-0.001897	-0.001205	-0.000559	0.000000	0.000439	0.000775	0.001036

x	12	12.5	13	13.5	14
y	0.001244	0.001419	0.001575	0.001722	0.001867



4-22 (**a**) Useful relations

$$k = \frac{F}{y} = \frac{48EI}{l^3}$$

$$I = \frac{kl^3}{48E} = \frac{1800\left(36^3\right)}{48(30)10^6} = 0.05832 \text{ in}^4$$

From $I = bh^3/12$, and $b = 10\,h$, then $I = 5\,h^4/6$, or,

$$h = \sqrt[4]{\frac{6I}{5}} = \sqrt[4]{\frac{6(0.05832)}{5}} = 0.514 \text{ in}$$

h is close to 1/2 in and 9/16 in, while b is close to 5.14 in. Changing the height drastically changes the spring rate, so changing the base will make finding a close solution easier. Trial and error was applied to find the combination of values from Table A-17 that yielded the closet desired spring rate.

h (in)	b (in)	b/h	k (lbf/in)
1/2	5	10	1608
1/2	5½	11	1768
1/2	5¾	11.5	1849
9/16	5	8.89	2289
9/16	4	7.11	1831

h = ½ in, b = 5 ½ in should be selected because it results in a close spring rate and b/h is still reasonably close to 10.

(b) $I = 5.5(0.5)^3/12 = 0.05729 \text{ in}^4$

$$\sigma = \frac{Mc}{I} = \frac{(Fl/4)c}{I} \quad \Rightarrow \quad F = \frac{4\sigma I}{lc} = \frac{4(60)10^3(0.05729)}{(36)(0.25)} = 1528 \text{ lbf}$$

$$y = \frac{Fl^3}{48EI} = \frac{(1528)(36^3)}{48(30)10^6(0.05729)} = 0.864 \text{ in} \quad \textit{Ans.}$$

4-23 From the solutions to Prob. 3-68, $T_1 = 60$ lbf and $T_2 = 400$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198 \text{ in}^4$$

From Table A-9, beam 6,

$$z_A = \left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]_{x=10\text{in}}$$

$$= \frac{(-575)(30)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 30^2 - 40^2\right)$$

$$+ \frac{460(12)(10)}{6(30)10^6(0.1198)(40)}\left(10^2 + 12^2 - 40^2\right) = 0.0332 \text{ in} \quad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=10\text{in}} = -\left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=10\text{in}}$$

$$= -\left\{\frac{F_1 b_1}{6EIl}(3x^2 + b_1^2 - l^2) + \frac{F_2 b_2}{6EIl}(3x^2 + b_2^2 - l^2)\right\}_{x=10\text{in}}$$

$$= -\frac{(575)(30)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 30^2 - 40^2\right]$$

$$-\frac{-460(12)}{6(30)10^6(0.1198)(40)}\left[3\left(10^2\right) + 12^2 - 40^2\right]$$

$$= 6.02(10^{-4}) \text{ rad} \quad \textit{Ans.}$$

4-24 From the solutions to Prob. 3-69, $T_1 = 2880$ N and $T_2 = 432$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(30)^4}{64} = 39.76\left(10^3\right) \text{ mm}^4$$

The load in between the supports supplies an angle to the overhanging end of the beam. That angle is found by taking the derivative of the deflection from that load. From Table A-9, beams 6 (subscript 1) and 10 (subscript 2),

$$y_A = \left[\theta_{BC}\big|_C (a_2)\right]_{\text{beam 6}} + (y_A)_{\text{beam 10}} \qquad (1)$$

$$\theta_{BC}\big|_C = \left\{\frac{d}{dx}\left[\frac{F_1 a_1 (l-x)}{6EIl}\left(x^2 + a_1{}^2 - 2lx\right)\right]\right\}_{x=l} = \left[\frac{F_1 a_1}{6EIl}\left(6lx - 3x^2 - a_1{}^2 - 2l^2\right)\right]_{x=l}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right)$$

Equation (1) is thus

$$y_A = \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right)a_2 - \frac{F_2 a_2{}^2}{3EI}(l + a_2)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)(300) - \frac{2070(300^2)}{3(207)10^3(39.76)10^3}(510 + 300)$$

$$= -7.99 \text{ mm} \qquad \textit{Ans.}$$

The slope at A, relative to the z axis is

$$(\theta_A)_z = \frac{F_1 a_1}{6EIl}(l^2 - a_1{}^2) + \left\{\frac{d}{dx}\left[\frac{F_2 (x-l)}{6EI}\left[(x-l)^2 - a_2(3x-l)\right]\right]\right\}_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}\left(l^2 - a_1{}^2\right) + \frac{F_2}{6EI}\left[3(x-l)^2 - 3a_2(x-l) - a_2(3x-l)\right]_{x=l+a_2}$$

$$= \frac{F_1 a_1}{6EIl}(l^2 - a_1{}^2) - \frac{F_2}{6EI}\left(3a_2{}^2 + 2la_2\right)$$

$$= \frac{-3312(230)}{6(207)10^3(39.76)10^3(510)}\left(510^2 - 230^2\right)$$

$$- \frac{2070}{6(207)10^3(39.76)10^3}\left[3(300^2) + 2(510)(300)\right]$$

$$= -0.0304 \text{ rad} \qquad \textit{Ans.}$$

4-25 From the solutions to Prob. 3-70, $T_1 = 392.16$ lbf and $T_2 = 58.82$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi (1)^4}{64} = 0.049\,09 \text{ in}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^{\,2} - l^2\right)\right]_{x=8\text{in}} = \frac{(-350)(14)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 14^2 - 22^2\right) = 0.0452 \text{ in} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=8\text{in}} = \frac{(-450.98)(6)(8)}{6(30)10^6(0.049\ 09)(22)}\left(8^2 + 6^2 - 22^2\right) = 0.0428 \text{ in} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0452^2 + 0.0428^2} = 0.0622$ in *Ans.*

$$(\theta_A)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=a_1} = \frac{F_1 b_1}{6EIl}(3a_1^2 + b_1^2 - l^2)$$

$$= \frac{(-350)(14)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 14^2 - 22^2\right] = 0.00242 \text{ rad} \quad \textit{Ans.}$$

$$(\theta_A)_y = \left(-\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{-F_2 b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]\right\}_{x=a_1} = \frac{F_2 b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= \frac{(450.98)(6)}{6(30)10^6(0.04909)(22)}\left[3\left(8^2\right) + 6^2 - 22^2\right] = -0.00356 \text{ rad} \quad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00242^2 + \left(-0.00356\right)^2} = 0.00430$ rad *Ans.*

4-26 From the solutions to Prob. 3-71, $T_1 = 250$ N and $T_2 = 37.5$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi(20)^4}{64} = 7\ 854 \text{ mm}^4$$

$$y_A = \left[\frac{F_{1y} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2)\right]_{x=300\text{mm}} = \frac{\left(-345\sin 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$= 1.60 \text{ mm} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z} b_1 x}{6EIl}(x^2 + b_1^{\,2} - l^2) + \frac{F_2 b_2 x}{6EIl}(x^2 + b_2^{\,2} - l^2)\right]_{x=300\text{mm}}$$

$$= \frac{\left(345\cos 45^\circ\right)(550)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 550^2 - 850^2\right)$$

$$+ \frac{-287.5(150)(300)}{6(207)10^3(7\ 854)(850)}\left(300^2 + 150^2 - 850^2\right) = -0.650 \text{ mm} \quad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{1.60^2 + \left(-0.650\right)^2} = 1.73$ mm *Ans.*

$$(\theta_A)_z = \left(\frac{dy}{dx}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=a_1} = \frac{F_{1y}b_1}{6EIl}(3a_1^2 + b_1^2 - l^2)$$

$$= \frac{-\left(345\sin 45^\circ\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right) + 550^2 - 850^2\right] = 0.00243 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1{}^2 - l^2\right) + \frac{F_2 b_2 x}{6EIl}\left(x^2 + b_2{}^2 - l^2\right)\right]\right\}_{x=a_1}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) - \frac{F_2 b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= -\frac{\left(345\cos 45^\circ\right)(550)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right) + 550^2 - 850^2\right]$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left[3\left(300^2\right) + 150^2 - 850^2\right] = 1.91\cdot 10^{-4} \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{0.00243^2 + 0.000191^2} = 0.00244$ rad *Ans.*

4-27 From the solutions to Prob. 3-72, $F_B = 750$ lbf

$$I = \frac{\pi d^4}{64} = \frac{\pi(1.25)^4}{64} = 0.1198 \text{ in}^4$$

From Table A-9, beams 6 (subscript 1) and 10 (subscript 2)

$$y_A = \left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2y}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(-300\cos 20^\circ\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2 + 14^2 - 30^2\right) + \frac{\left(750\sin 20^\circ\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2 - 16^2\right)$$

$$= 0.0805 \text{ in} \qquad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1{}^2 - l^2\right) + \frac{F_{2z}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]_{x=16\text{in}}$$

$$= \frac{\left(300\sin 20^\circ\right)(14)(16)}{6(30)10^6(0.119\ 8)(30)}\left(16^2 + 14^2 - 30^2\right) + \frac{\left(-750\cos 20^\circ\right)(9)(16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2 - 16^2\right)$$

$$= -0.1169 \text{ in} \qquad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{0.0805^2 + \left(-0.1169\right)^2} = 0.142$ in *Ans.*

$$(\theta_A)_z = \left(\frac{dy}{dx}\right)_{x=a_1} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2y}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=a_1}$$

$$= \frac{F_{1y}b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) + \frac{F_{2y}a_2}{6EIl}\left(l^2 - 3a_1^2\right)$$

$$= \frac{\left(-300\cos 20^\circ\right)(14)}{6(30)10^6(0.119\ 8)(30)}\left[3\left(16^2\right) + 14^2 - 30^2\right]$$

$$+ \frac{\left(750\sin 20^\circ\right)(9)}{6(30)10^6(0.119\ 8)(30)}\left[30^2 - 3\left(16^2\right)\right] = 8.06\left(10^{-5}\right)\text{rad} \quad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{dz}{dx}\right)_{x=a_1} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=a_1}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) - \frac{F_{2z}a_2}{6EIl}\left(l^2 - 3a_1^2\right)$$

$$= -\frac{\left(300\sin 20^\circ\right)(14)}{6(30)10^6(0.119\ 8)(30)}\left[3\left(16^2\right) + 14^2 - 30^2\right] - \frac{\left(-750\cos 20^\circ\right)(9)}{6(30)10^6(0.119\ 8)(30)}\left[30^2 - 3\left(16^2\right)\right]$$

$$= 0.00115 \text{ rad} \quad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{\left[8.06\left(10^{-5}\right)\right]^2 + 0.00115^2} = 0.00115$ rad *Ans.*

4-28 From the solutions to Prob. 3-73, $F_B = 22.8\ (10^3)$ N

$$I = \frac{\pi d^4}{64} = \frac{\pi\left(50^4\right)}{64} = 306.8\left(10^3\right)\ \text{mm}^4$$

From Table A-9, beam 6,

$$y_A = \left[\frac{F_{1y}b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_{2y}b_2 x}{6EIl}(x^2 + b_2^2 - l^2)\right]_{x=400\text{mm}}$$

$$= \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 650^2 - 1050^2\right)$$

$$+ \frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 300^2 - 1050^2\right)$$

$$= -3.735 \text{ mm} \quad \textit{Ans.}$$

$$z_A = \left[\frac{F_{1z} b_1 x}{6EIl}(x^2 + b_1{}^2 - l^2) + \frac{F_{2z} b_2 x}{6EIl}(x^2 + b_2{}^2 - l^2) \right]_{x=400\,\text{mm}}$$

$$= \frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 650^2 - 1050^2\right)$$

$$+ \frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)(400)}{6(207)10^3(306.8)10^3(1050)}\left(400^2 + 300^2 - 1050^2\right) = 1.791 \text{ mm} \qquad \textit{Ans.}$$

The displacement magnitude is $\delta = \sqrt{y_A^2 + z_A^2} = \sqrt{(-3.735)^2 + 1.791^2} = 4.14$ mm *Ans.*

$$(\theta_A)_z = \left(\frac{d\,y}{d\,x}\right)_{x=a_1} = \left\{ \frac{d}{dx}\left[\frac{F_{1z} b_1 x}{6EIl}\left(x^2 + b_1{}^2 - l^2\right) + \frac{F_{2z} b_2 x}{6EIl}\left(x^2 + b_2{}^2 - l^2\right) \right] \right\}_{x=a_1}$$

$$= \frac{F_{1y} b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) + \frac{F_{2y} b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 650^2 - 1050^2\right]$$

$$+ \frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 300^2 - 1050^2\right]$$

$$= -0.00507 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_A)_y = -\left(\frac{d\,z}{d\,x}\right)_{x=a_1} = -\left\{ \frac{d}{dx}\left[\frac{F_{1z} b_1 x}{6EIl}\left(x^2 + b_1{}^2 - l^2\right) + \frac{F_{2z} b_2 x}{6EIl}\left(x^2 + b_2{}^2 - l^2\right) \right] \right\}_{x=a_1}$$

$$= -\frac{F_{1z} b_1}{6EIl}\left(3a_1^2 + b_1^2 - l^2\right) - \frac{F_{2z} b_2}{6EIl}\left(3a_1^2 + b_2^2 - l^2\right)$$

$$= -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 650^2 - 1050^2\right]$$

$$- \frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left[3\left(400^2\right) + 300^2 - 1050^2\right]$$

$$= -0.00489 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_A = \sqrt{(-0.00507)^2 + (-0.00489)^2} = 0.00704$ rad *Ans.*

4-29 From the solutions to Prob. 3-68, $T_1 = 60$ lbf and $T_2 = 400$ lbf , and Prob. 4-23, $I = 0.119\ 8$ in^4. From Table A-9, beam 6,

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z}b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2 - l^2\right) - \frac{F_{2z}b_2}{6EIl}\left(b_2^2 - l^2\right) = -\frac{-575(30)}{6(30)10^6(0.119\ 8)(40)}\left(30^2 - 40^2\right)$$

$$-\frac{460(12)}{6(30)10^6(0.119\ 8)(40)}\left(12^2 - 40^2\right) = -0.00468 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right) + \frac{F_{2z}a_2}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2 - a_1^2\right) - \frac{F_{2z}a_2}{6EIl}\left(l^2 - a_2^2\right)$$

$$= -\frac{-575(10)\left(40^2 - 10^2\right)}{6(30)10^6(0.119\ 8)(40)} - \frac{460(28)\left(40^2 - 28^2\right)}{6(30)10^6(0.119\ 8)(40)} = -0.00219 \text{ rad} \qquad \textit{Ans.}$$

4-30 From the solutions to Prob. 3-69, $T_1 = 2\ 880$ N and $T_2 = 432$ N, and Prob. 4-24, $I = 39.76\ (10^3)$ mm^4. From Table A-9, beams 6 and 10

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_1 b_1 x}{6EIl}(x^2 + b_1^2 - l^2) + \frac{F_2 a_2 x}{6EIl}(l^2 - x^2)\right]\right\}_{x=0}$$

$$= \left[\frac{F_1 b_1}{6EIl}(3x^2 + b_1^2 - l^2) + \frac{F_2 a_2}{6EIl}(l^2 - 3x^2)\right]_{x=0} = \frac{F_1 b_1}{6EIl}(b_1^2 - l^2) + \frac{F_2 a_2 l}{6EI}$$

$$= \frac{-3\ 312(280)}{6(207)10^3(39.76)10^3(510)}\left(280^2 - 510^2\right) + \frac{2\ 070(300)(510)}{6(207)10^3(39.76)10^3}$$

$$= 0.0131 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_1 a_1 (l-x)}{6EIl}(x^2 + a_1^2 - 2lx) + \frac{F_2 a_2 x}{6EIl}(l^2 - x^2)\right]\right\}_{x=l}$$

$$= \left[\frac{F_1 a_1}{6EIl}(6lx - 2l^2 - 3x^2 - a_1^2) + \frac{F_2 a_2}{6EIl}(l^2 - 3x^2)\right]_{x=l} = \frac{F_1 a_1}{6EIl}(l^2 - a_1^2) - \frac{F_2 a_2 l}{3EI}$$

$$= \frac{-3\ 312(230)}{6(207)10^3(39.76)10^3(510)}(510^2 - 230^2) - \frac{2\ 070(300)(510)}{3(207)10^3(39.76)10^3}$$

$$= -0.0191 \text{ rad} \qquad \textit{Ans.}$$

4-31 From the solutions to Prob. 3-70, $T_1 = 392.19$ lbf and $T_2 = 58.82$ lbf , and Prob. 4-25, $I = 0.0490\ 9$ in^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^2 - l^2)$$

$$= \frac{-350(14)}{6(30)10^6(0.04909)(22)}\left(14^2 - 22^2\right) = 0.00726 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right)\right]\right\}_{x=0} = -\frac{F_{2z}b_2}{6EIl}\left(b_2^2 - l^2\right)$$

$$= -\frac{-450.98(6)}{6(30)10^6(0.04909)(22)}\left(6^2 - 22^2\right)$$

$$= -0.00624 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00726^2 + \left(-0.00624\right)^2} = 0.00957$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2 - a_1^2)$$

$$= \frac{-350(8)}{6(30)10^6(0.0491)(22)}\left(22^2 - 8^2\right) = -0.00605 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{2z}a_2}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_2^2\right)\right]_{x=l} = -\frac{F_{2z}a_2}{6EIl}\left(l^2 - a_2^2\right)$$

$$= -\frac{-450.98(16)}{6(30)10^6(0.04909)(22)}\left(22^2 - 16^2\right) = 0.00846 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{\left(-0.00605\right)^2 + 0.00846^2} = 0.0104$ rad *Ans.*

4-32 From the solutions to Prob. 3-71, T_1 =250 N and T_1 =37.5 N, and Prob. 4-26, I = 7 854 mm^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right)\right]\right\}_{x=0} = \frac{F_{1y}b_1}{6EIl}(b_1^2 - l^2)$$

$$= \frac{\left[-345\sin 45^\circ\right](550)}{6(207)10^3(7\ 854)(850)}\left(550^2 - 850^2\right) = 0.00680 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z}b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2 - l^2\right) - \frac{F_{2z}b_2}{6EIl}\left(b_2^2 - l^2\right) = -\frac{\left[345\cos 45^\circ\right](550)}{6(207)10^3(7\ 854)(850)}\left(550^2 - 850^2\right)$$

$$-\frac{-287.5(150)}{6(207)10^3(7\ 854)(850)}\left(150^2 - 850^2\right) = 0.00316 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00680^2 + 0.00316^2} = 0.00750$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right)\right]\right\}_{x=l} = \left[\frac{F_{1y}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}(l^2 - a_1^2) = \frac{\left[-345\sin 45^\circ\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2 - 300^2\right) = -0.00558 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2 - a_1^2\right) - \frac{F_{2z}a_2}{6EIl}\left(l^2 - a_2^2\right) = -\frac{\left[345\cos 45^\circ\right](300)}{6(207)10^3(7\ 854)(850)}\left(850^2 - 300^2\right)$$

$$-\frac{-287.5(700)}{6(207)10^3(7\ 854)(850)}\left(850^2 - 700^2\right) = 6.04\left(10^{-5}\right) \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{\left(-0.00558\right)^2 + \left[6.04\left(10^{-5}\right)\right]^2} = 0.00558$ rad *Ans.*

4-33 From the solutions to Prob. 3-72, $F_B = 750$ lbf, and Prob. 4-27, $I = 0.119\ 8$ in^4. From Table A-9, beams 6 and 10

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2y}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=0}$$

$$= \left[\frac{F_{1y}b_1}{6EIl}\left(3x^2 + b_1^2 - l^2\right) + \frac{F_{2y}a_2}{6EIl}\left(l^2 - 3x^2\right)\right]_{x=0} = \frac{F_{1y}b_1}{6EIl}\left(b_1^2 - l^2\right) + \frac{F_{2y}a_2 l}{6EI}$$

$$= \frac{\left[-300\cos 20^\circ\right](14)}{6(30)10^6(0.119\ 8)(30)}\left(14^2 - 30^2\right) + \frac{\left[750\sin 20^\circ\right](9)(30)}{6(30)10^6(0.119\ 8)} = 0.00751 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=0}$$

$$= -\left[\frac{F_{1z}b_1}{6EIl}\left(3x^2 + b_1^2 - l^2\right) + \frac{F_{2z}a_2}{6EIl}\left(l^2 - 3x^2\right)\right]_{x=0} = -\frac{F_{1z}b_1}{6EIl}\left(b_1^2 - l^2\right) - \frac{F_{2z}a_2 l}{6EI}$$

$$= -\frac{\left[300\sin 20^\circ\right](14)}{6(30)10^6(0.119\ 8)(30)}\left(14^2 - 30^2\right) - \frac{\left[-750\cos 20^\circ\right](9)(30)}{6(30)10^6(0.119\ 8)} = 0.0104 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{0.00751^2 + 0.0104^2} = 0.0128$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2y}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right) + \frac{F_{2y}a_2}{6EIl}\left(l^2 - 3x^2\right)\right]_{x=l} = \frac{F_{1y}a_1}{6EIl}(l^2 - a_1{}^2) - \frac{F_{2y}a_2 l}{3EI}$$

$$= \frac{\left[-300\cos 20^\circ\right](16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2 - 16^2\right) - \frac{\left[750\sin 20^\circ\right](9)(30)}{3(30)10^6(0.119\ 8)} = -0.0109 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2z}a_2 x}{6EIl}\left(l^2 - x^2\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right) + \frac{F_{2z}a_2}{6EIl}\left(l^2 - 3x^2\right)\right]_{x=l} = -\frac{F_{1z}a_1}{6EIl}\left(l^2 - a_1^2\right) + \frac{F_{2z}a_2 l}{3EI}$$

$$= -\frac{\left[300\sin 20^\circ\right](16)}{6(30)10^6(0.119\ 8)(30)}\left(30^2 - 16^2\right) + \frac{\left[-750\cos 20^\circ\right](9)(30)}{3(30)10^6(0.119\ 8)} = -0.0193 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{(-0.0109)^2 + (-0.0193)^2} = 0.0222$ rad *Ans.*

4-34 From the solutions to Prob. 3-73, $F_B = 22.8$ kN, and Prob. 4-28, $I = 306.8\ (10^3)$ mm^4. From Table A-9, beam 6

$$(\theta_O)_z = \left(\frac{dy}{dx}\right)_{x=0} = \left\{\frac{d}{dx}\left[\frac{F_{1y}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2y}b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right)\right]\right\}_{x=0}$$

$$= \frac{F_{1y}b_1}{6EIl}\left(b_1^2 - l^2\right) + \frac{F_{2y}b_2}{6EIl}\left(b_2^2 - l^2\right) = \frac{\left[11\left(10^3\right)\sin 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2 - 1050^2\right)$$

$$+ \frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2 - 1050^2\right) = -0.0115 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_O)_y = -\left(\frac{dz}{dx}\right)_{x=0} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) + \frac{F_{2z}b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right)\right]\right\}_{x=0}$$

$$= -\frac{F_{1z}b_1}{6EIl}\left(b_1^2 - l^2\right) - \frac{F_{2z}b_2}{6EIl}\left(b_2^2 - l^2\right)$$

$$= -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](650)}{6(207)10^3(306.8)10^3(1050)}\left(650^2 - 1050^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](300)}{6(207)10^3(306.8)10^3(1050)}\left(300^2 - 1050^2\right) = -0.00427 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_O = \sqrt{(-0.0115)^2 + (-0.00427)^2} = 0.0123$ rad *Ans.*

$$(\theta_C)_z = \left(\frac{dy}{dx}\right)_{x=l} = \left\{\frac{d}{dx}\left[\frac{F_{1y}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2y}a_2(l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= \left[\frac{F_{1y}a_1}{6EIl}(6lx - 2l^2 - 3x^2 - a_1^2) + \frac{F_{2y}a_2}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_2^2\right)\right]_{x=l}$$

$$= \frac{F_{1y}a_1}{6EIl}\left(l^2 - a_1^2\right) + \frac{F_{2y}a_2}{6EIl}\left(l^2 - a_2^2\right) = \frac{\left[11\left(10^3\right)\sin 20^\circ\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2 - 400^2\right)$$

$$+\frac{\left[22.8\left(10^3\right)\sin 25^\circ\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2 - 750^2\right) = 0.0133 \text{ rad} \qquad \textit{Ans.}$$

$$(\theta_C)_y = -\left(\frac{dz}{dx}\right)_{x=l} = -\left\{\frac{d}{dx}\left[\frac{F_{1z}a_1(l-x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) + \frac{F_{2z}a_2(l-x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right)\right]\right\}_{x=l}$$

$$= -\left[\frac{F_{1z}a_1}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_1^2\right) + \frac{F_{2z}a_2}{6EIl}\left(6lx - 2l^2 - 3x^2 - a_2^2\right)\right]_{x=l}$$

$$= -\frac{F_{1z}a_1}{6EIl}\left(l^2 - a_1^2\right) - \frac{F_{2z}a_2}{6EIl}\left(l^2 - a_2^2\right) = -\frac{\left[11\left(10^3\right)\cos 20^\circ\right](400)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2 - 400^2\right)$$

$$-\frac{\left[-22.8\left(10^3\right)\cos 25^\circ\right](750)}{6(207)10^3(306.8)10^3(1050)}\left(1050^2 - 750^2\right) = 0.0112 \text{ rad} \qquad \textit{Ans.}$$

The slope magnitude is $\Theta_C = \sqrt{0.0133^2 + 0.0112^2} = 0.0174$ rad *Ans.*

4-35 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-29, $I = 0.119\ 8$ in^4, and it was found that the greater angle occurs at the bearing at O where $(\theta_O)_y = -0.00468$ rad.

Since θ is inversely proportional to I,

$$\theta_{new}\, I_{new} = \theta_{old}\, I_{old} \quad \Rightarrow \quad I_{new} = \pi\, d_{new}^4/64 = \theta_{old}\, I_{old}\, /\theta_{new}$$

or,
$$d_{new} = \left(\frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

The absolute sign is used as the old slope may be negative.

$$d_{new} = \left(\frac{64}{\pi} \left| \frac{-0.00468}{0.00105} \right| 0.119\ 8 \right)^{1/4} = 1.82 \text{ in} \qquad \textit{Ans.}$$

4-36 The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-30, $I = 39.76\ (10^3)$ mm^4, and it was found that the greater angle occurs at the bearing at C where $(\theta_C)_y = -0.0191$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{new} = \left(\frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

$$d_{new} = \left(\frac{64}{\pi} \left| \frac{-0.0191}{0.00105} \right| 39.76\left(10^3\right) \right)^{1/4} = 62.0 \text{ mm} \qquad \textit{Ans.}$$

4-37 The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-31, $I = 0.0491$ in^4, and the maximum slope is $\Theta_C = 0.0104$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{new} = \left(\frac{64}{\pi} \left| \frac{\theta_{old}}{\theta_{new}} \right| I_{old} \right)^{1/4}$$

$$d_{new} = \left(\frac{64}{\pi} \left| \frac{0.0104}{0.00105} \right| 0.0491 \right)^{1/4} = 1.77 \text{ in} \qquad \textit{Ans.}$$

4-38 The required new slope in radians is $\theta_{new} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-32, $I = 7\ 854$ mm^4, and the maximum slope is $\Theta_O = 0.00750$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.00750}{0.00105} \right| 7\ 854 \right)^{1/4} = 32.7 \text{ mm} \qquad \textit{Ans.}$$

4-39 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-33, $I = 0.119\ 8$ in^4, and the maximum slope $\Theta = 0.0222$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0222}{0.00105} \right| 0.119\ 8 \right)^{1/4} = 2.68 \text{ in} \qquad \textit{Ans.}$$

4-40 The required new slope in radians is $\theta_{\text{new}} = 0.06(\pi/180) = 0.00105$ rad.
In Prob. 4-34, $I = 306.8\ (10^3)$ mm^4, and the maximum slope is $\Theta_C = 0.0174$ rad.

See the solution to Prob. 4-35 for the development of the equation

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{\theta_{\text{old}}}{\theta_{\text{new}}} \right| I_{\text{old}} \right)^{1/4}$$

$$d_{\text{new}} = \left(\frac{64}{\pi} \left| \frac{0.0174}{0.00105} \right| 306.8\left(10^3\right) \right)^{1/4} = 100.9 \text{ mm} \qquad \textit{Ans.}$$

4-41 $I_{AB} = \pi\, 1^4/64 = 0.04909$ in^4, $J_{AB} = 2\, I_{AB} = 0.09818$ in^4, $I_{BC} = (0.25)(1.5)^3/12 = 0.07031$ in^4, $I_{CD} = \pi\,(3/4)^4/64 = 0.01553$ in^4. For Eq. (3-41), p. 102, $b/c = 1.5/0.25 = 6 \Rightarrow \beta = 0.299$.

The deflection can be broken down into several parts

1. The vertical deflection of B due to force and moment acting on B (y_1).
2. The vertical deflection due to the slope at B, θ_{B1}, due to the force and moment acting on B ($y_2 = \overline{CD}\,\theta_{B1} = 2\theta_{B1}$).

3. The vertical deflection due to the rotation at B, θ_{B2}, due to the torsion acting at B ($y_3 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
4. The vertical deflection of C due to the force acting on C (y_4).
5. The rotation at C, θ_C, due to the torsion acting at C ($y_3 = \overline{CD}\,\theta_C = 2\theta_C$).
6. The vertical deflection of D due to the force acting on D (y_5).

1. From Table A-9, beams 1 and 4 with $F = -200$ lbf and $M_B = 2(200) = 400$ lbf·in

$$y_1 = -\frac{-200\left(6^3\right)}{3(30)10^6\left(0.04909\right)} + \frac{400\left(6^2\right)}{2(30)10^6\left(0.04909\right)} = 0.01467 \text{ in}$$

2. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{\frac{d}{dx}\left[\frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI}\right]\right\}_{x=l} = \left[\frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI}\right]_{x=l}$$

$$= \left\{\frac{l}{2EI}\left[-Fl + 2M_B\right]\right\} = \frac{6}{2(30)10^6\left(0.04909\right)}\left[-(-200)(6) + 2(400)\right] = 0.004074 \text{ rad}$$

$$y_2 = 2(0.004072) = 0.00815 \text{ in}$$

3. The torsion at B is $T_B = 5(200) = 1000$ lbf·in. From Eq. (4-5)

$$\theta_{B2} = \left(\frac{TL}{JG}\right)_{AB} = \frac{1000(6)}{0.09818(11.5)10^6} = 0.005314 \text{ rad}$$

$$y_3 = 5(0.005314) = 0.02657 \text{ in}$$

4. For bending of BC, from Table A-9, beam 1

$$y_4 = -\frac{-200\left(5^3\right)}{3(30)10^6\left(0.07031\right)} = 0.00395 \text{ in}$$

5. For twist of BC, from Eq. (3-41), p. 102, with $T = 2(200) = 400$ lbf·in

$$\theta_C = \frac{400(5)}{0.299(1.5)0.25^3(11.5)10^6} = 0.02482 \text{ rad}$$

$$y_5 = 2(0.02482) = 0.04964 \text{ in}$$

6. For bending of CD, from Table A-9, beam 1

$$y_6 = -\frac{-200\left(2^3\right)}{3(30)10^6\left(0.01553\right)} = 0.00114 \text{ in}$$

Summing the deflections results in

$$y_D = \sum_{i=1}^{6} y_i = 0.01467 + 0.00815 + 0.02657 + 0.00395 + 0.04964 + 0.00114 = 0.1041 \text{ in} \quad \textit{Ans.}$$

This problem is solved more easily using Castigliano's theorem. See Prob. 4-71.

4-42 The deflection of D in the x direction due to F_z is from:

1. The deflection due to the slope at B, θ_{B1}, due to the force and moment acting on B ($x_1 = \overline{BC}\,\theta_{B1} = 5\theta_{B1}$).
2. The deflection due to the moment acting on C (x_2).

1. For AB, $I_{AB} = \pi 1^4/64 = 0.04909$ in^4. From Table A-9, beams 1 and 4

$$\theta_{B1} = \left\{ \frac{d}{dx}\left[\frac{Fx^2}{6EI}(x-3l) + \frac{M_B x^2}{2EI} \right] \right\}_{x=l} = \left[\frac{Fx}{6EI}(3x-6l) + \frac{M_B x}{EI} \right]_{x=l}$$

$$= \left\{ \frac{l}{2EI}\left[-Fl + 2M_B \right] \right\} = \frac{6}{2(30)10^6(0.04909)}\left[-(100)(6) + 2(-200) \right] = -0.002037 \text{ rad}$$

$$x_1 = 5(-0.002037) = -0.01019 \text{ in}$$

2. For BC, $I_{BC} = (1.5)(0.25)^3/12 = 0.001953$ in^4. From Table A-9, beam 4

$$x_2 = \frac{M_C l^2}{2EI} = \frac{2(-100)5}{2(30)10^6(0.001953)} = -0.04267 \text{ in}$$

The deflection of D in the x direction due to F_x is from:

3. The elongation of AB due to the tension. For AB, the area is $A = \pi 1^2/4 = 0.7854$ in^2

$$x_3 = \left(\frac{Fl}{AE} \right)_{AB} = \frac{-150(6)}{0.7854(30)10^6} = -3.82\left(10^{-5}\right) \text{ in}$$

4. The deflection due to the slope at B, θ_{B2}, due to the moment acting on B ($x_1 = \overline{BC}\,\theta_{B2} = 5\theta_{B2}$). With $I_{AB} = 0.04907$ in^4,

$$\theta_{B2} = \frac{M_B l}{EI} = \frac{5(-150)6}{30\left(10^6\right)0.04909} = -0.003056 \text{ rad}$$

$x_4 = 5(-0.003056) = -0.01528$ in

5. The deflection at C due to the bending force acting on C. With $I_{BC} = 0.001953$ in^4

$$x_5 = \left(-\frac{Fl^3}{3EI}\right)_{BC} = -\frac{150\left(5^3\right)}{3(30)10^6\left(0.001953\right)} = -0.10667 \text{ in}$$

6. The elongation of CD due to the tension. For CD, the area is $A = \pi(0.75^2)/4 = 0.4418$ in^2

$$x_6 = \left(\frac{Fl}{AE}\right)_{CD} = \frac{-150(2)}{0.4418(30)10^6} = -2.26\left(10^{-5}\right) \text{ in}$$

Summing the deflections results in

$$x_D = \sum_{i=1}^{6} x_i = -0.01019 - 0.04267 - 3.82\left(10^{-5}\right)$$
$$- 0.01528 - 0.10667 - 2.26\left(10^{-5}\right) = -0.1749 \text{ in} \quad \textit{Ans.}$$

4-43 $J_{OA} = J_{BC} = \pi(1.5^4)/32 = 0.4970$ in^4, $J_{AB} = \pi(1^4)/32 = 0.09817$ in^4, $I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, and $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4.

$$\theta = \left(\frac{Tl}{GJ}\right)_{OA} + \left(\frac{Tl}{GJ}\right)_{AB} + \left(\frac{Tl}{GJ}\right)_{BC} = \frac{T}{G}\left(\frac{l_{OA}}{J_{OA}} + \frac{l_{AB}}{J_{AB}} + \frac{l_{BC}}{J_{BC}}\right)$$

$$= \frac{250(12)}{11.5(10^6)}\left(\frac{2}{0.4970} + \frac{9}{0.09817} + \frac{2}{0.4970}\right) = 0.0260 \text{ rad} \qquad \textit{Ans.}$$

Simplified

$$\theta_s = \frac{Tl}{GJ} = \frac{250(12)(13)}{11.5\left(10^6\right)\left(0.09817\right)}$$

$\theta_s = 0.0345$ rad *Ans.*

Simplified is 0.0345/0.0260 = 1.33 times greater *Ans.*

$$y_D = \frac{F_y {l_{OC}}^3}{3EI_{AB}} + \theta_s\left(l_{CD}\right) + \frac{F_y {l_{CD}}^3}{3EI_{CD}} = \frac{250\left(13^3\right)}{3(30)10^6\left(0.04909\right)} + 0.0345(12) + \frac{250\left(12^3\right)}{3(30)10^6\left(0.01553\right)}$$

$y_D = 0.847$ in *Ans.*

4-44 Reverse the deflection equation of beam 7 of Table A-9. Using units in lbf, inches

$$y = -\frac{wx}{24EI}\left(2lx^2 - x^3 - l^3\right) = -\frac{(3000/12)x}{24(30)10^6(485)}\left\{2(25)x^2 - x^3 - \left[25(12)\right]^3\right\}$$

$$= 7.159\left(10^{-10}\right)x\left[27\left(10^6\right) - 600x^2 + x^3\right] \qquad \textit{Ans.}$$

The maximum height occurs at $x = 25(12)/2 = 150$ in

$$y_{\max} = 7.159\left(10^{-10}\right)150\left[27\left(10^6\right) - 600\left(150^2\right) + 150^3\right] = 1.812 \text{ in} \qquad \textit{Ans.}$$

4-45 From Table A-9-6,

$$y_L = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right)$$

$$y_L = \frac{Fb}{6EIl}\left(x^3 + b^2x - l^2x\right)$$

$$\frac{dy_L}{dx} = \frac{Fb}{6EIl}\left(3x^2 + b^2 - l^2\right)$$

$$\left.\frac{dy_L}{dx}\right|_{x=0} = \frac{Fb\left(b^2 - l^2\right)}{6EIl}$$

Let $\xi = \left.\dfrac{dy_L}{dx}\right|_{x=0}$ and set $I = \dfrac{\pi d_L^4}{64}$. Thus,

$$d_L = \left|\frac{32Fb\left(b^2 - l^2\right)}{3\pi El\xi}\right|^{1/4} \qquad \textit{Ans.}$$

For the other end view, observe beam 6 of Table A-9 from the back of the page, noting that a and b interchange as do x and $-x$

$$d_R = \left|\frac{32Fa\left(l^2 - a^2\right)}{3\pi El\xi}\right|^{1/4} \qquad \textit{Ans.}$$

For a uniform diameter shaft the necessary diameter is the larger of d_L and d_R.

4-46 The maximum slope will occur at the left bearing. Incorporating a design factor into the solution for d_L of Prob. 4-45,

$$d = \left[\frac{32nFb\left(l^2 - b^2\right)}{3\pi El\xi}\right]^{1/4}$$

$$d = \sqrt[4]{\frac{32(1.28)(3000)(200)\left(300^2 - 200^2\right)}{3\pi(207)10^3(300)(0.001)}}$$

$$d = 38.1 \text{ mm} \quad \textit{Ans.}$$

$$I = \frac{\pi\left(38.1^4\right)}{64} = 103.4\left(10^3\right) \text{ mm}^4$$

From Table A-9, beam 6, the maximum deflection will occur in BC where $dy_{BC}/dx = 0$

$$\frac{d}{dx}\left[\frac{Fa\left(l-x\right)}{6EIl}\left(x^2 + a^2 - 2lx\right)\right] = 0 \quad \Rightarrow \quad 3x^2 - 6lx + \left(a^2 + 2l^2\right) = 0$$

$$3x^2 - 6\left(300\right)x + \left[100^2 + 2\left(300^2\right)\right] = 0 \quad \Rightarrow \quad x^2 - 600x + 63333 = 0$$

$$x = \frac{1}{2}\left[600 \pm \sqrt{600^2 - 4(1)63\,333}\right] = 463.3, 136.7 \text{ mm}$$

$x = 136.7$ mm is acceptable.

$$y_{\max} = \left[\frac{Fa\left(l-x\right)}{6EIl}\left(x^2 + a^2 - 2lx\right)\right]_{x=136.7\,\text{mm}}$$

$$= \frac{3\left(10^3\right)100\left(300-136.7\right)}{6\left(207\right)10^3\left(103.4\right)10^3\left(300\right)}\left[136.7^2 + 100^2 - 2\left(300\right)136.7\right] = -0.0678 \text{ mm} \quad \textit{Ans.}$$

4-47 $I = \pi(1.25^4)/64 = 0.1198$ in^4. From Table A-9, beam 6

$$\delta = \sqrt{\left[\frac{F_1 a_1 (l-x)}{6EIl}(x^2 + a_1{}^2 - 2lx)\right]^2 + \left[\frac{F_2 b_2 x}{6EIl}(x^2 + b_2{}^2 - l^2)\right]^2}$$

$$= \left\{\left[\frac{150(5)(20-8)}{6(30)10^6\left(0.1198\right)(20)}\left(8^2 + 5^2 - 2(20)(8)\right)\right]^2 + \left[\frac{250(10)(8)}{6(30)10^6\left(0.1198\right)(20)}\left(8^2 + 10^2 - 20^2\right)\right]^2\right\}^{1/2}$$

$$= 0.0120 \text{ in} \quad \textit{Ans.}$$

4-48 $I = \pi(1.25^4)/64 = 0.1198$ in^4. For both forces use beam 6 of Table A-9.

For $F_1 = 150$ lbf:

$0 \le x \le 5$

$$y = \frac{F_1 b_1 x}{6EIl}\left(x^2 + b_1^2 - l^2\right) = \frac{150(15)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 15^2 - 20^2\right)$$

$$= 5.217\left(10^{-6}\right)x\left(x^2 - 175\right) \qquad (1)$$

$5 \le x \le 20$

$$y = \frac{F_1 a_1 (l - x)}{6EIl}\left(x^2 + a_1^2 - 2lx\right) = \frac{150(5)(20 - x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 5^2 - 2(20)x\right]$$

$$= 1.739\left(10^{-6}\right)(20 - x)\left(x^2 - 40x + 25\right) \qquad (2)$$

For $F_2 = 250$ lbf:

$0 \le x \le 10$

$$z = \frac{F_2 b_2 x}{6EIl}\left(x^2 + b_2^2 - l^2\right) = \frac{250(10)x}{6(30)10^6(0.1198)(20)}\left(x^2 + 10^2 - 20^2\right)$$

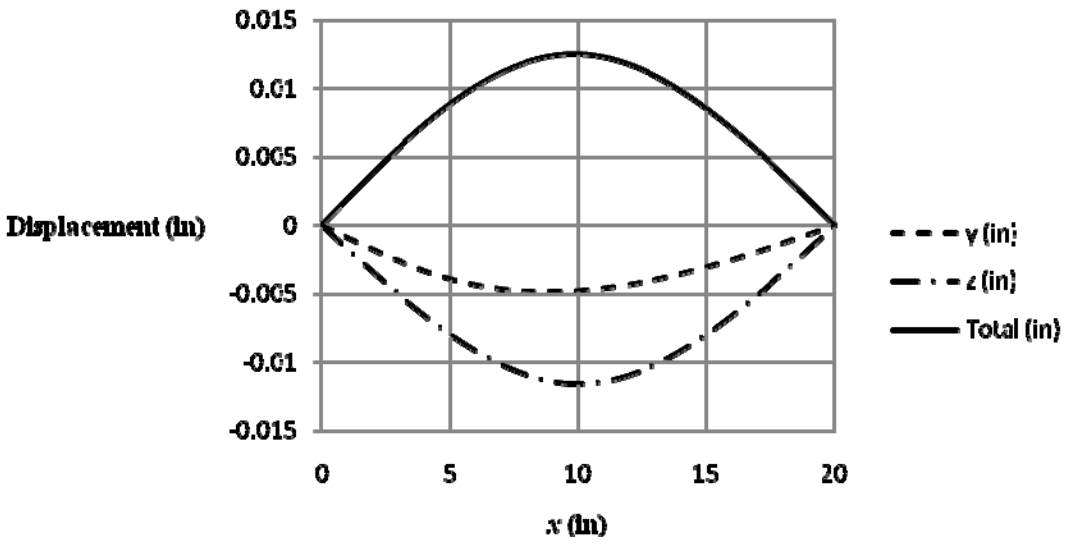
$$= 5.797\left(10^{-6}\right)x\left(x^2 - 300\right) \qquad (3)$$

$10 \le x \le 20$

$$z = \frac{F_2 a_2 (l - x)}{6EIl}\left(x^2 + a_2^2 - 2lx\right) = \frac{250(10)(20 - x)}{6(30)10^6(0.1198)(20)}\left[x^2 + 10^2 - 2(20)x\right]$$

$$= 5.797\left(10^{-6}\right)(20 - x)\left(x^2 - 40x + 100\right) \qquad (4)$$

Plot Eqs. (1) to (4) for each 0.1 in using a spreadsheet. There are 201 data points, too numerous to tabulate here but the plot is shown below, where the maximum deflection of $\delta = 0.01255$ in occurs at $x = 9.9$ in. *Ans.*



4-49 The larger slope will occur at the left end.
From Table A-9, beam 8

$$y_{AB} = \frac{M_B x}{6EIl}(x^2 + 3a^2 - 6al + 2l^2)$$

$$\frac{dy_{AB}}{dx} = \frac{M_B}{6EIl}(3x^2 + 3a^2 - 6al + 2l^2)$$

With $I = \pi d^4/64$, the slope at the left bearing is

$$\left.\frac{dy_{AB}}{dx}\right|_{x=0} = \theta_A = \frac{M_B}{6E\left(\pi d^4/64\right)l}(3a^2 - 6al + 2l^2)$$

Solving for d

$$d = \sqrt[4]{\frac{32M_B}{3\pi E\theta_A l}\left(3a^2 - 6al + 2l^2\right)} = \sqrt[4]{\frac{32(1000)}{3\pi(30)10^6(0.002)(10)}\left[3(4^2) - 6(4)(10) + 2\left(10^2\right)\right]}$$

$$= 0.461 \text{ in} \qquad \textit{Ans.}$$

4-50 From Table A-5, E = 10.4 Mpsi
$\Sigma M_O = 0 = 18\,F_{BC} - 6(100) \quad \Rightarrow \quad F_{BC} = 33.33$ lbf
The cross sectional area of rod BC is $A = \pi(0.5^2)/4 = 0.1963$ in^2.

The deflection at point B will be equal to the elongation of the rod BC.

$$y_B = \left(\frac{FL}{AE}\right)_{BC} = \frac{33.33(12)}{(0.1963)30\left(10^6\right)} = 6.79\left(10^{-5}\right) \text{ in} \qquad \textit{Ans.}$$

4-51 $\Sigma M_O = 0 = 6\,F_{AC} - 11(100) \quad \Rightarrow \quad F_{AC} = 183.3$ lbf

The deflection at point A in the negative y direction is equal to the elongation of the rod AC. From Table A-5, E_s = 30 Mpsi.

$$y_A = -\left(\frac{FL}{AE}\right)_{AC} = -\frac{183.3(12)}{\left[\pi\left(0.5^2\right)/4\right]30\left(10^6\right)} = -3.735\left(10^{-4}\right) \text{ in}$$

By similar triangles the deflection at B due to the elongation of the rod AC is

$$\frac{y_A}{6} = \frac{y_{B1}}{18} \quad \Rightarrow \quad y_{B1} = 3y_A = 3(-3.735)10^{-4} = -0.00112 \text{ in}$$

From Table A-5, E_a = 10.4 Mpsi

The bar can then be treated as a simply supported beam with an overhang AB. From Table A-9, beam 10

$$y_{B2} = \left(\overline{BD}\right)\left(\left.\frac{dy_{BC}}{dx}\right|_{x=l+a}\right) - \frac{Fa^2}{3EI}(l+a) = 7\left\{\frac{d}{dx}\left(\frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right)\right\}_{x=l+a} - \frac{Fa^2}{3EI}(l+a)$$

$$= 7\frac{F}{6EI}\left[3(x-l)^2 - 3a(x-l) - a(3x-l)\right]|_{x=l+a} - \frac{Fa^2}{3EI}(l+a) = -\frac{7Fa}{6EI}(2l+3a) - \frac{Fa^2}{3EI}(l+a)$$

$$= -\frac{7(100)5}{6(10.4)10^6\left(0.25(2^3)/12\right)}\left[2(6)+3(5)\right] - \frac{100\left(5^2\right)}{3(10.4)10^6\left(0.25(2^3)/12\right)}(6+5)$$

$$= -0.01438 \text{ in}$$

$y_B = y_{B1} + y_{B2} = -0.00112 - 0.01438 = -0.0155$ in *Ans.*

4-52 From Table A-5, $E = 207$ GPa, and $G = 79.3$ GPa.

$$|y_B| = \left(\frac{Tl}{GJ}\right)_{OC} l_{AB} + \left(\frac{Tl}{GJ}\right)_{AC} l_{AB} + \frac{Fl_{AB}^{\ 3}}{3EI_{AB}} = \frac{Fl_{OC}l_{AB}^{\ 2}}{G\left(\pi d_{OC}^{\ 4}/32\right)} + \frac{Fl_{AC}l_{AB}^{\ 2}}{G\left(\pi d_{AC}^{\ 4}/32\right)} + \frac{Fl_{AB}^{\ 3}}{3E\left(\pi d_3^{\ 4}/64\right)}$$

$$= \frac{32Fl_{AB}^{\ 2}}{\pi}\left[\frac{l_{OC}}{Gd_{OC}^{\ 4}} + \frac{l_{AC}}{Gd_{AC}^{\ 4}} + \frac{2l_{AB}}{3Ed_{AB}^{\ 4}}\right]$$

The spring rate is $k = F/|y_B|$. Thus

$$k = \left\{\frac{32l_{AB}^{\ 2}}{\pi}\left[\frac{l_{OC}}{Gd_{OC}^{\ 4}} + \frac{l_{AC}}{Gd_{AC}^{\ 4}} + \frac{2l_{AB}}{3Ed_{AB}^{\ 4}}\right]\right\}^{-1}$$

$$= \left\{\frac{32\left(200^2\right)}{\pi}\left[\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2(200)}{3(207)10^3\left(8^4\right)}\right]\right\}^{-1}$$

$$= 8.10 \text{ N/mm}$$ *Ans.*

4-53 For the beam deflection, use beam 5 of Table A-9.

$$R_1 = R_2 = \frac{F}{2}$$

$$\delta_1 = \frac{F}{2k_1}, \text{ and } \delta_2 = \frac{F}{2k_2}$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fx}{48EI}(4x^2 - 3l^3)$$

$$y_{AB} = F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{x}{48EI}(4x^2 - 3l^3)\right]$$ *Ans.*

For *BC*, since Table A-9 does not have an equation (because of symmetry) an equation will need to be developed as the problem is no longer symmetric. This can be done easily using beam 6 of Table A-9 with $a = l/2$

$$y_{BC} = \frac{-F}{2k_1} + \frac{Fk_2 - Fk_1}{2k_1k_2l}x + \frac{F(l/2)(l-x)}{EIl}\left(x^2 + \frac{l^2}{4} - 2lx\right)$$

$$= F\left[-\frac{1}{2k_1} + \frac{k_2 - k_1}{2k_1k_2l}x + \frac{(l-x)}{48EI}\left(4x^2 + l^2 - 8lx\right)\right] \qquad \textit{Ans.}$$

4-54

$$R_1 = \frac{Fa}{l}, \text{ and } R_2 = \frac{F}{l}(l+a)$$

$$\delta_1 = \frac{Fa}{lk_1}, \text{ and } \delta_2 = \frac{F}{lk_2}(l+a)$$

$$y_{AB} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{Fax}{6EIl}(l^2 - x^2)$$

$$y_{AB} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{ax}{6EIl}(l^2 - x^2)\right\} \qquad \textit{Ans.}$$

$$y_{BC} = -\delta_1 + \frac{\delta_1 - \delta_2}{l}x + \frac{F(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]$$

$$y_{BC} = F\left\{-\frac{a}{k_1l} + \frac{x}{k_1k_2l^2}\left[k_2a - k_1(l+a)\right] + \frac{(x-l)}{6EI}\left[(x-l)^2 - a(3x-l)\right]\right\} \qquad \textit{Ans.}$$

4-55 Let the load be at $x \geq l/2$. The maximum deflection will be in Section *AB* (Table A-9, beam 6)

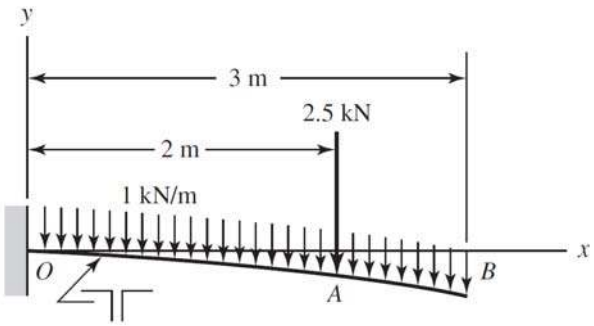
$$y_{AB} = \frac{Fbx}{6EIl}\left(x^2 + b^2 - l^2\right)$$

$$\frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}\left(3x^2 + b^2 - l^2\right) = 0 \qquad \Rightarrow \qquad 3x^2 + b^2 - l^2 = 0$$

$$x = \sqrt{\frac{l^2 - b^2}{3}}, \quad x_{\max} = \sqrt{\frac{l^2}{3}} = 0.577l \qquad \textit{Ans.}$$

For $x \leq l/2$, $x_{\min} = l - 0.577l = 0.423l$ *Ans.*

4-56



$$M_O = 1(3000)(1500) + 2500(2000)$$
$$= 9.5\left(10^6\right) \text{ N·mm}$$
$$R_O = 1(3000) + 2500 = 5\ 500 \text{ N}$$

From Prob. 4-10, $I = 4.14(10^6)$ mm^4

$$M = -9.5\left(10^6\right) + 5500x - \frac{x^2}{2} - 2500\langle x - 2000\rangle^1$$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000\rangle^2 + C_1$$

$\frac{dy}{dx} = 0$ at $x = 0$ $\quad \therefore C_1 = 0$

$$EI\frac{dy}{dx} = -9.5\left(10^6\right)x + 2750x^2 - \frac{x^3}{6} - 1250\langle x - 2000\rangle^2$$

$$EIy = -4.75\left(10^6\right)x^2 + 916.67x^3 - \frac{x^4}{24} - 416.67\langle x - 2000\rangle^3 + C_2$$

$y = 0$ at $x = 0$ $\quad \therefore C_2 = 0$, and therefore

$$y = -\frac{1}{24EI}\left[114\left(10^6\right)x^2 - 22\left(10^3\right)x^3 + x^4 + 10\left(10^3\right)\langle x - 2000\rangle^3\right]$$

$$y_B = -\frac{1}{24(207)10^3(4.14)10^6}\left[114\left(10^6\right)3000^2 - 22\left(10^3\right)3000^3 + 3000^4 + 10\left(10^3\right)(3000 - 2000)^3\right]$$

$= -25.4$ mm $\quad$ *Ans.*

$M_O = 9.5\ (10^6)$ N·m. The maximum stress is compressive at the bottom of the beam where $y = 29.0 - 100 = -71$ mm

$$\sigma_{\max} = -\frac{My}{I} = -\frac{-9.5\left(10^6\right)(-71)}{4.14(10^6)} = -163\left(10^6\right)\text{Pa} = -163\text{MPa} \quad \textit{Ans.}$$

The solutions are the same as Prob. 4-10.

4-57 See Prob. 4-11 for reactions: $R_O = 465$ lbf and $R_C = 285$ lbf. Using lbf and inch units

$$M = 465\,x - 450\,\langle x - 72\rangle^1 - 300\,\langle x - 120\rangle^1$$

$$EI\frac{dy}{dx} = 232.5x^2 - 225\left\langle x - 72\right\rangle^2 - 150\left\langle x - 120\right\rangle^2 + C_1$$

$$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - C_1 x$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0$ at $x = 240$ in

$0 = 77.5(240^3) - 75(240 - 72)^3 - 50(240 - 120)^3 + C_1\,x \quad \Rightarrow \quad C_1 = -2.622(10^6)$ lbf·in^2

and,

$$EIy = 77.5\,x^3 - 75\,\langle x - 72\rangle^3 - 50\,\langle x - 120\rangle^3 - 2.622(10^6)\,x$$

Substituting $y = -0.5$ in at $x = 120$ in gives

$$30(10^6)\,I\,(-0.5) = 77.5\,(120^3) - 75(120 - 72)^3 - 50(120 - 120)^3 - 2.622(10^6)(120)$$

$$I = 12.60 \text{ in}^4$$

Select two 5 in × 6.7 lbf/ft channels; from Table A-7, $I = 2(7.49) = 14.98$ in^4

$$y_{\text{midspan}} = \frac{12.60}{14.98}\left(-\frac{1}{2}\right) = -0.421 \text{ in} \qquad \textit{Ans.}$$

The maximum moment occurs at $x = 120$ in where $M_{\max} = 34.2(10^3)$ lbf·in

$$\sigma_{\max} = \frac{Mc}{I} = \frac{34.2(10^3)(2.5)}{14.98} = 5\,710 \text{ psi} \qquad \text{O.K.}$$

The solutions are the same as Prob. 4-17.

4-58 $I = \pi(1.5^4)/64 = 0.2485$ in^4, and $w = 150/12 = 12.5$ lbf/in.

$$R_O = \frac{1}{2}(12.5)39 + \frac{24}{39}(340) = 453.0 \text{ lbf}$$

$$M = 453.0x - \frac{12.5}{2}x^2 - 340\left\langle x - 15\right\rangle^1$$

$$EI\frac{dy}{dx} = 226.5x^2 - \frac{12.5}{6}x^3 - 170\left\langle x - 15\right\rangle^2 + C_1$$

$$EIy = 75.5x^3 - 0.5208x^4 - 56.67\left\langle x - 15\right\rangle^3 + C_1 x + C_2$$

$y = 0 \text{ at } x = 0 \quad \Rightarrow \quad C_2 = 0$

$y = 0$ at $x = 39$ in $\quad \Rightarrow \quad C_1 = -6.385(10^4)$ lbf · in^2 Thus,

$$y = \frac{1}{EI}\left[75.5x^3 - 0.5208x^4 - 56.67\left\langle x - 15\right\rangle^3 - 6.385\left(10^4\right)x\right]$$

Evaluating at $x = 15$ in,

$$y_A = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(15^3\right) - 0.5208\left(15^4\right) - 56.67\left(15-15\right)^3 - 6.385\left(10^4\right)(15)\right]$$

$$= -0.0978 \text{ in} \quad \textit{Ans.}$$

$$y_{\text{midspan}} = \frac{1}{30(10^6)(0.2485)}\left[75.5\left(19.5^3\right) - 0.5208\left(19.5^4\right) - 56.67\left(19.5-15\right)^3 - 6.385\left(10^4\right)(19.5)\right]$$

$$= -0.1027 \text{ in} \quad \textit{Ans.}$$

5 % difference *Ans.*

The solutions are the same as Prob. 4-12.

4-59 $I = 0.05 \text{ in}^4$, $R_A = \dfrac{3(14)100}{10} = 420 \text{ lbf} \uparrow$ and $R_B = \dfrac{7(14)100}{10} = 980 \text{ lbf} \uparrow$

$M = 420\,x - 50\,x^2 + 980\,\langle x - 10 \rangle^1$

$$EI\frac{dy}{dx} = 210x^2 - 16.667x^3 + 490\left\langle x-10\right\rangle^2 + C_1$$

$$EIy = 70x^3 - 4.167x^4 + 163.3\left\langle x-10\right\rangle^3 + C_1x + C_2$$

$y = 0$ at $x = 0 \;\Rightarrow\; C_2 = 0$

$y = 0$ at $x = 10$ in $\;\Rightarrow\; C_1 = -2\,833 \text{ lbf}\cdot\text{in}^2$. Thus,

$$y = \frac{1}{30\left(10^6\right)0.05}\left[70x^3 - 4.167x^4 + 163.3\left\langle x-10\right\rangle^3 - 2833x\right]$$

$$= 6.667\left(10^{-7}\right)\left[70x^3 - 4.167x^4 + 163.3\left\langle x-10\right\rangle^3 - 2833x\right] \qquad \textit{Ans.}$$

The tabular results and plot are exactly the same as Prob. 4-21.

4-60 $R_A = R_B = 400$ N, and $I = 6(32^3)/12 = 16\,384 \text{ mm}^4$.

First half of beam,

$M = -400\,x + 400\,\langle x - 300 \rangle^1$

$$EI\frac{dy}{dx} = -200x^2 + 200\left\langle x-300\right\rangle^2 + C_1$$

From symmetry, $dy/dx = 0$ at $x = 550$ mm $\quad\Rightarrow\quad 0 = -200(550^2) + 200(550 - 300)^2 + C_1$

$\Rightarrow\quad C_1 = 48(10^6) \text{ N}\cdot\text{mm}^2$

$EIy = -66.67\,x^3 + 66.67\,\langle x - 300 \rangle^3 + 48(10^6)\,x + C_2$

$y = 0$ at $x = 300$ mm $\Rightarrow$ $C_2 = -12.60(10^9)$ N·mm^3.

The term $(EI)^{-1} = [207(10^3)16\,384]^{-1} = 2.949\,(10^{-10})$ Thus

$y = 2.949\,(10^{-10})\,[-66.67\,x^3 + 66.67\,\langle x - 300\rangle^3 + 48(10^6)\,x - 12.60(10^9)]$

$y_O = -3.72$ mm *Ans.*

$y|_{x=550\text{ mm}} = 2.949\,(10^{-10})\,[-66.67\,(550^3) + 66.67\,(550 - 300)^3 + 48(10^6)\,550 - 12.60(10^9)] = 1.11$ mm *Ans.*

The solutions are the same as Prob. 4-13.

4-61

$$\sum M_B = 0 = R_1 l + Fa - M_A \quad \Rightarrow \quad R_1 = \frac{1}{l}(M_A - Fa)$$

$$\sum M_A = 0 = M_A + R_2 l - F(l + a) \quad \Rightarrow \quad R_2 = \frac{1}{l}(Fl + Fa - M_A)$$

$$M = R_1 x - M_A + R_2 \langle x - l\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_1 x^2 - M_A x + \frac{1}{2}R_2\langle x - l\rangle^2 + C_1$$

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2\langle x - l\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0$ $\Rightarrow$ $C_2 = 0$

$y = 0$ at $x = l$ $\Rightarrow$ $C_1 = -\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l$. Thus,

$$EIy = \frac{1}{6}R_1 x^3 - \frac{1}{2}M_A x^2 + \frac{1}{6}R_2\langle x - l\rangle^3 + \left(-\frac{1}{6}R_1 l^2 + \frac{1}{2}M_A l\right)x$$

$$y = \frac{1}{6EIl}\left[(M_A - Fa)x^3 - 3M_A x^2 l + (Fl + Fa - M_A)\langle x - l\rangle^3 + (Fal^2 + 2M_A l^2)x\right] \qquad \textit{Ans.}$$

In regions,

$$y_{AB} = \frac{1}{6EIl}\left[(M_A - Fa)x^3 - 3M_A x^2 l + (Fal^2 + 2M_A l^2)x\right]$$

$$= \frac{x}{6EIl}\left[M_A(x^2 - 3lx + 2l^2) + Fa(l^2 - x^2)\right] \qquad \textit{Ans.}$$

$$y_{BC} = \frac{1}{6EIl}\left[\left(M_A - Fa\right)x^3 - 3M_A x^2 l + \left(Fl + Fa - M_A\right)\left(x-l\right)^3 + \left(Fal^2 + 2M_A l^2\right)x\right]$$

$$= \frac{1}{6EIl}\left\{M_A\left[x^3 - 3x^2 l - \left(x-l\right)^3 + 2xl^2\right] + F\left[-ax^3 + \left(l+a\right)\left(x-l\right)^3 + axl^2\right]\right\}$$

$$= \frac{1}{6EIl}\left\{-M_A\left(x-l\right)l^2 + Fl\left(x-l\right)\left[\left(x-l\right)^2 - a\left(3x-l\right)\right]\right\}$$

$$= \frac{\left(x-l\right)}{6EI}\left\{-M_A l + F\left[\left(x-l\right)^2 - a\left(3x-l\right)\right]\right\} \qquad \textit{Ans.}$$

The solutions reduce to the same as Prob. 4-17.

4-62 $\sum M_D = 0 = R_1 l - w\left(b-a\right)\left[l - b + \frac{1}{2}\left(b-a\right)\right] \quad \Rightarrow \quad R_1 = \frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)$

$$M = R_1 x - \frac{w}{2}\left\langle x-a\right\rangle^2 + \frac{w}{2}\left\langle x-b\right\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_1 x^2 - \frac{w}{6}\left\langle x-a\right\rangle^3 + \frac{w}{6}\left\langle x-b\right\rangle^3 + C_1$$

$$EIy = \frac{1}{6}R_1 x^3 - \frac{w}{24}\left\langle x-a\right\rangle^4 + \frac{w}{24}\left\langle x-b\right\rangle^4 + C_1 x + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$
$y = 0$ at $x = l$

$$C_1 = -\frac{1}{l}\left[\frac{1}{6}R_1 l^3 - \frac{w}{24}\left(l-a\right)^4 + \frac{w}{24}\left(l-b\right)^4\right]$$

$$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)x^3 - \frac{w}{24}\left\langle x-a\right\rangle^4 + \frac{w}{24}\left\langle x-b\right\rangle^4 - x\frac{1}{l}\left[\frac{1}{6}\frac{w\left(b-a\right)}{2l}\left(2l-b-a\right)l^3 - \frac{w}{24}\left(l-a\right)^4 + \frac{w}{24}\left(l-b\right)^4\right]\right\}$$

$$= \frac{w}{24EIl}\left\{2\left(b-a\right)\left(2l-b-a\right)x^3 - l\left\langle x-a\right\rangle^4 + l\left\langle x-b\right\rangle^4 - x\left[2\left(b-a\right)\left(2l-b-a\right)l^2 - \left(l-a\right)^4 + \left(l-b\right)^4\right]\right\} \qquad \textit{Ans.}$$

The above answer is sufficient. In regions,

$$y_{AB} = \frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3 - x\left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4\right]\right\}$$
$$= \frac{wx}{24EIl}\left[2(b-a)(2l-b-a)x^2 - 2(b-a)(2l-b-a)l^2 + (l-a)^4 - (l-b)^4\right]$$

$$y_{BC} = \frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3 - l(x-a)^4 - x\left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4\right]\right\}$$

$$y_{CD} = \frac{w}{24EIl}\left\{2(b-a)(2l-b-a)x^3 - l(x-a)^4 + l(x-b)^4 - x\left[2(b-a)(2l-b-a)l^2 - (l-a)^4 + (l-b)^4\right]\right\}$$

These equations can be shown to be equivalent to the results found in Prob. 4-19.

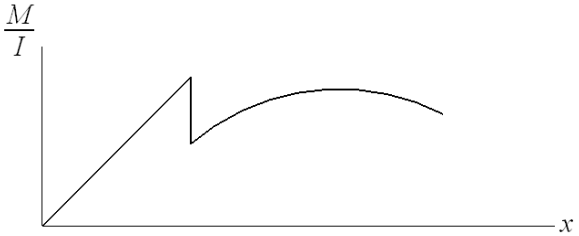
4-63 $I_1 = \pi(1.375^4)/64 = 0.1755$ in^4, $I_2 = \pi(1.75^4)/64 = 0.4604$ in^4,

$R_1 = 0.5(180)(10) = 900$ lbf

Since the loading and geometry are symmetric, we will only write the equations for half the beam

For $\underline{0 \le x \le 8 \text{ in}}$ $\quad M = 900x - 90\langle x-3\rangle^2$

At $x = 3$, $M = 2700$ lbf·in



Writing an equation for M / I, as seen in the figure, the magnitude and slope reduce since $I_2 > I_1$. To reduce the magnitude at $x = 3$ in, we add the term, $-2700(1/I_1 - 1/I_2)\langle x-3\rangle^0$. The slope of 900 at $x = 3$ in is also reduced. We account for this with a ramp function, $\langle x-3\rangle^1$. Thus,

$$\frac{M}{I} = \frac{900x}{I_1} - 2700\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-3\rangle^0 - 900\left(\frac{1}{I_1} - \frac{1}{I_2}\right)\langle x-3\rangle^1 - \frac{90}{I_2}\langle x-3\rangle^2$$
$$= 5128x - 9520\langle x-3\rangle^0 - 3173\langle x-3\rangle^1 - 195.5\langle x-3\rangle^2$$

$$E\frac{dy}{dx} = 2564x^2 - 9520\langle x-3\rangle^1 - 1587\langle x-3\rangle^2 - 65.17\langle x-3\rangle^3 + C_1$$

Boundary Condition: $\frac{dy}{dx} = 0$ at $x = 8$ in

$$0 = 2564(8)^2 - 9520(8-3) - 1587(8-3)^2 - 65.17(8-3)^3 + C_1 \Rightarrow$$
$$C_1 = -68.67\ (10^3)\ \text{lbf/in}^2$$

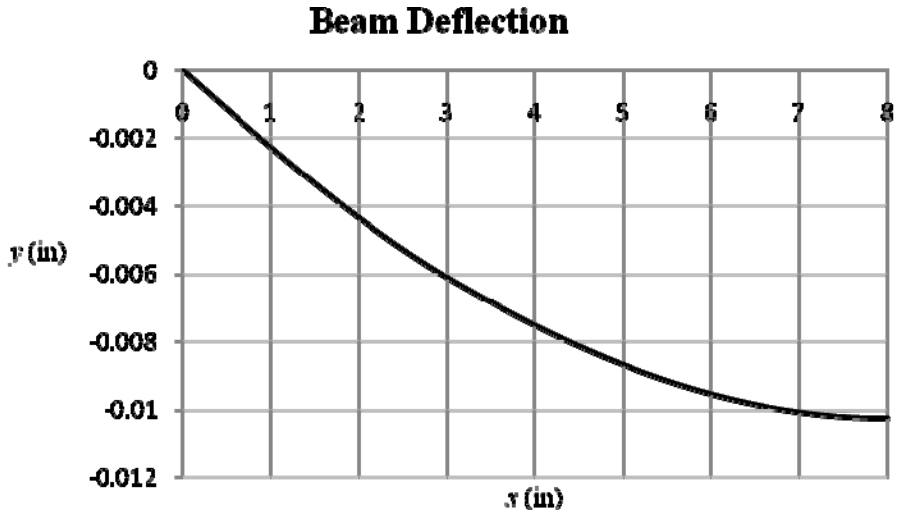
$$Ey = 854.7x^3 - 4760\langle x-3\rangle^2 - 529\langle x-3\rangle^3 - 16.29\langle x-3\rangle^4 - 68.67(10^3)x + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

Thus, for $\underline{0 \le x \le 8 \text{ in}}$

$$y = \frac{1}{30(10^6)}\left[854.7x^3 - 4760\langle x-3\rangle^2 - 529\langle x-3\rangle^3 - 16.29\langle x-3\rangle^4 - 68.7(10^3)x\right] \quad \textit{Ans.}$$

Using a spreadsheet, the following graph represents the deflection equation found above



The maximum is $y_{max} = -0.0102$ in at $x = 8$ in *Ans.*

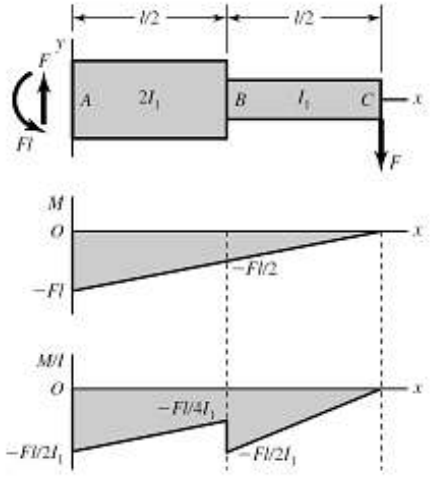
4-64 The force and moment reactions at the left support are F and Fl respectively. The bending moment equation is

$$M = Fx - Fl$$

Plots for M and M/I are shown.

M/I can be expressed using singularity functions

$$\frac{M}{I} = \frac{F}{2I_1}x - \frac{Fl}{2I_1} - \frac{Fl}{4I_1}\left\langle x - \frac{l}{2}\right\rangle^0 + \frac{F}{2I_1}\left\langle x - \frac{l}{2}\right\rangle^1$$



where the step down and increase in slope at $x = l/2$ are given by the last two terms. Integrate

$$E\frac{dy}{dx} = \frac{F}{4I_1}x^2 - \frac{Fl}{2I_1}x - \frac{Fl}{4I_1}\left\langle x-\frac{l}{2}\right\rangle^1 + \frac{F}{4I_1}\left\langle x-\frac{l}{2}\right\rangle^2 + C_1$$

$dy/dx = 0$ at $x = 0 \quad \Rightarrow \quad C_1 = 0$

$$Ey = \frac{F}{12I_1}x^3 - \frac{Fl}{4I_1}x^2 - \frac{Fl}{8I_1}\left\langle x-\frac{l}{2}\right\rangle^2 + \frac{F}{12I_1}\left\langle x-\frac{l}{2}\right\rangle^3 + C_2$$

$y = 0$ at $x = 0 \quad \Rightarrow \quad C_2 = 0$

$$y = \frac{F}{24EI_1}\left(2x^3 - 6lx^2 - 3l\left\langle x-\frac{l}{2}\right\rangle^2 + 2\left\langle x-\frac{l}{2}\right\rangle^3\right)$$

$$y\big|_{x=l/2} = \frac{F}{24EI_1}\left[2\left(\frac{l}{2}\right)^3 - 6l\left(\frac{l}{2}\right)^2 - 3l(0) + 2(0)\right] = -\frac{5Fl^3}{96EI_1} \qquad \textit{Ans.}$$

$$y\big|_{x=l} = \frac{F}{24EI_1}\left[2(l)^3 - 6l\left(l^2\right) - 3l\left(l-\frac{l}{2}\right)^2 + 2\left(x-\frac{l}{2}\right)^3\right] = -\frac{3Fl^3}{16EI_1} \qquad \textit{Ans.}$$

The answers are identical to Ex. 4-10.

4-65 Place a dummy force, Q, at the center. The reaction, $R_1 = wl/2 + Q/2$

$$M = \left(\frac{wl}{2} + \frac{Q}{2}\right)x - \frac{wx^2}{2} \qquad \frac{\partial M}{\partial Q} = \frac{x}{2}$$

Integrating for half the beam and doubling the results

$$y_{\max} = \left(2\frac{1}{EI}\int_0^{l/2} M\left(\frac{\partial M}{\partial Q}\right)dx\right)_{Q=0} = \frac{2}{EI}\int_0^{l/2}\left[\left(\frac{wl}{2}\right)x - \frac{wx^2}{2}\right]\left(\frac{x}{2}\right)dx$$

Note, after differentiating with respect to Q, it can be set to zero

$$y_{\max} = \frac{w}{2EI}\int_0^{l/2} x^2(l-x)dx = \frac{w}{2EI}\left(\frac{x^3 l}{3} - \frac{x^4}{4}\right)\Bigg|_0^{l/2} = \frac{5w}{384EI} \qquad \textit{Ans.}$$

4-66 Place a fictitious force Q pointing downwards at the end. Use the variable $\bar{x}$ originating at the free end at positive to the left

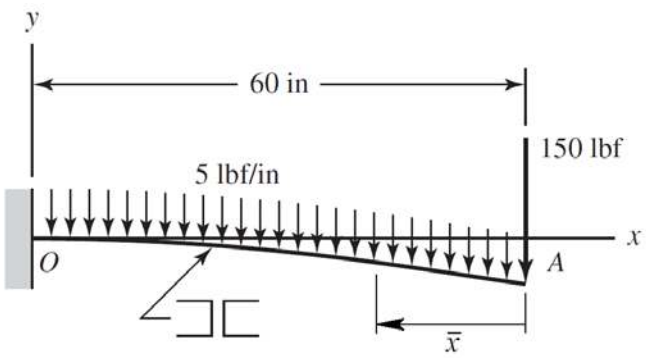
$$M = -Q\bar{x} - \frac{w\bar{x}^2}{2} \qquad \frac{\partial M}{\partial Q} = -\bar{x}$$

$$y_{\max} = \left[\frac{1}{EI} \int_0^l M \left(\frac{\partial M}{\delta Q} \right) dx \right]_{Q=0} = \frac{1}{EI} \int_0^l \left(-\frac{wx^2}{2} \right)(-x)\, dx = \frac{w}{2EI} \int_0^l x^3 dx$$

$$= \frac{wl^4}{8EI} \qquad Ans.$$

4-67 From Table A-7, $I_{1\text{-}1} = 1.85$ in^4. Thus, $I = 2(1.85) = 3.70$ in^4

First treat the end force as a variable, F. Adding weight of channels of 2(5)/12 = 0.833 lbf/in. Using the variable $\bar{x}$ as shown in the figure



$$M = -F\bar{x} - \frac{5.833}{2}\bar{x}^2 = -F\bar{x} - 2.917\bar{x}^2$$

$$\frac{\partial M}{\partial F} = -\bar{x}$$

$$\delta_A = \frac{1}{EI}\int_0^{60} M \frac{\partial M}{\partial F}\, d\bar{x} = \frac{1}{EI}\int_0^{60} (F\bar{x} + 2.917\bar{x}^2)(\bar{x})\, d\bar{x}$$

$$= \frac{(150/3)(60^3) + (2.917/4)(60^4)}{30(10^6)(3.70)} = 0.182 \text{ in} \quad \text{in the direction of the 150 lbf force}$$

$$\therefore y_A = -0.182 \text{ in} \qquad Ans.$$

4-68 The energy includes torsion in AC, torsion in CO, and bending in AB.

Neglecting transverse shear in AB

$$M = Fx, \quad \frac{\partial M}{\partial F} = x$$

In AC and CO,

$$T = Fl_{AB}, \quad \frac{\partial T}{\partial F} = l_{AB}$$

The total energy is

$$U = \left(\frac{T^2 l}{2GJ} \right)_{AC} + \left(\frac{T^2 l}{2GJ} \right)_{CO} + \int_0^{l_{AB}} \frac{M^2}{2EI_{AB}} dx$$

The deflection at the tip is

$$\delta = \frac{\partial U}{\partial F} = \frac{Tl_{AC}}{GJ_{AC}}\frac{\partial T}{\partial F} + \frac{Tl_{CO}}{GJ_{CO}}\frac{\partial T}{\partial F} + \int_0^{l_{AB}} \frac{M}{EI_3}\frac{\partial M}{\partial F}dx = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{1}{EI_{AB}}\int_0^{l_{AB}} Fx^2 dx$$

$$\delta = \frac{Tl_{AC}l_{AB}}{GJ_{AC}} + \frac{Tl_{CO}l_{AB}}{GJ_{CO}} + \frac{Fl_{AB}^3}{3EI_{AB}} = \frac{Fl_{AC}l_{AB}^2}{G\left(\pi d_{AC}^4/32\right)} + \frac{Fl_{CO}l_{AB}^2}{G\left(\pi d_{CO}^4/32\right)} + \frac{Fl_{AB}^3}{3E\left(\pi d_{AB}^4/64\right)}$$

$$= \frac{32Fl_{AB}^2}{\pi}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)$$

$$k = \frac{F}{\delta} = \frac{\pi}{32l_{AB}^2}\left(\frac{l_{AC}}{Gd_{AC}^4} + \frac{l_{CO}}{Gd_{CO}^4} + \frac{2l_{AB}}{3Ed_{AB}^4}\right)^{-1}$$

$$= \frac{\pi}{32\left(200^2\right)}\left(\frac{200}{79.3\left(10^3\right)18^4} + \frac{200}{79.3\left(10^3\right)12^4} + \frac{2\left(200\right)}{3\left(207\right)10^3\left(8^4\right)}\right)^{-1} = 8.10 \text{ N/mm} \quad \textit{Ans.}$$

4-69 $I_1 = \pi(1.375^4)/64 = 0.1755$ in^4, $I_2 = \pi(1.75^4)/64 = 0.4604$ in^4

Place a fictitious force Q pointing downwards at the midspan of the beam, $x = 8$ in

$$R_1 = \frac{1}{2}(10)180 + \frac{1}{2}Q = 900 + 0.5Q$$

For $\underline{0 \le x \le 3 \text{ in}}$ $\quad M = \left(900 + 0.5Q\right)x \qquad \dfrac{\partial M}{\partial Q} = 0.5x$

For $\underline{3 \le x \le 13 \text{ in}}$ $\quad M = \left(900 + 0.5Q\right)x - 90(x-3)^2 \qquad \dfrac{\partial M}{\partial Q} = 0.5x$

By symmetry it is equivalent to use twice the integral from 0 to 8

$$\delta = \left(2\int_0^8 \frac{M}{EI}\frac{\partial M}{\partial Q}dx\right)_{Q=0} = \frac{1}{EI_1}\int_0^3 900x^2 dx + \frac{1}{EI_2}\int_3^8 \left[900x - 90\left(x-3\right)^2\right]x\,dx$$

$$= \frac{300x^3}{EI_1}\bigg|_0^3 + \frac{1}{EI_2}\left[300x^3 - 90\left(\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right)\right]\bigg|_3^8$$

$$= \frac{8100}{EI_1} + \frac{1}{EI_2}\left[145.5\left(10^3\right) - 25.31\left(10^3\right)\right] = \frac{8100}{30\left(10^6\right)0.1755} + \frac{120.2\left(10^3\right)}{30\left(10^6\right)0.4604}$$

$$= 0.0102 \text{ in} \qquad \textit{Ans.}$$

4-70 $I = \pi(0.5^4)/64 = 3.068\ (10^{-3})$ in^4, $J = 2I = 6.136\ (10^{-3})$ in^4, $A = \pi(0.5^2)/4 = 0.1963$ in^2.

Consider x to be in the direction of OA, y vertically upward, and z in the direction of AB. Resolve the force F into components in the x and y directions obtaining 0.6 F in the horizontal direction and 0.8 F in the negative vertical direction. The 0.6 F force creates strain energy in the form of bending in AB and OA, and tension in OA. The 0.8 F force creates strain energy in the form of bending in AB and OA, and torsion in OA. Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

$\underline{0.6\ F}$: AB $\quad M = 0.6F\,\bar{x} \quad \dfrac{\partial M}{\partial F} = 0.6\,\bar{x}$

OA $\quad M = 4.2F \quad \dfrac{\partial M}{\partial F} = 4.2$

$\quad F_a = 0.6F \quad \dfrac{\partial F_a}{\partial F} = 0.6$

$\underline{0.8\ F}$: AB $\quad M = 0.8F\,\bar{x} \quad \dfrac{\partial M}{\partial F} = 0.8\,\bar{x}$

OA $\quad M = 0.8F\,\bar{x} \quad \dfrac{\partial M}{\partial F} = 0.8\,\bar{x}$

$\quad T = 5.6F \quad \dfrac{\partial T}{\partial F} = 5.6$

Once the derivatives are taken the value of F = 15 lbf can be substituted in. The deflection of B in the direction of F is*

$$(\delta_B)_F = \frac{\partial U}{\partial F} = \left(\frac{F_a L}{AE}\right)_{OA}\frac{\partial F_a}{\partial F} + \left(\frac{TL}{JG}\right)_{OA}\frac{\partial T}{\partial F} + \frac{1}{EI}\sum\int M\,\frac{\partial M}{\partial F}\,d\bar{x}$$

$$= \frac{0.6(15)15}{0.1963(30)10^6}(0.6) + \frac{5.6(15)15}{6.136(10^{-3})11.5(10^6)}(5.6)$$

$$+ \frac{15}{30(10^6)3.068(10^{-3})}\int_0^7 (0.6\bar{x})^2\,d\bar{x} + \frac{15(4.2^2)}{30(10^6)3.068(10^{-3})}\int_0^{15} d\bar{x} +$$

$$+ \frac{15}{30(10^6)3.068(10^{-3})}\int_0^7 (0.8\bar{x})^2\,d\bar{x} + \frac{15}{30(10^6)3.068(10^{-3})}\int_0^{15} (0.8\bar{x})^2\,d\bar{x}$$

$$= 1.38(10^{-5}) + 0.1000 + 6.71(10^{-3}) + 0.0431 + 0.0119 + 0.1173$$

$$= 0.279 \text{ in} \qquad \textit{Ans.}$$

*Note. This is not the actual deflection of point *B*. For this, dummy forces must be placed on *B* in the *x*, *y*, and *z* directions. Determine the energy due to each, take derivatives, and then substitute the values of $F_x = 9$ lbf, $F_y = -12$ lbf, and $F_z = 0$. This can be done separately and then use superposition. The actual deflections of *B* are

$$\boldsymbol{\delta_B} = 0.0831\,\mathbf{i} - 0.2862\,\mathbf{j} - 0.00770\,\mathbf{k} \text{ in}$$

From this, the deflection of *B* in the direction of *F* is

$$(\delta_B)_F = 0.6(0.0831) + 0.8(0.2862) = 0.279 \text{ in}$$

which agrees with our result.

4-71 Strain energy. *AB*: Bending and torsion, *BC*: Bending and torsion, *CD*: Bending.
$I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, $J_{AB} = 2\,I_{AB} = 0.09818$ in^4, $I_{BC} = 0.25(1.5^3)/12 = 0.07031$ in^4, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4.
For the torsion of bar *BC*, Eq. (3-41) is in the form of $\theta = TL/(JG)$, where the equivalent of J is $J_{\text{eq}} = \beta bc^3$. With $b/c = 1.5/0.25 = 6$, $J_{BC} = \beta bc^3 = 0.299(1.5)0.25^3 = 7.008\,(10^{-3})$ in^4.

Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

AB: Bending $\quad M = F\bar{x} + 2F \qquad \dfrac{\partial M}{\partial F} = \bar{x} + 2$

Torsion $\quad T = 5F \qquad \dfrac{\partial T}{\partial F} = 5$

BC: Bending $\quad M = F\bar{x} \qquad \dfrac{\partial M}{\partial F} = \bar{x}$

Torsion $\quad T = 2F \qquad \dfrac{\partial T}{\partial F} = 2$

CD: Bending $\quad M = F\bar{x} \qquad \dfrac{\partial M}{\partial F} = \bar{x}$

$$\delta_D = \frac{\partial U}{\partial F} = \sum \frac{Tl}{JG}\frac{\partial T}{\partial F} + \sum \frac{1}{EI}\int M\frac{\partial M}{\partial F}\,d\bar{x}$$

$$= \frac{5F(6)}{0.09818(11.5)10^6}(5) + \frac{2F(5)}{7.008(10^{-3})11.5(10^6)}2 + \frac{1}{30(10^6)0.04909}\int_0^6 F(\bar{x}+2)^2\,d\bar{x}$$

$$+ \frac{1}{30(10^6)0.07031}\int_0^5 F\bar{x}^2\,d\bar{x} + \frac{1}{30(10^6)0.01553}\int_0^2 F\bar{x}^2\,d\bar{x}$$

$$= 1.329(10^{-4})F + 2.482(10^{-4})F + 1.141(10^{-4})F + 1.98(10^{-5})F + 5.72(10^{-6})F$$

$$= 5.207(10^{-4})F = 5.207(10^{-4})200 = 0.104 \text{ in} \qquad \textit{Ans.}$$

4-72 $A_{AB} = \pi(1^2)/4 = 0.7854$ in^2, $I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, $I_{BC} = 1.5\,(0.25^3)/12 = 1.953\,(10^{-3})$ in^4, $A_{CD} = \pi(0.75^2)/4 = 0.4418$ in^2, $I_{AB} = \pi(0.75^4)/64 = 0.01553$ in^4. For $(\delta_D)_x$ let $F = F_x = -150$ lbf and $F_z = -100$ lbf . Use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

CD: $M_y = F_z\bar{x}$ $\quad \dfrac{\partial M_y}{\partial F} = 0$

$F_a = F$ $\quad \dfrac{\partial F_a}{\partial F} = 1$

BC: $M_y = F\,\bar{x} + 2F_z$ $\quad \dfrac{\partial M_y}{\partial F} = \bar{x}$

$F_a = F_z$ $\quad \dfrac{\partial F_a}{\partial F} = 0$

AB: $M_y = 5F + 2F_z + F_z\bar{x}$ $\quad \dfrac{\partial M_y}{\partial F} = 5$

$F_a = F$ $\quad \dfrac{\partial F_a}{\partial F} = 1$

$$(\delta_D)_x = \frac{\partial U}{\partial F} = \left(\frac{FL}{AE}\right)_{CD}\frac{\partial F_a}{\partial F} + \frac{1}{EI_{BC}}\int_0^5 (F\,\bar{x} + 2F_z)\,\bar{x}\,d\,\bar{x}$$

$$+ \frac{1}{EI_{AB}}\int_0^6 (5F + 2F_z + F_z\bar{x})(5)\,d\,\bar{x} + \left(\frac{FL}{AE}\right)_{AB}\frac{\partial F_a}{\partial F}$$

$$= \frac{F(2)}{0.4418(30)10^6}(1) + \frac{1}{30(10^6)1.953(10^{-3})}\left[\frac{F}{3}(5)^3 + F_z(5^2)\right]$$

$$+ \frac{1}{30(10^6)0.04909}\left[25F(6) + 10F_z(6) + \frac{F_z}{2}(6^2)5\right] + \frac{F(6)}{0.7854(30)10^6}(1)$$

$$= 1.509(10^{-7})F + 7.112(10^{-4})F + 4.267(10^{-4})F_z + 1.019(10^{-4})F$$

$$+ 1.019(10^{-4})F_z + 2.546(10^{-7})F = 8.135(10^{-4})F + 5.286(10^{-4})F_z$$

Substituting $F = F_x = -150$ lbf and $F_z = -100$ lbf gives

$$(\delta_D)_x = 8.135(10^{-4})(-150) + 5.286(10^{-4})(-100) = -0.1749 \text{ in} \qquad \textit{Ans.}$$

4-73 $I_{OA} = I_{BC} = \pi(1.5^4)/64 = 0.2485$ in^4, $J_{OA} = J_{BC} = 2\,I_{OA} = 0.4970$ in^4, $I_{AB} = \pi(1^4)/64 = 0.04909$ in^4, $J_{AB} = 2\,I_{AB} = 0.09818$ in^4, $I_{CD} = \pi(0.75^4)/64 = 0.01553$ in^4

Let $F_y = F$, and use the dummy variable $\bar{x}$ to originate at the end where the loads are applied on each segment,

OC: $\quad M = F\,\overline{x} \quad \dfrac{\partial M}{\partial F} = \overline{x}, \qquad T = 12F \quad \dfrac{\partial T}{\partial F} = 12$

DC: $\quad M = F\,\overline{x} \quad \dfrac{\partial M}{\partial F} = \overline{x}$

$$\left(\delta_D\right)_y = \frac{\partial U}{\partial F} = \sum\left(\frac{TL}{JG}\right)_{OC}\frac{\partial T}{\partial F} + \sum\frac{1}{EI}\int M\,\frac{\partial M}{\partial F}\,d\,\overline{x}$$

The terms involving the torsion and bending moments in OC must be split up because of the changing second-area moments.

$$\begin{aligned}\left(\delta_D\right)_y &= \frac{12F(4)}{0.4970(11.5)10^6}(12) + \frac{12F(9)}{0.09818(11.5)10^6}(12) + \frac{1}{30\left(10^6\right)0.2485}\int_0^2 F\,\overline{x}^2\,d\,\overline{x} \\ &\quad + \frac{1}{30\left(10^6\right)0.04909}\int_2^{11} F\,\overline{x}^2\,d\,\overline{x} + \frac{1}{30\left(10^6\right)0.2485}\int_{11}^{13} F\,\overline{x}^2\,d\,\overline{x} + \frac{1}{30\left(10^6\right)0.01553}\int_0^{12} F\,\overline{x}^2\,d\,\overline{x} \\ &= 1.008\left(10^{-4}\right)F + 1.148\left(10^{-3}\right)F + 3.58\left(10^{-7}\right)F \\ &\quad + 2.994\left(10^{-4}\right)F + 3.872\left(10^{-5}\right)F + 1.2363\left(10^{-3}\right)F \\ &= 2.824\left(10^{-3}\right)F = 2.824\left(10^{-3}\right)250 = 0.706 \text{ in} \qquad \textit{Ans.}\end{aligned}$$

For the simplified shaft OC,

$$\begin{aligned}\left(\delta_B\right)_y &= \frac{12F(13)}{0.09818(11.5)10^6}(12) + \frac{1}{30\left(10^6\right)0.04909}\int_0^{13} F\,\overline{x}^2\,d\,\overline{x} + \frac{1}{30\left(10^6\right)0.01553}\int_0^{12} F\,\overline{x}^2\,d\,\overline{x} \\ &= 1.6580\left(10^{-3}\right)F + 4.973\left(10^{-4}\right)F + 1.2363\left(10^{-3}\right)F = 3.392\left(10^{-3}\right)F = 3.392\left(10^{-3}\right)250 \\ &= 0.848 \text{ in} \qquad \textit{Ans.}\end{aligned}$$

Simplified is 0.848/0.706 = 1.20 times greater *Ans.*

4-74 Place a dummy force Q pointing downwards at point B. The reaction at C is

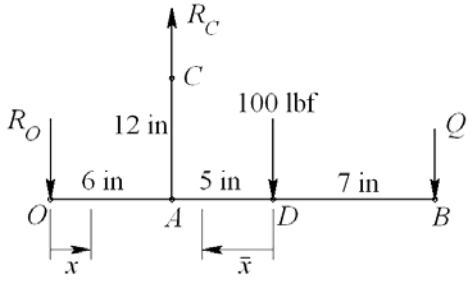
$$R_C = Q + (6/18)100 = Q + 33.33$$

This is the axial force in member BC. Isolating the beam, we find that the moment is not a function of Q, and thus does not contribute to the strain energy. Thus, only energy in the member BC needs to be considered. Let the axial force in BC be F, where

$$F = Q + 33.33 \qquad \frac{\partial F}{\partial Q} = 1$$

$$\delta_B = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \left[\left(\frac{FL}{AE}\right)_{BC}\frac{\partial F}{\partial Q}\right]_{Q=0} = \frac{(0+33.33)12}{\left[\pi\left(0.5^2\right)/4\right]30\left(10^6\right)}(1) = 6.79\left(10^{-5}\right)\text{ in} \qquad \textit{Ans.}$$

4-75 $I_{OB} = 0.25(2^3)/12 = 0.1667$ in^4



$A_{AC} = \pi(0.5^2)/4 = 0.1963$ in^2

$\Sigma M_O = 0 = 6\,R_C - 11(100) - 18\,Q$

$R_C = 3Q + 183.3$

$\Sigma M_A = 0 = 6\,R_O - 5(100) - 12\,Q \quad \Rightarrow \quad R_O = 2Q + 83.33$

Bending in *OB*.

BD: Bending in *BD* is only due to *Q* which when set to zero after differentiation gives no contribution.

AD: Using the variable $\bar{x}$ as shown in the figure above

$$M = -100\bar{x} - Q\left(7 + \bar{x}\right) \qquad \frac{\partial M}{\partial Q} = -\left(7 + \bar{x}\right)$$

OA: Using the variable *x* as shown in the figure above

$$M = -\left(2Q + 83.33\right)x \qquad \frac{\partial M}{\partial Q} = -2x$$

Axial in *AC*:

$$F = 3Q + 183.3 \qquad \frac{\partial F}{\partial Q} = 3$$

$$\delta_B = \left(\frac{\partial U}{\partial Q}\right)_{Q=0} = \left[\left(\frac{FL}{AE}\right)\frac{\partial F}{\partial Q}\right]_{Q=0} + \left(\frac{1}{EI}\sum M\frac{\partial M}{\partial Q}dx\right)_{Q=0}$$

$$= \frac{183.3(12)}{0.1963(30)10^6}(3) + \frac{1}{EI}\int_0^5 (100\,\bar{x})(7+\bar{x})\,d\,\bar{x} + \int_0^6 2(83.33)x^2 dx$$

$$= 1.121(10^{-3}) + \frac{1}{10.4(10^6)0.1667}\left[100\int_0^5 \bar{x}(7+\bar{x})\,d\,\bar{x} + 166.7\int_0^6 x^2 dx\right]$$

$$= 1.121(10^{-3}) + 5.768(10^{-7})\left[100(129.2) + 166.7(72)\right] = 0.0155 \text{ in} \qquad \textit{Ans.}$$

4-76 There is no bending in *AB*. Using the variable θ, rotating counterclockwise from *B*

$$M = PR\sin\theta \qquad \frac{\partial M}{\partial P} = R\sin\theta$$

$$F_r = P\cos\theta \qquad \frac{\partial F_r}{\partial P} = \cos\theta$$

$$F_\theta = P\sin\theta \qquad \frac{\partial F_\theta}{\partial P} = \sin\theta$$

$$\frac{\partial MF_\theta}{\partial P} = 2PR\sin^2\theta$$

$A = 6(4) = 24$ mm^2, $r_o = 40 + \frac{1}{2}(6) = 43$ mm, $r_i = 40 - \frac{1}{2}(6) = 37$ mm,

From Table 3-4, p.121, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489 \text{ mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n$ =40 – 39.92489 = 0.07511 mm

From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa

From Table 4-1, $C = 1.2$

From Eq. (4-38)

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial P}\right)d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial P}\right)d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial(MF_\theta)}{\partial P}d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial P}\right)d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{P(R\sin\theta)^2}{AeE}d\theta + \int_0^{\frac{\pi}{2}} \frac{PR(\sin\theta)^2}{AE}d\theta - \int_0^{\frac{\pi}{2}} \frac{2PR\sin^2\theta}{AE}d\theta + \int_0^{\frac{\pi}{2}} \frac{CPR(\cos\theta)^2}{AG}d\theta$$

$$= \frac{\pi PR}{4AE}\left(\frac{R}{e} + 1 - 2 + \frac{EC}{G}\right) = \frac{\pi(10)(40)}{4(24)(207\cdot 10^3)}\left(\frac{40}{0.07511} + 1 - 2 + \frac{(207\cdot 10^3)(1.2)}{79.3\cdot 10^3}\right)$$

$$\delta = 0.0338 \text{ mm} \qquad \textit{Ans.}$$

4-77 Place a dummy force Q pointing downwards at point A. Bending in AB is only due to Q which when set to zero after differentiation gives no contribution. For section BC use the variable θ, rotating counterclockwise from B

$$M = PR\sin\theta + Q\left(R + R\sin\theta\right) \qquad \frac{\partial M}{\partial Q} = R\left(1+\sin\theta\right)$$

$$F_r = \left(P+Q\right)\cos\theta \qquad \frac{\partial F_r}{\partial Q} = \cos\theta$$

$$F_\theta = \left(P+Q\right)\sin\theta \qquad \frac{\partial F_\theta}{\partial Q} = \sin\theta$$

$$MF_\theta = \left[PR\sin\theta + QR\left(1+\sin\theta\right)\right]\left(P+Q\right)\sin\theta$$

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin^2\theta + + PR\sin\theta\left(1+\sin\theta\right) + 2QR\sin\theta\left(1+\sin\theta\right)$$

But after differentiation, we can set $Q = 0$. Thus,

$$\frac{\partial MF_\theta}{\partial Q} = PR\sin\theta\left(1+2\sin\theta\right)$$

$A = 6(4) = 24\ \text{mm}^2$, $r_o = 40 + \frac{1}{2}(6) = 43$ mm, $r_i = 40 - \frac{1}{2}(6) = 37$ mm,

From Table 3-4, p.121, for a rectangular cross section

$$r_n = \frac{6}{\ln(43/37)} = 39.92489\ \text{mm}$$

From Eq. (4-33), the eccentricity is $e = R - r_n = 40 - 39.92489 = 0.07511$ mm

From Table A-5, $E = 207(10^3)$ MPa, $G = 79.3(10^3)$ MPa

From Table 4-1, $C = 1.2$

From Eq. (4-38)

$$\delta = \int_0^{\frac{\pi}{2}} \frac{M}{AeE}\left(\frac{\partial M}{\partial Q}\right)d\theta + \int_0^{\frac{\pi}{2}} \frac{F_\theta R}{AE}\left(\frac{\partial F_\theta}{\partial Q}\right)d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{AE}\frac{\partial\left(MF_\theta\right)}{\partial Q}d\theta + \int_0^{\frac{\pi}{2}} \frac{CF_r R}{AG}\left(\frac{\partial F_r}{\partial Q}\right)d\theta$$

$$= \frac{PR^2}{AeE}\int_0^{\frac{\pi}{2}} \sin\theta\left(1+\sin\theta\right)d\theta + \frac{PR}{AE}\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta - \frac{PR}{AE}\int_0^{\frac{\pi}{2}} \sin\theta\left(1+2\sin\theta\right)d\theta$$

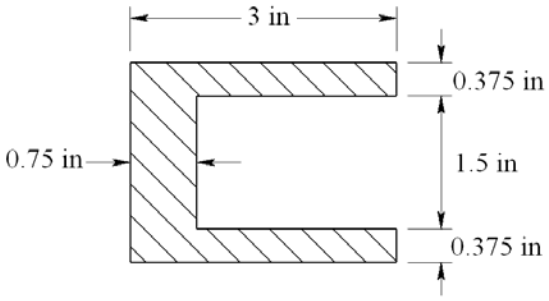
$$+ \frac{CPR}{AG}\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta$$

$$= \left(\frac{\pi}{4}+1\right)\frac{PR^2}{AeE} + \frac{\pi}{4}\frac{PR}{AE} - \left(\frac{\pi}{4}+2\right)\frac{PR}{AE} + \frac{\pi}{4}\frac{CPR}{AG} = \frac{PR}{AE}\left[\left(\frac{\pi}{4}+1\right)\frac{R}{e} - 2 + \frac{\pi}{4}\frac{CE}{G}\right]$$

$$= \frac{10(40)}{24(207)10^3}\left[\left(\frac{\pi}{4}+1\right)\frac{40}{0.07511} - 2 + \frac{\pi}{4}\frac{1.2(207)10^3}{79.3\left(10^3\right)}\right]$$

$$= 0.0766\ \text{mm} \qquad \textit{Ans.}$$

4-78 **Note to the Instructor**. The cross section shown in the first printing is incorrect and the solution presented here reflects the correction which will be made in subsequent printings. The corrected cross section should appear as shown in this figure. We apologize for any inconvenience.



Section *A-A*

$$A = 3(2.25) - 2.25(1.5) = 3.375 \text{ in}^2$$

$$R = \frac{(1+1.5)(3)(2.25) - (1+0.75+1.125)(1.5)(2.25)}{3.375} = 2.125 \text{ in}$$

Section is equivalent to the "T" section of Table 3-4, p. 121,

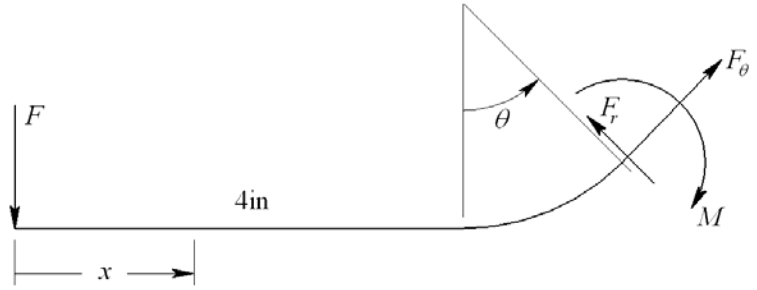
$$r_n = \frac{2.25(0.75) + 0.75(2.25)}{2.25\ln[(1+0.75)/1] + 0.75\ln[(1+3)/(1+0.75)]} = 1.7960 \text{ in}$$

$$e = R - r_n = 2.125 - 1.7960 = 0.329 \text{ in}$$

For the straight section

$$I_z = \frac{1}{12}(2.25)\left(3^3\right) + 2.25(3)(1.5-1.125)^2 - \left[\frac{1}{12}(1.5)\left(2.25^3\right) + 1.5(2.25)\left(0.75 + \frac{2.25}{2} - 1.125\right)^2\right]$$

$$= 2.689 \text{ in}^4$$



For $\underline{0 \le x \le 4 \text{ in}}$

$$M = -Fx \quad \frac{\partial M}{\partial F} = -x, \qquad V = F \quad \frac{\partial V}{\partial F} = 1$$

For $\underline{\theta \le \pi/2}$

$$F_r = F\cos\theta \quad \frac{\partial F_r}{\partial F} = \cos\theta, \qquad F_\theta = F\sin\theta \quad \frac{\partial F_\theta}{\partial F} = \sin\theta$$

$$M = F(4 + 2.125\sin\theta) \quad \frac{\partial M}{\partial F} = (4 + 2.125\sin\theta)$$

$$MF_\theta = F(4 + 2.125\sin\theta)F\sin\theta \qquad \frac{\partial MF_\theta}{\partial F} = 2F(4 + 2.365\sin\theta)\sin\theta$$

Use Eqs. (4-31) and (4-24) (with $C = 1$) for the straight part, and Eq. (4-38) for the curved part, integrating from 0 to π/2, and double the results

$$\delta = \frac{2}{E}\left\{\frac{1}{I}\int_0^4 Fx^2 dx + \frac{F(4)(1)}{3.375(G/E)} + \int_0^{\pi/2} F\frac{(4+2.125\sin\theta)^2}{3.375(0.329)}d\theta \right.$$
$$+\int_0^{\pi/2}\frac{F\sin^2\theta(2.125)}{3.375}d\theta - \int_0^{\pi/2}\frac{2F(4+2.125\sin\theta)\sin\theta}{3.375}d\theta$$
$$\left.+\int_0^{\pi/2}\frac{(1)F\cos^2\theta(2.125)}{3.375(G/E)}d\theta\right.$$

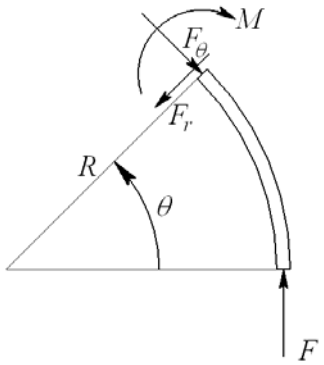
Substitute $I = 2.689$ in^4, $F = 6700$ lbf, $E = 30\ (10^6)$ psi, $G = 11.5\ (10^6)$ psi

$$\delta = \frac{2(6700)}{30(10^6)}\left\{\frac{4^3}{3(2.689)} + \frac{4}{3.375(11.5/30)} + \frac{1}{3.375(0.329)}\left[16\left(\frac{\pi}{2}\right)+17(1)+4.516\left(\frac{\pi}{4}\right)\right]\right.$$
$$\left.+\frac{2.125}{3.375}\left(\frac{\pi}{4}\right) - \frac{2}{3.375}\left[4(1)+2.125\left(\frac{\pi}{4}\right)\right] + \frac{2.125}{3.375(11.5/30)}\left(\frac{\pi}{4}\right)\right\}$$

$= 0.0226$ in *Ans.*

4-79 Since $R/h = 35/4.5 = 7.78$ use Eq. (4-38), integrate from 0 to π, and double the results

$$M = FR(1 - \cos\theta) \qquad \frac{\partial M}{\partial F} = R(1 - \cos\theta)$$
$$F_r = F\sin\theta \qquad \frac{\partial F_r}{\partial F} = \sin\theta$$
$$F_\theta = F\cos\theta \qquad \frac{\partial F_\theta}{\partial F} = \cos\theta$$
$$MF_\theta = F^2 R\cos\theta(1 - \cos\theta)$$
$$\frac{\partial(MF_\theta)}{\partial F} = 2FR\cos\theta(1 - \cos\theta)$$



From Eq. (4-38),

$$\delta = 2\left[\frac{FR^2}{AeE}\int_0^\pi (1-\cos\theta)^2 d\theta + \frac{FR}{AE}\int_0^\pi \cos^2\theta d\theta\right.$$
$$\left.-\frac{2FR}{AE}\int_0^\pi \cos\theta(1-\cos\theta)d\theta + \frac{1.2FR}{AG}\int_0^\pi \sin^2\theta d\theta\right]$$
$$= \frac{2FR}{AE}\left(\frac{3\pi}{2}\frac{R}{e} + \frac{3\pi}{2} + 0.6\pi\frac{E}{G}\right)$$

$A = 4.5(3) = 13.5$ mm^2, $E = 207\ (10^3)$ N/mm^2, $G = 79.3\ (10^3)$ N/mm^2, and from Table 3-4, p. 121,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{4.5}{\ln \frac{37.25}{32.75}} = 34.95173 \text{ mm}$$

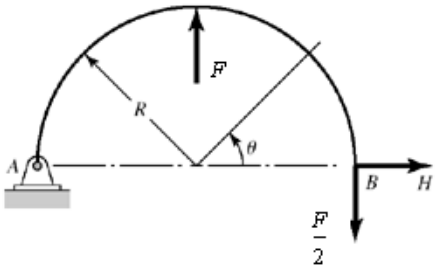
and $e = R - r_n = 35 - 34.95173 = 0.04827$ mm. Thus,

$$\delta = \frac{2F(35)}{13.5(207)10^3}\left(\frac{3\pi}{2}\frac{35}{0.04827} + \frac{3\pi}{2} + 0.6\pi\frac{207}{79.3}\right) = 0.08583F$$

where F is in N. For $\delta = 1$ mm, $F = \dfrac{1}{0.08583} = 11.65$ N *Ans.*

Note: The first term in the equation for δ dominates and this is from the bending moment. Try Eq. (4-41), and compare the results.

4-80 $R/h = 20 > 10$ so Eq. (4-41) can be used to determine deflections. Consider the horizontal reaction, to applied at B, subject to the constraint $(\delta_B)_H = 0$.



$$M = \frac{FR}{2}(1-\cos\theta) - HR\sin\theta \qquad \frac{\partial M}{\partial H} = -R\sin\theta \qquad 0 < \theta < \frac{\pi}{2}$$

By symmetry, we may consider only half of the wire form and use twice the strain energy Eq. (4-41) then becomes,

$$(\delta_B)_H = \frac{\partial U}{\partial H} \doteq \frac{2}{EI}\int_0^{\pi/2}\left(M\frac{\partial M}{\partial H}\right)Rd\theta = 0$$

$$\int_0^{\pi/2}\left[\frac{FR}{2}(1-\cos\theta) - HR\sin\theta\right](-R\sin\theta)R\ d\theta = 0$$

$$-\frac{F}{2} + \frac{F}{4} + H\frac{\pi}{4} = 0 \ \Rightarrow\ H = \frac{F}{\pi} = \frac{30}{\pi} = 9.55 \text{ N} \quad \textit{Ans.}$$

Reaction at A is the same where H goes to the left. Substituting H into the moment equation we get,

$$M = \frac{FR}{2\pi}\left[\pi(1-\cos\theta) - 2\sin\theta\right] \qquad \frac{\partial M}{\partial F} = \frac{R}{2\pi}[\pi(1-\cos\theta) - 2\sin\theta] \qquad 0 < \theta < \frac{\pi}{2}$$

$$\delta_P = \frac{\partial U}{\partial P} \doteq \int \frac{2}{EI}\left(M\,\frac{\partial M}{\partial F}\right)Rd\theta = \frac{2}{EI}\int_0^{\pi/2}\frac{FR^2}{4\pi^2}[\pi(1-\cos\theta)-2\sin\theta]^2 R\; d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\int_0^{\pi/2}(\pi^2+\pi^2\cos^2\theta+4\sin^2\theta-2\pi^2\cos\theta-4\pi\sin\theta+4\pi\sin\theta\cos\theta)\; d\theta$$

$$= \frac{FR^3}{2\pi^2 EI}\left[\pi^2\left(\frac{\pi}{2}\right)+\pi^2\left(\frac{\pi}{4}\right)+4\left(\frac{\pi}{4}\right)-2\pi^2-4\pi+2\pi\right]$$

$$= \frac{(3\pi^2-8\pi-4)}{8\pi}\frac{FR^3}{EI} = \frac{(3\pi^2-8\pi-4)}{8\pi}\frac{(30)(40^3)}{207\left(10^3\right)\left[\pi\left(2^4\right)/64\right]} = 0.224 \text{ mm} \qquad \textit{Ans.}$$

4-81 The radius is sufficiently large compared to the wire diameter to use Eq. (4-41) for the curved beam portion. The shear and axial components will be negligible compared to bending.

Place a fictitious force Q pointing to the left at point A.

$$M = PR\sin\theta + Q(R\sin\theta + l) \qquad \frac{\partial M}{\partial Q} = R\sin\theta + l$$

Note that the strain energy in the straight portion is zero since there is no real force in that section.

From Eq. (4-41),

$$\delta = \left[\int_0^{\pi/2}\frac{1}{EI}\left(M\,\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta\left(R\sin\theta + l\right)Rd\theta$$

$$= \frac{PR^2}{EI}\int_0^{\pi/2}\left(R\sin^2\theta + l\sin\theta\right)d\theta = \frac{PR^2}{EI}\left(\frac{\pi}{4}R + l\right) = \frac{1(5^2)}{30\left(10^6\right)\left[\pi\left(0.125^4\right)/64\right]}\left(\frac{\pi}{4}(5)+4\right)$$

$$= 0.551 \text{ in} \qquad \textit{Ans.}$$

4-82 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Straight portion: $M_{AB} = Px \qquad \dfrac{\partial M_{AB}}{\partial P} = x$

Curved portion: $M_{BC} = P\left[R(1-\cos\theta)+l\right] \qquad \dfrac{\partial M_{BC}}{\partial P} = \left[R(1-\cos\theta)+l\right]$

From Eq. (4-41) with the addition of the bending strain energy in the straight portion of the wire,

$$\delta = \int_0^l \frac{1}{EI}\left(M_{AB}\frac{\partial M_{AB}}{\partial P}\right)dx + \int_0^{\pi/2}\frac{1}{EI}\left(M_{BC}\frac{\partial M_{BC}}{\partial P}\right)Rd\theta$$

$$= \frac{P}{EI}\int_0^l x^2 dx + \frac{PR}{EI}\int_0^{\pi/2}\left[R(1-\cos\theta)+l\right]^2 d\theta$$

$$= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2(1-2\cos\theta+\cos^2\theta)+2Rl(1-\cos\theta)+l^2\right]d\theta$$

$$= \frac{Pl^3}{3EI} + \frac{PR}{EI}\int_0^{\pi/2}\left[R^2\cos^2\theta-\left(2R^2+2Rl\right)\cos\theta+(R+l)^2\right]d\theta$$

$$= \frac{Pl^3}{3EI} + \frac{PR}{EI}\left[\frac{\pi}{4}R^2-\left(2R^2+2Rl\right)+\frac{\pi}{2}(R+l)^2\right]$$

$$= \frac{P}{EI}\left[\frac{l^3}{3}+\frac{\pi}{4}R^3-R\left(2R^2+2Rl\right)+\frac{\pi}{2}R(R+l)^2\right]$$

$$= \frac{1}{30\left(10^6\right)\pi\left(0.125^4\right)/64}\left[\frac{4^3}{3}+\frac{\pi}{4}(5^3)-5\left[2(5^2)+2(5)(4)\right]+\frac{\pi}{2}(5)(5+4)^2\right]$$

$$= 0.850 \text{ in} \quad \textit{Ans.}$$

4-83 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, Q, at A vertically downward. The only load in the straight section is the axial force, Q. Since this will be zero, there is no contribution.

In the curved section

$$M = PR\sin\theta + QR(1-\cos\theta) \qquad \frac{\partial M}{\partial Q} = R(1-\cos\theta)$$

From Eq. (4-41)

$$\delta = \left[\int_0^{\pi/2}\frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2} PR\sin\theta\left[R(1-\cos\theta)\right]Rd\theta$$

$$= \frac{PR^3}{EI}\int_0^{\pi/2}(\sin\theta-\sin\theta\cos\theta)\,d\theta = \frac{PR^3}{EI}\left(1-\frac{1}{2}\right) = \frac{PR^3}{2EI}$$

$$= \frac{1\left(5^3\right)}{2(30)10^6\left[\pi\left(0.125^4\right)/64\right]} = 0.174 \text{ in} \qquad \textit{Ans.}$$

4-84 Both the radius and the length are sufficiently large to use Eq. (4-41) for the curved beam portion and to neglect transverse shear stress for the straight portion.

Place a dummy force, Q, at A vertically downward. The load in the straight section is the axial force, Q, whereas the bending moment is only a function of P and is not a function of Q. When setting $Q = 0$, there is no axial or bending contribution.

In the curved section

$$M = P\left[R\left(1-\cos\theta\right)+l\right]-QR\sin\theta \qquad \frac{\partial M}{\partial Q} = -R\sin\theta$$

From Eq. (4-41)

$$\delta = \left[\int_0^{\pi/2}\frac{1}{EI}\left(M\frac{\partial M}{\partial Q}\right)Rd\theta\right]_{Q=0} = \frac{1}{EI}\int_0^{\pi/2}P\left[R\left(1-\cos\theta\right)+l\right]\left(-R\sin\theta\right)Rd\theta$$

$$= -\frac{PR^2}{EI}\int_0^{\pi/2}\left(R\sin\theta - R\sin\theta\cos\theta + l\sin\theta\right)d\theta = -\frac{PR^2}{EI}\left(R+l-\frac{1}{2}R\right) = -\frac{PR^2}{2EI}\left(R+2l\right)$$

$$= -\frac{1\left(5^2\right)}{2\left(30\right)10^6\left[\pi\left(0.125^4\right)/64\right]}\left[5+2\left(4\right)\right] = -0.452 \text{ in}$$

Since the deflection is negative, δ is in the opposite direction of Q. Thus the deflection is

$$\delta = 0.452 \text{ in} \uparrow \qquad \textit{Ans.}$$

4-85 Consider the force of the mass to be F, where $F = 9.81(1) = 9.81$ N. The load in AB is tension

$$F_{AB} = F \qquad \frac{\partial F_{AB}}{\partial F} = 1$$

For the curved section, the radius is sufficiently large to use Eq. (4-41). There is no bending in section DE. For section BCD, let θ be counterclockwise originating at D

$$M = FR\sin\theta \qquad \frac{\partial M}{\partial F} = R\sin\theta \qquad 0 \le \theta \le \pi$$

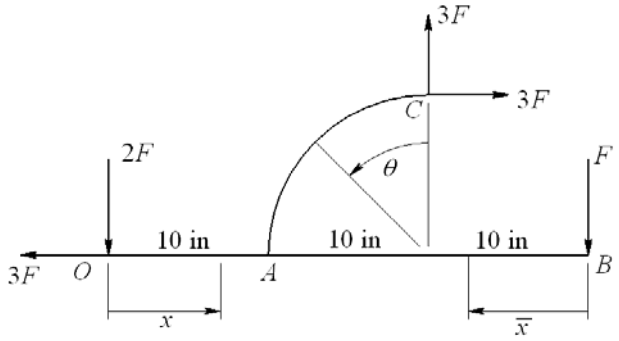
Using Eqs. (4-29) and (4-41)

$$\delta = \left(\frac{Fl}{AE}\right)_{AB}\frac{\partial F_{AB}}{\partial F} + \int_0^{\pi}\frac{1}{EI}\left(M\frac{\partial M}{\partial F}\right)Rd\theta = \frac{Fl}{AE}\left(1\right) + \int_0^{\pi}\frac{FR^3}{EI}\sin^2\theta\, d\theta$$

$$= \frac{Fl}{AE} + \frac{\pi FR^3}{2EI} = \frac{F}{E}\left(\frac{l}{A}+\frac{\pi R^3}{2I}\right) = \frac{9.81}{207\left(10^3\right)}\left[\frac{80}{\left[\pi\left(2^2\right)/4\right]} + \frac{\pi\left(40^3\right)}{2\left[\pi\left(2^4\right)/64\right]}\right]$$

$$= 6.067 \text{ mm} \qquad \textit{Ans.}$$

4-86 $A_{OA} = 2(0.25) = 0.5 \text{ in}^2$,
$I_{OAB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$,
$I_{AC} = \pi(0.5^4)/64 = 3.068\,(10^{-3}) \text{ in}^4$



Applying a force F at point B, using statics, the reaction forces at O and C are as shown.

OA: Axial $F_{OA} = 3F$ $\quad \dfrac{\partial F_{OA}}{\partial F} = 3$

Bending $M_{OA} = -2Fx$ $\quad \dfrac{\partial M_{OA}}{\partial F} = -2x$

AB: Bending $M_{AB} = -F\,\bar{x}$ $\quad \dfrac{\partial M_{AB}}{\partial F} = -\bar{x}$

AC: Isolating the upper curved section

$$M_{AC} = 3FR(\sin\theta + \cos\theta - 1) \qquad \frac{\partial M_{AC}}{\partial F} = 3R(\sin\theta + \cos\theta - 1)$$

$$\delta = \left(\frac{Fl}{AE}\right)_{OA}\frac{\partial F_{OA}}{\partial F} + \frac{1}{(EI)_{OAB}}\int_0^{10} 4Fx^2dx + \frac{1}{(EI)_{OAB}}\int_0^{20} F\,\bar{x}^2 d\,\bar{x}$$

$$+ \frac{9FR^3}{(EI)_{AC}}\int_0^{\pi/2}(\sin\theta + \cos\theta - 1)^2\,d\theta$$

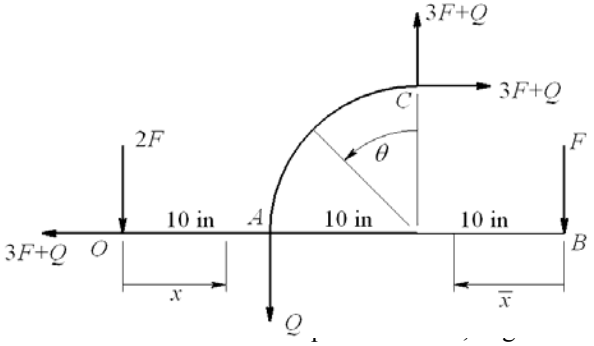
$$= \frac{3F(10)}{0.5(10.4)10^6}(3) + \frac{4F(10^3)}{3(10.4)10^6(0.1667)} + \frac{F(20^3)}{3(10.4)10^6(0.1667)}$$

$$+ \frac{9F(10^3)}{30(10^6)3.068(10^{-3})}\int_0^{\pi/2}(\sin^2\theta + 2\sin\theta\cos\theta - 2\sin\theta + \cos^2\theta - 2\cos\theta + 1)\,d\theta$$

$$= 1.731(10^{-5})F + 7.691(10^{-4})F + 1.538(10^{-3})F + 0.09778F\left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2}\right)$$

$$= 0.0162F = 0.0162(100) = 1.62 \text{ in} \qquad \textit{Ans.}$$

4-87 $A_{OA} = 2(0.25) = 0.5 \text{ in}^2$,
$I_{OAB} = 0.25(2^3)/12 = 0.1667 \text{ in}^4$,
$I_{AC} = \pi(0.5^4)/64 = 3.068\,(10^{-3}) \text{ in}^4$
Applying a vertical dummy force, Q, at A, from statics the reactions are as shown. The dummy force is transmitted through section



OA and member *AC*.

OA: $F_{OA} = 3F + Q$ $\qquad \dfrac{\partial F_{OA}}{\partial Q} = 1$

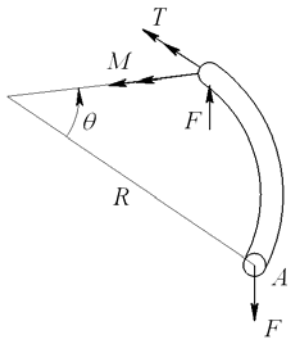
AC: $M_{AC} = (3F + Q)R\sin\theta - (3F + Q)R(1 - \cos\theta)$ $\qquad \dfrac{\partial M_{AC}}{\partial Q} = R(\sin\theta + \cos\theta - 1)$

$$\delta = \left[\left(\frac{Fl}{AE} \right)_{OA} \left(\frac{\partial F_{OA}}{\partial Q} \right) + \left(\frac{1}{EI} \right)_{AC} \int_0^{\pi/2} M_{AC} \frac{\partial M_{AC}}{\partial Q} R d\theta \right]_{Q=0}$$

$$= \frac{3Fl_{OA}}{(AE)_{OA}} + \frac{3FR^3}{(EI)_{AC}} \int_0^{\pi/2} (\sin\theta + \cos\theta - 1)^2 \, d\theta$$

$$= \frac{3(100)10}{10.4(10^6)0.5} + \frac{3(100)10^3}{30(10^6)3.068(10^{-3})}\left(\frac{\pi}{4} + 1 - 2 + \frac{\pi}{4} - 2 + \frac{\pi}{2} \right) = 0.462 \text{ in} \qquad \textit{Ans.}$$

4-88 $I = \pi(6^4)/64 = 63.62 \text{ mm}^4$



$\underline{0 \le \theta \le \pi/2}$

$M = FR\sin\theta$ $\qquad \dfrac{\partial M}{\partial F} = R\sin\theta$

$T = FR(1 - \cos\theta)$ $\qquad \dfrac{\partial T}{\partial F} = R(1 - \cos\theta)$

According to Castigliano's theorem, a positive $\partial U/\partial F$ will yield a deflection of *A* in the negative *y* direction. Thus the deflection in the positive *y* direction is

$$(\delta_A)_y = -\frac{\partial U}{\partial F} = -\left\{ \frac{1}{EI}\int_0^{\pi/2} F(R\sin\theta)^2 R \, d\theta + \frac{1}{GJ}\int_0^{\pi/2} F[R(1 - \cos\theta)]^2 R \, d\theta \right\}$$

Integrating and substituting $J = 2I$ and $G = E/\left[2(1+\nu)\right]$

$$(\delta_A)_y = -\frac{FR^3}{EI}\left[\frac{\pi}{4} + (1+\nu)\left(\frac{3\pi}{4} - 2 \right) \right] = -\left[4\pi - 8 + (3\pi - 8)\nu \right] \frac{FR^3}{4EI}$$

$$= -[4\pi - 8 + (3\pi - 8)(0.29)] \frac{(250)(80)^3}{4(200)10^3(63.62)} = -12.5 \text{ mm} \qquad \textit{Ans.}$$

4-89 The force applied to the copper and steel wire assembly is

$$F_c + F_s = 400 \text{ lbf} \qquad (1)$$

Since the deflections are equal, $\delta_c = \delta_s$

$$\left(\frac{Fl}{AE}\right)_c = \left(\frac{Fl}{AE}\right)_s$$

$$\frac{F_c l}{3(\pi/4)(0.1019)^2(17.2)10^6} = \frac{F_s l}{(\pi/4)(0.1055)^2(30)10^6}$$

Yields, $F_c = 1.6046F_s$. Substituting this into Eq. (1) gives

$1.604F_s + F_s = 2.6046F_s = 400 \quad \Rightarrow \quad F_s = 153.6$ lbf

$F_c = 1.6046F_s = 246.5$ lbf

$$\sigma_c = \frac{F_c}{A_c} = \frac{246.5}{3(\pi/4)(0.1019)^2} = 10\ 075 \text{ psi} = 10.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{153.6}{(\pi/4)(0.1055^2)} = 17\ 571 \text{ psi} = 17.6 \text{ kpsi} \quad \textit{Ans.}$$

$$\delta = \left(\frac{Fl}{AE}\right)_s = \frac{153.6(100)(12)}{(\pi/4)(0.1055)^2(30)10^6} = 0.703 \text{ in} \quad \textit{Ans.}$$

4-90 **(a)** Bolt stress $\sigma_b = 0.75(65) = 48.8$ kpsi *Ans.*

Total bolt force $F_b = 6\sigma_b A_b = 6(48.8)\left(\frac{\pi}{4}\right)(0.5^2) = 57.5$ kips

Cylinder stress $\sigma_c = -\frac{F_b}{A_c} = \frac{57.43}{(\pi/4)(5.5^2 - 5^2)} = -13.9$ kpsi *Ans.*

(b) Force from pressure

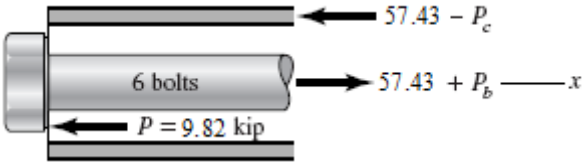
$$P = \frac{\pi D^2}{4} p = \frac{\pi(5^2)}{4}(500) = 9817 \text{ lbf} = 9.82 \text{ kip}$$

$\Sigma F_x = 0$

$P_b + P_c = 9.82 \qquad (1)$

Since $\delta_c = \delta_b$,

$$\frac{P_c l}{(\pi/4)(5.5^2 - 5^2)E} = \frac{P_b l}{6(\pi/4)(0.5^2)E}$$



$P_c = 3.5\ P_b \qquad (2)$

Substituting this into Eq. (1)

$P_b + 3.5\ P_b = 4.5\ P_b = 9.82 \Rightarrow P_b = 2.182$ kip. From Eq. (2), $P_c = 7.638$ kip

Using the results of (**a**) above, the total bolt and cylinder stresses are

$$\sigma_b = 48.8 + \frac{2.182}{6(\pi/4)(0.5^2)} = 50.7 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_c = -13.9 + \frac{7.638}{(\pi/4)(5.5^2 - 5^2)} = -12.0 \text{ kpsi} \quad \textit{Ans.}$$

4-91 $T_c + T_s = T$ (1)

$$\theta_c = \theta_s \quad \Rightarrow \quad \frac{T_c l}{(JG)_c} = \frac{T_s l}{(JG)_s} \quad \Rightarrow \quad T_c = \frac{(JG)_c}{(JG)_s} T_s \qquad (2)$$

Substitute this into Eq. (1)

$$\frac{(JG)_c}{(JG)_s} T_s + T_s = T \quad \Rightarrow \quad T_s = \frac{(JG)_s}{(JG)_s + (JG)_c} T$$

The percentage of the total torque carried by the shell is

$$\%\ \text{Torque} = \frac{100(JG)_s}{(JG)_s + (JG)_c} \qquad \textit{Ans.}$$

4-92 $R_O + R_B = W$ (1)

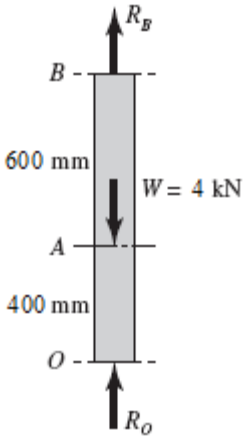
$\delta_{OA} = \delta_{AB}$

$$\left(\frac{Fl}{AE}\right)_{OA} = \left(\frac{Fl}{AE}\right)_{AB}$$

$$\frac{400R_O}{AE} = \frac{600R_B}{AE} \quad \Rightarrow \quad R_O = \frac{3}{2} R_B \qquad (2)$$

Substitute this unto Eq. (1)

$$\frac{3}{2} R_B + R_B = 4 \quad \Rightarrow \quad R_B = 1.6 \text{ kN} \quad \textit{Ans.}$$



From Eq. (2) $R_O = \frac{3}{2}1.6 = 2.4$ kN *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad \textit{Ans.}$$

4-93 See figure in Prob. 4-92 solution.

Procedure 1:

1. Let R_B be the redundant reaction.

2. Statics. $R_O + R_B = 4\,000$ N $\Rightarrow$ $R_O = 4\,000 - R_B$ (1)

3. Deflection of point B. $\delta_B = \dfrac{R_B(600)}{AE} + \dfrac{(R_B - 4000)(400)}{AE} = 0$ (2)

4. From Eq. (2), AE cancels and $R_B = 1\,600$ N *Ans.*
and from Eq. (1), $R_O = 4\,000 - 1\,600 = 2\,400$ N *Ans.*

$$\delta_A = \left(\frac{Fl}{AE}\right)_{OA} = \frac{2\,400(400)}{10(60)(71.7)(10^3)} = 0.0223 \text{ mm} \quad \textit{Ans.}$$

4-94 (**a**) Without the right-hand wall the deflection of point C would be

$$\delta_C = \sum \frac{Fl}{AE} = \frac{5(10^3)8}{(\pi/4)0.75^2(10.4)10^6} + \frac{2(10^3)5}{(\pi/4)0.5^2(10.4)10^6}$$
$$= 0.01360 \text{ in} > 0.005 \text{ in} \quad \therefore \text{ Hits wall} \quad \textit{Ans.}$$

(**b**) Let R_C be the reaction of the wall at C acting to the left ($\leftarrow$). Thus, the deflection of point C is now

$$\delta_C = \frac{\left[5(10^3) - R_C\right]8}{(\pi/4)0.75^2(10.4)10^6} + \frac{\left[2(10^3) - R_C\right]5}{(\pi/4)0.5^2(10.4)10^6}$$
$$= 0.01360 - \frac{4R_C}{\pi(10.4)10^6}\left(\frac{8}{0.75^2} + \frac{5}{0.5^2}\right) = 0.005$$

or,

$0.01360 - 4.190(10^{-6})R_C = 0.005 \quad \Rightarrow \quad R_C = 2\,053 \text{ lbf} = 2.05 \text{ kip} \leftarrow$ *Ans.*

Statics. Considering $\rightarrow +$, $5\,000 - R_A - 2\,053 = 0 \Rightarrow R_A = 2\,947$ lbf $= 2.95$ kip $\leftarrow$ *Ans.*

Deflection. AB is 2 947 lbf in tension. Thus

$$\delta_B = \delta_{AB} = \frac{R_A(8)}{A_{AB}E} = \frac{2\,947(8)}{(\pi/4)0.75^2(10.4)10^6} = 5.13(10^{-3}) \text{ in} \rightarrow \quad \textit{Ans.}$$

4-95 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{JG} = \frac{T_{AB}(6)}{JG} \qquad \Rightarrow \qquad T_{OA} = \frac{3}{2}T_{AB} \qquad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$\frac{3}{2}T_{AB} + T_{AB} = \frac{5}{2}T_{AB} = 200 \qquad \Rightarrow \qquad T_{AB} = 80 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

From Eq. (1) $\quad T_{OA} = \frac{3}{2}T_{AB} = \frac{3}{2}80 = 120 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$

$$\theta_A = \frac{80(6)}{(\pi/32)0.5^4(11.5)10^6}\frac{180}{\pi} = 0.390^0 \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \Rightarrow \quad \tau_{OA} = \frac{16(120)}{\pi(0.5^3)} = 4890 \text{ psi} = 4.89 \text{ kpsi} \qquad \textit{Ans.}$$

$$\tau_{AB} = \frac{16(80)}{\pi(0.5^3)} = 3260 \text{ psi} = 3.26 \text{ kpsi} \qquad \textit{Ans.}$$

4-96 Since $\theta_{OA} = \theta_{AB}$,

$$\frac{T_{OA}(4)}{(\pi/32)0.5^4 G} = \frac{T_{AB}(6)}{(\pi/32)0.75^4 G} \qquad \Rightarrow \qquad T_{OA} = 0.2963T_{AB} \qquad (1)$$

Statics. $T_{OA} + T_{AB} = 200$ (2)

Substitute Eq. (1) into Eq. (2),

$$0.2963T_{AB} + T_{AB} = 1.2963T_{AB} = 200 \qquad \Rightarrow \qquad T_{AB} = 154.3 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

From Eq. (1) $\quad T_{OA} = 0.2963T_{AB} = 0.2963(154.3) = 45.7 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$

$$\theta_A = \frac{154.3(6)}{(\pi/32)0.75^4(11.5)10^6}\frac{180}{\pi} = 0.148^0 \quad \textit{Ans.}$$

$$\tau_{\max} = \frac{16T}{\pi d^3} \qquad \Rightarrow \quad \tau_{OA} = \frac{16(45.7)}{\pi(0.5^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad \textit{Ans.}$$

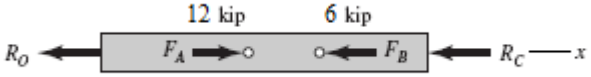
$$\tau_{AB} = \frac{16(154.3)}{\pi(0.75^3)} = 1862 \text{ psi} = 1.86 \text{ kpsi} \qquad \textit{Ans.}$$

4-97 Procedure 1.

1. Arbitrarily, choose R_C as a redundant reaction.
2. Statics. $\Sigma F_x = 0$,

$$12(10^3) - 6(10^3) - R_O - R_C = 0$$

$$R_O = 6(10^3) - R_C \qquad (1)$$



3. The deflection of point C.

$$\delta_C = \frac{\left[12(10^3) - 6(10^3) - R_C\right](20)}{AE} - \frac{\left[6(10^3) + R_C\right](10)}{AE} - \frac{R_C(15)}{AE} = 0$$

4. The deflection equation simplifies to

$$-45\,R_C + 60(10^3) = 0 \quad \Rightarrow \quad R_C = 1\,333 \text{ lbf} \doteq 1.33 \text{ kip} \quad \textit{Ans.}$$

From Eq. (1), $R_O = 6(10^3) - 1\,333 = 4\,667$ lbf $\doteq 4.67$ kip *Ans.*

$$F_{AB} = F_B + R_C = 6 + 1.333 = 7.333 \text{ kips compression}$$

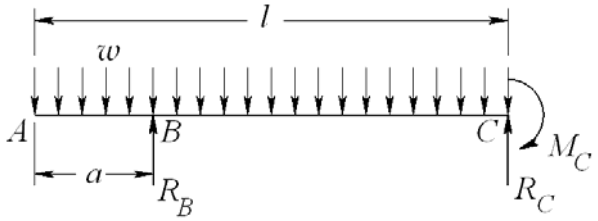
$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{-7.333}{(0.5)(1)} = -14.7 \text{ kpsi} \quad \textit{Ans.}$$

Deflection of A. Since OA is in tension,

$$\delta_A = \delta_{OA} = \frac{R_O l_{OA}}{AE} = \frac{4\,667(20)}{(0.5)(1)(30)10^6} = 0.00622 \text{ in} \quad \textit{Ans.}$$

4-98 Procedure 1.

1. Choose R_B as redundant reaction.



2. Statics. $R_C = wl - R_B \qquad (1)$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) \qquad (2)$$

3. Deflection equation for point B. Superposition of beams 2 and 3 of Table A-9,

$$y_B = \frac{R_B(l-a)^3}{3EI} + \frac{w(l-a)^2}{24EI}\left[4l(l-a) - (l-a)^2 - 6l^2\right] = 0$$

4. Solving for R_B.

$$R_B = \frac{w}{8(l-a)}\left[6l^2 - 4l(l-a) + (l-a)^2\right]$$

$$= \frac{w}{8(l-a)}\left(3l^2 + 2al + a^2\right) \qquad \textit{Ans.}$$

Substituting this into Eqs. (1) and (2) gives

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2 - 10al - a^2\right) \qquad \textit{Ans.}$$

$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}\left(l^2 - 2al - a^2\right) \qquad \textit{Ans.}$$

4-99 See figure in Prob. 4-98 solution.

Procedure 1.
1. Choose R_B as redundant reaction.

2. Statics. $R_C = wl - R_B$ (1)

$$M_C = \frac{1}{2}\omega l^2 - R_B(l-a) \qquad (2)$$

3. Deflection equation for point *B*. Let the variable *x* start at point *A* and to the right. Using singularity functions, the bending moment as a function of *x* is

$$M = -\frac{1}{2}wx^2 + R_B\langle x-a\rangle^1 \qquad \frac{\partial M}{\partial R_B} = \langle x-a\rangle^1$$

$$y_B = \frac{\partial U}{\partial R_B} = \frac{1}{EI}\int_0^l M\frac{\partial M}{\partial R_B}dx$$

$$= \frac{1}{EI}\int_0^l -\frac{1}{2}wx^2(0)\,dx + \frac{1}{EI}\int_a^l\left[-\frac{1}{2}wx^2 + R_B(x-a)\right](x-a)\,dx = 0$$

or,

$$-\frac{1}{2}w\left[\frac{1}{4}\left(l^4 - a^4\right) - \frac{a}{3}\left(l^3 - a^3\right)\right] + \frac{R_B}{3}\left[(l-a)^3 - (a-a)^3\right] = 0$$

Solving for R_B gives

$$R_B = \frac{w}{8(l-a)^3}\left[3\left(l^4 - a^4\right) - 4a\left(l^3 - a^3\right)\right] = \frac{w}{8(l-a)}\left(3l^2 + 2al + a^2\right) \qquad \textit{Ans.}$$

From Eqs. (1) and (2)

$$R_C = wl - R_B = \frac{w}{8(l-a)}\left(5l^2 - 10al - a^2\right) \qquad \textit{Ans.}$$

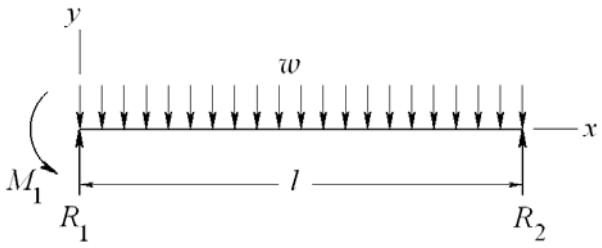
$$M_C = \frac{1}{2}wl^2 - R_B(l-a) = \frac{w}{8}\left(l^2 - 2al - a^2\right) \qquad \textit{Ans.}$$

4-100 **Note:** When setting up the equations for this problem, no rounding of numbers was made. It turns out that the deflection equation is very sensitive to rounding.

Procedure 2.

1. Statics. $R_1 + R_2 = wl$ (1)

$$R_2 l + M_1 = \frac{1}{2} w l^2 \qquad (2)$$



2. Bending moment equation.

$$M = R_1 x - \frac{1}{2} w x^2 - M_1$$

$$EI \frac{dy}{dx} = \frac{1}{2} R_1 x^2 - \frac{1}{6} w x^3 - M_1 x + C_1 \qquad (3)$$

$$EIy = \frac{1}{6} R_1 x^3 - \frac{1}{24} w x^4 - \frac{1}{2} M_1 x^2 + C_1 x + C_2 \qquad (4)$$

$EI = 30(10^6)(0.85) = 25.5(10^6)$ lbf·in^2.

3. Boundary condition 1. At $x = 0$, $y = -R_1/k_1 = -R_1/[1.5(10^6)]$. Substitute into Eq. (4) with value of EI yields $C_2 = -17\ R_1$.

Boundary condition 2. At $x = 0$, $dy/dx = -M_1/k_2 = -M_1/[2.5(10^6)]$. Substitute into Eq. (3) with value of EI yields $C_1 = -10.2\ M_1$.

Boundary condition 3. At $x = l$, $y = -R_2/k_3 = -R_1/[2.0(10^6)]$. Substitute into Eq. (4) with value of EI yields

$$-12.75R_2 = \frac{1}{6} R_1 l^3 - \frac{1}{24} w l^4 - \frac{1}{2} M_1 l^2 - 10.2 M_1 l - 17 R_1 \qquad (5)$$

Equations (1), (2), and (5), written in matrix form with $w = 500/12$ lbf/in and $l = 24$ in, are

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 24 & 1 \\ 2287 & 12.75 & -532.8 \end{pmatrix} \begin{Bmatrix} R_1 \\ R_2 \\ M_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 12 \\ 576 \end{Bmatrix} \left(10^3\right)$$

Solving, the simultaneous equations yields

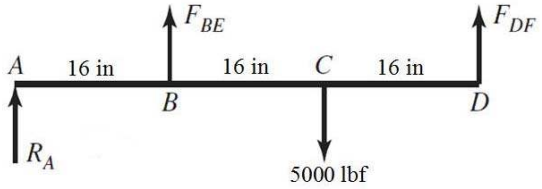
$R_1 = 554.59$ lbf, $R_2 = 445.41.59$ lbf, $M_1 = 1310.1$ lbf·in *Ans.*

For the deflection at $x = l/2 = 12$ in, Eq. (4) gives

$$y\big|_{x=12\text{ in}} = \frac{1}{25.5(10^6)}\left[\frac{1}{6}(554.59)12^3 - \frac{1}{24}\frac{500}{12}12^4 - \frac{1}{2}(1310.1)12^2 - 10.2(1310.1)12 - 17(554.59)\right]$$

$$= -5.51(10^{-3})\text{ in} \qquad \textit{Ans.}$$

4-101 Cable area, $A = \frac{\pi}{4}(0.5^2) = 0.1963\text{ in}^2$



Procedure 2.

1. Statics. $R_A + F_{BE} + F_{DF} = 5(10^3)$ (1)

$3\,F_{DF} + F_{BE} = 10(10^3)$ (2)

2. Bending moment equation.

$$M = R_A x + F_{BE}\langle x-16\rangle^1 - 5000\langle x-32\rangle^1$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + \frac{1}{2}F_{BE}\langle x-16\rangle^2 - 2500\langle x-32\rangle^2 + C_1 \qquad (3)$$

$$EIy = \frac{1}{6}R_A x^3 + \frac{1}{6}F_{BE}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-32\rangle^3 + C_1 x + C_2 \qquad (4)$$

3. B.C. 1: At $x = 0,\ y = 0 \quad \Rightarrow \quad C_2 = 0$

B.C. 2: At x = 16 in,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{BE}$$

Substituting into Eq. (4) and evaluating at x = 16 in

$$EIy_B = 30(10^6)(1.2)(-6.453)(10^{-6})F_{BE} = \frac{1}{6}R_A(16^3) + C_1(16)$$

Simplifying gives $682.7\,R_A + 232.3\,F_{BE} + 16\,C_1 = 0$ (5)

B.C. 2: At x = 48 in,

$$y_D = -\left(\frac{Fl}{AE}\right)_{DF} = -\frac{F_{DF}(38)}{0.1963(30)10^6} = -6.453(10^{-6})F_{DF}$$

Substituting into Eq. (4) and evaluating at x = 48 in,

$$EIy_D = -232.3F_{DF} = \frac{1}{6}R_A(48^3) + \frac{1}{6}F_{BE}(48-16)^3 - \frac{2500}{3}(48-32)^3 + 48C_1$$

Simplifying gives $18\,432\,R_A + 5\,461\,F_{BE} + 232.3\,F_{DF} + 48\,C_1 = 3.413(10^6)$ (6)

Equations (1), (2), (5) and (6) in matrix form are

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 682.7 & 232.3 & 0 & 16 \\ 18\,432 & 5\,461 & 232.3 & 48 \end{pmatrix} \begin{Bmatrix} R_A \\ F_{BE} \\ F_{DF} \\ C_1 \end{Bmatrix} = \begin{Bmatrix} 5\,000 \\ 10\,000 \\ 0 \\ 3.413(10^6) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_A = -970.5$ lbf, $F_{BE} = 3956$ lbf, $F_{DF} = 2015$ lbf, and $C_1 = -16\,020$ lbf·in^2.

$$\sigma_{BE} = \frac{3956}{0.1963} = 20.2 \text{ kpsi}, \quad \sigma_{DF} = \frac{2015}{0.1963} = 10.3 \text{ kpsi} \quad \textit{Ans.}$$

$EI = 30(10^6)(1.2) = 36(10^6)$ lbf·in^2

$$y = \frac{1}{36(10^6)}\left(-\frac{970.5}{6}x^3 + \frac{3956}{6}\langle x-16\rangle^3 - \frac{2500}{3}\langle x-32\rangle^3 - 16\,020x\right)$$

$$= \frac{1}{36(10^6)}\left(-161.8x^3 + 659.3\langle x-16\rangle^3 - 833.3\langle x-32\rangle^3 - 16\,020x\right)$$

B: $x = 16$ in, $\quad y_B = \dfrac{1}{36(10^6)}\left[-161.8(16^3) - 16\,020(16)\right] = -0.0255$ in $\quad$ *Ans.*

C: $x = 32$ in,

$$y_C = \frac{1}{36(10^6)}\left[-161.8(32^3) + 659.3(32-16)^3 - 16\,020(32)\right]$$

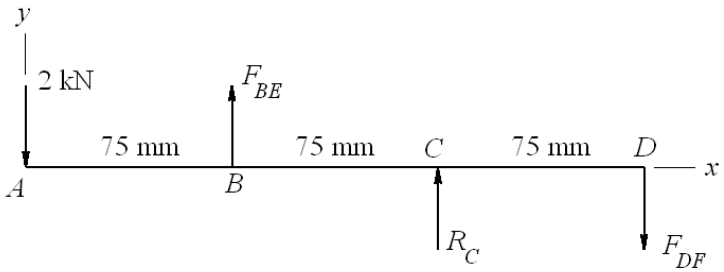
$= -0.0865$ in $\quad$ *Ans.*

D: $x = 48$ in,

$$y_D = \frac{1}{36(10^6)}\left[-161.8(48^3) + 659.3(48-16)^3 - 833.3(48-32)^3 - 16\,020(48)\right]$$

$= -0.0131$ in $\quad$ *Ans.*

4-102 Beam: $EI = 207(10^3)21(10^3)$
$= 4.347(10^9)$ N·mm^2.
Rods: $A = (\pi/4)8^2 = 50.27$ mm^2.



Procedure 2.

1. Statics.

$$R_C + F_{BE} - F_{DF} = 2\,000 \qquad (1)$$

$$R_C + 2F_{BE} = 6\,000 \qquad (2)$$

2. Bending moment equation.

$$M = -2\,000\,x + F_{BE}\langle x - 75\rangle^1 + R_C\langle x - 150\rangle^1$$

$$EI\frac{dy}{dx} = -1000x^2 + \frac{1}{2}F_{BE}\left\langle x-75\right\rangle^2 + \frac{1}{2}R_C\left\langle x-150\right\rangle^2 + C_1 \qquad (3)$$

$$EIy = -\frac{1000}{3}x^3 + \frac{1}{6}F_{BE}\left\langle x-75\right\rangle^3 + \frac{1}{6}R_C\left\langle x-150\right\rangle^3 + C_1x + C_2 \qquad (4)$$

3. B.C 1. At $x = 75$ mm,

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{F_{BE}(50)}{50.27(207)10^3} = -4.805\left(10^{-6}\right)F_{BE}$$

Substituting into Eq. (4) at $x = 75$ mm,

$$4.347\left(10^9\right)\left[-4.805\left(10^{-6}\right)F_{BE}\right] = -\frac{1000}{3}\left(75^3\right) + C_1(75) + C_2$$

Simplifying gives

$$20.89\left(10^3\right)F_{BE} + 75C_1 + C_2 = 140.6\left(10^6\right) \qquad (5)$$

B.C 2. At $x = 150$ mm, $y = 0$. From Eq. (4),

$$-\frac{1000}{3}\left(150^3\right) + \frac{1}{6}F_{BE}(150-75)^3 + C_1(150) + C_2 = 0$$

or,

$$70.31\left(10^3\right)F_{BE} + 150C_1 + C_2 = 1.125\left(10^9\right) \qquad (6)$$

B.C 3. At $x = 225$ mm,

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{F_{DF}(65)}{50.27(207)10^3} = 6.246\left(10^{-6}\right)F_{DF}$$

Substituting into Eq. (4) at $x = 225$ mm,

$$4.347\left(10^9\right)\left[6.246\left(10^{-6}\right)F_{DF}\right] = -\frac{1000}{3}\left(225^3\right) + \frac{1}{6}F_{BE}\left(225-75\right)^3 + \frac{1}{6}R_C\left(225-150\right)^3 + C_1\left(225\right) + C_2$$

Simplifying gives

$$70.31\left(10^3\right)R_C + 562.5\left(10^3\right)F_{BE} - 27.15\left(10^3\right)F_{DF} + 225C_1 + C_2 = 3.797\left(10^9\right) \qquad (7)$$

Equations (1), (2), (5), (6), and (7) in matrix form are

$$\begin{pmatrix} 1 & 1 & -1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ 0 & 20.89\left(10^3\right) & 0 & 75 & 1 \\ 0 & 70.31\left(10^3\right) & 0 & 150 & 1 \\ 70.31\left(10^3\right) & 562.5\left(10^3\right) & -27.15\left(10^3\right) & 225 & 1 \end{pmatrix} \begin{Bmatrix} R_C \\ F_{BE} \\ F_{DF} \\ C_1 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 2\left(10^3\right) \\ 6\left(10^3\right) \\ 140.6\left(10^6\right) \\ 1.125\left(10^9\right) \\ 3.797\left(10^9\right) \end{Bmatrix}$$

Solve simultaneously or use software. The results are

$R_C = -2378$ N, $F_{BE} = 4189$ N, $F_{DF} = -189.2$ N *Ans.*
and $C_1 = 1.036\ (10^7)$ N·mm^2, $C_2 = -7.243\ (10^8)$ N·mm^3.

The bolt stresses are $\sigma_{BE} = 4189/50.27 = 83.3$ MPa, $\sigma_{DF} = -189/50.27 = -3.8$ MPa *Ans.*

The deflections are

From Eq. (4) $y_A = \dfrac{1}{4.347\left(10^9\right)}\left[-7.243\left(10^8\right)\right] = -0.167$ mm *Ans.*

For points *B* and *D* use the axial deflection equations*.

$$y_B = -\left(\frac{Fl}{AE}\right)_{BE} = -\frac{4189(50)}{50.27(207)10^3} = -0.0201 \text{ mm} \qquad \textit{Ans.}$$

$$y_D = \left(\frac{Fl}{AE}\right)_{DF} = \frac{-189(65)}{50.27(207)10^3} = -1.18\left(10^{-3}\right) \text{ mm} \qquad \textit{Ans.}$$

*Note. The terms in Eq. (4) are quite large, and due to rounding are not very accurate for calculating the very small deflections, especially for point *D*.

4-103 (**a**) The cross section at *A* does not rotate. Thus, for a single quadrant we have

$$\frac{\partial U}{\partial M_A} = 0$$

The bending moment at an angle θ to the x axis is

$$M = M_A - \frac{FR}{2}(1-\cos\theta) \qquad \frac{\partial M}{\partial M_A} = 1$$

The rotation at A is

$$\theta_A = \frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^{\pi/2} M\,\frac{\partial M}{\partial M_A}\,Rd\theta = 0$$

Thus, $$\frac{1}{EI}\int_0^{\pi/2}\left[M_A - \frac{FR}{2}(1-\cos\theta)\right](1)\,Rd\theta = 0 \quad \Rightarrow \quad \left(M_A - \frac{FR}{2}\right)\frac{\pi}{2} + \frac{FR}{2} = 0$$

or,

$$M_A = \frac{FR}{2}\left(1 - \frac{2}{\pi}\right)$$

Substituting this into the equation for M gives

$$M = \frac{FR}{2}\left(\cos\theta - \frac{2}{\pi}\right) \quad (1)$$

The maximum occurs at B where $\theta = \pi/2$

$$M_{\max} = M_B = -\frac{FR}{\pi} \qquad Ans.$$

(**b**) Assume B is supported on a knife edge. The deflection of point D is $\partial U/\partial F$. We will deal with the quarter-ring segment and multiply the results by 4. From Eq. (1)

$$\frac{\partial M}{\partial F} = \frac{R}{2}\left(\cos\theta - \frac{2}{\pi}\right)$$

Thus,

$$\delta_D = \frac{\partial U}{\partial F} = \frac{4}{EI}\int_0^{\pi/2} M\,\frac{\partial M}{\partial F}\,Rd\theta = \frac{FR^3}{EI}\int_0^{\pi/2}\left(\cos\theta - \frac{2}{\pi}\right)^2 d\theta = \frac{FR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right)$$

$$= \frac{FR^3}{4\pi EI}\left(\pi^2 - 8\right) \quad Ans.$$

4-104

$$P_{\text{cr}} = \frac{C\pi^2 EI}{l^2}$$

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) = \frac{\pi D^4}{64}\left(1 - K^4\right) \qquad \text{where } K = \frac{d}{D}$$

$$P_{\text{cr}} = \frac{C\pi^2 E}{l^2}\left[\frac{\pi D^4}{64}\left(1 - K^4\right)\right]$$

$$D = \left[\frac{64 P_{\text{cr}} l^2}{\pi^3 C E \left(1 - K^4\right)} \right]^{1/4} \qquad \textit{Ans.}$$

4-105 $A = \dfrac{\pi}{4} D^2 \left(1 - K^2\right), \quad I = \dfrac{\pi}{64} D^4 \left(1 - K^4\right) = \dfrac{\pi}{64} D^4 \left(1 - K^2\right)\left(1 + K^2\right),$ where $K = d / D$.

The radius of gyration, k, is given by

$$k^2 = \frac{I}{A} = \frac{D^2}{16}\left(1 + K^2\right)$$

From Eq. (4-46)

$$\frac{P_{\text{cr}}}{(\pi/4) D^2 \left(1 - K^2\right)} = S_y - \frac{S_y^2 l^2}{4\pi^2 k^2 C E} = S_y - \frac{S_y^2 l^2}{4\pi^2 \left(D^2/16\right)\left(1 + K^2\right) C E}$$

$$4 P_{\text{cr}} = \pi D^2 \left(1 - K^2\right) S_y - \frac{4 S_y^2 l^2 \pi D^2 \left(1 - K^2\right)}{\pi^2 D^2 \left(1 + K^2\right) C E}$$

$$\pi D^2 \left(1 - K^2\right) S_y = 4 P_{\text{cr}} + \frac{4 S_y^2 l^2 \left(1 - K^2\right)}{\pi \left(1 + K^2\right) C E}$$

$$D = \left[\frac{4 P_{\text{cr}}}{\pi S_y \left(1 - K^2\right)} + \frac{4 S_y^2 l^2 \left(1 - K^2\right)}{\pi \left(1 + K^2\right) C E \pi \left(1 - K^2\right) S_y} \right]^{1/2}$$

$$= 2 \left[\frac{P_{\text{cr}}}{\pi S_y \left(1 - K^2\right)} + \frac{S_y l^2}{\pi^2 C E \left(1 + K^2\right)} \right]^{1/2} \qquad \textit{Ans.}$$

4-106 **(a)** $\Sigma M_A = 0, \quad (0.75)(800) - \dfrac{0.9}{\sqrt{0.9^2 + 0.5^2}} F_{BO}(0.5) = 0 \quad \Rightarrow \quad F_{BO} = 1373 \text{ N}$

Using $n_d = 4$, design for $F_{\text{cr}} = n_d F_{BO} = 4(1373) = 5492$ N

$l = \sqrt{0.9^2 + 0.5^2} = 1.03$ m, $\quad S_y = 165$ MPa

In-plane:

$$k = \left(\frac{I}{A}\right)^{1/2} = \left(\frac{bh^3/12}{bh}\right)^{1/2} = 0.2887h = 0.2887(0.025) = 0.007\,218 \text{ m}, \qquad C = 1.0$$

$$\frac{l}{k} = \frac{1.03}{0.007218} = 142.7$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 (207)(10^9)}{165(10^6)}\right)^{1/2} = 157.4$$

Since $(l/k)_1 > (l/k)$ use Johnson formula.

Try 25 mm x 12 mm,

$$P_{cr} = 0.025(0.012)\left\{165\left(10^6\right) - \left[\frac{165\left(10^6\right)}{2\pi}(142.7)\right]^2 \frac{1}{1(207)10^9}\right\} = 29.1 \text{ kN}$$

This is significantly greater than the design load of 5492 N found earlier. Check out-of-plane.

Out-of-plane: $k = 0.2887(0.012) = 0.003\,464$ in, $C = 1.2$

$$\frac{l}{k} = \frac{1.03}{0.003\,464} = 297.3$$

Since $(l/k)_1 < (l/k)$ use Euler equation.

$$P_{cr} = 0.025(0.012)\frac{1.2\pi^2\left(207\right)10^9}{297.3^2} = 8\,321 \text{ N}$$

This is greater than the design load of 5492 N found earlier. It is also significantly less than the in-plane P_{cr} found earlier, so the out-of-plane condition will dominate. Iterate the process to find the minimum h that gives P_{cr} greater than the design load.

With $h = 0.010$, $P_{cr} = 4815$ N (too small)
$h = 0.011$, $P_{cr} = 6409$ N (acceptable)

Use 25 mm x 11 mm. If standard size is preferred, use 25 mm x 12 mm. *Ans.*

(**b**) $\sigma_b = -\dfrac{P}{dh} = -\dfrac{1373}{0.012(0.011)} = -10.4\left(10^6\right)$ Pa $= -10.4$ MPa

No, bearing stress is not significant. *Ans.*

4-107 This is an open-ended design problem with no one distinct solution.

4-108 $F = 1500(\pi/4)2^2 = 4712$ lbf. From Table A-20, $S_y = 37.5$ kpsi
$P_{cr} = n_d F = 2.5(4712) = 11\,780$ lbf

(**a**) Assume Euler with $C = 1$

$$I = \frac{\pi}{64}d^4 = \frac{P_{cr}l^2}{C\pi^2 E} \quad \Rightarrow \quad d = \left(\frac{64P_{cr}l^2}{\pi^3 CE}\right)^{1/4} = \left[\frac{64\left(11790\right)50^2}{\pi^3\left(1\right)30\left(10^6\right)}\right]^{1/4} = 1.193 \text{ in}$$

Use $d = 1.25$ in. The radius of gyration, $k = (I/A)^{1/2} = d/4 = 0.3125$ in

$$\frac{l}{k}=\frac{50}{0.3125}=160$$

$$\left(\frac{l}{k}\right)_1=\left(\frac{2\pi^2CE}{S_y}\right)^{1/2}=\left(\frac{2\pi^2(1)30\left(10^6\right)}{37.5\left(10^3\right)}\right)^{1/2}=126 \qquad \therefore \text{ use Euler}$$

$$P_{\text{cr}}=\frac{\pi^2(30)10^6(\pi/64)1.25^4}{50^2}=14194 \text{ lbf}$$

Since 14 194 lbf > 11 780 lbf, $d = 1.25$ in is satisfactory. *Ans.*

(**b**)
$$d=\left[\frac{64(11780)16^2}{\pi^3(1)30\left(10^6\right)}\right]^{1/4}=0.675 \text{ in, so use } d=0.750 \text{ in}$$

$k = 0.750/4 = 0.1875$ in

$$\frac{l}{k}=\frac{16}{0.1875}=85.33 \qquad \text{use Johnson}$$

$$P_{\text{cr}}=\frac{\pi}{4}\left(0.750^2\right)\left\{37.5\left(10^3\right)-\left[\frac{37.5\left(10^3\right)}{2\pi}85.33\right]^2\frac{1}{1(30)10^6}\right\}=12748 \text{ lbf}$$

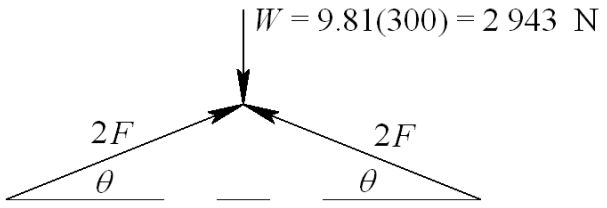
Use $d = 0.75$ in.

(**c**)

$$n_{(a)}=\frac{14194}{4712}=3.01 \qquad \textit{Ans.}$$

$$n_{(b)}=\frac{12748}{4712}=2.71 \qquad \textit{Ans.}$$

4-109 From Table A-20, $S_y = 180$ MPa



$$4F\sin\theta = 2\,943$$

$$F=\frac{735.8}{\sin\theta}$$

In range of operation, F is maximum when $\theta = 15°$

$$F_{\text{max}}=\frac{735.8}{\sin 15^\circ}=2843 \text{ N per bar}$$

$$P_{\text{cr}} = n_d F_{\text{max}} = 3.50\ (2\ 843) = 9\ 951 \text{ N}$$

$l = 350$ mm, $h = 30$ mm

Try $b = 5$ mm. Out of plane, $k = b/\sqrt{12} = 5/\sqrt{12} = 1.443$ mm

$$\frac{l}{k} = \frac{350}{1.443} = 242.6$$

$$\left(\frac{l}{k}\right)_1 = \left[\frac{2\pi^2(1.4)207(10^9)}{180(10^6)}\right]^{1/2} = 178.3 \qquad \therefore \text{ use Euler}$$

$$P_{cr} = A\frac{C\pi^2 E}{(l/k)^2} = 5(30)\frac{1.4\pi^2(207)10^3}{(242.6)^2} = 7\,290 \text{ N}$$

Too low. Try $b = 6$ mm. $k = 6/\sqrt{12} = 1.732$ mm

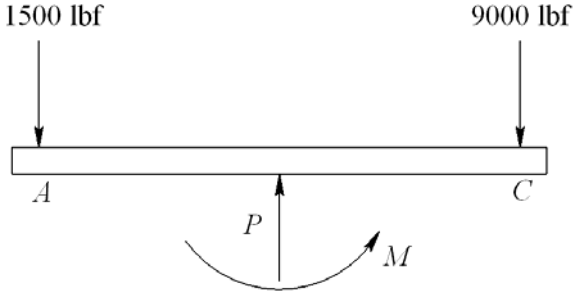
$$\frac{l}{k} = \frac{350}{1.732} = 202.1$$

$$P_{cr} = A\frac{C\pi^2 E}{(l/k)^2} = 6(30)\frac{1.4\pi^2(207)10^3}{(202.1)^2} = 12\,605 \text{ N}$$

O.K. Use 25 × 6 mm bars *Ans.* The factor of safety is

$$n = \frac{12\,605}{2\,843} = 4.43 \quad \textit{Ans.}$$

4-110 $P = 1\,500 + 9\,000 = 10\,500$ lbf *Ans.*



$\Sigma M_A = 10\,500\,(4.5/2) - 9\,000\,(4.5) + M = 0$

$M = 16\,874$ lbf·in

$e = M/P = 16\,874/10\,500 = 1.607$ in *Ans.*

From Table A-8, $A = 2.160$ in^2, and $I = 2.059$ in^4. The stresses are determined using Eq. (4-55)

$$k^2 = \frac{I}{A} = \frac{2.059}{2.160} = 0.953 \text{ in}^2$$

$$\sigma_c = -\frac{P}{A}\left(1 + \frac{ec}{k^2}\right) = -\frac{10\,500}{2.160}\left[1 + \frac{1.607(3/2)}{0.953}\right] = -17157 \text{ psi} = -17.16 \text{ kpsi} \quad \textit{Ans.}$$

4-111 This is a design problem which has no single distinct solution.

4-112 Loss of potential energy of weight = $W(h+\delta)$

Increase in potential energy of spring = $\frac{1}{2}k\delta^2$

$$W(h+\delta) = \frac{1}{2}k\delta^2$$

or, $\delta^2 - \frac{2W}{k}\delta - \frac{2W}{k}h = 0$. W = 30 lbf, k = 100 lbf/in, h = 2 in yields

$$\delta^2 - 0.6\,\delta - 1.2 = 0$$

Taking the positive root (see discussion on p. 192)

$$\delta_{\max} = \frac{1}{2}\left[0.6 + \sqrt{(-0.6)^2 + 4(1.2)}\right] = 1.436 \text{ in} \qquad \textit{Ans.}$$

$$F_{\max} = k\,\delta_{\max} = 100\,(1.436) = 143.6 \text{ lbf} \qquad \textit{Ans.}$$

4-113 The drop of weight W_1 converts potential energy, $W_1\ h$, to kinetic energy $\frac{1}{2}\frac{W_1}{g}v_1^2$.

Equating these provides the velocity of W_1 at impact with W_2.

$$W_1 h = \frac{1}{2}\frac{W_1}{g}v_1^2 \qquad \Rightarrow \qquad v_1 = \sqrt{2gh} \qquad (1)$$

Since the collision is inelastic, momentum is conserved. That is, $(m_1 + m_2)\,v_2 = m_1\,v_1$, where v_2 is the velocity of $W_1 + W_2$ after impact. Thus

$$\frac{W_1 + W_2}{g}v_2 = \frac{W_1}{g}v_1 \qquad \Rightarrow \qquad v_2 = \frac{W_1}{W_1 + W_2}v_1 = \frac{W_1}{W_1 + W_2}\sqrt{2gh} \qquad (2)$$

The kinetic and potential energies of $W_1 + W_2$ are then converted to potential energy of the spring. Thus,

$$\frac{1}{2}\frac{W_1 + W_2}{g}v_2^2 + \left(W_1 + W_2\right)\delta = \frac{1}{2}k\delta^2$$

Substituting in Eq. (1) and rearranging results in

$$\delta^2 - 2\frac{W_1 + W_2}{k}\delta - 2\frac{W_1^2}{W_1 + W_2}\frac{h}{k} = 0 \qquad (3)$$

Solving for the positive root (see discussion on p. 192)

$$\delta = \frac{1}{2}\left[2\frac{W_1 + W_2}{k} + \sqrt{4\left(\frac{W_1 + W_2}{k}\right)^2 + 8\frac{W_1^2}{W_1 + W_2}\frac{h}{k}}\right] \qquad (4)$$

$W_1 = 40$ N, $W_2 = 400$ N, $h = 200$ mm, $k = 32$ kN/m = 32 N/mm.

$$\delta = \frac{1}{2}\left[2\left(\frac{40+400}{32}\right) + \sqrt{4\left(\frac{40+400}{32}\right)^2 + 8\frac{40^2}{40+400}\frac{200}{32}}\right] = 29.06 \text{ mm} \qquad \textit{Ans.}$$

$F_{\max} = k\delta = 32(29.06) = 930$ N *Ans.*

4-114 The initial potential energy of the k_1 spring is $V_i = \frac{1}{2}k_1a^2$. The movement of the weight W the distance y gives a final potential of $V_f = \frac{1}{2}k_1(a-y)^2 + \frac{1}{2}k_2y^2$. Equating the two energies give

$$\frac{1}{2}k_1a^2 = \frac{1}{2}k_1(a-y)^2 + \frac{1}{2}k_2y^2$$

Simplifying gives

$$(k_1 + k_2)y^2 - 2ak_1y = 0$$

This has two roots, $y = 0$, $\frac{2k_1a}{k_1+k_2}$. Without damping the weight will vibrate between these two limits. The maximum displacement is thus $y_{\max} = \frac{2k_1a}{k_1+k_2}$ *Ans.*

With $W = 5$ lbf, $k_1 = 10$ lbf/in, $k_2 = 20$ lbf/in, and $a = 0.25$ in

$$y_{\max} = \frac{2(0.25)10}{10+20} = 0.1667 \text{ in} \qquad \textit{Ans.}$$

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p. 238 $$n = \frac{K_{Ic}}{K_I} \tag{5–38}$$

where K_{Ic} is found in Table 5–1 (p. 238)

Stochastic Analysis

Mean factor of safety defined as $\bar{n} = \mu_S/\mu_\sigma$ (μ_S and μ_σ are mean strength and stress, respectively)

Normal-Normal Case

p. 241 $$\bar{n} = \frac{1 \pm \sqrt{1 - (1 - z^2 C_s^2)(1 - z^2 C_\sigma^2)}}{1 - z^2 C_s^2} \tag{5–42}$$

where z can be found in Table A–10, $C_S = \hat{\sigma}_S/\mu_S$, and $C_\sigma = \hat{\sigma}_\sigma/\mu_\sigma$.

Lognormal-Lognormal Case

p. 242 $$\bar{n} = \exp\left[-z\sqrt{\ln(1 + C_n^2)} + \ln\sqrt{1 + C_n^2}\right] \doteq \exp\left[C_n\left(-z + \frac{C_n}{2}\right)\right] \tag{5–45}$$

where

$$C_n = \sqrt{\frac{C_S^2 + C_\sigma^2}{1 + C_\sigma^2}}$$

(See other definitions in normal-normal case.)

Extracted form 8th ed.

PROBLEMS

solutions to chap. 5 are missing; best substitute available is used (chap. 5 of 8th edition)

5–1 A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 50 kpsi. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:
(*a*) $\sigma_x = 12$ kpsi, $\sigma_y = 6$ kpsi
(*b*) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi
(*c*) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi
(*d*) $\sigma_x = 12$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi

5–2 Repeat Prob. 5–1 for:
(*a*) $\sigma_A = 12$ kpsi, $\sigma_B = 12$ kpsi
(*b*) $\sigma_A = 12$ kpsi, $\sigma_B = 6$ kpsi
(*c*) $\sigma_A = 12$ kpsi, $\sigma_B = -12$ kpsi
(*d*) $\sigma_A = -6$ kpsi, $\sigma_B = -12$ kpsi

5–3 Repeat Prob. 5–1 for a bar of AISI 1020 cold-drawn steel and:
(*a*) $\sigma_x = 180$ MPa, $\sigma_y = 100$ MPa
(*b*) $\sigma_x = 180$ MPa, $\tau_{xy} = 100$ MPa
(*c*) $\sigma_x = -160$ MPa, $\tau_{xy} = 100$ MPa
(*d*) $\tau_{xy} = 150$ MPa

5–4 Repeat Prob. 5–1 for a bar of AISI 1018 hot-rolled steel and:
(*a*) $\sigma_A = 100$ MPa, $\sigma_B = 80$ MPa
(*b*) $\sigma_A = 100$ MPa, $\sigma_B = 10$ MPa
(*c*) $\sigma_A = 100$ MPa, $\sigma_B = -80$ MPa
(*d*) $\sigma_A = -80$ MPa, $\sigma_B = -100$ MPa

5–5 Repeat Prob. 5–3 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

5–6 Repeat Prob. 5–4 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

5–7 An ASTM cast iron has minimum ultimate strengths of 30 kpsi in tension and 100 kpsi in compression. Find the factors of safety using the MNS, BCM, and MM theories for each of the following stress states. Plot the failure diagrams in the σ_A, σ_B plane to scale and locate the coordinates of each stress state.
(a) $\sigma_x = 20$ kpsi, $\sigma_y = 6$ kpsi
(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi
(c) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi
(d) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

5–8 For Prob. 5–7, case (d), estimate the factors of safety from the three theories by graphical measurements of the load line.

5–9 Among the decisions a designer must make is selection of the failure criteria that is applicable to the material and its static loading. A 1020 hot-rolled steel has the following properties: $S_y = 42$ kpsi, $S_{ut} = 66.2$ kpsi, and true strain at fracture $\varepsilon_f = 0.90$. Plot the failure locus and, for the static stress states at the critical locations listed below, plot the load line and estimate the factor of safety analytically and graphically.
(a) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi.
(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = 3$ kpsi ccw.
(c) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi cw.
(d) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi cw.

5–10 A 4142 steel Q&T at 80°F exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi, and $\varepsilon_f = 0.06$. Choose and plot the failure locus and, for the static stresses at the critical locations, which are 10 times those in Prob. 5–9, plot the load lines and estimate the factors of safety analytically and graphically.

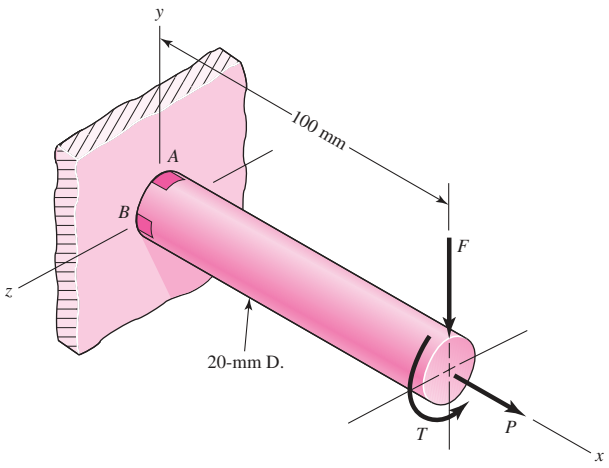
5–11 For grade 20 cast iron, Table A–24 gives $S_{ut} = 22$ kpsi, $S_{uc} = 83$ kpsi. Choose and plot the failure locus and, for the static loadings inducing the stresses at the critical locations of Prob. 5–9, plot the load lines and estimate the factors of safety analytically and graphically.

5–12 A cast aluminum 195-T6 has an ultimate strength in tension of $S_{ut} = 36$ kpsi and ultimate strength in compression of $S_{uc} = 35$ kpsi, and it exhibits a true strain at fracture $\varepsilon_f = 0.045$. Choose and plot the failure locus and, for the static loading inducing the stresses at the critical locations of Prob. 5–9, plot the load lines and estimate the factors of safety analytically and graphically.

5–13 An ASTM cast iron, grade 30 (see Table A–24), carries static loading resulting in the stress state listed below at the critical locations. Choose the appropriate failure locus, plot it and the load lines, and estimate the factors of safety analytically and graphically.
(a) $\sigma_A = 20$ kpsi, $\sigma_B = 20$ kpsi.
(b) $\tau_{xy} = 15$ kpsi.
(c) $\sigma_A = \sigma_B = -80$ kpsi.
(d) $\sigma_A = 15$ kpsi, $\sigma_B = -25$ kpsi.

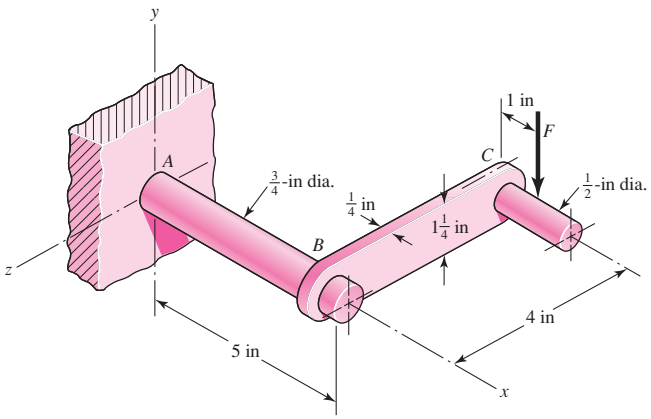
5–14 This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. Here you are to compute factors of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces $F = 0.55$ kN, $P = 8.0$ kN, and $T = 30$ N · m.

Problem 5–14



5–15 The figure shows a crank loaded by a force $F = 190$ lbf which causes twisting and bending of the $\frac{3}{4}$-in-diameter shaft fixed to a support at the origin of the reference system. In actuality, the support may be an inertia which we wish to rotate, but for the purposes of a strength analysis we can consider this to be a statics problem. The material of the shaft *AB* is hot-rolled AISI 1018 steel (Table A–20). Using the maximum-shear-stress theory, find the factor of safety based on the stress at point *A*.

Problem 5–15



5–16 Solve Prob. 5–15 using the distortion energy theory. If you have solved Prob. 5–15, compare the results and discuss the difference.

5–17* Design the lever arm *CD* of Fig. 5–16 by specifying a suitable size and material.

5–18 A spherical pressure vessel is formed of 18-gauge (0.05-in) cold-drawn AISI 1018 sheet steel. If the vessel has a diameter of 8 in, estimate the pressure necessary to initiate yielding. What is the estimated bursting pressure?

*The asterisk indicates a problem that may not have a unique result or may be a particularly challenging problem.

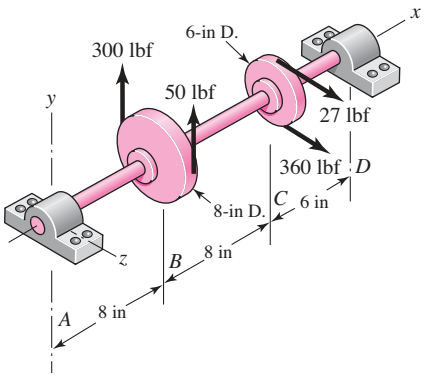
5–19 This problem illustrates that the strength of a machine part can sometimes be measured in units other than those of force or moment. For example, the maximum speed that a flywheel can reach without yielding or fracturing is a measure of its strength. In this problem you have a rotating ring made of hot-forged AISI 1020 steel; the ring has a 6-in inside diameter and a 10-in outside diameter and is 1.5 in thick. What speed in revolutions per minute would cause the ring to yield? At what radius would yielding begin? [*Note:* The maximum radial stress occurs at $r = (r_o r_i)^{1/2}$; see Eq. (3–55).]

5–20 A light pressure vessel is made of 2024-T3 aluminum alloy tubing with suitable end closures. This cylinder has a $3\frac{1}{2}$-in OD, a 0.065-in wall thickness, and $\nu = 0.334$. The purchase order specifies a minimum yield strength of 46 kpsi. What is the factor of safety if the pressure-release valve is set at 500 psi?

5–21 A cold-drawn AISI 1015 steel tube is 300 mm OD by 200 mm ID and is to be subjected to an external pressure caused by a shrink fit. What maximum pressure would cause the material of the tube to yield?

5–22 What speed would cause fracture of the ring of Prob. 5–19 if it were made of grade 30 cast iron?

5–23 The figure shows a shaft mounted in bearings at A and D and having pulleys at B and C. The forces shown acting on the pulley surfaces represent the belt tensions. The shaft is to be made of ASTM grade 25 cast iron using a design factor $n_d = 2.8$. What diameter should be used for the shaft?



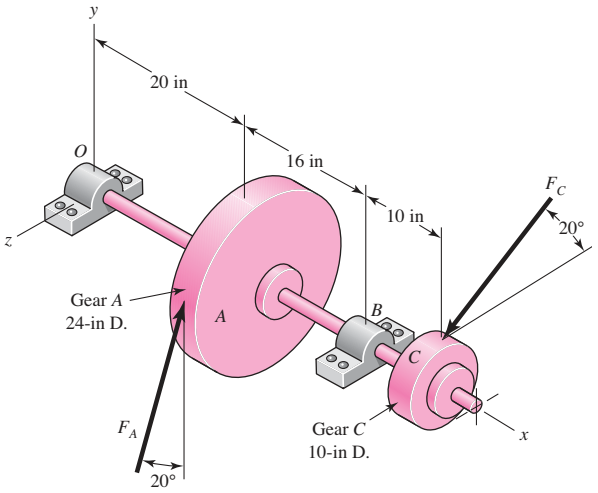
Problem 5–23

5–24 By modern standards, the shaft design of Prob. 5–23 is poor because it is so long. Suppose it is redesigned by halving the length dimensions. Using the same material and design factor as in Prob. 5–23, find the new shaft diameter.

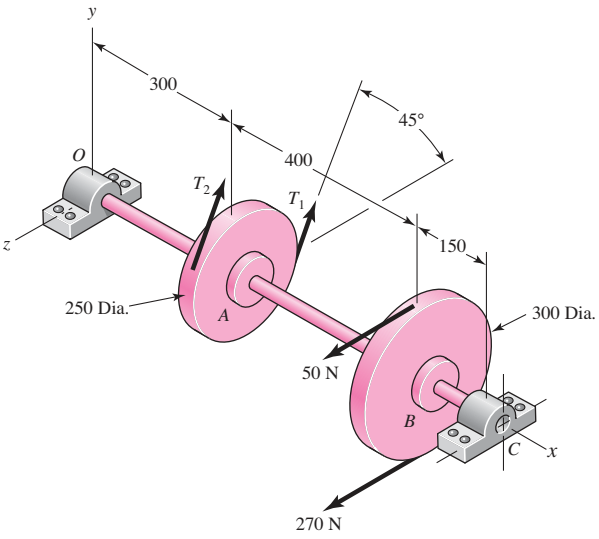
5–25 The gear forces shown act in planes parallel to the yz plane. The force on gear A is 300 lbf. Consider the bearings at O and B to be simple supports. For a static analysis and a factor of safety of 3.5, use distortion energy to determine the minimum safe diameter of the shaft. Consider the material to have a yield strength of 60 kpsi.

5–26 Repeat Prob. 5–25 using maximum-shear-stress.

5–27 The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. For each pulley, the belt tensions are parallel. For pulley A consider the loose belt tension is 15 percent of the tension on the tight side. A cold-drawn UNS G10180 steel shaft of uniform diameter is to be selected for this application. For a static analysis with a factor of safety of 3.0, determine the minimum preferred size diameter. Use the distortion-energy theory.



Problem 5–25



Problem 5–27
Dimensions in millimeters

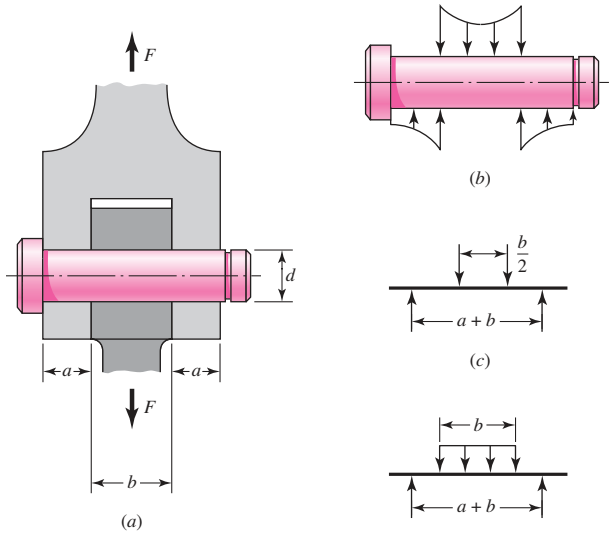
5–28 Repeat Prob. 5–27 using maximum shear stress.

5–29 The clevis pin shown in the figure is 12 mm in diameter and has the dimensions $a = 12$ mm and $b = 18$ mm. The pin is machined from AISI 1018 hot-rolled steel (Table A–20) and is to be loaded to no more than 4.4 kN. Determine whether or not the assumed loading of figure *c* yields a factor of safety any different from that of figure *d*. Use the maximum-shear-stress theory.

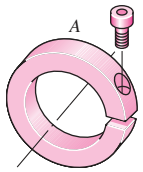
5–30 Repeat Prob. 5–29, but this time use the distortion-energy theory.

5–31 A split-ring clamp-type shaft collar is shown in the figure. The collar is 2 in OD by 1 in ID by $\frac{1}{2}$ in wide. The screw is designated as $\frac{1}{4}$-28 UNF. The relation between the screw tightening torque T, the nominal screw diameter d, and the tension in the screw F_i is approximately $T = 0.2\,F_i d$. The shaft is sized to obtain a close running fit. Find the axial holding force F_x of the collar as a function of the coefficient of friction and the screw torque.

Problem 5–29



Problem 5–31



5–32 Suppose the collar of Prob. 5–31 is tightened by using a screw torque of 190 lbf · in. The collar material is AISI 1040 steel heat-treated to a minimum tensile yield strength of 63 kpsi.

(*a*) Estimate the tension in the screw.

(*b*) By relating the tangential stress to the hoop tension, find the internal pressure of the shaft on the ring.

(*c*) Find the tangential and radial stresses in the ring at the inner surface.

(*d*) Determine the maximum shear stress and the von Mises stress.

(*e*) What are the factors of safety based on the maximum-shear-stress hypothesis and the distortion-energy theory?

5–33 In Prob. 5–31, the role of the screw was to induce the hoop tension that produces the clamping. The screw should be placed so that no moment is induced in the ring. Just where should the screw be located?

5–34 A tube has another tube shrunk over it. The specifications are:

	Inner Member	Outer Member
ID	1.000 ± 0.002 in	1.999 ± 0.0004 in
OD	2.000 ± 0.0004 in	3.000 ± 0.004 in

Both tubes are made of a plain carbon steel.

(*a*) Find the nominal shrink-fit pressure and the von Mises stresses at the fit surface.

(*b*) If the inner tube is changed to solid shafting with the same outside dimensions, find the nominal shrink-fit pressure and the von Mises stresses at the fit surface.

5–35 Steel tubes with a Young's modulus of 207 GPa have the specifications:

	Inner Tube	Outer Tube
ID	25 ± 0.050 mm	49.98 ± 0.010 mm
OD	50 ± 0.010 mm	75 ± 0.10 mm

These are shrink-fitted together. Find the nominal shrink-fit pressure and the von Mises stress in each body at the fit surface.

5–36 Repeat Prob. 5–35 for maximum shrink-fit conditions.

5–37 A 2-in-diameter solid steel shaft has a gear with ASTM grade 20 cast-iron hub ($E = 14.5$ Mpsi) shrink-fitted to it. The specifications for the shaft are

$$2.000 \quad \begin{matrix} +0.0000 \\ -0.0004 \end{matrix} \text{ in}$$

The hole in the hub is sized at 1.999 ± 0.0004 in with an OD of $4.00 \pm \frac{1}{32}$ in. Using the midrange values and the modified Mohr theory, estimate the factor of safety guarding against fracture in the gear hub due to the shrink fit.

5–38 Two steel tubes are shrink-fitted together where the nominal diameters are 1.50, 1.75, and 2.00 in. Careful measurement before fitting revealed that the diametral interference between the tubes to be 0.00246 in. After the fit, the assembly is subjected to a torque of 8000 lbf · in and a bending-moment of 6000 lbf · in. Assuming no slipping between the cylinders, analyze the outer cylinder at the inner and outer radius. Determine the factor of safety using distortion energy with $S_y = 60$ kpsi.

5–39 Repeat Prob. 5–38 for the inner tube.

5–40 For Eqs. (5–36) show that the principal stresses are given by

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right)$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right)$$

$$\sigma_3 = \begin{cases} 0 & \text{(plane stress)} \\ \sqrt{\dfrac{2}{\pi r}}\, \nu K_I \cos\dfrac{\theta}{2} & \text{(plane strain)} \end{cases}$$

5–41 Use the results of Prob. 5–40 for plane strain near the tip with $\theta = 0$ and $\nu = \frac{1}{3}$. If the yield strength of the plate is S_y, what is σ_1 when yield occurs?
(a) Use the distortion-energy theory.
(b) Use the maximum-shear-stress theory. Using Mohr's circles, explain your answer.

5–42 A plate 4 in wide, 8 in long, and 0.5 in thick is loaded in tension in the direction of the length. The plate contains a crack as shown in Fig. 5–26 with the crack length of 0.625 in. The material is steel with $K_{Ic} = 70$ kpsi · $\sqrt{\text{in}}$, and $S_y = 160$ kpsi. Determine the maximum possible load that can be applied before the plate (a) yields, and (b) has uncontrollable crack growth.

5–43 A cylinder subjected to internal pressure p_i has an outer diameter of 350 mm and a 25-mm wall thickness. For the cylinder material, $K_{Ic} = 80$ MPa · $\sqrt{\text{m}}$, $S_y = 1200$ MPa, and $S_{ut} = 1350$ MPa.

If the cylinder contains a radial crack in the longitudinal direction of depth 12.5 mm determine the pressure that will cause uncontrollable crack growth.

5–44 A carbon steel collar of length 1 in is to be machined to inside and outside diameters, respectively, of

$$D_i = 0.750 \pm 0.0004 \text{ in} \qquad D_o = 1.125 \pm 0.002 \text{ in}$$

This collar is to be shrink-fitted to a hollow steel shaft having inside and outside diameters, respectively, of

$$d_i = 0.375 \pm 0.002 \text{ in} \qquad d_o = 0.752 \pm 0.0004 \text{ in}$$

These tolerances are assumed to have a normal distribution, to be centered in the spread interval, and to have a total spread of ± 4 standard deviations. Determine the means and the standard deviations of the tangential stress components for both cylinders at the interface.

5–45 Suppose the collar of Prob. 5–44 has a yield strength of $\mathbf{S}_y = \mathbf{N}(95.5, 6.59)$ kpsi. What is the probability that the material will not yield?

5–46 A carbon steel tube has an outside diameter of 1 in and a wall thickness of $\frac{1}{8}$ in. The tube is to carry an internal hydraulic pressure given as $\mathbf{p} = \mathbf{N}(6000,\ 500)$ psi. The material of the tube has a yield strength of $\mathbf{S}_y = \mathbf{N}(50,\ 4.1)$ kpsi. Find the reliability using thin-wall theory.

Extracted from 8th ed.

Chapter 5

5-1

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $n = \dfrac{S_y}{\sigma'}$

$$\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$$

(a) MSS: $\sigma_1 = 12,\ \sigma_2 = 6,\ \sigma_3 = 0$ kpsi

$$n = \frac{50}{12} = 4.17 \quad \textit{Ans.}$$

DE: $\sigma' = (12^2 - 6(12) + 6^2)^{1/2} = 10.39$ kpsi, $\quad n = \dfrac{50}{10.39} = 4.81 \quad \textit{Ans.}$

(b) $\sigma_A, \sigma_B = \dfrac{12}{2} \pm \sqrt{\left(\dfrac{12}{2}\right)^2 + (-8)^2} = 16, -4$ kpsi

$\sigma_1 = 16,\ \sigma_2 = 0,\ \sigma_3 = -4$ kpsi

MSS: $n = \dfrac{50}{16 - (-4)} = 2.5 \quad \textit{Ans.}$

DE: $\sigma' = (12^2 + 3(-8^2))^{1/2} = 18.33$ kpsi, $\quad n = \dfrac{50}{18.33} = 2.73 \quad \textit{Ans.}$

(c) $\sigma_A, \sigma_B = \dfrac{-6 - 10}{2} \pm \sqrt{\left(\dfrac{-6 + 10}{2}\right)^2 + (-5)^2} = -2.615, -13.385$ kpsi

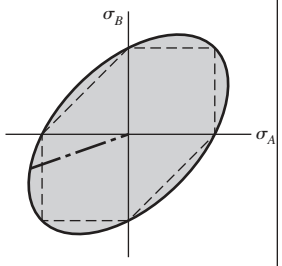
$\sigma_1 = 0,\ \sigma_2 = -2.615,\ \sigma_3 = -13.385$ kpsi

MSS: $n = \dfrac{50}{0 - (-13.385)} = 3.74 \quad \textit{Ans.}$

DE: $\sigma' = [(-6)^2 - (-6)(-10) + (-10)^2 + 3(-5)^2]^{1/2}$

$= 12.29$ kpsi

$$n = \frac{50}{12.29} = 4.07 \quad \textit{Ans.}$$



(d) $\sigma_A, \sigma_B = \dfrac{12 + 4}{2} \pm \sqrt{\left(\dfrac{12 - 4}{2}\right)^2 + 1^2} = 12.123, 3.877$ kpsi

$\sigma_1 = 12.123,\ \sigma_2 = 3.877,\ \sigma_3 = 0$ kpsi

MSS: $n = \dfrac{50}{12.123 - 0} = 4.12 \quad \textit{Ans.}$

DE: $\sigma' = [12^2 - 12(4) + 4^2 + 3(1^2)]^{1/2} = 10.72$ kpsi

$$n = \frac{50}{10.72} = 4.66 \quad \textit{Ans.}$$

5-2 $S_y = 50$ kpsi

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12 - 0} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(12) + 12^2]^{1/2}} = 4.17$ *Ans.*

(b) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = 0$, $n = \dfrac{50}{12} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(6) + 6^2]^{1/2}} = 4.81$ *Ans.*

(c) MSS: $\sigma_1 = 12$ kpsi, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{12 - (-12)} = 2.08$ *Ans.*

DE: $n = \dfrac{50}{[12^2 - (12)(-12) + (-12)^2]^{1/3}} = 2.41$ *Ans.*

(d) MSS: $\sigma_1 = 0$, $\sigma_3 = -12$ kpsi, $n = \dfrac{50}{-(-12)} = 4.17$ *Ans.*

DE: $n = \dfrac{50}{[(-6)^2 - (-6)(-12) + (-12)^2]^{1/2}} = 4.81$

5-3 $S_y = 390$ MPa

MSS: $\sigma_1 - \sigma_3 = S_y/n \quad \Rightarrow \quad n = \dfrac{S_y}{\sigma_1 - \sigma_3}$

DE: $\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = S_y/n \quad \Rightarrow \quad n = S_y/\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$

(a) MSS: $\sigma_1 = 180$ MPa, $\sigma_3 = 0$, $n = \dfrac{390}{180} = 2.17$ *Ans.*

DE: $n = \dfrac{390}{[180^2 - 180(100) + 100^2]^{1/2}} = 2.50$ *Ans.*

(b) $\sigma_A, \sigma_B = \dfrac{180}{2} \pm \sqrt{\left(\dfrac{180}{2}\right)^2 + 100^2} = 224.5, -44.5 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{224.5 - (-44.5)} = 1.45$ *Ans.*

DE: $n = \dfrac{390}{[180^2 + 3(100^2)]^{1/2}} = 1.56$ *Ans.*

(c) $\sigma_A, \sigma_B = -\dfrac{160}{2} \pm \sqrt{\left(-\dfrac{160}{2}\right)^2 + 100^2} = 48.06, -208.06 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{48.06 - (-208.06)} = 1.52$ *Ans.*

DE: $n = \dfrac{390}{[-160^2 + 3(100^2)]^{1/2}} = 1.65$ *Ans.*

(d) $\sigma_A, \sigma_B = 150, -150 \text{ MPa} = \sigma_1, \sigma_3$

MSS: $n = \dfrac{390}{150 - (-150)} = 1.30$ *Ans.*

DE: $n = \dfrac{390}{[3(150)^2]^{1/2}} = 1.50$ *Ans.*

5-4 $S_y = 220$ MPa

(a) $\sigma_1 = 100, \sigma_2 = 80, \sigma_3 = 0$ MPa

MSS: $n = \dfrac{220}{100 - 0} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(80) + 80^2]^{1/2} = 91.65 \text{ MPa}$

$n = \dfrac{220}{91.65} = 2.40$ *Ans.*

(b) $\sigma_1 = 100, \sigma_2 = 10, \sigma_3 = 0$ MPa

MSS: $n = \dfrac{220}{100} = 2.20$ *Ans.*

DET: $\sigma' = [100^2 - 100(10) + 10^2]^{1/2} = 95.39 \text{ MPa}$

$n = \dfrac{220}{95.39} = 2.31$ *Ans.*

(c) $\sigma_1 = 100, \sigma_2 = 0, \sigma_3 = -80$ MPa

MSS: $n = \dfrac{220}{100 - (-80)} = 1.22$ *Ans.*

DE: $\sigma' = [100^2 - 100(-80) + (-80)^2]^{1/2} = 156.2 \text{ MPa}$

$n = \dfrac{220}{156.2} = 1.41$ *Ans.*

(d) $\sigma_1 = 0, \sigma_2 = -80, \sigma_3 = -100$ MPa

MSS: $n = \dfrac{220}{0 - (-100)} = 2.20$ *Ans.*

DE: $\sigma' = [(-80)^2 - (-80)(-100) + (-100)^2] = 91.65 \text{ MPa}$

$n = \dfrac{220}{91.65} = 2.40$ *Ans.*

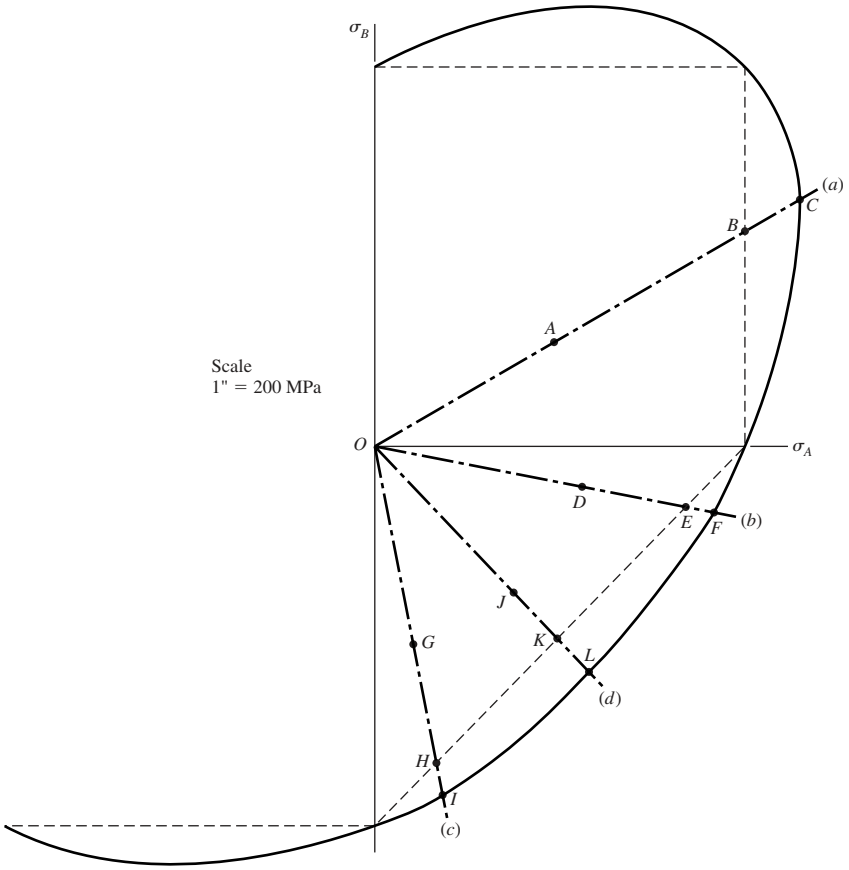
5-5

(a) MSS: $n = \dfrac{OB}{OA} = \dfrac{2.23}{1.08} = 2.1$

DE: $n = \dfrac{OC}{OA} = \dfrac{2.56}{1.08} = 2.4$

(b) MSS: $n = \dfrac{OE}{OD} = \dfrac{1.65}{1.10} = 1.5$

DE: $n = \dfrac{OF}{OD} = \dfrac{1.8}{1.1} = 1.6$



(c) MSS: $n = \dfrac{OH}{OG} = \dfrac{1.68}{1.05} = 1.6$

DE: $n = \dfrac{OI}{OG} = \dfrac{1.85}{1.05} = 1.8$

(d) MSS: $n = \dfrac{OK}{OJ} = \dfrac{1.38}{1.05} = 1.3$

DE: $n = \dfrac{OL}{OJ} = \dfrac{1.62}{1.05} = 1.5$

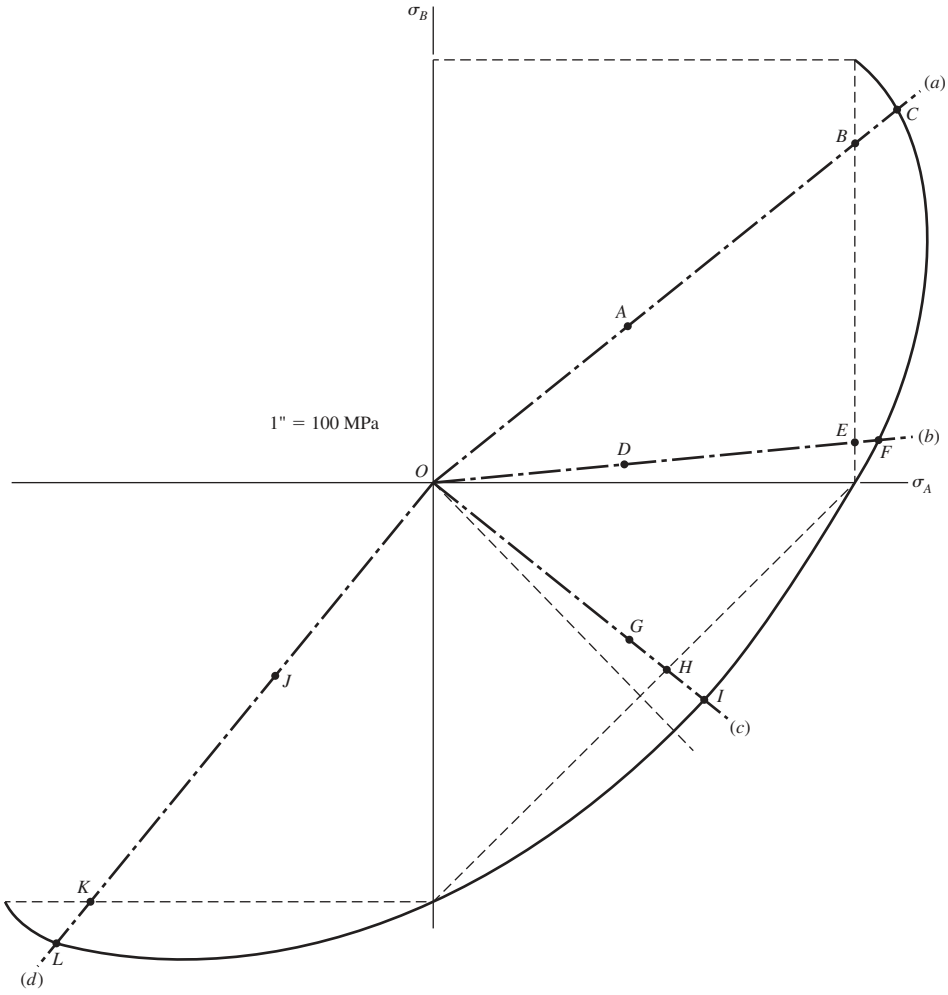
5-6 $S_y = 220$ MPa

(a) MSS: $n = \dfrac{OB}{OA} = \dfrac{2.82}{1.3} = 2.2$

DE: $n = \dfrac{OC}{OA} = \dfrac{3.1}{1.3} = 2.4$

(b) MSS: $n = \dfrac{OE}{OD} = \dfrac{2.2}{1} = 2.2$

DE: $n = \dfrac{OF}{OD} = \dfrac{2.33}{1} = 2.3$



(c) MSS: $n = \dfrac{OH}{OG} = \dfrac{1.55}{1.3} = 1.2$

DE: $n = \dfrac{OI}{OG} = \dfrac{1.8}{1.3} = 1.4$

(d) MSS: $n = \dfrac{OK}{OJ} = \dfrac{2.82}{1.3} = 2.2$

DE: $n = \dfrac{OL}{OJ} = \dfrac{3.1}{1.3} = 2.4$

5-7 $S_{ut} = 30$ kpsi, $S_{uc} = 100$ kpsi; $\sigma_A = 20$ kpsi, $\sigma_B = 6$ kpsi

(a) MNS: Eq. (5-30a) $$n = \frac{S_{ut}}{\sigma_x} = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

BCM: Eq. (5-31a) $$n = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

MM: Eq. (5-32a) $$n = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi

$$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + (-8)^2} = 16, -4 \text{ kpsi}$$

MNS: Eq. (5-30a) $$n = \frac{30}{16} = 1.88 \quad \textit{Ans.}$$

BCM: Eq. (5-31b) $$\frac{1}{n} = \frac{16}{30} - \frac{(-4)}{100} \quad \Rightarrow \quad n = 1.74 \quad \textit{Ans.}$$

MM: Eq. (5-32a) $$n = \frac{30}{16} = 1.88 \quad \textit{Ans.}$$

(c) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi

$$\sigma_A, \sigma_B = \frac{-6 - 10}{2} \pm \sqrt{\left(\frac{-6 + 10}{2}\right)^2 + (-5)^2} = -2.61, -13.39 \text{ kpsi}$$

MNS: Eq. (5-30b) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

BCM: Eq. (5-31c) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

MM: Eq. (5-32c) $$n = -\frac{100}{-13.39} = 7.47 \quad \textit{Ans.}$$

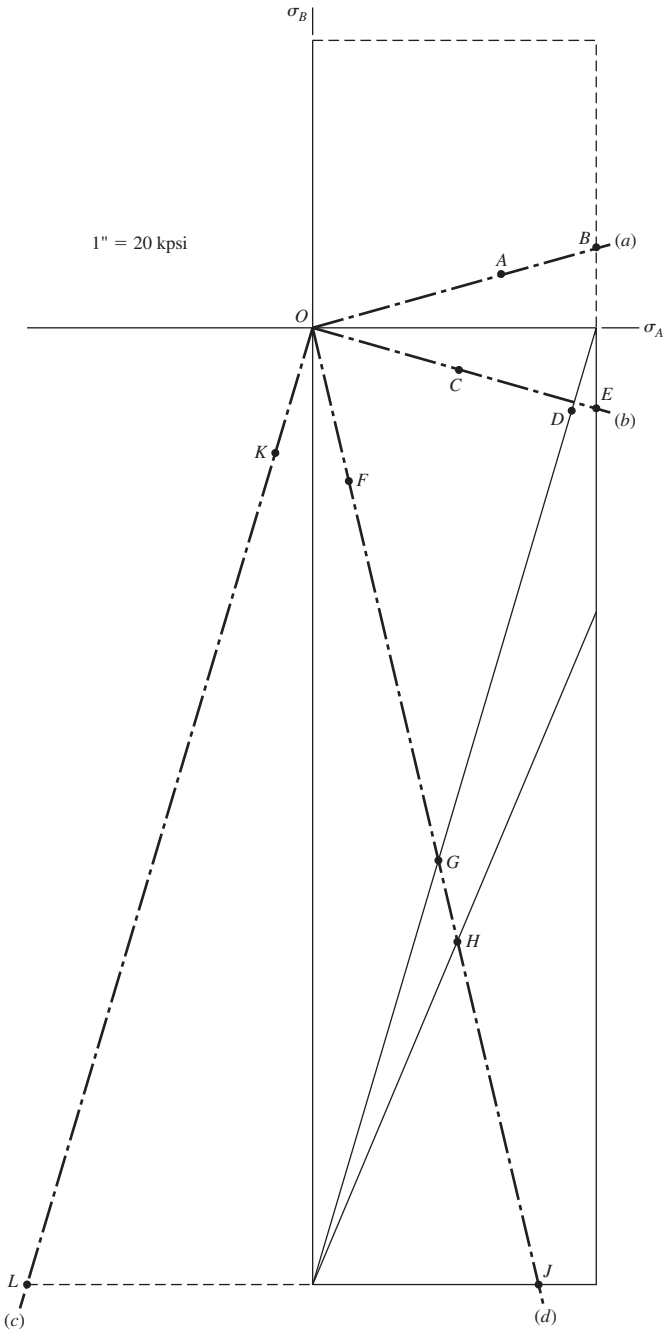
(d) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

$$\sigma_A, \sigma_B = -\frac{12}{2} \pm \sqrt{\left(-\frac{12}{2}\right)^2 + 8^2} = 4, -16 \text{ kpsi}$$

MNS: Eq. (5-30b) $$n = \frac{-100}{-16} = 6.25 \quad \textit{Ans.}$$

BCM: Eq. (5-31*b*) $$\frac{1}{n} = \frac{4}{30} - \frac{(-16)}{100} \quad \Rightarrow \quad n = 3.41 \quad \textit{Ans.}$$

MM: Eq. (5-32*b*) $$\frac{1}{n} = \frac{(100 - 30)4}{100(30)} - \frac{-16}{100} \quad \Rightarrow \quad n = 3.95 \quad \textit{Ans.}$$



5-8 See Prob. 5-7 for plot.

(a) For all methods: $n = \dfrac{OB}{OA} = \dfrac{1.55}{1.03} = 1.5$

(b) BCM: $n = \dfrac{OD}{OC} = \dfrac{1.4}{0.8} = 1.75$

All other methods: $n = \dfrac{OE}{OC} = \dfrac{1.55}{0.8} = 1.9$

(c) For all methods: $n = \dfrac{OL}{OK} = \dfrac{5.2}{0.68} = 7.6$

(d) MNS: $n = \dfrac{OJ}{OF} = \dfrac{5.12}{0.82} = 6.2$

BCM: $n = \dfrac{OG}{OF} = \dfrac{2.85}{0.82} = 3.5$

MM: $n = \dfrac{OH}{OF} = \dfrac{3.3}{0.82} = 4.0$

5-9 Given: $S_y = 42\,\text{kpsi}$, $S_{ut} = 66.2\,\text{kpsi}$, $\varepsilon_f = 0.90$. Since $\varepsilon_f > 0.05$, the material is ductile and thus we may follow convention by setting $S_{yc} = S_{yt}$.

Use DE theory for analytical solution. For σ', use Eq. (5-13) or (5-15) for plane stress and Eq. (5-12) or (5-14) for general 3-D.

(a) $\sigma' = [9^2 - 9(-5) + (-5)^2]^{1/2} = 12.29\,\text{kpsi}$

$$n = \frac{42}{12.29} = 3.42 \quad \textit{Ans.}$$

(b) $\sigma' = [12^2 + 3(3^2)]^{1/2} = 13.08\,\text{kpsi}$

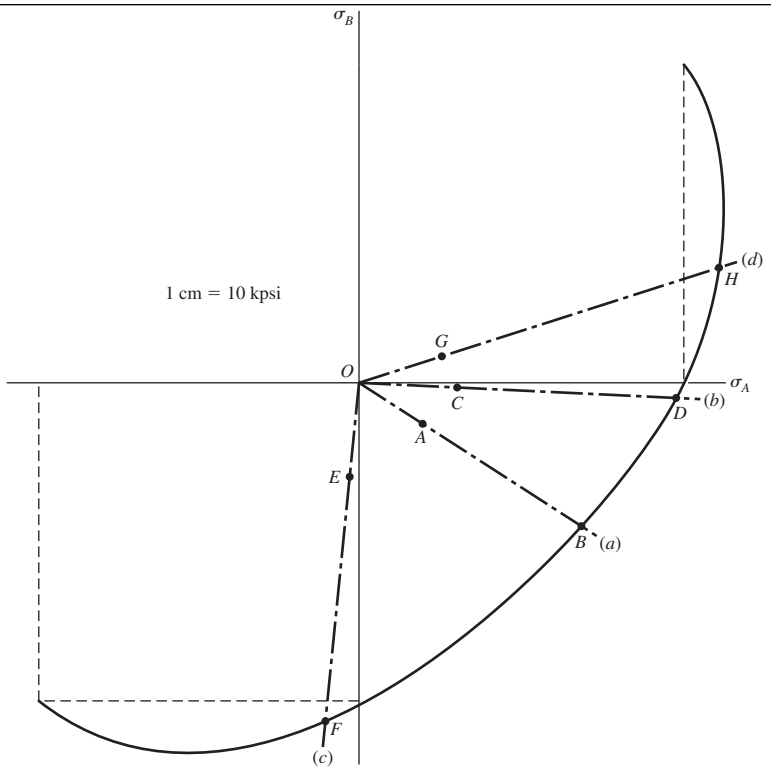
$$n = \frac{42}{13.08} = 3.21 \quad \textit{Ans.}$$

(c) $\sigma' = [(-4)^2 - (-4)(-9) + (-9)^2 + 3(5^2)]^{1/2} = 11.66\,\text{kpsi}$

$$n = \frac{42}{11.66} = 3.60 \quad \textit{Ans.}$$

(d) $\sigma' = [11^2 - (11)(4) + 4^2 + 3(1^2)]^{1/2} = 9.798$

$$n = \frac{42}{9.798} = 4.29 \quad \textit{Ans.}$$



For graphical solution, plot load lines on DE envelope as shown.

(a) $\sigma_A = 9, \sigma_B = -5$ kpsi

$$n = \frac{OB}{OA} = \frac{3.5}{1} = 3.5 \quad \textit{Ans.}$$

(b) $$\sigma_A, \sigma_B = \frac{12}{2} \pm \sqrt{\left(\frac{12}{2}\right)^2 + 3^2} = 12.7, -0.708 \text{ kpsi}$$

$$n = \frac{OD}{OC} = \frac{4.2}{1.3} = 3.23$$

(c) $$\sigma_A, \sigma_B = \frac{-4-9}{2} \pm \sqrt{\left(\frac{4-9}{2}\right)^2 + 5^2} = -0.910, -12.09 \text{ kpsi}$$

$$n = \frac{OF}{OE} = \frac{4.5}{1.25} = 3.6 \quad \textit{Ans.}$$

(d) $$\sigma_A, \sigma_B = \frac{11+4}{2} \pm \sqrt{\left(\frac{11-4}{2}\right)^2 + 1^2} = 11.14, 3.86 \text{ kpsi}$$

$$n = \frac{OH}{OG} = \frac{5.0}{1.15} = 4.35 \quad \textit{Ans.}$$

5-10 This heat-treated steel exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi and $\varepsilon_f = 0.06$. The steel is ductile ($\varepsilon_f > 0.05$) but of unequal yield strengths. The Ductile Coulomb-Mohr hypothesis (DCM) of Fig. 5-19 applies — confine its use to first and fourth quadrants.

(a) $\sigma_x = 90$ kpsi, $\sigma_y = -50$ kpsi, $\sigma_z = 0$ $\therefore \sigma_A = 90$ kpsi and $\sigma_B = -50$ kpsi. For the fourth quadrant, from Eq. (5-31b)

$$n = \frac{1}{(\sigma_A/S_{yt}) - (\sigma_B/S_{uc})} = \frac{1}{(90/235) - (-50/275)} = 1.77 \quad \textit{Ans.}$$

(b) $\sigma_x = 120$ kpsi, $\tau_{xy} = -30$ kpsi ccw. $\sigma_A, \sigma_B = 127.1, -7.08$ kpsi. For the fourth quadrant

$$n = \frac{1}{(127.1/235) - (-7.08/275)} = 1.76 \quad \textit{Ans.}$$

(c) $\sigma_x = -40$ kpsi, $\sigma_y = -90$ kpsi, $\tau_{xy} = 50$ kpsi. $\sigma_A, \sigma_B = -9.10, -120.9$ kpsi. Although no solution exists for the third quadrant, use

$$n = -\frac{S_{yc}}{\sigma_y} = -\frac{275}{-120.9} = 2.27 \quad \textit{Ans.}$$

(d) $\sigma_x = 110$ kpsi, $\sigma_y = 40$ kpsi, $\tau_{xy} = 10$ kpsi cw. $\sigma_A, \sigma_B = 111.4, 38.6$ kpsi. For the first quadrant

$$n = \frac{S_{yt}}{\sigma_A} = \frac{235}{111.4} = 2.11 \quad \textit{Ans.}$$

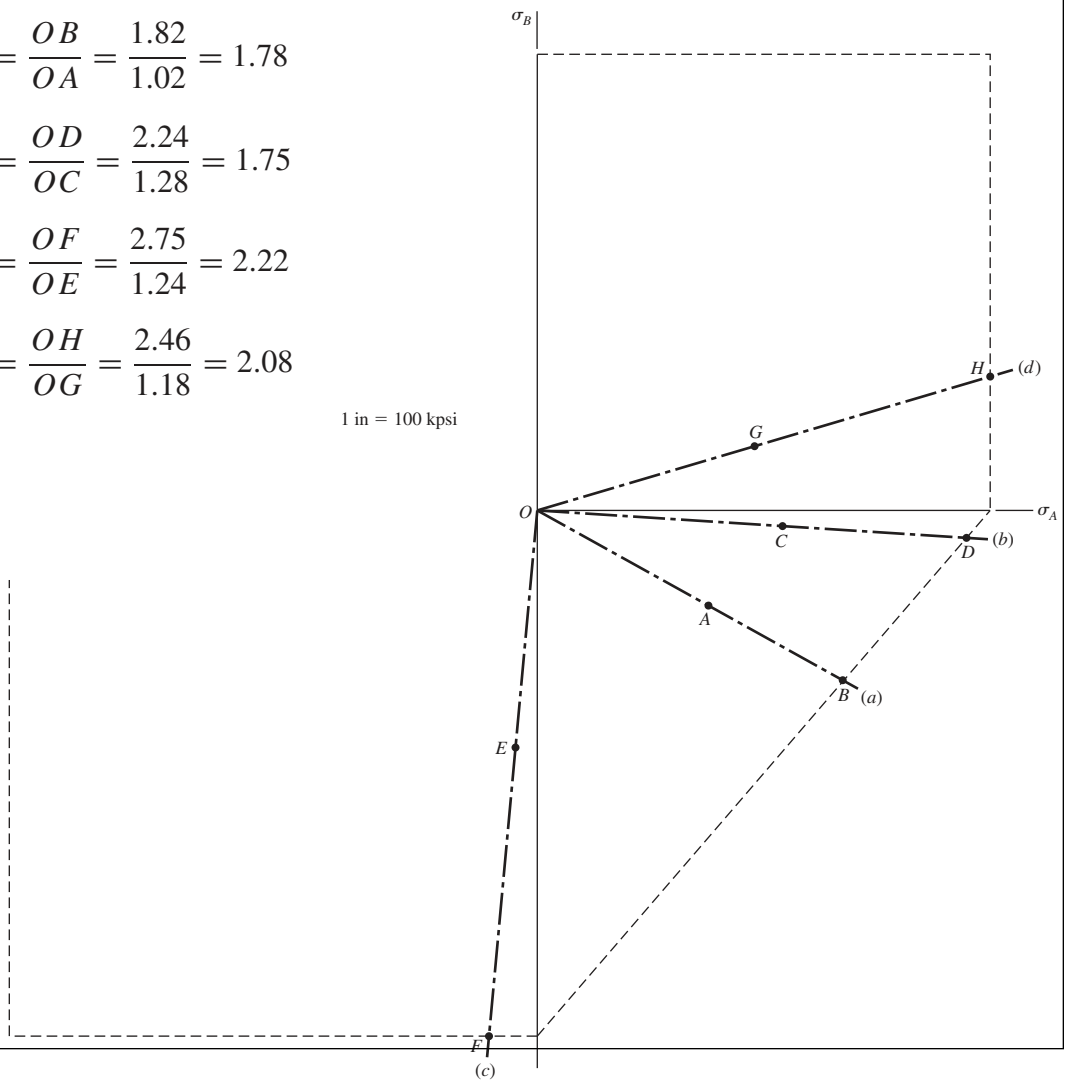
Graphical Solution:

(a) $n = \dfrac{OB}{OA} = \dfrac{1.82}{1.02} = 1.78$

(b) $n = \dfrac{OD}{OC} = \dfrac{2.24}{1.28} = 1.75$

(c) $n = \dfrac{OF}{OE} = \dfrac{2.75}{1.24} = 2.22$

(d) $n = \dfrac{OH}{OG} = \dfrac{2.46}{1.18} = 2.08$



5-11 The material is brittle and exhibits unequal tensile and compressive strengths. *Decision:* Use the Modified Mohr theory.

$$S_{ut} = 22 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

(a) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi. $\sigma_A, \sigma_B = 9, -5$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{5}{9} < 1$, use Eq. (5-32*a*)

$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{9} = 2.44 \quad \textit{Ans.}$$

(b) $\sigma_x = 12$ kpsi, $\tau_{xy} = -3$ kpsi ccw. $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi. For the fourth quadrant, $|\frac{\sigma_B}{\sigma_A}| = \frac{0.708}{12.7} < 1$,

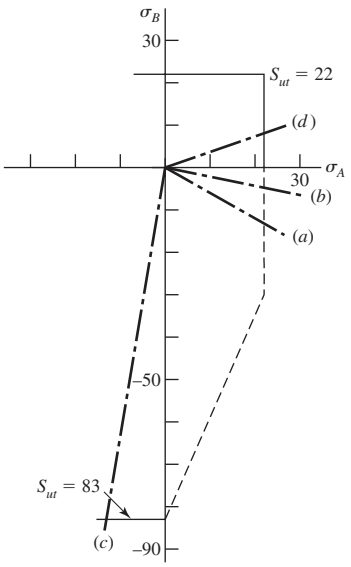
$$n = \frac{S_{ut}}{\sigma_A} = \frac{22}{12.7} = 1.73 \quad \textit{Ans.}$$

(c) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi. $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi. For the third quadrant, no solution exists; however, use Eq. (6-32*c*)

$$n = \frac{-83}{-12.09} = 6.87 \quad \textit{Ans.}$$

(d) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi. $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi. For the first quadrant

$$n = \frac{S_A}{\sigma_A} = \frac{S_{yt}}{\sigma_A} = \frac{22}{11.14} = 1.97 \quad \textit{Ans.}$$



5-12 Since $\varepsilon_f < 0.05$, the material is brittle. Thus, $S_{ut} \doteq S_{uc}$ and we may use MM which is basically the same as MNS.

(a) $\sigma_A, \sigma_B = 9, -5$ kpsi

$$n = \frac{35}{9} = 3.89 \quad \textit{Ans.}$$

(b) $\sigma_A, \sigma_B = 12.7, -0.708$ kpsi

$$n = \frac{35}{12.7} = 2.76 \quad \textit{Ans.}$$

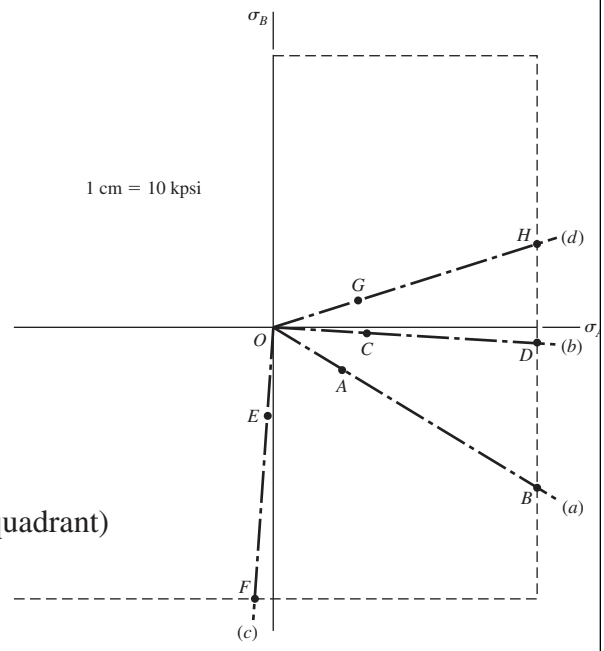
(c) $\sigma_A, \sigma_B = -0.910, -12.09$ kpsi (3rd quadrant)

$$n = \frac{36}{12.09} = 2.98 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = 11.14, 3.86$ kpsi

$$n = \frac{35}{11.14} = 3.14 \quad \textit{Ans.}$$

Graphical Solution:



(a) $n = \dfrac{OB}{OA} = \dfrac{4}{1} = 4.0 \quad \textit{Ans.}$

(b) $n = \dfrac{OD}{OC} = \dfrac{3.45}{1.28} = 2.70 \quad \textit{Ans.}$

(c) $n = \dfrac{OF}{OE} = \dfrac{3.7}{1.3} = 2.85 \quad \textit{Ans.}$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{3.6}{1.15} = 3.13 \quad \textit{Ans.}$

5-13 $S_{ut} = 30$ kpsi, $S_{uc} = 109$ kpsi

Use MM:

(a) $\sigma_A, \sigma_B = 20, 20$ kpsi

Eq. (5-32*a*): $$n = \frac{30}{20} = 1.5 \quad \textit{Ans.}$$

(b) $\sigma_A, \sigma_B = \pm\sqrt{(15)^2} = 15, -15$ kpsi

Eq. (5-32*a*) $$n = \frac{30}{15} = 2 \quad \textit{Ans.}$$

(c) $\sigma_A, \sigma_B = -80, -80$ kpsi

For the 3rd quadrant, there is no solution but use Eq. (5-32*c*).

Eq. (5-32*c*): $$n = -\frac{109}{-80} = 1.36 \quad \textit{Ans.}$$

(d) $\sigma_A, \sigma_B = 15, -25$ kpsi, $|\sigma_B|\sigma_A| = 25/15 > 1$,

Eq. (5-32*b*): $$\frac{(109-30)15}{109(30)} - \frac{-25}{109} = \frac{1}{n}$$

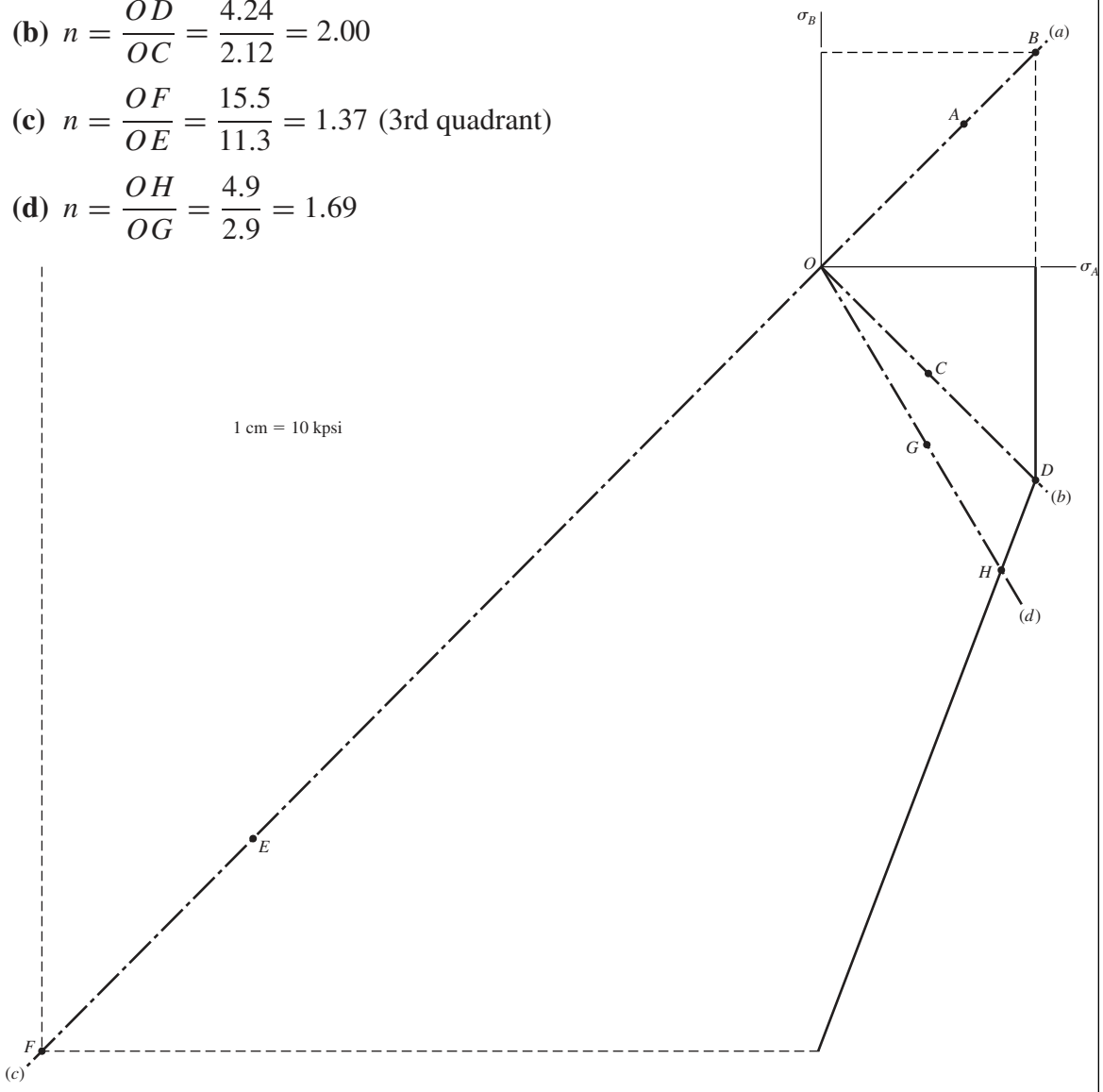
$$n = 1.69 \quad \textit{Ans.}$$

(a) $n = \dfrac{OB}{OA} = \dfrac{4.25}{2.83} = 1.50$

(b) $n = \dfrac{OD}{OC} = \dfrac{4.24}{2.12} = 2.00$

(c) $n = \dfrac{OF}{OE} = \dfrac{15.5}{11.3} = 1.37$ (3rd quadrant)

(d) $n = \dfrac{OH}{OG} = \dfrac{4.9}{2.9} = 1.69$



5-14 Given: AISI 1006 CD steel, $F = 0.55$ N, $P = 8.0$ kN, and $T = 30$ N · m, applying the DE theory to stress elements A and B with $S_y = 280$ MPa

A: $$\sigma_x = \frac{32Fl}{\pi d^3} + \frac{4P}{\pi d^2} = \frac{32(0.55)(10^3)(0.1)}{\pi(0.020^3)} + \frac{4(8)(10^3)}{\pi(0.020^2)}$$

$$= 95.49(10^6)\ \text{Pa} = 95.49\ \text{MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(30)}{\pi(0.020^3)} = 19.10(10^6)\,\text{Pa} = 19.10\,\text{MPa}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = [95.49^2 + 3(19.1)^2]^{1/2} = 101.1\,\text{MPa}$$

$$n = \frac{S_y}{\sigma'} = \frac{280}{101.1} = 2.77 \quad \textit{Ans.}$$

B:

$$\sigma_x = \frac{4P}{\pi d^3} = \frac{4(8)(10^3)}{\pi(0.020^2)} = 25.47(10^6)\,\text{Pa} = 25.47\,\text{MPa}$$

$$\tau_{xy} = \frac{16T}{\pi d^3} + \frac{4}{3}\frac{V}{A} = \frac{16(30)}{\pi(0.020^3)} + \frac{4}{3}\left[\frac{0.55(10^3)}{(\pi/4)(0.020^2)}\right]$$

$$= 21.43(10^6)\,\text{Pa} = 21.43\,\text{MPa}$$

$$\sigma' = [25.47^2 + 3(21.43^2)]^{1/2} = 45.02\,\text{MPa}$$

$$n = \frac{280}{45.02} = 6.22 \quad \textit{Ans.}$$

5-15 $S_y = 32\,\text{kpsi}$

At A, $M = 6(190) = 1\,140\ \text{lbf}\cdot\text{in}$, $T = 4(190) = 760\ \text{lbf}\cdot\text{in}$.

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(1140)}{\pi(3/4)^3} = 27\,520\,\text{psi}$$

$$\tau_{zx} = \frac{16T}{\pi d^3} = \frac{16(760)}{\pi(3/4)^3} = 9175\,\text{psi}$$

$$\tau_{\max} = \sqrt{\left(\frac{27\,520}{2}\right)^2 + 9175^2} = 16\,540\,\text{psi}$$

$$n = \frac{S_y}{2\tau_{\max}} = \frac{32}{2(16.54)} = 0.967 \quad \textit{Ans.}$$

MSS predicts yielding

5-16 From Prob. 4-15, $\sigma_x = 27.52\,\text{kpsi}$, $\tau_{zx} = 9.175\,\text{kpsi}$. For Eq. (5-15), adjusted for coordinates,

$$\sigma' = \left[27.52^2 + 3(9.175)^2\right]^{1/2} = 31.78\,\text{kpsi}$$

$$n = \frac{S_y}{\sigma'} = \frac{32}{31.78} = 1.01 \quad \textit{Ans.}$$

DE predicts no yielding, but it is extremely close. Shaft size should be increased.

5-17 Design decisions required:

- Material and condition
- Design factor
- Failure model
- Diameter of pin

Using $F = 416$ lbf from Ex. 5-3

$$\sigma_{\max} = \frac{32M}{\pi d^3}$$

$$d = \left(\frac{32M}{\pi \sigma_{\max}}\right)^{1/3}$$

Decision 1: Select the same material and condition of Ex. 5-3 (AISI 1035 steel, $S_y = 81\,000$).

Decision 2: Since we prefer the pin to yield, set n_d a little larger than 1. Further explanation will follow.

Decision 3: Use the Distortion Energy static failure theory.

Decision 4: Initially set $n_d = 1$

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{S_y}{1} = 81\,000 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(81\,000)}\right]^{1/3} = 0.922 \text{ in}$$

Choose preferred size of $d = 1.000$ in

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274$$

Set design factor to $n_d = 1.274$

Adequacy Assessment:

$$\sigma_{\max} = \frac{S_y}{n_d} = \frac{81\,000}{1.274} = 63\,580 \text{ psi}$$

$$d = \left[\frac{32(416)(15)}{\pi(63\,580)}\right]^{1/3} = 1.000 \text{ in} \quad (OK)$$

$$F = \frac{\pi(1)^3(81\,000)}{32(15)} = 530 \text{ lbf}$$

$$n = \frac{530}{416} = 1.274 \quad \text{(OK)}$$

5-18 For a thin walled cylinder made of AISI 1018 steel, $S_y = 54$ kpsi, $S_{ut} = 64$ kpsi.

The state of stress is

$$\sigma_t = \frac{pd}{4t} = \frac{p(8)}{4(0.05)} = 40p, \quad \sigma_l = \frac{pd}{8t} = 20p, \quad \sigma_r = -p$$

These three are all principal stresses. Therefore,

$$\begin{aligned}\sigma' &= \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \\ &= \frac{1}{\sqrt{2}}[(40p - 20p)^2 + (20p + p)^2 + (-p - 40p)^2] \\ &= 35.51p = 54 \quad \Rightarrow \quad p = 1.52 \text{ kpsi} \quad \text{(for yield)} \quad \textit{Ans.}\end{aligned}$$

For rupture, $35.51p \doteq 64 \Rightarrow p \doteq 1.80$ kpsi *Ans.*

5-19 For hot-forged AISI steel $w = 0.282\,\text{lbf/in}^3$, $S_y = 30$ kpsi and $\nu = 0.292$. Then $\rho = w/g = 0.282/386\,\text{lbf} \cdot \text{s}^2/\text{in}$; $r_i = 3$ in; $r_o = 5$ in; $r_i^2 = 9$; $r_o^2 = 25$; $3 + \nu = 3.292$; $1 + 3\nu = 1.876$.

Eq. (3-55) for $r = r_i$ becomes

$$\sigma_t = \rho\omega^2 \left(\frac{3+\nu}{8}\right)\left[2r_o^2 + r_i^2\left(1 - \frac{1+3\nu}{3+\nu}\right)\right]$$

Rearranging and substituting the above values:

$$\begin{aligned}\frac{S_y}{\omega^2} &= \frac{0.282}{386}\left(\frac{3.292}{8}\right)\left[50 + 9\left(1 - \frac{1.876}{3.292}\right)\right] \\ &= 0.016\,19\end{aligned}$$

Setting the tangential stress equal to the yield stress,

$$\omega = \left(\frac{30\,000}{0.016\,19}\right)^{1/2} = 1361 \text{ rad/s}$$

or

$$\begin{aligned}n &= 60\omega/2\pi = 60(1361)/(2\pi) \\ &= 13\,000 \text{ rev/min}\end{aligned}$$

Now check the stresses at $r = (r_o r_i)^{1/2}$, or $r = [5(3)]^{1/2} = 3.873$ in

$$\begin{aligned}\sigma_r &= \rho\omega^2\left(\frac{3+\nu}{8}\right)(r_o - r_i)^2 \\ &= \frac{0.282\omega^2}{386}\left(\frac{3.292}{8}\right)(5-3)^2 \\ &= 0.001\,203\omega^2\end{aligned}$$

Applying Eq. (3-55) for σ_t

$$\begin{aligned}\sigma_t &= \omega^2\left(\frac{0.282}{386}\right)\left(\frac{3.292}{8}\right)\left[9 + 25 + \frac{9(25)}{15} - \frac{1.876(15)}{3.292}\right] \\ &= 0.012\,16\omega^2\end{aligned}$$

Using the Distortion-Energy theory

$$\sigma' = \left(\sigma_t^2 - \sigma_r\sigma_t + \sigma_r^2\right)^{1/2} = 0.011\,61\omega^2$$

Solving $$\omega = \left(\frac{30\,000}{0.011\,61}\right)^{1/2} = 1607\,\text{rad/s}$$

So the inner radius governs and $n = 13\,000\,\text{rev/min}$ *Ans.*

5-20 For a thin-walled pressure vessel,

$$d_i = 3.5 - 2(0.065) = 3.37\,\text{in}$$

$$\sigma_t = \frac{p(d_i + t)}{2t}$$

$$\sigma_t = \frac{500(3.37 + 0.065)}{2(0.065)} = 13\,212\,\text{psi}$$

$$\sigma_l = \frac{pd_i}{4t} = \frac{500(3.37)}{4(0.065)} = 6481\,\text{psi}$$

$$\sigma_r = -p_i = -500\,\text{psi}$$

These are all principal stresses, thus,

$$\sigma' = \frac{1}{\sqrt{2}}\{(13\,212 - 6481)^2 + [6481 - (-500)]^2 + (-500 - 13\,212)^2\}^{1/2}$$

$$\sigma' = 11\,876\,\text{psi}$$

$$n = \frac{S_y}{\sigma'} = \frac{46\,000}{\sigma'} = \frac{46\,000}{11\,876}$$

$$= 3.87 \quad \textit{Ans.}$$

5-21 Table A-20 gives S_y as 320 MPa. The maximum significant stress condition occurs at r_i where $\sigma_1 = \sigma_r = 0$, $\sigma_2 = 0$, and $\sigma_3 = \sigma_t$. From Eq. (3-49) for $r = r_i$, $p_i = 0$,

$$\sigma_t = -\frac{2r_o^2 p_o}{r_o^2 - r_i^2} = -\frac{2(150^2)\,p_o}{150^2 - 100^2} = -3.6p_o$$

$$\sigma' = 3.6p_o = S_y = 320$$

$$p_o = \frac{320}{3.6} = 88.9\,\text{MPa} \quad \textit{Ans.}$$

5-22 $S_{ut} = 30\,\text{kpsi}$, $w = 0.260\,\text{lbf/in}^3$, $\nu = 0.211$, $3 + \nu = 3.211$, $1 + 3\nu = 1.633$. At the inner radius, from Prob. 5-19

$$\frac{\sigma_t}{\omega^2} = \rho\left(\frac{3+\nu}{8}\right)\left(2r_o^2 + r_i^2 - \frac{1+3\nu}{3+\nu}r_i^2\right)$$

Here $r_o^2 = 25,\ r_i^2 = 9,$ and so

$$\frac{\sigma_t}{\omega^2} = \frac{0.260}{386}\left(\frac{3.211}{8}\right)\left(50 + 9 - \frac{1.633(9)}{3.211}\right) = 0.0147$$

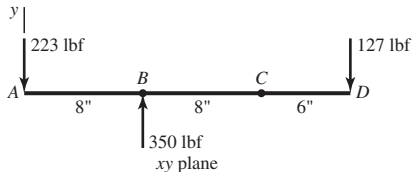
Since σ_r is of the same sign, we use M2M failure criteria in the first quadrant. From Table A-24, $S_{ut} = 31\,\text{kpsi}$, thus,

$$\omega = \left(\frac{31\,000}{0.0147}\right)^{1/2} = 1452\,\text{rad/s}$$

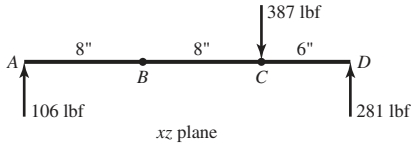
$$\text{rpm} = 60\omega/(2\pi) = 60(1452)/(2\pi)$$
$$= 13\,866\,\text{rev/min}$$

Using the grade number of 30 for $S_{ut} = 30\,000\,\text{kpsi}$ gives a bursting speed of 13 640 rev/min.

5-23 $T_C = (360 - 27)(3) = 1000\,\text{lbf}\cdot\text{in}, \quad T_B = (300 - 50)(4) = 1000\,\text{lbf}\cdot\text{in}$



In xy plane, $M_B = 223(8) = 1784\,\text{lbf}\cdot\text{in}$ and $M_C = 127(6) = 762\,\text{lbf}\cdot\text{in}$.



In the xz plane, $M_B = 848\,\text{lbf}\cdot\text{in}$ and $M_C = 1686\,\text{lbf}\cdot\text{in}$. The resultants are

$$M_B = [(1784)^2 + (848)^2]^{1/2} = 1975\,\text{lbf}\cdot\text{in}$$

$$M_C = [(1686)^2 + (762)^2]^{1/2} = 1850\,\text{lbf}\cdot\text{in}$$

So point B governs and the stresses are

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(1000)}{\pi d^3} = \frac{5093}{d^3}\,\text{psi}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(1975)}{\pi d^3} = \frac{20\,120}{d^3}\,\text{psi}$$

Then

$$\sigma_A, \sigma_B = \frac{\sigma_x}{2} \pm \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\frac{20.12}{2} \pm \left[\left(\frac{20.12}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$= \frac{(10.06 \pm 11.27)}{d^3}\,\text{kpsi}\cdot\text{in}^3$$

Then

$$\sigma_A = \frac{10.06 + 11.27}{d^3} = \frac{21.33}{d^3}\,\text{kpsi}$$

and

$$\sigma_B = \frac{10.06 - 11.27}{d^3} = -\frac{1.21}{d^3}\,\text{kpsi}$$

For this state of stress, use the Brittle-Coulomb-Mohr theory for illustration. Here we use $S_{ut}(\text{min}) = 25\,\text{kpsi}$, $S_{uc}(\text{min}) = 97\,\text{kpsi}$, and Eq. (5-31*b*) to arrive at

$$\frac{21.33}{25d^3} - \frac{-1.21}{97d^3} = \frac{1}{2.8}$$

Solving gives $d = 1.34$ in. So use $d = 1\,3/8$ in *Ans.*

Note that this has been solved as a statics problem. Fatigue will be considered in the next chapter.

5-24 As in Prob. 5-23, we will assume this to be statics problem. Since the proportions are unchanged, the bearing reactions will be the same as in Prob. 5-23. Thus

xy plane: $M_B = 223(4) = 892\,\text{lbf}\cdot\text{in}$

xz plane: $M_B = 106(4) = 424\,\text{lbf}\cdot\text{in}$

So

$$M_{\max} = [(892)^2 + (424)^2]^{1/2} = 988\,\text{lbf}\cdot\text{in}$$

$$\sigma_x = \frac{32M_B}{\pi d^3} = \frac{32(988)}{\pi d^3} = \frac{10\,060}{d^3}\,\text{psi}$$

Since the torsional stress is unchanged,

$$\tau_{xz} = 5.09/d^3\,\text{kpsi}$$

$$\sigma_A, \sigma_B = \frac{1}{d^3}\left\{\left(\frac{10.06}{2}\right) \pm \left[\left(\frac{10.06}{2}\right)^2 + (5.09)^2\right]^{1/2}\right\}$$

$$\sigma_A = 12.19/d^3 \quad \text{and} \quad \sigma_B = -2.13/d^3$$

Using the Brittle-Coulomb-Mohr, as was used in Prob. 5-23, gives

$$\frac{12.19}{25d^3} - \frac{-2.13}{97d^3} = \frac{1}{2.8}$$

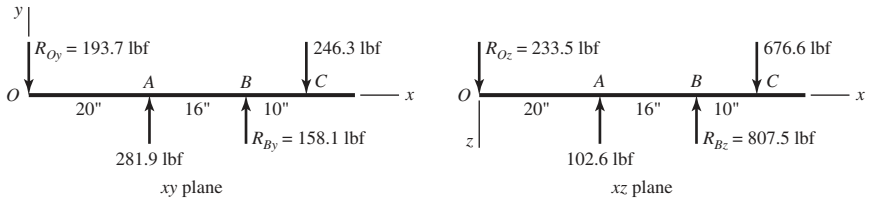
Solving gives $d = 1\,1/8$ in. *Ans.*

5-25

$$(F_A)_t = 300\cos 20 = 281.9\,\text{lbf}, \quad (F_A)_r = 300\sin 20 = 102.6\,\text{lbf}$$

$$T = 281.9(12) = 3383\,\text{lbf}\cdot\text{in}, \quad (F_C)_t = \frac{3383}{5} = 676.6\,\text{lbf}$$

$$(F_C)_r = 676.6\tan 20 = 246.3\,\text{lbf}$$



$$M_A = 20\sqrt{193.7^2 + 233.5^2} = 6068\,\text{lbf} \cdot \text{in}$$

$$M_B = 10\sqrt{246.3^2 + 676.6^2} = 7200\,\text{lbf} \cdot \text{in} \quad \text{(maximum)}$$

$$\sigma_x = \frac{32(7200)}{\pi d^3} = \frac{73\,340}{d^3}$$

$$\tau_{xy} = \frac{16(3383)}{\pi d^3} = \frac{17\,230}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \frac{S_y}{n}$$

$$\left[\left(\frac{73\,340}{d^3}\right)^2 + 3\left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{79\,180}{d^3} = \frac{60\,000}{3.5}$$

$d = 1.665\,\text{in}$ so use a standard diameter size of 1.75 in *Ans.*

5-26 From Prob. 5-25,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

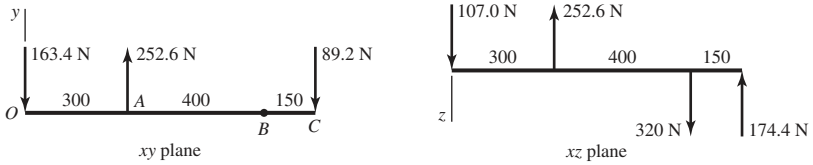
$$\left[\left(\frac{73\,340}{2d^3}\right)^2 + \left(\frac{17\,230}{d^3}\right)^2\right]^{1/2} = \frac{40\,516}{d^3} = \frac{60\,000}{2(3.5)}$$

$d = 1.678\,\text{in}$ so use 1.75 in *Ans.*

5-27 $T = (270 - 50)(0.150) = 33\,\text{N} \cdot \text{m}$, $S_y = 370\,\text{MPa}$

$$(T_1 - 0.15T_1)(0.125) = 33 \quad \Rightarrow \quad T_1 = 310.6\,\text{N}, \quad T_2 = 0.15(310.6) = 46.6\,\text{N}$$

$$(T_1 + T_2)\cos 45 = 252.6\,\text{N}$$



$$M_A = 0.3\sqrt{163.4^2 + 107^2} = 58.59\,\text{N}\cdot\text{m} \qquad \text{(maximum)}$$

$$M_B = 0.15\sqrt{89.2^2 + 174.4^2} = 29.38\,\text{N}\cdot\text{m}$$

$$\sigma_x = \frac{32(58.59)}{\pi d^3} = \frac{596.8}{d^3}$$

$$\tau_{xy} = \frac{16(33)}{\pi d^3} = \frac{168.1}{d^3}$$

$$\sigma' = \left(\sigma_x^2 + 3\tau_{xy}^2\right)^{1/2} = \left[\left(\frac{596.8}{d^3}\right)^2 + 3\left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{664.0}{d^3} = \frac{370(10^6)}{3.0}$$

$d = 17.5(10^{-3})\,\text{m} = 17.5\,\text{mm}$, so use 18 mm *Ans.*

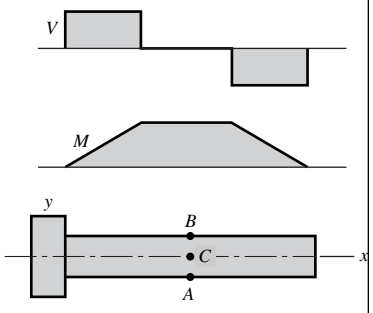
5-28 From Prob. 5-27,

$$\tau_{\max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2\right]^{1/2} = \frac{S_y}{2n}$$

$$\left[\left(\frac{596.8}{2d^3}\right)^2 + \left(\frac{168.1}{d^3}\right)^2\right]^{1/2} = \frac{342.5}{d^3} = \frac{370(10^6)}{2(3.0)}$$

$d = 17.7(10^{-3})\,\text{m} = 17.7\,\text{mm}$, so use 18 mm *Ans.*

5-29 For the loading scheme shown in Figure (*c*),



$$M_{\max} = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right) = \frac{4.4}{2}(6 + 4.5)$$
$$= 23.1\,\text{N}\cdot\text{m}$$

For a stress element at *A*:

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32(23.1)(10^3)}{\pi(12)^3} = 136.2\,\text{MPa}$$

The shear at *C* is

$$\tau_{xy} = \frac{4(F/2)}{3\pi d^2/4} = \frac{4(4.4/2)(10^3)}{3\pi(12)^2/4} = 25.94\,\text{MPa}$$

$$\tau_{\max} = \left[\left(\frac{136.2}{2}\right)^2\right]^{1/2} = 68.1\,\text{MPa}$$

Since $S_y = 220\,\text{MPa}$, $S_{sy} = 220/2 = 110\,\text{MPa}$, and

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{110}{68.1} = 1.62 \quad \textit{Ans.}$$

For the loading scheme depicted in Figure (*d*)

$$M_{\max} = \frac{F}{2}\left(\frac{a+b}{2}\right) - \frac{F}{2}\left(\frac{1}{2}\right)\left(\frac{b}{2}\right)^2 = \frac{F}{2}\left(\frac{a}{2} + \frac{b}{4}\right)$$

This result is the same as that obtained for Figure (*c*). At point *B*, we also have a surface compression of

$$\sigma_y = \frac{-F}{A} = \frac{-F}{bd} - \frac{-4.4(10^3)}{18(12)} = -20.4 \text{ MPa}$$

With $\sigma_x = -136.2$ MPa. From a Mohrs circle diagram, $\tau_{\max} = 136.2/2 = 68.1$ MPa.

$$n = \frac{110}{68.1} = 1.62 \text{ MPa} \quad \textit{Ans.}$$

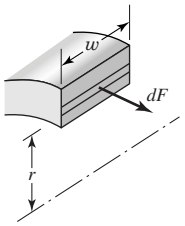
5-30 Based on Figure (*c*) and using Eq. (5-15)

$$\begin{aligned}\sigma' &= \left(\sigma_x^2\right)^{1/2} \\ &= (136.2^2)^{1/2} = 136.2 \text{ MPa} \\ n &= \frac{S_y}{\sigma'} = \frac{220}{136.2} = 1.62 \quad \textit{Ans.}\end{aligned}$$

Based on Figure (*d*) and using Eq. (5-15) and the solution of Prob. 5-29,

$$\begin{aligned}\sigma' &= \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2\right)^{1/2} \\ &= [(-136.2)^2 - (-136.2)(-20.4) + (-20.4)^2]^{1/2} \\ &= 127.2 \text{ MPa} \\ n &= \frac{S_y}{\sigma'} = \frac{220}{127.2} = 1.73 \quad \textit{Ans.}\end{aligned}$$

5-31



When the ring is set, the hoop tension in the ring is equal to the screw tension.

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

We have the hoop tension at any radius. The differential hoop tension dF is

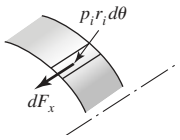
$$dF = w\sigma_t\, dr$$

$$F = \int_{r_i}^{r_o} w\sigma_t\, dr = \frac{wr_i^2 p_i}{r_o^2 - r_i^2}\int_{r_i}^{r_o}\left(1 + \frac{r_o^2}{r^2}\right) dr = wr_i p_i \qquad (1)$$

The screw equation is

$$F_i = \frac{T}{0.2d} \tag{2}$$

From Eqs. (1) and (2)



$$p_i = \frac{F}{wr_i} = \frac{T}{0.2dwr_i}$$

$$dF_x = fp_i r_i\, d\theta$$

$$F_x = \int_o^{2\pi} fp_i wr_i\, d\theta = \frac{fTw}{0.2dwr_i} r_i \int_o^{2\pi} d\theta$$

$$= \frac{2\pi fT}{0.2d} \quad \textit{Ans.}$$

5-32

(a) From Prob. 5-31, $T = 0.2F_i d$

$$F_i = \frac{T}{0.2d} = \frac{190}{0.2(0.25)} = 3800\,\text{lbf} \quad \textit{Ans.}$$

(b) From Prob. 5-31, $F = wr_i p_i$

$$p_i = \frac{F}{wr_i} = \frac{F_i}{wr_i} = \frac{3800}{0.5(0.5)} = 15\,200\,\text{psi} \quad \textit{Ans.}$$

(c)

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r}\right)_{r=r_i} = \frac{p_i\left(r_i^2 + r_o^2\right)}{r_o^2 - r_i^2}$$

$$= \frac{15\,200(0.5^2 + 1^2)}{1^2 - 0.5^2} = 25\,333\,\text{psi} \quad \textit{Ans.}$$

$$\sigma_r = -p_i = -15\,200\,\text{psi}$$

(d)

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_t - \sigma_r}{2}$$

$$= \frac{25\,333 - (-15\,200)}{2} = 20\,267\,\text{psi} \quad \textit{Ans.}$$

$$\sigma' = \left(\sigma_A^2 + \sigma_B^2 - \sigma_A\sigma_B\right)^{1/2}$$

$$= [25\,333^2 + (-15\,200)^2 - 25\,333(-15\,200)]^{1/2}$$

$$= 35\,466\,\text{psi} \quad \textit{Ans.}$$

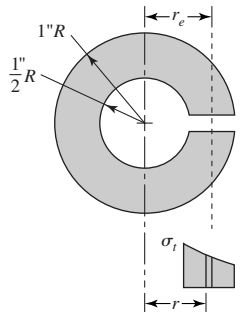
(e) Maximum Shear hypothesis

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.5S_y}{\tau_{\max}} = \frac{0.5(63)}{20.267} = 1.55 \quad \textit{Ans.}$$

Distortion Energy theory

$$n = \frac{S_y}{\sigma'} = \frac{63}{35\,466} = 1.78 \quad \textit{Ans.}$$

5-33



The moment about the center caused by force F is Fr_e where r_e is the effective radius. This is balanced by the moment about the center caused by the tangential (hoop) stress.

$$Fr_e = \int_{r_i}^{r_o} r\sigma_t w\, dr$$

$$= \frac{wp_i r_i^2}{r_o^2 - r_i^2} \int_{r_i}^{r_o} \left(r + \frac{r_o^2}{r}\right) dr$$

$$r_e = \frac{wp_i r_i^2}{F\left(r_o^2 - r_i^2\right)} \left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln \frac{r_o}{r_i}\right)$$

From Prob. 5-31, $F = wr_i p_i$. Therefore,

$$r_e = \frac{r_i}{r_o^2 - r_i^2} \left(\frac{r_o^2 - r_i^2}{2} + r_o^2 \ln \frac{r_o}{r_i}\right)$$

For the conditions of Prob. 5-31, $r_i = 0.5$ and $r_o = 1$ in

$$r_e = \frac{0.5}{1^2 - 0.5^2} \left(\frac{1^2 - 0.5^2}{2} + 1^2 \ln \frac{1}{0.5}\right) = 0.712 \text{ in}$$

5-34 $\delta_{\text{nom}} = 0.0005$ in

(a) From Eq. (3-57)

$$p = \frac{30(10^6)(0.0005)}{(1^3)} \left[\frac{(1.5^2 - 1^2)(1^2 - 0.5^2)}{2(1.5^2 - 0.5^2)}\right] = 3516 \text{ psi} \quad \textit{Ans.}$$

Inner member:

Eq. (3-58) $$(\sigma_t)_i = -p\frac{R^2 + r_i^2}{R^2 - r_i^2} = -3516\left(\frac{1^2 + 0.5^2}{1^2 - 0.5^2}\right) = -5860 \text{ psi}$$

$$(\sigma_r)_i = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_i' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}$$

$$= [(-5860)^2 - (-5860)(-3516) + (-3516)^2]^{1/2}$$

$$= 5110 \text{ psi} \quad \textit{Ans.}$$

Outer member:

Eq. (3-59) $$(\sigma_t)_o = 3516\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 9142 \text{ psi}$$

$$(\sigma_r)_o = -p = -3516 \text{ psi}$$

Eq. (5-13) $$\sigma_o' = [9142^2 - 9142(-3516) + (-3516)^2]^{1/2}$$

$$= 11\,320 \text{ psi} \quad \textit{Ans.}$$

(b) For a solid inner tube,

$$p = \frac{30(10^6)(0.0005)}{1}\left[\frac{(1.5^2 - 1^2)(1^2)}{2(1^2)(1.5^2)}\right] = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_i = -p = -4167 \text{ psi}, \quad (\sigma_r)_i = -4167 \text{ psi}$$

$$\sigma_i' = [(-4167)^2 - (-4167)(-4167) + (-4167)^2]^{1/2} = 4167 \text{ psi} \quad \textit{Ans.}$$

$$(\sigma_t)_o = 4167\left(\frac{1.5^2 + 1^2}{1.5^2 - 1^2}\right) = 10\,830 \text{ psi}, \quad (\sigma_r)_o = -4167 \text{ psi}$$

$$\sigma_o' = [10\,830^2 - 10\,830(-4167) + (-4167)^2]^{1/2} = 13\,410 \text{ psi} \quad \textit{Ans.}$$

5-35 Using Eq. (3-57) with diametral values,

$$p = \frac{207(10^3)(0.02)}{(50^3)}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 19.41 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-58) $$(\sigma_t)_i = -19.41\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -32.35 \text{ MPa}$$

$$(\sigma_r)_i = -19.41 \text{ MPa}$$

Eq. (5-13) $$\sigma_i' = [(-32.35)^2 - (-32.35)(-19.41) + (-19.41)^2]^{1/2}$$

$$= 28.20 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-59) $$(\sigma_t)_o = 19.41\left(\frac{75^2 + 50^2}{75^2 - 50^2}\right) = 50.47 \text{ MPa},$$

$$(\sigma_r)_o = -19.41 \text{ MPa}$$

$$\sigma_o' = [50.47^2 - 50.47(-19.41) + (-19.41)^2]^{1/2} = 62.48 \text{ MPa} \quad \textit{Ans.}$$

5-36 Max. shrink-fit conditions: Diametral interference $\delta_d = 50.01 - 49.97 = 0.04$ mm. Equation (3-57) using diametral values:

$$p = \frac{207(10^3)0.04}{50^3}\left[\frac{(75^2 - 50^2)(50^2 - 25^2)}{2(75^2 - 25^2)}\right] = 38.81 \text{ MPa} \quad \textit{Ans.}$$

Eq. (3-58): $$(\sigma_t)_i = -38.81\left(\frac{50^2 + 25^2}{50^2 - 25^2}\right) = -64.68 \text{ MPa}$$

$$(\sigma_r)_i = -38.81 \text{ MPa}$$

Eq. (5-13):

$$\sigma_i' = \left[(-64.68)^2 - (-64.68)(-38.81) + (-38.81)^2\right]^{1/2}$$

$$= 56.39 \text{ MPa} \quad \textit{Ans.}$$

5-37

$$\delta = \frac{1.9998}{2} - \frac{1.999}{2} = 0.0004 \text{ in}$$

Eq. (3-56)

$$0.0004 = \frac{p(1)}{14.5(10^6)}\left[\frac{2^2+1^2}{2^2-1^2} + 0.211\right] + \frac{p(1)}{30(10^6)}\left[\frac{1^2+0}{1^2-0} - 0.292\right]$$

$$p = 2613 \text{ psi}$$

Applying Eq. (4-58) at R,

$$(\sigma_t)_o = 2613\left(\frac{2^2+1^2}{2^2-1^2}\right) = 4355 \text{ psi}$$

$$(\sigma_r)_o = -2613 \text{ psi}, \quad S_{ut} = 20 \text{ kpsi}, \quad S_{uc} = 83 \text{ kpsi}$$

$$\left|\frac{\sigma_o}{\sigma_A}\right| = \frac{2613}{4355} < 1, \ \therefore \text{ use Eq. (5-32a)}$$

$$h = S_{ut}/\sigma_A = 20/4.355 = 4.59 \quad \textit{Ans.}$$

5-38 $E = 30(10^6)$ psi, $\nu = 0.292$, $I = (\pi/64)(2^4 - 1.5^4) = 0.5369 \text{ in}^4$

Eq. (3-57) can be written in terms of diameters,

$$p = \frac{E\delta_d}{D}\left[\frac{\left(d_o^2 - D^2\right)\left(D^2 - d_i^2\right)}{2D^2\left(d_o^2 - d_i^2\right)}\right] = \frac{30(10^6)}{1.75}(0.002\,46)\left[\frac{(2^2 - 1.75^2)(1.75^2 - 1.5^2)}{2(1.75^2)(2^2 - 1.5^2)}\right]$$

$$= 2997 \text{ psi} = 2.997 \text{ kpsi}$$

Outer member:

Outer radius: $(\sigma_t)_o = \dfrac{1.75^2(2.997)}{2^2 - 1.75^2}(2) = 19.58 \text{ kpsi}, (\sigma_r)_o = 0$

Inner radius: $(\sigma_t)_i = \dfrac{1.75^2(2.997)}{2^2 - 1.75^2}\left(1 + \dfrac{2^2}{1.75^2}\right) = 22.58 \text{ kpsi}, (\sigma_r)_i = -2.997 \text{ kpsi}$

Bending:

r_o: $(\sigma_x)_o = \dfrac{6.000(2/2)}{0.5369} = 11.18 \text{ kpsi}$

r_i: $(\sigma_x)_i = \dfrac{6.000(1.75/2)}{0.5369} = 9.78 \text{ kpsi}$

Torsion: $J = 2I = 1.0738 \text{ in}^4$

r_o: $(\tau_{xy})_o = \dfrac{8.000(2/2)}{1.0738} = 7.45 \text{ kpsi}$

r_i: $(\tau_{xy})_i = \dfrac{8.000(1.75/2)}{1.0738} = 6.52 \text{ kpsi}$

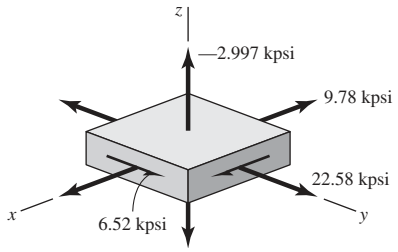
Outer radius is plane stress

$$\sigma_x = 11.18 \text{ kpsi}, \quad \sigma_y = 19.58 \text{ kpsi}, \quad \tau_{xy} = 7.45 \text{ kpsi}$$

Eq. (5-15) $$\sigma' = [11.18^2 - (11.18)(19.58) + 19.58^2 + 3(7.45^2)]^{1/2} = \frac{S_y}{n_o} = \frac{60}{n_o}$$

$$21.35 = \frac{60}{n_o} \quad \Rightarrow \quad n_o = 2.81 \quad \textit{Ans.}$$

Inner radius, 3D state of stress



From Eq. (5-14) with $\tau_{yz} = \tau_{zx} = 0$

$$\sigma' = \frac{1}{\sqrt{2}}[(9.78 - 22.58)^2 + (22.58 + 2.997)^2 + (-2.997 - 9.78)^2 + 6(6.52)^2]^{1/2} = \frac{60}{n_i}$$

$$24.86 = \frac{60}{n_i} \quad \Rightarrow \quad n_i = 2.41 \quad \textit{Ans.}$$

5-39 From Prob. 5-38: $p = 2.997$ kpsi, $I = 0.5369$ in^4, $J = 1.0738$ in^4

Inner member:

Outer radius: $$(\sigma_t)_o = -2.997\left[\frac{(0.875^2 + 0.75^2)}{(0.875^2 - 0.75^2)}\right] = -19.60 \text{ kpsi}$$

$$(\sigma_r)_o = -2.997 \text{ kpsi}$$

Inner radius: $$(\sigma_t)_i = -\frac{2(2.997)(0.875^2)}{0.875^2 - 0.75^2} = -22.59 \text{ kpsi}$$

$$(\sigma_r)_i = 0$$

Bending:

r_o: $$(\sigma_x)_o = \frac{6(0.875)}{0.5369} = 9.78 \text{ kpsi}$$

r_i: $$(\sigma_x)_i = \frac{6(0.75)}{0.5369} = 8.38 \text{ kpsi}$$

Torsion:

r_o: $$(\tau_{xy})_o = \frac{8(0.875)}{1.0738} = 6.52 \text{ kpsi}$$

r_i: $$(\tau_{xy})_i = \frac{8(0.75)}{1.0738} = 5.59 \text{ kpsi}$$

The inner radius is in plane stress: $\sigma_x = 8.38$ kpsi, $\sigma_y = -22.59$ kpsi, $\tau_{xy} = 5.59$ kpsi

$$\sigma_i' = [8.38^2 - (8.38)(-22.59) + (-22.59)^2 + 3(5.59^2)]^{1/2} = 29.4 \text{ kpsi}$$

$$n_i = \frac{S_y}{\sigma_i'} = \frac{60}{29.4} = 2.04 \quad \textit{Ans.}$$

Outer radius experiences a radial stress, σ_r

$$\sigma_o' = \frac{1}{\sqrt{2}}\left[(-19.60 + 2.997)^2 + (-2.997 - 9.78)^2 + (9.78 + 19.60)^2 + 6(6.52)^2\right]^{1/2}$$
$$= 27.9 \text{ kpsi}$$

$$n_o = \frac{60}{27.9} = 2.15 \quad \textit{Ans.}$$

5-40

$$\sigma_p = \frac{1}{2}\left(2\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\right) \pm \left[\left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\sin\frac{3\theta}{2}\right)^2 + \left(\frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)^2\right]^{1/2}$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left[\cos\frac{\theta}{2} \pm \left(\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\sin^2\frac{3\theta}{2} + \sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\cos^2\frac{3\theta}{2}\right)^{1/2}\right]$$

$$= \frac{K_I}{\sqrt{2\pi r}}\left(\cos\frac{\theta}{2} \pm \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right) = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 \pm \sin\frac{\theta}{2}\right)$$

Plane stress: The third principal stress is zero and

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\right), \quad \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right), \quad \sigma_3 = 0 \quad \textit{Ans.}$$

Plane strain: σ_1 and σ_2 equations still valid however,

$$\sigma_3 = \nu(\sigma_x + \sigma_y) = 2\nu\frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2} \quad \textit{Ans.}$$

5-41 For $\theta = 0$ and plane strain, the principal stress equations of Prob. 5-40 give

$$\sigma_1 = \sigma_2 = \frac{K_I}{\sqrt{2\pi r}}, \quad \sigma_3 = 2\nu\frac{K_I}{\sqrt{2\pi r}} = 2\nu\sigma_1$$

(a) DE: $$\frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_1)^2 + (\sigma_1 - 2\nu\sigma_1)^2 + (2\nu\sigma_1 - \sigma_1)^2]^{1/2} = S_y$$

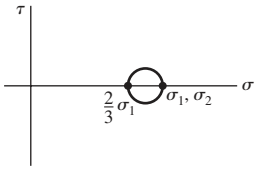
$$\sigma_1 - 2\nu\sigma_1 = S_y$$

For $\nu = \frac{1}{3}$, $$\left[1 - 2\left(\frac{1}{3}\right)\right]\sigma_1 = S_y \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

(b) MSS: $\sigma_1 - \sigma_3 = S_y \quad \Rightarrow \quad \sigma_1 - 2\nu\sigma_1 = S_y$

$$\nu = \frac{1}{3} \quad \Rightarrow \quad \sigma_1 = 3S_y \quad \textit{Ans.}$$

$$\sigma_3 = \frac{2}{3}\sigma_1$$



Radius of largest circle

$$R = \frac{1}{2}\left[\sigma_1 - \frac{2}{3}\sigma_1\right] = \frac{\sigma_1}{6}$$

5-42 **(a)** Ignoring stress concentration

$$F = S_y A = 160(4)(0.5) = 320 \text{ kips} \quad \textit{Ans.}$$

(b) From Fig. 6-36: $h/b = 1$, $a/b = 0.625/4 = 0.1563$, $\beta = 1.3$

Eq. (6-51) $$70 = 1.3\frac{F}{4(0.5)}\sqrt{\pi(0.625)}$$

$$F = 76.9 \text{ kips} \quad \textit{Ans.}$$

5-43 Given: $a = 12.5$ mm, $K_{Ic} = 80$ MPa $\cdot \sqrt{m}$, $S_y = 1200$ MPa, $S_{ut} = 1350$ MPa

$$r_o = \frac{350}{2} = 175 \text{ mm}, \quad r_i = \frac{350 - 50}{2} = 150 \text{ mm}$$

$$a/(r_o - r_i) = \frac{12.5}{175 - 150} = 0.5$$

$$r_i/r_o = \frac{150}{175} = 0.857$$

Fig. 5-30: $\beta \doteq 2.5$

Eq. (5-37): $K_{Ic} = \beta\sigma\sqrt{\pi a}$

$$80 = 2.5\sigma\sqrt{\pi(0.0125)}$$

$$\sigma = 161.5 \text{ MPa}$$

Eq. (3-50) at $r = r_o$:

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}(2)$$

$$161.5 = \frac{150^2 p_i(2)}{175^2 - 150^2}$$

$$p_i = 29.2 \text{ MPa} \quad \textit{Ans.}$$

5-44

(a) First convert the data to radial dimensions to agree with the formulations of Fig. 3-33. Thus

$$r_o = 0.5625 \pm 0.001 \text{ in}$$
$$r_i = 0.1875 \pm 0.001 \text{ in}$$
$$R_o = 0.375 \pm 0.0002 \text{ in}$$
$$R_i = 0.376 \pm 0.0002 \text{ in}$$

The stochastic nature of the dimensions affects the $\boldsymbol{\delta} = |\mathbf{R}_i| - |\mathbf{R}_o|$ relation in Eq. (3-57) but not the others. Set $R = (1/2)(R_i + R_o) = 0.3755$. From Eq. (3-57)

$$\mathbf{p} = \frac{E\boldsymbol{\delta}}{R}\left[\frac{\left(r_o^2 - R^2\right)\left(R^2 - r_i^2\right)}{2R^2\left(r_o^2 - r_i^2\right)}\right]$$

Substituting and solving with $E = 30$ Mpsi gives

$$\mathbf{p} = 18.70(10^6)\,\boldsymbol{\delta}$$

Since $\boldsymbol{\delta} = \mathbf{R}_i - \mathbf{R}_o$

$$\bar{\delta} = \bar{R}_i - \bar{R}_o = 0.376 - 0.375 = 0.001 \text{ in}$$

and

$$\hat{\sigma}_\delta = \left[\left(\frac{0.0002}{4}\right)^2 + \left(\frac{0.0002}{4}\right)^2\right]^{1/2}$$
$$= 0.000\,070\,7 \text{ in}$$

Then

$$C_\delta = \frac{\hat{\sigma}_\delta}{\bar{\delta}} = \frac{0.000\,070\,7}{0.001} = 0.0707$$

The tangential inner-cylinder stress at the shrink-fit surface is given by

$$\boldsymbol{\sigma}_{it} = -\mathbf{p}\frac{\bar{R}^2 + \bar{r}_i^2}{\bar{R}^2 - \bar{r}_i^2}$$
$$= -18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.3755^2 + 0.1875^2}{0.3755^2 - 0.1875^2}\right)$$
$$= -31.1(10^6)\,\boldsymbol{\delta}$$
$$\bar{\sigma}_{it} = -31.1(10^6)\,\bar{\delta} = -31.1(10^6)(0.001)$$
$$= -31.1(10^3) \text{ psi}$$

Also

$$\hat{\sigma}_{\sigma_{it}} = |C_\delta \bar{\sigma}_{it}| = 0.0707(-31.1)10^3$$
$$= 2899 \text{ psi}$$
$$\boldsymbol{\sigma}_{it} = \mathbf{N}(-31\,100, 2899) \text{ psi} \quad \textit{Ans.}$$

(b) The tangential stress for the outer cylinder at the shrink-fit surface is given by

$$\begin{aligned}
\boldsymbol{\sigma}_{ot} &= \mathbf{p}\left(\frac{\bar{r}_o^2 + \bar{R}^2}{\bar{r}_o^2 - \bar{R}^2}\right) \\
&= 18.70(10^6)\,\boldsymbol{\delta}\left(\frac{0.5625^2 + 0.3755^2}{0.5625^2 - 0.3755^2}\right) \\
&= 48.76(10^6)\,\boldsymbol{\delta} \text{ psi} \\
\bar{\sigma}_{ot} &= 48.76(10^6)(0.001) = 48.76(10^3) \text{ psi} \\
\hat{\sigma}_{\sigma_{ot}} &= C_\delta \bar{\sigma}_{ot} = 0.0707(48.76)(10^3) = 34.45 \text{ psi} \\
\therefore \boldsymbol{\sigma}_{ot} &= \mathbf{N}(48\,760, 3445) \text{ psi} \quad \textit{Ans.}
\end{aligned}$$

5-45 From Prob. 5-44, at the fit surface $\boldsymbol{\sigma}_{ot} = \mathbf{N}(48.8, 3.45)$ kpsi. The radial stress is the fit pressure which was found to be

$$\begin{aligned}
\mathbf{p} &= 18.70(10^6)\,\boldsymbol{\delta} \\
\bar{p} &= 18.70(10^6)(0.001) = 18.7(10^3) \text{ psi} \\
\hat{\sigma}_p &= C_\delta \bar{p} = 0.0707(18.70)(10^3) \\
&= 1322 \text{ psi}
\end{aligned}$$

and so

$$\mathbf{p} = \mathbf{N}(18.7, 1.32) \text{ kpsi}$$

and

$$\boldsymbol{\sigma}_{or} = -\mathbf{N}(18.7, 1.32) \text{ kpsi}$$

These represent the principal stresses. The von Mises stress is next assessed.

$$\begin{aligned}
\bar{\sigma}_A &= 48.8 \text{ kpsi}, \quad \bar{\sigma}_B = -18.7 \text{ kpsi} \\
k &= \bar{\sigma}_B / \bar{\sigma}_A = -18.7/48.8 = -0.383 \\
\bar{\sigma}' &= \bar{\sigma}_A (1 - k + k^2)^{1/2} \\
&= 48.8[1 - (-0.383) + (-0.383)^2]^{1/2} \\
&= 60.4 \text{ kpsi} \\
\hat{\sigma}_{\sigma'} &= C_p \bar{\sigma}' = 0.0707(60.4) = 4.27 \text{ kpsi}
\end{aligned}$$

Using the interference equation

$$\begin{aligned}
z &= -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}} \\
&= -\frac{95.5 - 60.4}{[(6.59)^2 + (4.27)^2]^{1/2}} = -4.5 \\
p_f &= \alpha = 0.000\,003\,40,
\end{aligned}$$

or about 3 chances in a million. *Ans.*

5-46

$$\boldsymbol{\sigma}_t = \frac{\mathbf{p}d}{2t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{2(0.125)}$$

$$= 18\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_l = \frac{\mathbf{p}d}{4t} = \frac{6000\mathbf{N}(1, 0.083\,33)(0.75)}{4(0.125)}$$

$$= 9\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

$$\boldsymbol{\sigma}_r = -\mathbf{p} = -6000\mathbf{N}(1, 0.083\,33) \text{ kpsi}$$

These three stresses are principal stresses whose variability is due to the loading. From Eq. (5-12), we find the von Mises stress to be

$$\sigma' = \left\{ \frac{(18-9)^2 + [9-(-6)]^2 + (-6-18)^2}{2} \right\}^{1/2}$$

$$= 21.0 \text{ kpsi}$$

$$\hat{\sigma}_{\sigma'} = C_p \bar{\sigma}' = 0.083\,33(21.0) = 1.75 \text{ kpsi}$$

$$z = -\frac{\bar{S} - \bar{\sigma}'}{\left(\hat{\sigma}_S^2 + \hat{\sigma}_{\sigma'}^2\right)^{1/2}}$$

$$= \frac{50 - 21.0}{(4.1^2 + 1.75^2)^{1/2}} = -6.5$$

The reliability is very high

$$R = 1 - \Phi(6.5) = 1 - 4.02(10^{-11}) \doteq 1 \quad \textit{Ans.}$$

Chapter 6

6-1 Eq. (2-21): $S_{ut} = 3.4H_B = 3.4(300) = 1020$ MPa

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(1020) = 510$ MPa

Table 6-2: $a = 1.58,\ b = -0.085$

Eq. (6-19): $k_a = aS_{ut}^b = 1.58(1020)^{-0.085} = 0.877$

Eq. (6-20): $k_b = 1.24d^{-0.107} = 1.24(10)^{-0.107} = 0.969$

Eq. (6-18): $S_e = k_a k_b S'_e = (0.877)(0.969)(510) = 433$ MPa *Ans.*

6-2 **(a)** Table A-20: $S_{ut} = 80$ kpsi

Eq. (6-8): $S'_e = 0.5(80) = 40$ kpsi *Ans.*

(b) Table A-20: $S_{ut} = 90$ kpsi

Eq. (6-8): $S'_e = 0.5(90) = 45$ kpsi *Ans.*

(c) Aluminum has no endurance limit. *Ans.*

(d) Eq. (6-8): $S_{ut} > 200$ kpsi, $S'_e = 100$ kpsi *Ans.*

6-3 $S_{ut} = 120$ kpsi, $\sigma_{\text{rev}} = 70$ kpsi

Fig. 6-18: $f = 0.82$

Eq. (6-8): $S'_e = S_e = 0.5(120) = 60$ kpsi

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.82(120)\right]^2}{60} = 161.4 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{60}\right) = -0.0716$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{70}{161.4}\right)^{\frac{1}{-0.0716}} = 116\,700 \text{ cycles}$$ *Ans.*

6-4 $S_{ut} = 1600$ MPa, $\sigma_{\text{rev}} = 900$ MPa

Fig. 6-18: $S_{ut} = 1600$ MPa = 232 kpsi. Off the graph, so estimate $f = 0.77$.

Eq. (6-8): $S_{ut} > 1400$ MPa, so $S_e = 700$ MPa

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(1600)\right]^2}{700} = 2168.3 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(1600)}{700}\right) = -0.081838$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{900}{2168.3}\right)^{\frac{1}{-0.081838}} = 46\,400 \text{ cycles} \quad \textit{Ans.}$$

6-5 $S_{ut} = 230$ kpsi, $N = 150\,000$ cycles

Fig. 6-18, point is off the graph, so estimate: $f = 0.77$

Eq. (6-8): $S_{ut} > 200$ kpsi, so $S'_e = S_e = 100$ kpsi

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.77(230)\right]^2}{100} = 313.6 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.77(230)}{100}\right) = -0.08274$$

Eq. (6-13): $$S_f = aN^b = 313.6(150\,000)^{-0.08274} = 117.0 \text{ kpsi} \quad \textit{Ans.}$$

6-6 $S_{ut} = 1100$ MPa = 160 kpsi

Fig. 6-18: $f = 0.79$

Eq. (6-8): $S'_e = S_e = 0.5(1100) = 550$ MPa

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.79(1100)\right]^2}{550} = 1373 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.79(1100)}{550}\right) = -0.06622$$

Eq. (6-13): $$S_f = aN^b = 1373(150\,000)^{-0.06622} = 624 \text{ MPa} \quad \textit{Ans.}$$

6-7 $S_{ut} = 150$ kpsi, $S_{yt} = 135$ kpsi, $N = 500$ cycles

Fig. 6-18: $f = 0.798$

From Fig. 6-10, we note that below 10^3 cycles on the *S*-*N* diagram constitutes the low-cycle region, in which Eq. (6-17) is applicable.

Eq. (6-17): $S_f = S_{ut} N^{(\log f)/3} = 150\left(500\right)^{\left[\log(0.798)\right]/3} = 122$ kpsi *Ans.*

The testing should be done at a completely reversed stress of 122 kpsi, which is below the yield strength, so it is possible. *Ans.*

6-8 The general equation for a line on a log S_f - log N scale is $S_f = aN^b$, which is Eq. (6-13). By taking the log of both sides, we can get the equation of the line in slope-intercept form.

$$\log S_f = b \log N + \log a$$

Substitute the two known points to solve for unknowns a and b. Substituting point (1, S_{ut}),

$$\log S_{ut} = b \log(1) + \log a$$

From which $a = S_{ut}$. Substituting point $(10^3, f\, S_{ut})$ and $a = S_{ut}$

$$\log f\, S_{ut} = b \log 10^3 + \log S_{ut}$$

From which $b = \left(1/3\right) \log f$

$$\therefore S_f = S_{ut} N^{(\log f)/3} \qquad 1 \le N \le 10^3$$

6-9 Read from graph: $\left(10^3, 90\right)$ and $(10^6, 50)$. From $S = aN^b$

$$\log S_1 = \log a + b \log N_1$$
$$\log S_2 = \log a + b \log N_2$$

From which

$$\log a = \frac{\log S_1 \log N_2 - \log S_2 \log N_1}{\log N_2 / N_1}$$
$$= \frac{\log 90 \log 10^6 - \log 50 \log 10^3}{\log 10^6 / 10^3}$$
$$= 2.2095$$

$$a = 10^{\log a} = 10^{2.2095} = 162.0 \text{ kpsi}$$
$$b = \frac{\log 50/90}{3} = -0.0851$$
$$(S_f)_{ax} = 162 N^{-0.0851} \qquad 10^3 \le N \le 10^6 \text{ in kpsi} \qquad \textit{Ans.}$$

Check:

$$\left[(S_f)_{ax}\right]_{10^3} = 162(10^3)^{-0.0851} = 90 \text{ kpsi}$$
$$\left[(S_f)_{ax}\right]_{10^6} = 162(10^6)^{-0.0851} = 50 \text{ kpsi}$$

The end points agree.

6-10 $d = 1.5$ in, $S_{ut} = 110$ kpsi

Eq. (6-8): $S'_e = 0.5(110) = 55$ kpsi
Table 6-2: $a = 2.70$, $b = -0.265$
Eq. (6-19): $k_a = aS_{ut}^{\ b} = 2.70(110)^{-0.265} = 0.777$

Since the loading situation is not specified, we'll assume rotating bending or torsion so Eq. (6-20) is applicable. This would be the worst case.

$$k_b = 0.879d^{-0.107} = 0.879(1.5)^{-0.107} = 0.842$$
Eq. (6-18): $S_e = k_a k_b S'_e = 0.777(0.842)(55) = 36.0$ kpsi *Ans.*

6-11 For AISI 4340 as-forged steel,

Eq. (6-8): $S_e = 100$ kpsi
Table 6-2: $a = 39.9$, $b = -0.995$
Eq. (6-19): $k_a = 39.9(260)^{-0.995} = 0.158$
Eq. (6-20): $k_b = \left(\dfrac{0.75}{0.30}\right)^{-0.107} = 0.907$

Each of the other modifying factors is unity.

$$S_e = 0.158(0.907)(100) = 14.3 \text{ kpsi}$$

For AISI 1040:

$$S'_e = 0.5(113) = 56.5 \text{ kpsi}$$
$$k_a = 39.9(113)^{-0.995} = 0.362$$
$$k_b = 0.907 \text{ (same as 4340)}$$

Each of the other modifying factors is unity

$$S_e = 0.362(0.907)(56.5) = 18.6 \text{ kpsi}$$

Not only is AISI 1040 steel a contender, it has a superior endurance strength.

6-12 $D = 1$ in, $d = 0.8$ in, $T = 1800$ lbf·in, $f = 0.9$, and from Table A-20 for AISI 1020 CD, $S_{ut} = 68$ kpsi, and $S_y = 57$ kpsi.

(a) Fig. A-15-15: $\dfrac{r}{d} = \dfrac{0.1}{0.8} = 0.125, \dfrac{D}{d} = \dfrac{1}{0.8} = 1.25, K_{ts} = 1.40$

Get the notch sensitivity either from Fig. 6-21, or from the curve-fit Eqs. (6-34) and (6-35b). We'll use the equations.

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(68) + 1.35\left(10^{-5}\right)(68)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.07335$$

$$q_s = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07335}{\sqrt{0.1}}} = 0.812$$

Eq. (6-32): $K_{fs} = 1 + q_s\,(K_{ts} - 1) = 1 + 0.812(1.40 - 1) = 1.32$

For a purely reversing torque of $T = 1800$ lbf·in,

$$\tau_a = K_{fs}\frac{Tr}{J} = \frac{K_{fs}16T}{\pi d^3} = \frac{1.32(16)(1800)}{\pi(0.8)^3} = 23\,635 \text{ psi} = 23.6 \text{ kpsi}$$

Eq. (6-8): $S'_e = 0.5(68) = 34$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Eq. (6-20): $k_b = 0.879(0.8)^{-0.107} = 0.900$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18) (labeling for shear): $S_{se} = 0.883(0.900)(0.59)(34) = 15.9$ kpsi

For purely reversing torsion, use Eq. (6-54) for the ultimate strength in shear.

Eq. (6-54): $S_{su} = 0.67\,S_{ut} = 0.67(68) = 45.6$ kpsi

Adjusting the fatigue strength equations for shear,

Eq. (6-14): $$a = \frac{\left(f\,S_{su}\right)^2}{S_{se}} = \frac{\left[0.9(45.6)\right]^2}{15.9} = 105.9 \text{ kpsi}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{15.9}\right) = -0.137\;27$$

Eq. (6-16): $$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.3}{105.9}\right)^{\frac{1}{-0.137\;27}} = 61.7\left(10^3\right) \text{ cycles} \quad \textit{Ans.}$$

(b) For an operating temperature of 750°F, the temperature modification factor, from Table 6-4 is $k_d = 0.90$.

$$S_{se} = 0.883(0.900)(0.59)(0.9)(34) = 14.3 \text{ kpsi}$$

$$a = \frac{(f S_{su})^2}{S_{se}} = \frac{[0.9(45.6)]^2}{14.3} = 117.8 \text{ kpsi}$$

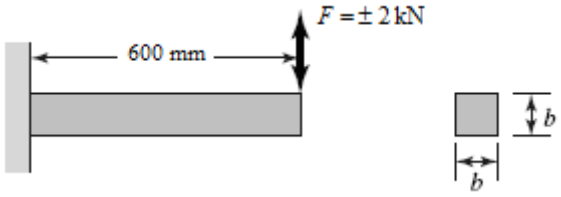
$$b = -\frac{1}{3}\log\left(\frac{f S_{su}}{S_{se}}\right) = -\frac{1}{3}\log\left(\frac{0.9(45.6)}{14.3}\right) = -0.152\,62$$

$$N = \left(\frac{\tau_a}{a}\right)^{\frac{1}{b}} = \left(\frac{23.3}{117.8}\right)^{\frac{1}{-0.152\,62}} = 40.9\left(10^3\right) \text{ cycles} \qquad \textit{Ans.}$$

6-13 $L = 0.6$ m, $F_a = 2$ kN, $n = 1.5$, $N = 10^4$ cycles, $S_{ut} = 770$ MPa, $S_y = 420$ MPa (Table A-20)

First evaluate the fatigue strength.

$$S'_e = 0.5(770) = 385 \text{ MPa}$$
$$k_a = 57.7(770)^{-0.718} = 0.488$$



Since the size is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

Eq. (6-18): $S_e = 0.488(0.85)(385) = 160$ MPa

In kpsi, $S_{ut} = 770/6.89 = 112$ kpsi

Fig. 6-18: $f = 0.83$

Eq. (6-14): $$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.83(770)]^2}{160} = 2553 \text{ MPa}$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.83(770)}{160}\right) = -0.2005$$

Eq. (6-13): $$S_f = aN^b = 2553(10^4)^{-0.2005} = 403 \text{ MPa}$$

Now evaluate the stress.

$$M_{\max} = (2000 \text{ N})(0.6 \text{ m}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_a = \sigma_{\max} = \frac{Mc}{I} = \frac{M(b/2)}{b(b^3)/12} = \frac{6M}{b^3} = \frac{6(1200)}{b^3} = \frac{7200}{b^3} \text{ Pa, with } b \text{ in m.}$$

Compare strength to stress and solve for the necessary b.

$$n = \frac{S_f}{\sigma_a} = \frac{403\left(10^6\right)}{7200/b^3} = 1.5$$

$$b = 0.0299 \text{ m} \quad \text{Select } b = 30 \text{ mm.}$$

Since the size factor was guessed, go back and check it now.

Eq. (6-25): $d_e = 0.808\left(hb\right)^{1/2} = 0.808b = 0.808\left(30\right) = 24.24 \text{ mm}$

Eq. (6-20): $k_b = \left(\frac{24.2}{7.62}\right)^{-0.107} = 0.88$

Our guess of 0.85 was slightly conservative, so we will accept the result of

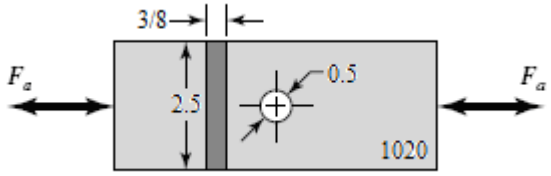
$$b = 30 \text{ mm.} \quad \textit{Ans.}$$

Checking yield,

$$\sigma_{\max} = \frac{7200}{0.030^3}\left(10^{-6}\right) = 267 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{420}{267} = 1.57$$

6-14 Given: w =2.5 in, t = 3/8 in, d = 0.5 in, n_d = 2. From Table A-20, for AISI 1020 CD, S_{ut} = 68 kpsi and S_y = 57 kpsi.



Eq. (6-8): $S_e' = 0.5(68) = 34 \text{ kpsi}$

Table 6-2: $k_a = 2.70(68)^{-0.265} = 0.88$

Eq. (6-21): $k_b = 1$ (axial loading)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = 0.88(1)(0.85)(34) = 25.4 \text{ kpsi}$

Table A-15-1: $d/w = 0.5/2.5 = 0.2,\ K_t = 2.5$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). The relatively large radius is off the graph of Fig. 6-20, so we'll assume the curves continue according to the same trend and use the equations to estimate the notch sensitivity.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)\left(68\right) + 1.51\left(10^{-5}\right)\left(68\right)^2 - 2.67\left(10^{-8}\right)\left(68^3\right) = 0.09799$$

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \frac{0.09799}{\sqrt{0.25}}} = 0.836$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.836(2.5 - 1) = 2.25$

$$\sigma_a = K_f \frac{F_a}{A} = \frac{2.25F_a}{(3/8)(2.5-0.5)} = 3F_a$$

Since a finite life was not mentioned, we'll assume infinite life is desired, so the completely reversed stress must stay below the endurance limit.

$$n_f = \frac{S_e}{\sigma_a} = \frac{25.4}{3F_a} = 2$$

$$F_a = 4.23 \text{ kips} \quad \textit{Ans.}$$

6-15 Given: $D = 2$ in, $d = 1.8$ in, $r = 0.1$ in, $M_{\max} = 25\,000$ lbf · in, $M_{\min} = 0$.
From Table A-20, for AISI 1095 HR, S_{ut} = 120 kpsi and S_y = 66 kpsi.

Eq. (6-8): $S'_e = 0.5S_{ut} = 0.5(120) = 60$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(120)^{-0.265} = 0.76$

Eq. (6-24): $d_e = 0.370d = 0.370(1.8) = 0.666$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.666)^{-0.107} = 0.92$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S'_e = (0.76)(0.92)(1)(60) = 42.0$ kpsi

Fig. A-15-14: $D/d = 2/1.8 = 1.11, \quad r/d = 0.1/1.8 = 0.056 \quad \therefore K_t = 2.1$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We'll use the equations.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(120) + 1.51\left(10^{-5}\right)(120)^2 - 2.67\left(10^{-8}\right)\left(120^3\right) = 0.04770$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.04770}{\sqrt{0.1}}} = 0.87$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.87(2.1 - 1) = 1.96$

$$I = (\pi/64)d^4 = (\pi/64)(1.8)^4 = 0.5153 \text{ in}^4$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{25\,000(1.8/2)}{0.5153} = 43\,664 \text{ psi} = 43.7 \text{ kpsi}$$

$$\sigma_{\min} = 0$$

Eq. (6-36): $$\sigma_m = K_f \frac{\sigma_{\max} + \sigma_{\min}}{2} = (1.96)\frac{(43.7+0)}{2} = 42.8 \text{ kpsi}$$

$$\sigma_a = K_f \left|\frac{\sigma_{\max} - \sigma_{\min}}{2}\right| = (1.96)\left|\frac{(43.7-0)}{2}\right| = 42.8 \text{ kpsi}$$

Eq. (6-46): $$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{42.8}{42.0} + \frac{42.8}{120}$$

$$n_f = 0.73 \quad \textit{Ans.}$$

A factor of safety less than unity indicates a finite life.

Check for yielding. It is not necessary to include the stress concentration for static yielding of a ductile material.

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{66}{43.7} = 1.51 \quad \textit{Ans.}$$

6-16 From a free-body diagram analysis, the bearing reaction forces are found to be 2.1 kN at the left bearing and 3.9 kN at the right bearing. The critical location will be at the shoulder fillet between the 35 mm and the 50 mm diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists. The bending moment at this point is M = 2.1(200) = 420 kN·mm. With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{420\,(35/2)}{(\pi/64)(35)^4} = 0.09978 \text{ kN/mm}^2 = 99.8 \text{ MPa}$$

This stress is far below the yield strength of 390 MPa, so yielding is not predicted. Find the stress concentration factor for the fatigue analysis.

Fig. A-15-9: r/d = 3/35 = 0.086, D/d = 50/35 = 1.43, K_t =1.7

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We'll use the equations, with S_{ut} = 470 MPa = 68.2 kpsi and r = 3 mm = 0.118 in.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(68.2) + 1.51\left(10^{-5}\right)(68.2)^2 - 2.67\left(10^{-8}\right)(68.2)^3 = 0.09771$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.09771}{\sqrt{0.118}}} = 0.78$$

Eq. (6-32): $$K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.55$$

Eq. (6-8): $S_e^{'} = 0.5S_{ut} = 0.5(470) = 235$ MPa

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(470)^{-0.265} = 0.88$

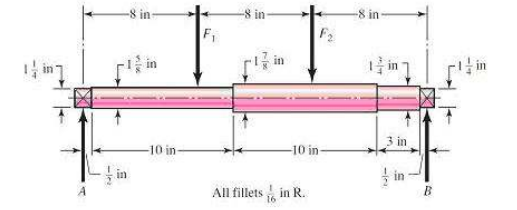
Eq. (6-24): $k_b = 1.24d^{-0.107} = 1.24(35)^{-0.107} = 0.85$

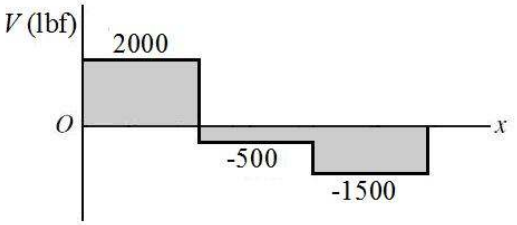
Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.88)(0.85)(1)(235) = 176$ MPa

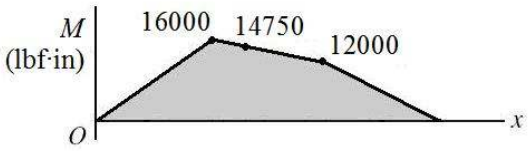
$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{176}{1.55(99.8)} = 1.14$$ Infinite life is predicted. *Ans.*

6-17 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 2000 lbf and R_B = 1500 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.





$M = 16\,000 - 500\,(2.5) = 14\,750$ lbf · in



With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{14\,750(1.625/2)}{(\pi/64)(1.625)^4} = 35.0 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We will use the equations.

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07690}{\sqrt{0.0625}}} = 0.76.$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-8): $S_e^{'} = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$

Eq. (6-20): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.832)(0.835)(1)(42.5) = 29.5 \text{ kpsi}$

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72(35.0)} = 0.49 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

Fig. 6-18: $f = 0.867$

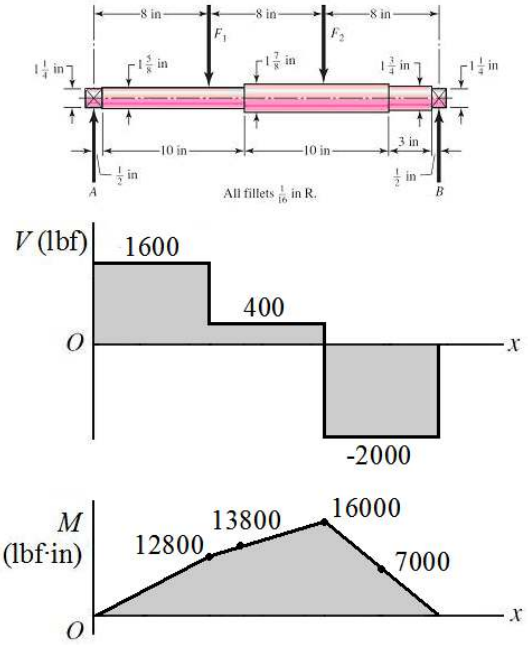
Eq. (6-14): $$a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.867(85)]^2}{29.5} = 184.1$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$$

Eq. (6-16): $$N = \left(\frac{K_f \sigma_{\text{rev}}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(35.0)}{184.1}\right)^{\frac{1}{-0.1325}} = 4611 \text{ cycles}$$

$N = 4600$ cycles *Ans.*

6-18 From a free-body diagram analysis, the bearing reaction forces are found to be R_A = 1600 lbf and R_B = 2000 lbf. The shear-force and bending-moment diagrams are shown. The critical location will be at the shoulder fillet between the 1-5/8 in and the 1-7/8 in diameters, where the bending moment is large, the diameter is smaller, and the stress concentration exists.



$$M = 12\,800 + 400\,(2.5) = 13\,800 \text{ lbf} \cdot \text{in}$$

With a rotating shaft, the bending stress will be completely reversed.

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{13\,800(1.625/2)}{(\pi/64)(1.625)^4} = 32.8 \text{ kpsi}$$

This stress is far below the yield strength of 71 kpsi, so yielding is not predicted.

Fig. A-15-9: $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_t = 1.95$

Get the notch sensitivity either from Fig. 6-20, or from the curve-fit Eqs. (6-34) and (6-35a). We will use the equations

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(85) + 1.51\left(10^{-5}\right)(85)^2 - 2.67\left(10^{-8}\right)(85)^3 = 0.07690$$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.07690}{\sqrt{0.0625}}} = 0.76$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.76(1.95 - 1) = 1.72$

Eq. (6-8): $S_e^{'} = 0.5S_{ut} = 0.5(85) = 42.5$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(85)^{-0.265} = 0.832$

Eq. (6-20): $k_b = 0.879d^{-0.107} = 0.879(1.625)^{-0.107} = 0.835$

Eq. (6-26): $k_c = 1$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.832)(0.835)(1)(42.5) = 29.5$ kpsi

$$n_f = \frac{S_e}{K_f \sigma_{\text{rev}}} = \frac{29.5}{1.72\left(32.8\right)} = 0.52 \qquad \textit{Ans.}$$

Infinite life is not predicted. Use the *S-N* diagram to estimate the life.

Fig. 6-18: $f = 0.867$

Eq. (6-14): $a = \frac{\left(f\,S_{ut}\right)^2}{S_e} = \frac{\left[0.867(85)\right]^2}{29.5} = 184.1$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$

Eq. (6-16): $N = \left(\frac{K_f \sigma_{\text{rev}}}{a}\right)^{\frac{1}{b}} = \left(\frac{(1.72)(32.8)}{184.1}\right)^{\frac{1}{-0.1325}} = 7527$ cycles

$N = 7500$ cycles *Ans.*

6-19 Table A-20: $S_{ut} = 120$ kpsi, $S_y = 66$ kpsi

N = (950 rev/min)(10 hr)(60 min/hr) = 570 000 cycles

One approach is to guess a diameter and solve the problem as an iterative analysis problem. Alternatively, we can estimate the few modifying parameters that are dependent on the diameter and solve the stress equation for the diameter, then iterate to check the estimates. We'll use the second approach since it should require only one iteration, since the estimates on the modifying parameters should be pretty close.

First, we'll evaluate the stress. From a free-body diagram analysis, the reaction forces at the bearings are $R_1 = 2$ kips and $R_2 = 6$ kips. The critical stress location is in the middle of the span at the shoulder, where the bending moment is high, the shaft diameter is smaller, and a stress concentration factor exists. If the critical location is not obvious, prepare a complete bending moment diagram and evaluate at any potentially critical locations. Evaluating at the critical shoulder,

$$M = 2 \text{ kip}(10 \text{ in}) = 20 \text{ kip} \cdot \text{in}$$

$$\sigma_{\text{rev}} = \frac{Mc}{I} = \frac{M(d/2)}{\pi d^4/64} = \frac{32M}{\pi d^3} = \frac{32(20)}{\pi d^3} = \frac{203.7}{d^3} \text{ kpsi}$$

Now we'll get the notch sensitivity and stress concentration factor. The notch sensitivity depends on the fillet radius, which depends on the unknown diameter. For now, we'll estimate a value for $q = 0.85$ from observation of Fig. 6-20, and check it later.

Fig. A-15-9: $D/d = 1.4 d/d = 1.4, \quad r/d = 0.1 d/d = 0.1, \quad K_t = 1.65$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.85(1.65 - 1) = 1.55$

Now we will evaluate the fatigue strength.

$$S_e^{'} = 0.5(120) = 60 \text{ kpsi}$$

$$k_a = 2.70(120)^{-0.265} = 0.76$$

Since the diameter is not yet known, assume a typical value of $k_b = 0.85$ and check later. All other modifiers are equal to one.

$$S_e = (0.76)(0.85)(60) = 38.8 \text{ kpsi}$$

Determine the desired fatigue strength from the *S-N* diagram.

Fig. 6-18: $f = 0.82$

Eq. (6-14): $a = \frac{(f S_{ut})^2}{S_e} = \frac{[0.82(120)]^2}{38.8} = 249.6$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.82(120)}{38.8}\right) = -0.1347$

Eq. (6-13): $S_f = aN^b = 249.6(570\,000)^{-0.1347} = 41.9 \text{ kpsi}$

Compare strength to stress and solve for the necessary *d*.

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{41.9}{(1.55)\left(203.7/d^3\right)} = 1.6$$

$$d = 2.29 \text{ in}$$

Since the size factor and notch sensitivity were guessed, go back and check them now.

Eq. (6-20): $k_b = 0.91 d^{-0.157} = 0.91(2.29)^{-0.157} = 0.80$

Our guess of 0.85 was conservative. From Fig. 6-20 with $r = d/10 = 0.229$ in, we are off the graph, but it appears our guess for q is low. Assuming the trend of the graph continues, we'll choose $q = 0.91$ and iterate the problem with the new values of k_b and q. Intermediate results are $S_e = 36.5$ kpsi, $S_f = 39.6$ kpsi, and $K_f = 1.59$. This gives

$$n_f = \frac{S_f}{K_f \sigma_{\text{rev}}} = \frac{39.6}{(1.59)\left(203.7/d^3\right)} = 1.6$$

$$d = 2.36 \text{ in} \qquad \textit{Ans.}$$

A quick check of k_b and q show that our estimates are still reasonable for this diameter.

6-20 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 25$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[25^2 + 3(0)^2\right]^{1/2} = 25.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[25^2 + 3\left(15^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \qquad \textit{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.00/40) + (25.98/80)} = 1.05 \qquad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2 \left(\frac{25.00}{40}\right)\left[-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(25.00)}\right)^2}\right] = 1.31 \qquad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.00/40)^2 + (25.98/60)^2}} = 1.32 \quad \textit{Ans.}$$

6-21 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_m = 20$ kpsi, $\sigma_a = 10$ kpsi, $\sigma_m = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[10^2 + 3(0)^2\right]^{1/2} = 10.00 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(20)^2\right]^{1/2} = 34.64 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[10^2 + 3\left(20^2\right)\right]^{1/2} = 36.06 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{36.06} = 1.66 \quad \textit{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(10.00/40) + (34.64/80)} = 1.46 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{34.64}\right)^2\left(\frac{10.00}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(34.64)(40)}{80(10.00)}\right)^2}\right\} = 1.74 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(10.00/40)^2 + (34.64/60)^2}} = 1.59 \quad \textit{Ans.}$$

6-22 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 10$ kpsi, $\tau_m = 15$ kpsi, $\sigma_a = 12$ kpsi, $\sigma_m = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[12^2 + 3(10)^2\right]^{1/2} = 21.07 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma^2_{\max} + 3\tau^2_{\max}\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[(12+0)^2 + 3(10+15)^2\right]^{1/2} = 44.93 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{44.93} = 1.34 \quad \textit{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(21.07/40) + (25.98/80)} = 1.17 \quad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{25.98}\right)^2\left(\frac{21.07}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(25.98)(40)}{80(21.07)}\right)^2}\right\} = 1.47 \quad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(21.07/40)^2 + (25.98/60)^2}} = 1.47 \quad \textit{Ans.}$$

6-23 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 30$ kpsi, $\sigma_m = \sigma_a = \tau_a = 0$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3(30)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = 0 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma^2_{\max} + 3\tau^2_{\max}\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[3(30)^2\right]^{1/2} = 51.96 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{51.96} = 1.15 \quad \textit{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(51.96/40)} = 0.77 \quad \textit{Ans.}$$

(b) Gerber criterion of Table 6-7 is only valid for $\sigma_m > 0$; therefore use Eq. (6-47).

$$n_f \frac{\sigma'_a}{S_e} = 1 \quad \Rightarrow \quad n_f = \frac{S_e}{\sigma'_a} = \frac{40}{51.96} = 0.77 \qquad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(51.96/40)^2}} = 0.77 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. Since $\sigma'_m = 0$, the stress state is completely reversed and the *S-N* diagram is applicable for σ'_a.
Fig. 6-18: $f = 0.875$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.875(80)\right]^2}{40} = 122.5$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.875(80)}{40}\right) = -0.08101$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{51.96}{122.5}\right)^{\frac{1}{-0.08101}} = 39\,600 \text{ cycles} \qquad \textit{Ans.}$$

6-24 $S_e = 40$ kpsi, $S_y = 60$ kpsi, $S_{ut} = 80$ kpsi, $\tau_a = 15$ kpsi, $\sigma_m = 15$ kpsi, $\tau_m = \sigma_a = 0$
Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[0^2 + 3(15)^2\right]^{1/2} = 25.98 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[15^2 + 3(0)^2\right]^{1/2} = 15.00 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(\sigma_a + \sigma_m\right)^2 + 3\left(\tau_a + \tau_m\right)^2\right]^{1/2}$$

$$= \left[(15)^2 + 3(15)^2\right]^{1/2} = 30.00 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{60}{30} = 2.00 \qquad \textit{Ans.}$$

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(25.98/40) + (15.00/80)} = 1.19 \qquad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{80}{15.00}\right)^2\left(\frac{25.98}{40}\right)\left\{-1 + \sqrt{1 + \left(\frac{2(15.00)(40)}{80(25.98)}\right)^2}\right\} = 1.43 \qquad \textit{Ans.}$$

(c) ASME-Elliptic, Table 6-8

$$n_f = \sqrt{\frac{1}{(25.98/40)^2 + (15.00/60)^2}} = 1.44 \quad \textit{Ans.}$$

6-25 Given: $F_{\max} = 28$ kN, $F_{\min} = -28$ kN . From Table A-20, for AISI 1040 CD, $S_{ut} = 590$ MPa, $S_y = 490$ MPa,

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life

Eq. (6-8): $S_e^{'} = 0.5(590) = 295$ MPa

Eq. (6-19): $k_a = aS_{ut}^b = 4.51(590)^{-0.265} = 0.832$

Eq. (6-21): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = k_a k_b k_c S_e^{'} = (0.832)(1)(0.85)(295) = 208.6$ MPa

Fig. 6-20: $q = 0.83$

Fig. A-15-1: $d/w = 0.24$, $K_t = 2.44$

$$K_f = 1 + q(K_t - 1) = 1 + 0.83(2.44 - 1) = 2.20$$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - (-28\,000)}{2(10)(25-6)} \right| = 324.2 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 0$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{324.2}{208.6} + \frac{0}{590}$$

$$n_f = 0.64 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. Since $\sigma_m = 0$, the stress state is completely reversed and the *S-N* diagram is applicable for σ_a.

S_{ut} = 590/6.89 = 85.6 kpsi

Fig. 6-18: $f = 0.87$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.87(590)\right]^2}{208.6} = 1263$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{208.6}\right) = -0.1304$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{324.2}{1263}\right)^{\frac{1}{-0.1304}} = 33\,812 \text{ cycles}$$

$N = 34\,000$ cycles *Ans.*

6-26 $S_{ut} = 590$ MPa, $S_y = 490$ MPa, $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

Determine the fatigue factor of safety based on infinite life.
From Prob. 6-25: $S_e = 208.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000 - (12\,000)}{2(10)(25-6)}\right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000 + 12\,000}{2(10)(25-6)}\right] = 231.6 \text{ MPa}$$

Modified Goodman criteria:

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$$n_f = 1.20 \quad \textit{Ans.}$$

Gerber criteria:

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{590}{231.6}\right)^2 \frac{92.63}{208.6}\left[-1 + \sqrt{1 + \left(\frac{2(231.6)(208.6)}{590(92.63)}\right)^2}\right]$$

$$n_f = 1.49 \quad \textit{Ans.}$$

ASME-Elliptic criteria:

$$n_f = \sqrt{\frac{1}{(\sigma_a / S_e)^2 + (\sigma_m / S_y)^2}} = \sqrt{\frac{1}{(92.63/208.6)^2 + (231.6/490)^2}}$$

$$= 1.54 \quad \textit{Ans.}$$

The results are consistent with Fig. 6-27, where for a mean stress that is about half of the yield strength, the Modified Goodman line should predict failure significantly before the other two.

6-27 $S_{ut} = 590$ MPa, $S_y = 490$ MPa

(a) $F_{\max} = 28$ kN, $F_{\min} = 0$ kN

Check for yielding

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{28\,000}{10(25-6)} = 147.4 \text{ N/mm}^2 = 147.4 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{490}{147.4} = 3.32 \quad \textit{Ans.}$$

From Prob. 6-25: $S_e = 208.6$ MPa, $K_f = 2.2$

$$\sigma_a = K_f \left| \frac{F_{\max} - F_{\min}}{2A} \right| = 2.2 \left| \frac{28\,000 - 0}{2(10)(25-6)} \right| = 162.1 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2 \left[\frac{28\,000 + 0}{2(10)(25-6)} \right] = 162.1 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{162.1}{208.6} + \frac{162.1}{590}$$

$$n_f = 0.95 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})} = \frac{162.1}{1 - (162.1/590)} = 223.5 \text{ MPa}$$

Fig. 6-18: $f = 0.87$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.87(590)]^2}{208.6} = 1263$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.87(590)}{208.6}\right) = -0.1304$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{223.5}{1263}\right)^{\frac{1}{-0.1304}} = 586\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) $F_{\max} = 28$ kN, $F_{\min} = 12$ kN

The maximum load is the same as in part (a), so

$$\sigma_{\max} = 147.4 \text{ MPa}$$

$$n_y = 3.32 \qquad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{28\,000 - 12\,000}{2(10)(25-6)}\right| = 92.63 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{28\,000 + 12\,000}{2(10)(25-6)}\right] = 231.6 \text{ MPa}$$

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{92.63}{208.6} + \frac{231.6}{590}$$

$$n_f = 1.20 \qquad \textit{Ans.}$$

(c) $F_{\max} = 12$ kN, $F_{\min} = -28$ kN

The compressive load is the largest, so check it for yielding.

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{-28\,000}{10(25-6)} = -147.4 \text{ MPa}$$

$$n_y = \frac{S_{yc}}{\sigma_{\min}} = \frac{-490}{-147.4} = 3.32 \qquad \textit{Ans.}$$

Factor of safety based on infinite life:

$$\sigma_a = K_f \left|\frac{F_{\max} - F_{\min}}{2A}\right| = 2.2\left|\frac{12\,000 - (-28\,000)}{2(10)(25-6)}\right| = 231.6 \text{ MPa}$$

$$\sigma_m = K_f \frac{F_{\max} + F_{\min}}{2A} = 2.2\left[\frac{12\,000 + (-28\,000)}{2(10)(25-6)}\right] = -92.63 \text{ MPa}$$

For $\sigma_m < 0$, $$n_f = \frac{S_e}{\sigma_a} = \frac{208.6}{231.6} = 0.90 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. For a negative mean stress, we shall assume the equivalent completely reversed stress is the same as the actual alternating stress. Get *a* and *b* from part (a).

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a} \right)^{1/b} = \left(\frac{231.6}{1263} \right)^{\frac{1}{-0.1304}} = 446\,000 \text{ cycles} \qquad \textit{Ans.}$$

6-28 Eq. (2-21): $S_{ut} = 0.5(400) = 200$ kpsi

Eq. (6-8): $$S_e^{'} = 0.5(200) = 100 \text{ kpsi}$$

Eq. (6-19): $$k_a = aS_{ut}^b = 14.4(200)^{-0.718} = 0.321$$

Eq. (6-25): $$d_e = 0.37d = 0.37(0.375) = 0.1388 \text{ in}$$

Eq. (6-20): $$k_b = 0.879 d_e^{-0.107} = 0.879(0.1388)^{-0.107} = 1.09$$

Since we have used the equivalent diameter method to get the size factor, and in doing so introduced greater uncertainties, we will choose not to use a size factor greater than one. Let $k_b = 1$.

Eq. (6-18): $$S_e = (0.321)(1)(100) = 32.1 \text{ kpsi}$$

$$F_a = \frac{40-20}{2} = 10 \text{ lb} \qquad F_m = \frac{40+20}{2} = 30 \text{ lb}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} = \frac{32(10)(12)}{\pi(0.375)^3} = 23.18 \text{ kpsi}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(30)(12)}{\pi(0.375)^3} = 69.54 \text{ kpsi}$$

(a) Modified Goodman criterion

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{23.18}{32.1} + \frac{69.54}{200}$$

$$n_f = 0.94 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a}{1-(\sigma_m / S_{ut})} = \frac{23.18}{1-(69.54/200)} = 35.54 \text{ kpsi}$$

Fig. 6-18: $f = 0.775$

Eq. (6-14): $$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.775(200)\right]^2}{32.1} = 748.4$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.775(200)}{32.1}\right) = -0.228$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{35.54}{748.4}\right)^{\frac{1}{-0.228}} = 637\,000 \text{ cycles} \qquad \textit{Ans.}$$

(b) Gerber criterion, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1+\sqrt{1+\left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{200}{69.54}\right)^2 \frac{23.18}{32.1}\left[-1+\sqrt{1+\left(\frac{2(69.54)(32.1)}{200(23.18)}\right)^2}\right]$$

$$= 1.16 \qquad \text{Infinite life is predicted} \qquad \textit{Ans.}$$

6-29 $E = 207.0$ GPa

(a) $I = \frac{1}{12}(20)(4^3) = 106.7 \text{ mm}^4$

$$y = \frac{Fl^3}{3EI} \quad \Rightarrow \quad F = \frac{3EIy}{l^3}$$

$$F_{\min} = \frac{3(207)(10^9)(106.7)(10^{-12})(2)(10^{-3})}{140^3(10^{-9})} = 48.3 \text{ N} \qquad \textit{Ans.}$$

$$F_{\max} = \frac{3(207)(10^9)(106.7)(10^{-12})(6)(10^{-3})}{140^3(10^{-9})} = 144.9 \text{ N} \qquad \textit{Ans.}$$

(b) Get the fatigue strength information.

Eq. (2-21): $S_{ut} = =3.4H_B = 3.4(490) = 1666$ MPa

From problem statement: $S_y = 0.9S_{ut} = 0.9(1666) = 1499$ MPa

Eq. (6-8): $S'_e = 700$ MPa

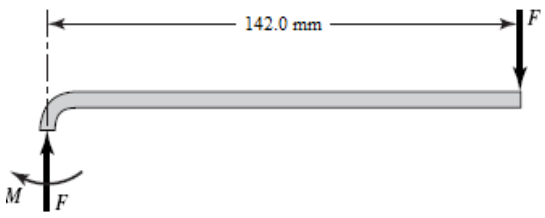
Eq. (6-19): $k_a = 1.58(1666)^{-0.085} = 0.84$

Eq. (6-25): $d_e = 0.808[20(4)]^{1/2} = 7.23$ mm

Eq. (6-20): $k_b = 1.24(7.23)^{-0.107} = 1.00$

Eq. (6-18): $S_e = 0.84(1)(700) = 588$ MPa

This is a relatively thick curved beam, so use the method in Sect. 3-18 to find the stresses. The maximum bending moment will be to the centroid of the section as shown.



$M = 142F$ N·mm, $A = 4(20) = 80$ mm^2, $h = 4$ mm, $r_i = 4$ mm, $r_o = r_i + h = 8$ mm, $r_c = r_i + h/2 = 6$ mm

Table 3-4: $$r_n = \frac{h}{\ln(r_o / r_i)} = \frac{4}{\ln(8/4)} = 5.7708 \text{ mm}$$

$$e = r_c - r_n = 6 - 5.7708 = 0.2292 \text{ mm}$$

$$c_i = r_n - r_i = 5.7708 - 4 = 1.7708 \text{ mm}$$

$$c_o = r_o - r_n = 8 - 5.7708 = 2.2292 \text{ mm}$$

Get the stresses at the inner and outer surfaces from Eq. (3-65) with the axial stresses added. The signs have been set to account for tension and compression as appropriate.

$$\sigma_i = -\frac{Mc_i}{Aer_i} - \frac{F}{A} = -\frac{(142F)(1.7708)}{80(0.2292)(4)} - \frac{F}{80} = -3.441F \text{ MPa}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} - \frac{F}{A} = \frac{(142F)(2.2292)}{80(0.2292)(8)} - \frac{F}{80} = 2.145F \text{ MPa}$$

$$(\sigma_i)_{\min} = -3.441(144.9) = -498.6 \text{ MPa}$$

$$(\sigma_i)_{\max} = -3.441(48.3) = -166.2 \text{ MPa}$$

$$(\sigma_o)_{\min} = 2.145(48.3) = 103.6 \text{ MPa}$$

$$(\sigma_o)_{\max} = 2.145(144.9) = 310.8 \text{ MPa}$$

$$(\sigma_i)_a = \left| \frac{-166.2 - (-498.6)}{2} \right| = 166.2 \text{ MPa}$$

$$(\sigma_i)_m = \frac{-166.2 + (-498.6)}{2} = -332.4 \text{ MPa}$$

$$(\sigma_o)_a = \left| \frac{310.8 - 103.6}{2} \right| = 103.6 \text{ MPa}$$

$$(\sigma_o)_m = \frac{310.8 + 103.6}{2} = 207.2 \text{ MPa}$$

To check for yielding, we note that the largest stress is –498.6 MPa (compression) on the inner radius. This is considerably less than the estimated yield strength of 1499 MPa, so yielding is not predicted.

Check for fatigue on both inner and outer radii since one has a compressive mean stress and the other has a tensile mean stress.
Inner radius:

Since $\sigma_m < 0$, $n_f = \frac{S_e}{\sigma_a} = \frac{588}{166.2} = 3.54$

Outer radius:

Since $\sigma_m > 0$, we will use the Modified Goodman line.

$$1/n_f = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{103.6}{588} + \frac{207.2}{1666}$$

$$n_f = 3.33$$

Infinite life is predicted at both inner and outer radii. *Ans.*

6-30 From Table A-20, for AISI 1018 CD, $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-8): $S_e^{'} = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = (0.897)(1)(0.85)(32) = 24.4$ kpsi

Fillet:

Fig. A-15-5: $D/d = 3.5/3 = 1.17, \quad r/d = 0.25/3 = 0.083, \quad K_t = 1.85$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(1.85 - 1) = 1.72$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{5}{3.0(0.5)} = 3.33 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-16}{3.0(0.5)} = -10.67 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.72 \left| \frac{3.33 - (-10.67)}{2} \right| = 12.0 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.72 \left(\frac{3.33 + (-10.67)}{2} \right) = -6.31 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.67} \right| = 5.06 \quad \therefore \text{ Does not yield.}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{12.0} = 2.03$$

Hole:

Fig. A-15-1: $d / w_1 = 0.4 / 3.5 = 0.11 \quad \therefore K_t = 2.68$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.68 - 1) = 2.43$$

$$\sigma_{\max} = \frac{F_{\max}}{h(w_1 - d)} = \frac{5}{0.5(3.5 - 0.4)} = 3.226 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h(w_1 - d)} = \frac{-16}{0.5(3.5 - 0.4)} = -10.32 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.43 \left| \frac{3.226 - (-10.32)}{2} \right| = 16.5 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.43 \left(\frac{3.226 + (-10.32)}{2} \right) = -8.62 \text{ kpsi}$$

$$n_y = \left| \frac{S_y}{\sigma_{\min}} \right| = \left| \frac{54}{-10.32} \right| = 5.23 \quad \therefore \text{does not yield}$$

Since the midrange stress is negative,

$$n_f = \frac{S_e}{\sigma_a} = \frac{24.4}{16.5} = 1.48$$

Thus the design is controlled by the threat of fatigue at the hole with a minimum factor of safety of $n_f = 1.48$. *Ans.*

6-31 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-8): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

Eq. (6-20): $k_b = 1$ (axial)

Eq. (6-26): $k_c = 0.85$

Eq. (6-18): $S_e = (0.897)(1)(0.85)(32) = 24.4$ kpsi

Fillet:

Fig. A-15-5: $D / d = 2.5 / 1.5 = 1.67, \quad r / d = 0.25 / 1.5 = 0.17, \quad K_t \doteq 2.1$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.1 - 1) = 1.94$$

$$\sigma_{\max} = \frac{F_{\max}}{w_2 h} = \frac{16}{1.5(0.5)} = 21.3 \text{ kpsi}$$

$$\sigma_{\min} = \frac{-4}{1.5(0.5)} = -5.33 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 1.94 \left| \frac{21.3 - (-5.33)}{2} \right| = 25.8 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 1.94 \left(\frac{21.3 + (-5.33)}{2} \right) = 15.5 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{21.3} = 2.54 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{25.8}{24.4} + \frac{15.5}{64}$$

$$n_f = 0.77$$

Hole:

Fig. A-15-1: $d / w_1 = 0.4 / 2.5 = 0.16 \quad \therefore K_t = 2.55$

Use Fig. 6-20 or Eqs. (6-34) and (6-35a) for q. Estimate a little high since it is off the graph. $q = 0.85$

$$K_f = 1 + 0.85(2.55 - 1) = 2.32$$

$$\sigma_{\max} = \frac{F_{\max}}{h\left(w_1 - d\right)} = \frac{16}{0.5(2.5 - 0.4)} = 15.2 \text{ kpsi}$$

$$\sigma_{\min} = \frac{F_{\min}}{h\left(w_1 - d\right)} = \frac{-4}{0.5(2.5 - 0.4)} = -3.81 \text{ kpsi}$$

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = 2.32 \left(\frac{15.2 - (-3.81)}{2} \right) = 22.1 \text{ kpsi}$$

$$\sigma_m = K_f \left(\frac{\sigma_{\max} + \sigma_{\min}}{2} \right) = 2.32 \left(\frac{15.2 + (-3.81)}{2} \right) = 13.2 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma_{\max}} = \frac{54}{15.2} = 3.55 \quad \therefore \text{Does not yield.}$$

Using Modified Goodman criteria

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{22.1}{24.4} + \frac{13.2}{64}$$

$$n_f = 0.90$$

Thus the design is controlled by the threat of fatigue at the fillet with a minimum factor of safety of $n_f = 0.77$ *Ans.*

6-32 $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 6-30, the fatigue factor of safety at the hole is $n_f = 1.48$. To match this at the fillet,

$$n_f = \frac{S_e}{\sigma_a} \quad \Rightarrow \quad \sigma_a = \frac{S_e}{n_f} = \frac{24.4}{1.48} = 16.5 \text{ kpsi}$$

where S_e is unchanged from Prob. 6-30. The only aspect of σ_a that is affected by the fillet radius is the fatigue stress concentration factor. Obtaining σ_a in terms of K_f,

$$\sigma_a = K_f \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right| = K_f \left| \frac{3.33 - (-10.67)}{2} \right| = 7.00 K_f$$

Equating to the desired stress, and solving for K_f,

$$\sigma_a = 7.00 K_f = 16.5 \qquad \Rightarrow \qquad K_f = 2.36$$

Assume since we are expecting to get a smaller fillet radius than the original, that q will be back on the graph of Fig. 6-20, so we'll estimate $q = 0.8$.

$$K_f = 1 + 0.80(K_t - 1) = 2.36 \quad \Rightarrow \qquad K_t = 2.7$$

From Fig. A-15-5, with $D / d = 3.5/3 = 1.17$ and $K_t = 2.6$, find r / d. Choosing $r / d = 0.03$, and with $d = w_2 = 3.0$,

$$r = 0.03 w_2 = 0.03(3.0) = 0.09 \text{ in}$$

At this small radius, our estimate for q is too high. From Fig. 6-20, with $r = 0.09$, q should be about 0.75. Iterating, we get $K_t = 2.8$. This is at a difficult range on Fig. A-15-5 to read the graph with any confidence, but we'll estimate $r / d = 0.02$, giving $r = 0.06$ in. This is a very rough estimate, but it clearly demonstrates that the fillet radius can be relatively sharp to match the fatigue factor of safety of the hole. *Ans.*

6-33 $S_y = 60$ kpsi, $S_{ut} = 110$ kpsi

Inner fiber where $r_c = 3/4$ in

$$r_o = \frac{3}{4} + \frac{3}{16(2)} = 0.84375$$

$$r_i = \frac{3}{4} - \frac{3}{32} = 0.65625$$

Table 3-4, p. 121,

$$r_n = \frac{h}{\ln \frac{r_o}{r_i}} = \frac{3/16}{\ln \frac{0.84375}{0.65625}} = 0.74608 \text{ in}$$

$$e = r_c - r_n = 0.75 - 0.74608 = 0.00392 \text{ in}$$
$$c_i = r_n - r_i = 0.74608 - 0.65625 = 0.08983$$
$$A = \left(\frac{3}{16}\right)\left(\frac{3}{16}\right) = 0.035156 \text{ in}^2$$

Eq. (3-65), p. 119,

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-T(0.08983)}{(0.035156)(0.00392)(0.65625)} = -993.3T$$

where T is in lbf·in and σ_i is in psi.

$$\sigma_m = \frac{1}{2}(-993.3)T = -496.7T$$
$$\sigma_a = 496.7T$$

Eq. (6-8): $S_e^{'} = 0.5(110) = 55$ kpsi

Eq. (6-19): $k_a = 2.70(110)^{-0.265} = 0.777$

Eq. (6-25): $d_e = 0.808\left[(3/16)(3/16)\right]^{1/2} = 0.1515$ in

Eq. (6-20): $k_b = 0.879(0.1515)^{-0.107} = 1.08$ (round to 1)

Eq. (6-19): $S_e = (0.777)(1)(55) = 42.7$ kpsi

For a compressive midrange component, $\sigma_a = S_e / n_f$. Thus,

$$0.4967T = \frac{42.7}{3}$$
$$T = 28.7 \text{ lbf} \cdot \text{in}$$

Outer fiber where $r_c = 2.5$ in

$$r_o = 2.5 + \frac{3}{32} = 2.59375$$
$$r_i = 2.5 - \frac{3}{32} = 2.40625$$
$$r_n = \frac{3/16}{\ln \frac{2.59375}{2.40625}} = 2.49883$$
$$e = 2.5 - 2.49883 = 0.00117 \text{ in}$$
$$c_o = 2.59375 - 2.49883 = 0.09492 \text{ in}$$

$$\sigma_o = \frac{Mc_o}{Aer_o} = \frac{T(0.09492)}{(0.035156)(0.00117)(2.59375)} = 889.7T \text{ psi}$$

$$\sigma_m = \sigma_a = \frac{1}{2}(889.7T) = 444.9T \text{ psi}$$

(a) Using Eq. (6-46), for modified Goodman, we have

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

$$\frac{0.4449T}{42.7} + \frac{0.4449T}{110} = \frac{1}{3}$$

$$T = 23.0 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Gerber, Eq. (6-47), at the outer fiber,

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$\frac{3(0.4449T)}{42.7} + \left(\frac{3(0.4449T)}{110}\right)^2 = 1$$

$$T = 28.2 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(c) To guard against yield, use T of part (b) and the inner stress.

$$n_y = \frac{S_y}{\sigma_i} = \frac{60}{0.9933(28.2)} = 2.14 \quad \textit{Ans.}$$

6-34 From Prob. 6-33, $S_e = 42.7$ kpsi, $S_y = 60$ kpsi, and $S_{ut} = 110$ kpsi

(a) Assuming the beam is straight,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{M(h/2)}{bh^3/12} = \frac{6M}{bh^2} = \frac{6T}{(3/16)^3} = 910.2T$$

Goodman: $$\frac{0.4551T}{42.7} + \frac{0.4551T}{110} = \frac{1}{3}$$

$$T = 22.5 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(b) Gerber: $$\frac{3(0.4551T)}{42.7} + \left(\frac{3(0.4551T)}{110}\right)^2 = 1$$

$$T = 27.6 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(c) $n_y = \dfrac{S_y}{\sigma_{\max}} = \dfrac{60}{0.9102(27.6)} = 2.39$ *Ans.*

6-35 $K_{f,\text{bend}} = 1.4,\ K_{f,\text{axial}} = 1.1,\ K_{f,\text{tors}} = 2.0,\ S_y = 300 \text{ MPa},\ S_{ut} = 400 \text{ MPa},\ S_e = 200 \text{ MPa}$

Bending: $\sigma_m = 0,\ \sigma_a = 60 \text{ MPa}$

Axial: $\sigma_m = 20 \text{ MPa},\ \sigma_a = 0$

Torsion: $\tau_m = 25 \text{ MPa},\ \tau_a = 25 \text{ MPa}$

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{\left[1.4(60) + 0\right]^2 + 3\left[2.0(25)\right]^2} = 120.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{\left[0 + 1.1(20)\right]^2 + 3\left[2.0(25)\right]^2} = 89.35 \text{ MPa}$$

Using Modified Goodman, Eq. (6-46),

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{120.6}{200} + \frac{89.35}{400}$$

$$n_f = 1.21 \quad \textit{Ans.}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{120.6 + 89.35} = 1.43 \quad \textit{Ans.}$$

6-36 $K_{f,\text{bend}} = 1.4,\ K_{f,\text{tors}} = 2.0,\ S_y = 300 \text{ MPa},\ S_{ut} = 400 \text{ MPa},\ S_e = 200 \text{ MPa}$

Bending: $\sigma_{\max} = 150 \text{ MPa},\ \sigma_{\min} = -40 \text{ MPa},\ \sigma_m = 55 \text{ MPa},\ \sigma_a = 95 \text{ MPa}$

Torsion: $\tau_m = 90 \text{ MPa},\ \tau_a = 9 \text{ MPa}$

Eqs. (6-55) and (6-56):

$$\sigma'_a = \sqrt{\left[1.4(95)\right]^2 + 3\left[2.0(9)\right]^2} = 136.6 \text{ MPa}$$

$$\sigma'_m = \sqrt{\left[1.4(55)\right]^2 + 3\left[2.0(90)\right]^2} = 321.1 \text{ MPa}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{136.6}{200} + \frac{321.1}{400}$$

$$n_f = 0.67 \quad \textit{Ans.}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{300}{136.6 + 321.1} = 0.66 \qquad \textit{Ans.}$$

Since the conservative yield check indicates yielding, we will check more carefully with with $\sigma'_{\max}$ obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\max} = \sqrt{(\sigma_{\max})^2 + 3(\tau_{\max})^2} = \sqrt{(150)^2 + 3(90+9)^2} = 227.8 \text{ MPa}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{300}{227.8} = 1.32 \qquad \textit{Ans.}$$

Since yielding is not predicted, and infinite life is not predicted, we would like to estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{136.6}{1 - (321.1/400)} = 692.5 \text{ MPa}$$

This stress is much higher than the ultimate strength, rendering it impractical for the *S-N* diagram. We must conclude that the stresses from the combination loading, when increased by the stress concentration factors, produce such a high midrange stress that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

6-37 Table A-20: $S_{\text{ut}} = 64$ kpsi, $S_{\text{y}} = 54$ kpsi

From Prob. 3-68, the critical stress element experiences $\sigma = 15.3$ kpsi and $\tau = 4.43$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 15.3$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 4.43$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[15.3^2 + 3(0)^2\right]^{1/2} = 15.3 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(4.43)^2\right]^{1/2} = 7.67 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[15.3^2 + 3(4.43)^2\right]^{1/2} = 17.11 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{17.11} = 3.16$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $\quad S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(32) = 24.8$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{15.3}{24.8} + \frac{7.67}{64}$$

$$n_f = 1.36 \quad \textit{Ans.}$$

6-38 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-69, the critical stress element experiences $\sigma = 263$ MPa and $\tau = 57.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 263$ MPa, $\sigma_m = 0$, $\tau_a = 0$ MPa, $\tau_m = 57.7$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[263^2 + 3(0)^2\right]^{1/2} = 263 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(57.7)^2\right]^{1/2} = 99.9 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[263^2 + 3(57.7)^2\right]^{1/2} = 281 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{281} = 1.32$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(220) = 170$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{263}{170} + \frac{99.9}{440}$$

$n_f = 0.56$ Infinite life is not predicted. *Ans.*

6-39 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-70, the critical stress element experiences $\sigma = 21.5$ kpsi and $\tau = 5.09$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 21.5$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 5.09$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[21.5^2 + 3(0)^2\right]^{1/2} = 21.5 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(5.09)^2\right]^{1/2} = 8.82 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[21.5^2 + 3(5.09)^2\right]^{1/2} = 23.24 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{23.24} = 2.32$$

Obtain the modifying factors and endurance limit.

$$k_a = 2.70(64)^{-0.265} = 0.90$$

$$k_b = 0.879(1)^{-0.107} = 0.88$$

$$S_e = 0.90(0.88)(0.5)(64) = 25.3 \text{ kpsi}$$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{21.5}{25.3} + \frac{8.82}{64}$$

$$n_f = 1.01 \quad \textit{Ans.}$$

6-40 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-71, the critical stress element experiences $\sigma = 72.9$ MPa and $\tau = 20.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 72.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 20.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[72.9^2 + 3(0)^2\right]^{1/2} = 72.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(20.3)^2\right]^{1/2} = 35.2 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[72.9^2 + 3(20.3)^2\right]^{1/2} = 80.9 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{80.9} = 4.57$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(20)^{-0.107} = 0.90$

Eq. (6-18): $S_e = 0.90(0.90)(220) = 178.2$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{72.9}{178.2} + \frac{35.2}{440}$$

$$n_f = 2.04 \quad \textit{Ans.}$$

6-41 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-72, the critical stress element experiences $\sigma = 35.2$ kpsi and $\tau = 7.35$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 35.2$ kpsi, $\sigma_m = 0$ kpsi, $\tau_a = 0$ kpsi, $\tau_m = 7.35$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[35.2^2 + 3(0)^2\right]^{1/2} = 35.2 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(7.35)^2\right]^{1/2} = 12.7 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[35.2^2 + 3(7.35)^2\right]^{1/2} = 37.4 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{37.4} = 1.44$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.25)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(32) = 24.8$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{35.2}{24.8} + \frac{12.7}{64}$$

$n_f = 0.62$ Infinite life is not predicted. *Ans.*

6-42 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-73, the critical stress element experiences $\sigma = 333.9$ MPa and $\tau = 126.3$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 333.9$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 126.3$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[333.9^2 + 3(0)^2\right]^{1/2} = 333.9 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(126.3)^2\right]^{1/2} = 218.8 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[333.9^2 + 3(126.3)^2\right]^{1/2} = 399.2 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{399.2} = 0.93$$

The sample fails by yielding, infinite life is not predicted. *Ans.*

The fatigue analysis will be continued only to obtain the requested fatigue factor of safety, though the yielding failure will dictate the life.

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(50)^{-0.107} = 0.82$

Eq. (6-18): $S_e = 0.90(0.82)(220) = 162.4$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{333.9}{162.4} + \frac{218.8}{440}$$

$n_f = 0.39$ Infinite life is not predicted. *Ans.*

6-43 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-74, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 9.495 \text{ kpsi}, \quad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$
$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \quad \sigma_{m,\text{axial}} = -0.362 \text{ kpsi}$$
$$\tau_a = 0 \text{ kpsi}, \quad \tau_m = 11.07 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[(9.495)^2 + 3(0)^2\right]^{1/2} = 9.495 \text{ kpsi}$$
$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[(-0.362)^2 + 3(11.07)^2\right]^{1/2} = 19.18 \text{ kpsi}$$
$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[(-9.495 - 0.362)^2 + 3(11.07)^2\right]^{1/2} = 21.56 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{21.56} = 2.50$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.13)^{-0.107} = 0.87$

Eq. (6-18): $S_e = 0.90(0.87)(32) = 25.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{9.495}{25.1} + \frac{19.18}{64}$$

$$n_f = 1.47 \quad \textit{Ans.}$$

6-44 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-76, the critical stress element experiences completely reversed bending stress due to the rotation, and steady torsional and axial stresses.

$$\sigma_{a,\text{bend}} = 33.99 \text{ kpsi}, \quad \sigma_{m,\text{bend}} = 0 \text{ kpsi}$$
$$\sigma_{a,\text{axial}} = 0 \text{ kpsi}, \quad \sigma_{m,\text{axial}} = -0.153 \text{ kpsi}$$
$$\tau_a = 0 \text{ kpsi}, \quad \tau_m = 7.847 \text{ kpsi}$$

Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(33.99\right)^2 + 3\left(0\right)^2\right]^{1/2} = 33.99 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(-0.153\right)^2 + 3\left(7.847\right)^2\right]^{1/2} = 13.59 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[\left(-33.99 - 0.153\right)^2 + 3\left(7.847\right)^2\right]^{1/2} = 36.75 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{36.75} = 1.47$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(0.88)^{-0.107} = 0.89$

Eq. (6-18): $S_e = 0.90(0.89)(32) = 25.6$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{33.99}{25.6} + \frac{13.59}{64}$$

$n_f = 0.65$ Infinite life is not predicted. *Ans.*

6-45 Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

From Prob. 3-77, the critical stress element experiences $\sigma = 68.6$ MPa and $\tau = 37.7$ MPa. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 68.6$ MPa, $\sigma_m = 0$ MPa, $\tau_a = 0$ MPa, $\tau_m = 37.7$ MPa. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[68.6^2 + 3\left(0\right)^2\right]^{1/2} = 68.6 \text{ MPa}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3\left(37.7\right)^2\right]^{1/2} = 65.3 \text{ MPa}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[68.6^2 + 3\left(37.7\right)^2\right]^{1/2} = 94.7 \text{ MPa}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{370}{94.7} = 3.91$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(440) = 220$ MPa

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 1.24(30)^{-0.107} = 0.86$

Eq. (6-18): $S_e = 0.90(0.86)(220) = 170$ MPa

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{68.6}{170} + \frac{65.3}{440}$$

$$n_f = 1.81 \quad \textit{Ans.}$$

6-46 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-79, the critical stress element experiences $\sigma = 3.46$ kpsi and $\tau = 0.882$ kpsi. The bending is completely reversed due to the rotation, and the torsion is steady, giving $\sigma_a = 3.46$ kpsi, $\sigma_m = 0$, $\tau_a = 0$ kpsi, $\tau_m = 0.882$ kpsi. Obtain von Mises stresses for the alternating, mid-range, and maximum stresses.

$$\sigma'_a = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[3.46^2 + 3(0)^2\right]^{1/2} = 3.46 \text{ kpsi}$$

$$\sigma'_m = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[0^2 + 3(0.882)^2\right]^{1/2} = 1.53 \text{ kpsi}$$

$$\sigma'_{\max} = \left(\sigma_{\max}^2 + 3\tau_{\max}^2\right)^{1/2} = \left[3.46^2 + 3(0.882)^2\right]^{1/2} = 3.78 \text{ kpsi}$$

Check for yielding, using the distortion energy failure theory.

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{3.78} = 14.3$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.90$

Eq. (6-20): $k_b = 0.879(1.375)^{-0.107} = 0.85$

Eq. (6-18): $S_e = 0.90(0.85)(32) = 24.5$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{3.46}{24.5} + \frac{1.53}{64}$$

$$n_f = 6.06 \qquad \textit{Ans.}$$

6-47 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-80, the critical stress element experiences $\sigma = 16.3$ kpsi and $\tau = 5.09$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.38)(8.15)\right]^2 + 3\left[(1.88)(2.55)\right]^2\right\}^{1/2} = 13.98 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.98 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.98 + 13.98} = 1.93$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879 d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{13.98}{28.2} + \frac{13.98}{64}$$

$$n_f = 1.40 \quad \textit{Ans.}$$

6-48 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-81, the critical stress element experiences $\sigma = 16.4$ kpsi and $\tau = 4.46$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 16.4$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 4.46$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 8.20$ kpsi, $\tau_m = \tau_a = 2.23$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.38)(8.20)\right]^2 + 3\left[(1.88)(2.23)\right]^2\right\}^{1/2} = 13.45 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 13.45 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{13.45 + 13.45} = 2.01$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{13.45}{28.2} + \frac{13.45}{64}$$

$$n_f = 1.46 \quad \textit{Ans.}$$

6-49 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-82, the critical stress element experiences repeatedly applied bending, axial, and torsional stresses of $\sigma_{x,\text{bend}} = 20.2$ kpsi, $\sigma_{x,\text{axial}} = 0.1$ kpsi, and $\tau = 5.09$ kpsi.. Since the axial stress is practically negligible compared to the bending stress, we will simply combine the two and not treat the axial stress separately for stress concentration factor and load factor. This gives $\sigma_{\max} = 20.3$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 5.09$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 10.15$ kpsi, $\tau_m = \tau_a = 2.55$ kpsi.

For bending, from Eqs. (6-34) and (6-35a),

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right)(64) + 1.51\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.10373$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.10373}{\sqrt{0.1}}} = 0.75$$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.75(1.5 - 1) = 1.38$

For torsion, from Eqs. (6-34) and (6-35b),

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right)(64) + 1.35\left(10^{-5}\right)(64)^2 - 2.67\left(10^{-8}\right)(64)^3 = 0.07800$$

$$q = \frac{1}{1 + \dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1 + \dfrac{0.07800}{\sqrt{0.1}}} = 0.80$$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.80(2.1 - 1) = 1.88$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.38)(10.15) \right]^2 + 3\left[(1.88)(2.55) \right]^2 \right\}^{1/2} = 16.28 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 16.28 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{16.28 + 16.28} = 1.66$$

Obtain the modifying factors and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.98$

Eq. (6-18): $S_e = (0.90)(0.98)(32) = 28.2 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{16.28}{28.2} + \frac{16.28}{64}$$

$$n_f = 1.20 \quad \textit{Ans.}$$

6-50 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-83, the critical stress element on the neutral axis in the middle of the longest side of the rectangular cross section experiences a repeatedly applied shear stress of $\tau_{\max} = 14.3$ kpsi, $\tau_{\min} = 0$ kpsi. Thus, $\tau_m = \tau_a = 7.15$ kpsi. Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{54/2}{14.3} = 1.89$$

Find the modifiers and endurance limit.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.90$

The size factor for a torsionally loaded rectangular cross section is not readily available. Following the procedure on p. 289, we need an equivalent diameter based on the 95 percent stress area. However, the stress situation in this case is nonlinear, as described on p. 102. Noting that the maximum stress occurs at the middle of the longest side, or with a radius from the center of the cross section equal to half of the shortest side, we will simply choose an equivalent diameter equal to the length of the shortest side.

$$d_e = 0.25 \text{ in}$$

Eq. (6-20): $$k_b = 0.879 d_e^{-0.107} = 0.879(0.25)^{-0.107} = 1.02$$

We will round down to $k_b = 1$.

Eq. (6-26): $$k_c = 0.59$$

Eq. (6-18): $$S_{se} = 0.9(1)(0.59)(32) = 17.0 \text{ kpsi}$$

Since the stress is entirely shear, we choose to use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67 S_u = 0.67\ (64) = 42.9$ kpsi.

Using Modified Goodman,

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(7.15/17.0) + (7.15/42.9)} = 1.70 \qquad \textit{Ans.}$$

6-51 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-84, the critical stress element experiences $\sigma = 28.0$ kpsi and $\tau = 15.3$ kpsi. Since the load is applied and released repeatedly, this gives $\sigma_{\max} = 28.0$ kpsi, $\sigma_{\min} = 0$ kpsi, $\tau_{\max} = 15.3$ kpsi, $\tau_{\min} = 0$ kpsi. Consequently, $\sigma_m = \sigma_a = 14.0$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60 - 1) = 1.47$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.47)(14.0)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 27.0 \text{ kpsi}$$

$$\sigma'_m = \sigma'_a = 27.0 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{27.0 + 27.0} = 1.00$$

Since stress concentrations are included in this quick yield check, the low factor of safety is acceptable.

Eq. (6-8): $S'_e = 0.5(64) = 32 \text{ kpsi}$

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370 \text{ in}$

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1 \text{ kpsi}$

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{27.0}{28.1} + \frac{27.0}{64}$$

$$n_f = 0.72 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{27.0}{1 - (27.0/64)} = 46.7 \text{ kpsi}$$

Fig. 6-18: $f = 0.9$

Eq. (6-14): $a = \frac{(f\,S_{ut})^2}{S_e} = \frac{\left[0.9(64)\right]^2}{28.1} = 118.07$

Eq. (6-15): $b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9(64)}{28.1}\right) = -0.1039$

Eq. (6-16): $N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{46.7}{118.07}\right)^{\frac{1}{-0.1039}} = 7534 \text{ cycles} \doteq 7500 \text{ cycles} \quad \textit{Ans.}$

6-52 Table A-20: $S_{ut} = 64 \text{ kpsi}$, $S_y = 54 \text{ kpsi}$

From Prob. 3-85, the critical stress element experiences $\sigma_{x,\text{bend}} = 46.1$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}} = 46.1$ kpsi, $\sigma_{\text{min,bend}} = 0$ kpsi, $\sigma_{\text{max,axial}} = 0.382$ kpsi, $\sigma_{\text{min,axial}} = 0$ kpsi, $\tau_{\text{max}} = 15.3$ kpsi, $\tau_{\text{min}} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 23.05$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \qquad r/d = 0.125/1 = 0.125$$
$$K_{t,\text{bend}} = 1.60, \qquad K_{t,\text{tors}} = 1.39, \qquad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.78(1.60 - 1) = 1.47$$
$$K_{f,\text{axial}} = 1 + q_{\text{axial}}\left(K_{t,\text{axial}} - 1\right) = 1 + 0.78(1.75 - 1) = 1.59$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.47)(23.05) + (1.59)\frac{(0.191)}{0.85}\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.45 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.47)(23.05) + (1.59)(0.191)\right]^2 + 3\left[(1.32)(7.65)\right]^2\right\}^{1/2} = 38.40 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\text{max}} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{54}{38.45 + 38.40} = 0.70$$

Since the conservative yield check indicates yielding, we will check more carefully with with σ'_{max} obtained directly from the maximum stresses, using the distortion energy failure theory, without stress concentrations. Note that this is exactly the method used for static failure in Ch. 5.

$$\sigma'_{\text{max}} = \sqrt{\left(\sigma_{\text{max,bend}} + \sigma_{\text{max,axial}}\right)^2 + 3\left(\tau_{\text{max}}\right)^2} = \sqrt{\left(46.1 + 0.382\right)^2 + 3\left(15.3\right)^2} = 53.5 \text{ kpsi}$$
$$n_y = \frac{S_y}{\sigma'_{\text{max}}} = \frac{54}{53.5} = 1.01 \qquad \textit{Ans.}$$

This shows that yielding is imminent, and further analysis of fatigue life should not be interpreted as a guarantee of more than one cycle of life.

Eq. (6-8): $S_e' = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{38.45}{28.1} + \frac{38.40}{64}$$

$$n_f = 0.51 \quad Ans.$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma_a'}{1-(\sigma_m'/S_{ut})} = \frac{38.45}{1-(38.40/64)} = 96.1 \text{ kpsi}$$

This stress is much higher than the ultimate strength, rendering it impractical for the *S-N* diagram. We must conclude that the fluctuating stresses from the combination loading, when increased by the stress concentration factors, are so far from the Goodman line that the equivalent completely reversed stress method is not practical to use. Without testing, we are unable to predict a life.

6-53 Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

From Prob. 3-86, the critical stress element experiences $\sigma_{x,\text{bend}} = 55.5$ kpsi, $\sigma_{x,\text{axial}} = 0.382$ kpsi and $\tau = 15.3$ kpsi. The axial load is practically negligible, but we'll include it to demonstrate the process. Since the load is applied and released repeatedly, this gives $\sigma_{\text{max,bend}} = 55.5$ kpsi, $\sigma_{\text{min,bend}} = 0$ kpsi, $\sigma_{\text{max,axial}} = 0.382$ kpsi, $\sigma_{\text{min,axial}} = 0$ kpsi, $\tau_{\text{max}} = 15.3$ kpsi, $\tau_{\text{min}} = 0$ kpsi. Consequently, $\sigma_{m,\text{bend}} = \sigma_{a,\text{bend}} = 27.75$ kpsi, $\sigma_{m,\text{axial}} = \sigma_{a,\text{axial}} = 0.191$ kpsi, $\tau_m = \tau_a = 7.65$ kpsi. From Table A-15-7, A-15-8 and A-15-9,

$$D/d = 1.5/1 = 1.5, \quad r/d = 0.125/1 = 0.125$$

$$K_{t,\text{bend}} = 1.60, \quad K_{t,\text{tors}} = 1.39, \quad K_{t,\text{axial}} = 1.75$$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = q_{\text{axial}} = 0.78$, $q_{\text{tors}} = 0.82$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.78(1.60 - 1) = 1.47$$

$$K_{f,\text{axial}} = 1 + q_{\text{axial}}(K_{t,\text{axial}} - 1) = 1 + 0.78(1.75 - 1) = 1.59$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.82(1.39 - 1) = 1.32$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{ \left[(1.47)(27.75) + (1.59)\frac{(0.191)}{0.85} \right]^2 + 3\left[(1.32)(7.65) \right]^2 \right\}^{1/2} = 44.71 \text{ kpsi}$$

$$\sigma'_m = \left\{ \left[(1.47)(27.75) + (1.59)(0.191) \right]^2 + 3\left[(1.32)(7.65) \right]^2 \right\}^{1/2} = 44.66 \text{ kpsi}$$

Since these stresses are relatively high compared to the yield strength, we will go ahead and check for yielding using the distortion energy failure theory.

$$\sigma'_{\max} = \sqrt{\left(\sigma_{\max,\text{bend}} + \sigma_{\max,\text{axial}}\right)^2 + 3\left(\tau_{\max}\right)^2} = \sqrt{(55.5 + 0.382)^2 + 3(15.3)^2} = 61.8 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{54}{61.8} = 0.87 \qquad \textit{Ans.}$$

This shows that yielding is predicted. Further analysis of fatigue life is just to be able to report the fatigue factor of safety, though the life will be dictated by the static yielding failure, i.e. $N = 1/2$ cycle. *Ans.*

Eq. (6-8): $S'_e = 0.5(64) = 32$ kpsi

Eq. (6-19): $k_a = aS_{ut}^b = 2.70(64)^{-0.265} = 0.897$

Eq. (6-24): $d_e = 0.370d = 0.370(1) = 0.370$ in

Eq. (6-20): $k_b = 0.879d_e^{-0.107} = 0.879(0.370)^{-0.107} = 0.978$

Eq. (6-18): $S_e = (0.897)(0.978)(0.5)(64) = 28.1$ kpsi

Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{44.71}{28.1} + \frac{44.66}{64}$$

$$n_f = 0.44 \qquad \textit{Ans.}$$

6-54 From Table A-20, for AISI 1040 CD, $S_{ut} = 85$ kpsi and $S_y = 71$ kpsi. From the solution to Prob. 6-17 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{\text{rev}} = 35.0$ kpsi, producing $\sigma_a = 35.0$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2500(1.625/2)}{\pi(1.625^4)/32} = 2967 \text{ psi} = 2.97 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,\text{bend}} = 1.95$, $K_{t,\text{tors}} = 1.60$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.76,\ q_{\text{tors}} = 0.81$

Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}\left(K_{t,\text{bend}} - 1\right) = 1 + 0.76\left(1.95 - 1\right) = 1.72$$

$$K_{f,\text{tors}} = 1 + q_{\text{tors}}\left(K_{t,\text{tors}} - 1\right) = 1 + 0.81\left(1.60 - 1\right) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[\left(1.72\right)\left(35.0\right)\right]^2 + 3\left[\left(1.49\right)\left(0\right)\right]^2\right\}^{1/2} = 60.2 \text{ kpsi}$$

$$\sigma'_m = \left\{\left[\left(1.72\right)\left(0\right)\right]^2 + 3\left[\left(1.49\right)\left(2.97\right)\right]^2\right\}^{1/2} = 7.66 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{60.2 + 7.66} = 1.05$$

From the solution to Prob. 6-17, $S_e = 29.5$ kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{60.2}{29.5} + \frac{7.66}{85}$$

$$n_f = 0.47 \qquad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{60.2}{1 - (7.66/85)} = 66.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.867$

Eq. (6-14): $$a = \frac{\left(f\, S_{ut}\right)^2}{S_e} = \frac{\left[0.867(85)\right]^2}{29.5} = 184.1$$

Eq. (6-15): $$b = -\frac{1}{3}\log\left(\frac{f\, S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.867(85)}{29.5}\right) = -0.1325$$

Eq. (6-16): $$N = \left(\frac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\frac{66.2}{184.1}\right)^{\frac{1}{-0.1325}} = 2251 \text{ cycles}$$

$$N = 2300 \text{ cycles} \qquad \textit{Ans.}$$

6-55 From the solution to Prob. 6-18 we find the completely reversed stress at the critical shoulder fillet to be $\sigma_{\text{rev}} = 32.8$ kpsi, producing $\sigma_a = 32.8$ kpsi and $\sigma_m = 0$ kpsi. This problem adds a steady torque which creates torsional stresses of

$$\tau_m = \frac{Tr}{J} = \frac{2200(1.625/2)}{\pi(1.625^4)/32} = 2611 \text{ psi} = 2.61 \text{ kpsi}, \quad \tau_a = 0 \text{ kpsi}$$

From Table A-15-8 and A-15-9, $r/d = 0.0625/1.625 = 0.04$, $D/d = 1.875/1.625 = 1.15$, $K_{t,\text{bend}} = 1.95$, $K_{t,\text{tors}} = 1.60$

Eqs. (6-34) and (6-35), or Figs. 6-20 and 6-21: $q_{\text{bend}} = 0.76$, $q_{\text{tors}} = 0.81$
Eq. (6-32):

$$K_{f,\text{bend}} = 1 + q_{\text{bend}}(K_{t,\text{bend}} - 1) = 1 + 0.76(1.95 - 1) = 1.72$$
$$K_{f,\text{tors}} = 1 + q_{\text{tors}}(K_{t,\text{tors}} - 1) = 1 + 0.81(1.60 - 1) = 1.49$$

Obtain von Mises stresses for the alternating and mid-range stresses from Eqs. (6-55) and (6-56).

$$\sigma'_a = \left\{\left[(1.72)(32.8)\right]^2 + 3\left[(1.49)(0)\right]^2\right\}^{1/2} = 56.4 \text{ kpsi}$$
$$\sigma'_m = \left\{\left[(1.72)(0)\right]^2 + 3\left[(1.49)(2.61)\right]^2\right\}^{1/2} = 6.74 \text{ kpsi}$$

Check for yielding, using the conservative $\sigma'_{\max} = \sigma'_a + \sigma'_m$,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m} = \frac{71}{56.4 + 6.74} = 1.12$$

From the solution to Prob. 6-18, $S_e = 29.5$ kpsi. Using Modified Goodman,

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{56.4}{29.5} + \frac{6.74}{85}$$

$$n_f = 0.50 \quad \textit{Ans.}$$

Since infinite life is not predicted, estimate a life from the *S-N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{56.4}{1 - (6.74/85)} = 61.3 \text{ kpsi}$$

Fig. 6-18: $f = 0.867$

Eq. (6-14): $a = \dfrac{\left(f\,S_{ut}\right)^2}{S_e} = \dfrac{\left[0.867(85)\right]^2}{29.5} = 184.1$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.867(85)}{29.5}\right) = -0.1325$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{61.3}{184.1}\right)^{\frac{1}{-0.1325}} = 4022 \text{ cycles}$

$N = 4000$ cycles *Ans.*

6-56 $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi, $K_{ts} = 1.6$, $L = 2$ ft, $F_{\min} = 150$ lbf, $F_{\max} = 500$ lbf

Eqs. (6-34) and (6-35*b*), or Fig. 6-21: $q_s = 0.80$

Eq. (6-32): $K_{fs} = 1 + q_s\left(K_{ts} - 1\right) = 1 + 0.80\left(1.6 - 1\right) = 1.48$

$$T_{\max} = 500(2) = 1000 \text{ lbf} \cdot \text{in}, \quad T_{\min} = 150(2) = 300 \text{ lbf} \cdot \text{in}$$

$$\tau_{\max} = \frac{16K_{fs}T_{\max}}{\pi d^3} = \frac{16(1.48)(1000)}{\pi(0.875)^3} = 11\,251 \text{ psi} = 11.25 \text{ kpsi}$$

$$\tau_{\min} = \frac{16K_{fs}T_{\min}}{\pi d^3} = \frac{16(1.48)(300)}{\pi(0.875)^3} = 3375 \text{ psi} = 3.38 \text{ kpsi}$$

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} = \frac{11.25 + 3.38}{2} = 7.32 \text{ kpsi}$$

$$\tau_a = \frac{\tau_{\max} - \tau_{\min}}{2} = \frac{11.25 - 3.38}{2} = 3.94 \text{ kpsi}$$

Since the stress is entirely shear, it is convenient to check for yielding using the standard Maximum Shear Stress theory.

$$n_y = \frac{S_y / 2}{\tau_{\max}} = \frac{30/2}{11.25} = 1.33$$

Find the modifiers and endurance limit.

Eq. (6-8): $S'_e = 0.5(55) = 27.5$ kpsi

Eq. (6-19): $k_a = 14.4(55)^{-0.718} = 0.81$

Eq. (6-24): $d_e = 0.370(0.875) = 0.324$ in

Eq. (6-20): $k_b = 0.879(0.324)^{-0.107} = 0.99$

Eq. (6-26): $k_c = 0.59$

Eq. (6-18): $S_{se} = 0.81(0.99)(0.59)(27.5) = 13.0$ kpsi

Since the stress is entirely shear, we will use a load factor $k_c = 0.59$, and convert the ultimate strength to a shear value rather than using the combination loading method of Sec. 6-14. From Eq. (6-54), $S_{su} = 0.67S_u = 0.67\ (55) = 36.9$ kpsi.

(a) Modified Goodman, Table 6-6

$$n_f = \frac{1}{(\tau_a / S_{se}) + (\tau_m / S_{su})} = \frac{1}{(3.94/13.0) + (7.32/36.9)} = 1.99 \qquad \textit{Ans.}$$

(b) Gerber, Table 6-7

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{36.9}{7.32}\right)^2 \left(\frac{3.94}{13.0}\right)\left[-1 + \sqrt{1 + \left(\frac{2(7.32)(13.0)}{36.9(3.94)}\right)^2}\right]$$

$$n_f = 2.49 \qquad \textit{Ans.}$$

6-57 $S_{ut} = 145$ kpsi, $S_y = 120$ kpsi

From Eqs. (6-34) and (6-35*a*), or Fig. 6-20, with a notch radius of 0.1 in, $q = 0.9$. Thus, with $K_t = 3$ from the problem statement,

$$K_f = 1 + q(K_t - 1) = 1 + 0.9(3 - 1) = 2.80$$

$$\sigma_{\max} = -K_f \frac{4P}{\pi d^2} = \frac{-2.80(4)(P)}{\pi(1.2)^2} = -2.476P$$

$$\sigma_m = -\sigma_a = \frac{1}{2}(-2.476P) = -1.238P$$

$$T_{\max} = \frac{f\,P(D+d)}{4} = \frac{0.3P(6+1.2)}{4} = 0.54P$$

From Eqs. (6-34) and (6-35*b*), or Fig. 6-21, with a notch radius of 0.1 in, $q_s = 0.92$. Thus, with $K_{ts} = 1.8$ from the problem statement,

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.92(1.8 - 1) = 1.74$$

$$\tau_{\max} = \frac{16K_{fs}T}{\pi d^3} = \frac{16(1.74)(0.54P)}{\pi(1.2)^3} = 2.769P$$

$$\tau_a = \tau_m = \frac{\tau_{\max}}{2} = \frac{2.769P}{2} = 1.385P$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(1.238P / 0.85)^2 + 3(1.385P)^2]^{1/2} = 2.81P$$
$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-1.238P)^2 + 3(1.385P)^2]^{1/2} = 2.70P$$

Eq. (6-8): $S'_e = 0.5(145) = 72.5$ kpsi

Eq. (6-19): $k_a = 2.70(145)^{-0.265} = 0.722$

Eq. (6-20): $k_b = 0.879(1.2)^{-0.107} = 0.862$

Eq. (6-18): $S_e = (0.722)(0.862)(72.5) = 45.12$ kpsi

Modified Goodman: $\dfrac{1}{n_f} = \dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}} = \dfrac{2.81P}{45.12} + \dfrac{2.70P}{145} = \dfrac{1}{3}$

$P = 4.12$ kips *Ans.*

Yield (conservative): $n_y = \dfrac{S_y}{\sigma'_a + \sigma'_m} = \dfrac{120}{(2.81)(4.12) + (2.70)(4.12)} = 5.29$ *Ans.*

6-58 From Prob. 6-57, $K_f = 2.80$, $K_{fs} = 1.74$, $S_e = 45.12$ kpsi

$$\sigma_{\max} = -K_f \frac{4P_{\max}}{\pi d^2} = -2.80\frac{4(18)}{\pi(1.2^2)} = -44.56 \text{ kpsi}$$
$$\sigma_{\min} = -K_f \frac{4P_{\min}}{\pi d^2} = -2.80\frac{4(4.5)}{\pi(1.2)^2} = -11.14 \text{ kpsi}$$
$$T_{\max} = f\,P_{\max}\left(\frac{D+d}{4}\right) = 0.3(18)\left(\frac{6+1.2}{4}\right) = 9.72 \text{ kip}\cdot\text{in}$$
$$T_{\min} = f\,P_{\min}\left(\frac{D+d}{4}\right) = 0.3(4.5)\left(\frac{6+1.2}{4}\right) = 2.43 \text{ kip}\cdot\text{in}$$
$$\tau_{\max} = K_{fs}\frac{16T_{\max}}{\pi d^3} = 1.74\frac{16(9.72)}{\pi(1.2)^3} = 49.85 \text{ kpsi}$$
$$\tau_{\min} = K_{fs}\frac{16T_{\min}}{\pi d^3} = 1.74\frac{16(2.43)}{\pi(1.2)^3} = 12.46 \text{ kpsi}$$
$$\sigma_a = \left|\frac{-44.56-(-11.14)}{2}\right| = 16.71 \text{ kpsi}$$
$$\sigma_m = \frac{-44.56+(-11.14)}{2} = -27.85 \text{ kpsi}$$
$$\tau_a = \frac{49.85-12.46}{2} = 18.70 \text{ kpsi}$$
$$\tau_m = \frac{49.85+12.46}{2} = 31.16 \text{ kpsi}$$

Eqs. (6-55) and (6-56):

$$\sigma'_a = [(\sigma_a / 0.85)^2 + 3\tau_a^2]^{1/2} = [(16.71/0.85)^2 + 3(18.70)^2]^{1/2} = 37.89 \text{ kpsi}$$
$$\sigma'_m = [\sigma_m{}^2 + 3\tau_m^2]^{1/2} = [(-27.85)^2 + 3(31.16)^2]^{1/2} = 60.73 \text{ kpsi}$$

Modified Goodman: $\dfrac{1}{n_f} = \dfrac{\sigma'_a}{S_e} + \dfrac{\sigma'_m}{S_{ut}} = \dfrac{37.89}{45.12} + \dfrac{60.73}{145}$

$n_f = 0.79$

Since infinite life is not predicted, estimate a life from the *S*-*N* diagram. First, find an equivalent completely reversed stress (See Ex. 6-12).

$$\sigma_{\text{rev}} = \frac{\sigma'_a}{1 - (\sigma'_m / S_{ut})} = \frac{37.89}{1 - (60.73/145)} = 65.2 \text{ kpsi}$$

Fig. 6-18: $f = 0.8$

Eq. (6-14): $a = \dfrac{(f\,S_{ut})^2}{S_e} = \dfrac{[0.8(145)]^2}{45.12} = 298.2$

Eq. (6-15): $b = -\dfrac{1}{3}\log\left(\dfrac{f\,S_{ut}}{S_e}\right) = -\dfrac{1}{3}\log\left(\dfrac{0.8(145)}{45.12}\right) = -0.1367$

Eq. (6-16): $N = \left(\dfrac{\sigma_{\text{rev}}}{a}\right)^{1/b} = \left(\dfrac{65.2}{298.2}\right)^{\frac{1}{-0.1367}} = 67\,607 \text{ cycles}$

$N = 67\,600$ cycles *Ans.*

6-59 For AISI 1020 CD, From Table A-20, $S_y = 390$ MPa, $S_{ut} = 470$ MPa. Given: $S_e = 175$ MPa.

First Loading: $(\sigma_m)_1 = \dfrac{360+160}{2} = 260 \text{ MPa}, \quad (\sigma_a)_1 = \dfrac{360-160}{2} = 100 \text{ MPa}$

Goodman: $(\sigma_a)_{e1} = \dfrac{(\sigma_a)_1}{1 - (\sigma_m)_1 / S_{ut}} = \dfrac{100}{1 - 260/470} = 223.8 \text{ MPa} > S_e \therefore \text{finite life}$

$$a = \frac{\left[0.9(470)\right]^2}{175} = 1022.5 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(470)}{175} = -0.127\,767$$

$$N = \left(\frac{223.8}{1022.5}\right)^{-1/0.127767} = 145\,920 \text{ cycles}$$

Second loading: $(\sigma_m)_2 = \dfrac{320+(-200)}{2} = 60$ MPa, $\quad (\sigma_a)_2 = \dfrac{320-(-200)}{2} = 260$ MPa

$$(\sigma_a)_{e2} = \frac{260}{1-60/470} = 298.0 \text{ MPa}$$

(**a**) Miner's method: $N_2 = \left(\dfrac{298.0}{1022.5}\right)^{-1/0.127767} = 15\,520$ cycles

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad \Rightarrow \quad \frac{80\,000}{145\,920} + \frac{n_2}{15\,520} = 1 \quad \Rightarrow \quad n_2 = 7000 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method: The number of cycles remaining after the first loading

$$N_{\text{remaining}} = 145\,920 - 80\,000 = 65\,920 \text{ cycles}$$

Two data points: 0.9(470) MPa, 10^3 cycles
223.8 MPa, 65 920 cycles

$$\frac{0.9(470)}{223.8} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(65\,920\right)^{b_2}}$$

$$1.8901 = (0.015170)^{b_2}$$

$$b_2 = \frac{\log 1.8901}{\log 0.015170} = -0.151\,997$$

$$a_2 = \frac{223.8}{(65\,920)^{-0.151\,997}} = 1208.7 \text{ MPa}$$

$$n_2 = \left(\frac{298.0}{1208.7}\right)^{1/-0.151\,997} = 10\,000 \text{ cycles} \qquad \textit{Ans.}$$

6-60 Given: S_e = 50 kpsi, S_{ut} = 140 kpsi, f =0.8. Using Miner's method,

$$a = \frac{\left[0.8(140)\right]^2}{50} = 250.88 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.8(140)}{50} = -0.116\,749$$

$$\sigma_1 = 95 \text{ kpsi}, \quad N_1 = \left(\frac{95}{250.88}\right)^{1/-0.116\,749} = 4100 \text{ cycles}$$

$$\sigma_2 = 80 \text{ kpsi}, \quad N_2 = \left(\frac{80}{250.88}\right)^{1/-0.116\,749} = 17\,850 \text{ cycles}$$

$$\sigma_3 = 65 \text{ kpsi}, \quad N_3 = \left(\frac{65}{250.88}\right)^{1/-0.116\,749} = 105\,700 \text{ cycles}$$

$$\frac{0.2N}{4100} + \frac{0.5N}{17\,850} + \frac{0.3N}{105\,700} = 1 \quad \Rightarrow \quad N = 12\,600 \text{ cycles} \quad \textit{Ans.}$$

6-61 Given: $S_{ut} = 530$ MPa, $S_e = 210$ MPa, and $f = 0.9$.

(a) Miner's method

$$a = \frac{\left[0.9(530)\right]^2}{210} = 1083.47 \text{ MPa}$$

$$b = -\frac{1}{3}\log\frac{0.9(530)}{210} = -0.118\,766$$

$$\sigma_1 = 350 \text{ MPa}, \quad N_1 = \left(\frac{350}{1083.47}\right)^{1/-0.118\,766} = 13\,550 \text{ cycles}$$

$$\sigma_2 = 260 \text{ MPa}, \quad N_2 = \left(\frac{260}{1083.47}\right)^{1/-0.118\,766} = 165\,600 \text{ cycles}$$

$$\sigma_3 = 225 \text{ MPa}, \quad N_3 = \left(\frac{225}{1083.47}\right)^{1/-0.118\,766} = 559\,400 \text{ cycles}$$

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

$$\frac{5000}{13\,550} + \frac{50\,000}{165\,600} + \frac{n_3}{559\,400} = 184\,100 \text{ cycles} \quad \textit{Ans.}$$

(**b**) Manson's method:

The life remaining after the first series of cycling is $N_{R1} = 13\,550 - 5000 = 8550$ cycles. The two data points required to define $S'_{e,1}$ are [0.9(530), 10^3] and (350, 8550).

$$\frac{0.9(530)}{350} = \frac{a_2\left(10^3\right)^{b_2}}{a_2\left(8550\right)^{b_2}} \quad \Rightarrow \quad 1.3629 = \left(0.11696\right)^{b_2}$$

$$b_2 = \frac{\log(1.362\ 9)}{\log(0.116\ 96)} = -0.144\,280$$

$$a_2 = \frac{350}{(8550)^{-0.144\,280}} = 1292.3 \text{ MPa}$$

$$N_2 = \left(\frac{260}{1292.3}\right)^{-1/0.144\,280} = 67\ 090 \text{ cycles}$$

$$N_{R2} = 67\ 090 - 50\ 000 = 17\ 090 \text{ cycles}$$

$$\frac{0.9(530)}{260} = \frac{a_3\left(10^3\right)^{b_3}}{a_3\left(17\ 090\right)^{b_3}} \quad \Rightarrow \quad 1.834\ 6 = \left(0.058\ 514\right)^{b_2}$$

$$b_3 = \frac{\log(1.834\ 6)}{\log(0.058\ 514)} = -0.213\ 785, \quad a_3 = \frac{260}{(17\ 090)^{-0.213\,785}} = 2088.7 \text{ MPa}$$

$$N_3 = \left(\frac{225}{2088.7}\right)^{-1/0.213\,785} = 33\ 610 \text{ cycles} \quad \textit{Ans.}$$

6-62 Given: S_e = 45 kpsi, S_{ut} = 85 kpsi, f = 0.86, and σ_a = 35 kpsi and σ_m = 30 kpsi for 12 (10^3) cycles.

Gerber equivalent reversing stress: $\sigma_{\text{rev}} = \dfrac{\sigma_a}{1-\left(\sigma_m / S_{ut}\right)^2} = \dfrac{35}{1-\left(30/85\right)^2} = 39.98$ kpsi

(**a**) Miner's method: $\sigma_{\text{rev}} < S_e$. According to the method, this means that the endurance limit has not been reduced and the new endurance limit is $S_e' = 45$ kpsi. Ans.

(**b**) Manson's method: Again, $\sigma_{\text{rev}} < S_e$. According to the method, this means that the material has not been damaged and the endurance limit has not been reduced. Thus, the new endurance limit is $S_e' = 45$ kpsi. Ans.

6-63 Given: S_e = 45 kpsi, S_{ut} = 85 kpsi, f = 0.86, and σ_a = 35 kpsi and σ_m = 30 kpsi for 12 (10^3) cycles.

Goodman equivalent reversing stress: $\sigma_{\text{rev}} = \dfrac{\sigma_a}{1-\left(\sigma_m / S_{ut}\right)} = \dfrac{35}{1-\left(30/85\right)} = 54.09$ kpsi

Initial cycling

$$a = \frac{\left[0.86(85)\right]^2}{45} = 116.00 \text{ kpsi}$$

$$b = -\frac{1}{3}\log\frac{0.86(85)}{45} = -0.070\ 235$$

$$\sigma_1 = 54.09 \text{ kpsi}, \qquad N_1 = \left(\frac{54.09}{116.00}\right)^{1/-0.070\ 235} = 52\ 190 \text{ cycles}$$

(**a**) Miner's method (see discussion on p. 325): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190$ cycles. The new coefficients are $b' = b$, and $a' = S_f / N^b = 54.09/(40\ 190)^{-0.070\ 235} = 113.89$ kpsi. The new endurance limit is

$$S'_{e,1} = a' N_e^{b'} = 113.89\left(10^6\right)^{-0.070\ 235} = 43.2 \text{ kpsi} \qquad \textit{Ans.}$$

(**b**) Manson's method (see discussion on p. 326): The number of remaining cycles at 54.09 kpsi is $N_{\text{remaining}} = 52\ 190 - 12\ 000 = 40\ 190$ cycles. At 10^3 cycles, $S_f = 0.86(85) = 73.1$ kpsi. The new coefficients are $b' = [\log(73.1/54.09)]/\log(10^3/40\ 190) = -0.081\ 540$ and $a' = \sigma_1 / (N_{\text{remaining}})^{b'} = 54.09/(40\ 190)^{-0.081\ 540} = 128.39$ kpsi. The new endurance limit is

$$S'_{e,1} = a' N_e^{b'} = 128.39\left(10^6\right)^{-0.081\ 540} = 41.6 \text{ kpsi} \qquad \textit{Ans.}$$

6-64 Given $\mathbf{S}_{ut}$ =1030**LN**(1, 0.0508) MPa

From Table 6-10: $a = 1.58$, $b = -0.086$, $C = 0.120$

Eq. (6-72) and Table 6-10): $\mathbf{k}_a = 1.58(1030)^{-0.086}\,\mathbf{LN}(1,\ 0.120) = 0.870\mathbf{LN}(1,\ 0.120)$

From Prob. 6-1: $k_b = 0.97$

Eqs. (6-70) and (6-71): $\mathbf{S}_e$ = [0.870**LN**(1, 0.120)] (0.97) [0.506(1030)**LN**(1, 0.138)]

$$\bar{S}_e = 0.870\ (0.97)(0.506)(1030) = 440 \text{ MPa}$$

and, $C_{Se} \doteq (0.12^2 + 0.138^2)^{1/2} = 0.183$

$$\mathbf{S}_e = 440\mathbf{LN}(1,\ 0.183) \text{ MPa} \qquad \textit{Ans.}$$

6-65 A Priori Decisions:

- Material and condition: 1020 CD, $\mathbf{S}_{ut}$ = 68 **LN**(1, 0.28), and $\mathbf{S}_y$ = 57 **LN**(1, 0.058) kpsi
- Reliability goal: $R = 0.99$ ($z = -2.326$, Table A-10)
- Function:
  Critical location—hole
- Variabilities:

$$C_{ka} = 0.058$$
$$C_{kc} = 0.125$$
$$C_{S'_e} = 0.138$$
$$C_{Se} = \left(C_{ka}^2 + C_{kc}^2 + C_{S'_e}^2\right)^{1/2} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$
$$C_{Kf} = 0.10$$
$$C_{Fa} = 0.20$$
$$C_{\sigma a} = (0.10^2 + 0.20^2)^{1/2} = 0.234$$
$$C_n = \sqrt{\frac{C_{Se}^2 + C_{\sigma a}^2}{1 + C_{\sigma a}^2}} = \sqrt{\frac{0.195^2 + 0.234^2}{1 + 0.234^2}} = 0.297$$

Resulting in a design factor n_f of,

Eq. (6-59): $n_f = \exp[-(-2.326)\sqrt{\ln(1 + 0.297^2)} + \ln\sqrt{1 + 0.297^2}\,] = 2.05$

- Decision: Set $n_f = 2.05$

Now proceed deterministically using the mean values:

Table 6-10: $\bar{k}_a = 2.67(68)^{-0.265} = 0.873$

Eq. (6-21): $k_b = 1$

Table 6-11: $\bar{k}_c = 1.23(68)^{-0.0778} = 0.886$

Eq. (6-70): $\bar{S}'_e = 0.506(68) = 34.4$ kpsi

Eq. (6-71): $\bar{S}_e = 0.873(1)(0.886)34.4 = 26.6$ kpsi

From Prob. 6-14, $K_f = 2.26$. Thus,

$$\bar{\sigma}_a = \bar{K}_f \frac{\bar{F}_a}{A} = \bar{K}_f \frac{\bar{F}_a}{t(2.5-0.5)} = \bar{K}_f \frac{\bar{F}_a}{2t} = \frac{\bar{S}_e}{\bar{n}_f}$$

$$\therefore t = \frac{\bar{n}_f \bar{K}_f \bar{F}_a}{2\bar{S}_e} = \frac{2.05(2.26)3.8}{2(26.6)} = 0.331 \text{ in}$$

Decision: Use $t = \frac{3}{8}$ in *Ans.*

6-66 Rotation is presumed. M and S_{ut} are given as deterministic, but notice that $\boldsymbol{\sigma}$ is not; therefore, a reliability estimation can be made.

From Eq. (6-70): $\mathbf{S}'_e = 0.506(780)\mathbf{LN}(1, 0.138) = 394.7\ \mathbf{LN}(1, 0.138)$

Table 6-13: $\mathbf{k}_a = 4.45(780)^{-0.265}\mathbf{LN}(1, 0.058) = 0.762\ \mathbf{LN}(1, 0.058)$
Based on $d = 32 - 6 = 26$ mm, Eq. (6-20) gives

$$k_b = \left(\frac{26}{7.62}\right)^{-0.107} = 0.877$$

Conservatism is not necessary

$$\mathbf{S}_e = \left[0.762\mathbf{LN}(1,\ 0.058)\right](0.877)(394.7)\left[\mathbf{LN}(1,\ 0.138)\right]$$
$$\bar{S}_e = 263.8 \text{ MPa}$$
$$C_{Se} = (0.058^2 + 0.138^2)^{1/2} = 0.150$$
$$\mathbf{S}_e = 263.8\mathbf{LN}(1,\ 0.150) \text{ MPa}$$

Fig. A-15-14: $D/d = 32/26 = 1.23$, $r/d = 3/26 = 0.115$. Thus, $K_t \doteq 1.75$, and Eq. (6-78) and Table 6-15 gives

$$\bar{K}_f = \frac{K_t}{1 + \dfrac{2(K_t - 1)}{K_t}\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1.75}{1 + \dfrac{2(1.75-1)}{1.75}\dfrac{104/780}{\sqrt{3}}} = 1.64$$

From Table 6-15, $C_{Kf} = 0.15$. Thus,

$$\mathbf{K}_f = 1.64\mathbf{LN}(1, 0.15)$$

The bending stress is

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{32M}{\pi d^3} = 1.64\mathbf{LN}(1,\ 0.15)\left[\frac{32(160)}{\pi(0.026)^3}\right]$$
$$= 152\left(10^6\right)\mathbf{LN}(1,\ 0.15) \text{ Pa} = 152\mathbf{LN}(1,\ 0.15) \text{ MPa}$$

From Eq. (5-43), p. 250,

$$z = -\frac{\ln\left(\dfrac{\mu_S}{\mu_\sigma}\sqrt{\dfrac{1+C_\sigma^2}{1+C_S^2}}\right)}{\sqrt{\ln\left[\left(1+C_S^2\right)\left(1+C_\sigma^2\right)\right]}}$$

$$= -\frac{\ln\left[\left(263.8/152\right)\sqrt{\left(1+0.15^2\right)/\left(1+0.15^2\right)}\right]}{\sqrt{\ln\left[\left(1+0.15^2\right)\left(1+0.15^2\right)\right]}} = -2.61$$

From Table A-10, $p_f = 0.004\,53$. Thus, $R = 1 - 0.004\,53 = 0.995$ *Ans.*

Note: The correlation method uses only the mean of $\mathbf{S}_{ut}$; its variability is already included in the 0.138. When a deterministic load, in this case M, is used in a reliability estimate, engineers state, "For a *Design* Load of M, the reliability is 0.995." They are, in fact, referring to a Deterministic Design Load.

6-67 For completely reversed torsion, $\mathbf{k}_a$ and k_b of Prob. 6-66 apply, but $\mathbf{k}_c$ must also be considered. $\overline{S}_{ut} = 780/6.89 = 113$ kpsi

Eq. 6-74: $\mathbf{k}_c = 0.328(113)^{0.125}\mathbf{LN}(1,\ 0.125) = 0.592\mathbf{LN}(1,\ 0.125)$

Note 0.590 is close to 0.577.

$$\mathbf{S}_e = \mathbf{k}_a k_b \mathbf{k}_c \mathbf{S}'_e$$

$$= 0.762[\mathbf{LN}(1,\ 0.058)](0.877)[0.592\mathbf{LN}(1,\ 0.125)][394.7\mathbf{LN}(1,\ 0.138)]$$

$$\overline{S}_e = 0.762(0.877)(0.592)(394.7) = 156.2 \text{ MPa}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

$$\mathbf{S}_e = 156.2\mathbf{LN}(1,\ 0.195) \text{ MPa}$$

Fig. A-15-15: $D/d = 1.23$, $r/d = 0.115$, then $K_{ts} \doteq 1.40$. From Eq. (6-78) and Table 7-8

$$\overline{K}_{fs} = \frac{K_{ts}}{1 + \dfrac{2\left(K_{ts}-1\right)}{K_{ts}}\dfrac{\sqrt{a}}{\sqrt{r}}} = \frac{1.40}{1 + \dfrac{2\left(1.40-1\right)}{1.40}\dfrac{104/780}{\sqrt{3}}} = 1.34$$

From Table 6-15, $C_{Kf} = 0.15$. Thus,

$$\mathbf{K}_{fs} = 1.34\mathbf{LN}(1,\ 0.15)$$

The torsional stress is

$$\boldsymbol{\tau} = \mathbf{K}_{fs}\frac{16T}{\pi d^3} = 1.34\mathbf{LN}(1,\ 0.15)\left[\frac{16(160)}{\pi(0.026)^3}\right]$$

$$= 62.1\left(10^6\right)\mathbf{LN}(1,\ 0.15) \text{ Pa} = 62.1\mathbf{LN}(1,\ 0.15) \text{ MPa}$$

From Eq. (5-43), p. 250,

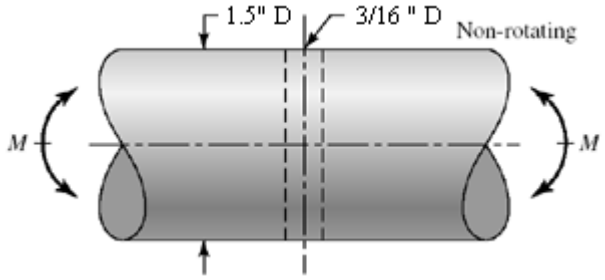
$$z = -\frac{\ln\left[(156.2/62.1)\sqrt{(1+0.15^2)/(1+0.195^2)}\right]}{\sqrt{\ln[(1+0.195^2)(1+0.15^2)]}} = -3.75$$

From Table A-10, $p_f = 0.000\ 09$

$$R = 1 - p_f = 1 - 0.000\ 09 = 0.999\ 91 \quad \textit{Ans.}$$

For a design with completely-reversed torsion of 160 N · m, the reliability is 0.999 91. The improvement over bending comes from a smaller stress-concentration factor in torsion. See the note at the end of the solution of Prob. 6-66 for the reason for the phraseology.

6-68



Given: $S_{ut} = 58$ kpsi.

Eq. (6-70): $\mathbf{S}'_e = 0.506(76)\ \mathbf{LN}(1, 0.138) = 38.5\ \mathbf{LN}(1, 0.138)$ kpsi

Table 6-13: $\mathbf{k}_a = 14.5(76)^{-0.719}\ \mathbf{LN}(1, 0.11) = 0.644\ \mathbf{LN}(1, 0.11)$

Eq. (6-24): $d_e = 0.370(1.5) = 0.555$ in

Eq. (6-20): $k_b = (0.555/0.3)^{-0.107} = 0.936$

Eq. (6-70): $\mathbf{S}_e = [0.644\ \mathbf{LN}(1, 0.11)](0.936)[38.5\ \mathbf{LN}(1, 0.138)]$

$$\bar{S}_e = 0.644(0.936)(38.5) = 23.2 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.138^2)^{1/2} = 0.176$$

$$\mathbf{S}_e = 23.2\ \mathbf{LN}(1, 0.176) \text{ kpsi}$$

Table A-16: $d/D = 0$, $a/D = (3/16)/1.5 = 0.125$, $A = 0.80$ $\therefore$ $K_t = 2.20$.

From Eqs. (6-78) and (6-79) and Table 6-15

$$K_f = \frac{2.20\mathbf{LN}(1,\ 0.10)}{1+\dfrac{2(2.20-1)}{2.20}\dfrac{5/76}{\sqrt{0.125}}} = 1.83\mathbf{LN}(1,\ 0.10)$$

Table A-16:

$$Z_{\text{net}} = \frac{\pi AD^3}{32} = \frac{\pi(0.80)(1.5^3)}{32} = 0.265 \text{ in}^3$$

$$\boldsymbol{\sigma} = \mathbf{K}_f \frac{M}{Z_{\text{net}}} = 1.83\mathbf{LN}(1,\ 0.10)\left(\frac{1.5}{0.265}\right)$$

$$= 10.4\mathbf{LN}(1,\ 0.10) \text{ kpsi}$$

$$\bar{\sigma} = 10.4 \text{ kpsi}$$

$$C_\sigma = 0.10$$

Eq. (5-43), p. 250: $$z = -\frac{\ln\left[(23.2/10.4)\sqrt{(1+0.10^2)/(1+0.176^2)}\right]}{\sqrt{\ln[(1+0.176^2)(1+0.10^2)]}} = -3.94$$

Table A-10: $p_f = 0.000\ 041\ 5 \Rightarrow R = 1 - p_f = 1 - 0.000\ 041\ 5 = 0.999\ 96$ *Ans.*

6-69 From Prob. 6-68: $\mathbf{S}'_e = 23.2\ \mathbf{LN}(1, 0.138)$ kpsi

$\mathbf{k}_a = 0.644\mathbf{LN}(1, 0.11)$

$k_b = 0.936$

Eq. (6-74): $\mathbf{k}_c = 0.328(76)^{0.125}\mathbf{LN}(1, 0.125) = 0.564\ \mathbf{LN}(1, 0.125)$

Eq. (6-71): $\mathbf{S}_e = [0.644\mathbf{LN}(1, 0.11)](0.936)[\ 0.564\ \mathbf{LN}(1, 0.125)][\ 23.2\ \mathbf{LN}(1, 0.138)]$

$$\bar{S}_e = 0.644(0.936)(0.564)(23.2) = 7.89 \text{ kpsi}$$

$$C_{Se} = (0.11^2 + 0.125^2 + 0.138^3)^{1/2} = 0.216$$

Table A-16: $d/D = 0$, $a/D = (3/16)/1.5 = 0.125$, $A = 0.89$, $\boldsymbol{K}_{ts} = 1.64$
From Eqs. (6-78) and(7-79), and Table 6-15

$$\mathbf{K}_{fs} = \frac{1.64\mathbf{LN}(1,\ 0.10)}{1+\dfrac{2(1.64-1)}{1.64}\dfrac{5/76}{\sqrt{3/32}}} = 1.40\mathbf{LN}(1,\ 0.10)$$

Table A-16:

$$J_{\text{net}} = \frac{\pi A D^4}{32} = \frac{\pi(0.89)(1.5^4)}{32} = 0.4423 \text{ in}^4$$

$$\tau_a = \mathbf{K}_{fs}\frac{T_a D}{2J_{\text{net}}} = 1.40[\mathbf{LN}(1,\ 0.10)]\frac{2(1.5)}{2(0.4423)} = 4.75\mathbf{LN}(1,\ 0.10) \text{ kpsi}$$

From Eq. (6-57):

$$z = -\frac{\ln(7.89/4.75)\sqrt{(1+0.10^2)/(1+0.216^2)}}{\sqrt{\ln[(1+0.10^2)(1+0.216^2)]}} = -2.08$$

Table A-10, $p_f = 0.0188$, $\quad R = 1 - p_f = 1 - 0.0188 = 0.981$ *Ans.*

6-70 This is a very important task for the student to attempt before starting Part 3. It illustrates the drawback of the deterministic factor of safety method. It also identifies the a priori decisions and their consequences.

The range of force fluctuation in Prob. 6-30 is – 16 to + 5 kip, or 21 kip. Let the repeatedly-applied F_a be 10.5 kip. The stochastic properties of this heat of AISI 1018 CD are given in the problem statement.

Function	Consequences
Axial	$F_a = 10.5$ kip
Fatigue load	$C_{Fa} = 0$
	$C_{kc} = 0.125$
Overall reliability $R \geq 0.998$;	$z = -3.09$
with twin fillets	$C_{Kf} = 0.11$
$R \geq \sqrt{0.998} = 0.999$	
Cold rolled or machined surfaces	$C_{ka} = 0.058$
Ambient temperature	$C_{kd} = 0$
Use correlation method	$C_{\phi} = 0.138$
Stress amplitude	$C_{Kf} = 0.11$
	$C_{\sigma a} = 0.11$
Significant strength S_e	$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$

Choose the mean design factor which will meet the reliability goal. From Eq. (6-88)

$$C_n = \sqrt{\frac{0.195^2 + 0.11^2}{1 + 0.11^2}} = 0.223$$

$$\overline{n} = \exp\left[-(-3.09)\sqrt{\ln(1+0.223^2)} + \ln\sqrt{1+0.223^2}\right]$$

$$\overline{n} = 2.02$$

In Prob. 6-30, it was found that the hole was the significant location that controlled the analysis. Thus,

$$\boldsymbol{\sigma}_a = \frac{\mathbf{S}_e}{\mathbf{n}}$$

$$\bar{\sigma}_a = \frac{\bar{S}_e}{\bar{n}} \quad \Rightarrow \quad \bar{K}_f \frac{F_a}{h(w_1 - d)} = \frac{\bar{S}_e}{\bar{n}}$$

We need to determine $\bar{S}_e$

$$\bar{k}_a = 2.67\bar{S}_{ut}^{-0.265} = 2.67(64)^{-0.265} = 0.887$$
$$k_b = 1$$
$$\bar{k}_c = 1.23\bar{S}_{ut}^{-0.0778} = 1.23(64)^{-0.0778} = 0.890$$
$$\bar{k}_d = \bar{k}_e = 1$$
$$\bar{S}_e = 0.887(1)(0.890)(1)(1)(0.506)(64) = 25.6 \text{ kpsi}$$

From the solution to Prob. 6-30, the stress concentration factor at the hole is $K_t = 2.68$. From Eq. (6-78) and Table 6-15

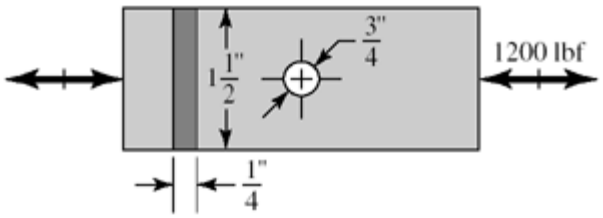
$$\bar{K}_f = \frac{2.68}{1 + \dfrac{2(2.68-1)}{2.68}\dfrac{5/64}{\sqrt{0.2}}} = 2.20$$

$$h = \frac{\bar{K}_f \bar{n} F_a}{(w_1 - d)\bar{S}_e} = \frac{2.20(2.02)(10.5)}{(3.5 - 0.4)(25.6)} = 0.588 \quad \textit{Ans.}$$

6-71

$F_a = 1200$ lbf

$S_{ut} = 80$ kpsi



(a) *Strength*

$$\mathbf{k}_a = 2.67(80)^{-0265}\mathbf{LN}(1, 0.058) = 0.836\ \mathbf{LN}(1, 0.058)$$

$$k_b = 1$$

$$\mathbf{k}_c = 1.23(80)^{-0.0778}\mathbf{LN}(1, 0.125) = 0.875\ \mathbf{LN}(1, 0.125)$$

$$\mathbf{S}'_e = 0.506(80)\mathbf{LN}(1,\ 0.138) = 40.5\mathbf{LN}(1,\ 0.138)\ \text{kpsi}$$

$$\mathbf{S}_e = \left[0.836\mathbf{LN}(1,\ 0.058)\right](1)\left[0.875\mathbf{LN}(1,\ 0.125)\right]\left[40.5\mathbf{LN}(1,\ 0.138)\right]$$

$$\bar{S}_e = 0.836(1)(0.875)(40.5) = 29.6\ \text{kpsi}$$

$$C_{Se} = (0.058^2 + 0.125^2 + 0.138^2)^{1/2} = 0.195$$

Stress: Fig. A-15-1; $d/w = 0.75/1.5 = 0.5$, $K_t = 2.18$. From Eqs. (6-78), (6-79) and Table 6-15

$$\mathbf{K}_f = \frac{2.18\mathbf{LN}(1,\ 0.10)}{1 + \dfrac{2(2.18-1)}{2.18}\dfrac{5/80}{\sqrt{0.375}}} = 1.96\mathbf{LN}(1,\ 0.10)$$

$$\boldsymbol{\sigma}_a = \mathbf{K}_f \frac{F_a}{(w-d)t}, \quad C_\sigma = 0.10$$

$$\bar{\sigma}_a = \frac{\bar{K}_f F_a}{(w-d)t} = \frac{1.96(1.2)}{(1.5-0.75)(0.25)} = 12.54\ \text{kpsi}$$

$$\bar{S}_a = \bar{S}_e = 29.6\ \text{kpsi}$$

$$z = -\frac{\ln\left[(\bar{S}_a/\bar{\sigma}_a)\sqrt{\left(1+C_\sigma^2\right)/\left(1+C_S^2\right)}\right]}{\ln\left[\left(1+C_\sigma^2\right)\left(1+C_S^2\right)\right]}$$

$$= -\frac{\ln\left[(29.6/12.48)\sqrt{\left(1+0.10^2\right)/\left(1+0.195^2\right)}\right]}{\sqrt{\ln\left[\left(1+0.10^2\right)\left(1+0.195^2\right)\right]}} = -3.9$$

From Table A-20, $p_f = 4.81(10^{-5}) \quad \Rightarrow \quad R = 1 - 4.81(10^{-5}) = 0.999\ 955$ *Ans.*

(b) All computer programs will differ in detail.

6-72 to 6-78 Computer programs are very useful for automating specific tasks in the design process. All computer programs will differ in detail.

Chapter 7

7-1 **(a)** DE-Gerber, Eq. (7-10):

$$A=\sqrt{4\left(K_f M_a\right)^2+3\left(K_{fs}T_a\right)^2}=\sqrt{4\left[(2.2)(70)\right]^2+3\left[(1.8)(45)\right]^2}=338.4\ \text{N}\cdot\text{m}$$

$$B=\sqrt{4\left(K_f M_m\right)^2+3\left(K_{fs}T_m\right)^2}=\sqrt{4\left[(2.2)(55)\right]^2+3\left[(1.8)(35)\right]^2}=265.5\ \text{N}\cdot\text{m}$$

$$d=\left\{\frac{8(2)(338.4)}{\pi(210)(10^6)}\left[1+\left(1+\left[\frac{2(265.5)(210)(10^6)}{338.4(700)(10^6)}\right]^2\right)^{1/2}\right]\right\}^{1/3}$$

$d = 25.85\ (10^{-3})\ \text{m} = 25.85\ \text{mm}$ *Ans.*

(b) DE-elliptic, Eq. (7-12) can be shown to be

$$d=\left(\frac{16n}{\pi}\sqrt{\frac{A^2}{S_e^2}+\frac{B^2}{S_y^2}}\right)^{1/3}=\left(\frac{16(2)}{\pi}\sqrt{\frac{(338.4)^2}{\left[(210)(10^6)\right]^2}+\frac{(265.5)^2}{\left[(560)(10^6)\right]^2}}\right)^{1/3}$$

$d = 25.77\ (10^{-3})\ \text{m} = 25.77\ \text{mm}$ *Ans.*

(c) DE-Soderberg, Eq. (7-14) can be shown to be

$$d=\left[\frac{16n}{\pi}\left(\frac{A}{S_e}+\frac{B}{S_y}\right)\right]^{1/3}=\left[\frac{16(2)}{\pi}\left(\frac{338.4}{210(10^6)}+\frac{265.5}{560(10^6)}\right)\right]^{1/3}$$

$d = 27.70\ (10^{-3})\ \text{m} = 27.70\ \text{mm}$ *Ans.*

(d) DE-Goodman: Eq. (7-8) can be shown to be

$$d=\left[\frac{16n}{\pi}\left(\frac{A}{S_e}+\frac{B}{S_{ut}}\right)\right]^{1/3}=\left[\frac{16(2)}{\pi}\left(\frac{338.4}{210(10^6)}+\frac{265.5}{700(10^6)}\right)\right]^{1/3}$$

$d = 27.27\ (10^{-3})\ \text{m} = 27.27\ \text{mm}$ *Ans.*

Criterion	d (mm)	Compared to DE-Gerber	
DE-Gerber	25.85		
DE-Elliptic	25.77	0.31% Lower	Less conservative
DE-Soderberg	27.70	7.2% Higher	More conservative
DE-Goodman	27.27	5.5% Higher	More conservative

7-2 This problem has to be done by successive trials, since S_e is a function of shaft size. The material is SAE 2340 for which S_{ut} = 175 kpsi, S_y = 160 kpsi, and $H_B \geq 370$.

Eq. (6-19), p. 287: $k_a = 2.70(175)^{-0.265} = 0.69$

Trial #1: Choose $d_r = 0.75$ in

Eq. (6-20), p. 288: $k_b = 0.879(0.75)^{-0.107} = 0.91$

Eq. (6-8), p.282: $S_e' = 0.5S_{ut} = 0.5(175) = 87.5$ kpsi

Eq. (6-18), p. 287: $S_e = 0.69\ (0.91)(87.5) = 54.9$ kpsi

$$d_r = d - 2r = 0.75D - 2D/20 = 0.65D$$

$$D = \frac{d_r}{0.65} = \frac{0.75}{0.65} = 1.15 \text{ in}$$

$$r = \frac{D}{20} = \frac{1.15}{20} = 0.058 \text{ in}$$

Fig. A-15-14:

$$d = d_r + 2r = 0.75 + 2(0.058) = 0.808 \text{ in}$$

$$\frac{d}{d_r} = \frac{0.808}{0.75} = 1.08$$

$$\frac{r}{d_r} = \frac{0.058}{0.75} = 0.077$$

$$K_t = 1.9$$

Fig. 6-20, p. 295: $r = 0.058$ in, $q = 0.90$

Eq. (6-32), p. 295: $K_f = 1 + 0.90\ (1.9 - 1) = 1.81$

Fig. A-15-15: $K_{ts} = 1.5$

Fig. 6-21, p. 296: $r = 0.058$ in, $q_s = 0.92$

Eq. (6-32), p. 295: $K_{fs} = 1 + 0.92\ (1.5 - 1) = 1.46$

We select the DE-ASME Elliptic failure criteria, Eq. (7-12), with d as d_r, and $M_m = T_a = 0$,

$$d_r = \left\{ \frac{16(2.5)}{\pi} \left[4\left(\frac{1.81(600)}{54.9\left(10^3\right)} \right)^2 + 3\left(\frac{1.46(400)}{160\left(10^3\right)} \right)^2 \right]^{1/2} \right\}^{1/3}$$

$$d_r = 0.799 \text{ in}$$

Trial #2: Choose $d_r = 0.799$ in.

$$k_b = 0.879(0.799)^{-0.107} = 0.90$$

$$S_e = 0.69\ (0.90)(0.5)(175) = 54.3 \text{ kpsi}$$

$$D = \frac{d_r}{0.65} = \frac{0.799}{0.65} = 1.23 \text{ in}$$

$$r = D / 20 = 1.23/20 = 0.062 \text{ in}$$

Figs. A-15-14 and A-15-15:

$$d = d_r + 2r = 0.799 + 2(0.062) = 0.923 \text{ in}$$

$$\frac{d}{d_r} = \frac{0.923}{0.799} = 1.16$$

$$\frac{r}{d_r} = \frac{0.062}{0.799} = 0.078$$

With these ratios only slightly different from the previous iteration, we are at the limit of readability of the figures. We will keep the same values as before.

$$K_t = 1.9, \quad K_{ts} = 1.5, \quad q = 0.90, \quad q_s = 0.92$$

$$\therefore K_f = 1.81, \quad K_{fs} = 1.46$$

Using Eq. (7-12) produces d_r = 0.802 in. Further iteration produces no change. With d_r = 0.802 in,

$$D = \frac{0.802}{0.65} = 1.23 \text{ in}$$

$$d = 0.75(1.23) = 0.92 \text{ in}$$

A look at a bearing catalog finds that the next available bore diameter is 0.9375 in. In nominal sizes, we select d = 0.94 in, D = 1.25 in, r = 0.0625 in *Ans.*

7-3 $F \cos 20°(d / 2) = T_A$, $F = 2\ T_A / (\ d \cos 20°) = 2(340) / (0.150 \cos 20°) = 4824$ N.
The maximum bending moment will be at point C, with $M_C = 4824(0.100) = 482.4$ N·m.
Due to the rotation, the bending is completely reversed, while the torsion is constant.
Thus, $M_a = 482.4$ N·m, $T_m = 340$ N·m, $M_m = T_a = 0$.

For sharp fillet radii at the shoulders, from Table 7-1, $K_t = 2.7$, and $K_{ts} = 2.2$. Examining Figs. 6-20 and 6-21 (pp. 295 and 296 respectively) with $S_{ut} = 560$ MPa, conservatively estimate $q = 0.8$ and $q_s = 0.9$. These estimates can be checked once a specific fillet radius is determined.

Eq. (6-32): $K_f = 1 + 0.8(2.7 - 1) = 2.4$

$$K_{fs} = 1 + 0.9(2.2 - 1) = 2.1$$

(a) We will choose to include fatigue stress concentration factors even for the static analysis to avoid localized yielding.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

Eq. (7-16): $$n = \frac{S_y}{\sigma'_{\max}} = \frac{\pi d^3 S_y}{16}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{-1/2}$$

Solving for d,

$$d = \left\{\frac{16n}{\pi S_y}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2}\right\}^{1/3}$$

$$= \left(\frac{16(2.5)}{\pi(420)(10^6)}\left\{4\left[(2.4)(482.4)\right]^2 + 3\left[(2.1)(340)\right]^2\right\}^{1/2}\right)^{1/3}$$

$d = 0.0430$ m $= 43.0$ mm *Ans.*

(b) $k_a = 4.51(560)^{-0.265} = 0.84$

Assume $k_b = 0.85$ for now. Check later once a diameter is known.

$S_e = 0.84(0.85)(0.5)(560) = 200$ MPa

Selecting the DE-ASME Elliptic criteria, use Eq. (7-12) with $M_m = T_a = 0$.

$$d = \left\{\frac{16(2.5)}{\pi}\left[4\left(\frac{2.4(482.4)}{200(10^6)}\right)^2 + 3\left(\frac{2.1(340)}{420(10^6)}\right)^2\right]^{1/2}\right\}^{1/3}$$

$$= 0.0534 \text{ m} = 53.4 \text{ mm}$$

With this diameter, we can refine our estimates for k_b and q.

Eq. (6-20): $k_b = 1.51 d^{-0.157} = 1.51(53.4)^{-0.157} = 0.81$

Assuming a sharp fillet radius, from Table 7-1, $r = 0.02d = 0.02\ (53.4) = 1.07$ mm.

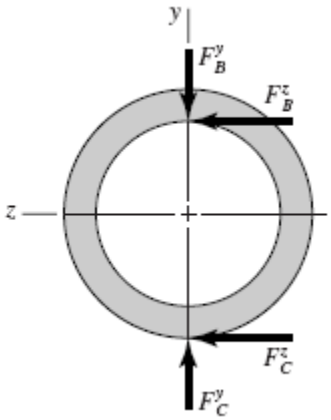
Fig. (6-20): $q = 0.72$
Fig. (6-21): $q_s = 0.77$

Iterating with these new estimates,

Eq. (6-32): $K_f = 1 + 0.72\ (2.7 - 1) = 2.2$
$K_{fs} = 1 + 0.77\ (2.2 - 1) = 1.9$
Eq. (6-18): $S_e = 0.84(0.81)(0.5)(560) = 191$ MPa
Eq. (7-12): $d = 53$ mm *Ans.*

Further iteration does not change the results.

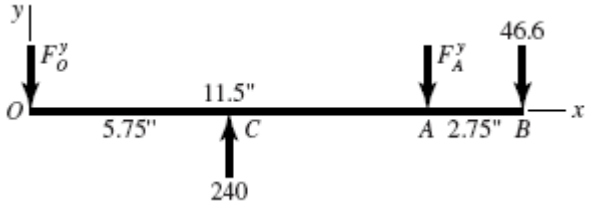
7-4 We have a design task of identifying bending moment and torsion diagrams which are preliminary to an industrial roller shaft design. Let point *C* represent the center of the span of the roller.



$$F_C^y = 30(8) = 240 \text{ lbf}$$
$$F_C^z = 0.4(240) = 96 \text{ lbf}$$
$$T = F_C^z(2) = 96(2) = 192 \text{ lbf} \cdot \text{in}$$
$$F_B^z = \frac{T}{1.5} = \frac{192}{1.5} = 128 \text{ lbf}$$

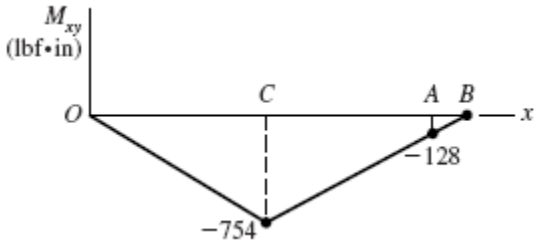
$$F_B^y = F_B^z \tan 20^\circ = 128 \tan 20^\circ = 46.6 \text{ lbf}$$

(a) *xy-plane*

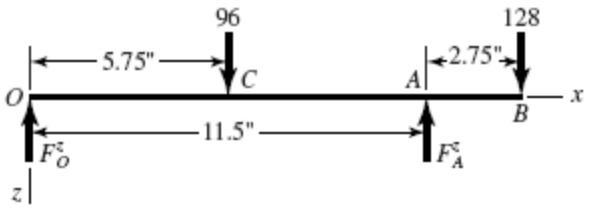


$$\Sigma M_O = 240(5.75) - F_A^y(11.5) - 46.6(14.25) = 0$$
$$F_A^y = \frac{240(5.75) - 46.6(14.25)}{11.5} = 62.3 \text{ lbf}$$
$$\Sigma M_A = F_O^y(11.5) - 46.6(2.75) - 240(5.75) = 0$$
$$F_O^y = \frac{240(5.75) + 46.6(2.75)}{11.5} = 131.1 \text{ lbf}$$

Bending moment diagram:



xz-plane



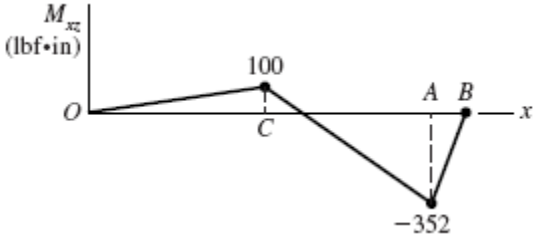
$$\Sigma M_O = 0 = 96(5.75) - F_A^z(11.5) + 128(14.25)$$

$$F_A^z = \frac{96(5.75) + 128(14.25)}{11.5} = 206.6 \text{ lbf}$$

$$\Sigma M_A = 0 = F_O^z(11.5) + 128(2.75) - 96(5.75)$$

$$F_O^z = \frac{96(5.75) - 128(2.75)}{11.5} = 17.4 \text{ lbf}$$

Bending moment diagram:

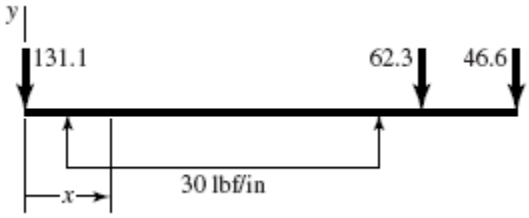


$$M_C = \sqrt{100^2 + (-754)^2} = 761 \text{ lbf} \cdot \text{in}$$

$$M_A = \sqrt{(-128)^2 + (-352)^2} = 375 \text{ lbf} \cdot \text{in}$$

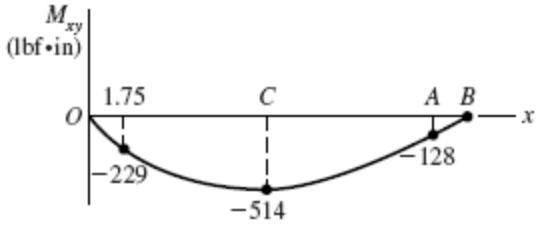
Torque: The torque is constant from *C* to *B*, with a magnitude previously obtained of 192 lbf·in.

(b) *xy-plane*



$$M_{xy} = -131.1x + 15\langle x - 1.75\rangle^2 - 15\langle x - 9.75\rangle^2 - 62.3\langle x - 11.5\rangle^1$$

Bending moment diagram:

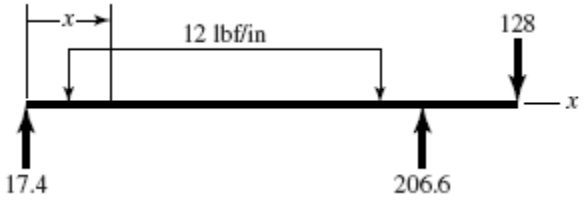


$M_{\max} = -516$ lbf · in and occurs at 6.12 in.

$$M_C = 131.1(5.75) - 15(5.75 - 1.75)^2 = 514 \text{ lbf} \cdot \text{in}$$

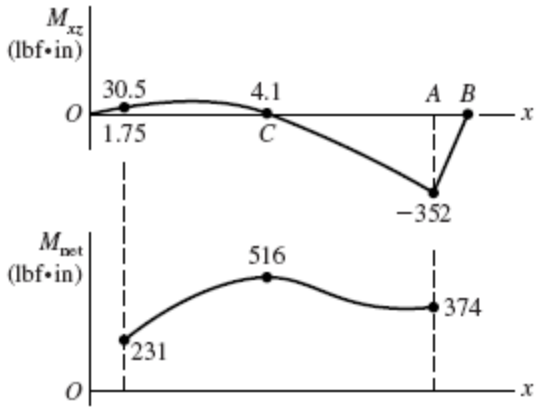
This is reduced from 754 lbf · in found in part (a). The maximum occurs at $x = 6.12$ in rather than *C*, but it is close enough.

xz-plane



$$M_{xz} = 17.4x - 6\langle x - 1.75\rangle^2 + 6\langle x - 9.75\rangle^2 + 206.6\langle x - 11.5\rangle^1$$

Bending moment diagram:



Let $M_{\text{net}} = \sqrt{M_{xy}^2 + M_{xz}^2}$

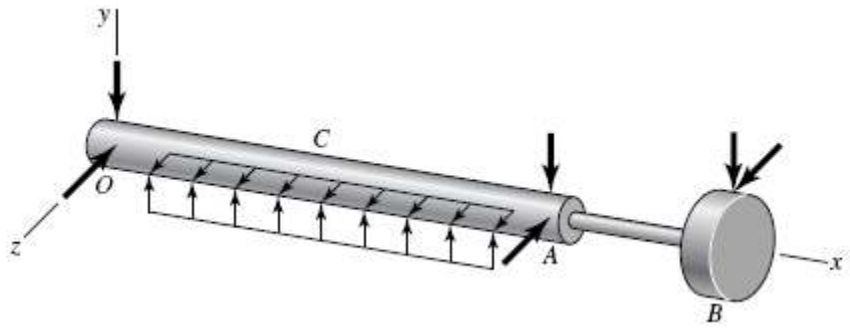
Plot $M_{\text{net}}(x)$, $1.75 \le x \le 11.5$ in

$M_{\max} = 516$ lbf · in at $x = 6.25$ in

Torque: The torque rises from 0 to 192 lbf·in linearly across the roller, then is constant to *B*. *Ans.*

7-5 This is a design problem, which can have many acceptable designs. See the solution for Prob. 7-17 for an example of the design process.

7-6 If students have access to finite element or beam analysis software, have them model the shaft to check deflections. If not, solve a simpler version of shaft for deflection. The 1 in diameter sections will not affect the deflection results much, so model the 1 in diameter as 1.25 in. Also, ignore the step in *AB*.



From Prob. 7-4, integrate M_{xy} and M_{xz}.

xy plane, with *dy/dx = y'*

$$EIy' = -\frac{131.1}{2}\left(x^2\right) + 5\left\langle x-1.75\right\rangle^3 - 5\left\langle x-9.75\right\rangle^3 - \frac{62.3}{2}\left\langle x-11.5\right\rangle^2 + C_1 \qquad (1)$$

$$EIy = -\frac{131.1}{6}\left(x^3\right) + \frac{5}{4}\left\langle x-1.75\right\rangle^4 - \frac{5}{4}\left\langle x-9.75\right\rangle^4 - \frac{62.3}{6}\left\langle x-11.5\right\rangle^3 + C_1 x + C_2$$

$y = 0$ at $x = 0$ $\Rightarrow$ $C_2 = 0$

$y = 0$ at $x = 11.5$ $\Rightarrow$ $C_1 = 1908.4 \text{ lbf}\cdot\text{in}^3$

From (1), $x = 0$: $EIy' = 1908.4$

$x = 11.5$: $EIy' = -2153.1$

xz plane (treating $z\uparrow +$)

$$EIz' = \frac{17.4}{2}\left(x^2\right) - 2\left\langle x-1.75\right\rangle^3 + 2\left\langle x-9.75\right\rangle^3 + \frac{206.6}{2}\left\langle x-11.5\right\rangle^2 + C_3 \qquad (2)$$

$$EIz = \frac{17.4}{6}\left(x^3\right) - \frac{1}{2}\left\langle x-1.75\right\rangle^4 + \frac{1}{2}\left\langle x-9.75\right\rangle^4 + \frac{206.6}{6}\left\langle x-11.5\right\rangle^3 + C_3 x + C_4$$

$z = 0$ at $x = 0$ $\Rightarrow$ $C_4 = 0$

$z = 0$ at $x = 11.5$ $\Rightarrow$ $C_3 = 8.975 \text{ lbf}\cdot\text{in}^3$

From (2), $x = 0$: $EIz' = 8.975$

$x = 11.5$: $EIz' = -683.5$

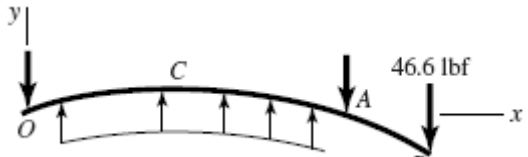
At *O*: $EI\theta = \sqrt{1908.4^2 + 8.975^2} = 1908.4 \text{ lbf}\cdot\text{in}^3$

At *A*: $EI\theta = \sqrt{(-2153.1)^2 + (-683.5)^2} = 2259.0 \text{ lbf} \cdot \text{in}^3$ (dictates size)

$$\theta = \frac{2259}{30\left(10^6\right)\left(\pi / 64\right)\left(1.25^4\right)} = 0.000\,628 \text{ rad}$$

$$n = \frac{0.001}{0.000\,628} = 1.59$$

At gear mesh, *B*
xy plane



With $I = I_1$ in section *OCA*,

$$y'_A = -2153.1 / EI_1$$

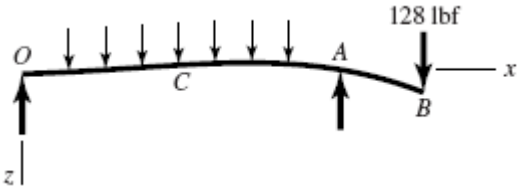
Since $y'_{B/A}$ is a cantilever, from Table A-9-1, with $I = I_2$ in section *AB*

$$y'_{B/A} = \frac{Fx(x-2l)}{2EI_2} = \frac{46.6}{2EI_2}(2.75)[2.75 - 2(2.75)] = -176.2 / EI_2$$

$$\therefore y'_B = y'_A + y'_{B/A} = -\frac{2153.1}{30\left(10^6\right)\left(\pi / 64\right)\left(1.25^4\right)} - \frac{176.2}{30\left(10^6\right)\left(\pi / 64\right)\left(0.875^4\right)}$$

$$= -0.000\,803 \text{ rad} \quad \text{(magnitude greater than 0.0005 rad)}$$

xz plane



$$z'_A = -\frac{683.5}{EI_1}, \qquad z'_{B/A} = -\frac{128\left(2.75^2\right)}{2EI_2} = -\frac{484}{EI_2}$$

$$z'_B = -\frac{683.5}{30\left(10^6\right)\left(\pi / 64\right)\left(1.25^4\right)} - \frac{484}{30\left(10^6\right)\left(\pi / 64\right)\left(0.875^4\right)} = -0.000\,751 \text{ rad}$$

$$\theta_B = \sqrt{(-0.000\,803)^2 + (-0.000\,751)^2} = 0.001\,10 \text{ rad}$$

Crowned teeth must be used.

Finite element results:	Error in simplified model
$\theta_O = 5.47(10^{-4})$ rad	3.0%
$\theta_A = 7.09(10^{-4})$ rad	11.4%
$\theta_B = 1.10(10^{-3})$ rad	0.0%

The simplified model yielded reasonable results.

Strength $S_{ut} = 72$ kpsi, $S_y = 39.5$ kpsi

At the shoulder at A, $x = 10.75$ in. From Prob. 7-4,

$$M_{xy} = -209.3 \text{ lbf} \cdot \text{in}, \quad M_{xz} = -293.0 \text{ lbf} \cdot \text{in}, \quad T = 192 \text{ lbf} \cdot \text{in}$$

$$M = \sqrt{(-209.3)^2 + (-293)^2} = 360.0 \text{ lbf} \cdot \text{in}$$

$$S'_e = 0.5(72) = 36 \text{ kpsi}$$

$$k_a = 2.70(72)^{-0.265} - 0.869$$

$$k_b = \left(\frac{1}{0.3}\right)^{-0.107} = 0.879$$

$$k_c = k_d = k_e = k_f = 1$$

$$S_e = 0.869(0.879)(36) = 27.5 \text{ kpsi}$$

$D / d = 1.25$, $r / d = 0.03$

Fig. A-15-8: $K_{ts} = 1.8$

Fig. A-15-9: $K_t = 2.3$

Fig. 6-20: $q = 0.65$

Fig. 6-21: $q_s = 0.70$

Eq. (6-32): $K_f = 1 + 0.65(2.3 - 1) = 1.85$

$$K_{fs} = 1 + 0.70(1.8 - 1) = 1.56$$

Using DE-ASME Elliptic, Eq. (7-11) with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi\left(1^3\right)}\left\{4\left[\frac{1.85(360)}{27\,500}\right]^2 + 3\left[\frac{1.56(192)}{39\,500}\right]^2\right\}^{1/2}$$

$$n = 3.91$$

Perform a similar analysis at the profile keyway under the gear.

The main problem with the design is the undersized shaft overhang with excessive slope at the gear. The use of crowned-teeth in the gears will eliminate this problem.

7-7 through 7-16

These are design problems, which can have many acceptable designs. See the solution for Prob. 7-17 for an example of the design process.

7-17 **(a)** One possible shaft layout is shown in part (e). Both bearings and the gear will be located against shoulders. The gear and the motor will transmit the torque through the

keys. The bearings can be lightly pressed onto the shaft. The left bearing will locate the shaft in the housing, while the right bearing will float in the housing.

(b) From summing moments around the shaft axis, the tangential transmitted load through the gear will be

$$W_t = T/(d/2) = 2500/(4/2) = 1250 \text{ lbf}$$

The radial component of gear force is related by the pressure angle.

$$W_r = W_t \tan\phi = 1250 \tan 20^\circ = 455 \text{ lbf}$$

$$W = \left(W_r^2 + W_t^2\right)^{1/2} = \left(455^2 + 1250^2\right)^{1/2} = 1330 \text{ lbf}$$

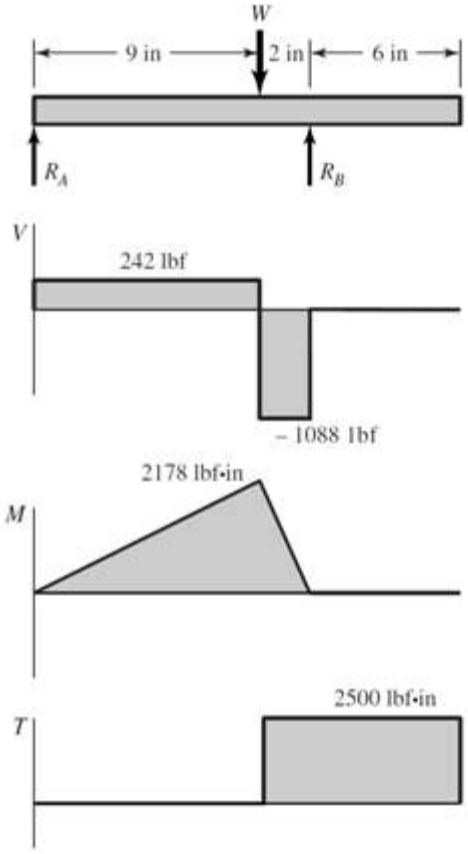
Reactions R_A and R_B, and the load W are all in the same plane. From force and moment balance,

$$R_A = 1330(2/11) = 242 \text{ lbf}$$

$$R_B = 1330(9/11) = 1088 \text{ lbf}$$

$$M_{\max} = R_A(9) = 242(9) = 2178 \text{ lbf} \cdot \text{in}$$

Shear force, bending moment, and torque diagrams can now be obtained.



Ans.

(c) Potential critical locations occur at each stress concentration (shoulders and keyways). To be thorough, the stress at each potentially critical location should be evaluated. For

now, we will choose the most likely critical location, by observation of the loading situation, to be in the keyway for the gear. At this point there is a large stress concentration, a large bending moment, and the torque is present. The other locations either have small bending moments, or no torque. The stress concentration for the keyway is highest at the ends. For simplicity, and to be conservative, we will use the maximum bending moment, even though it will have dropped off a little at the end of the keyway.

(d) At the gear keyway, approximately 9 in from the left end of the shaft, the bending is completely reversed and the torque is steady.

$$M_a = 2178 \text{ lbf} \cdot \text{in} \qquad T_m = 2500 \text{ lbf} \cdot \text{in} \qquad M_m = T_a = 0$$

From Table 7-1, estimate stress concentrations for the end-milled keyseat to be $K_t = 2.14$ and $K_{ts} = 3.0$. For the relatively low strength steel specified (AISI 1020 CD), roughly estimate notch sensitivities of $q = 0.75$ and $q_s = 0.80$, obtained by observation of Figs. 6-20 and 6-21, assuming a typical radius at the bottom of the keyseat of $r / d = 0.02$ (p. 373), and a shaft diameter of up to 3 inches.

Eq. (6-32): $K_f = 1 + 0.75(2.14 - 1) = 1.9$

$K_{fs} = 1 + 0.8(3.0 - 1) = 2.6$

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

For estimating k_b, guess $d = 2$ in.

Eq. (6-20) $k_b = (2 / 0.3)^{-0.107} = 0.816$

Eq. (6-18) $S_e = 0.883(0.816)(0.5)(68) = 24.5 \text{ kpsi}$

Selecting the DE-Goodman criteria for a conservative first design,

Eq. (7-8):
$$d = \left[\frac{16n}{\pi} \left\{ \frac{\left[4\left(K_f M_a \right)^2 \right]^{1/2}}{S_e} + \frac{\left[3\left(K_{fs} T_m \right)^2 \right]^{1/2}}{S_{ut}} \right\} \right]^{1/3}$$

$$d = \left[\frac{16(1.5)}{\pi} \left\{ \frac{\left[4\left(1.9 \cdot 2178 \right)^2 \right]^{1/2}}{24\,500} + \frac{\left[3\left(2.6 \cdot 2500 \right)^2 \right]^{1/2}}{68\,000} \right\} \right]^{1/3}$$

$$d = 1.57 \text{ in} \quad \textit{Ans.}$$

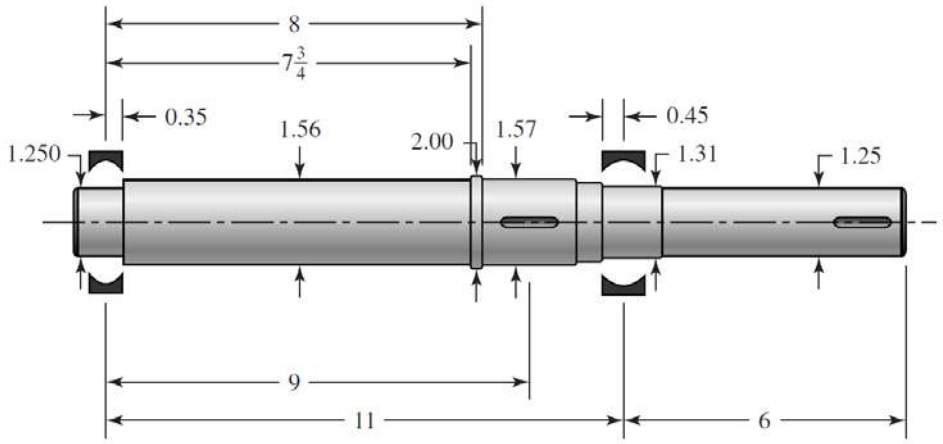
With this diameter, the estimates for notch sensitivity and size factor were conservative, but close enough for a first iteration until deflections are checked. Check yielding with this diameter.

Eq. (7-15): $$\sigma'_{\max} = \left[\left(\frac{32 K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16 K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma'_{\max} = \left[\left(\frac{32(1.9)(2178)}{\pi (1.57)^3} \right)^2 + 3 \left(\frac{16(2.6)(2500)}{\pi (1.57)^3} \right)^2 \right]^{1/2} = 18389 \text{ psi} = 18.4 \text{ kpsi}$$

$$n_y = S_y / \sigma'_{\max} = 57 / 18.4 = 3.1 \quad \textit{Ans.}$$

(e) Now estimate other diameters to provide typical shoulder supports for the gear and bearings (p. 372). Also, estimate the gear and bearing widths.



(f) Entering this shaft geometry into beam analysis software (or Finite Element software), the following deflections are determined:

Left bearing slope:	0.000 532 rad
Right bearing slope:	– 0.000 850 rad
Gear slope:	– 0.000 545 rad
Right end of shaft slope:	– 0.000 850 rad
Gear deflection:	– 0.001 45 in
Right end of shaft deflection:	0.005 10 in

Comparing these deflections to the recommendations in Table 7-2, everything is within typical range except the gear slope is a little high for an uncrowned gear.

(g) To use a non-crowned gear, the gear slope is recommended to be less than 0.0005 rad. Since all other deflections are acceptable, we will target an increase in diameter only for the long section between the left bearing and the gear. Increasing this diameter from the proposed 1.56 in to 1.75 in, produces a gear slope of – 0.000 401 rad. All other deflections are improved as well.

7-18

(a) Use the distortion-energy elliptic failure locus. The torque and moment loadings on the shaft are shown in the solution to Prob. 7-17.

Candidate critical locations for strength:

- Left seat keyway
- Right bearing shoulder
- Right keyway

Table A-20 for 1030 HR: $S_{ut} = 68$ kpsi, $S_y = 37.5$ kpsi, $H_B = 137$

Eq. (6-8): $S'_e = 0.5(68) = 34.0$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

$k_c = k_d = k_e = 1$

Left keyway

See Table 7-1 for keyway stress concentration factors,

$$\left.\begin{aligned} K_t &= 2.14 \\ K_{ts} &= 3.0 \end{aligned}\right\} \text{ Profile keyway}$$

For an end-mill profile keyway cutter of 0.010 in radius, estimate notch sensitivities.

Fig. 6-20: $q = 0.51$

Fig. 6-21: $q_s = 0.57$

Eq. (6-32): $K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.57(3.0 - 1) = 2.1$

$K_f = 1 + 0.51(2.14 - 1) = 1.6$

Eq. (6-20): $k_b = \left(\dfrac{1.875}{0.30}\right)^{-0.107} = 0.822$

Eq. (6-18): $S_e = 0.883(0.822)(34.0) = 24.7$ kpsi

Eq. (7-11):
$$\frac{1}{n_f} = \frac{16}{\pi(1.875^3)}\left\{4\left[\frac{1.6(2178)}{24\,700}\right]^2 + 3\left[\frac{2.1(2500)}{37\,500}\right]^2\right\}^{\frac{1}{2}}$$

$n_f = 3.5$ *Ans.*

Right bearing shoulder

The text does not give minimum and maximum shoulder diameters for 03-series bearings (roller). Use $D = 1.75$ in.

$$\frac{r}{d} = \frac{0.030}{1.574} = 0.019, \quad \frac{D}{d} = \frac{1.75}{1.574} = 1.11$$

Fig. A-15-9: $K_t = 2.4$

Fig. A-15-8: $K_{ts} = 1.6$

Fig. 6-20: $q = 0.65$

Fig. 6-21: $q_s = 0.70$

Eq. (6-32): $K_f = 1 + 0.65(2.4 - 1) = 1.91$

$K_{fs} = 1 + 0.70(1.6 - 1) = 1.42$

$$M = 2178\left(\frac{0.453}{2}\right) = 493 \text{ lbf} \cdot \text{in}$$

Eq. (7-11):
$$\frac{1}{n_f} = \frac{16}{\pi(1.574^3)}\left[4\left(\frac{1.91(493)}{24\,700}\right)^2 + 3\left(\frac{1.42(2500)}{37\,500}\right)^2\right]^{1/2}$$

$$n_f = 4.2 \qquad \textit{Ans.}$$

Right keyway

Use the same stress concentration factors as for the left keyway. There is no bending moment, thus Eq. (7-11) reduces to:

$$\frac{1}{n_f} = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3 S_y} = \frac{16\sqrt{3}(2.1)(2500)}{\pi\left(1.5^3\right)(37\,500)}$$

$$n_f = 2.7 \qquad \textit{Ans.}$$

Yielding

Check for yielding at the left keyway, where the completely reversed bending is maximum, and the steady torque is present. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.6)(2178)}{\pi(1.875)^3}\right)^2 + 3\left(\frac{16(2.1)(2500)}{\pi(1.875)^3}\right)^2\right]^{1/2}$$

$$= 8791 \text{ psi} = 8.79 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{37.5}{8.79} = 4.3 \qquad \textit{Ans.}$$

Check in smaller diameter at right end of shaft where only steady torsion exists.

$$\sigma'_{\max} = \left[3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[3\left(\frac{16(2.1)(2500)}{\pi(1.5)^3}\right)^2\right]^{1/2}$$

$$= 13\,722 \text{ psi} = 13.7 \text{ kpsi}$$

$$n_y = \frac{S_y}{\sigma'_{\max}} = \frac{37.5}{13.7} = 2.7 \qquad \textit{Ans.}$$

(b) One could take pains to model this shaft exactly, using finite element software. However, for the bearings and the gear, the shaft is basically of uniform diameter, 1.875 in. The reductions in diameter at the bearings will change the results insignificantly. Use E = 30 Mpsi for steel.

To the left of the load, from Table A-9, case 6, p. 1015,

$$\theta_{AB} = \frac{dy_{AB}}{dx} = \frac{Fb}{6EIl}(3x^2 + b^2 - l^2) = \frac{1449(2)(3x^2 + 2^2 - 11^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)}$$

$$= 2.4124(10^{-6})(3x^2 - 117)$$

At x = 0 in: $\theta = -2.823(10^{-4})$ rad

At x = 9 in: $\theta = 3.040(10^{-4})$ rad

To the right of the load, from Table A-9, case 6, p. 1015,

$$\theta_{BC} = \frac{dy_{BC}}{dx} = \frac{Fa}{6EIl}\left(-3x^2 + 6xl - 2l^2 - a^2\right)$$

At $x = l$ = 11 in:

$$\theta = \frac{Fa}{6EIl}\left(l^2 - a^2\right) = \frac{1449(9)(11^2 - 9^2)}{6(30)(10^6)(\pi/64)(1.875^4)(11)} = 4.342(10^{-4}) \text{ rad}$$

Obtain allowable slopes from Table 7-2.

Left bearing:

$$n_{fs} = \frac{\text{Allowable slope}}{\text{Actual slope}} = \frac{0.001}{0.000\,282\,3} = 3.5 \qquad \textit{Ans.}$$

Right bearing:

$$n_{fs} = \frac{0.0008}{0.000\,434\,2} = 1.8 \qquad \textit{Ans.}$$

Gear mesh slope:
Table 7-2 recommends a minimum relative slope of 0.0005 rad. While we don't know the slope on the next shaft, we know that it will need to have a larger diameter and be stiffer. At the moment we can say

$$n_{fs} < \frac{0.0005}{0.000\,304} = 1.6 \qquad \textit{Ans.}$$

7-19 The most likely critical locations for fatigue are at locations where the bending moment is high, the cross section is small, stress concentration exists, and torque exists. The two-plane bending moment diagrams, shown in the solution to Prob. 3-72, indicate decreasing moments in both planes to the left of A and to the right of C, with combined values at A and C of $M_A = 5324$ lbf·in and $M_C = 6750$ lbf·in. The torque is constant between A and B, with $T = 2819$ lbf·in. The most likely critical locations are at the stress concentrations near A and C. The two shoulders near A can be eliminated since the shoulders near C have the same geometry but a higher bending moment. We will consider the following potentially critical locations:

- keyway at A
- shoulder to the left of C
- shoulder to the right of C

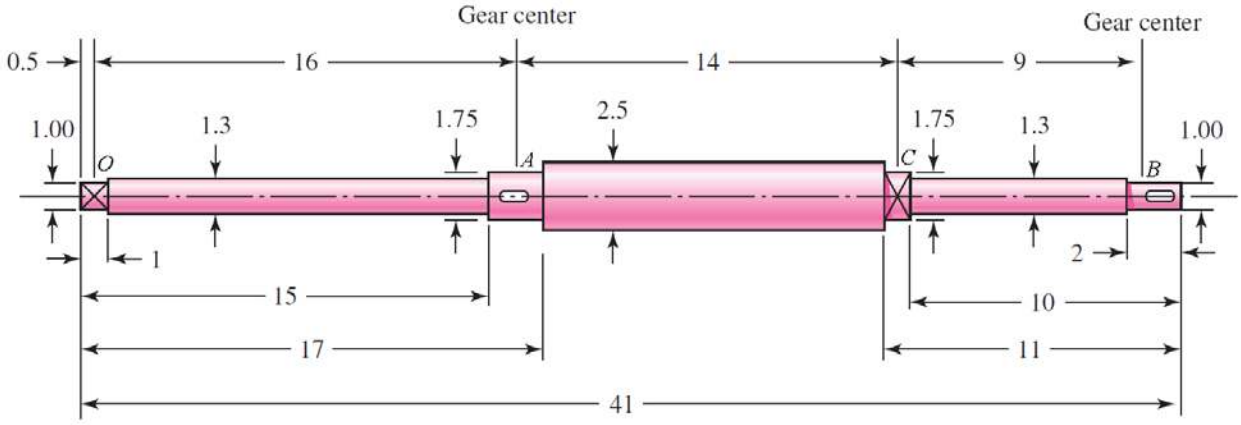


Table A-20: $S_{ut} = 64$ kpsi, $S_y = 54$ kpsi

Eq. (6-8): $S'_e = 0.5(64) = 32.0$ kpsi

Eq. (6-19): $k_a = 2.70(64)^{-0.265} = 0.897$

$k_c = k_d = k_e = 1$

Keyway at A

Assuming $r / d = 0.02$ for typical end-milled keyway cutter (p. 373), with $d = 1.75$ in, $r = 0.02d = 0.035$ in.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$

Fig. 6-20: $q = 0.65$

Fig. 6-21: $q_s = 0.71$

Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.65(2.14 - 1) = 1.7$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.71(3.0 - 1) = 2.4$

Eq. (6-20): $k_b = \left(\dfrac{1.75}{0.30}\right)^{-0.107} = 0.828$

Eq. (6-18): $S_e = 0.897(0.828)(32) = 23.8$ kpsi

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations.
Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.7)(5324)\right]^2} = 18\,102 \text{ lbf} \cdot \text{in} = 18.10 \text{ kip} \cdot \text{in}$$

$$B = \sqrt{3\left(K_{fs} T_m\right)^2} = \sqrt{3\left[(2.4)(2819)\right]^2} = 11\,718 \text{ lbf} \cdot \text{in} = 11.72 \text{ kip} \cdot \text{in}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(18.10)}{\pi\left(1.75^3\right)(23.8)}\left\{1 + \left[1 + \left(\frac{2(11.72)(23.8)}{(18.10)(64)}\right)^2\right]^{1/2}\right\}$$

$$n = 1.3$$

Shoulder to the left of C

$r / d = 0.0625 / 1.75 = 0.036$, $D / d = 2.5 / 1.75 = 1.43$

Fig. A-15-9: $K_t = 2.2$
Fig. A-15-8: $K_{ts} = 1.8$
Fig. 6-20: $q = 0.71$
Fig. 6-21: $q_s = 0.76$
Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.71(2.2 - 1) = 1.9$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.76(1.8 - 1) = 1.6$$

Eq. (6-20): $$k_b = \left(\frac{1.75}{0.30}\right)^{-0.107} = 0.828$$

Eq. (6-18): $S_e = 0.897(0.828)(32) = 23.8$ kpsi

For convenience, we will use the full value of the bending moment at *C*, even though it will be slightly less at the shoulder. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.9)(6750)\right]^2} = 25\,650 \text{ lbf} \cdot \text{in} = 25.65 \text{ kip} \cdot \text{in}$$

$$B = \sqrt{3\left(K_{fs} T_m\right)^2} = \sqrt{3\left[(1.6)(2819)\right]^2} = 7812 \text{ lbf} \cdot \text{in} = 7.812 \text{ kip} \cdot \text{in}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(25.65)}{\pi\left(1.75^3\right)(23.8)}\left\{1 + \left[1 + \left(\frac{2(7.812)(23.8)}{(25.65)(64)}\right)^2\right]^{1/2}\right\}$$

$n = 0.96$

Shoulder to the right of C

$r / d = 0.0625 / 1.3 = 0.048,\ D / d = 1.75 / 1.3 = 1.35$

Fig. A-15-9: $K_t = 2.0$
Fig. A-15-8: $K_{ts} = 1.7$
Fig. 6-20: $q = 0.71$
Fig. 6-21: $q_s = 0.76$
Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.71(2.0 - 1) = 1.7$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.76(1.7 - 1) = 1.5$

Eq. (6-20): $k_b = \left(\dfrac{1.3}{0.30}\right)^{-0.107} = 0.855$

Eq. (6-18): $S_e = 0.897(0.855)(32) = 24.5 \text{ kpsi}$

For convenience, we will use the full value of the bending moment at *C*, even though it will be slightly less at the shoulder. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.7)(6750)\right]^2} = 22\,950 \text{ lbf} \cdot \text{in} = 22.95 \text{ kip} \cdot \text{in}$$

$$B = \sqrt{3\left(K_{fs} T_m\right)^2} = \sqrt{3\left[(1.5)(2819)\right]^2} = 7324 \text{ lbf} \cdot \text{in} = 7.324 \text{ kip} \cdot \text{in}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(22.95)}{\pi(1.3^3)(24.5)}\left\{1 + \left[1 + \left(\frac{2(7.324)(24.5)}{(22.95)(64)}\right)^2\right]^{1/2}\right\}$$

$n = 0.45$

The critical location is at the shoulder to the right of *C*, where $n = 0.45$ and finite life is predicted. *Ans.*

Though not explicitly called for in the problem statement, a static check for yielding is especially warranted with such a low fatigue factor of safety. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.7)(6750)}{\pi(1.3)^3}\right)^2 + 3\left(\frac{16(1.5)(2819)}{\pi(1.3)^3}\right)^2\right]^{1/2} = 55\,845 \text{ psi} = 55.8 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma'_{\max}} = \frac{54}{55.8} = 0.97$$

This indicates localized yielding is predicted at the stress-concentration, though after localized cold-working it may not be a problem. The finite fatigue life is still likely to be the failure mode that will dictate whether this shaft is acceptable.

It is interesting to note the impact of stress concentration on the acceptability of the proposed design. This problem is linked with several previous problems (see Table 1-1, p. 24) in which the shaft was considered to have a constant diameter of 1.25 in. In each of the previous problems, the 1.25 in diameter was more than adequate for deflection, static, and fatigue considerations. In this problem, even though practically the entire shaft has diameters larger than 1.25 in, the stress concentrations significantly reduce the anticipated fatigue life.

7-20 For a shaft with significantly varying diameters over its length, we will choose to use shaft analysis software or finite element software to calculate the deflections. Entering the geometry from the shaft as defined in Prob. 7-19, and the loading as defined in Prob. 3-72, the following deflection magnitudes are determined:

Location	Slope (rad)	Deflection (in)
Left bearing *O*	0.00640	0.00000
Right bearing *C*	0.00434	0.00000
Left Gear *A*	0.00260	0.04839
Right Gear *B*	0.01078	0.07517

Comparing these values to the recommended limits in Table 7-2, we find that they are all out of the desired range. This is not unexpected since the stress analysis of Prob. 7-19 also indicated the shaft is undersized for infinite life. The slope at the right gear is the most excessive, so we will attempt to increase all diameters to bring it into compliance. Using Eq. (7-18) at the right gear,

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d \left(dy/dx \right)_{\text{old}}}{\left(\text{slope} \right)_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(0.01078)}{0.0005} \right|^{1/4} = 2.15$$

Multiplying all diameters by 2.15, we obtain the following deflections:

Location	Slope (rad)	Deflection (in)
Left bearing *O*	0.00030	0.00000
Right bearing *C*	0.00020	0.00000
Left Gear *A*	0.00012	0.00225
Right Gear *B*	0.00050	0.00350

This brings the slope at the right gear just to the limit for an uncrowned gear, and all other slopes well below the recommended limits. For the gear deflections, the values are below recommended limits as long as the diametral pitch is less than 20.

7-21 The most likely critical locations for fatigue are at locations where the bending moment is high, the cross section is small, stress concentration exists, and torque exists. The two-plane bending moment diagrams, shown in the solution to Prob. 3-73, indicate both planes have a maximum bending moment at *B*. At this location, the combined bending moment from both planes is $M = 4097$ N·m, and the torque is $T = 3101$ N·m. The shoulder to the right of *B* will be eliminated since its diameter is only slightly smaller, and there is no torque. Comparing the shoulder to the left of *B* with the keyway at *B*, the primary difference between the two is the stress concentration, since they both have essentially the same bending moment, torque, and size. We will check the stress concentration factors for both to determine which is critical.

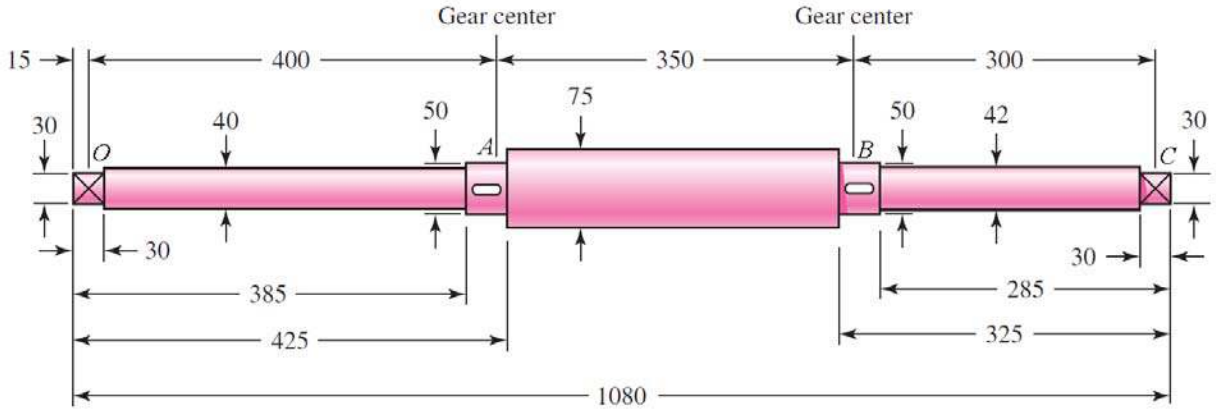


Table A-20: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

Keyway at A
Assuming $r / d = 0.02$ for typical end-milled keyway cutter (p. 373), with $d = 50$ mm, $r = 0.02d = 1$ mm.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$
Fig. 6-20: $q = 0.66$
Fig. 6-21: $q_s = 0.72$
Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.66(2.14 - 1) = 1.8$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.72(3.0 - 1) = 2.4$$

Shoulder to the left of B

$$r / d = 2 / 50 = 0.04, \quad D / d = 75 / 50 = 1.5$$

Fig. A-15-9: $K_t = 2.2$

Fig. A-15-8: $K_{ts} = 1.8$
Fig. 6-20: $q = 0.73$
Fig. 6-21: $q_s = 0.78$
Eq. (6-32): $K_f = 1 + q\left(K_t - 1\right) = 1 + 0.73(2.2 - 1) = 1.9$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.78(1.8 - 1) = 1.6$$

Examination of the stress concentration factors indicates the keyway will be the critical location.

Eq. (6-8): $S'_e = 0.5(440) = 220 \text{ MPa}$

Eq. (6-19): $k_a = 4.51(440)^{-0.265} = 0.899$

Eq. (6-20): $k_b = \left(\dfrac{50}{7.62}\right)^{-0.107} = 0.818$

$$k_c = k_d = k_e = 1$$

Eq. (6-18): $S_e = 0.899(0.818)(220) = 162 \text{ MPa}$

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations. Using Eq. (7-9) with $M_m = T_a = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.8)(4097)\right]^2} = 14\,750 \text{ N} \cdot \text{m}$$

$$B = \sqrt{3\left(K_{fs} T_m\right)^2} = \sqrt{3\left[(2.4)(3101)\right]^2} = 12\,890 \text{ N} \cdot \text{m}$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(14\,750)}{\pi\left(0.050^3\right)(162)\left(10^6\right)}\left\{1 + \left[1 + \left(\frac{2(12\,890)(162)\left(10^6\right)}{(14\,750)(440)\left(10^6\right)}\right)^2\right]^{1/2}\right\}$$

$n = 0.25$ Infinite life is not predicted. *Ans.*

Though not explicitly called for in the problem statement, a static check for yielding is especially warranted with such a low fatigue factor of safety. Using Eq. (7-15), with $M_m = T_a = 0$,

$$\sigma'_{\max} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

$$= \left[\left(\frac{32(1.8)(4097)}{\pi(0.050)^3}\right)^2 + 3\left(\frac{16(2.4)(3101)}{\pi(0.050)^3}\right)^2\right]^{1/2} = 7.98\left(10^8\right) \text{ Pa} = 798 \text{ MPa}$$

$$n = \frac{S_y}{\sigma'_{\max}} = \frac{370}{798} = 0.46$$

This indicates localized yielding is predicted at the stress-concentration. Even without the stress concentration effects, the static factor of safety turns out to be 0.93. Static failure is predicted, rendering this proposed shaft design unacceptable.

This problem is linked with several previous problems (see Table 1-1, p. 24) in which the shaft was considered to have a constant diameter of 50 mm. The results here are consistent with the previous problems, in which the 50 mm diameter was found to slightly undersized for static, and significantly undersized for fatigue. Though in the current problem much of the shaft has larger than 50 mm diameter, the added contribution of stress concentration limits the fatigue life.

7-22 For a shaft with significantly varying diameters over its length, we will choose to use shaft analysis software or finite element software to calculate the deflections. Entering the geometry from the shaft as defined in Prob. 7-21, and the loading as defined in Prob. 3-73, the following deflection magnitudes are determined:

Location	Slope (rad)	Deflection (mm)
Left bearing *O*	0.01445	0.000
Right bearing *C*	0.01843	0.000
Left Gear *A*	0.00358	3.761
Right Gear *B*	0.00366	3.676

Comparing these values to the recommended limits in Table 7-2, we find that they are all well out of the desired range. This is not unexpected since the stress analysis in Prob. 7-21 also indicated the shaft is undersized for infinite life. The transverse deflection at the left gear is the most excessive, so we will attempt to increase all diameters to bring it into compliance. Using Eq. (7-17) at the left gear, assuming from Table 7-2 an allowable deflection of $y_{\text{all}} = 0.01$ in = 0.254 mm,

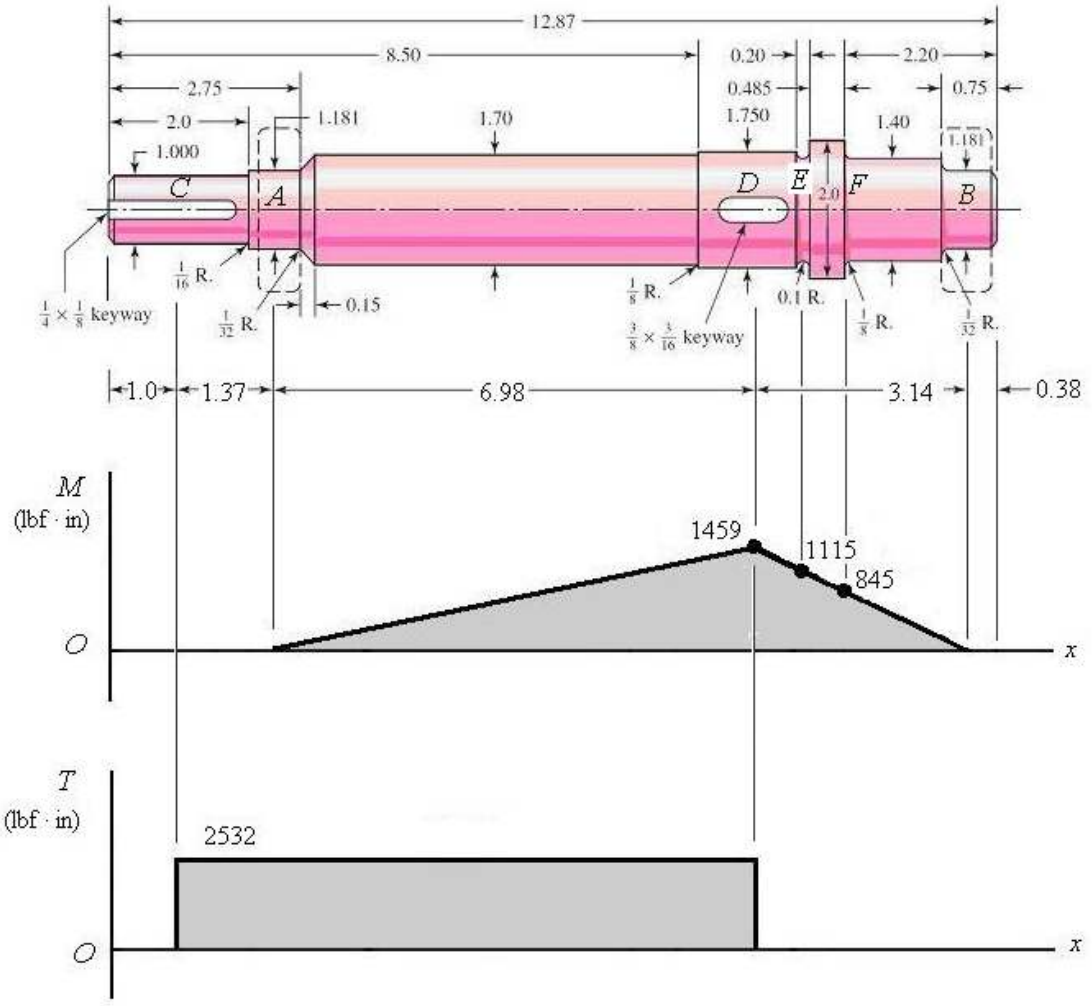
$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d y_{\text{old}}}{y_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(3.761)}{0.254} \right|^{1/4} = 1.96$$

Multiplying all diameters by 2, we obtain the following deflections:

Location	Slope (rad)	Deflection (mm)
Left bearing *O*	0.00090	0.000
Right bearing *C*	0.00115	0.000
Left Gear *A*	0.00022	0.235
Right Gear *B*	0.00023	0.230

This brings the deflection at the gears just within the limit for a spur gear (assuming $P <$ 10 teeth/in), and all other deflections well below the recommended limits.

7-23 **(a)** Label the approximate locations of the effective centers of the bearings as *A* and *B*, the fan as *C*, and the gear as *D*, with axial dimensions as shown. Since there is only one gear, we can combine the radial and tangential gear forces into a single resultant force with an accompanying torque, and handle the statics problem in a single plane. From statics, the resultant reactions at the bearings can be found to be $R_A = 209.9$ lbf and $R_B = 464.5$ lbf. The bending moment and torque diagrams are shown, with the maximum bending moment at *D* of $M_D = 209.9(6.98) = 1459$ lbf·in and a torque transmitted from *D* to *C* of $T = 633\ (8/2) = 2532$ lbf·in. Due to the shaft rotation, the bending stress on any stress element will be completely reversed, while the torsional stress will be steady. Since we do not have any information about the fan, we will ignore any axial load that it would introduce. It would not likely contribute much compared to the bending anyway.



Potentially critical locations are identified as follows:

- Keyway at *C*, where the torque is high, the diameter is small, and the keyway creates a stress concentration.

- Keyway at *D*, where the bending moment is maximum, the torque is high, and the keyway creates a stress concentration.
- Groove at *E*, where the diameter is smaller than at *D*, the bending moment is still high, and the groove creates a stress concentration. There is no torque here, though.
- Shoulder at *F*, where the diameter is smaller than at *D* or *E*, the bending moment is still moderate, and the shoulder creates a stress concentration. There is no torque here, though.
- The shoulder to the left of *D* can be eliminated since the change in diameter is very slight, so that the stress concentration will undoubtedly be much less than at *D*.

Table A-20: $S_{ut} = 68$ kpsi, $S_y = 57$ kpsi

Eq. (6-8): $S_e' = 0.5(68) = 34.0$ kpsi

Eq. (6-19): $k_a = 2.70(68)^{-0.265} = 0.883$

Keyway at C

Since there is only steady torsion here, only a static check needs to be performed. We'll use the maximum shear stress theory.

$$\tau = \frac{Tr}{J} = \frac{2532\left(1.00/2\right)}{\pi\left(1.00^4\right)/32} = 12.9 \text{ kpsi}$$

Eq. (5-3): $$n_y = \frac{S_y/2}{\tau} = \frac{57/2}{12.9} = 2.21$$

Keyway at D

Assuming $r/d = 0.02$ for typical end-milled keyway cutter (p. 373), with $d = 1.75$ in, $r = 0.02d = 0.035$ in.

Table 7-1: $K_t = 2.14$, $K_{ts} = 3.0$

Fig. 6-20: $q = 0.66$

Fig. 6-21: $q_s = 0.72$

Eq. (6-32): $$K_f = 1 + q\left(K_t - 1\right) = 1 + 0.66(2.14 - 1) = 1.8$$

$$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.72(3.0 - 1) = 2.4$$

Eq. (6-20): $$k_b = \left(\frac{1.75}{0.30}\right)^{-0.107} = 0.828$$

Eq. (6-18): $S_e = 0.883(0.828)(34.0) = 24.9$ kpsi

We will choose the DE-Gerber criteria since this is an analysis problem in which we would like to evaluate typical expectations.

Using Eq. (7-9) with $M_m = T_a = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.8)(1459)\right]^2}=5252 \text{ lbf}\cdot\text{in}=5.252 \text{ kip}\cdot\text{in}$$

$$B=\sqrt{3\left(K_{fs} T_m\right)^2}=\sqrt{3\left[(2.4)(2532)\right]^2}=10\,525 \text{ lbf}\cdot\text{in}=10.53 \text{ kip}\cdot\text{in}$$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(5.252)}{\pi\left(1.75^3\right)(24.9)}\left\{1+\left[1+\left(\frac{2(10.53)(24.9)}{(5.252)(68)}\right)^2\right]^{1/2}\right\}$$

$n = 3.59$ *Ans.*

Groove at E

We will assume Figs. A-15-14 is applicable since the 2 in diameter to the right of the groove is relatively narrow and will likely not allow the stress flow to fully develop. (See Fig.7-9 for the stress flow concept.)

$r / d = 0.1 / 1.55 = 0.065$, $D / d = 1.75 / 1.55 = 1.13$

Fig. A-15-14: $K_t = 2.1$

Fig. 6-20: $q = 0.76$

Eq. (6-32): $K_f = 1+q\left(K_t-1\right)=1+0.76(2.1-1)=1.8$

Eq. (6-20): $k_b=\left(\frac{1.55}{0.30}\right)^{-0.107}=0.839$

Eq. (6-18): $S_e = 0.883(0.839)(34) = 25.2$ kpsi

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A=\sqrt{4\left(K_f M_a\right)^2}=\sqrt{4\left[(1.8)(1115)\right]^2}=4122 \text{ lbf}\cdot\text{in}=4.122 \text{ kip}\cdot\text{in}$$

$B = 0$

$$\frac{1}{n}=\frac{8A}{\pi d^3 S_e}\left\{1+\left[1+\left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$=\frac{8(4.122)}{\pi\left(1.55^3\right)(25.2)}\left\{1+\left[1+(0)^2\right]^{1/2}\right\}$$

$n = 4.47$ *Ans.*

Shoulder at F

$r / d = 0.125 / 1.40 = 0.089$, $D / d = 2.0 / 1.40 = 1.43$

Fig. A-15-9: $K_t = 1.7$

Fig. 6-20: $q = 0.78$

Eq. (6-32): $K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = 1.5$

Eq. (6-20): $k_b = \left(\frac{1.40}{0.30}\right)^{-0.107} = 0.848$

Eq. (6-18): $S_e = 0.883(0.848)(34) = 25.5 \text{ kpsi}$

Using Eq. (7-9) with $M_m = T_a = T_m = 0$,

$$A = \sqrt{4\left(K_f M_a\right)^2} = \sqrt{4\left[(1.5)(845)\right]^2} = 2535 \text{ lbf} \cdot \text{in} = 2.535 \text{ kip} \cdot \text{in}$$

$$B = 0$$

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

$$= \frac{8(2.535)}{\pi\left(1.40^3\right)(25.5)}\left\{1 + \left[1 + (0)^2\right]^{1/2}\right\}$$

$n = 5.42$ *Ans.*

(b) The deflection will not be much affected by the details of fillet radii, grooves, and keyways, so these can be ignored. Also, the slight diameter changes, as well as the narrow 2.0 in diameter section, can be neglected. We will model the shaft with the following three sections:

Section	Diameter (in)	Length (in)
1	1.00	2.90
2	1.70	7.77
3	1.40	2.20

The deflection problem can readily (though tediously) be solved with singularity functions. For examples, see Ex. 4-7, p. 159, or the solution to Prob. 7-24. Alternatively, shaft analysis software or finite element software may be used. Using any of the methods, the results should be as follows:

Location	Slope (rad)	Deflection (in)
Left bearing *A*	0.000290	0.000000
Right bearing *B*	0.000400	0.000000
Fan *C*	0.000290	0.000404
Gear *D*	0.000146	0.000928

Comparing these values to the recommended limits in Table 7-2, we find that they are all within the recommended range.

7-24 Shaft analysis software or finite element software can be utilized if available. Here we will demonstrate how the problem can be simplified and solved using singularity functions.

Deflection: First we will ignore the steps near the bearings where the bending moments are low. Thus let the 30 mm dia. be 35 mm. Secondly, the 55 mm dia. is very thin, 10 mm. The full bending stresses will not develop at the outer fibers so full stiffness will not develop either. Thus, ignore this step and let the diameter be 45 mm.

Statics: Left support: $R_1 = 7(315-140)/315 = 3.889$ kN

Right support: $R_2 = 7(140)/315 = 3.111$ kN

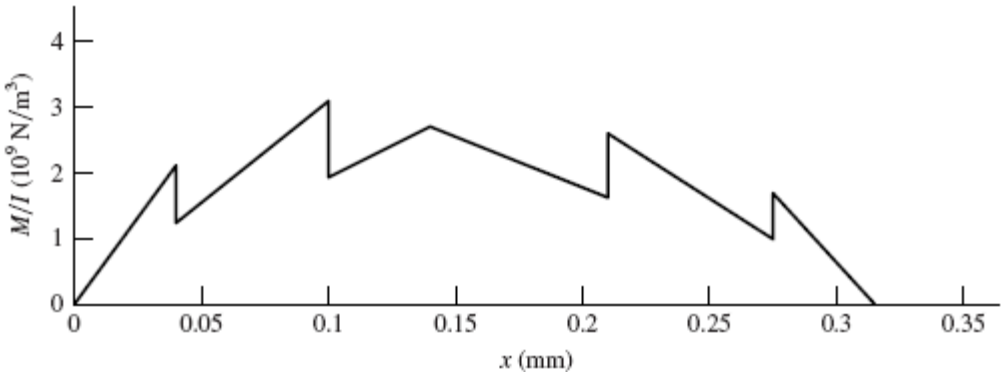
Determine the bending moment at each step.

x(mm)	0	40	100	140	210	275	315
M(N · m)	0	155.56	388.89	544.44	326.67	124.44	0

$I_{35} = (\pi/64)(0.035^4) = 7.366(10^{-8})$ m^4, $I_{40} = 1.257(10^{-7})$ m^4, $I_{45} = 2.013(10^{-7})$ m^4

Plot M/I as a function of x.

x(m)	M/I (10^9 N/m^3)	Step	Slope	ΔSlope
0	0		52.8	
0.04	2.112			
0.04	1.2375	–0.8745	30.942	–21.86
0.1	3.094			
0.1	1.932	–1.162	19.325	–11.617
0.14	2.705			
0.14	2.705	0	–15.457	–34.78
0.21	1.623			
0.21	2.6	0.977	-24.769	-9.312
0.275	0.99			
0.275	1.6894	0.6994	-42.235	-17.47
0.315	0			



The steps and the change of slopes are evaluated in the table. From these, the function *M/I* can be generated:

$$M\ /\ I = \Big[52.8x - 0.8745\langle x-0.04\rangle^0 - 21.86\langle x-0.04\rangle^1 - 1.162\langle x-0.1\rangle^0$$
$$-11.617\langle x-0.1\rangle^1 - 34.78\langle x-0.14\rangle^1 + 0.977\langle x-0.21\rangle^0$$
$$-9.312\langle x-0.21\rangle^1 + 0.6994\langle x-0.275\rangle^0 - 17.47\langle x-0.275\rangle^1\Big]10^9$$

Integrate twice:

$$E\frac{dy}{dx} = \Big[26.4x^2 - 0.8745\langle x-0.04\rangle^1 - 10.93\langle x-0.04\rangle^2 - 1.162\langle x-0.1\rangle^1$$
$$-5.81\langle x-0.1\rangle^2 - 17.39\langle x-0.14\rangle^2 + 0.977\langle x-0.21\rangle^1$$
$$-4.655\langle x-0.21\rangle^2 + 0.6994\langle x-0.275\rangle^1 - 8.735\langle x-0.275\rangle^2 + C_1\Big]10^9 \quad (1)$$
$$Ey = \Big[8.8x^3 - 0.4373\langle x-0.04\rangle^2 - 3.643\langle x-0.04\rangle^3 - 0.581\langle x-0.1\rangle^2$$
$$-1.937\langle x-0.1\rangle^3 - 5.797\langle x-0.14\rangle^3 + 0.4885\langle x-0.21\rangle^2$$
$$-1.552\langle x-0.21\rangle^3 + 0.3497\langle x-0.275\rangle^2 - 2.912\langle x-0.275\rangle^3 + C_1x + C_2\Big]10^9$$

Boundary conditions: $y = 0$ at $x = 0$ yields $C_2 = 0$;
$y = 0$ at $x = 0.315$ m yields $C_1 = -0.295\ 25$ N/m^2.

Equation (1) with $C_1 = -0.295\ 25$ provides the slopes at the bearings and gear. The following table gives the results in the second column. The third column gives the results from a similar finite element model. The fourth column gives the results of a full model which models the 35 and 55 mm diameter steps.

x (mm)	θ (rad)	F.E. Model	Full F.E. Model
0	–0.001 4260	–0.001 4270	–0.001 4160
140	–0.000 1466	–0.000 1467	–0.000 1646
315	0.001 3120	0.001 3280	0.001 3150

The main discrepancy between the results is at the gear location (x = 140 mm). The larger value in the full model is caused by the stiffer 55 mm diameter step. As was stated earlier, this step is not as stiff as modeling implicates, so the exact answer is somewhere between the full model and the simplified model which in any event is a small value. As expected, modeling the 30 mm dia. as 35 mm does not affect the results much.

It can be seen that the allowable slopes at the bearings are exceeded. Thus, either the load has to be reduced or the shaft "beefed" up. If the allowable slope is 0.001 rad, then the maximum load should be $F_{\max}$ = (0.001/0.001 426)7 = 4.91 kN. With a design factor this would be reduced further.

To increase the stiffness of the shaft, apply Eq. (7-18) to the most offending deflection (at x = 0) to determine a multiplier to be used for all diameters.

$$\frac{d_{\text{new}}}{d_{\text{old}}} = \left| \frac{n_d \left(dy/dx \right)_{\text{old}}}{\left(\text{slope} \right)_{\text{all}}} \right|^{1/4} = \left| \frac{(1)(0.0014260)}{0.001} \right|^{1/4} = 1.093$$

Form a table:

Old d, mm	20.00	30.00	35.00	40.00	45.00	55.00
New ideal d, mm	21.86	32.79	38.26	43.72	49.19	60.12
Rounded up d, mm	22.00	34.00	40.00	44.00	50.00	62.00

Repeating the full finite element model results in

$x = 0$: $\theta = -9.30 \times 10^{-4}$ rad
x = 140 mm: $\theta = -1.09 \times 10^{-4}$ rad
x = 315 mm: $\theta = 8.65 \times 10^{-4}$ rad

This is well within our goal. Have the students try a goal of 0.0005 rad at the gears.

Strength: Due to stress concentrations and reduced shaft diameters, there are a number of locations to look at. A table of nominal stresses is given below. Note that torsion is only to the right of the 7 kN load. Using $\sigma = 32M/(\pi d^3)$ and $\tau = 16T/(\pi d^3)$,

x (mm)	0	15	40	100	110	140	210	275	300	330
σ (MPa)	0	22.0	37.0	61.9	47.8	60.9	52.0	39.6	17.6	0
τ (MPa)	0	0	0	0	0	6	8.5	12.7	20.2	68.1
σ'(MPa)	0	22.0	37.0	61.9	47.8	61.8	53.1	45.3	39.2	118.0

Table A-20 for AISI 1020 CD steel: S_{ut} = 470 MPa, S_y = 390 MPa

At x = 210 mm:
Eq. (6-19): $k_a = 4.51(470)^{-0.265} = 0.883$

Eq. (6-20): $k_b = (40 / 7.62)^{-0.107} = 0.837$

Eq. (6-18): $S_e = 0.883\ (0.837)(0.5)(470) = 174$ MPa

$D / d = 45 / 40 = 1.125, \quad r / d = 2 / 40 = 0.05$

Fig. A-15-8: $K_{ts} = 1.4$

Fig. A-15-9: $K_t = 1.9$

Fig. 6-20: $q = 0.75$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_f = 1 + 0.75(1.9 - 1) = 1.68$

$K_{fs} = 1 + 0.79(1.4 - 1) = 1.32$

Choosing DE-ASME Elliptic to inherently include the yield check, from Eq. (7-11), with $M_m = T_a = 0$,

$$\frac{1}{n} = \frac{16}{\pi\left(0.04^3\right)}\left\{4\left[\frac{1.68(326.67)}{174\left(10^6\right)}\right]^2 + 3\left[\frac{1.32(107)}{390\left(10^6\right)}\right]^2\right\}^{1/2}$$

$$n = 1.98$$

At $x = 330$ mm:

The von Mises stress is the highest but it comes from the steady torque only.

$D / d = 30 / 20 = 1.5, \quad r / d = 2 / 20 = 0.1$

Fig. A-15-9: $K_{ts} = 1.42$

Fig. 6-21: $q_s = 0.79$

Eq. (6-32): $K_{fs} = 1 + 0.79(1.42 - 1) = 1.33$

Eq. (7-11):

$$\frac{1}{n} = \frac{16}{\pi\left(0.02^3\right)}\left(\sqrt{3}\right)\left[\frac{1.33(107)}{390\left(10^6\right)}\right]$$

$$n = 2.49$$

Note that since there is only a steady torque, Eq. (7-11) reduces to essentially the equivalent of the distortion energy failure theory.

Check the other locations.

If worse-case is at $x = 210$ mm, the changes discussed for the slope criterion will improve the strength issue.

7-25 and 7-26 With these design tasks each student will travel different paths and almost all details will differ. The important points are

- The student gets a blank piece of paper, a statement of function, and some constraints – explicit and implied. At this point in the course, this is a good experience.
- It is a good preparation for the capstone design course.

- The adequacy of their design must be demonstrated and possibly include a designer's notebook.
- Many of the fundaments of the course, based on this text and this course, are useful. The student will find them useful and notice that he/she is doing it.
- Don't let the students create a time sink for themselves. Tell them how far you want them to go.

7-27 This task was once given as a final exam problem. This problem is a learning experience. Following the task statement, the following guidance was added.

- Take the first half hour, resisting the temptation of putting pencil to paper, and decide what the problem really is.
- Take another twenty minutes to list several possible remedies.
- Pick one, and show your instructor how you would implement it.

The students' initial reaction is that he/she does not know much from the problem statement. Then, slowly the realization sets in that they do know some important things that the designer did not. They knew how it failed, where it failed, and that the design wasn't good enough; it was close, though.

Also, a fix at the bearing seat lead-in could transfer the problem to the shoulder fillet, and the problem may not be solved.

To many students' credit, they chose to keep the shaft geometry, and selected a new material to realize about twice the Brinell hardness.

7-28 In Eq. (7-22) set

$$I = \frac{\pi d^4}{64}, \quad A = \frac{\pi d^2}{4}$$

to obtain

$$\omega = \left(\frac{\pi}{l}\right)^2 \left(\frac{d}{4}\right) \sqrt{\frac{gE}{\gamma}} \qquad (1)$$

or

$$d = \frac{4l^2\omega}{\pi^2} \sqrt{\frac{\gamma}{gE}} \qquad (2)$$

(a) From Eq. (1) and Table A-5

$$\omega = \left(\frac{\pi}{0.6}\right)^2 \left(\frac{0.025}{4}\right) \sqrt{\frac{9.81(207)(10^9)}{76.5\left(10^3\right)}} = 883 \text{ rad/s} \quad \textit{Ans.}$$

(b) From Eq. (1), we observe that the critical speed is linearly proportional to the diameter. Thus, to double the critical speed, we should double the diameter to $d = 50$ mm. *Ans.*

(c) From Eq. (2),

$$l\omega = \frac{\pi^2}{4}\frac{d}{l}\sqrt{\frac{gE}{\gamma}}$$

Since d / l is the same regardless of the scale,

$$l\omega = \text{constant} = 0.6(883) = 529.8$$
$$\omega = \frac{529.8}{0.3} = 1766 \text{ rad/s} \quad \textit{Ans.}$$

Thus the first critical speed doubles.

7-29 From Prob. 7-28, $\omega = 883$ rad/s

$$A = 4.909\left(10^{-4}\right) \text{ m}^2, \quad I = 1.917\left(10^{-8}\right) \text{m}^4, \quad \gamma = 7.65\left(10^4\right) \text{ N/m}^3$$
$$E = 207(10^9) \text{ Pa}, \quad w = A\gamma l = 4.909\left(10^{-4}\right)7.65\left(10^4\right)(0.6) = 22.53 \text{ N}$$

One element:

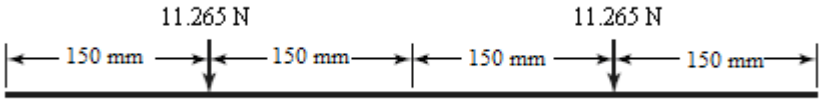
Eq. (7-24):

$$\delta_{11} = \frac{0.3(0.3)\left(0.6^2 - 0.3^2 - 0.3^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 1.134\left(10^{-6}\right) \text{ m/N}$$

$$y_1 = w_1\delta_{11} = 22.53(1.134)\left(10^{-6}\right) = 2.555\left(10^{-5}\right) \text{ m}$$
$$y_1^2 = 6.528\left(10^{-10}\right)$$
$$\Sigma wy = 22.53(2.555)\left(10^{-5}\right) = 5.756\left(10^{-4}\right)$$
$$\Sigma wy^2 = 22.53(6.528)\left(10^{-10}\right) = 1.471\left(10^{-8}\right)$$

$$\omega_1 = \sqrt{g\frac{\Sigma wy}{\Sigma wy^2}} = \sqrt{9.81\frac{5.756\left(10^{-4}\right)}{1.471\left(10^{-8}\right)}} = 620 \text{ rad/s} \quad (30\% \text{ low})$$

Two elements:



$$\delta_{11} = \delta_{22} = \frac{0.45(0.15)\left(0.6^2 - 0.45^2 - 0.15^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 6.379\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{12} = \delta_{21} = \frac{0.15(0.15)(0.6^2 - 0.15^2 - 0.15^2)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 4.961\left(10^{-7}\right) \text{ m/N}$$

$$y_1 = y_2 = w_1\delta_{11} + w_2\delta_{12} = 11.265(6.379)\left(10^{-7}\right) + 11.265(4.961)\left(10^{-7}\right) = 1.277\left(10^{-5}\right) \text{ m}$$

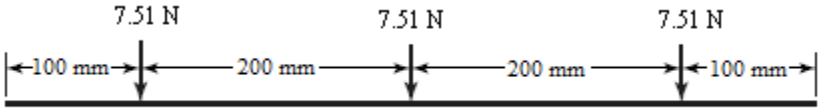
$$y_1^2 = y_2^2 = 1.632\left(10^{-10}\right)$$

$$\Sigma wy = 2(11.265)(1.277)\left(10^{-5}\right) = 2.877\left(10^{-4}\right)$$

$$\Sigma wy^2 = 2(11.265)(1.632)\left(10^{-10}\right) = 3.677\left(10^{-9}\right)$$

$$\omega_1 = \sqrt{9.81\left[\frac{2.877\left(10^{-4}\right)}{3.677\left(10^{-9}\right)}\right]} = 876 \text{ rad/s} \qquad (0.8\% \text{ low})$$

Three elements:



$$\delta_{11} = \delta_{33} = \frac{0.5(0.1)\left(0.6^2 - 0.5^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 3.500\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{22} = \frac{0.3(0.3)\left(0.6^2 - 0.3^2 - 0.3^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 1.134\left(10^{-6}\right) \text{ m/N}$$

$$\delta_{12} = \delta_{32} = \frac{0.3(0.1)\left(0.6^2 - 0.3^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 5.460\left(10^{-7}\right) \text{ m/N}$$

$$\delta_{13} = \frac{0.1(0.1)\left(0.6^2 - 0.1^2 - 0.1^2\right)}{6(207)\left(10^9\right)(1.917)\left(10^{-8}\right)(0.6)} = 2.380\left(10^{-7}\right) \text{ m/N}$$

$$y_1 = 7.51\left[3.500\left(10^{-7}\right) + 5.460\left(10^{-7}\right) + 2.380\left(10^{-7}\right)\right] = 8.516\left(10^{-6}\right)$$

$$y_2 = 7.51\left[5.460\left(10^{-7}\right) + 1.134\left(10^{-6}\right) + 5.460\left(10^{-7}\right)\right] = 1.672\left(10^{-5}\right)$$

$$y_3 = 7.51\left[2.380\left(10^{-7}\right) + 5.460\left(10^{-7}\right) + 3.500\left(10^{-7}\right)\right] = 8.516\left(10^{-6}\right)$$

$$\Sigma wy = 7.51\left[8.516\left(10^{-6}\right) + 1.672\left(10^{-5}\right) + 8.516\left(10^{-6}\right)\right] = 2.535\left(10^{-4}\right)$$

$$\Sigma wy^2 = 7.51\left\{\left[8.516\left(10^{-6}\right)\right]^2 + \left[1.672\left(10^{-5}\right)\right]^2 + \left[8.516\left(10^{-6}\right)\right]^2\right\} = 3.189\left(10^{-9}\right)$$

$$\omega_1 = \sqrt{9.81\left[\frac{2.535\left(10^{-4}\right)}{3.189\left(10^{-9}\right)}\right]} = 883 \text{ rad/s}$$

The result is the same as in Prob. 7-28. The point was to show that convergence is rapid using a static deflection beam equation. The method works because:

- If a deflection curve is chosen which meets the boundary conditions of moment-free and deflection-free ends, as in this problem, the strain energy is not very sensitive to the equation used.
- Since the static bending equation is available, and meets the moment-free and deflection-free ends, it works.

7-30 **(a)** For two bodies, Eq. (7-26) is

$$\begin{vmatrix} (m_1\delta_{11} - 1/\omega^2) & m_2\delta_{12} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) \end{vmatrix} = 0$$

Expanding the determinant yields,

$$\left(\frac{1}{\omega^2}\right)^2 - (m_1\delta_{11} + m_2\delta_{22})\left(\frac{1}{\omega_1^2}\right) + m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) = 0 \qquad (1)$$

Eq. (1) has two roots $1/\omega_1^2$ and $1/\omega_2^2$. Thus

$$\left(\frac{1}{\omega^2} - \frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega^2} - \frac{1}{\omega_2^2}\right) = 0$$

or,

$$\left(\frac{1}{\omega^2}\right)^2 + \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\left(\frac{1}{\omega}\right)^2 + \left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = 0 \qquad (2)$$

Equate the third terms of Eqs. (1) and (2), which must be identical.

$$\frac{1}{\omega_1^2}\frac{1}{\omega_2^2} = m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21}) \quad \Rightarrow \quad \frac{1}{\omega_2^2} = \omega_1^2 m_1m_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})$$

and it follows that

$$\omega_2 = \frac{1}{\omega_1}\sqrt{\frac{g^2}{w_1 w_2(\delta_{11}\delta_{22} - \delta_{12}\delta_{21})}} \qquad \textit{Ans.}$$

(b) In Ex. 7-5, part (*b*), the first critical speed of the two-disk shaft (w_1 = 35 lbf, w_2 = 55 lbf) is ω_1 = 124.8 rad/s. From part (a), using influence coefficients,

$$\omega_2 = \frac{1}{124.8}\sqrt{\frac{386^2}{35(55)\left[2.061(3.534) - 2.234^2\right]\left(10^{-8}\right)}} = 466 \text{ rad/s} \qquad \textit{Ans.}$$

7-31 In Eq. (7-22), for ω_1, the term $\sqrt{I/A}$ appears. For a hollow uniform diameter shaft,

$$\omega_1 \propto \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi\left(d_o^4 - d_i^4\right)/64}{\pi\left(d_o^2 - d_i^2\right)/4}} = \sqrt{\frac{1}{16}\frac{\left(d_o^2 + d_i^2\right)\left(d_o^2 - d_i^2\right)}{d_o^2 - d_i^2}} = \frac{1}{4}\sqrt{d_o^2 + d_i^2}$$

This means that when a solid shaft is hollowed out, the critical speed increases beyond that of the solid shaft of the same size. By how much?

$$\frac{(1/4)\sqrt{d_o^2 + d_i^2}}{(1/4)\sqrt{d_o^2}} = \sqrt{1 + \left(\frac{d_i}{d_o}\right)^2}$$

The possible values of d_i are $0 \le d_i \le d_o$, so the range of the critical speeds is

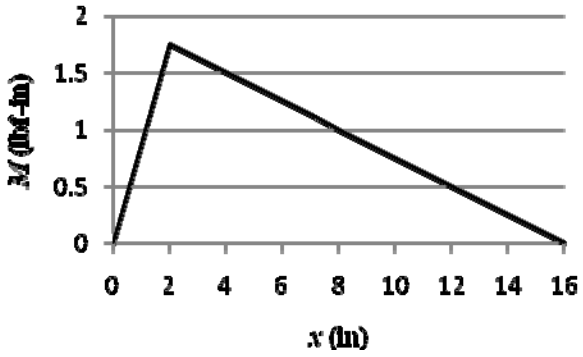
$\omega_1\sqrt{1+0}$ to about $\omega_1\sqrt{1+1}$

or from ω_1 to $\sqrt{2}\,\omega_1$. *Ans.*

7-32 All steps will be modeled using singularity functions with a spreadsheet. Programming both loads will enable the user to first set the left load to 1, the right load to 0 and calculate δ_{11} and δ_{21}. Then set the left load to 0 and the right to 1 to get δ_{12} and δ_{22}. The spreadsheet shows the δ_{11} and δ_{21} calculation. A table for M/I vs. x is easy to make. First, draw the bending-moment diagram as shown with the data.

x	0	1	2	3	4	5	6	7	8
M	0	0.875	1.75	1.625	1.5	1.375	1.25	1.125	1

x	9	10	11	12	13	14	15	16	
M	0.875	0.75	0.625	0.5	0.375	0.25	0.125	0	



The second-area moments are:

$$0 \le x \le 1 \text{ in and } 15 \le x \le 16 \text{ in}, \quad I_1 = \pi\left(2^4\right)/64 = 0.7854 \text{ in}^4$$

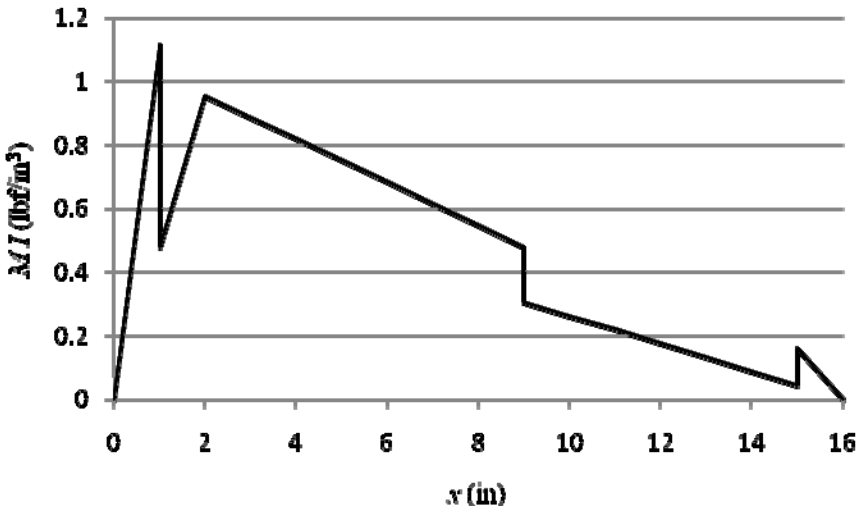
$$1 \le x \le 9 \text{ in}, \quad I_2 = \pi\left(2.472^4\right)/64 = 1.833 \text{ in}^4$$

$$9 \le x \le 15 \text{ in}, \quad I_3 = \pi\left(2.763^4\right)/64 = 2.861 \text{ in}^4$$

Divide *M* by *I* at the key points *x* = 0, 1, 2, 9, 14, 15, and 16 in and plot

x	0	1	1	2	2	3	4	5	6	7	8
M/I	0	1.1141	0.4774	0.9547	0.9547	0.8865	0.8183	0.7501	0.6819	0.6137	0.5456

x	9	9	10	11	12	13	14	14	15	15	16
M/I	0.4774	0.3058	0.2621	0.2185	0.1748	0.1311	0.0874	0.0874	0.0437	0.1592	0



From this diagram, one can see where changes in value (steps) and slope occur. Using a spreadsheet, one can form a table of these changes. An example of a step is, at *x* = 1 in, *M/I* goes from 0.875/0.7854 = 1.1141 lbf/in^3 to 0.875/1.833 = 0.4774 lbf/in^3, a step change of 0.4774 – 1.1141 = – 0.6367 lbf/in^3. A slope change also occurs at at *x* = 1 in.

The slope for $0 \le x \le 1$ in is 1.1141/1 = 1.1141 lbf/in^2, which changes to (0.9547 – 0.4774)/1 = 0.4774 lbf/in^2, a change of 0.4774 – 1.1141 = – 0.6367 lbf/in^2. Following this approach, a table is made of all the changes. The table shown indicates the column letters and row numbers for the spreadsheet.

	A	B	C	D	E	F
1	*x*	*M*	*M/I*	step	Slope	Δ Slope
2	1a	0.875	1.114085	0.000000	1.114085	0.000000
3	1b	0.875	0.477358	-0.636727	0.477358	-0.636727
4	2	1.75	0.954716	0.000000	0.477358	0.000000
5	2	1.75	0.954716	0.000000	-0.068194	-0.545552
6	9a	0.875	0.477358	0.000000	-0.068194	0.000000
7	9b	0.875	0.305854	-0.171504	-0.043693	0.024501
8	14	0.25	0.087387	0.000000	-0.043693	0.000000
9	14	0.25	0.087387	0.000000	-0.043693	0.000000
10	15a	0.125	0.043693	0.000000	-0.043693	0.000000
11	15b	0.125	0.159155	0.115461	-0.159155	-0.115461
12	16	0	0.000000	0.000000	-0.159155	0.000000

The equation for *M / I* in terms of the spreadsheet cell locations is:

$$M / I = \mathrm{E2}\ (x) + \mathrm{D3}\ \langle x-1\rangle^0 + \mathrm{F3}\langle x-1\rangle^1 + \mathrm{F5}\langle x-2\rangle^1$$
$$+\mathrm{D7}\ \langle x-9\rangle^0 + \mathrm{F7}\ \langle x-9\rangle^1 + \mathrm{D11}\ \langle x-15\rangle^0 + \mathrm{F11}\ \langle x-15\rangle^1$$

Integrating twice gives the equation for *Ey*. Assume the shaft is steel. Boundary conditions $y = 0$ at $x = 0$ and at $x = 16$ inches provide integration constants ($C_1 = -4.906$ lbf/in and $C_2 = 0$). Substitution back into the deflection equation at $x = 2$ and 14 in provides the δ's. The results are: $\delta_{11} = 2.917(10^{-7})$ and $\delta_{12} = 1.627(10^{-7})$. Repeat for $F_1 = 0$ and $F_2 = 1$, resulting in $\delta_{21} = 1.627(10^{-7})$ and $\delta_{22} = 2.231(10^{-7})$. This can be verified by finite element analysis.

$$y_1 = 18(2.917)(10^{-7}) + 32(1.627)(10^{-7}) = 1.046(10^{-5})$$
$$y_2 = 18(1.627)(10^{-7}) + 32(2.231)(10^{-7}) = 1.007(10^{-5})$$
$$y_1^2 = 1.093(10^{-10}), \quad y_2^2 = 1.014(10^{-10})$$
$$\sum wy = 5.105(10^{-4}), \quad \sum wy^2 = 5.212(10^{-9})$$

Neglecting the shaft, Eq. (7-23) gives

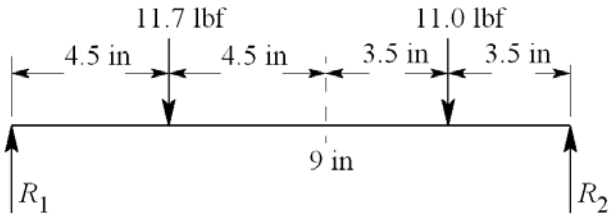
$$\omega_1 = \sqrt{386\frac{5.105(10^{-4})}{5.212(10^{-9})}} = 6149 \text{ rad/s} \quad \text{or} \quad 58\,720 \text{ rev/min} \quad \textit{Ans.}$$

Without the loads, we will model the shaft using 2 elements, one between $0 \le x \le 9$ in, and one between $0 \le x \le 16$ in. As an approximation, we will place their weights at $x = 9/2 = 4.5$ in, and $x = 9 + (16 - 9)/2 = 12.5$ in. From Table A-5, the weight density of steel is $\gamma = 0.282$ lbf/in^3. The weight of the left element is

$$w_1 = \gamma \frac{\pi}{4} \sum d^2 l = 0.282 \left(\frac{\pi}{4} \right) \left[2^2 (1) + 2.472^2 (8) \right] = 11.7 \text{ lbf}$$

The right element is

$$w_2 == 0.282 \left(\frac{\pi}{4} \right) \left[2.763^2 (6) + 2^2 (1) \right] = 11.0 \text{ lbf}$$



The spreadsheet can be easily modified to give

$$\delta_{11} = 9.605(10^{-7}), \quad \delta_{12} = \delta_{21} = 5.718(10^{-7}), \delta_{22} = 5.472(10^{-7})$$
$$y_1 = 1.753(10^{-5}), \quad y_2 = 1.271(10^{-5})$$
$$y_1^2 = 3.072(10^{-10}), \quad y_2^2 = 1.615(10^{-10})$$
$$\sum wy = 3.449(10^{-4}), \quad \sum wy^2 = 5.371(10^{-9})$$
$$\omega_1 = \sqrt{386 \left[\frac{3.449(10^{-4})}{5.371(10^{-9})} \right]} = 4980 \text{ rad/s}$$

A finite element model of the exact shaft gives $\omega_1 = 5340$ rad/s. The simple model is 6.8% low.

Combination: Using Dunkerley's equation, Eq. (7-32):

$$\frac{1}{\omega_1^2} \doteq \frac{1}{6149^2} + \frac{1}{4980^2} \quad \Rightarrow \quad \omega_1 \doteq 3870 \text{ rad/s} \quad \textit{Ans.}$$

7-33 We must not let the basis of the stress concentration factor, as presented, impose a viewpoint on the designer. Table A-16 shows K_{ts} as a decreasing monotonic as a function of a/D. All is not what it seems. Let us change the basis for data presentation to the full section rather than the net section.

$$\tau = K_{ts} \tau_0 = K'_{ts} \tau'_0$$

$$K_{ts} = \frac{32T}{\pi A D^3} = K'_{ts}\left(\frac{32T}{\pi D^3}\right)$$

Therefore

$$K'_{ts} = \frac{K_{ts}}{A}$$

Form a table:

(a/D)	A	K_{ts}	K'_{ts}
0.050	0.95	1.77	1.86
0.075	0.93	1.71	1.84
0.100	0.92	1.68	1.83 ← minimum
0.125	0.89	1.64	1.84
0.150	0.87	1.62	1.86
0.175	0.85	1.60	1.88
0.200	0.83	1.58	1.90

K'_{ts} has the following attributes:

- It exhibits a minimum;
- It changes little over a wide range;
- Its minimum is a stationary point minimum at $a / D \doteq 0.100$;
- Our knowledge of the minima location is

$$0.075 \le (a / D) \le 0.125$$

We can form a design rule: In torsion, the pin diameter should be about 1/10 of the shaft diameter, for greatest shaft capacity. However, it is not catastrophic if one forgets the rule.

7-34 From the solution to Prob. 3-72, the torque to be transmitted through the key from the gear to the shaft is $T = 2819$ lbf·in. From Prob. 7-19, the nominal shaft diameter supporting the gear is 1.00 in. From Table 7-6, a 0.25 in square key is appropriate for a 1.00 in shaft diameter. The force applied to the key is

$$F = \frac{T}{r} = \frac{2819}{1.00/2} = 5638 \text{ lbf}$$

Selecting 1020 CD steel for the key, with $S_y = 57$ kpsi, and using the distortion-energy theory, $S_{sy} = 0.577\, S_y = (0.577)(57) = 32.9$ kpsi.

Failure by shear across the key:

$$\tau = \frac{F}{A} = \frac{F}{tl}$$

$$n = \frac{S_{sy}}{\tau} = \frac{S_{sy}}{F/tl} \quad \Rightarrow \quad l = \frac{nF}{tS_{sy}} = \frac{1.1(5638)}{0.25(32\,900)} = 0.754 \text{ in}$$

Failure by crushing:

$$\sigma = \frac{F}{A} = \frac{F}{(t/2)l} \quad n = \frac{S_y}{\sigma} = \frac{S_y}{2F/(tl)} \quad \Rightarrow \quad l = \frac{2Fn}{tS_y} = \frac{2(5638)(1.1)}{0.25(57)(10^3)} = 0.870 \text{ in}$$

Select ¼-in square key, 7/8 in long, 1020 CD steel. *Ans.*

7-35 From the solution to Prob. 3-73, the torque to be transmitted through the key from the gear to the shaft is $T = 3101$ N·m. From Prob. 7-21, the nominal shaft diameter supporting the gear is 50 mm. To determine an appropriate key size for the shaft diameter, we can either convert to inches and use Table 7-6, or we can look up standard metric key sizes from the internet or a machine design handbook. It turns out that the recommended metric key for a 50 mm shaft is 14 x 9 mm. Since the problem statement specifies a square key, we will use a 14 x 14 mm key. For comparison, using Table 7-6 as a guide, for $d = 50$ mm = 1.97 in, a 0.5 in square key is appropriate. This is equivalent to 12.7 mm. A 14 x 14 mm size is conservative, but reasonable after rounding up to standard sizes.

The force applied to the key is

$$F = \frac{T}{r} = \frac{3101}{0.050/2} = 124(10^3) \text{ N}$$

Selecting 1020 CD steel for the key, with $S_y = 390$ MPa, and using the distortion-energy theory, $S_{sy} = 0.577\, S_y = 0.577(390) = 225$ MPa.

Failure by shear across the key:

$$\tau = \frac{F}{A} = \frac{F}{tl}$$

$$n = \frac{S_{sy}}{\tau} = \frac{S_{sy}}{F/(tl)} \quad \Rightarrow \quad l = \frac{nF}{tS_{sy}} = \frac{1.1(124)(10^3)}{(0.014)(225)(10^6)} = 0.0433 \text{ m} = 43.3 \text{ mm}$$

Failure by crushing:

$$\sigma = \frac{F}{A} = \frac{F}{(t/2)l}$$

$$n = \frac{S_y}{\sigma} = \frac{S_y}{2F/(tl)} \quad \Rightarrow \quad l = \frac{2Fn}{tS_y} = \frac{2(124)(10^3)(1.1)}{(0.014)(390)(10^6)} = 0.0500 \text{ m} = 50.0 \text{ mm}$$

Select 14 mm square key, 50 mm long, 1020 CD steel. *Ans.*

7-36 Choose basic size $D = d = 15$ mm. From Table 7-9, a locational clearance fit is designated as 15H7/h6. From Table A-11, the tolerance grades are $\Delta D = 0.018$ mm and $\Delta d = 0.011$ mm. From Table A-12, the fundamental deviation is $\delta_F = 0$ mm.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 15 + 0.018 = 15.018$ mm *Ans.*

$D_{min} = D = 15.000$ mm *Ans.*

Shaft:

Eq. (7-37): $d_{max} = d + \delta_F = 15.000 + 0 = 15.000$ mm *Ans.*

$d_{min} = d + \delta_F - \Delta d = 15.000 + 0 - 0.011 = 14.989$ mm *Ans.*

7-37 Choose basic size $D = d = 1.75$ in. From Table 7-9, a medium drive fit is designated as H7/s6. From Table A-13, the tolerance grades are $\Delta D = 0.0010$ in and $\Delta d = 0.0006$ in. From Table A-14, the fundamental deviation is $\delta_F = 0.0017$ in.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 1.75 + 0.0010 = 1.7510$ in *Ans.*

$D_{min} = D = 1.7500$ in *Ans.*

Shaft:

Eq. (7-38): $d_{min} = d + \delta_F = 1.75 + 0.0017 = 1.7517$ in *Ans.*

$d_{max} = d + \delta_F + \Delta d = 1.75 + 0.0017 + 0.0006 = 1.7523$ in *Ans.*

7-38 Choose basic size $D = d = 45$ mm. From Table 7-9, a sliding fit is designated as H7/g6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm. From Table A-12, the fundamental deviation is $\delta_F = -0.009$ mm.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 45 + 0.025 = 45.025$ mm *Ans.*

$D_{min} = D = 45.000$ mm *Ans.*

Shaft:

Eq. (7-37): $d_{max} = d + \delta_F = 45.000 + (-0.009) = 44.991$ mm *Ans.*

$d_{min} = d + \delta_F - \Delta d = 45.000 + (-0.009) - 0.016 = 44.975$ mm *Ans.*

7-39 Choose basic size $D = d = 1.250$ in. From Table 7-9, a close running fit is designated as H8/f7. From Table A-13, the tolerance grades are $\Delta D = 0.0015$ in and $\Delta d = 0.0010$ in. From Table A-14, the fundamental deviation is $\delta_F = -0.0010$ in.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 1.250 + 0.0015 = 1.2515$ in *Ans.*

$D_{min} = D = 1.2500$ in *Ans.*

Shaft:

Eq. (7-37): $d_{max} = d + \delta_F = 1.250 + (-0.0010) = 1.2490$ in *Ans.*

$d_{min} = d + \delta_F - \Delta d = 1.250 + (-0.0010) - 0.0010 = 1.2480$ in *Ans.*

7-40 Choose basic size $D = d = 35$ mm. From Table 7-9, a locational interference fit is designated as H7/p6. From Table A-11, the tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm. From Table A-12, the fundamental deviation is $\delta_F = 0.026$ mm.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 35 + 0.025 = 35.025$ mm

$D_{min} = D = 35.000$ mm

The bearing bore specifications are within the hole specifications for a locational interference fit. Now find the necessary shaft sizes.

Shaft:

Eq. (7-38): $d_{min} = d + \delta_F = 35 + 0.026 = 35.026$ mm *Ans.*

$d_{max} = d + \delta_F + \Delta d = 35 + 0.026 + 0.016 = 35.042$ mm *Ans.*

7-41 Choose basic size $D = d = 1.5$ in. From Table 7-9, a locational interference fit is designated as H7/p6. From Table A-13, the tolerance grades are $\Delta D = 0.0010$ in and $\Delta d = 0.0006$ in. From Table A-14, the fundamental deviation is $\delta_F = 0.0010$ in.

Hole:

Eq. (7-36): $D_{max} = D + \Delta D = 1.5000 + 0.0010 = 1.5010$ in

$D_{min} = D = 1.5000$ in

The bearing bore specifications exactly match the requirements for a locational interference fit. Now check the shaft.

Shaft:

Eq. (7-38): $d_{min} = d + \delta_F = 1.5000 + 0.0010 = 1.5010$ in

$d_{max} = d + \delta_F + \Delta d = 1.5000 + 0.0010 + 0.0006 = 1.5016$ in

The shaft diameter of 1.5020 in is greater than the maximum allowable diameter of 1.5016 in, and therefore does not meet the specifications for the locational interference fit. *Ans.*

7-42 **(a)** Basic size is $D = d = 35$ mm.

Table 7-9: H7/s6 is specified for medium drive fit.

Table A-11: Tolerance grades are $\Delta D = 0.025$ mm and $\Delta d = 0.016$ mm.

Table A-12: Fundamental deviation is $\delta_F = +0.043$ mm.

Eq. (7-36): $D_{\max} = D + \Delta D = 35 + 0.025 = 35.025$ mm

$D_{\min} = D = 35.000$ mm

Eq. (7-38): $d_{\min} = d + \delta_F = 35 + 0.043 = 35.043$ mm *Ans.*

$d_{\max} = d + \delta_F + \Delta d = 35 + 0.043 + 0.016 = 35.059$ mm *Ans.*

(b)

Eq. (7-42): $$\delta_{\min} = d_{\min} - D_{\max} = 35.043 - 35.025 = 0.018 \text{ mm}$$

Eq. (7-43): $$\delta_{\max} = d_{\max} - D_{\min} = 35.059 - 35.000 = 0.059 \text{ mm}$$

Eq. (7-40): $$p_{\max} = \frac{E\delta_{\max}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{207\left(10^9\right)(0.059)}{2\left(35^3\right)}\left[\frac{\left(60^2 - 35^2\right)\left(35^2 - 0\right)}{60^2 - 0}\right] = 115 \text{ MPa} \quad \textit{Ans.}$$

$$p_{\min} = \frac{E\delta_{\min}}{2d^3}\left[\frac{\left(d_o^2 - d^2\right)\left(d^2 - d_i^2\right)}{d_o^2 - d_i^2}\right]$$

$$= \frac{207\left(10^9\right)(0.018)}{2\left(35^3\right)}\left[\frac{\left(60^2 - 35^2\right)\left(35^2 - 0\right)}{60^2 - 0}\right] = 35.1 \text{ MPa} \quad \textit{Ans.}$$

(c) For the shaft:

Eq. (7-44): $$\sigma_{t,\text{shaft}} = -p = -115 \text{ MPa}$$

Eq. (7-46): $$\sigma_{r,\text{shaft}} = -p = -115 \text{ MPa}$$

Eq. (5-13): $$\sigma' = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)^{1/2}$$

$$= \left[(-115)^2 - (-115)(-115) + (-115)^2\right]^{1/2} = 115 \text{ MPa}$$

$$n = S_y / \sigma' = 390/115 = 3.4 \quad \textit{Ans.}$$

For the hub:

Eq. (7-45): $$\sigma_{t,\text{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2} = 115\left(\frac{60^2 + 35^2}{60^2 - 35^2}\right) = 234 \text{ MPa}$$

Eq. (7-46): $$\sigma_{r,\text{hub}} = -p = -115 \text{ MPa}$$

Eq. (5-13): $\sigma' = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)^{1/2}$

$= \left[(234)^2 - (234)(-115) + (-115)^2\right]^{1/2} = 308 \text{ MPa}$

$n = S_y / \sigma' = 600 / 308 = 1.9$ *Ans.*

(d) A value for the static coefficient of friction for steel to steel can be obtained online or from a physics textbook as approximately $f = 0.8$.

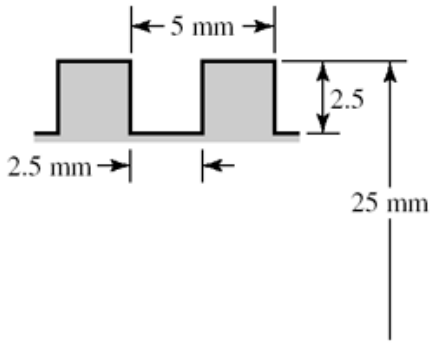
Eq. (7-49) $T = (\pi / 2) f p_{\min} l d^2$

$= (\pi / 2)(0.8)(35.1)\left(10^6\right)(0.050)(0.035)^2 = 2700 \text{ N} \cdot \text{m}$ *Ans.*

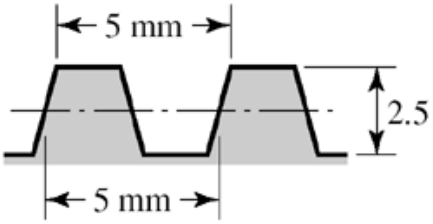
Chapter 8

Note to the Instructor for Probs. 8-41 to 8-44. These problems, as well as many others in this chapter are best implemented using a spreadsheet.

8-1 **(a)** Thread depth= 2.5 mm *Ans.*
Width = 2.5 mm *Ans.*
$d_m = 25 - 1.25 - 1.25 = 22.5$ mm
$d_r = 25 - 5 = 20$ mm
$l = p = 5$ mm *Ans.*



(b) Thread depth = 2.5 mm *Ans.*
Width at pitch line = 2.5 mm *Ans.*
$d_m = 22.5$ mm
$d_r = 20$ mm
$l = p = 5$ mm *Ans.*



8-2 From Table 8-1,

$$d_r = d - 1.226\,869p$$
$$d_m = d - 0.649\,519p$$
$$\bar{d} = \frac{d - 1.226\,869p + d - 0.649\,519p}{2} = d - 0.938\,194p$$

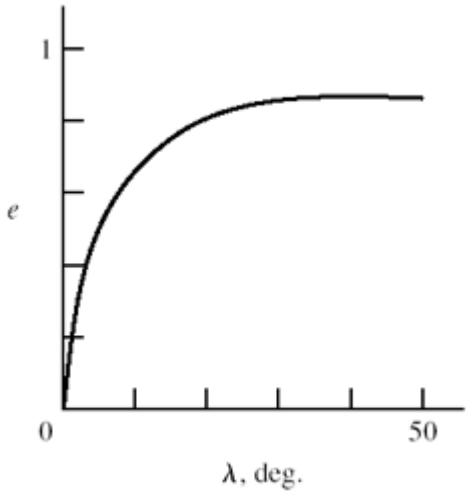
$$A_t = \frac{\pi \bar{d}^2}{4} = \frac{\pi}{4}(d - 0.938\,194p)^2 \quad \textit{Ans.}$$

8-3 From Eq. (*c*) of Sec. 8-2,

$$P_R = F\frac{\tan\lambda + f}{1 - f\tan\lambda}$$
$$T_R = \frac{P_R d_m}{2} = \frac{F d_m}{2}\frac{\tan\lambda + f}{1 - f\tan\lambda}$$
$$e = \frac{T_0}{T_R} = \frac{Fl\ /\ (2\pi)}{Fd_m\ /\ 2}\frac{1 - f\tan\lambda}{\tan\lambda + f} = \tan\lambda\frac{1 - f\tan\lambda}{\tan\lambda + f} \quad \textit{Ans.}$$

Using $f = 0.08$, form a table and plot the efficiency curve.

λ, deg.	e
0	0
0	0.678
20	0.796
30	0.838
40	0.8517
45	0.8519



8-4 Given $F = 5$ kN, $l = 5$ mm, and $d_m = d - p/2 = 25 - 5/2 = 22.5$ mm, the torque required to raise the load is found using Eqs. (8-1) and (8-6)

$$T_R = \frac{5(22.5)}{2}\left[\frac{5+\pi(0.09)22.5}{\pi(22.5)-0.09(5)}\right]+\frac{5(0.06)45}{2} = 15.85 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

The torque required to lower the load, from Eqs. (8-2) and (8-6) is

$$T_L = \frac{5(22.5)}{2}\left[\frac{\pi(0.09)22.5-5}{\pi(22.5)+0.09(5)}\right]+\frac{5(0.06)45}{2} = 7.83 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Since T_L is positive, the thread is self-locking. From Eq.(8-4) the efficiency is

$$e = \frac{5(5)}{2\pi(15.85)} = 0.251 \qquad \textit{Ans.}$$

8-5 Collar (thrust) bearings, at the bottom of the screws, must bear on the collars. The bottom segment of the screws must be in compression. Whereas, tension specimens and their grips must be in tension. Both screws must be of the same-hand threads.

8-6 Screws rotate at an angular rate of

$$n = \frac{1720}{60} = 28.67 \text{ rev/min}$$

(a) The lead is 0.25 in, so the linear speed of the press head is

$$V = 28.67(0.25) = 7.17 \text{ in/min} \quad \textit{Ans.}$$

(b) $F = 2500$ lbf/screw

$$d_m = 2 - 0.25 / 2 = 1.875 \text{ in}$$
$$\sec \alpha = 1 / \cos(29^\circ / 2) = 1.033$$

Eq. (8-5):

$$T_R = \frac{2500(1.875)}{2}\left(\frac{0.25 + \pi(0.05)(1.875)(1.033)}{\pi(1.875) - 0.05(0.25)(1.033)}\right) = 221.0 \text{ lbf} \cdot \text{in}$$

Eq. (8-6):

$$T_c = 2500(0.08)(3.5 / 2) = 350 \text{ lbf} \cdot \text{in}$$
$$T_{total} = 350 + 221.0 = 571 \text{ lbf} \cdot \text{in/screw}$$
$$T_{motor} = \frac{571(2)}{60(0.95)} = 20.04 \text{ lbf} \cdot \text{in}$$
$$H = \frac{Tn}{63\,025} = \frac{20.04(1720)}{63\,025} = 0.547 \text{ hp} \quad \textit{Ans.}$$

8-7 **Note to the Instructor:** The statement for this problem in the first printing of this edition was vague regarding the effective handle length. For the printings to follow the statement "The overall length is 4.25 in." will be replaced by " A force will be applied to the handle at a radius of $3\frac{1}{2}$ in from the screw centerline." We apologize if this has caused any inconvenience.

$$L = 3.5 \text{ in}$$
$$T = 3.5F$$
$$M = \left(L - \frac{3}{8}\right)F = \left(3.5 - \frac{3}{8}\right)F = 3.125F$$
$$S_y = 41 \text{ kpsi}$$
$$\sigma = S_y = \frac{32M}{\pi d^3} = \frac{32(3.125)F}{\pi(0.1875)^3} = 41\,000$$
$$F = 8.49 \text{ lbf}$$
$$T = 3.5(8.49) = 29.7 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

(**b**) Eq. (8-5), $2\alpha = 60^\circ$, $l = 1/10 = 0.1$ in, $f = 0.15$, $\sec \alpha = 1.155$, $p = 0.1$ in

$$d_m = \frac{3}{4} - 0.649\,519(0.1) = 0.6850 \text{ in}$$

$$T_R = \frac{F_{\text{clamp}}(0.6850)}{2}\left(\frac{0.1 + \pi(0.15)(0.6850)(1.155)}{\pi(0.6850) - 0.15(0.1)(1.155)}\right)$$

$$T_R = 0.075\,86F_{\text{clamp}}$$

$$F_{\text{clamp}} = \frac{T_R}{0.075\,86} = \frac{29.7}{0.075\,86} = 392 \text{ lbf} \quad \textit{Ans.}$$

(**c**) The column has one end fixed and the other end pivoted. Base the decision on the mean diameter column. Input: $C = 1.2$, $D = 0.685$ in, $A = \pi(0.685^2)/4 = 0.369$ in^2, $S_y = 41$ kpsi, $E = 30(10^6)$ psi, $L = 6$ in, $k = D/4 = 0.171\,25$ in, $L/k = 35.04$. From Eq. (4-45),

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2} = \left[\frac{2\pi^2(1.2)30(10^6)}{41\,000}\right]^{1/2} = 131.7$$

From Eq. (4-46), the limiting clamping force for buckling is

$$F_{\text{clamp}} = P_{\text{cr}} = A\left[S_y - \left(\frac{S_y}{2\pi}\frac{l}{k}\right)^2 \frac{1}{CE}\right]$$

$$= 0.369\left\{41(10^3) - \left[\frac{41(10^3)}{2\pi}35.04\right]^2 \frac{1}{1.2(30)10^6}\right\} = 14.6(10^3) \text{ lbf} \qquad \textit{Ans}$$

(**d**) This is a subject for class discussion.

8-8 $T = 8(3.5) = 28 \text{ lbf} \cdot \text{in}$

$$d_m = \frac{3}{4} - \frac{1}{12} = 0.6667 \text{ in}$$

$$l = \frac{1}{6} = 0.1667 \text{ in}, \qquad \alpha = \frac{29^0}{2} = 14.5^0, \quad \sec 14.5^0 = 1.033$$

From Eqs. (8-5) and (8-6)

$$T_{\text{total}} = \frac{0.6667F}{2}\left[\frac{0.1667 + \pi(0.15)(0.6667)(1.033)}{\pi(0.6667) - 0.15(0.1667)(1.033)}\right] + \frac{0.15(1)F}{2} = 0.1542F$$

$$F = \frac{28}{0.1542} = 182 \text{ lbf} \qquad \textit{Ans.}$$

8-9 $d_m = 1.5 - 0.25/2 = 1.375$ in, $l = 2(0.25) = 0.5$ in

From Eq. (8-1) and Eq. (8-6)

$$T_R = \frac{2.2\left(10^3\right)(1.375)}{2}\left[\frac{0.5 + \pi(0.10)(1.375)}{\pi(1.375) - 0.10(0.5)}\right] + \frac{2.2\left(10^3\right)(0.15)(2.25)}{2}$$
$$= 330 + 371 = 701 \text{ lbf} \cdot \text{in}$$

Since $n = V/l = 2/0.5 = 4$ rev/s $= 240$ rev/min

so the power is

$$H = \frac{Tn}{63\,025} = \frac{701(240)}{63\,025} = 2.67 \text{ hp} \qquad \textit{Ans.}$$

8-10 $d_m = 40 - 4 = 36$ mm, $l = p = 8$ mm

From Eqs. (8-1) and (8-6)

$$T = \frac{36F}{2}\left[\frac{8 + \pi(0.14)(36)}{\pi(36) - 0.14(8)}\right] + \frac{0.09(100)F}{2}$$
$$= (3.831 + 4.5)F = 8.33F \text{ N} \cdot \text{m} \quad (F \text{ in kN})$$
$$\omega = 2\pi n = 2\pi(1) = 2\pi \text{ rad/s}$$
$$H = T\omega$$
$$T = \frac{H}{\omega} = \frac{3000}{2\pi} = 477 \text{ N} \cdot \text{m}$$
$$F = \frac{477}{8.33} = 57.3 \text{ kN} \qquad \textit{Ans.}$$

$$e = \frac{Fl}{2\pi T} = \frac{57.3(8)}{2\pi(477)} = 0.153 \qquad \textit{Ans.}$$

8-11 (**a**) Table A-31, nut height $H = 12.8$ mm. $L \geq l + H = 2(15) + 12.8 = 42.8$ mm. Rounding up,

$L = 45$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 45 - 34 = 11$ mm, $l_t = l - l_d = 2(15) - 11 = 19$ mm,

$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(19)+115(11)} = 874.6 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**) From Eq. (8-22), with $l = 2(15) = 30$ mm

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(30)+0.5(14)}{0.5774(30)+2.5(14)}\right]} = 3\,116.5 \text{ MN/m} \qquad \textit{Ans.}$$

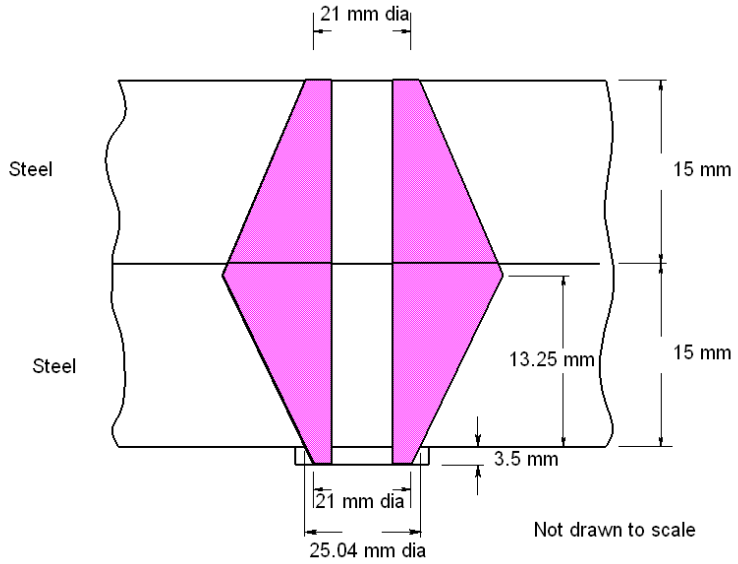
8-12 (**a**) Table A-31, nut height $H = 12.8$ mm. Table A-33, washer thickness $t = 3.5$ mm. Thus, the grip is $l = 2(15) + 3.5 = 33.5$ mm. $L \geq l + H = 33.5 + 12.8 = 46.3$ mm. Rounding up

$L = 50$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 50 - 34 = 16$ mm, $l_t = l - l_d = 33.5 - 16 = 17.5$ mm,
$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(17.5)+115(16)} = 808.2 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



From Eq. (8-22)

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(33.5)+0.5(14)}{0.5774(33.5)+2.5(14)}\right]} = 2\,969 \text{ MN/m} \qquad \textit{Ans.}$$

8-13 (**a**) Table 8-7, $l = h + d/2 = 15 + 14/2 = 22$ mm. $L \geq h + 1.5d = 36$ mm. Rounding up
$L = 40$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(14) + 6 = 34$ mm
From Table 8-7, $l_d = L - L_T = 40 - 34 = 6$ mm, $l_t = l - l_d = 22 - 6 = 16$ mm

$A_d = \pi(14^2)/4 = 153.9$ mm^2. From Table 8-1, $A_t = 115$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.9(115)207}{153.9(16) + 115(6)} = 1\,162.2 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**) From Eq. (8-22), with $l = 22$ mm

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(22) + 0.5(14)}{0.5774(22) + 2.5(14)}\right]} = 3\,624.4 \text{ MN/m} \qquad \textit{Ans.}$$

8-14 (**a**) From Table A-31, the nut height is $H = 7/16$ in. $L \geq l + H = 2 + 1 + 7/16 = 3\ 7/16$ in.
Rounding up, $L = 3.5$ in *Ans.*

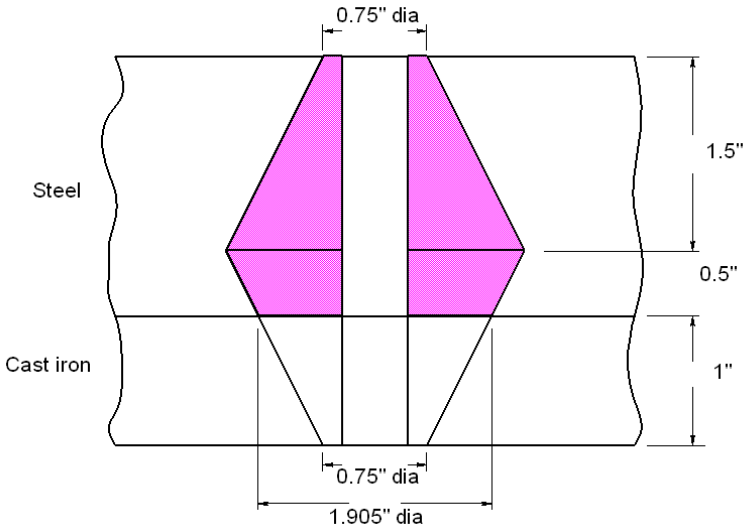
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 3.5 - 1.25 = 2.25$ in, $l_t = l - l_d = 3 - 2.25 = 0.75$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.75) + 0.1419(2.25)} = 1.79 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Top steel frustum: $t = 1.5$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{\left[1.155(1.5)+0.75-0.5\right](0.75+0.5)}{\left[1.155(1.5)+0.75+0.5\right](0.75-0.5)}} = 22.65 \text{ Mlbf/in}$$

Lower steel frustum: $t = 0.5$ in, $d = 0.5$ in, $D = 0.75 + 2(1)\tan 30° = 1.905$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_2 = 210.7$ Mlbf/in

Cast iron: $t = 1$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow k_3 = 12.27$ Mlbf/in

From Eq. (8-18)

$$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/22.65 + 1/210.7 + 1/12.27)^{-1} = 7.67 \text{ Mlbf/in} \quad \textit{Ans.}$$

8-15 (**a**) From Table A-32, the washer thickness is 0.095 in. Thus, $l = 2 + 1 + 2(0.095) = 3.19$ in. From Table A-31, the nut height is $H = 7/16$ in. $L \geq l + H = 3.19 + 7/16 = 3.63$ in. Rounding up, $L = 3.75$ in *Ans.*

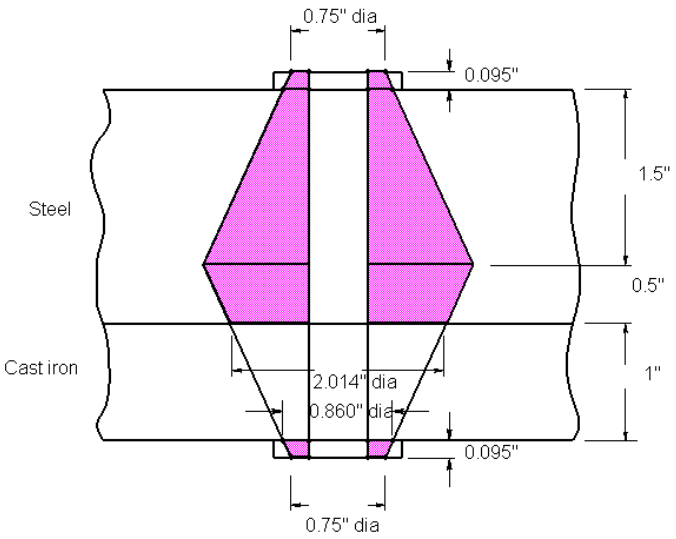
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 3.75 - 1.25 = 2.5$ in, $l_t = l - l_d = 3.19 - 2.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69) + 0.1419(2.5)} = 1.705 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Each steel washer frustum: $t = 0.095$ in, $d = 0.531$ in (Table A-32), $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.531}{\ln\dfrac{\left[1.155(0.095) + 0.75 - 0.531\right](0.75 + 0.531)}{\left[1.155(0.095) + 0.75 + 0.531\right](0.75 - 0.531)}} = 89.20 \text{ Mlbf/in}$$

Top plate, top steel frustum: $t = 1.5$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095)\tan 30° = 0.860$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_2 = 28.99$ Mlbf/in

Top plate, lower steel frustum: $t = 0.5$ in, $d = 0.5$ in, $D = 0.860 + 2(1)\tan 30° = 2.015$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_3 = 234.08$ Mlbf/in

Cast iron: $t = 1$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095)\tan 30° = 0.860$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow k_4 = 15.99$ Mlbf/in
From Eq. (8-18)

$k_m = (2/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (2/89.20 + 1/28.99 + 1/234.08 + 1/15.99)^{-1}$
$= 8.08$ Mlbf/in *Ans.*

8-16 (**a**) From Table 8-7, $l = h + d/2 = 2 + 0.5/2 = 2.25$ in.
$L \geq h + 1.5d = 2 + 1.5(0.5) = 2.75$ in *Ans.*

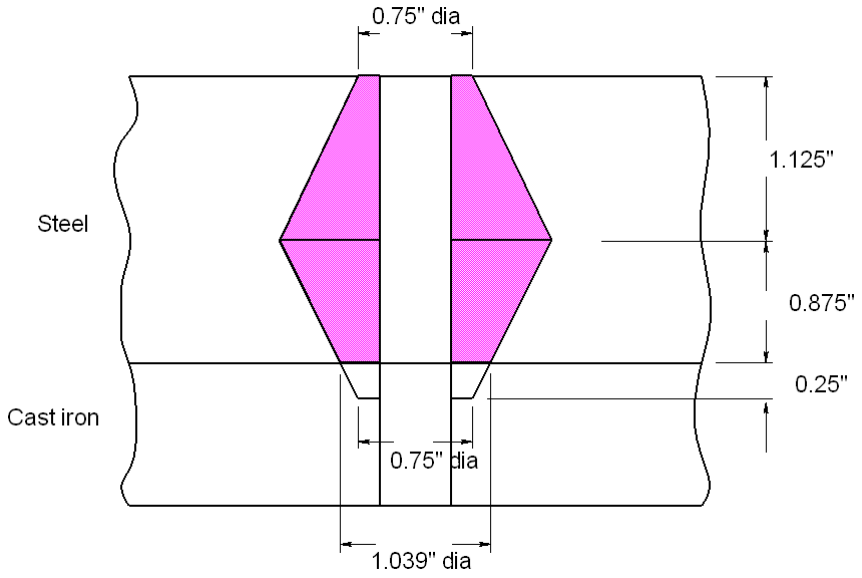
(**b**) From Table 8-7, $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

$l_d = L - L_T = 2.75 - 1.25 = 1.5$ in, $l_t = l - l_d = 2.25 - 1.5 = 0.75$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.75) + 0.1419(1.5)} = 2.321 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Top steel frustum: $t = 1.125$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{\left[1.155(1.125) + 0.75 - 0.5\right](0.75 + 0.5)}{\left[1.155(1.125) + 0.75 + 0.5\right](0.75 - 0.5)}} = 24.48 \text{ Mlbf/in}$$

Lower steel frustum: $t = 0.875$ in, $d = 0.5$ in, $D = 0.75 + 2(0.25)\tan 30° = 1.039$ in, $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 49.36$ Mlbf/in

Cast iron: $t = 0.25$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi (Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 23.49$ Mlbf/in

From Eq. (8-18)

$$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/24.48 + 1/49.36 + 1/23.49)^{-1} = 9.645 \text{ Mlbf/in} \qquad \textit{Ans.}$$

8-17 **a**) Grip, $l = 2(2 + 0.095) = 4.19$ in. $L \geq 4.19 + 7/16 = 4.628$ in.
Rounding up, $L = 4.75$ in *Ans.*

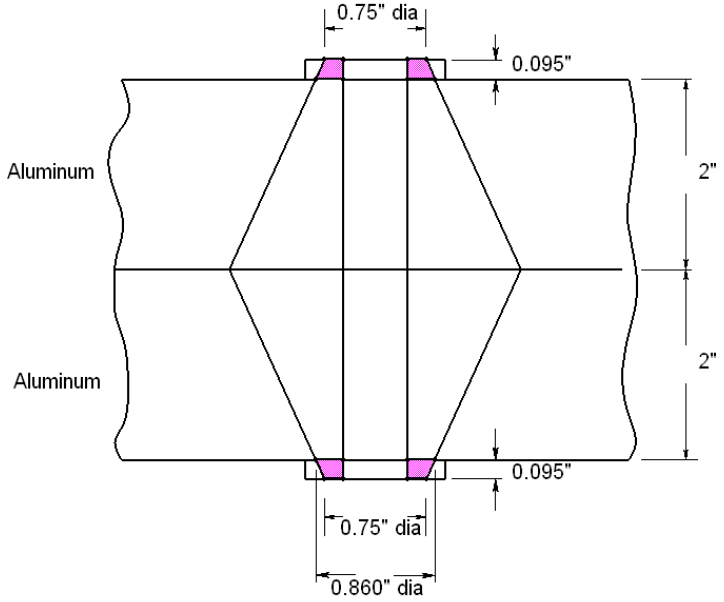
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 4.75 - 1.25 = 3.5$ in, $l_t = l - l_d = 4.19 - 3.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69) + 0.1419(3.5)} = 1.322 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Upper and lower halves are the same. For the upper half,
Steel frustum: $t = 0.095$ in, $d = 0.531$ in, $D = 0.75$ in, and $E = 30$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(30)0.531}{\ln\dfrac{[1.155(0.095) + 0.75 - 0.531](0.75 + 0.531)}{[1.155(0.095) + 0.75 + 0.531](0.75 - 0.531)}} = 89.20 \text{ Mlbf/in}$$

Aluminum: $t = 2$ in, $d = 0.5$ in, $D = 0.75 + 2(0.095)\tan 30° = 0.860$ in, and $E = 10.3$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_2 = 9.24$ Mlbf/in
For the top half, $k'_m = (1/k_1 + 1/k_2)^{-1} = (1/89.20 + 1/9.24)^{-1} = 8.373$ Mlbf/in
Since the bottom half is the same, the overall stiffness is given by

$$k_m = (1/k'_m + 1/k'_m)^{-1} = k'_m/2 = 8.373/2 = 4.19 \text{ Mlbf/in} \qquad \textit{Ans}$$

8-18 (**a**) Grip, $l = 2(2 + 0.095) = 4.19$ in. $L \geq 4.19 + 7/16 = 4.628$ in.
Rounding up, $L = 4.75$ in *Ans.*

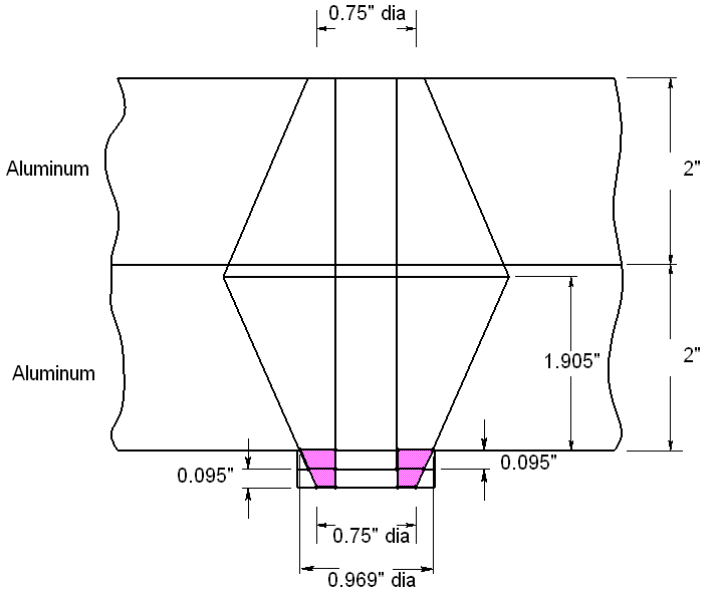
(**b**) From Eq. (8-13), $L_T = 2d + 1/4 = 2(0.5) + 0.25 = 1.25$ in

From Table 8-7, $l_d = L - L_T = 4.75 - 1.25 = 3.5$ in, $l_t = l - l_d = 4.19 - 3.5 = 0.69$ in

$A_d = \pi(0.5^2)/4 = 0.1963$ in^2. From Table 8-2, $A_t = 0.1419$ in^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963(0.1419)30}{0.1963(0.69) + 0.1419(3.5)} = 1.322 \text{ Mlbf/in} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: $t = [4 + 2(0.095)]/2 = 2.095$ in, $d = 0.5$ in, $D = 0.75$ in, and $E = 10.3$ Mpsi. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)0.5}{\ln\dfrac{\left[1.155(2.095) + 0.75 - 0.5\right](0.75 + 0.5)}{\left[1.155(2.095) + 0.75 + 0.5\right](0.75 - 0.5)}} = 7.23 \text{ Mlbf/in}$$

Lower aluminum frustum: $t = 4 - 2.095 = 1.905$ in, $d = 0.5$ in,
$D = 0.75 + 4(0.095)\tan 30° = 0.969$ in, and $E = 10.3$ Mpsi. Eq. (8-20) $\Rightarrow k_2 = 11.34$ Mlbf/in

Steel washers frustum: $t = 2(0.095) = 0.190$ in, $d = 0.531$ in, $D = 0.75$ in, and $E = 30$ Mpsi. Eq. (8-20) $\Rightarrow k_3 = 53.91$ Mlbf/in

From Eq. (8-18)
$k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1} = (1/7.23 + 1/11.34 + 1/53.91)^{-1} = 4.08$ Mlbf/in *Ans.*

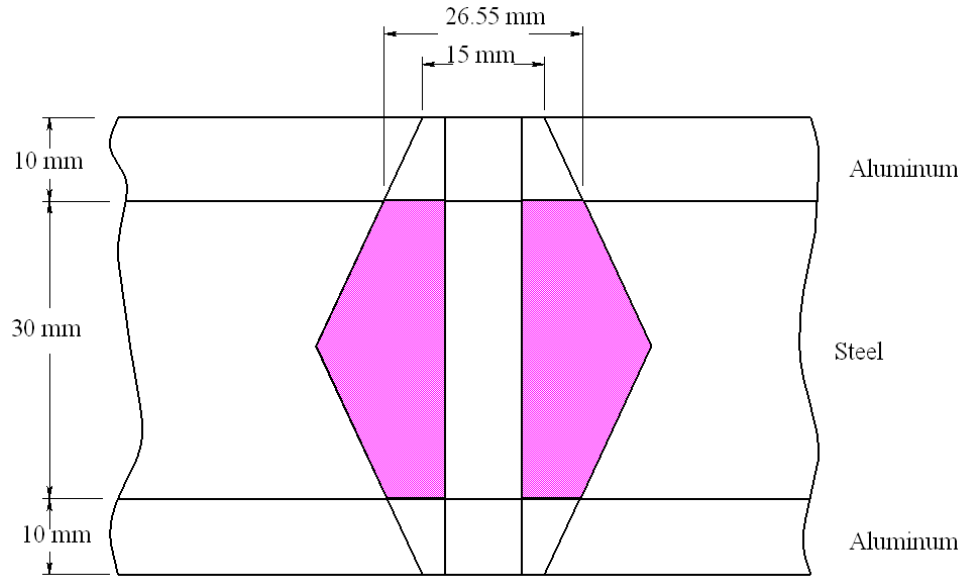
8-19 (**a**) From Table A-31, the nut height is $H = 8.4$ mm. $L > l + H = 50 + 8.4 = 58.4$ mm.

Rounding up, $L = 60$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 60 - 26 = 34$ mm, $l_t = l - l = 50 - 34 = 16$ mm. $A_d = \pi(10^2) / 4 = 78.54$ mm^2. From Table 8-1, $A_t = 58$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16) + 58.0(34)} = 292.1 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



Upper and lower frustums are the same. For the upper half,

Aluminum: $t = 10$ mm, $d = 10$ mm, $D = 15$ mm, and from Table 8-8, $E = 71$ GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(71)10}{\ln\dfrac{[1.155(10)+15-10](15+10)}{[1.155(10)+15+10](15-10)}} = 1576 \text{ MN/m}$$

Steel: $t = 15$ mm, $d = 10$ mm, $D = 15 + 2(10)\tan 30° = 26.55$ mm, and $E = 207$ GPa. From Eq. (8-20)

$$k_2 = \frac{0.5774\pi(207)10}{\ln\dfrac{[1.155(15)+26.55-10](26.55+10)}{[1.155(15)+26.55+10](26.55-10)}} = 11\,440 \text{ MN/m}$$

For the top half, $k'_m = (1/k_1 + 1/k_2)^{-1} = (1/1576 + 1/11\,440)^{-1} = 1385$ MN/m

Since the bottom half is the same, the overall stiffness is given by

$$k_m = (1/k'_m + 1/k'_m)^{-1} = k'_m/2 = 1385/2 = 692.5 \text{ MN/m} \qquad \textit{Ans.}$$

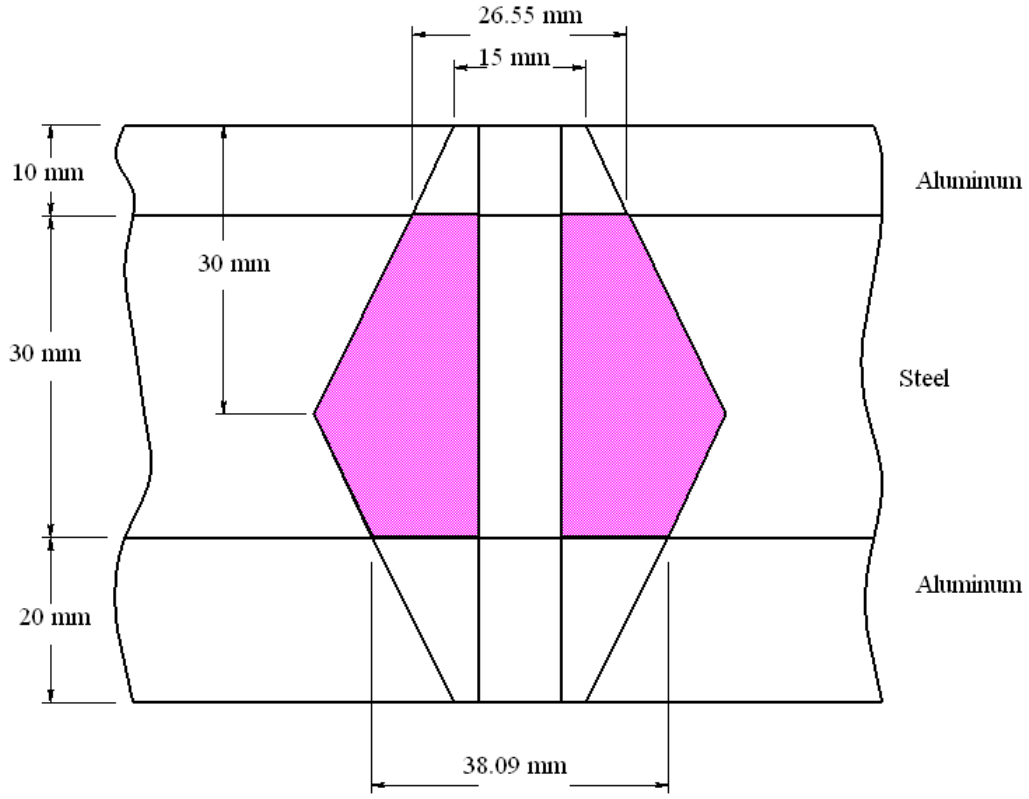
8-20 (**a**) From Table A-31, the nut height is $H = 8.4$ mm. $L > l + H = 60 + 8.4 = 68.4$ mm.

Rounding up, $L = 70$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 70 - 26 = 44$ mm, $l_t = l - l_d = 60 - 44 = 16$ mm. $A_d = \pi(10^2)/4 = 78.54$ mm^2. From Table 8-1, $A_t = 58$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16) + 58.0(44)} = 247.6 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: $t = 10$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)71}{\ln\dfrac{\left[1.155(2.095)+15-10\right](15+10)}{\left[1.155(2.095)+15+10\right](15-10)}} = 1576 \text{ MN/m}$$

Lower aluminum frustum: $t = 20$ mm, $d = 10$ mm, $D = 15$ mm, and $E = 71$ GPa. Eq. (8-20) $\Rightarrow k_2 = 1\,201$ MN/m

Top steel frustum: $t = 20$ mm, $d = 10$ mm, $D = 15 + 2(10)\tan 30° = 26.55$ mm, and $E = 207$ GPa. Eq. (8-20) $\Rightarrow k_3 = 9\,781$ MN/m

Lower steel frustum: $t = 10$ mm, $d = 10$ mm, $D = 15 + 2(20)\tan 30° = 38.09$ mm, and $E = 207$ GPa. Eq. (8-20) $\Rightarrow k_4 = 29\,070$ MN/m

From Eq. (8-18)

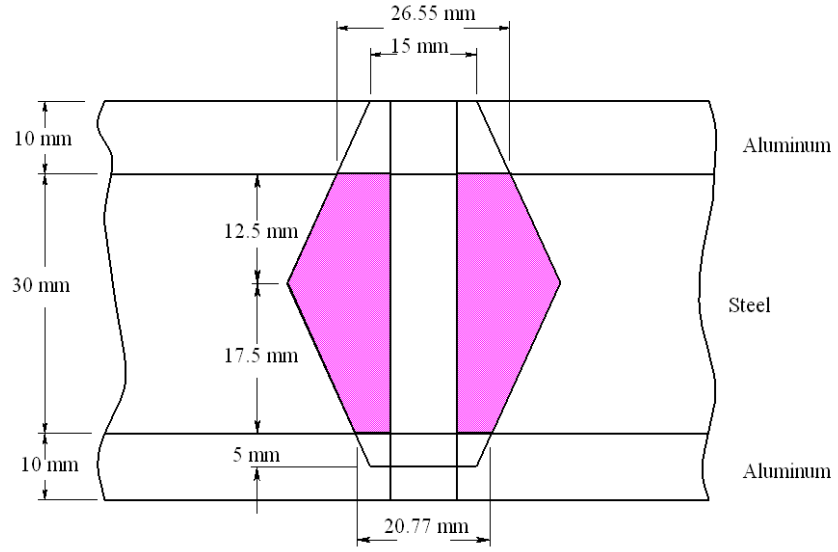
$$k_m = (1/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (1/1\,576 + 1/1\,201 + 1/9\,781 + 1/29\,070)^{-1}$$
$$= 623.5 \text{ MN/m} \quad \textit{Ans.}$$

8-21 (**a**) From Table 8-7, $l = h + d/2 = 10 + 30 + 10/2 = 45$ mm. $L \geq h + 1.5\,d = 10 + 30 + 1.5(10) = 55$ mm *Ans.*

(**b**) From Eq. (8-14), $L_T = 2d + 6 = 2(10) + 6 = 26$ mm, $l_d = L - L_T = 55 - 26 = 29$ mm, $l_t = l - l_d = 45 - 29 = 16$ mm. $A_d = \pi(10^2)/4 = 78.54$ mm^2. From Table 8-1, $A_t = 58$ mm^2. From Eq. (8-17)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.54(58.0)207}{78.54(16)+58.0(29)} = 320.9 \text{ MN/m} \qquad \textit{Ans.}$$

(**c**)



Upper aluminum frustum: t = 10 mm, d = 10 mm, D = 15 mm, and E = 71 GPa. From Eq. (8-20)

$$k_1 = \frac{0.5774\pi(10.3)71}{\ln\dfrac{\left[1.155(2.095)+15-10\right](15+10)}{\left[1.155(2.095)+15+10\right](15-10)}} = 1576 \text{ MN/m}$$

Lower aluminum frustum: t = 5 mm, d = 10 mm, D = 15 mm, and E = 71 GPa. Eq. (8-20) $\Rightarrow k_2$ = 2 300 MN/m

Top steel frustum: t = 12.5 mm, d = 10 mm, D = 15 + 2(10) tan 30° = 26.55 mm, and E = 207 GPa. Eq. (8-20) $\Rightarrow k_3$ = 12 759 MN/m
Lower steel frustum: t = 17.5 mm, d = 10 mm, D = 15 + 2(5) tan 30° = 20.77 mm, and E = 207 GPa. Eq. (8-20) $\Rightarrow k_4$ = 6 806 MN/m

From Eq. (8-18)
$k_m = (1/k_1 + 1/k_2 + 1/k_3 + 1/k_4)^{-1} = (1/1\,576 + 1/2\,300 + 1/12\,759 + 1/6\,806)^{-1}$
$= 772.4$ MN/m *Ans.*

8-22 Equation (*f*), p. 436: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

Eq. (8-22): $k_m = \dfrac{0.5774\pi(207)d}{2\ln\left[5\dfrac{0.5774(40)+0.5d}{0.5774(40)+2.5d}\right]}$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are mm, mm^2, MN/m):

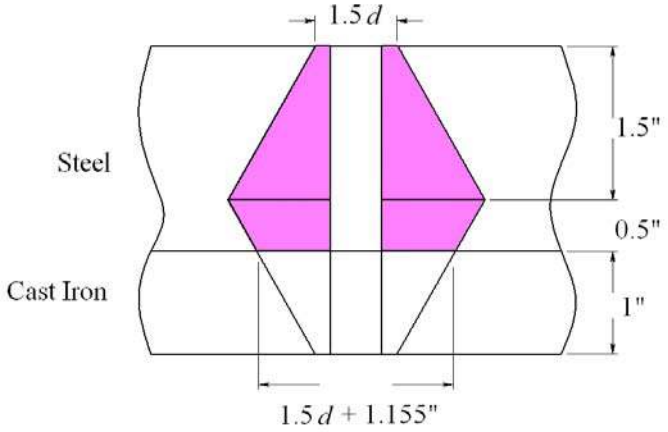
d	A_t	A_d	H	$L >$	L	L_T
10	58	78.53982	8.4	48.4	50	26
12	84.3	113.0973	10.8	50.8	55	30
14	115	153.938	12.8	52.8	55	34
16	157	201.0619	14.8	54.8	55	38
20	245	314.1593	18	58	60	46
24	353	452.3893	21.5	61.5	65	54
30	561	706.8583	25.6	65.6	70	66

d	l	l_d	l_t	k_b	k_m	C
10	40	24	16	356.0129	1751.566	0.16892
12	40	25	15	518.8172	2235.192	0.188386
14	40	21	19	686.2578	2761.721	0.199032
16	40	17	23	895.9182	3330.796	0.211966
20	40	14	26	1373.719	4595.515	0.230133
24	40	11	29	1944.24	6027.684	0.243886
30	40	4	36	2964.343	8487.533	0.258852

The 14 mm would probably be ok, but to satisfy the question, use a 16 mm bolt *Ans.*

8-23 Equation (*f*), p. 436: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$



For upper frustum, Eq. (8-20), with $D = 1.5\,d$ and $t = 1.5$ in:

$$k_1 = \frac{0.5774\pi(30)d}{\ln\left[\dfrac{\left[1.155(1.5)+0.5d\right](2.5d)}{\left[1.155(1.5)+2.5d\right](0.5d)}\right]} = \frac{0.5774\pi(30)d}{\ln\left[5\dfrac{(1.733+0.5d)}{(1.733+2.5d)}\right]}$$

Lower steel frustum, with $D = 1.5d + 2(1)\tan 30° = 1.5d + 1.155$, and $t = 0.5$ in:

$$k_2 = \frac{0.5774\pi(30)d}{\ln\left[\dfrac{(1.733+0.5d)(2.5d+1.155)}{(1.733+2.5d)(0.5d+1.155)}\right]}$$

For cast iron frustum, let E = 14. 5 Mpsi, and D = 1.5 d, and t = 1 in:

$$k_3 = \frac{0.5774\pi(14.5)d}{\ln\left[5\dfrac{(1.155+0.5d)}{(1.155+2.5d)}\right]}$$

Overall, $k_m = (1/k_1 + 1/k_2 + 1/k_3)^{-1}$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are in, in^2, Mlbf/in):

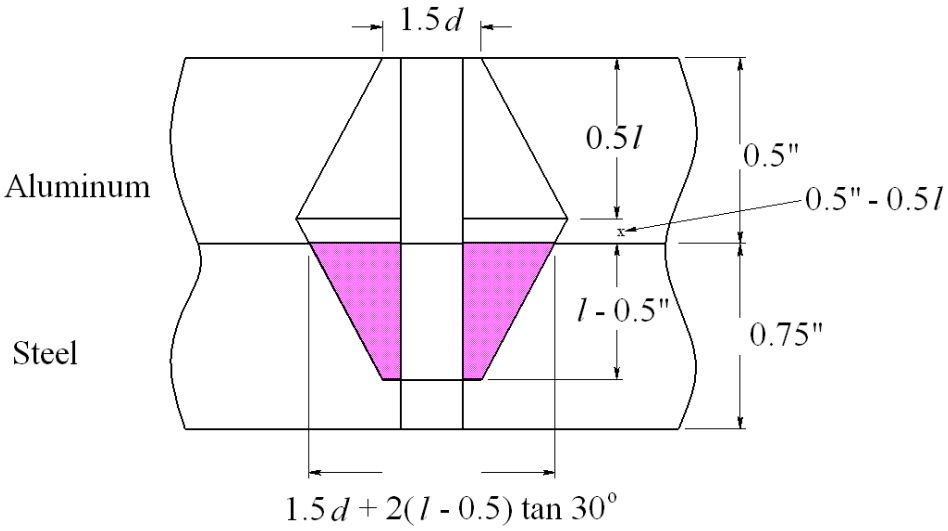
d	A_t	A_d	H	$L >$	L	L_T	l
0.375	0.0775	0.110447	0.328125	3.328125	3.5	1	3
0.4375	0.1063	0.15033	0.375	3.375	3.5	1.125	3
0.5	0.1419	0.19635	0.4375	3.4375	3.5	1.25	3
0.5625	0.182	0.248505	0.484375	3.484375	3.5	1.375	3
0.625	0.226	0.306796	0.546875	3.546875	3.75	1.5	3
0.75	0.334	0.441786	0.640625	3.640625	3.75	1.75	3
0.875	0.462	0.60132	0.75	3.75	3.75	2	3

d	l_d	l_t	k_b	k_1	k_2	k_3	k_m	C
0.375	2.5	0.5	1.031389	15.94599	178.7801	8.461979	5.362481	0.161309
0.4375	2.375	0.625	1.383882	19.21506	194.465	10.30557	6.484256	0.175884
0.5	2.25	0.75	1.791626	22.65332	210.6084	12.26874	7.668728	0.189383
0.5625	2.125	0.875	2.245705	26.25931	227.2109	14.35052	8.915294	0.20121
0.625	2.25	0.75	2.816255	30.03179	244.2728	16.55009	10.22344	0.215976
0.75	2	1	3.988786	38.07191	279.7762	21.29991	13.02271	0.234476
0.875	1.75	1.25	5.341985	46.7663	317.1203	26.51374	16.06359	0.24956

Use a $\frac{9}{16}$–12 UNC × 3.5 in long bolt *Ans.*

8-24 Equation (*f*), p. 436: $C = \dfrac{k_b}{k_b + k_m}$

Eq. (8-17): $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$



Top frustum, Eq. (8-20), with E = 10.3Mpsi, D = 1.5 d, and $t = l/2$:

$$k_1 = \frac{0.5774\pi(10.3)d}{\ln\left[5\dfrac{1.155\,l/2+0.5d}{1.155\,l/2+2.5d}\right]}$$

Middle frustum, with E = 10.3 Mpsi, $D = 1.5d + 2(l - 0.5)\tan 30°$, and $t = 0.5 - l/2$

$$k_2 = \frac{0.5774\pi(10.3)d}{\ln\dfrac{\left\{\left[1.155(0.5-0.5l)+0.5d+2(l-0.5)\tan 30^0\right]\left[2.5d+2(l-0.5)\tan 30^0\right]\right\}}{\left\{\left[1.155(0.5-0.5l)+2.5d+2(l-0.5)\tan 30^0\right]\left[0.5d+2(l-0.5)\tan 30^0\right]\right\}}}$$

Lower frustum, with E = 30Mpsi, D = 1.5 d, $t = l - 0.5$

$$k_3 = \frac{0.5774\pi(30)d}{\ln 5\left\{\dfrac{\left[1.155(l-0.5)+0.5d\right]}{\left[1.155(l-0.5)+2.5d\right]}\right\}}$$

See Table 8-7 for other terms used.
Using a spreadsheet, with coarse-pitch bolts (units are in, in^2, Mlbf/in)

Size	d	A_t	A_d	$L >$	L	L_T	l	l_d
1	0.073	0.00263	0.004185	0.6095	0.75	0.396	0.5365	0.354
2	0.086	0.0037	0.005809	0.629	0.75	0.422	0.543	0.328
3	0.099	0.00487	0.007698	0.6485	0.75	0.448	0.5495	0.302
4	0.112	0.00604	0.009852	0.668	0.75	0.474	0.556	0.276
5	0.125	0.00796	0.012272	0.6875	0.75	0.5	0.5625	0.25
6	0.138	0.00909	0.014957	0.707	0.75	0.526	0.569	0.224
8	0.164	0.014	0.021124	0.746	0.75	0.578	0.582	0.172
10	0.19	0.0175	0.028353	0.785	1	0.63	0.595	0.37
Size	d	l_t	k_b	k_1	k_2	k_3	k_m	C
1	0.073	0.1825	0.194841	1.084468	1.954599	7.09432	0.635049	0.23478
2	0.086	0.215	0.261839	1.321595	2.449694	8.357692	0.778497	0.251687
3	0.099	0.2475	0.333134	1.570439	2.993366	9.621064	0.930427	0.263647
4	0.112	0.28	0.403377	1.830494	3.587564	10.88444	1.090613	0.27
5	0.125	0.3125	0.503097	2.101297	4.234381	12.14781	1.258846	0.285535
6	0.138	0.345	0.566787	2.382414	4.936066	13.41118	1.434931	0.28315
8	0.164	0.41	0.801537	2.974009	6.513824	15.93792	1.809923	0.306931
10	0.19	0.225	1.15799	3.602349	8.342138	18.46467	2.214214	0.343393

The lowest coarse series screw is a 1–64 UNC × 0.75 in long up to a 6–32 UNC × 0.75 in long. *Ans.*

8-25 For half of joint, Eq. (8-20): $t = 20$ mm, $d = 14$ mm, $D = 21$ mm, and $E = 207$ GPa

$$k_1 = \frac{0.5774\pi(207)14}{\ln\dfrac{\left[1.155(20)+21-14\right](21+14)}{\left[1.155(20)+21+14\right](21-14)}} = 5523 \text{ MN/m}$$

$k_m = (1/k_1 + 1/k_1)^{-1} = k_1/2 = 5523/2 = 2762$ MN/m *Ans.*

From Eq. (8-22) with $l = 40$ mm

$$k_m = \frac{0.5774\pi(207)14}{2\ln\left[5\dfrac{0.5774(40)+0.5(14)}{0.5774(40)+2.5(14)}\right]} = 2762 \text{ MN/m} \qquad \textit{Ans.}$$

which agrees with the earlier calculation.

For Eq. (8-23), from Table 8-8, $A = 0.787\ 15$, $B = 0.628\ 73$

$$k_m = 207(14)(0.78\ 715)\exp[0.628\ 73(14)/40] = 2843 \text{ MN/m} \qquad \textit{Ans.}$$

This is 2.9% higher than the earlier calculations.

8-26 (**a**) Grip, $l = 10$ in. Nut height, $H = 41/64$ in (Table A-31).
$L \geq l + H = 10 + 41/64 = 10.641$ in. Let $L = 10.75$ in.
Table 8-7, $L_T = 2d + 0.5 = 2(0.75) + 0.5 = 2$ in, $l_d = L - L_T = 10.75 - 2 = 8.75$ in,
$l_t = l - l_d = 10 - 8.75 = 1.25$ in
$A_d = \pi(0.75^2)/4 = 0.4418$ in^2, $A_t = 0.373$ in^2 (Table 8-2)
Eq. (8-17),

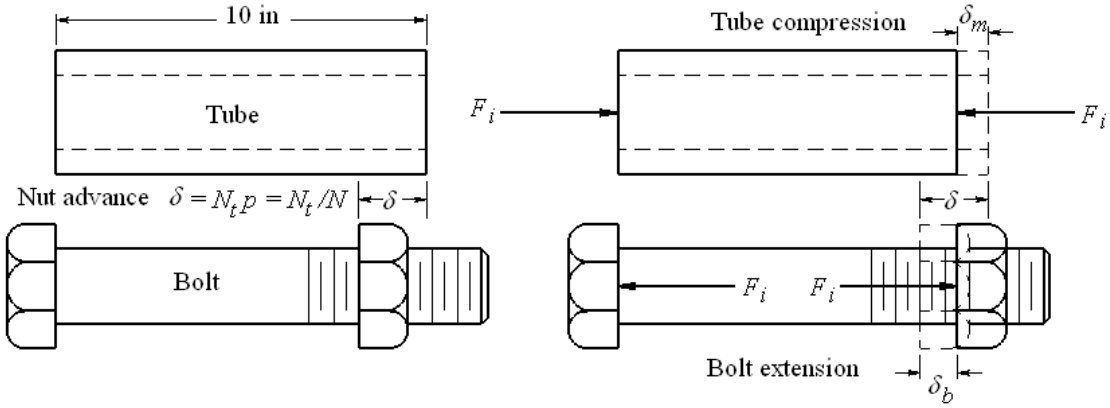
$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.4418(0.373)30}{0.4418(1.25) + 0.373(8.75)} = 1.296 \quad \text{Mlbf/in} \qquad \textit{Ans.}$$

Eq. (4-4), p. 149,

$$k_m = \frac{A_m E_m}{l} = \frac{(\pi/4)(1.125^2 - 0.75^2)30}{10} = 1.657 \quad \text{Mlbf/in} \qquad \textit{Ans.}$$

Eq. (*f*), p. 436, $C = k_b/(k_b + k_m) = 1.296/(1.296 + 1.657) = 0.439$ *Ans.*

(**b**)



Let: N_t = no. of turns, p = pitch of thread (in), N = no. of threads per in = $1/p$. Then,

$$\delta = \delta_b + \delta_m = N_t\, p = N_t / N \qquad (1)$$

But, $\delta_b = F_i / k_b$, and, $\delta_m = F_i / k_m$. Substituting these into Eq. (1) and solving for F_i gives

$$F_i = \frac{k_b k_m}{k_b + k_m}\frac{N_t}{N} \qquad (2)$$

$$= \frac{1.296(1.657)10^6}{1.296 + 1.657}\frac{1/3}{16} = 15\,150 \quad \text{lbf} \qquad \textit{Ans.}$$

8-27 Proof for the turn-of-nut equation is given in the solution of Prob. 8-26, Eq. (2), where $N_t = \theta / 360°$.

The relationship between the turn-of-nut method and the torque-wrench method is as follows.

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right) F_i N \qquad \text{(turn-of-nut)}$$

$$T = K F_i d \qquad \text{(torque-wrench)}$$

Eliminate F_i

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right)\frac{NT}{Kd} = \frac{\theta}{360^\circ} \qquad \textit{Ans.}$$

8-28 **(a)** From Ex. 8-4, F_i = 14.4 kip, $k_b = 5.21(10^6)$ lbf/in, $k_m = 8.95(10^6)$ lbf/in
Eq. (8-27): $T = kF_i d = 0.2(14.4)(10^3)(5/8) = 1800$ lbf · in *Ans.*
From Prob. 8-27,

$$N_t = \left(\frac{k_b + k_m}{k_b k_m}\right) F_i N = \left[\frac{5.21 + 8.95}{5.21(8.95)10^6}\right](14.4)(10^3)11$$

$$= 0.0481 \text{ turns} = 17.3^\circ \qquad \textit{Ans.}$$

Bolt group is (1.5) / (5/8) = 2.4 diameters. Answer is much lower than RB&W recommendations.

8-29 $C = k_b / (k_b + k_m) = 3/(3+12) = 0.2$, $P = P_{\text{total}} / N = 80/6 = 13.33$ kips/bolt
Table 8-2, $A_t = 0.141\,9$ in^2; Table 8-9, $S_p = 120$ kpsi; Eqs. (8-31) and (8-32),
$F_i = 0.75\, A_t S_p = 0.75(0.141\,9)(120) = 12.77$ kips
(**a**) From Eq. (8-28), the factor of safety for yielding is

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.141\,9)}{0.2(13.33) + 12.77} = 1.10 \qquad \textit{Ans.}$$

(**b**) From Eq. (8-29), the overload factor is

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.141\,9) - 12.77}{0.2(13.33)} = 1.60 \qquad \textit{Ans.}$$

(**c**) From Eq. (803), the joint separation factor of safety is

$$n_0 = \frac{F_i}{P(1-C)} = \frac{12.77}{13.33(1-0.2)} = 1.20 \qquad \textit{Ans.}$$

8-30 1/2 – 13 UNC Grade 8 bolt, $K = 0.20$

(**a**) Proof strength, Table 8-9, $S_p = 120$ kpsi

Table 8-2, $A_t = 0.141\,9$ in^2

Maximum, $F_i = S_p A_t = 120(0.141\,9) = 17.0$ kips *Ans.*

(**b**) From Prob. 8-29, $C = 0.2$, $P = 13.33$ kips

Joint separation, Eq. (8-30) with $n_0 = 1$

Minimum $F_i = P(1 - C) = 13.33(1 - 0.2) = 10.66$ kips *Ans.*

(**c**) $\overline{F}_i = (17.0 + 10.66)/2 = 13.8$ kips

Eq. (8-27), $T = KF_i d = 0.2(13.8)10^3(0.5)/12 = 115$ lbf · ft *Ans.*

8-31 (**a**) Table 8-1, $A_t = 20.1$ mm^2. Table 8-11, $S_p = 380$ MPa.

Eq. (8-31), $F_i = 0.75 F_p = 0.75 A_t S_p = 0.75(20.1)380(10^{-3}) = 5.73$ kN

Eq. (*f*), p. 436, $$C = \frac{k_b}{k_b + k_m} = \frac{1}{1+2.6} = 0.278$$

Eq. (8-28) with $n_p = 1$,

$$P = \frac{S_p A_t - F_i}{C} = \frac{0.25 S_p A_t}{C} = \frac{0.25(20.1)380(10^{-3})}{0.278} = 6.869 \text{ kN}$$

$P_{\text{total}} = NP = 8(6.869) = 55.0$ kN *Ans.*

(**b**) Eq. (8-30) with $n_0 = 1$,

$$P = \frac{F_i}{1-C} = \frac{5.73}{1-0.278} = 7.94 \text{ kN}$$

$P_{\text{total}} = NP = 8(7.94) = 63.5$ kN *Ans.* Bolt stress would exceed proof strength

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8-32 (**a**) Table 8-2, $A_t = 0.141\,9$ in^2. Table 8-9, $S_p = 120$ kpsi.

Eq. (8-31), $F_i = 0.75 F_p = 0.75 A_t S_p = 0.75(0.141\,9)120 = 12.77$ kips

Eq. (*f*), p. 436, $$C = \frac{k_b}{k_b + k_m} = \frac{4}{4+12} = 0.25$$

Eq. (8-28) with $n_p = 1$,

$$P_{\text{total}} = N\left(\frac{S_p A_t - F_i}{C}\right) = \frac{0.25 N S_p A_t}{C}$$

$$N = \frac{P_{\text{total}} C}{0.25 S_p A_t} = \frac{80(0.25)}{0.25(120)0.141\,9} = 4.70$$

Round to $N = 5$ bolts *Ans.*

(**b**) Eq. (8-30) with $n_0 = 1$,

$$P_{\text{total}} = N\left(\frac{F_i}{1 - C}\right)$$

$$N = \frac{P_{\text{total}}(1 - C)}{F_i} = \frac{80(1 - 0.25)}{12.77} = 4.70$$

Round to $N = 5$ bolts *Ans.*

8-33 Bolts: From Table A-31, the nut height is $H = 10.8$ mm. $L \geq l + H = 40 + 10.8 = 50.8$ mm. Although Table A-17 indicates to go to 60 mm, 55 mm is readily available

Round up to $L = 55$ mm *Ans.*

Eq. (8-14): $L_T = 2d + 6 = 2(12) + 6 = 30$ mm

Table 8-7: $l_d = L - L_T = 55 - 30 = 25$ mm, $l_t = l - l_d = 40 - 25 = 15$ mm

$A_d = \pi(12^2)/4 = 113.1$ mm^2, Table 8-1: $A_t = 84.3$ mm^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{113.1(84.3)207}{113.1(15) + 84.3(25)} = 518.8 \text{ MN/m}$$

Members: Steel cyl. head: $t = 20$ mm, $d = 12$ mm, $D = 18$ mm, $E = 207$ GPa. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)12}{\ln\dfrac{[1.155(20) + 18 - 12](18 + 12)}{[1.155(20) + 18 + 12](18 - 12)}} = 4470 \text{ MN/m}$$

Cast iron: $t = 20$ mm, $d = 12$ mm, $D = 18$ mm, $E = 100$ GPa (from Table 8-8). The only difference from k_1 is the material

$$k_2 = (100/207)(4470) = 2159 \text{ MN/m}$$

Eq. (8-18): $k_m = (1/4470 + 1/2159)^{-1} = 1456$ MN/m

$C = k_b / (k_b + k_m) = 518.8/(518.8+1456) = 0.263$

Table 8-11: $S_p = 650$ MPa
Assume non-permanent connection. Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(84.3)(650)10^{-3} = 41.1 \text{ kN}$$

The total external load is $P_{\text{total}} = p_g\, A_c$, where A_c is the diameter of the cylinder which is 100 mm. The external load per bolt is $P = P_{\text{total}} / N$. Thus

$$P = [6\pi(100^2)/4](10^{-3})/10 = 4.712 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{650(84.3)10^{-3}}{0.263(4.712) + 41.10} = 1.29 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{650(84.3)10^{-3} - 41.10}{0.263(4.712)} = 11.1 \qquad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{41.10}{4.712(1-0.263)} = 11.8 \qquad \textit{Ans.}$$

8-34 Bolts: Grip, $l = 1/2 + 5/8 = 1.125$ in. From Table A-31, the nut height is $H = 7/16$ in.
$L \geq l + H = 1.125 + 7/16 = 1.563$ in.

Round up to $L = 1.75$ in *Ans.*

Eq. (8-13): $L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in

Table 8-7: $l_d = L - L_T = 1.75 - 1.25 = 0.5$ in, $l_t = l - l_d = 1.125 - 0.5 = 0.625$ in

$A_d = \pi(0.5^2)/4 = 0.196\,3$ in^2, Table 8-2: $A_t = 0.141\,9$ in^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.196\,3(0.141\,9)30}{0.196\,3(0.625) + 0.141\,9(0.5)} = 4.316 \text{ Mlbf/in}$$

Members: Steel cyl. head: $t = 0.5$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{\left[1.155(0.5)+0.75-0.5\right](0.75+0.5)}{\left[1.155(0.5)+0.75+0.5\right](0.75-0.5)}} = 33.30 \text{ Mlbf/in}$$

Cast iron: Has two frusta. Midpoint of complete joint is at (1/2 + 5/8)/2 = 0.5625 in.

Upper frustum, $t = 0.5625 - 0.5 = 0.0625$ in, $d = 0.5$ in,
$D = 0.75 + 2(0.5)\tan 30° = 1.327$ in, $E = 14.5$ Mpsi (from Table 8-8)

Eq. (8-20) $\Rightarrow k_2 = 292.7$ Mlbf/in

Lower frustum, $t = 0.5625$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi

Eq. (8-20) $\Rightarrow k_3 = 15.26$ Mlbf/in

Eq. (8-18): $k_m = (1/33.30 + 1/292.7 + 1/15.26)^{-1} = 10.10$ Mlbf/in

$C = k_b / (k_b + k_m) = 4.316/(4.316+10.10) = 0.299$

Table 8-9: $S_p = 85$ kpsi
Assume non-permanent connection. Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(0.141\,9)(85) = 9.05 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g\, A_c$, where A_c is the diameter of the cylinder which is 3.5 in. The external load per bolt is $P = P_{\text{total}}/N$. Thus

$$P = [1\,500\pi(3.5^2)/4](10^{-3})/10 = 1.443 \text{ kips/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85(0.141\,9)}{0.299(1.443)+9.05} = 1.27 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{85(0.141\,9)-9.05}{0.299(1.443)} = 6.98 \qquad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.05}{1.443(1-0.299)} = 8.95 \qquad \textit{Ans.}$$

8-35 Bolts: Grip: $l = 20 + 25 = 45$ mm. From Table A-31, the nut height is $H = 8.4$ mm. $L \geq l + H = 45 + 8.4 = 53.4$ mm. Although Table A-17 indicates to go to 60 mm, 55 mm is readily available

Round up to $L = 55$ mm *Ans.*

Eq. (8-14): $L_T = 2d + 6 = 2(10) + 6 = 26$ mm

Table 8-7: $l_d = L - L_T = 55 - 26 = 29$ mm, $l_t = l - l_d = 45 - 29 = 16$ mm

$A_d = \pi(10^2)/4 = 78.5$ mm^2, Table 8-1: $A_t = 58.0$ mm^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58.0)207}{78.5(16) + 58.0(29)} = 320.8 \text{ MN/m}$$

Members: Steel cyl. head: $t = 20$ mm, $d = 10$ mm, $D = 15$ mm, $E = 207$ GPa. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)10}{\ln\dfrac{\left[1.155(20)+15-10\right](15+10)}{\left[1.155(20)+15+10\right](15-10)}} = 3503 \text{ MN/m}$$

Cast iron: Has two frusta. Midpoint of complete joint is at $(20 + 25)/2 = 22.5$ mm

Upper frustum, $t = 22.5 - 20 = 2.5$ mm, $d = 10$ mm,
$D = 15 + 2(20)\tan 30° = 38.09$ mm, $E = 100$ GPa (from Table 8-8),
Eq. (8-20) $\Rightarrow$ $k_2 = 45\,880$ MN/m

Lower frustum, $t = 22.5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 100$ GPa

Eq. (8-20) $\Rightarrow$ $k_3 = 1632$ MN/m

Eq. (8-18): $k_m = (1/3503 + 1/45\,880 + 1/1632)^{-1} = 1087$ MN/m

$C = k_b / (k_b + k_m) = 320.8/(320.8+1087) = 0.228$

Table 8-11: $S_p = 830$ MPa
Assume non-permanent connection. Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(58.0)(830)10^{-3} = 36.1 \text{ kN}$$

The total external load is $P_{\text{total}} = p_g A_c$, where A_c is the diameter of the cylinder which is 0.8 m. The external load per bolt is $P = P_{\text{total}} / N$. Thus

$$P = [550\pi(0.8^2)/4]/36 = 7.679 \text{ kN/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{830(58.0)10^{-3}}{0.228(7.679) + 36.1} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{830(58.0)10^{-3} - 36.1}{0.228(7.679)} = 6.88 \; Ans.$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{36.1}{7.679(1-0.228)} = 6.09 \qquad Ans.$$

8-36 Bolts: Grip, $l = 3/8 + 1/2 = 0.875$ in. From Table A-31, the nut height is $H = 3/8$ in. $L \geq l + H = 0.875 + 3/8 = 1.25$ in.

Let $L = 1.25$ in *Ans.*

Eq. (8-13): $L_T = 2d + 0.25 = 2(7/16) + 0.25 = 1.125$ in

Table 8-7: $l_d = L - L_T = 1.25 - 1.125 = 0.125$ in, $l_t = l - l_d = 0.875 - 0.125 = 0.75$ in

$A_d = \pi(7/16)^2/4 = 0.150\,3$ in^2, Table 8-2: $A_t = 0.106\,3$ in^2

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.150\,3(0.106\,3)30}{0.150\,3(0.75) + 0.106\,3(0.125)} = 3.804 \text{ Mlbf/in}$$

Members: Steel cyl. head: $t = 0.375$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 30$ Mpsi. Eq. (8-20),

$$k_1 = \frac{0.5774\pi(30)0.4375}{\ln\dfrac{\left[1.155(0.375)+0.65625-0.4375\right](0.65625+0.4375)}{\left[1.155(0.375)+0.65625+0.4375\right](0.65625-0.4375)}} = 31.40 \text{ Mlbf/in}$$

Cast iron: Has two frusta. Midpoint of complete joint is at (3/8 + 1/2)/2 = 0.4375 in.
Upper frustum, $t = 0.4375 - 0.375 = 0.0625$ in, $d = 0.4375$ in,
$D = 0.65625 + 2(0.375)\tan 30° = 1.089$ in, $E = 14.5$ Mpsi (from Table 8-8)

Eq. (8-20) $\Rightarrow$ $k_2 = 195.5$ Mlbf/in

Lower frustum, $t = 0.4375$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 14.5$ Mpsi

Eq. (8-20) $\Rightarrow$ $k_3 = 14.08$ Mlbf/in

Eq. (8-18): $k_m = (1/31.40 + 1/195.5 + 1/14.08)^{-1} = 9.261$ Mlbf/in

$C = k_b / (k_b + k_m) = 3.804/(3.804 + 9.261) = 0.291$

Table 8-9: $S_p = 120$ kpsi
Assume non-permanent connection. Eqs. (8-31) and (8-32)

$$F_i = 0.75\, A_t\, S_p = 0.75(0.106\ 3)(120) = 9.57 \text{ kips}$$

The total external load is $P_{\text{total}} = p_g\, A_c$, where A_c is the diameter of the cylinder which is 3.25 in. The external load per bolt is $P = P_{\text{total}}/N$. Thus

$$P = [1\ 200\pi(3.25^2)/4](10^{-3})/8 = 1.244 \text{ kips/bolt}$$

Yielding factor of safety, Eq. (8-28):

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120(0.106\ 3)}{0.291(1.244)+9.57} = 1.28 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29):

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.106\ 3)-9.57}{0.291(1.244)} = 8.80 \quad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30):

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.57}{1.244(1-0.291)} = 10.9 \qquad \textit{Ans.}$$

8-37 From Table 8-7, $h = t_1 = 20$ mm
For $t_2 > d$, $l = h + d/2 = 20 + 12/2 = 26$ mm
$L \geq h + 1.5\,d = 20 + 1.5(12) = 38$ mm. Round up to $L = 40$ mm
$L_T = 2d + 6 = 2(12) + 6 = 30$ mm
$l_d = L - L_T = 40 - 20 = 10$ mm
$l_t = l - l_d = 26 - 10 = 16$ mm

From Table 8-1, $A_t = 84.3$ mm^2. $A_d = \pi(12^2)/4 = 113.1$ mm^2
Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{113.1(84.3)207}{113.1(16) + 84.3(10)} = 744.0 \text{ MN/m}$$

Similar to Fig. 8-21, we have three frusta.
Top frusta, steel: $t = l/2 = 13$ mm, $d = 12$ mm, $D = 18$ mm, $E = 207$ GPa. Eq. (8-20)

$$k_1 = \frac{0.5774\pi(207)12}{\ln\dfrac{\left[1.155(13)+18-12\right](18+12)}{\left[1.155(13)+18+12\right](18-12)}} = 5\,316 \text{ MN/m}$$

Middle frusta, steel: $t = 20 - 13 = 7$ mm, $d = 12$ mm, $D = 18 + 2(13 - 7)\tan 30° = 24.93$ mm, $E = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 15\,660$ MN/m

Lower frusta, cast iron: $t = 26 - 20 = 6$ mm, $d = 12$ mm, $D = 18$ mm, $E = 100$ GPa (see Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 3\,887$ MN/m

Eq. (8-18), $k_m = (1/5\,316 + 1/15\,660 + 1/3\,887)^{-1} = 1\,964$ MN/m

$$C = k_b / (k_b + k_m) = 744.0/(744.0 + 1\,964) = 0.275$$

Table 8-11: $S_p = 650$ MPa. From Prob. 8-33, $P = 4.712$ kN. Assume a non-permanent connection. Eqs. (8-31) and (8-32),

$$F_i = 0.75\,A_t\,S_p = 0.75(84.3)(650)10^{-3} = 41.1 \text{ kN}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{650(84.3)10^{-3}}{0.275(4.712) + 41.1} = 1.29 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{650(84.3)10^{-3} - 41.1}{0.275(4.712)} = 10.7 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{41.1}{4.712(1-0.275)} = 12.0 \qquad Ans.$$

8-38 From Table 8-7, $h = t_1 = 0.5$ in
For $t_2 > d$, $l = h + d/2 = 0.5 + 0.5/2 = 0.75$ in
$L \geq h + 1.5\,d = 0.5 + 1.5(0.5) = 1.25$ in. Let $L = 1.25$ in
$L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in. All threaded.
From Table 8-1, $A_t = 0.141\,9$ in^2. The bolt stiffness is $k_b = A_t\,E / l = 0.141\,9(30)/0.75 = 5.676$ Mlbf/in
Similar to Fig. 8-21, we have three frusta.
Top frusta, steel: $t = l / 2 = 0.375$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 30$ Mpsi

$$k_1 = \frac{0.5774\pi(30)0.5}{\ln\dfrac{\left[1.155(0.375)+0.75-0.5\right](0.75+0.5)}{\left[1.155(0.375)+0.75+0.5\right](0.75-0.5)}} = 38.45 \text{ Mlbf/in}$$

Middle frusta, steel: $t = 0.5 - 0.375 = 0.125$ in, $d = 0.5$ in,
$D = 0.75 + 2(0.75 - 0.5)\tan 30° = 1.039$ in, $E = 30$ Mpsi.
Eq. (8-20) $\Rightarrow$ $k_2 = 184.3$ Mlbf/in

Lower frusta, cast iron: $t = 0.75 - 0.5 = 0.25$ in, $d = 0.5$ in, $D = 0.75$ in, $E = 14.5$ Mpsi.
Eq. (8-20) $\Rightarrow$ $k_3 = 23.49$ Mlbf/in

Eq. (8-18), $k_m = (1/38.45 + 1/184.3 + 1/23.49)^{-1} = 13.51$ Mlbf/in

$$C = k_b / (k_b + k_m) = 5.676 / (5.676 + 13.51) = 0.296$$

Table 8-9, $S_p = 85$ kpsi. From Prob. 8-34, $P = 1.443$ kips/bolt. Assume a non-permanent connection. Eqs. (8-31) and (8-32),

$$F_i = 0.75\,A_t\,S_p = 0.75(0.141\,9)(85) = 9.05 \text{ kips}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85(0.141\,9)}{0.296(1.443) + 9.05} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{85(0.141\,9) - 9.05}{0.296(1.443)} = 7.05 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{9.05}{1.443(1-0.296)} = 8.91 \qquad Ans.$$

8-39 From Table 8-7, $h = t_1 = 20$ mm
For $t_2 > d$, $l = h + d/2 = 20 + 10/2 = 25$ mm
$L \geq h + 1.5\,d = 20 + 1.5(10) = 35$ mm. Let $L = 35$ mm
$L_T = 2d + 6 = 2(10) + 6 = 26$ mm
$l_d = L - L_T = 35 - 26 = 9$ mm
$l_t = l - l_d = 25 - 9 = 16$ mm

From Table 8-1, $A_t = 58.0$ mm^2. $A_d = \pi(10^2)/4 = 78.5$ mm^2
Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{78.5(58.0)207}{78.5(16) + 58.0(9)} = 530.1 \text{ MN/m}$$

Similar to Fig. 8-21, we have three frusta.
Top frusta, steel: $t = l/2 = 12.5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 207$ GPa. Eq. (8-20)

$$k_1 = \frac{0.5774\pi(207)10}{\ln\dfrac{[1.155(12.5)+15-10](15+10)}{[1.155(12.5)+15+10](15-10)}} = 4\,163 \text{ MN/m}$$

Middle frusta, steel: $t = 20 - 12.5 = 7.5$ mm, $d = 10$ mm, $D = 15 + 2(12.5 - 7.5)\tan 30° = 20.77$ mm, $E = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 10\,975$ MN/m

Lower frusta, cast iron: $t = 25 - 20 = 5$ mm, $d = 10$ mm, $D = 15$ mm, $E = 100$ GPa (see Table 8-8). Eq. (8-20) $\Rightarrow$ $k_3 = 3\,239$ MN/m

Eq. (8-18), $k_m = (1/4\,163 + 1/10\,975 + 1/3\,239)^{-1} = 1\,562$ MN/m

$$C = k_b/(k_b + k_m) = 530.1/(530.1 + 1\,562) = 0.253$$

Table 8-11: $S_p = 830$ MPa. From Prob. 8-35, $P = 7.679$ kN/bolt. Assume a non-permanent connection. Eqs. (8-31) and (8-32),

$$F_i = 0.75\,A_t S_p = 0.75(58.0)(830)10^{-3} = 36.1 \text{ kN}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{830(58.0)10^{-3}}{0.253(7.679) + 36.1} = 1.27 \qquad Ans.$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{830(58.0)10^{-3} - 36.1}{0.253(7.679)} = 6.20 \qquad Ans.$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{36.1}{7.679(1-0.253)} = 6.29 \qquad Ans.$$

8-40 From Table 8-7, $h = t_1 = 0.375$ in
For $t_2 > d$, $l = h + d/2 = 0.375 + 0.4375/2 = 0.59375$ in
$L \geq h + 1.5\,d = 0.375 + 1.5(0.4375) = 1.031$ in. Round up to $L = 1.25$ in
$L_T = 2d + 0.25 = 2(0.4375) + 0.25 = 1.125$ in
$l_d = L - L_T = 1.25 - 1.125 = 0.125$
$l_t = l - l_d = 0.59375 - 0.125 = 0.46875$ in
$A_d = \pi(7/16)^2/4 = 0.150\ 3$ in^2, Table 8-2: $A_t = 0.106\ 3$ in^2

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.150\ 3(0.106\ 3)30}{0.150\ 3(0.46875) + 0.106\ 3(0.125)} = 5.724 \text{ Mlbf/in}$$

Similar to Fig. 8-21, we have three frusta.
Top frusta, steel: $t = l/2 = 0.296875$ in, $d = 0.4375$ in, $D = 0.65625$ in, $E = 30$ Mpsi

$$k_1 = \frac{0.5774\pi(30)0.4375}{\ln\dfrac{\left[1.155(0.296875) + 0.656255 - 0.4375\right](0.75 + 0.656255)}{\left[1.155(0.296875) + 0.75 + 0.656255\right](0.75 - 0.656255)}} = 35.52 \text{ Mlbf/in}$$

Middle frusta, steel: $t = 0.375 - 0.296875 = 0.078125$ in, $d = 0.4375$ in,
$D = 0.65625 + 2(0.59375 - 0.375)\tan 30° = 0.9088$ in, $E = 30$ Mpsi.
Eq. (8-20) $\Rightarrow$ $k_2 = 215.8$ Mlbf/in

Lower frusta, cast iron: $t = 0.59375 - 0.375 = 0.21875$ in, $d = 0.4375$ in, $D = 0.65625$ in,
$E = 14.5$ Mpsi. Eq. (8-20) $\Rightarrow$ $k_3 = 20.55$ Mlbf/in

Eq. (8-18), $k_m = (1/35.52 + 1/215.8 + 1/20.55)^{-1} = 12.28$ Mlbf/in

$$C = k_b / (k_b + k_m) = 5.724/(5.724 + 12.28) = 0.318$$

Table 8-9, S_p = 120 kpsi. From Prob. 8-34, P = 1.244 kips/bolt. Assume a non-permanent connection. Eqs. (8-31) and (8-32),

$$F_i = 0.75\, A_t\, S_p = 0.75(0.106\ 3)(120) = 9.57 \text{ kips}$$

Yielding factor of safety, Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120\left(0.106\ 3\right)}{0.318\left(1.244\right) + 9.57} = 1.28 \qquad \textit{Ans.}$$

Overload factor of safety, Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120\left(0.106\ 3\right) - 9.57}{0.318\left(1.244\right)} = 8.05 \qquad \textit{Ans.}$$

Separation factor of safety, Eq. (8-30)

$$n_0 = \frac{F_i}{P\left(1 - C\right)} = \frac{9.57}{1.244\left(1 - 0.318\right)} = 11.3 \qquad \textit{Ans.}$$

8-41 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let d = 10 mm. Using Eq. (8-20) on members, and combining using Eq. (8-18), yields k_m = 1 141 MN/m (see Prob. 8-33 for method of calculation.

2. Look up the nut height in Table A-31. For the example, H = 8.4 mm. From this, L is rounded up from the calculation of $l + H$ = 40 + 8.4 = 48.4 mm to 50 mm. Next, calculations are made for L_T = 2(10) + 6 = 26 mm, l_d = 50 – 26 = 24 mm, l_t = 40 – 24 = 16 mm. From step 1, $A_d = \pi(10^2)/4$ = 78.54 mm^2. Next, from Table 8-1, A_t = 78.54 mm^2. From Eq. (8-17), k_b = 356 MN/m. Finally, from Eq. (*e*), p. 421, C = 0.238.

3. From Prob. 8-33, the bolt circle diameter is E = 200 mm. Substituting this for D_b in Eq. (8-34), the number of bolts are

$$N = \frac{\pi D_b}{4d} = \frac{\pi\left(200\right)}{4\left(10\right)} = 15.7$$

Rounding this up gives N = 16.

4. Next, select a grade bolt. Based on the solution to Prob. 8-33, the strength of ISO 9.8 was so high to give very large factors of safety for overload and separation. Try ISO 4.6

with S_p = 225 MPa. From Eqs. (8-31) and (8-32) for a non-permanent connection, F_i = 9.79 kN.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_c/N$, where from Prob 8-33, p_g = 6 MPa, and $A_c = \pi(100^2)/4$. This gives P = 3.39 kN/bolt.

6. Using Eqs. (8-28) to (8-30) yield n_p = 1.23, n_L = 4.05, and n_0 = 3.79.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. The results for four bolt sizes are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
8	854	6.8	50	22	28	12	50.26	36.6	233.9
10	1141	8.4	50	26	24	16	78.54	58	356
12	1456	10.8	55	30	25	15	113.1	84.3	518.8
14	1950	12.8	55	34	21	19	153.9	115	686.3

d	C	N	S_p	F_i	P	n_p	n_L	n_0
8	0.215	20	225	6.18	2.71	1.22	3.53	2.90
10	0.238	16	225	9.79	3.39	1.23	4.05	3.79
12	0.263	13*	225	14.23	4.17	1.24	4.33	4.63
14	0.276	12	225	19.41	4.52	1.25	5.19	5.94

*Rounded down from13.08997, so spacing is slightly greater than four diameters.

Any one of the solutions is acceptable. A decision-maker might be cost such as N × cost/bolt, and/or N × cost per hole, etc.

8-42 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.
1. Select the diameter, d. For this example, let d = 0.5 in. Using Eq. (8-20) on three frusta (see Prob. 8-34 solution), and combining using Eq. (8-19), yields k_m = 10.10 Mlbf/in.

2. Look up the nut height in Table A-31. For the example, H = 0.4375 in. From this, L is rounded up from the calculation of $l + H$ = 1.125 + 0.4375 = 1.5625 in to 1.75 in. Next, calculations are made for $L_T = 2(0.5) + 0.25$ = 1.25 in, l_d = 1.75 – 1.25 = 0.5 in, l_t = 1.125 – 0.5 = 0.625 in. From step 1, $A_d = \pi(0.5^2)/4$ = 0.1963 in^2. Next, from Table 8-1, A_t = 0.141 9 in^2. From Eq. (8-17), k_b = 4.316 Mlbf/in. Finally, from Eq. (*e*), p. 421, C = 0.299.

3. From Prob. 8-34, the bolt circle diameter is E = 6 in. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(6)}{4(0.5)} = 9.425$$

Rounding this up gives $N = 10$.

4. Next, select a grade bolt. Based on the solution to Prob. 8-34, the strength of SAE grade 5 was adequate. Use this with S_p = 85 kpsi. From Eqs. (8-31) and (8-32) for a non-permanent connection, F_i = 9.046 kips.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_c/N$, where from Prob 8-34, p_g = 1 500 psi, and $A_c = \pi(3.5^2)/4$. This gives P = 1.660 kips/bolt.

6. Using Eqs. (8-28) to (8-30) yield n_p = 1.26, n_L = 6.07, and n_0 = 7.78.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
0.375	6.75	0.3281	1.5	1	0.5	0.625	0.1104	0.0775	2.383
0.4375	9.17	0.375	1.5	1.125	0.375	0.75	0.1503	0.1063	3.141
0.5	10.10	0.4375	1.75	1.25	0.5	0.625	0.1963	0.1419	4.316
0.5625	11.98	0.4844	1.75	1.375	0.375	0.75	0.2485	0.182	5.329

d	C	N	S_p	F_i	P	n_p	n_L	n_0
0.375	0.261	13	85	4.941	1.277	1.25	4.95	5.24
0.4375	0.273	11	85	6.777	1.509	1.26	5.48	6.18
0.5	0.299	10	85	9.046	1.660	1.26	6.07	7.78
0.5625	0.308	9	85	11.6	1.844	1.27	6.81	9.09

Any one of the solutions is acceptable. A decision-maker might be cost such as N × cost/bolt, and/or N × cost per hole, etc.

8-43 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let d = 10 mm. Using Eq. (8-20) on three frusta (see Prob. 8-35 solution), and combining using Eq. (8-19), yields k_m = 1 087 MN/m.

2. Look up the nut height in Table A-31. For the example, H = 8.4 mm. From this, L is rounded up from the calculation of $l + H$ = 45 + 8.4 = 53.4 mm to 55 mm. Next, calculations are made for $L_T = 2(10) + 6$ = 26 mm, l_d = 55 – 26 = 29 mm, l_t = 45 – 29 = 16 mm. From step 1, $A_d = \pi(10^2)/4$ = 78.54 mm^2. Next, from Table 8-1, A_t = 58.0 mm^2. From Eq. (8-17), k_b = 320.9 MN/m. Finally, from Eq. (*e*), p. 421, C = 0.228.

3. From Prob. 8-35, the bolt circle diameter is E = 1000 mm. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(1000)}{4(10)} = 78.5$$

Rounding this up gives $N = 79$. A rather large number, since the bolt circle diameter, E is so large. Try larger bolts.

4. Next, select a grade bolt. Based on the solution to Prob. 8-35, the strength of ISO 9.8 was so high to give very large factors of safety for overload and separation. Try ISO 5.8 with $S_p = 380$ MPa. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i = 16.53$ kN.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_c/N$, where from Prob 8-35, $p_g = 0.550$ MPa, and $A_c = \pi(800^2)/4$. This gives $P = 4.024$ kN/bolt.

6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.26$, $n_L = 6.01$, and $n_0 = 5.32$.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. The results for three bolt sizes are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
10	1087	8.4	55	26	29	16	78.54	58	320.9
20	3055	18	65	46	19	26	314.2	245	1242
36	6725	31	80	78	2	43	1018	817	3791

d	C	N	S_p	F_i	P	n_p	n_L	n_0
10	0.228	79	380	16.53	4.024	1.26	6.01	5.32
20	0.308	40	380	69.83	7.948	1.29	9.5	12.7
36	0.361	22	380	232.8	14.45	1.3	14.9	25.2

A large range is presented here. Any one of the solutions is acceptable. A decision-maker might be cost such as $N \times$ cost/bolt, and/or $N \times$ cost per hole, etc.

8-44 This is a design problem and there is no closed-form solution path or a unique solution. What is presented here is one possible iterative approach. We will demonstrate this with an example.

1. Select the diameter, d. For this example, let $d = 0.375$ in. Using Eq. (8-20) on three frusta (see Prob. 8-36 solution), and combining using Eq. (8-19), yields $k_m = 7.42$ Mlbf/in.

2. Look up the nut height in Table A-31. For the example, $H = 0.3281$ in. From this, $L \geq l + H = 0.875 + 0.3281 = 1.2031$ in. Rounding up, $L = 1.25$. Next, calculations are made for $L_T = 2(0.375) + 0.25 = 1$ in, $l_d = 1.25 - 1 = 0.25$ in, $l_t = 0.875 - 0.25 = 0.625$ in.

From step 1, $A_d = \pi(0.375^2)/4 = 0.1104$ in^2. Next, from Table 8-1, $A_t = 0.0775$ in^2. From Eq. (8-17), $k_b = 2.905$ Mlbf/in. Finally, from Eq. (*e*), p. 421, $C = 0.263$.

3. From Prob. 8-36, the bolt circle diameter is $E = 6$ in. Substituting this for D_b in Eq. (8-34), for the number of bolts

$$N = \frac{\pi D_b}{4d} = \frac{\pi(6)}{4(0.375)} = 12.6$$

Rounding this up gives $N = 13$.

4. Next, select a grade bolt. Based on the solution to Prob. 8-36, the strength of SAE grade 8 seemed high for overload and separation. Try SAE grade 5 with $S_p = 85$ kpsi. From Eqs. (8-31) and (8-32) for a non-permanent connection, $F_i = 4.941$ kips.

5. The external load requirement per bolt is $P = 1.15\, p_g\, A_c/N$, where from Prob 8-34, $p_g = 1\,200$ psi, and $A_c = \pi(3.25^2)/4$. This gives $P = 0.881$ kips/bolt.

6. Using Eqs. (8-28) to (8-30) yield $n_p = 1.27$, $n_L = 6.65$, and $n_0 = 7.81$.

Steps 1 - 6 can be easily implemented on a spreadsheet with lookup tables for the tables used from the text. For this solution we only looked at one bolt size, $\frac{3}{8}-16$, but evaluated changing the bolt grade. The results for four bolt grades are shown below. The dimension of each term is consistent with the example given above.

d	k_m	H	L	L_T	l_d	l_t	A_d	A_t	k_b
0.375	7.42	0.3281	1.25	1	0.25	0.625	0.1104	0.0775	2.905

d	C	N	SAE grade	S_p	F_i	P	n_p	n_L	n_0
0.375	0.281	13	1	33	1.918	0.881	1.18	2.58	3.03
0.375	0.281	13	2	55	3.197	0.881	1.24	4.30	5.05
0.375	0.281	13	4	65	3.778	0.881	1.25	5.08	5.97
0.375	0.281	13	5	85	4.941	0.881	1.27	6.65	7.81

Note that changing the bolt grade only affects S_p, F_i , n_p, n_L, and n_0. Any one of the solutions is acceptable, especially the lowest grade bolt.

8-45 (**a**) $F_b' = RF_{b,\max}' \sin\theta$

Half of the external moment is contributed by the line load in the interval $0 \le \theta \le \pi$

$$\frac{M}{2} = \int_0^{\pi} F_b' R^2 \sin\theta \, d\theta = \int_0^{\pi} F_{b,\max}' R^2 \sin^2\theta \, d\theta$$

$$\frac{M}{2} = \frac{\pi}{2} F_{b,\max}' R^2$$

from which $F_{b,\max}' = \dfrac{M}{\pi R^2}$

$$F_{\max} = \int_{\phi_1}^{\phi_2} F_b' R \sin\theta \, d\theta = \frac{M}{\pi R^2} \int_{\phi_1}^{\phi_2} R \sin\theta \, d\theta = \frac{M}{\pi R} (\cos\phi_1 - \cos\phi_2)$$

Noting $\phi_1 = 75°$, $\phi_2 = 105°$,

$$F_{\max} = \frac{12\ 000}{\pi(8/2)} (\cos 75^\circ - \cos 105^\circ) = 494 \text{ lbf} \quad \textit{Ans.}$$

(b)

$$F_{\max} = F_{b,\max}' R \Delta\phi = \frac{M}{\pi R^2}(R)\left(\frac{2\pi}{N}\right) = \frac{2M}{RN}$$

$$F_{\max} = \frac{2(12\ 000)}{(8/2)(12)} = 500 \text{ lbf} \quad \textit{Ans.}$$

(c) $F = F_{\max} \sin\theta$

$M = 2 F_{\max} R [(1) \sin^2 90° + 2 \sin^2 60° + 2 \sin^2 30° + (1) \sin^2 (0)] = 6F_{\max} R$

from which,

$$F_{\max} = \frac{M}{6R} = \frac{12\ 000}{6(8/2)} = 500 \text{ lbf} \quad \textit{Ans.}$$

The simple general equation resulted from part (*b*)

$$F_{\max} = \frac{2M}{RN}$$

8-46

(**a**) From Table 8-11, $S_p = 600$ MPa. From Table 8-1, $A_t = 353$ mm^2.

Eq. (8-31): $F_i = 0.9 A_t S_p = 0.9(353)(600)(10^{-3}) = 190.6$ kN

Table 8-15: $K = 0.18$

Eq. (8-27): $T = K F_i d = 0.18(190.6)(24) = 823$ N·m *Ans.*

(**b**) Washers: $t = 4.6$ mm, $d = 24$ mm, $D = 1.5(24) = 36$ mm, $E = 207$ GPa.

Eq. (8-20),

$$k_1 = \frac{0.5774\pi(207)24}{\ln\dfrac{\left[1.155(4.6)+36-24\right](36+24)}{\left[1.155(4.6)+36+24\right](36-24)}} = 31\,990 \text{ MN/m}$$

Cast iron: $t = 20$ mm, $d = 24$ mm, $D = 36 + 2(4.6)\tan 30° = 41.31$ mm, $E = 135$ GPa. Eq. (8-20) $\Rightarrow$ $k_2 = 10\,785$ MN/m

Steel joist: $t = 20$ mm, $d = 24$ mm, $D = 41.31$ mm, $E = 207$ GPa. Eq. (8-20) $\Rightarrow$ $k_3 = 16\,537$ MN/m

Eq. (8-18): $k_m = (2 / 31\,990 + 1 / 10\,785 + 1 / 16\,537)^{-1} = 4\,636$ MN/m

Bolt: $l = 2(4.6) + 2(20) = 49.2$ mm. Nut, Table A-31, $H = 21.5$ mm. $L > 49.2 + 21.5 = 70.7$ mm. From Table A-17, use $L = 80$ mm. From Eq. (8-14)

$L_T = 2(24) + 6 = 54$ mm, $l_d = 80 - 54 = 26$ mm, $l_t = 49.2 - 26 = 23.2$ mm

From Table (8-1), $A_t = 353$ mm^2, $A_d = \pi(24^2) / 4 = 452.4$ mm^2

Eq. (8-17):

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{452.4(353)207}{452.4(23.2)+353(26)} = 1680 \text{ MN/m}$$

$C = k_b / (k_b + k_m) = 1680 / (1680 + 4636) = 0.266$, $S_p = 600$ MPa, $F_i = 190.6$ kN, $P = P_{\text{total}} / \text{N} = 18/4 = 4.5$ kN

Yield: From Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{600(353)10^{-3}}{0.266(4.5)+190.6} = 1.10 \quad Ans.$$

Load factor: From Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{600(353)10^{-3} - 190.6}{0.266(4.5)} = 17.7 \qquad Ans.$$

Separation: From Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{190.6}{4.5(1-0.266)} = 57.7 \qquad Ans.$$

As was stated in the text, bolts are typically preloaded such that the yielding factor of safety is not much greater than unity which is the case for this problem. However, the other load factors indicate that the bolts are oversized for the external load.

8-47 (**a**) ISO M 20 × 2.5 grade 8.8 coarse pitch bolts, lubricated.

Table 8-2, $A_t = 245$ mm^2

Table 8-11, $S_p = 600$ MPa

$F_i = 0.90\, A_t\, S_p = 0.90(245)600(10^{-3}) = 132.3$ kN

Table 8-15, $K = 0.18$

Eq. (8-27), $T = KF_i\, d = 0.18(132.3)20 = 476$ N · m *Ans.*

(**b**) Table A-31, $H = 18$ mm, $L \geq L_G + H = 48 + 18 = 66$ mm. Round up to $L = 80$ mm per Table A-17.

$$L_T = 2d + 6 = 2(20) + 6 = 46 \text{ mm}$$
$$l_d = L - L_T = 80 - 46 = 34 \text{ mm}$$
$$l_t = l - l_d = 48 - 34 = 14 \text{ mm}$$

$A_d = \pi(20^2)/4 = 314.2$ mm^2,

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{314.2(245)(207)}{314.2(14) + 245(34)} = 1251.9 \text{ MN/m}$$

Members: Since all members are steel use Eq. (8-22) with $E = 207$ MPa, $l = 48$ mm, $d = 20$mm

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(207)20}{2\ln\left[5\dfrac{0.5774(48) + 0.5(20)}{0.5774(48) + 2.5(20)}\right]} = 4236 \text{ MN/m}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{1251.9}{1251.9 + 4236} = 0.228$$

$P = P_{\text{total}} / N = 40/2 = 20$ kN,

Yield: From Eq. (8-28)

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{600(245)10^{-3}}{0.228(20) + 132.3} = 1.07 \qquad Ans.$$

Load factor: From Eq. (8-29)

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{600(245)10^{-3} - 132.3}{0.228(20)} = 3.22 \qquad Ans.$$

Separation: From Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = \frac{132.3}{20(1-0.228)} = 8.57 \qquad Ans.$$

8-48 From Prob. 8-29 solution, $P_{\max}$ =13.33 kips, $C = 0.2$, $F_i = 12.77$ kips, $A_t = 0.141\,9$ in^2

$$\sigma_i = \frac{F_i}{A_t} = \frac{12.77}{0.141\,9} = 90.0 \text{ kpsi}$$

Eq. (8-39), $$\sigma_a = \frac{CP}{2A_t} = \frac{0.2(13.33)}{2(0.141\,9)} = 9.39 \text{ kpsi}$$

Eq. (8-41), $$\sigma_m = \sigma_a + \sigma_i = 9.39 + 90.0 = 99.39 \text{ kpsi}$$

(**a**) Goodman Eq. (8-45) for grade 8 bolts, $S_e = 23.2$ kpsi (Table 8-17), $S_{ut} = 150$ kpsi (Table 8-9)

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{23.2(150 - 90.0)}{9.39(150 + 23.2)} = 0.856 \qquad Ans.$$

(**b**) Gerber Eq. (8-46)

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(9.39)23.2}\left[150\sqrt{150^2 + 4(23.2)(23.2 + 90.0)} - 150^2 - 2(90.0)23.2\right] = 1.32 \qquad Ans.$$

(**c**) ASME-elliptic Eq. (8-47) with $S_p = 120$ kpsi (Table 8-9)

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{23.2}{9.39\left(120^2 + 23.2^2\right)}\left[120\sqrt{120^2 + 23.2^2 - 90^2} - 90(23.2)\right] = 1.30 \qquad Ans.$$

8-49 **Attention to the Instructor**. Part (**d**) requires the determination of the endurance strength, S_e, of a class 5.8 bolt. Table 8-17 does not provide this and the student will be required to estimate it by other means [see the solution of part (**d**)].

Per bolt, $P_{b\max} = 60/8 = 7.5$ kN, $P_{b\min} = 20/8 = 2.5$ kN

$$C = \frac{k_b}{k_b + k_m} = \frac{1}{1+2.6} = 0.278$$

(**a**) Table 8-1, $A_t = 20.1$ mm^2; Table 8-11, $S_p = 380$ MPa

Eqs. (8-31) and (8-32), $F_i = 0.75\, A_t\, S_p = 0.75(20.1)380(10^{-3}) = 5.73$ kN

Yield, Eq. (8-28), $$n_p = \frac{S_p A_t}{CP + F_i} = \frac{380(20.1)10^{-3}}{0.278(7.5) + 5.73} = 0.98 \qquad Ans.$$

(**b**) Overload, Eq. (8-29), $$n_L = \frac{S_p A_t - F_i}{CP} = \frac{380(20.1)10^{-3} - 5.73}{0.278(7.5)} = 0.915 \qquad Ans.$$

(**c**) Separation, Eq. (8-30), $$n_0 = \frac{F_i}{P(1-C)} = \frac{5.73}{7.5(1-0.278)} = 1.06 \qquad Ans.$$

(**d**) Goodman, Eq. (8-35), $$\sigma_a = \frac{C(P_{b\max} - P_{b\min})}{2A_t} = \frac{0.278(7.5-2.5)10^3}{2(20.1)} = 34.6 \text{ MPa}$$

Eq. (8-36), $$\sigma_m = \frac{C(P_{b\max} + P_{b\min})}{2A_t} + \frac{F_i}{A_t} = \frac{0.278(7.5+2.5)10^3}{2(20.1)} + \frac{5.73(10^3)}{20.1} = 354.2 \text{ MPa}$$

Table 8-11, $S_{ut} = 520$ MPa, $\sigma_i = F_i / A_t = 5.73(10^3)/20.1 = 285$ MPa

We have a problem for S_e. Table 8-17 does not list S_e for class 5.8 bolts. Here, we will estimate S_e using the methods of Chapter 6. Estimate S'_e from the,

Eq. (6-8), p. 282, $S'_e = 0.5 S_{ut} = 0.5(520) = 260$ MPa .

Table 6-2, p. 288, $a = 4.51$, $b = -0.265$

Eq. (6-19), p. 287, $k_a = a S_{ut}^b = 4.51(520^{-0.265}) = 0.860$

Eq. (6-21), p. 288, $k_b = 1$

Eq. (6-26), p.290, $k_c = 0.85$

The fatigue stress-concentration factor, from Table 8-16, is $K_f = 2.2$. For simple axial loading and infinite-life it is acceptable to reduce the endurance limit by K_f and use the nominal stresses in the stress/strength/design factor equations. Thus,

Eq. (6-18), p. 287, $S_e = k_a\, k_b\, k_c\, S'_e\, / K_f = 0.86(1)0.85(260) / 2.2 = 86.4$ MPa

Eq. (8-38),

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = \frac{86.4(520-285)}{520(34.6) + 86.4(354.2-285)} = 0.847 \qquad Ans.$$

It is obvious from the various answers obtained, the bolted assembly is undersized. This can be rectified by a one or more of the following: more bolts, larger bolts, higher class bolts.

8-50 Per bolt, $P_{b\max} = P_{\max}/N = 80/10 = 8$ kips, $P_{b\min} = P_{\min}/N = 20/10 = 2$ kips

$C = k_b / (k_b + k_m) = 4/(4 + 12) = 0.25$

(**a**) Table 8-2, $A_t = 0.141\ 9$ in^2, Table 8-9, $S_p = 120$ kpsi and $S_{ut} = 150$ kpsi

Table 8-17, S_e = 23.2 kpsi

Eqs. (8-31) and (8-32), $F_i = 0.75\, A_t S_p \;\Rightarrow \sigma_i = F_i / A_t = 0.75\, S_p = 0.75(120) = 90$ kpsi

Eq. (8-35), $$\sigma_a = \frac{C\left(P_{b\max} - P_{b\min}\right)}{2A_t} = \frac{0.25(8-2)}{2(0.141\,9)} = 5.29 \text{ kpsi}$$

Eq. (8-36), $$\sigma_m = \frac{C\left(P_{b\max} + P_{b\min}\right)}{2A_t} + \sigma_i = \frac{0.25(8+2)}{2(0.141\,9)} + 90 = 98.81 \text{ kpsi}$$

Eq. (8-38),

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{23.2(150-90)}{150(5.29) + 23.2(98.81-90)} = 1.39 \qquad \textit{Ans.}$$

8-51 From Prob. 8-33, C = 0.263, $P_{\max}$ = 4.712 kN / bolt, F_i = 41.1 kN, S_p = 650 MPa, and A_t = 84.3 mm^2

$$\sigma_i = 0.75\, S_p = 0.75(650) = 487.5 \text{ MPa}$$

Eq. (8-39): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.263(4.712)10^3}{2(84.3)} = 7.350 \text{ MPa}$$

Eq. (8-40) $$\sigma_m = \frac{CP}{2A_t} + \frac{F_i}{A_t} = 7.350 + 487.5 = 494.9 \text{ MPa}$$

(**a**) Goodman: From Table 8-11, S_{ut} = 900 MPa, and from Table 8-17, S_e = 140 MPa

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)} = \frac{140(900-487.5)}{7.350(900+140)} = 7.55 \qquad \textit{Ans.}$$

(**b**) Gerber:

Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(7.350)140}\left[900\sqrt{900^2 + 4(140)(140+487.5)} - 900^2 - 2(487.5)(140)\right]$$

$$= 11.4 \qquad \textit{Ans.}$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{140}{7.350\left(650^2 + 140^2\right)}\left[650\sqrt{650^2 + 140^2 - 487.5^2} - 487.5(140)\right] = 9.73 \qquad \textit{Ans.}$$

8-52 From Prob. 8-34, $C = 0.299$, $P_{\max} = 1.443$ kips/bolt, $F_i = 9.05$ kips, $S_p = 85$ kpsi, and $A_t = 0.141\,9$ in^2

$$\sigma_i = 0.75S_p = 0.75(85) = 63.75 \text{ kpsi}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.299(1.443)}{2(0.141\,9)} = 1.520 \text{ kpsi}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 1.520 + 63.75 = 65.27 \text{ kpsi}$$

(**a**) Goodman: From Table 8-9, $S_{ut} = 120$ kpsi, and from Table 8-17, $S_e = 18.8$ kpsi

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)} = \frac{18.8(120 - 63.75)}{1.520(120 + 18.8)} = 5.01 \qquad \textit{Ans.}$$

(**b**) Gerber:

Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(1.520)18.6}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.75)} - 120^2 - 2(63.75)(18.6)\right]$$

$$= 7.45 \qquad \textit{Ans.}$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{18.6}{1.520\left(85^2 + 18.6^2\right)}\left[85\sqrt{85^2 + 18.6^2 - 63.75^2} - 63.75(18.6)\right] = 6.22 \qquad \textit{Ans.}$$

8-53 From Prob. 8-35, $C = 0.228$, $P_{\max} = 7.679$ kN/bolt, $F_i = 36.1$ kN, $S_p = 830$ MPa, and $A_t = 58.0$ mm^2

$$\sigma_i = 0.75\, S_p = 0.75(830) = 622.5 \text{ MPa}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.228(7.679)10^3}{2(58.0)} = 15.09 \text{ MPa}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 15.09 + 622.5 = 637.6 \text{ MPa}$$

(**a**) Goodman: From Table 8-11, $S_{ut} = 1040$ MPa, and from Table 8-17, $S_e = 162$ MPa

Eq. (8-45): $$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{162(1040 - 622.5)}{15.09(1040 + 162)} = 3.73 \qquad \textit{Ans.}$$

(**b**) Gerber:

Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(15.09)162}\left[1040\sqrt{1040^2 + 4(162)(162 + 622.5)} - 1040^2 - 2(622.5)(162)\right]$$

$$= 5.74 \qquad \textit{Ans.}$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a(S_p^2 + S_e^2)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{162}{15.09(830^2 + 162^2)}\left[830\sqrt{830^2 + 162^2 - 622.5^2} - 622.5(162)\right] = 5.62 \qquad \textit{Ans.}$$

8-54 From Prob. 8-36, $C = 0.291$, $P_{\max} = 1.244$ kips/bolt, $F_i = 9.57$ kips, $S_p = 120$ kpsi, and $A_t = 0.106\,3$ in^2

$$\sigma_i = 0.75 S_p = 0.75(120) = 90 \text{ kpsi}$$

Eq. (8-37): $$\sigma_a = \frac{CP}{2A_t} = \frac{0.291(1.244)}{2(0.106\,3)} = 1.703 \text{ kpsi}$$

Eq. (8-38) $$\sigma_m = \frac{CP}{2A_t} + \sigma_i = 1.703 + 90 = 91.70 \text{ kpsi}$$

(**a**) Goodman: From Table 8-9, S_{ut} = 150 kpsi, and from Table 8-17, S_e = 23.2 kpsi

Eq. (8-45): $$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)} = \frac{23.2(150-90)}{1.703(150+23.2)} = 4.72 \qquad \textit{Ans.}$$

(**b**) Gerber:

Eq. (8-46):

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(1.703)23.2}\left[150\sqrt{150^2 + 4(23.2)(23.2+90)} - 150^2 - 2(90)(23.2)\right]$$

$$= 7.28 \qquad \textit{Ans.}$$

(**c**) ASME-elliptic:

Eq. (8-47):

$$n_f = \frac{S_e}{\sigma_a\left(S_p^2 + S_e^2\right)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right)$$

$$= \frac{23.2}{1.703\left(120^2 + 18.6^2\right)}\left[120\sqrt{120^2 + 23.2^2 - 90^2} - 90(23.2)\right] = 7.24 \qquad \textit{Ans.}$$

8-55 From Prob. 8-51, C = 0.263, S_e = 140 MPa, S_{ut} = 900 MPa, A_t = 84.4 mm^2, σ_i = 487.5 MPa, and $P_{\max}$ = 4.712 kN.

$$P_{\min} = P_{\max} / 2 = 4.712/2 = 2.356 \text{ kN}$$

Eq. (8-35): $$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.263(4.712 - 2.356)10^3}{2(84.3)} = 3.675 \text{ MPa}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.263\left(4.712 + 2.356\right)10^3}{2\left(84.3\right)} + 487.5 = 498.5 \text{ MPa}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{140\left(900 - 487.5\right)}{900\left(3.675\right) + 140\left(498.5 - 487.5\right)} = 11.9 \qquad \textit{Ans.}$$

8-56 From Prob. 8-52, $C = 0.299$, $S_e = 18.8$ kpsi, $S_{ut} = 120$ kpsi, $A_t = 0.141\,9$ in^2, $\sigma_i = 63.75$ kpsi, and $P_{\max} = 1.443$ kips

$$P_{\min} = P_{\max} / 2 = 1.443/2 = 0.722 \text{ kips}$$

Eq. (8-35):
$$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.299\left(1.443 - 0.722\right)}{2\left(0.141\,9\right)} = 0.760 \text{ kpsi}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.299\left(1.443 + 0.722\right)}{2\left(0.141\,9\right)} + 63.75 = 66.03 \text{ kpsi}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{18.8\left(120 - 63.75\right)}{120\left(0.760\right) + 18.8\left(66.03 - 63.75\right)} = 7.89 \qquad \textit{Ans.}$$

8-57 From Prob. 8-53, $C = 0.228$, $S_e = 162$ MPa, $S_{ut} = 1040$ MPa, $A_t = 58.0$ mm^2, $\sigma_i = 622.5$ MPa, and $P_{\max} = 7.679$ kN.

$$P_{\min} = P_{\max} / 2 = 7.679/2 = 3.840 \text{ kN}$$

Eq. (8-35):
$$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.228\left(7.679 - 3.840\right)10^3}{2\left(58.0\right)} = 7.546 \text{ MPa}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.228\left(7.679 + 3.840\right)10^3}{2\left(58.0\right)} + 622.5 = 645.1 \text{ MPa}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{162\left(1040 - 622.5\right)}{1040\left(7.546\right) + 162\left(645.1 - 622.5\right)} = 5.88 \qquad \textit{Ans.}$$

8-58 From Prob. 8-54, $C = 0.291$, $S_e = 23.2$ kpsi, $S_{ut} = 150$ kpsi, $A_t = 0.106\ 3$ in^2, $\sigma_i = 90$ kpsi, and $P_{\max} = 1.244$ kips

$$P_{\min} = P_{\max} / 2 = 1.244/2 = 0.622 \text{ kips}$$

Eq. (8-35):
$$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.291\left(1.244 - 0.622\right)}{2\left(0.106\ 3\right)} = 0.851 \text{ kpsi}$$

Eq. (8-36):

$$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i$$

$$= \frac{0.291\left(1.244 + 0.622\right)}{2\left(0.106\ 3\right)} + 90 = 92.55 \text{ kpsi}$$

Eq. (8-38):

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{23.2\left(150 - 90\right)}{150\left(0.851\right) + 23.2\left(92.55 - 90\right)} = 7.45 \qquad \textit{Ans.}$$

8-59 Let the repeatedly-applied load be designated as P. From Table A-22, S_{ut} = 93.7 kpsi. Referring to the Figure of Prob. 3-122, the following notation will be used for the radii of Section AA.

$$r_i = 1.5 \text{ in}, \quad r_o = 2.5 \text{ in}, \quad r_c = 2.0 \text{ in}$$

From Table 3-4, p. 121, with R = 0.5 in

$$r_n = \frac{R^2}{2\left(r_c - \sqrt{r_c^2 - R^2}\right)} = \frac{0.5^2}{2\left(2 - \sqrt{2^2 - 0.5^2}\right)} = 1.968\ 246 \text{ in}$$

$$e = r_c - r_n = 2.0 - 1.968\ 246 = 0.031\ 754 \text{ in}$$

$$c_o = r_o - r_n = 2.5 - 1.968\ 246 = 0.531\ 754 \text{ in}$$

$$c_i = r_n - r_i = 1.968\ 246 - 1.5 = 0.468\ 246 \text{ in}$$

$$A = \pi(1^2) / 4 = 0.7854 \text{ in}^2$$

If P is the maximum load

$$M = Pr_c = 2P$$

$$\sigma_i = \frac{P}{A}\left(1 + \frac{r_c c_i}{e r_i}\right) = \frac{P}{0.785\ 4}\left(1 + \frac{2(0.468)}{0.031\ 754(1.5)}\right) = 26.29P$$

$$\sigma_a = \sigma_m = \frac{\sigma_i}{2} = \frac{26.294P}{2} = 13.15P$$

(**a**) *Eye:* Section AA,
Table 6-2, p. 288, $a = 14.4$ kpsi, $b = -0.718$
Eq. (6-19), p. 287,

$$k_a = 14.4(93.7)^{-0.718} = 0.553$$

Eq. (6-23), p. 289,

$$d_e = 0.370\, d$$

Eq. (6-20), p. 288,

$$k_b = \left(\frac{0.37}{0.30}\right)^{-0.107} = 0.978$$

Eq. (6-26), p. 290,

$$k_c = 0.85$$

Eq. (6-8), p. 282,

$$S'_e = 0.5S_{ut} = 0.5\left(93.7\right) = 46.85 \text{ kpsi}$$

Eq. (6-18) p. 287,

$$S_e = 0.553(0.978)0.85(46.85) = 21.5 \text{ kpsi}$$

From Table 6-7, p. 307, for Gerber

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

With $\sigma_m = \sigma_a$,

$$n_f = \frac{1}{2}\frac{S_{ut}^2}{\sigma_a S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2}\right] = \frac{1}{2}\frac{93.7^2}{13.15P(21.5)}\left[-1 + \sqrt{1 + \left(\frac{2(21.5)}{93.7}\right)^2}\right] = \frac{1.557}{P}$$

where P is in kips.

Thread: Die cut. Table 8-17 gives S_e = 18.6 kpsi for rolled threads. Use Table 8-16 to find S_e for die cut threads

$$S_e = 18.6(3.0/3.8) = 14.7 \text{ kpsi}$$

Table 8-2, $A_t = 0.663$ in^2, $\sigma = P/A_t = P/0.663 = 1.51\,P$, $\sigma_a = \sigma_m = \sigma/2 = 0.755\,P$

From Table 6-7, Gerber

$$n_f = \frac{1}{2}\frac{S_{ut}^2}{\sigma_a S_e}\left[-1+\sqrt{1+\left(\frac{2S_e}{S_{ut}}\right)^2}\right] = \frac{1}{2}\frac{93.7^2}{0.755P(14.7)}\left[-1+\sqrt{1+\left(\frac{2(14.7)}{93.7}\right)^2}\right] = \frac{19.01}{P}$$

Comparing 1910/*P* with 19 200/*P*, we conclude that the *eye* is weaker in fatigue. *Ans.*

(**b**) Strengthening steps can include heat treatment, cold forming, cross section change (a round is a poor cross section for a curved bar in bending because the bulk of the material is located where the stress is small). *Ans.*

(**c**) For $n_f = 2$

$$P = \frac{1.557(10^3)}{2} = 779 \text{ lbf, max. load} \quad \textit{Ans.}$$

8-60 Member, Eq. (8-22) with E =16 Mpsi, d = 0.75 in, and l = 1.5 in

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\dfrac{0.5774l+0.5d}{0.5774l+2.5d}\right)} = \frac{0.5774\pi(16)0.75}{2\ln\left[5\dfrac{0.5774(1.5)+0.5(0.75)}{0.5774(1.5)+2.5(0.75)}\right]} = 13.32 \text{ Mlbf/in}$$

Bolt, Eq. (8-13),

$$L_T = 2d + 0.25 = 2(0.75) + 0.25 = 1.75 \text{ in}$$

$$l = 1.5 \text{ in}$$

$$l_d = L - L_T = 2.5 - 1.75 = 0.75 \text{ in}$$

$$l_t = l - l_d = 1.5 - 0.75 = 0.75 \text{ in}$$

Table 8-2,

$$A_t = 0.373 \text{ in}^2$$

$$A_d = \pi(0.75^2)/4 = 0.442 \text{ in}^2$$

Eq. (8-17),

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.442(0.373)30}{0.442(0.75)+0.373(0.75)} = 8.09 \text{ Mlbf/in}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{8.09}{8.09+13.32} = 0.378$$

Eq. (8-35),

$$\sigma_a = \frac{C(P_{\max} - P_{\min})}{2A_t} = \frac{0.378(6-4)}{2(0.373)} = 1.013 \text{ kpsi}$$

Eq.(8-36),

$$\sigma_m = \frac{C(P_{\max} + P_{\min})}{2A_t} + \frac{F_i}{A_t} = \frac{0.378(6+4)}{2(0.373)} + \frac{25}{0.373} = 72.09 \text{ kpsi}$$

(**a**) From Table 8-9, S_p = 85 kpsi, and Eq. (8-51), the yielding factor of safety is

$$n_p = \frac{S_p}{\sigma_m + \sigma_a} = \frac{85}{72.09+1.013} = 1.16 \qquad \textit{Ans.}$$

(**b**) From Eq. (8-29), the overload factor of safety is

$$n_L = \frac{S_p A_t - F_i}{CP_{\max}} = \frac{85(0.373)-25}{0.378(6)} = 2.96 \qquad \textit{Ans.}$$

(**c**) From Eq. (8-30), the factor of safety based on joint separation is

$$n_0 = \frac{F_i}{P_{\max}(1-C)} = \frac{25}{6(1-0.378)} = 6.70 \qquad \textit{Ans.}$$

(**d**) From Table 8-17, S_e = 18.6 kpsi; Table 8-9, S_{ut} = 120 kps; the preload stress is $\sigma_i = F_i / A_t$ = 25/0.373 = 67.0 kpsi; and from Eq. (8-38)

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = \frac{18.6(120-67.0)}{120(1.013)+18.6(72.09-67.0)} = 4.56 \qquad \textit{Ans.}$$

8-61 (**a**) Table 8-2, A_t = 0.1419 in^2
Table 8-9, S_p = 120 kpsi, S_{ut} = 150 kpsi
Table 8-17, S_e = 23.2 kpsi
Eqs. (8-31) and (8-32), $\sigma_i = 0.75\, S_p = 0.75(120) = 90$ kpsi

$$C = \frac{k_b}{k_b + k_m} = \frac{4}{4 + 16} = 0.2$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.2P}{2(0.141\,9)} = 0.705P \text{ kpsi}$$

Eq. (8-45) for the Goodman criterion,

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{\sigma_a\left(S_{ut} + S_e\right)} = \frac{23.2(150 - 90)}{0.705P(150 + 23.2)} = \frac{11.4}{P} = 2 \quad \Rightarrow \quad P = 5.70 \text{ kips} \quad \textit{Ans.}$$

(**b**) $F_i = 0.75A_t S_p = 0.75(0.141\,9)120 = 12.77$ kips

Yield, Eq. (8-28),

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{120\left(0.141\,9\right)}{0.2\left(5.70\right) + 12.77} = 1.22 \qquad \textit{Ans.}$$

Load factor, Eq. (8-29),

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{120(0.141\,9) - 12.77}{0.2(5.70)} = 3.74 \quad \textit{Ans.}$$

Separation load factor, Eq. (8-30)

$$n_0 = \frac{F_i}{P(1 - C)} = \frac{12.77}{5.70(1 - 0.2)} = 2.80 \quad \textit{Ans.}$$

8-62 Table 8-2, $A_t = 0.969$ in^2 (coarse), $A_t = 1.073$ in^2 (fine)
Table 8-9, $S_p = 74$ kpsi, $S_{ut} = 105$ kpsi
Table 8-17, $S_e = 16.3$ kpsi

Coarse thread,

$$F_i = 0.75\,A_t\,S_p = 0.75(0.969)74 = 53.78 \text{ kips}$$
$$\sigma_i = 0.75\,S_p = 0.75(74) = 55.5 \text{ kpsi}$$
$$\sigma_a = \frac{CP}{2A_t} = \frac{0.30P}{2(0.969)} = 0.155P \text{ kpsi}$$

Gerber, Eq. (8-46),

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e\left(S_e + \sigma_i\right)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2\left(0.155P\right)16.3}\left[105\sqrt{105^2 + 4\left(16.3\right)\left(16.3 + 55.5\right)} - 105^2 - 2\left(55.5\right)16.3\right] = \frac{64.28}{P}$$

With $n_f = 2$,

$$P = \frac{64.28}{2} = 32.14 \text{ kip} \quad \textit{Ans.}$$

Fine thread,

$$F_i = 0.75\, A_t\, S_p = 0.75(1.073)74 = 59.55 \text{kips}$$
$$\sigma_i = 0.75\, S_p = 0.75(74) = 55.5 \text{ kpsi}$$
$$\sigma_a = \frac{CP}{2A_t} = \frac{0.32P}{2(1.073)} = 0.149P \text{ kpsi}$$

The only thing that changes in Eq. (8-46) is σ_a. Thus,

$$n_f = \frac{0.155}{0.149}\frac{64.28}{P} = \frac{66.87}{P} = 2 \quad \Rightarrow \quad P = 33.43 \text{ kips} \qquad \textit{Ans.}$$

Percent improvement,

$$\frac{33.43 - 32.14}{32.14}(100) \doteq 4\% \quad \textit{Ans.}$$

8-63 For an M 30 × 3.5 ISO 8.8 bolt with P = 65 kN/bolt and C = 0.28

Table 8-1, A_t = 561 mm^2
Table 8-11, S_p = 600 MPa, S_{ut} = 830 MPa
Table 8-17, S_e = 129 MPa

Eq. (8-31), $F_i = 0.75F_p = 0.75\, A_t\, S_p$
$= 0.75(5610600(10^{-3})) = 252.45$ kN

$$\sigma_i = 0.75\, S_p = 0.75(600) = 450 \text{ MPa}$$

Eq. (8-39), $$\sigma_a = \frac{CP}{2A_t} = \frac{0.28(65)10^3}{2(561)} = 16.22 \text{ MPa}$$

Gerber, Eq. (8-46),

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$
$$= \frac{1}{2(16.22)129}\left[830\sqrt{830^2 + 4(129)(129 + 450)} - 830^2 - 2(450)129\right]$$
$$= 4.75 \qquad \textit{Ans.}$$

The yielding factor of safety, from Eq. (8-28) is

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{600(561)10^{-3}}{0.28(65) + 252.45} = 1.24 \qquad Ans.$$

From Eq. (8-29), the load factor is

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{600(561)10^{-3} - 252.45}{0.28(65)} = 4.62 \qquad Ans.$$

The separation factor, from Eq. (8-30) is

$$n_0 = \frac{F_i}{P(1-C)} = \frac{252.45}{65(1-0.28)} = 5.39 \qquad Ans.$$

8-64 (**a**) Table 8-2, $A_t = 0.077\ 5\ \text{in}^2$
Table 8-9, $S_p = 85$ kpsi, $S_{ut} = 120$ kpsi
Table 8-17, $S_e = 18.6$ kpsi
Unthreaded grip,

$$k_b = \frac{A_d E}{l} = \frac{\pi(0.375)^2(30)}{4(13.5)} = 0.245 \text{ Mlbf/in per bolt} \qquad Ans.$$

$$A_m = \frac{\pi}{4}[(D + 2t)^2 - D^2] = \frac{\pi}{4}(4.75^2 - 4^2) = 5.154 \text{ in}^2$$

$$k_m = \frac{A_m E}{l} = \frac{5.154(30)}{12}\left(\frac{1}{6}\right) = 2.148 \text{ Mlbf/in/bolt.} \qquad Ans.$$

(**b**) $F_i = 0.75\, A_t\, S_p = 0.75(0.0775)(85) = 4.94$ kip

$$\sigma_i = 0.75 S_p = 0.75(85) = 63.75 \text{ kpsi}$$

$$P = pA = \frac{2000}{6}\left[\frac{\pi}{4}(4)^2\right] = 4189 \text{ lbf/bolt}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{0.245}{0.245 + 2.148} = 0.102$$

$$\sigma_a = \frac{CP}{2A_t} = \frac{0.102(4.189)}{2(0.0775)} = 2.77 \text{ kpsi}$$

From Eq. (8-46) for Gerber fatigue criterion,

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2(2.77)18.6}\left[120\sqrt{120^2 + 4(18.6)(18.6 + 63.75)} - 120^2 - 2(63.75)18.6\right] = 4.09 \qquad Ans.$$

(**c**) Pressure causing joint separation from Eq. (8-30)

$$n_0 = \frac{F_i}{P(1-C)} = 1$$

$$P = \frac{F_i}{1-C} = \frac{4.94}{1-0.102} = 5.50 \text{ kip}$$

$$p = \frac{P}{A} = \frac{5.50}{\pi(4^2)\,/\,4}6 = 2.63 \text{ kpsi} \quad \textit{Ans.}$$

8-65 From the solution of Prob. 8-64, $A_t = 0.077\,5$ in^2, $S_{ut} = 120$ kpsi, $S_e = 18.6$ kpsi, $C = 0.102$, $\sigma_i = 63.75$ kpsi

$P_{\max} = p_{\max}A = 2\,\pi(4^2)/4 = 25.13$ kpsi, $P_{\min} = p_{\min}A = 1.2\,\pi(4^2)/4 = 15.08$ kpsi,

Eq. (8-35), $$\sigma_a = \frac{C\left(P_{\max} - P_{\min}\right)}{2A_t} = \frac{0.102\left(25.13-15.08\right)}{2\left(0.077\,5\right)} = 6.61 \text{ kpsi}$$

Eq. (8-36), $$\sigma_m = \frac{C\left(P_{\max} + P_{\min}\right)}{2A_t} + \sigma_i = \frac{0.102\left(25.13+15.08\right)}{2\left(0.077\,5\right)} + 63.75 = 90.21 \text{ kpsi}$$

Eq. (8-38),

$$n_f = \frac{S_e\left(S_{ut} - \sigma_i\right)}{S_{ut}\sigma_a + S_e\left(\sigma_m - \sigma_i\right)} = \frac{18.6\left(120-63.75\right)}{120\left(6.61\right)+18.6\left(90.21-63.75\right)} = 0.814 \quad \textit{Ans.}$$

This predicts a fatigue failure.

8-66 Members: $S_y = 57$ kpsi, $S_{sy} = 0.577(57) = 32.89$ kpsi.
Bolts: SAE grade 5, $S_y = 92$ kpsi, $S_{sy} = 0.577(92) = 53.08$ kpsi

Shear in bolts,

$$A_s = 2\left[\frac{\pi(0.25^2)}{4}\right] = 0.0982 \text{ in}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{0.0982(53.08)}{2} = 2.61 \text{ kips}$$

Bearing on bolts,

$$A_b = 2(0.25)0.25 = 0.125 \text{ in}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{0.125(92)}{2} = 5.75 \text{ kips}$$

Bearing on member,

$$F_b = \frac{0.125(57)}{2} = 3.56 \text{ kips}$$

Tension of members,

$$A_t = (1.25 - 0.25)(0.25) = 0.25 \text{ in}^2$$

$$F_t = \frac{0.25(57)}{2} = 7.13 \text{ kip}$$

$$F = \min(2.61,\ 5.75,\ 3.56,\ 7.13) = 2.61 \text{ kip} \quad \textit{Ans.}$$

The shear in the bolts controls the design.

8-67 Members, Table A-20, $S_y = 42$ kpsi
Bolts, Table 8-9, $S_y = 130$ kpsi, $S_{sy} = 0.577(130) = 75.01$ kpsi

Shear of bolts,

$$A_s = 2\left[\frac{\pi(5/16)^2}{4}\right] = 0.1534 \text{ in}^2$$

$$\tau = \frac{F_s}{A_s} = \frac{5}{0.1534} = 32.6 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau} = \frac{75.01}{32.6} = 2.30 \quad \textit{Ans.}$$

Bearing on bolts,

$$A_b = 2(0.25)(5/16) = 0.1563 \text{ in}^2$$

$$\sigma_b = -\frac{5}{0.1563} = -32.0 \text{ kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{130}{32.0} = 4.06 \quad \textit{Ans.}$$

Bearing on members,

$$n = \frac{S_y}{|\sigma_b|} = \frac{42}{32} = 1.31 \quad \textit{Ans.}$$

Tension of members,

$$A_t = [2.375 - 2(5/16)](1/4) = 0.4375 \text{ in}^2$$

$$\sigma_t = \frac{5}{0.4375} = 11.4 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{42}{11.4} = 3.68 \qquad Ans.$$

8-68 Members: Table A-20, $S_y = 490$ MPa, $S_{sy} = 0.577(490) = 282.7$ MPa
Bolts: Table 8-11, ISO class 5.8, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Shear in bolts,

$$A_s = 2\left[\frac{\pi(20^2)}{4}\right] = 628.3 \text{ mm}^2$$

$$F_s = \frac{A_s S_{sy}}{n} = \frac{628.3(242.3)10^{-3}}{2.5} = 60.9 \text{ kN}$$

Bearing on bolts,

$$A_b = 2(20)20 = 800 \text{ mm}^2$$

$$F_b = \frac{A_b S_{yc}}{n} = \frac{800(420)10^{-3}}{2.5} = 134 \text{ kN}$$

Bearing on member,

$$F_b = \frac{800(490)10^{-3}}{2.5} = 157 \text{ kN}$$

Tension of members,

$$A_t = (80 - 20)(20) = 1\,200 \text{ mm}^2$$

$$F_t = \frac{1\,200(490)10^{-3}}{2.5} = 235 \text{ kN}$$

$$F = \min(60.9,\ 134,\ 157,\ 235) = 60.9 \text{ kN} \qquad Ans.$$

The shear in the bolts controls the design.

8-69 Members: Table A-20, $S_y = 320$ MPa
Bolts: Table 8-11, ISO class 5.8, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Shear of bolts,

$$A_s = \pi(20^2)/4 = 314.2 \text{ mm}^2$$

$$\tau_s = \frac{90\left(10^3\right)}{3\left(314.2\right)} = 95.48 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{242.3}{95.48} = 2.54 \qquad Ans.$$

Bearing on bolt,

$$A_b = 3(20)15 = 900 \text{ mm}^2$$

$$\sigma_b = -\frac{90\left(10^3\right)}{900} = -100 \text{ MPa}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{420}{100} = 4.2 \quad \textit{Ans.}$$

Bearing on members,

$$n = \frac{S_y}{|\sigma_b|} = \frac{320}{100} = 3.2 \quad \textit{Ans.}$$

Tension on members,

$$\sigma_t = \frac{F}{A} = \frac{90\left(10^3\right)}{15[190 - 3(20)]} = 46.15 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_t} = \frac{320}{46.15} = 6.93 \quad \textit{Ans.}$$

8-70 Members: $S_y = 57$ kpsi
Bolts: $S_y = 100$ kpsi, $S_{sy} = 0.577(100) = 57.7$ kpsi
Shear of bolts,

$$A = 3\left[\frac{\pi(1/4)^2}{4}\right] = 0.1473 \text{ in}^2$$

$$\tau_s = \frac{F}{A_s} = \frac{5}{0.1473} = 33.94 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_s} = \frac{57.7}{33.94} = 1.70 \quad \textit{Ans.}$$

Bearing on bolts,

$$A_b = 3(1/4)(5/16) = 0.2344 \text{ in}^2$$

$$\sigma_b = -\frac{F}{A_b} = -\frac{5}{0.2344} = -21.3 \text{ kpsi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{21.3} = 4.69 \quad \textit{Ans.}$$

Bearing on members,

$A_b = 0.2344 \text{ in}^2$ (From bearing on bolts calculation)

$\sigma_b = -21.3$ kpsi (From bearing on bolts calculation)

$$n = \frac{S_y}{|\sigma_b|} = \frac{57}{21.3} = 2.68 \qquad Ans.$$

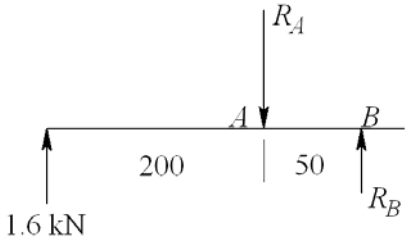
Tension in members, failure across two bolts,

$$A_t = \frac{5}{16}\left[2.375 - 2(1/4)\right] = 0.5859 \text{ in}^2$$

$$\sigma_t = \frac{F}{A_t} = \frac{5}{0.5859} = 8.534 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma_t} = \frac{57}{8.534} = 6.68 \qquad Ans.$$

8-71 By symmetry, the reactions at each support is 1.6 kN. The free-body diagram for the left member is



$$\sum M_B = 0 \quad 1.6(250) - 50R_A = 0 \quad \Rightarrow \quad R_A = 8 \text{ kN}$$
$$\sum M_A = 0 \quad 200(1.6) - 50R_B = 0 \quad \Rightarrow \quad R_B = 6.4 \text{ kN}$$

Members: Table A-20, $S_y = 370$ MPa
Bolts: Table 8-11, $S_y = 420$ MPa, $S_{sy} = 0.577(420) = 242.3$ MPa

Bolt shear, $A_s = \frac{\pi}{4}(12^2) = 113.1 \text{ mm}^2$

$$\tau = \frac{F_{\max}}{A_s} = \frac{8(10^3)}{113.1} = 70.73 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{70.73} = 3.43$$

Bearing on member, $A_b = td = 10(12) = 120 \text{ mm}^2$

$$\sigma_b = -\frac{8(10^3)}{120} = -66.67 \text{ MPa}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{370}{66.67} = 5.55$$

Strength of member. The bending moments at the hole locations are:
in the left member at A, $M_A = 1.6(200) = 320$ N · m. In the right member at B, $M_B = 8(50) = 400$ N · m. The bending moment is greater at B

$$I_B = \frac{1}{12}[10(50^3) - 10(12^3)] = 102.7(10^3) \text{ mm}^4$$

$$\sigma_B = \frac{M_A c}{I_A} = \frac{400(25)}{102.7(10^3)}(10^3) = 97.37 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_A} = \frac{370}{97.37} = 3.80$$

At the center, call it point C,

$$M_C = 1.6(350) = 560 \text{ N} \cdot \text{m}$$

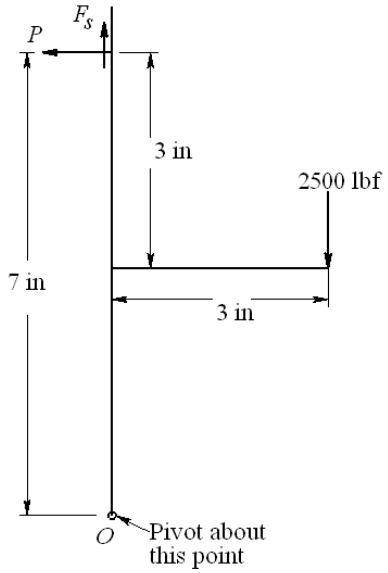
$$I_C = \frac{1}{12}(10)(50^3) = 104.2(10^3) \text{ mm}^4$$

$$\sigma_C = \frac{M_C c}{I_C} = \frac{560(25)}{104.2(10^3)}(10^3) = 134.4 \text{ MPa}$$

$$n = \frac{S_y}{\sigma_C} = \frac{370}{134.4} = 2.75 < 3.80 \quad \text{more critical at } C$$

$$n = \min(3.04,\ 3.80,\ 2.75) = 2.72 \quad \textit{Ans.}$$

8-72 The free-body diagram of the bracket, assuming the upper bolt takes all the shear and tensile load is



$$F_s = 2500 \text{ lbf}$$

$$P = \frac{2500(3)}{7} = 1071 \text{ lbf}$$

Table A-31, $H = 7/16 = 0.4375$ in. Grip, $l = 2(1/2) = 1$ in. $L \geq l + H = 1.4375$ in. Use 1.5 in bolts.
Eq. (8-13), $L_T = 2d + 0.25 = 2(0.5) + 0.25 = 1.25$ in
Table 8-7, $l_d = L - L_T = 1.5 - 1.25 = 0.25$ in

$l_t = l - l_d = 1 - 0.25 = 0.75$ in

Table 8-2, $A_t = 0.141\ 9$ in^2

$A_d = \pi(0.5^2)/4 = 0.196\ 3$ in^2

Eq. (8-17), $$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.196\ 3(0.141\ 9)30}{0.196\ 3(0.75) + 0.141\ 9(0.25)} = 4.574 \text{ Mlbf/in}$$

Eq. (8-22),

$$k_m = \frac{0.5774\pi Ed}{2\ln\left(5\,\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi(30)0.5}{2\ln\left(5\,\frac{0.5774(1) + 0.5(0.5)}{0.5774(1) + 2.5(0.5)}\right)} = 16.65 \text{ Mlbf/in}$$

$$C = \frac{k_b}{k_b + k_m} = \frac{4.574}{4.574 + 16.65} = 0.216$$

Table 8-9, $S_p = 65$ kpsi

Eqs. (8-31) and (8-32), $F_i = 0.75\, A_t\, S_p = 0.75(0.141\ 9)65 = 6.918$ kips

$\sigma_i = 0.75\, S_p = 0.75(65) = 48.75$ kips

Eq. (*a*), p. 440, $$\sigma_b = \frac{CP + F_i}{A_t} = \frac{0.216(1.071) + 6.918}{0.141\ 9} = 50.38 \text{ kpsi}$$

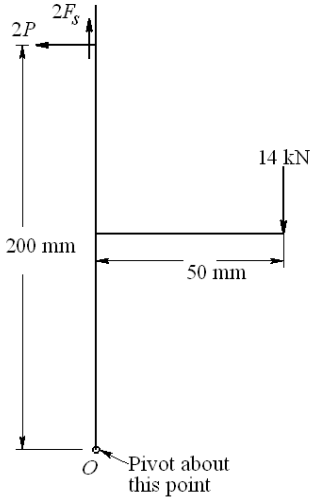
Direct shear, $$\tau_s \doteq \frac{F_s}{A_t} = \frac{3}{0.141\ 9} = 21.14 \text{ kpsi}$$

von Mises stress, Eq. (5-15), p. 223

$$\sigma' = \left(\sigma_b^2 + 3\tau_s^2\right)^{1/2} = \left[50.38^2 + 3\left(21.14^2\right)\right]^{1/2} = 62.3 \text{ kpsi}$$

Stress margin, $m = S_p - \sigma' = 65 - 62.3 = 3.7$ kpsi *Ans.*

8-73



$$2P(200) = 14(50)$$

$$P = \frac{14(50)}{2(200)} = 1.75 \text{ kN per bolt}$$

$F_s = 7$ kN/bolt

$S_p = 380$ MPa

$A_t = 245$ mm^2, $A_d = \frac{\pi}{4}(20^2) = 314.2$ mm^2

$F_i = 0.75(245)(380)(10^{-3}) = 69.83$ kN

$\sigma_i = 0.75(380) = 285$ MPa

$$\sigma_b = \frac{CP + F_i}{A_t} = \left(\frac{0.25(1.75) + 69.83}{245}\right)(10^3) = 287 \text{ MPa}$$

$$\tau = \frac{F_s}{A_d} = \frac{7(10^3)}{314.2} = 22.3 \text{ MPa}$$

$$\sigma' = [287^2 + 3(22.3^2)]^{1/2} = 290 \text{ MPa}$$

$$m = S_p - \sigma' = 380 - 290 = 90 \text{ MPa}$$

Stress margin, $m = S_p - \sigma' = 380 - 90 = 90$ MPa *Ans.*

8-74 Using the result of Prob. 5-67 for lubricated assembly (replace 0.2 with 0.18 per Table 8-15)

$$F_x = \frac{2\pi f\ T}{0.18d}$$

With a design factor of n_d gives

$$T = \frac{0.18 n_d\ F_x d}{2\pi f} = \frac{0.18(3)(1000)d}{2\pi(0.12)} = 716d$$

or $T/d = 716$. Also,

$$\frac{T}{d} = K(0.75 S_p A_t)$$
$$= 0.18(0.75)(85\ 000) A_t$$
$$= 11\ 475 A_t$$

Form a table

Size	A_t	$T/d = 11\ 475 A_t$	n
$\frac{1}{4}$ - 28	0.0364	417.70	1.75
$\frac{5}{16}$ - 24	0.058	665.55	2.8
$\frac{3}{8}$ – 24	0.0878	1007.50	4.23

where the factor of safety in the last column of the table comes from

$$n = \frac{2\pi f(T/d)}{0.18F_x} = \frac{2\pi(0.12)(T/d)}{0.18(1000)} = 0.0042(T/d)$$

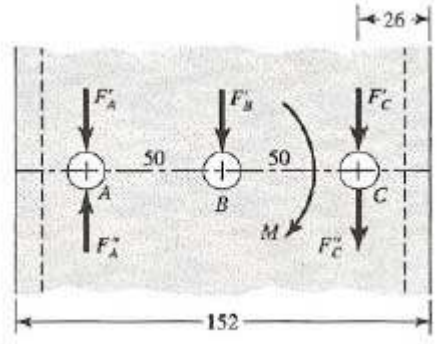
Select a $\frac{3}{8}$" - 24 UNF cap screw. The setting is given by

$$T = (11\ 475 A_t)d = 1007.5(0.375) = 378 \text{ lbf} \cdot \text{in}$$

Given the coarse scale on a torque wrench, specify a torque wrench setting of 400 lbf · in. Check the factor of safety

$$n = \frac{2\pi f\ T}{0.18F_x d} = \frac{2\pi(0.12)(400)}{0.18(1000)(0.375)} = 4.47$$

8-75



Bolts, from Table 8-11, S_y = 420 MPa
Channel, From Table A-20, S_y = 170 MPa. From Table A-7, t = 6.4 mm
Cantilever, from Table A-20, S_y = 190 MPa

$$F'_A = F'_B = F'_C = F/3$$

$$M = (50 + 26 + 125)\,F = 201\,F$$

$$F''_A = F''_C = \frac{201F}{2(50)} = 2.01F$$

Max. force, $$F_C = F'_C + F''_C = \left(\frac{1}{3} + 2.01\right)F = 2.343F \qquad (1)$$

Shear on Bolts: The shoulder bolt shear area, $A_s = \pi(10^2)/4 = 78.54$ mm^2

$$S_{sy} = 0.577(420) = 242.3 \text{ KPa}$$

$$\tau_{\max} = \frac{F_C}{A_s} = \frac{S_{sy}}{n}$$

From Eq. (1), $F_C = 2.343\,F$. Thus

$$F = \frac{S_{sy}}{n}\left(\frac{A_s}{2.343}\right) = \frac{242.3}{2.0}\left(\frac{78.54}{2.343}\right)10^{-3} = 4.06 \text{ kN}$$

Bearing on bolt: The bearing area is $A_b = td = 6.4(10) = 64$ mm^2. Similar to shear

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{420}{2.0}\left(\frac{64}{2.343}\right)10^{-3} = 5.74 \text{ kN}$$

Bearing on channel: $A_b = 64 \text{ mm}^2$, $S_y = 170$ MPa.

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{170}{2.0}\left(\frac{64}{2.343}\right)10^{-3} = 2.32 \text{ kN}$$

Bearing on cantilever: $A_b = 12(10) = 120 \text{ mm}^2$, $S_y = 190$ MPa.

$$F = \frac{S_y}{n}\left(\frac{A_b}{2.343}\right) = \frac{190}{2.0}\left(\frac{120}{2.343}\right)10^{-3} = 4.87 \text{ kN}$$

Bending of cantilever: At C

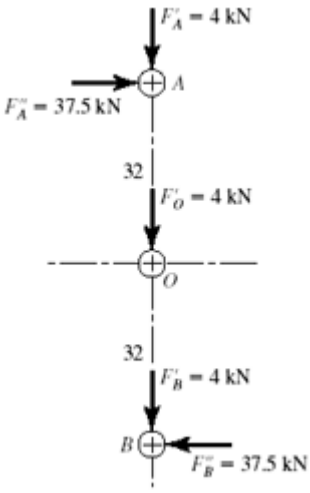
$$I = \frac{1}{12}(12)\left(50^3 - 10^3\right) = 1.24\left(10^5\right) \text{ mm}^4$$

$$\sigma_{\max} = \frac{S_y}{n} = \frac{Mc}{I} = \frac{151Fc}{I} \quad \Rightarrow \quad F = \frac{S_y}{n}\left(\frac{I}{151c}\right)$$

$$F = \frac{190}{2.0}\left[\frac{1.24\left(10^5\right)}{151(25)}\right]10^{-3} = 3.12 \text{ kN}$$

So $F = 2.32$ kN based on bearing on channel. *Ans.*

8-76 Bolts, from Table 8-11, $S_y = 420$ MPa
Bracket, from Table A-20, $S_y = 210$ MPa



$$F' = \frac{12}{3} = 4 \text{ kN}; \; M = 12(200) = 2400 \text{ N} \cdot \text{m}$$
$$F''_A = F''_B = \frac{2400}{64} = 37.5 \text{ kN}$$
$$F_A = F_B = \sqrt{(4)^2 + (37.5)^2} = 37.7 \text{ kN}$$
$$F_O = 4 \text{ kN}$$

Bolt shear:
The shoulder bolt shear area, $A_s = \pi(12^2) / 4 = 113.1 \text{ mm}^2$

$$S_{sy} = 0.577(420) = 242.3 \text{ KPa}$$

$$\tau = \frac{37.7(10)^3}{113} = 333 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau} = \frac{242.3}{333} = 0.728 \qquad \textit{Ans.}$$

Bearing on bolts:

$$A_b = 12(8) = 96 \text{ mm}^2$$

$$\sigma_b = -\frac{37.7(10)^3}{96} = -393 \text{ MPa}$$

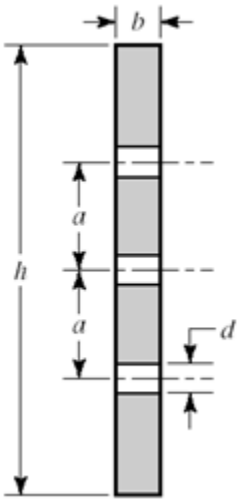
$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{420}{393} = 1.07 \qquad \textit{Ans.}$$

Bearing on member:

$$\sigma_b = -393 \text{ MPa}$$

$$n = \frac{S_{yc}}{|\sigma_b|} = \frac{210}{393} = 0.534 \qquad \textit{Ans.}$$

Bending stress in plate:



$$I = \frac{bh^3}{12} - \frac{bd^3}{12} - 2\left(\frac{bd^3}{12} + a^2bd\right)$$

$$= \frac{8(136)^3}{12} - \frac{8(12)^3}{12} - 2\left[\frac{8(12)^3}{12} + (32)^2(8)(12)\right]$$

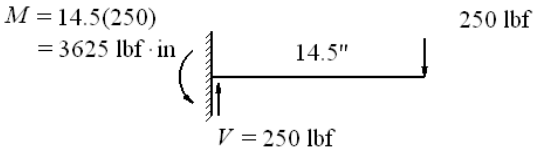
$$= 1.48(10)^6 \text{ mm}^4 \qquad \textit{Ans.}$$

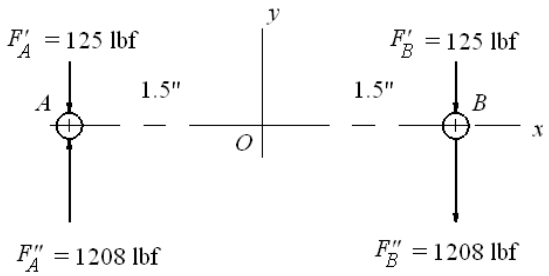
$$\sigma = \frac{Mc}{I} = \frac{2400(68)}{1.48(10)^6}(10)^3 = 110 \text{ MPa}$$

$$n = \frac{S_y}{\sigma} = \frac{210}{110} = 1.91 \qquad \textit{Ans.}$$

Failure is predicted for bolt shear and bearing on member.

8-77





$$F''_A = F''_B = \left(\frac{3625}{3}\right) = 1208 \text{ lbf}$$

$$F_A = 1208 - 125 = 1083 \text{ lbf}, \qquad F_B = 1208 + 125 = 1333 \text{ lbf}$$

Bolt shear:

$$A_s = (\pi / 4)(0.375^2) \;=\; 0.1104 \text{ in}^2$$

$$\tau_{\max} = \frac{F_{\max}}{A_s} = \frac{1333}{0.1104} = 12\,070 \text{ psi}$$

From Table 8-10, S_y = 100 kpsi, $S_{sy} \;=\; 0.577(100) =\; 57.7$ kpsi

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{57.7}{12.07} = 4.78 \qquad \textit{Ans.}$$

Bearing on bolt: Bearing area is $A_b = td$ = 0.375 (0.375) = 0.1406 in^2.

$$\sigma_b = -\frac{F}{A_b} = -\frac{1333}{0.1406} = -9\,481 \text{ psi}$$

$$n = \frac{S_y}{|\sigma_b|} = \frac{100}{9.481} = 10.55 \qquad \textit{Ans.}$$

Bearing on member: From Table A-20, S_y = 54 kpsi. Bearing stress same as bolt

$$n = \frac{S_y}{|\sigma_b|} = \frac{54}{9.481} = 5.70 \qquad \textit{Ans.}$$

Bending of member: At B, $M = 250(13) \;=\; 3250$ lbf·in

$$I = \frac{1}{12}\left(\frac{3}{8}\right)\left[2^3 - \left(\frac{3}{8}\right)^3\right] = 0.2484 \text{ in}^4$$

$$\sigma = \frac{Mc}{I} = \frac{3250(1)}{0.2484} = 13\,080 \text{ psi}$$

$$n = \frac{S_y}{\sigma} = \frac{54}{13.08} = 4.13 \qquad \textit{Ans.}$$

8-78 The direct shear load per bolt is $F' = 2000/6 = 333.3$ lbf. The moment is taken only by the four outside bolts. This moment is $M = 2000(5) = 10\,000$ lbf · in.

Thus $F'' = \dfrac{10\,000}{2(5)} = 1000$ lbf and the resultant bolt load is

$$F = \sqrt{(333.3)^2 + (1000)^2} = 1054 \text{ lbf}$$

Bolt strength, Table 8-9, $S_y = 100$ kpsi; Channel and Plate strength, $S_y = 42$ kpsi

Shear of bolt: $A_s = \pi(0.5)^2/4 = 0.1963$ in^2

$$n = \frac{S_{sy}}{\tau} = \frac{(0.577)(100)}{1.054 \,/\, 0.1963} = 10.7 \qquad \textit{Ans.}$$

Bearing on bolt: Channel thickness is $t = 3/16$ in, $A_b = 0.5(3/16) = 0.09375$ in^2

$$n = \frac{100}{1.054 \,/\, 0.09375} = 8.89 \qquad \textit{Ans.}$$

Bearing on channel: $n = \dfrac{42}{1.054 \,/\, 0.09375} = 3.74 \qquad \textit{Ans.}$

Bearing on plate: $A_b = 0.5(0.25) = 0.125$ in^2

$$n = \frac{42}{1.054 \,/\, 0.125} = 4.98 \qquad \textit{Ans.}$$

Strength of plate:

$$I = \frac{0.25(7.5)^3}{12} - \frac{0.25(0.5)^3}{12} - 2\left[\frac{0.25(0.5)^3}{12} + (0.25)(0.5)(2.5)^2\right] = 7.219 \text{ in}^4$$

$$M = 5000 \text{ lbf} \cdot \text{in per plate}$$

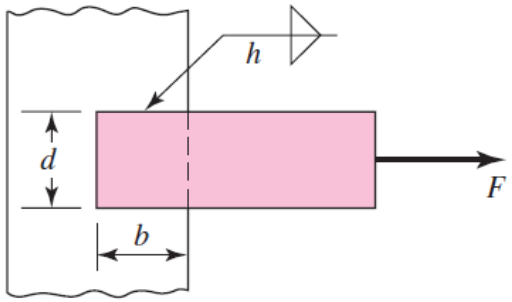
$$\sigma = \frac{Mc}{I} = \frac{5000(3.75)}{7.219} = 2597 \text{ psi}$$

$$n = \frac{42}{2.597} = 16.2 \quad \textit{Ans.}$$

8-79 to 8-81 Specifying bolts, screws, dowels and rivets is the way a student learns about such components. However, choosing an array a priori is based on experience. Here is a chance for students to build some experience.

Chapter 9

Figure for Probs. 9-1 to 9-4



9-1 Given, b = 50 mm, d = 50 mm, h = 5 mm, τ_{allow} = 140 MPa.

$$F = 0.707\, hl\, \tau_{allow} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \quad \textit{Ans}.$$

9-2 Given, b = 2 in, d = 2 in, h = 5/16 in, τ_{allow} = 25 kpsi.

$$F = 0.707\, hl\, \tau_{allow} = 0.707(5/16)[2(2)](25) = 22.1 \text{ kip} \quad \textit{Ans}.$$

9-3 Given, b = 50 mm, d = 30 mm, h = 5 mm, τ_{allow} = 140 MPa.

$$F = 0.707\, hl\, \tau_{allow} = 0.707(5)[2(50)](140)(10^{-3}) = 49.5 \text{ kN} \quad \textit{Ans}.$$

9-4 Given, b = 4 in, d = 2 in, h = 5/16 in, τ_{allow} = 25 kpsi.

$$F = 0.707\, hl\, \tau_{allow} = 0.707(5/16)[2(4)](25) = 44.2 \text{ kip} \quad \textit{Ans}.$$

9-5 Prob. 9-1 with E7010 Electrode.

Table 9-6: f = 14.85 h kip/in = 14.85 [5 mm/(25.4 mm/in)] = 2.923 kip/in

= 2.923(4.45/25.4) = 0.512 kN/mm

$$F = f\, l = 0.512[2(50)] = 51.2 \text{ kN} \quad \textit{Ans}.$$

9-6 Prob. 9-2 with E6010 Electrode.

Table 9-6: f = 14.85 h kip/in = 14.85(5/16) = 4.64 kip/in

$F = f\,l = 4.64[2(2)] = 18.6$ kip *Ans*.

9-7 Prob. 9-3 with E7010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in = 14.85 [5 mm/(25.4 mm/in)] = 2.923 kip/in

= 2.923(4.45/25.4) = 0.512 kN/mm

$F = f\,l = 0.512[2(50)] = 51.2$ kN *Ans*.

9-8 Prob. 9-4 with E6010 Electrode.

Table 9-6: $f = 14.85\,h$ kip/in = 14.85(5/16) = 4.64 kip/in

$F = f\,l = 4.64[2(4)] = 37.1$ kip *Ans*.

9-9 Table A-20:

1018 CD: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

1018 HR: $S_{ut} = 400$ MPa, $S_y = 220$ MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(400),\ 0.40(220)]\\ &= \min(120,\ 88) = 88\text{ MPa}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](88)(10^{-3}) = 31.1$ kN *Ans.*

9-10 Table A-20:

1020 CD: $S_{ut} = 68$ kpsi, $S_y = 57$ kpsi

1020 HR: $S_{ut} = 55$ kpsi, $S_y = 30$ kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0\text{ kpsi}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(2)](12.0) = 10.6$ kip *Ans.*

9-11 Table A-20:

1035 HR: S_{ut} = 500 MPa, S_y = 270 MPa

1035 CD: S_{ut} = 550 MPa, S_y = 460 MPa

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(500),\ 0.40(270)]\\ &= \min(150,\ 108) = 108 \text{ MPa}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5)[2(50)](108)(10^{-3}) = 38.2$ kN *Ans.*

9-12 Table A-20:

1035 HR: S_{ut} = 72 kpsi, S_y = 39.5 kpsi

1020 CD: S_{ut} = 68 kpsi, S_y = 57 kpsi, 1020 HR: S_{ut} = 55 kpsi, S_y = 30 kpsi

Cold-rolled properties degrade to hot-rolled properties in the neighborhood of the weld.

Table 9-4:

$$\begin{aligned}\tau_{\text{all}} &= \min(0.30S_{ut},\ 0.40S_y)\\ &= \min[0.30(55),\ 0.40(30)]\\ &= \min(16.5,\ 12.0) = 12.0 \text{ kpsi}\end{aligned}$$

for both materials.

Eq. (9-3): $F = 0.707hl\tau_{\text{all}} = 0.707(5/16)[2(4)](12.0) = 21.2$ kip *Ans.*

9-13

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+50)]} = 141 \text{ MPa}$$ *Ans.*

9-14

Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(2+2)]} = 22.6 \text{ kpsi}$$ *Ans.*

9-15 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(100)(10^3)}{5[2(50+30)]} = 177 \text{ MPa}$$ *Ans.*

9-16 Eq. (9-3): $$\tau = \frac{\sqrt{2}F}{hl} = \frac{\sqrt{2}(40)}{(5/16)[2(4+2)]} = 15.1 \text{ kpsi}$$ *Ans.*

9-17 $b = d$ =50 mm, c = 150 mm, h = 5 mm, and τ_{allow} = 140 MPa.
(a) *Primary shear*, Table 9-1, Case 2 (Note: b and d are interchanged between problem figure and table figure. Note, also, F in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{50\left[3\left(50^2\right) + 50^2\right]}{6} = 83.33\left(10^3\right) \text{ mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)(83.33)(10^3) = 294.6(10^3) \text{ mm}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(25)}{294.6\left(10^3\right)} = 14.85F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{14.85^2 + \left(2.829 + 14.85\right)^2} = 23.1F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{140}{23.1} = 6.06 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi = 21(6.89) = 145 MPa

1020 HR bar: S_{ut} = 380 MPa, S_y = 210 MPa

1015 HR support: S_{ut} = 340 MPa, S_y = 190 MPa

Table 9-3, E7010 Electrode: S_{ut} = 482 MPa, S_y = 393 MPa

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190)] = \min(102, 76)$

$= 76$ MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{23.1} = \frac{76}{23.1} = 3.29 \text{ kN} \quad \textit{Ans.}$$

9-18 $b = d$ =2 in, c = 6 in, h = 5/16 in, and τ_{allow} = 25 kpsi.

(a) *Primary shear*, Table 9-1(Note: *b* and *d* are interchanged between problem figure and table figure. Note, also, *F* in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2)} = 1.132F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{2\left[3\left(2^2\right) + 2^2\right]}{6} = 5.333 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)(5.333) = 1.178 \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{1.178} = 5.942F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{5.942^2 + \left(1.132 + 5.942\right)^2} = 9.24F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{25}{9.24} = 2.71 \text{ kip} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi

1020 HR bar: S_{ut} = 55 kpsi, S_y = 30 kpsi

1015 HR support: S_{ut} = 50 kpsi, S_y = 27.5 kpsi

Table 9-3, E7010 Electrode: S_{ut} = 70 kpsi, S_y = 57 kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5) = \min(15, 11)$
$= 11$ kpsi

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{9.24} = \frac{11}{9.24} = 1.19 \text{ kip} \quad \textit{Ans.}$$

9-19 b =50 mm, c = 150 mm, d = 30 mm, h = 5 mm, and τ_{allow} = 140 MPa.
(a) *Primary shear*, Table 9-1, Case 2 (Note: *b* and *d* are interchanged between problem figure and table figure. Note, also, *F* in kN and τ in MPa):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50)} = 2.829F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{50\left[3\left(30^2\right) + 50^2\right]}{6} = 43.33\left(10^3\right) \text{ mm}^3$$

$$J = 0.707\, h\, J_u = 0.707(5)(43.33)(10^3) = 153.2(10^3) \text{ mm}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(15)}{153.2\left(10^3\right)} = 17.13F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F\left(10^3\right)(25)}{153.2\left(10^3\right)} = 28.55F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{17.13^2 + \left(2.829 + 28.55\right)^2} = 35.8F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{140}{35.8} = 3.91 \text{ kN} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi = 21(6.89) = 145 MPa

1020 HR bar: S_{ut} = 380 MPa, S_y = 210 MPa

1015 HR support: S_{ut} = 340 MPa, S_y = 190 MPa

Table 9-3, E7010 Electrode: S_{ut} = 482 MPa, S_y = 393 MPa

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(340), 0.40(190)] = \min(102, 76)$

$= 76$ MPa

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{35.8} = \frac{76}{35.8} = 2.12 \text{ kN} \quad \textit{Ans.}$$

9-20 b = 4 in, c = 6 in, d = 2 in, h = 5/16 in, and τ_{allow} = 25 kpsi.

(a) *Primary shear*, Table 9-1(Note: *b* and *d* are interchanged between problem figure and table figure. Note, also, *F* in kip and τ in kpsi):

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4)} = 0.5658F$$

Secondary shear, Table 9-1:

$$J_u = \frac{d\left(3b^2 + d^2\right)}{6} = \frac{4\left[3\left(2^2\right) + 4^2\right]}{6} = 18.67 \text{ in}^3$$

$$J = 0.707\, h\, J_u = 0.707(5/16)(18.67) = 4.125 \text{ in}^4$$

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{4.125} = 1.939F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{4.125} = 3.879F$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{1.939^2 + \left(0.5658 + 3.879\right)^2} = 4.85F \quad (1)$$

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{25}{4.85} = 5.15 \text{ kip} \quad \textit{Ans.}$$

(**b**) For E7010 from Table 9-6, τ_{allow} = 21 kpsi

1020 HR bar: S_{ut} = 55 kpsi, S_y = 30 kpsi

1015 HR support: S_{ut} = 50 kpsi, S_y = 27.5 kpsi

Table 9-3, E7010 Electrode: S_{ut} = 70 kpsi, S_y = 57 kpsi

The support controls the design.

Table 9-4: $\tau_{\text{allow}} = \min(0.30S_{ut}, 0.40S_y) = \min[0.30(50), 0.40(27.5) = \min(15, 11)$

$= 11$ kpsi

The allowable load, from Eq. (1) is

$$F = \frac{\tau_{\text{allow}}}{4.85} = \frac{11}{4.85} = 2.27 \text{ kip} \quad \textit{Ans.}$$

9-21 Given, b = 50 mm, c = 150 mm, d = 50 mm, h = 5 mm, τ_{allow} = 140 MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50+50)} = 1.414F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(50+50)^3}{6} = 166.7\left(10^3\right) \text{ mm}^3$$

$J = 0.707\, h\, J_u = 0.707(5)166.7(10^3) = 589.2(10^3)$ mm^4

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(25)}{589.2\left(10^3\right)} = 7.425F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{7.425^2 + \left(1.414 + 7.425\right)^2} = 11.54F$$

$$F = \frac{\tau_{allow}}{11.54} = \frac{140}{11.54} = 12.1 \text{ kN} \quad \textit{Ans.}$$

9-22 Given, b = 2 in, c = 6 in, d = 2 in, h = 5/16 in, τ_{allow} = 25 kpsi.

Primary shear:

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(2+2)} = 0.5658F$$

Secondary shear:

Table 9-1: $$J_u = \frac{(b+d)^3}{6} = \frac{(2+2)^3}{6} = 10.67 \text{ in}^3$$

$J = 0.707\, h\, J_u = 0.707(5/16)10.67 = 2.357$ in^4

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{7F(1)}{2.357} = 2.970F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{2.970^2 + \left(0.566 + 2.970\right)^2} = 4.618F$$

$$F = \frac{\tau_{allow}}{4.618} = \frac{25}{4.618} = 5.41 \text{ kip} \quad \textit{Ans.}$$

9-23 Given, b = 50 mm, c = 150 mm, d = 30 mm, h = 5 mm, τ_{allow} = 140 MPa.

Primary shear (F in kN, τ in MPa, A in mm^2):

$$\tau'_y = \frac{V}{A} = \frac{F\left(10^3\right)}{1.414(5)(50+30)} = 1.768F$$

Secondary shear:

Table 9-1: $J_u = \dfrac{(b+d)^3}{6} = \dfrac{(50+30)^3}{6} = 85.33\left(10^3\right)$ mm^3

$J = 0.707\, h\, J_u = 0.707(5)85.33(10^3) = 301.6(10^3)$ mm^4

$$\tau''_x = \frac{Mr_y}{J} = \frac{175F\left(10^3\right)(15)}{301.6\left(10^3\right)} = 8.704F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{175F\left(10^3\right)(25)}{301.6\left(10^3\right)} = 14.51F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{8.704^2 + (1.768+14.51)^2} = 18.46F$$

$$F = \frac{\tau_{allow}}{18.46} = \frac{140}{18.46} = 7.58 \text{ kN} \quad \textit{Ans.}$$

9-24 Given, b = 4 in, c = 6 in, d = 2 in, h = 5/16 in, τ_{allow} = 25 kpsi.

Primary shear:

$$\tau'_y = \frac{V}{A} = \frac{F}{1.414(5/16)(4+2)} = 0.3772F$$

Secondary shear:

Table 9-1: $J_u = \dfrac{(b+d)^3}{6} = \dfrac{(4+2)^3}{6} = 36$ in^3

$J = 0.707\, h\, J_u = 0.707(5/16)36 = 7.954$ in^4

$$\tau''_x = \frac{Mr_y}{J} = \frac{8F(1)}{7.954} = 1.006F$$

$$\tau''_y = \frac{Mr_x}{J} = \frac{8F(2)}{7.954} = 2.012F$$

Maximum shear:

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = F\sqrt{1.006^2 + (0.3772+2.012)^2} = 2.592F$$

$$F = \frac{\tau_{allow}}{2.592} = \frac{25}{2.592} = 9.65\,\text{kip} \quad Ans.$$

9-25 Given, b = 50 mm, d = 50 mm, h = 5 mm, E6010 electrode.

$$A = 0.707(5)(50 + 50 + 50) = 530.3\ \text{mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 320 MPa.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(320)^{-0.995} = 0.875$

k_b = 1 (uniform shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-8) and (6-18): $S_e = 0.875(1)(0.59)(1)(0.5)(320) = 82.6$ MPa

Electrode endurance: E6010, Table 9-3, S_{ut} = 427 MPa

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(427)^{-0.995} = 0.657$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.657(1)(0.59)(1)(0.5)(427) = 82.8\ \text{MPa}$$

The members and electrode are basically of equal strength. We will use S_e = 82.6 MPa. For a factor of safety of 1, and with K_{fs} = 2.7 (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{82.6(530.3)}{2.7} = 16.2\left(10^3\right)\ \text{N} = 16.2\ \text{kN} \quad Ans.$$

9-26 Given, b = 2 in, d = 2 in, h = 5/16 in, E6010 electrode.

$$A = 0.707(5/16)(2 + 2 + 2) = 1.326\ \text{in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 47 kpsi.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(47)^{-0.995} = 0.865$

k_b = 1 (uniform shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-8) and (6-18): $S_e = 0.865(1)(0.59)(1)(0.5)(47) = 12.0$ kpsi

Electrode endurance: E6010, Table 9-3, S_{ut} = 62 kpsi

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(62)^{-0.995} = 0.657$

As before, k_b = 1 (uniform shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.657(1)(0.59)(1)(0.5)(62) = 12.0 \text{ kpsi}$$

Thus the members and electrode are of equal strength. For a factor of safety of 1, and with K_{fs} = 2.7 (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{12.0(1.326)}{2.7} = 5.89 \text{ kip} \qquad \textit{Ans.}$$

9-27 Given, b = 50 mm, d = 30 mm, h = 5 mm, E7010 electrode.

$$A = 0.707(5)(50 + 50 + 30) = 459.6 \text{ mm}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 320 MPa.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(320)^{-0.995} = 0.875$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-8) and (6-18): $S_e = 0.875(1)(0.59)(1)(0.5)(320) = 82.6$ MPa

Electrode endurance: E6010, Table 9-3, S_{ut} = 482 MPa

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 272(482)^{-0.995} = 0.582$

As before, k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

$$S_e = 0.582(1)(0.59)(1)(0.5)(482) = 82.7 \text{ MPa}$$

The members and electrode are basically of equal strength. We will use S_e =82.6 MPa. For a factor of safety of 1, and with K_{fs} = 2.7 (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{82.6(459.6)}{2.7} = 14.1(10^3) \text{ N} = 14.1 \text{ kN} \qquad \textit{Ans.}$$

9-28 Given, b = 4 in, d = 2 in, h = 5/16 in, E7010 electrode.

$$A = 0.707(5/16)(4 + 4 + 2) = 2.209 \text{ in}^2$$

Member endurance limit: From Table A-20 for AISI 1010 HR, S_{ut} = 47 kpsi.

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(47)^{-0.995} = 0.865$

k_b = 1 (direct shear), k_c = 0.59 (torsion, shear), k_d = 1

Eqs. (6-8) and (6-18): $S_e = 0.865(1)(0.59)(1)(0.5)(47) = 12.0$ kpsi

Electrode endurance: E7010, Table 9-3, $S_{ut} = 70$ kpsi

Eq. 6-19 and Table 6-2, pp. 287, 288: $k_a = 39.9(70)^{-0.995} = 0.582$

As before, $k_b = 1$ (direct shear), $k_c = 0.59$ (torsion, shear), $k_d = 1$

$$S_e = 0.582(1)(0.59)(1)(0.5)(70) = 12.0 \text{ kpsi}$$

Thus the members and electrode are of equal strength. For a factor of safety of 1, and with $K_{fs} = 2.7$ (Table 9-5)

$$F = \frac{\tau_{\text{allow}} A}{K_{fs}} = \frac{12.0(2.209)}{2.7} = 9.82 \text{ kip} \qquad \textit{Ans.}$$

9-29 *Primary shear*: $\tau' = 0$ (why?)

Secondary shear:

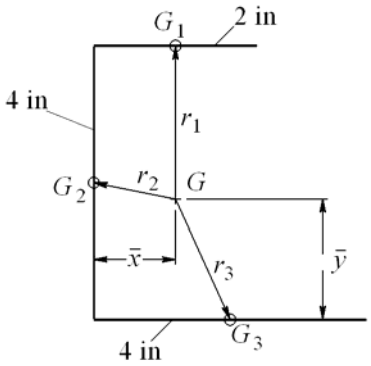
Table 9-1: $J_u = 2\pi r^3 = 2\pi(1.5)^3 = 21.21 \text{ in}^3$

$$J = 0.707\, h\, J_u = 0.707(1/4)(21.21) = 3.749 \text{ in}^4$$

2 welds: $\tau'' = \dfrac{Mr}{2J} = \dfrac{8F(1.5)}{2(3.749)} = 1.600F$

$$\tau'' = \tau_{\text{allow}} \quad \Rightarrow \quad 1.600F = 20 \quad \Rightarrow \quad F = 12.5 \text{ kip} \qquad \textit{Ans.}$$

9-30 $l = 2 + 4 + 4 = 10$ in



$$\bar{x} = \frac{2(1)+4(0)+4(2)}{10} = 1 \text{ in}$$

$$\bar{y} = \frac{2(4)+4(2)+4(0)}{10} = 1.6 \text{ in}$$

$$M = FR = F(10 - 1) = 9F$$

$$r_1 = \sqrt{(1-1)^2 + (4-1.6)^2} = 2.4 \text{ in}, \qquad r_2 = \sqrt{1^2 + (2-1.6)^2} = 1.077 \text{ in}$$

$$r_3 = \sqrt{(2-1)^2 + 1.6^2} = 1.887 \text{ in}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5/16)\left(2^3\right) = 0.1473 \text{ in}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5/16)\left(4^3\right) = 1.178 \text{ in}^4$$

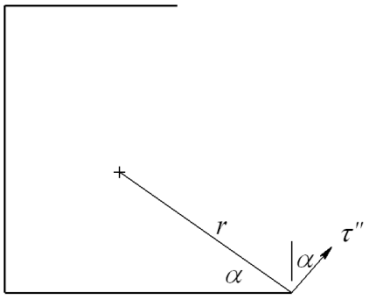
$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$

$$= 0.1473 + 0.707(5/16)(2)\left(2.4^2\right) + 1.178 + 0.707(5/16)(4)\left(1.077^2\right)$$

$$+ 1.178 + 0.707(5/16)(4)\left(1.887^2\right) = 9.220 \text{ in}^4$$

$$\alpha = \tan^{-1}\left(\frac{1.6}{4-1}\right) = 28.07^\circ$$

$$r = \sqrt{1.6^2 + (4-1)^2} = 3.4 \text{ in}$$



Primary shear (τ in kpsi, F in kip) :

$$\tau' = \frac{V}{A} = \frac{F}{0.707(5/16)(10)} = 0.4526F$$

Secondary shear:

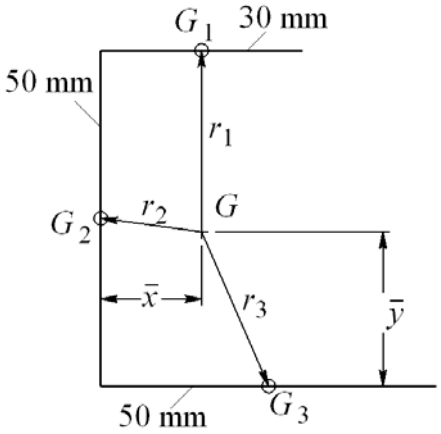
$$\tau'' = \frac{Mr}{J} = \frac{9F(3.4)}{9.220} = 3.319F$$

$$\tau_{\max} = \sqrt{\left(3.319F \sin 28.07^\circ\right)^2 + \left(3.319F \cos 28.07^\circ + 0.4526F\right)^2}$$

$$= 3.724F$$

$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad 3.724\,F = 25 \quad \Rightarrow \quad F = 6.71$ kip *Ans*.

9-31 $l = 30 + 50 + 50 = 130$ mm



$$\bar{x} = \frac{30(15)+50(0)+50(25)}{130} = 13.08 \text{ mm}$$

$$\bar{y} = \frac{30(50)+50(25)+50(0)}{130} = 21.15 \text{ mm}$$

$M = FR = F(200 - 13.08)$

$= 186.92\,F$ (M in N·m, F in kN)

$$r_1 = \sqrt{(15-13.08)^2+(50-21.15)^2} = 28.92 \text{ mm}, \quad r_2 = \sqrt{13.08^2+(25-21.15)^2} = 13.63 \text{ mm}$$

$$r_3 = \sqrt{(25-13.08)^2+21.15^2} = 24.28 \text{ mm}$$

$$J_{G_1} = \frac{1}{12}(0.707)(5)(30^3) = 7.954(10^3) \text{ mm}^4$$

$$J_{G_2} = J_{G_3} = \frac{1}{12}(0.707)(5)(50^3) = 36.82(10^3) \text{ mm}^4$$

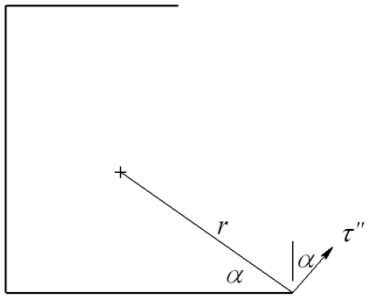
$$J = \sum_{i=1}^{3}\left(J_i + A_i r_{G_i}^2\right)$$

$$= 7.954(10^3) + 0.707(5)(30)(28.92^2) + 36.82(10^3) + 0.707(5)(50)(13.63^2)$$

$$+ 36.82(10^3) + 0.707(5)(50)(24.28^2) = 307.3(10^3) \text{ mm}^4$$

$$\alpha = \tan^{-1}\left(\frac{21.15}{50-13.08}\right) = 29.81^\circ$$

$$r = \sqrt{21.15^2+(50-13.08)^2} = 42.55 \text{ mm}$$



Primary shear (τ in MPa, F in kN) :

$$\tau' = \frac{V}{A} = \frac{F(10^3)}{0.707(5)(130)} = 2.176F$$

Secondary shear:

$$\tau'' = \frac{Mr}{J} = \frac{186.92F(10^3)(42.55)}{307.3(10^3)} = 25.88F$$

$$\tau_{\max} = \sqrt{\left(25.88F \sin 29.81^{\circ}\right)^2 + \left(25.88F \cos 29.81^{\circ} + 2.176F\right)^2}$$
$$= 27.79F$$

$\tau_{\max} = \tau_{\text{allow}} \quad \Rightarrow \quad 27.79\,F = 140 \quad \Rightarrow \quad F = 5.04$ kN *Ans.*

9-32

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{J_u}{lh} = \frac{a^3/12}{ah} = \frac{a^2}{12h} = 0.0833\left(\frac{a^2}{h}\right)$	**5**
2.	$\text{fom}' = \frac{a\left(3a^2 + a^2\right)}{6(2a)h} = \frac{a^2}{3h} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
3.	$\text{fom}' = \frac{(2a)^4 - 6a^2a^2}{12(a+a)2ah} = \frac{5a^2}{24h} = 0.2083\left(\frac{a^2}{h}\right)$	**4**
4.	$\text{fom}' = \frac{1}{3ah}\left(\frac{8a^3 + 6a^3 + a^3}{12} - \frac{a^4}{2a+a}\right) = 0.3056\left(\frac{a^2}{h}\right)$	**2**
5.	$\text{fom}' = \frac{(2a)^3}{6h}\frac{1}{4a} = \frac{8a^3}{24ah} = 0.3333\left(\frac{a^2}{h}\right)$	**1**
6.	$\text{fom}' = \frac{2\pi(a/2)^3}{\pi ah} = \frac{a^3}{4ah} = 0.25\left(\frac{a^2}{h}\right)$	**3**

9-33

Weld Pattern	Figure of merit	Rank
1.	$\text{fom}' = \frac{I_u}{lh} = \frac{\left(a^3/12\right)}{ah} = 0.0833\left(\frac{a^2}{h}\right)$	**6**
2.	$\text{fom}' = \frac{\left(a^3/6\right)}{2ah} = 0.0833\left(\frac{a^2}{h}\right)$	**6**
3.	$\text{fom}' = \frac{\left(aa^2/2\right)}{2ah} = 0.25\left(\frac{a^2}{h}\right)$	**1**
4.*	$\text{fom}' = \frac{\left(a^2/12\right)\left(6a+a\right)}{3ah} = \frac{7a^2}{36h} = 0.1944\left(\frac{a^2}{h}\right)$	**2**
5. & 7.	$\bar{x} = \frac{a}{2}, \quad \bar{y} = \frac{a^2}{a+2a} = \frac{a}{3}$	
	$I_u = \frac{2a^3}{3} - 2a^2\frac{a}{3} + \left(a+2a\right)\left(\frac{a}{3}\right)^2 = \frac{a^3}{3}$	
	$\text{fom}' = \frac{I_u}{lh} = \frac{\left(a^3/3\right)}{3ah} = \frac{1}{9}\left(\frac{a^2}{h}\right) = 0.1111\left(\frac{a^2}{h}\right)$	**5**
6. & 8.	$\text{fom}' = \frac{\left(a^2/6\right)\left(3a+a\right)}{4ah} = \frac{1}{6}\left(\frac{a^2}{h}\right) = 0.1667\left(\frac{a^2}{h}\right)$	**3**
9.	$\text{fom}' = \frac{\pi\left(a/2\right)^3}{\pi ah} = \frac{a^2}{8h} = 0.125\left(\frac{a^2}{h}\right)$	**4**

*Note. Because this section is not symmetric with the vertical axis, out-of-plane deflection may occur unless special precautions are taken. See the topic of "shear center" in books with more advanced treatments of mechanics of materials.

9-34 Attachment and member (1018 HR), S_y = 220 MPa and S_{ut} = 400 MPa.

The member and attachment are weak compared to the properties of the lowest electrode.

Decision Specify the E6010 electrode

Controlling property, Table 9-4: τ_{all} = min[0.3(400), 0.4(220)] = min(120, 88) = 88 MPa

For a static load, the parallel and transverse fillets are the same. Let the length of a bead be l = 75 mm, and n be the number of beads.

$$\tau = \frac{F}{n(0.707)hl} = \tau_{all}$$

$$nh = \frac{F}{0.707l\tau_{\text{all}}} = \frac{100(10^3)}{0.707(75)(88)} = 21.43$$

where h is in millimeters. Make a table

Number of beads, n	Leg size, h (mm)
1	21.43
2	10.71
3	7.14
4	5.36 $\rightarrow$ 6 mm

Decision Specify h = 6 mm on all four sides.

Weldment specification:

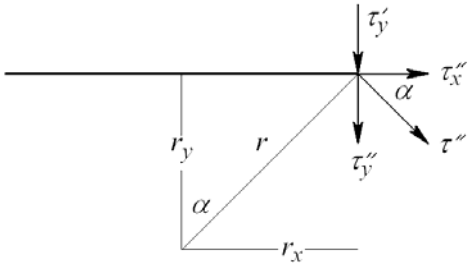
Pattern: All-around square, four beads each side, 75 mm long

Electrode: E6010

Leg size: h = 6 mm

9-35 *Decision*: Choose a parallel fillet weldment pattern. By so-doing, we've chosen an optimal pattern (see Prob. 9-32) and have thus reduced a synthesis problem to an analysis problem:
Table 9-1, case 2, rotated 90°: $A = 1.414hd = 1.414(h)(75) = 106.05h$ mm^2
Primary shear

$$\tau'_y = \frac{V}{A} = \frac{12(10^3)}{106.05h} = \frac{113.2}{h}$$



Secondary shear:

$$J_u = \frac{d(3b^2 + d^2)}{6}$$

$$= \frac{75[3(75^2) + 75^2]}{6} = 281.3(10^3) \text{ mm}^3$$

$$J = 0.707(h)(281.3)(10^3) = 198.8(10^3)h \text{ mm}^4$$

With $\alpha = 45°$,

$$\tau''_x = \frac{Mr\cos 45^\circ}{J} = \frac{Mr_y}{J} = \frac{12\left(10^3\right)(187.5)(37.5)}{198.8\left(10^3\right)h} = \frac{424.4}{h} = \tau''_y$$

$$\tau_{\max} = \sqrt{\tau''^2_x + \left(\tau'_y + \tau''_y\right)^2} = \frac{1}{h}\sqrt{424.4^2 + (113.2 + 424.4)^2} = \frac{684.9}{h}$$

Attachment and member (1018 HR): S_y = 220 MPa, S_{ut} = 400 MPa
Decision: Use E60XX electrode which is stronger

$$\tau_{\text{all}} = \min[0.3(400),\ 0.4(220)] = 88 \text{ MPa}$$

$$\tau_{\max} = \tau_{\text{all}} = \frac{684.9}{h} = 88 \text{ MPa}$$

$$h = \frac{684.9}{88} = 7.78 \text{ mm}$$

Decision: Specify 8 mm leg size
Weldment Specifications:
Pattern: Parallel horizontal fillet welds
Electrode: E6010
Type: Fillet
Length of each bead: 75 mm
Leg size: 8 mm

9-36 Problem 9-35 solves the problem using parallel horizontal fillet welds, each 75 mm long obtaining a leg size rounded up to 8 mm.
For this problem, since the width of the plate is fixed and the length has not been determined, we will explore reducing the leg size by using two vertical beads 75 mm long and two horizontal beads such that the beads have a leg size of 6 mm.

Decision: Use a rectangular weld bead pattern with a leg size of 6 mm (case 5 of Table 9-1 with b unknown and d = 75 mm).
Materials:
Attachment and member (1018 HR): S_y = 220 MPa, S_{ut} = 400 MPa
From Table 9-4, AISC welding code,

$$\tau_{\text{all}} = \min[0.3(400),\ 0.4(220)] = \min(120,\ 88) = 88 \text{ MPa}$$

Select a stronger electrode material from Table 9-3.
Decision: Specify E6010

Solving for b: In Prob. 9-35, every term was linear in the unknown h. This made solving for h relatively easy. In this problem, the terms will not be linear in b, and so we will use an iterative solution with a spreadsheet.

Throat area and other properties from Table 9-1:

$$A = 1.414(6)(b + 75) = 8.484(b + 75) \qquad (1)$$

$$J_u = \frac{(b+75)^3}{6}, \quad J = 0.707\,(6)\,J_u = 0.707(b+75)^3 \qquad (2)$$

Primary shear (τ in MPa, h in mm):

$$\tau'_y = \frac{V}{A} = \frac{12\left(10^3\right)}{A} \qquad (3)$$

Secondary shear (See Prob. 9-35 solution for the definition of α) :

$$\tau'' = \frac{Mr}{J}$$

$$\tau''_x = \tau'' \cos\alpha = \frac{Mr}{J}\cos\alpha = \frac{Mr_y}{J} = \frac{12\left(10^3\right)(150 + b/2)(37.5)}{0.707(b+75)^3} \qquad (4)$$

$$\tau''_y = \tau'' \sin\alpha = \frac{Mr}{J}\sin\alpha = \frac{Mr_x}{J} = \frac{12\left(10^3\right)(150 + b/2)(b/2)}{0.707(b+75)^3} \qquad (5)$$

$$\tau_{\max} = \sqrt{\tau'^2_y + \left(\tau''_x + \tau''_y\right)^2} \qquad (6)$$

Enter Eqs. (1) to (6) into a spreadsheet and iterate for various values of b. A portion of the spreadsheet is shown below.

b (mm)	A (mm^2)	J (mm^4)	τ'_y (Mpa)	τ''_y (Mpa)	τ''_x (Mpa)	$\tau_{\max}$ (Mpa)	
41	984.144	1103553.5	12.19334	69.5254	38.00722	90.12492	
42	992.628	1132340.4	12.08912	67.9566	38.05569	88.63156	
43	1001.112	1161623.6	11.98667	66.43718	38.09065	87.18485	< 88 Mpa
44	1009.596	1191407.4	11.88594	64.96518	38.11291	85.7828	

We see that $b \geq 43$ mm meets the strength goal.

Weldment Specifications:
Pattern: Horizontal parallel weld tracks 43 mm long, vertical parallel weld tracks 75 mm long
Electrode: E6010
Leg size: 6 mm

9-37 *Materials:*

Member and attachment (1018 HR): $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi

Table 9-4: $\tau_{\text{all}} = \min[0.3(58), 0.4(32)] = 12.8$ kpsi

Decision: Use E6010 electrode. From Table 9-3: $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi,

$$\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = 20 \text{ kpsi}$$

Decision: Since 1018 HR is weaker than the E6010 electrode, use $\tau_{\text{all}} = 12.8$ kpsi

Decision: Use an all-around square weld bead track.

$l_1 = 6 + a = 6 + 6.25 = 12.25$ in

Throat area and other properties from Table 9-1:

$$A = 1.414h(b + d) = 1.414(h)(6 + 6) = 16.97h$$

Primary shear

$$\tau'_y = \frac{V}{A} = \frac{F}{A} = \frac{20\left(10^3\right)}{16.97h} = \frac{1179}{h} \text{ psi}$$

Secondary shear

$$J_u = \frac{(b+d)^3}{6} = \frac{(6+6)^3}{6} = 288 \text{ in}^3$$

$$J = 0.707h(288) = 203.6h \text{ in}^4$$

$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{20\left(10^3\right)(6.25+3)(3)}{203.6h} = \frac{2726}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau''^2_x + (\tau'_y + \tau''_y)^2} = \frac{1}{h}\sqrt{2726^2 + (1179 + 2726)^2} = \frac{4762}{h} \text{ psi}$$

Relate stress to strength

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{4762}{h} = 12.8\left(10^3\right) \quad \Rightarrow \quad h = \frac{4762}{12.8\left(10^3\right)} = 0.372 \text{ in}$$

Decision:

Specify $3/8$ in leg size

Specifications:

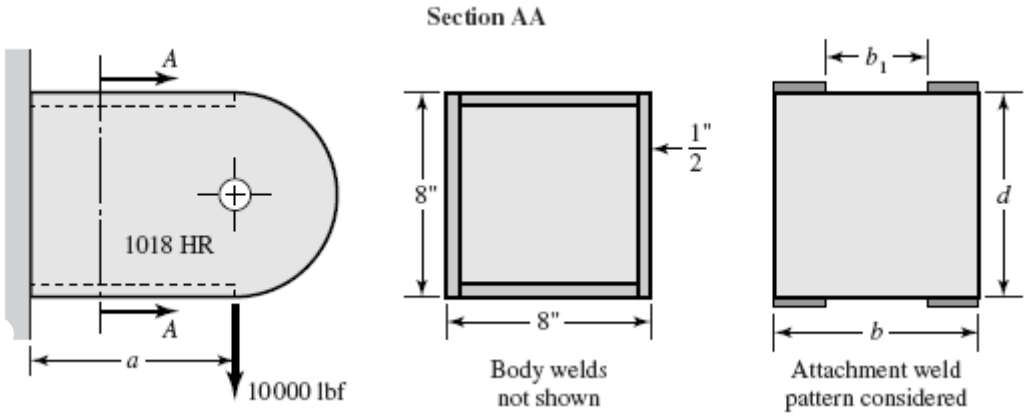
Pattern: All-around square weld bead track
Electrode: E6010
Type of weld: Fillet
Weld bead length: 24 in
Leg size: $3/8$ in
Attachment length: 12.25 in

9-38 This is a good analysis task to test a student's understanding.

(1) Solicit information related to a priori decisions.
(2) Solicit design variables *b* and *d*.
(3) Find *h* and round and output all parameters on a single screen. Allow return to Step 1 or Step 2.
(4) When the iteration is complete, the final display can be the bulk of your adequacy assessment.

Such a program can teach too.

9-39 The objective of this design task is to have the students teach themselves that the weld patterns of Table 9-2 can be added or subtracted to obtain the properties of a contemplated weld pattern. The instructor can control the level of complication. We have left the presentation of the drawing to you. Here is *one* possibility. Study the problem's opportunities, and then present this (or your sketch) with the problem assignment.



Use b_1 as the design variable. Express properties as a function of b_1. From Table 9-3, case 3:

$$A = 1.414h(b - b_1)$$

$$I_u = \frac{bd^2}{2} - \frac{b_1 d^2}{2} = \frac{(b - b_1)d^2}{2}$$

$$I = 0.707hI_u$$

$$\tau' = \frac{V}{A} = \frac{F}{1.414h(b - b_1)}$$

$$\tau'' = \frac{Mc}{I} = \frac{Fa(d/2)}{0.707hI_u}$$

Parametric study

Let $a = 10$ in, $b = 8$ in, $d = 8$ in, $b_1 = 2$ in, $\tau_{all} = 12.8$ kpsi, $l = 2(8 - 2) = 12$ in

$$A = 1.414h(8-2) = 8.48h \text{ in}^2$$
$$I_u = (8-2)(8^2/2) = 192 \text{ in}^3$$
$$I = 0.707(h)(192) = 135.7h \text{ in}^4$$
$$\tau' = \frac{10\,000}{8.48h} = \frac{1179}{h} \text{ psi}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{135.7h} = \frac{2948}{h} \text{ psi}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{1179^2 + 2948^2} = \frac{3175}{h} = 12\,800 \text{ psi}$$

from which $h = 0.248$ in. Do not round off the leg size – something to learn.

$$\text{fom}' = \frac{I_u}{hl} = \frac{192}{0.248(12)} = 64.5 \text{ in}$$
$$A = 8.48(0.248) = 2.10 \text{ in}^2$$
$$I = 135.7(0.248) = 33.65 \text{ in}^4$$
$$\text{vol} = \frac{h^2}{2}l = \frac{0.248^2}{2}12 = 0.369 \text{ in}^3$$
$$\text{eff} = \frac{I}{\text{vol}} = \frac{33.65}{0.369} = 91.2 \text{ in}$$
$$\tau' = \frac{1179}{0.248} = 4754 \text{ psi}$$
$$\tau'' = \frac{2948}{0.248} = 11\,887 \text{ psi}$$
$$\tau_{\max} = \frac{3175}{0.248} = 12\,800 \text{ psi}$$

Now consider the case of uninterrupted welds,

$$b_1 = 0$$
$$A = 1.414(h)(8-0) = 11.31h$$
$$I_u = (8-0)(8^2/2) = 256 \text{ in}^3$$
$$I = 0.707(256)h = 181h \text{ in}^4$$
$$\tau' = \frac{10\,000}{11.31h} = \frac{884}{h}$$
$$\tau'' = \frac{10\,000(10)(8/2)}{181h} = \frac{2210}{h}$$
$$\tau_{\max} = \frac{1}{h}\sqrt{884^2 + 2210^2} = \frac{2380}{h} = \tau_{\text{all}}$$
$$h = \frac{\tau_{\max}}{\tau_{\text{all}}} = \frac{2380}{12\,800} = 0.186 \text{ in}$$

Do not round off *h*.

$$A = 11.31(0.186) = 2.10 \text{ in}^2$$

$$I = 181(0.186) = 33.67 \text{ in}^4$$

$$\tau' = \frac{884}{0.186} = 4753 \text{ psi}, \quad \text{vol} = \frac{0.186^2}{2}16 = 0.277 \text{ in}^3$$

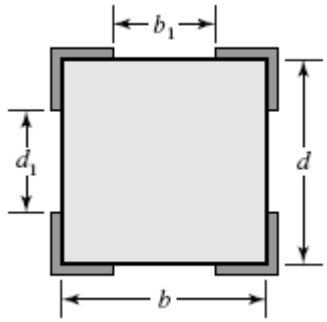
$$\tau'' = \frac{2210}{0.186} = 11\,882 \text{ psi}$$

$$\text{fom}' = \frac{I_u}{hl} = \frac{256}{0.186(16)} = 86.0 \text{ in}$$

$$\text{eff} = \frac{I}{(h^2/2)l} = \frac{33.67}{(0.186^2/2)16} = 121.7 \text{ in}$$

Conclusions: To meet allowable stress limitations, I and A do not change, nor do τ and σ. To meet the shortened bead length, h is increased proportionately. However, volume of bead laid down increases as h^2. The uninterrupted bead is superior. In this example, we did not round h and as a result we learned something. Our measures of merit are also sensitive to rounding. When the design decision is made, rounding to the next larger standard weld fillet size will decrease the merit.

Had the weld bead gone around the corners, the situation would change. Here is a follow up task analyzing an alternative weld pattern.



9-40 From Table 9-2

For the box $A = 1.414h(b+d)$

Subtracting b_1 from b and d_1 from d

$$A = 1.414h\left(b - b_1 + d - d_1\right)$$

$$I_u = \frac{d^2}{6}(3b+d) - \frac{d_1^3}{6} - \frac{b_1 d^2}{2} = \frac{1}{2}\left(b - b_1\right)d^2 + \frac{1}{6}\left(d^3 - d_1^3\right)$$

Length of bead $l = 2(b - b_1 + d - d_1)$

$\text{fom} = I_u / hl$

9-41 Computer programs will vary.

9-42 **Note to the Instructor**. In the first printing of the ninth edition, the loading was stated incorrectly. In the fourth line, "bending moment of 100 kip · in in" should read, "10 kip bending load 10 in from". This will be corrected in the printings that follow. We apologize if this has caused any inconvenience.

τ_{all} = 12 kpsi. Use Fig. 9-17(*a*) for general geometry, but employ ⎺ beads and then || beads.

Horizontal parallel weld bead pattern
b = 3 in, d = 6 in

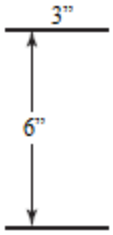


Table 9-2: $A = 1.414hb = 1.414(h)(3) = 4.24h \text{ in}^2$

$$I_u = \frac{bd^2}{2} = \frac{3(6)^2}{2} = 54 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(54) = 38.2h \text{ in}^4$$

$$\tau' = \frac{10}{4.24h} = \frac{2.358}{h} \text{ kpsi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{38.2h} = \frac{7.853}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{2.358^2 + 7.853^2} = \frac{8.199}{h} \text{ kpsi}$$

Equate the maximum and allowable shear stresses.

$$\tau_{\max} = \tau_{\text{all}} = \frac{8.199}{h} = 12$$

from which $h = 0.683$ in. It follows that

$$I = 38.2(0.683) = 26.1 \text{ in}^4$$

The volume of the weld metal is

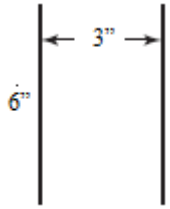
$$\text{vol} = \frac{h^2 l}{2} = \frac{(0.683)^2(3+3)}{2} = 1.40 \text{ in}^3$$

The effectiveness, $(\text{eff})_{\text{H}}$, is

$$(\text{eff})_{\text{H}} = \frac{I}{\text{vol}} = \frac{26.1}{1.4} = 18.6 \text{ in}$$

$$(\text{fom}')_{\text{H}} = \frac{I_u}{hl} = \frac{54}{0.683(3+3)} = 13.2 \text{ in}$$

Vertical parallel weld beads



$b = 3$ in

$d = 6$ in

From Table 9-2, case 2

$$A = 1.414hd = 1.414(h)(6) = 8.48h \text{ in}^2$$

$$I_u = \frac{d^3}{6} = \frac{6^3}{6} = 72 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(h)(72) = 50.9h$$

$$\tau' = \frac{10}{8.48h} = \frac{1.179}{h} \text{ psi}$$

$$\tau'' = \frac{Mc}{I} = \frac{10(10)(6/2)}{50.9h} = \frac{5.894}{h} \text{ psi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \frac{1}{h}\sqrt{1.179^2 + 5.894^2} = \frac{6.011}{h} \text{ kpsi}$$

Equating $\tau_{\max}$ to τ_{all} gives $h = 0.501$ in. It follows that

$$I = 50.9(0.501) = 25.5 \text{ in}^4$$

$$\text{vol} = \frac{h^2 l}{2} = \frac{0.501^2}{2}(6+6) = 1.51 \text{ in}^3$$

$$(\text{eff})_{\text{V}} = \frac{I}{\text{vol}} = \frac{25.5}{1.51} = 16.7 \text{ in}$$

$$(\text{fom}')_{\text{V}} = \frac{I_u}{hl} = \frac{72}{0.501(6+6)} = 12.0 \text{ in}$$

The ratio of $(\text{eff})_{\text{V}} / (\text{eff})_{\text{H}}$ is $16.7/18.6 = 0.898$. The ratio $(\text{fom}')_{\text{V}} / (\text{fom}')_{\text{H}}$ is $12.0/13.2 = 0.909$. This is not surprising since

$$\text{eff} = \frac{I}{vol} = \frac{I}{(h^2/2)l} = \frac{0.707hI_u}{(h^2/2)l} = 1.414\frac{I_u}{hl} = 1.414\text{fom}'$$

The ratios $(\text{eff})_{\text{V}} / (\text{eff})_{\text{H}}$ and $(\text{fom}')_{\text{V}} / (\text{fom}')_{\text{H}}$ give the same information.

9-43 $F = 0$, $T = 15$ kip·in.

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283$ in^3, $J = 0.707(1/4)\ 6.283 = 1.111$ in^4

$$\tau_{\max} = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi} \quad \textit{Ans.}$$

9-44 $F = 2$ kip, $T = 0$.

Table 9-2: $A = 1.414\ \pi h\ r = 1.414\ \pi(1/4)(1) = 1.111$ in^2

$I_u = \pi r^3 = \pi(1)^3 = 3.142$ in^3, $I = 0.707(1/4)\ 3.142 = 0.5553$ in^4

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$\tau'' = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

$$\tau_{\max} = (\tau'^2 + \tau''^2)^{1/2} = (1.80^2 + 21.6^2)^{1/2} = 21.7 \text{ kpsi} \quad \textit{Ans.}$$

9-45 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\ \pi h\ r = 1.414\ \pi(1/4)(1) = 1.111$ in^2

$I_u = \pi r^3 = \pi(1)^3 = 3.142$ in^3, $I = 0.707(1/4)\ 3.142 = 0.5553$ in^4

$$\tau' = \frac{V}{A} = \frac{2}{1.111} = 1.80 \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{0.5553} = 21.6 \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283$ in^3, $J = 0.707(1/4)\ 6.283 = 1.111$ in^4

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{1.111} = 13.5 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{1.80^2 + 21.6^2 + 13.5^2} = 25.5 \text{ kpsi} \quad \textit{Ans.}$$

9-46 $F = 2$ kip, $T = 15$ kip·in.

Bending:

Table 9-2: $A = 1.414\,\pi h\, r = 1.414\,\pi h\,(1) = 4.442h$ in^2

$$I_u = \pi r^3 = \pi(1)^3 = 3.142 \text{ in}^3, \quad I = 0.707\,h\,(3.142) = 2.221h \text{ in}^4$$

$$\tau' = \frac{V}{A} = \frac{2}{4.442h} = \frac{0.4502}{h} \text{ kpsi}$$

$$(\tau'')_M = \frac{Mr}{I} = \frac{2(6)(1)}{2.221h} = \frac{5.403}{h} \text{ kpsi}$$

Torsion:

Table 9-1: $J_u = 2\pi r^3 = 2\pi(1)^3 = 6.283 \text{ in}^3, \quad J = 0.707\,h\,(6.283) = 4.442 \text{ in}^4$

$$(\tau'')_T = \frac{Tr}{J} = \frac{15(1)}{4.442h} = \frac{3.377}{h} \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + (\tau'')_M^2 + (\tau'')_T^2} = \sqrt{\left(\frac{0.4502}{h}\right)^2 + \left(\frac{5.403}{h}\right)^2 + \left(\frac{3.377}{h}\right)^2} = \frac{6.387}{h} \text{ kpsi}$$

$$\tau_{\max} = \tau_{\text{all}} \quad \Rightarrow \quad \frac{6.387}{h} = 20 \quad \Rightarrow \quad h = 0.319 \text{ in} \quad \textit{Ans.}$$

Should specify a $\frac{3}{8}$ in weld. *Ans.*

9-47 $h = 9$ mm, $d = 200$ mm, $b = 25$ mm

From Table 9-2, case 2:

$A = 1.414(9)(200) = 2.545(10^3)$ mm^2

$$I_u = \frac{d^3}{6} = \frac{200^3}{6} = 1.333\left(10^6\right) \text{ mm}^3$$

$I = 0.707h\, I_u = 0.707(9)(1.333)(10^6) = 8.484(10^6)$ mm^4

$$\tau' = \frac{F}{A} = \frac{25\left(10^3\right)}{2.545(10^3)} = 9.82 \text{ MPa}$$

$$M = 25(150) = 3750 \text{ N·m}$$

$$\tau'' = \frac{Mc}{I} = \frac{3750(100)}{8.484(10^6)}\left(10^3\right) = 44.20 \text{ MPa}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{9.82^2 + 44.20^2} = 45.3 \text{ MPa} \quad \textit{Ans.}$$

9-48 **Note to the Instructor**. In the first printing of the ninth edition, the vertical dimension of 5 in should be to the top of the top plate. This will be corrected in the printings that follow. We apologize if this has caused any inconvenience.

h = 0.25 in, b = 2.5 in, d = 5 in.

Table 9-2, case 5: $A = 0.707h\,(b + 2d) = 0.707(0.25)[2.5 + 2(5)] = 2.209 \text{ in}^2$

$$\bar{y} = \frac{d^2}{b + 2d} = \frac{5^2}{2.5 + 2(5)} = 2 \text{ in}$$

$$I_u = \frac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\,\bar{y}^2$$

$$= \frac{2\left(5^3\right)}{3} - 2\left(5^2\right)(2) + \left[2.5 + 2(5)\right]\left(2^2\right) = 33.33 \text{ in}^3$$

$$I = 0.707\,h\,I_u = 0.707(1/4)(33.33) = 5.891 \text{ in}^4$$

Primary shear:

$$\tau' = \frac{F}{A} = \frac{2}{2.209} = 0.905 \text{ kpsi}$$

Secondary shear (the critical location is at the bottom of the bracket):

$$y = 5 - 2 = 3 \text{ in}$$

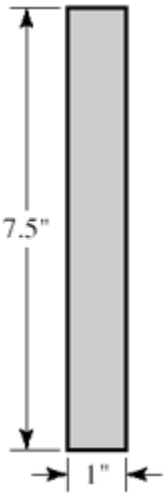
$$\tau'' = \frac{My}{I} = \frac{2(5)(3)}{5.891} = 5.093 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{0.905^2 + 5.093^2} = 5.173 \text{ kpsi}$$

$$n = \frac{\tau_{\text{all}}}{\tau_{\max}} = \frac{18}{5.173} = 3.48 \qquad \textit{Ans.}$$

9-49 The largest possible weld size is 1/16 in. This is a small weld and thus difficult to accomplish. The bracket's load-carrying capability is not known. There are geometry problems associated with sheet metal folding, load-placement and location of the center of twist. This is not available to us. We will identify the strongest possible weldment. Use a rectangular, weld-all-around pattern – Table 9-2, case 6:

$$A = 1.414\,h(b + d) = 1.414(1/16)(1 + 7.5)$$
$$= 0.7512 \text{ in}^2$$
$$\bar{x} = b / 2 = 0.5 \text{ in}$$
$$\bar{y} = d / 2 = 7.5 / 2 = 3.75 \text{ in}$$
$$I_u = \frac{d^2}{6}(3b + d) = \frac{7.5^2}{6}[3(1) + 7.5] = 98.44 \text{ in}^3$$
$$I = 0.707 h I_u = 0.707(1/16)(98.44) = 4.350 \text{ in}^4$$
$$M = (3.75 + 0.5)W = 4.25W$$
$$\tau' = \frac{V}{A} = \frac{W}{0.7512} = 1.331W$$
$$\tau'' = \frac{Mc}{I} = \frac{4.25W(7.5/2)}{4.350} = 3.664W$$
$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = W\sqrt{1.331^2 + 3.664^2} = 3.90W$$



Material properties: The allowable stress given is low. Let's demonstrate that. For the 1020 CD bracket, use HR properties of S_y = 30 kpsi and S_{ut} = 55. The 1030 HR support, S_y = 37.5 kpsi and S_{ut} = 68. The E6010 electrode has strengths of S_y = 50 and S_{ut} = 62 kpsi.

Allowable stresses:

1020 HR: $\tau_{\text{all}} = \min[0.3(55), 0.4(30)] = \min(16.5, 12) = 12$ kpsi

1020 HR: $\tau_{\text{all}} = \min[0.3(68), 0.4(37.5)] = \min(20.4, 15) = 15$ kpsi

E6010: $\tau_{\text{all}} = \min[0.3(62), 0.4(50)] = \min(18.6, 20) = 18.6$ kpsi

Since Table 9-6 gives 18.0 kpsi as the allowable shear stress, use this lower value. Therefore, the allowable shear stress is

$$\tau_{\text{all}} = \min(14.4, 12, 18.0) = 12 \text{ kpsi}$$

However, the allowable stress in the problem statement is 1.5 kpsi which is low from the weldment perspective. The load associated with this strength is

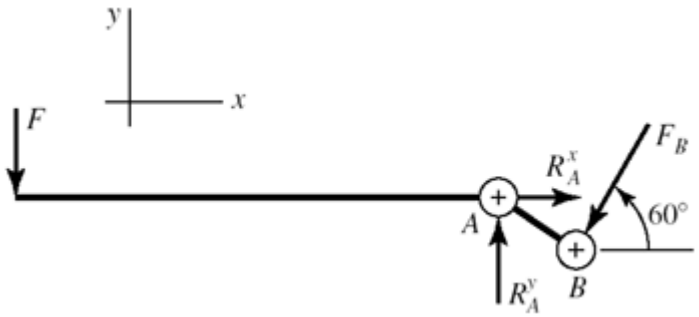
$$\tau_{\max} = \tau_{\text{all}} = 3.90W = 1500$$
$$W = \frac{1500}{3.90} = 385 \text{ lbf}$$

If the welding can be accomplished (1/16 leg size is a small weld), the weld strength is 12 000 psi and the load associated with this strength is W = 12 000/3.90 = 3077 lbf. Can the bracket carry such a load?

There are geometry problems associated with sheet metal folding. Load placement is important and the center of twist has not been identified. Also, the load-carrying capability of the top bend is unknown.

These uncertainties may require the use of a different weld pattern. Our solution provides the best weldment and thus insight for comparing a welded joint to one which employs screw fasteners.

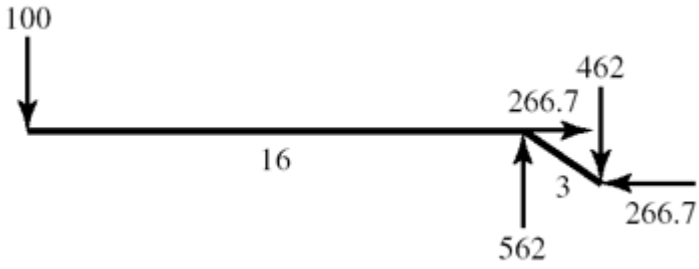
9-50



$$F = 100 \text{ lbf}, \quad \tau_{\text{all}} = 3 \text{ kpsi}$$
$$F_B = 100(16 / 3) = 533.3 \text{ lbf}$$
$$F_B^x = -533.3 \cos 60^\circ = -266.7 \text{ lbf}$$
$$F_B^y = -533.3 \cos 30^\circ = -462 \text{ lbf}$$

It follows that $R_A^y = 562$ lbf and $R_A^x = 266.7$ lbf, $R_A = 622$ lbf

$$M = 100(16) = 1600 \text{ lbf} \cdot \text{in}$$



The OD of the tubes is 1 in. From Table 9-1, case 6:

$$A = 2\left[1.414(\pi hr)\right] = 2(1.414)(\pi h)(1 / 2) = 4.442h \text{ in}^2$$
$$J_u = 2\pi r^3 = 2\pi(1 / 2)^3 = 0.7854 \text{ in}^3$$
$$J = 2(0.707)hJ_u = 1.414(0.7854)h = 1.111h \text{ in}^4$$

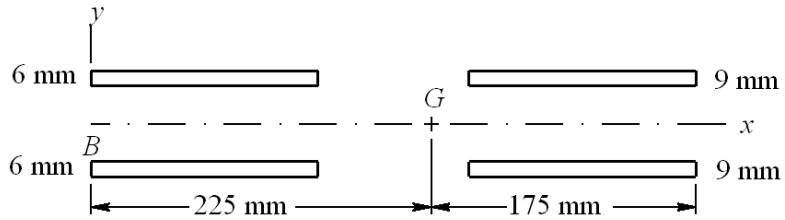
$$\tau' = \frac{V}{A} = \frac{622}{4.442h} = \frac{140.0}{h}$$
$$\tau'' = \frac{Tc}{J} = \frac{Mc}{J} = \frac{1600(0.5)}{1.111h} = \frac{720.1}{h}$$

The shear stresses, τ' and τ'', are additive algebraically

$$\tau_{\max} = \frac{1}{h}(140.0 + 720.1) = \frac{860}{h}\text{ psi}$$
$$\tau_{\max} = \tau_{\text{all}} = \frac{860}{h} = 3000$$
$$h = \frac{860}{3000} = 0.287 \rightarrow 5/16\text{ in}$$

Decision: Use 5/16 in fillet welds *Ans.*

9-51



For the pattern in bending shown, find the centroid *G* of the weld group.

$$\bar{x} = \frac{75(6)(150) + 325(9)(150)}{(6)(150) + (9)(150)} = 225\text{ mm}$$

$$I_{6\text{mm}} = 2\left(I_G + A\bar{x}^2\right)_{6\text{mm}}$$
$$= 2\left[\frac{0.707(6)(150^3)}{12} + 0.707(6)(150)(225-75)^2\right] = 31.02(10^6)\text{ mm}^4$$

$$I_{9\text{mm}} = 2\left[\frac{0.707(9)(150^3)}{12} + 0.707(9)(150)(175-75)^2\right] = 22.67(10^6)\text{ mm}^4$$

$$I = I_{6\text{ mm}} + I_{9\text{ mm}} = (31.02 + 22.67)(10^6) = 53.69(10^6)\text{ mm}^4$$

The critical location is at *B*. With τ in MPa, and *F* in kN

$$\tau' = \frac{V}{A} = \frac{F\left(10^3\right)}{2\left[0.707\left(6+9\right)\left(150\right)\right]} = 0.3143F$$

$$\tau'' = \frac{Mc}{I} = \frac{200F\left(10^3\right)\left(225\right)}{53.69\left(10^6\right)} = 0.8381F$$

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = F\sqrt{0.3143^2 + 0.8381^2} = 0.8951F$$

Materials:

1015 HR (Table A-20): S_y = 190 MPa, E6010 Electrode(Table 9-3): S_y = 345 MPa

Eq. (5-21), p. 225 $\tau_{\text{all}} = 0.577(190) = 109.6$ MPa

$$F = \frac{\tau_{\text{all}} / n}{0.8951} = \frac{109.6 / 2}{0.8951} = 61.2 \text{ kN} \qquad \textit{Ans.}$$

9-52 In the textbook, Fig. Problem 9-52*b* is a free-body diagram of the bracket. Forces and moments that act on the welds are equal, but of opposite sense.

(a) $M = 1200(0.366) = 439$ lbf · in *Ans.*

(b) $F_y = 1200 \sin 30° = 600$ lbf *Ans.*

(c) $F_x = 1200 \cos 30° = 1039$ lbf *Ans.*

(d) From Table 9-2, case 6:

$$A = 1.414(0.25)(0.25 + 2.5) = 0.972 \text{ in}^2$$

$$I_u = \frac{d^2}{6}(3b + d) = \frac{2.5^2}{6}[3(0.25) + 2.5] = 3.39 \text{ in}^3$$

The second area moment about an axis through G and parallel to z is

$$I = 0.707hI_u = 0.707(0.25)(3.39) = 0.599 \text{ in}^4 \qquad \textit{Ans.}$$

(e) Refer to Fig. Problem 9-52*b*. The shear stress due to F_y is

$$\tau_1 = \frac{F_y}{A} = \frac{600}{0.972} = 617 \text{ psi}$$

The shear stress along the throat due to F_x is

$$\tau_2 = \frac{F_x}{A} = \frac{1039}{0.972} = 1069 \text{ psi}$$

The resultant of τ_1 and τ_2 is in the throat plane

$$\tau' = \sqrt{\tau_1^2 + \tau_2^2} = \sqrt{617^2 + 1069^2} = 1234 \text{ psi}$$

The bending of the throat gives

$$\tau'' = \frac{Mc}{I} = \frac{439(1.25)}{0.599} = 916 \text{ psi}$$

The maximum shear stress is

$$\tau_{\max} = \sqrt{\tau'^2 + \tau''^2} = \sqrt{1234^2 + 916^2} = 1537 \text{ psi} \quad \textit{Ans.}$$

(f) *Materials:*

1018 HR Member: $S_y = 32$ kpsi, $S_{ut} = 58$ kpsi (Table A-20)
E6010 Electrode: $S_y = 50$ kpsi (Table 9-3)

$$n = \frac{S_{sy}}{\tau_{\max}} = \frac{0.577 S_y}{\tau_{\max}} = \frac{0.577(32)}{1.537} = 12.0 \quad \textit{Ans.}$$

(g) Bending in the attachment near the base. The cross-sectional area is approximately equal to *bh.*

$$A_1 \doteq bh = 0.25(2.5) = 0.625 \text{ in}^2$$

$$\tau_{xy} = \frac{F_x}{A_1} = \frac{1039}{0.625} = 1662 \text{ psi}$$

$$\frac{I}{c} = \frac{bd^2}{6} = \frac{0.25(2.5)^2}{6} = 0.260 \text{ in}^3$$

At location *A*,

$$\sigma_y = \frac{F_y}{A_1} + \frac{M}{I/c}$$

$$\sigma_y = \frac{600}{0.625} + \frac{439}{0.260} = 2648 \text{ psi}$$

The von Mises stress σ' is

$$\sigma' = \sqrt{\sigma_y^2 + 3\tau_{xy}^2} = \sqrt{2648^2 + 3(1662)^2} = 3912 \text{ psi}$$

Thus, the factor of safety is,

$$n = \frac{S_y}{\sigma'} = \frac{32}{3.912} = 8.18 \quad \textit{Ans.}$$

The clip on the mooring line bears against the side of the 1⁄2-in hole. If the clip fills the hole

$$\sigma = \frac{F}{td} = \frac{-1200}{0.25(0.50)} = -9600 \text{ psi}$$

$$n = -\frac{S_y}{\sigma'} = -\frac{32(10^3)}{-9600} = 3.33 \quad \textit{Ans.}$$

Further investigation of this situation requires more detail than is included in the task statement.

(h) In shear fatigue, the weakest constituent of the weld melt is 1018 HR with S_{ut} = 58 kpsi, Eq. (6-8), p. 282, gives

$$S'_e = 0.504 S_{ut} = 0.504(58) = 29.2 \text{ kpsi}$$

Eq. (6-19), p. 287: $k_a = 14.4(58)^{-0.718} = 0.780$

For the size factor estimate, we first employ Eq. (6-25), p. 289, for the equivalent diameter

$$d_e = 0.808\sqrt{0.707hb} = 0.808\sqrt{0.707(2.5)(0.25)} = 0.537 \text{ in}$$

Eq. (6-20), p. 288, is used next to find k_b

$$k_b = \left(\frac{d_e}{0.30}\right)^{-0.107} = \left(\frac{0.537}{0.30}\right)^{-0.107} = 0.940$$

Eq.(6-26), p. 290: $k_c = 0.59$

From Eq. (6-18), p. 287, the endurance strength in shear is

$$S_{se} = 0.780(0.940)(0.59)(29.2) = 12.6 \text{ kpsi}$$

From Table 9-5, the shear stress-concentration factor is $K_{fs} = 2.7$. The loading is repeatedly-applied

$$\tau_a = \tau_m = K_{fs}\frac{\tau_{\max}}{2} = 2.7\frac{1.537}{2} = 2.07 \text{ kpsi}$$

Table 6-7, p. 307: Gerber factor of safety n_f, adjusted for shear, with $S_{su} = 0.67 S_{ut}$

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{0.67(58)}{2.07}\right]^2\left(\frac{2.07}{12.6}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(2.07)(12.6)}{0.67(58)(2.07)}\right]^2}\right\} = 5.55 \quad \textit{Ans.}$$

Attachment metal should be checked for bending fatigue.

9-53 (**a**) Use $b = d = 4$ in. Since $h = 5/8$ in, the primary shear is

$$\tau' = \frac{F}{1.414(5/8)(4)} = 0.2829F$$

The secondary shear calculations, for a moment arm of 14 in give

$$J_u = \frac{4[3(4^2) + 4^2]}{6} = 42.67 \text{ in}^3$$
$$J = 0.707hJ_u = 0.707(5/8)42.67 = 18.85 \text{ in}^4$$
$$\tau''_x = \tau''_y = \frac{Mr_y}{J} = \frac{14F(2)}{18.85} = 1.485F$$

Thus, the maximum shear and allowable load are:

$$\tau_{\max} = F\sqrt{1.485^2 + (0.2829 + 1.485)^2} = 2.309F$$
$$F = \frac{\tau_{\text{all}}}{2.309} = \frac{25}{2.309} = 10.8 \text{ kip} \quad \textit{Ans.}$$

The load for part (a) has increased by a factor of 10.8/2.71 = 3.99 *Ans*.

(**b**) From Prob. 9-18*b*, $\tau_{\text{all}} = 11$ kpsi

$$F_{\text{all}} = \frac{\tau_{\text{all}}}{2.309} = \frac{11}{2.309} = 4.76 \text{ kip}$$

The allowable load in part (b) has increased by a factor of 4.76/1.19 = 4 *Ans.*

9-54 Purchase the hook having the design shown in Fig. Problem 9-54*b*. Referring to text Fig. 9-29*a*, this design reduces peel stresses.

9-55 **(a)**

$$\bar{\tau} = \frac{1}{l}\int_{-l/2}^{l/2} \frac{P\omega\cosh(\omega x)}{4b\sinh(\omega l/2)}dx = A_1\int_{-l/2}^{l/2}\cosh(\omega x)\,dx = \frac{A_1}{\omega}\sinh(\omega x)\Big|_{-l/2}^{l/2}$$
$$= \frac{A_1}{\omega}[\sinh(\omega l/2) - \sinh(-\omega l/2)] = \frac{A_1}{\omega}[\sinh(\omega l/2) - (-\sinh(\omega l/2))]$$
$$= \frac{2A_1\sinh(\omega l/2)}{\omega} = \frac{P\omega}{4bl\sinh(\omega l/2)}[2\sinh(\omega l/2)] = \frac{P}{2bl} \quad \textit{Ans.}$$

(b) $$\tau(l/2) = \frac{P\omega\cosh(\omega l/2)}{4b\sinh(\omega l/2)} = \frac{P\omega}{4b\tanh(\omega l/2)} \quad \textit{Ans.}$$

(c) $$K = \frac{\tau(l/2)}{\bar{\tau}} = \frac{P\omega}{4b\tanh(\omega l/2)}\left(\frac{2bl}{P}\right) = \frac{\omega l/2}{\tanh(\omega l/2)} \qquad Ans.$$

For computer programming, it can be useful to express the hyperbolic tangent in terms of exponentials:

$$K = \frac{\omega l}{2}\frac{\exp(\omega l/2) - \exp(-\omega l/2)}{\exp(\omega l/2) + \exp(-\omega l/2)} \qquad Ans.$$

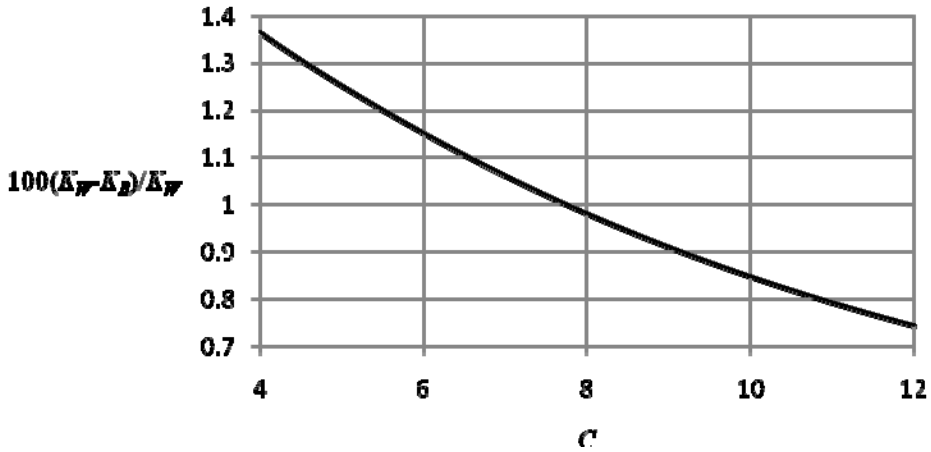
9-56 This is a computer programming exercise. All programs will vary.

Chapter 10

10-1 From Eqs. (10-4) and (10-5)

$$K_W - K_B = \frac{4C-1}{4C-4} + \frac{0.615}{C} - \frac{4C+2}{4C-3}$$

Plot $100(K_W - K_B)/K_W$ vs. C for $4 \le C \le 12$ obtaining



We see the maximum and minimum occur at C = 4 and 12 respectively where

Maximum = 1.36 % *Ans*., and Minimum = 0.743 % *Ans*.

10-2 $A = Sd^m$

$$\dim(A_{\text{uscu}}) = [\dim(S)\,\dim(d^{\text{m}})]_{\text{uscu}} = \text{kpsi}\cdot\text{in}^m$$

$$\dim(A_{\text{SI}}) = [\dim(S)\,\dim(d^{m})]_{\text{SI}} = \text{MPa}\cdot\text{mm}^m$$

$$A_{\text{SI}} = \frac{\text{MPa}}{\text{kpsi}} \cdot \frac{\text{mm}^m}{\text{in}^m} A_{\text{uscu}} = 6.894\,757\left(25.4\right)^m A_{\text{uscu}} \doteq 6.895\left(25.4\right)^m A_{\text{uscu}} \quad \textit{Ans}.$$

For music wire, from Table 10-4:

A_{uscu} = 201 kpsi·in^m, m = 0.145; what is A_{SI}?

$A_{\text{SI}} = 6.895(25.4)^{0.145}\,(201) = 2215$ MPa·mm^m *Ans*.

10-3 Given: Music wire, d = 2.5 mm, OD = 31 mm, plain ground ends, N_t = 14 coils.

(**a**) Table 10-1: $N_a = N_t - 1 = 14 - 1 = 13$ coils

$L_s = d\,N_t = 2.5(14) = 35$ mm

Table 10-4: $m = 0.145$, $A = 2211$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{2211}{2.5^{0.145}} = 1936 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.45(1936) = 871.2$ MPa

$D = \text{OD} - d = 31 - 2.5 = 28.5$ mm

$C = D/d = 28.5/2.5 = 11.4$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(11.4)+2}{4(11.4)-3} = 1.117$$

Eq. (10-7): $$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(2.5^3)871.2}{8(1.117)28.5} = 167.9 \text{ N}$$

Table 10-5): $d = 2.5/25.4 = 0.098$ in $\Rightarrow$ $G = 81.0(10^3)$ MPa

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{2.5^4(81)10^3}{8(28.5^3)13} = 1.314 \text{ N/mm}$$

$$L_0 = \frac{F_s}{k} + L_s = \frac{167.9}{1.314} + 35 = 162.8 \text{ mm} \qquad \textit{Ans.}$$

(**b**) $F_s = 167.9$ N *Ans.*

(**c**) $k = 1.314$ N/mm *Ans.*

(**d**) $(L_0)_{\text{cr}} = \dfrac{2.63(28.5)}{0.5} = 149.9$ mm. Spring needs to be supported. *Ans.*

10-4 Given: Design load, $F_1 = 130$ N.

Referring to Prob. 10-3 solution, $C = 11.4$, $N_a = 13$ coils, $S_{sy} = 871.2$ MPa, $F_s = 167.9$ N, $L_0 = 162.8$ mm and $(L_0)_{\text{cr}} = 149.9$ mm.
Eq. (10-18): $4 \le C \le 12$ $C = 11.4$ *O.K.*
Eq. (10-19): $3 \le N_a \le 15$ $N_a = 13$ *O.K.*

Eq. (10-17): $\xi = \dfrac{F_s}{F_1} - 1 = \dfrac{167.9}{130} - 1 = 0.29$

Eq. (10-20): $\xi \geq 0.15, \quad \xi = 0.29 \quad O.K.$

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.117\frac{8(130)(28.5)}{\pi(2.5)^3} = 674 \text{ MPa}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{871.2}{674} = 1.29$$

Eq. (10-21): $n_s \geq 1.2, \quad n = 1.29 \quad O.K.$

$$\tau_s = \tau_1\left(\frac{167.9}{130}\right) = 674\left(\frac{167.9}{130}\right) = 870.5 \text{ MPa}$$

$$S_{sy} \,/\, \tau_s = 871.2 \,/\, 870.5 \doteq 1$$

$S_{sy}/\tau_s \geq (n_s)_d$: Not solid-safe (but was the basis of the design). *Not O.K.*

$L_0 \leq (L_0)_{cr}$: $162.8 \geq 149.9$ *Not O.K.*

Design is unsatisfactory. Operate over a rod? *Ans.*

10-5 Given: Oil-tempered wire, $d = 0.2$ in, $D = 2$ in, $N_t = 12$ coils, $L_0 = 5$ in, squared ends.

(**a**) Table 10-1: $L_s = d\,(N_t + 1) = 0.2(12 + 1) = 2.6$ in *Ans*.

(**b**) Table 10-1: $N_a = N_t - 2 = 12 - 2 = 10$ coils

Table 10-5: $G = 11.2$ Mpsi

Eq. (10-9): $$k = \frac{d^4G}{8D^3N} = \frac{0.2^4\,(11.2)10^6}{8\left(2^3\right)10} = 28 \text{ lbf/in}$$

$$F_s = k\,y_s = k\,(L_0 - L_s) = 28(5 - 2.6) = 67.2 \text{ lbf} \qquad Ans.$$

(**c**) Eq. (10-1): $C = D/d = 2/0.2 = 10$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7): $$\tau_s = K_B\frac{8FD}{\pi d^3} = 1.135\frac{8(67.2)2}{\pi\left(0.2^3\right)} = 48.56\left(10^3\right) \text{ psi}$$

Table 10-4: $m = 0.187$, $A = 147$ kpsi·in^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2^{0.187}} = 198.6 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(198.6) = 99.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{99.3}{48.56} = 2.04 \qquad \textit{Ans.}$$

10-6 Given: Oil-tempered wire, $d = 4$ mm, $C = 10$, plain ends, $L_0 = 80$ mm, and at $F = 50$ N, $y = 15$ mm.

(**a**) $k = F/y = 50/15 = 3.333$ N/mm *Ans.*

(**b**) $D = Cd = 10(4) = 40$ mm

OD $= D + d = 40 + 4 = 44$ mm *Ans.*

(**c**) From Table 10-5, $G = 77.2$ GPa

Eq. (10-9): $$N_a = \frac{d^4 G}{8kD^3} = \frac{4^4(77.2)10^3}{8(3.333)40^3} = 11.6 \text{ coils}$$

Table 10-1: $N_t = N_a = 11.6$ coils *Ans.*

(**d**) Table 10-1: $L_s = d\,(N_t + 1) = 4(11.6 + 1) = 50.\,4$ mm *Ans.*

(**e**) Table 10-4: $m = 0.187$, $A = 1855$ MPa·mm^m

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1855}{4^{0.187}} = 1431 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1431) = 715.5$ MPa

$$y_s = L_0 - L_s = 80 - 50.4 = 29.6 \text{ mm}$$

$$F_s = k\, y_s = 3.333(29.6) = 98.66 \text{ N}$$

Eq. (10-5): $$K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.135\frac{8(98.66)40}{\pi(4^3)} = 178.2 \text{ MPa}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{715.5}{178.2} = 4.02 \qquad \textit{Ans.}$$

10-7 Static service spring with: HD steel wire, d = 0.080 in, OD = 0.880 in, N_t = 8 coils, plain and ground ends.

Preliminaries

Table 10-5: A = 140 kpsi · in^m, m = 0.190

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{140}{0.080^{0.190}} = 226.2 \text{ kpsi}$$

Table 10-6: S_{sy} = 0.45(226.2) = 101.8 kpsi

Then,

D = OD – d = 0.880 – 0.080 = 0.8 in

Eq. (10-1): $C = D/d$ = 0.8/0.08 = 10

Eq. (10-5): $$K_B = \frac{4C + 2}{4C - 3} = \frac{4(10) + 2}{4(10) - 3} = 1.135$$

Table 10-1: $N_a = N_t - 1 = 8 - 1 = 7$ coils

$L_s = dN_t$ = 0.08(8) = 0.64 in

Eq. (10-7) For solid-safe, n_s = 1.2 :

$$F_s = \frac{\pi d^3 S_{sy} / n_s}{8K_B D} = \frac{\pi(0.08^3)\left[101.8(10^3)/1.2\right]}{8(1.135)(0.8)} = 18.78 \text{ lbf}$$

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.08^4(11.5)10^6}{8(0.8^3)7} = 16.43 \text{ lbf/in}$$

$$y_s = \frac{F_s}{k} = \frac{18.78}{16.43} = 1.14 \text{ in}$$

(a) $L_0 = y_s + L_s$ = 1.14 + 0.64 = 1.78 in *Ans.*

(b) Table 10-1: $$p = \frac{L_0}{N_t} = \frac{1.78}{8} = 0.223 \text{ in} \qquad \textit{Ans.}$$

(c) From above: F_s = 18.78 lbf *Ans.*

(d) From above: k = 16.43 lbf/in *Ans.*

(e) Table 10-2 and Eq. (10-13): $$(L_0)_{cr} = \frac{2.63D}{\alpha} = \frac{2.63(0.8)}{0.5} = 4.21 \text{ in}$$

Since $L_0 < (L_0)_{cr}$, buckling is unlikely *Ans.*

10-8 Given: Design load, F_1 = 16.5 lbf.

Referring to Prob. 10-7 solution, C = 10, N_a = 7 coils, S_{sy} = 101.8 kpsi, F_s = 18.78 lbf, y_s = 1.14 in, L_0 = 1.78 in, and $(L_0)_{cr}$ = 4.208 in.

Eq. (10-18): $4 \le C \le 12$ $C = 10$ *O.K.*
Eq. (10-19): $3 \le N_a \le 15$ $N_a = 7$ *O.K.*

Eq. (10-17): $\xi = \dfrac{F_s}{F_1} - 1 = \dfrac{18.78}{16.5} - 1 = 0.14$

Eq. (10-20): $\xi \ge 0.15, \quad \xi = 0.14$ *not O.K.*, but probably acceptable.

From Eq. (10-7) for static service

$$\tau_1 = K_B\left(\frac{8F_1D}{\pi d^3}\right) = 1.135\frac{8(16.5)(0.8)}{\pi(0.080)^3} = 74.5\left(10^3\right) \text{ psi} = 74.5 \text{ kpsi}$$

$$n = \frac{S_{sy}}{\tau_1} = \frac{101.8}{74.5} = 1.37$$

Eq. (10-21): $n_s \ge 1.2, \quad n = 1.37$ *O.K.*

$$\tau_s = \tau_1\left(\frac{18.78}{16.5}\right) = 74.5\left(\frac{18.78}{16.5}\right) = 84.8 \text{ kpsi}$$

$$n_s = S_{sy} / \tau_s = 101.8 / 84.8 = 1.20$$

Eq. (10-21): $n_s \ge 1.2, \quad n_s = 1.2$ It is solid-safe (basis of design). *O.K.*

Eq. (10-13) and Table 10-2: $L_0 \le (L_0)_{cr}$ $1.78 \text{ in} \le 4.208 \text{ in}$ *O.K.*

10-9 Given: A228 music wire, sq. and grd. ends, $d = 0.007$ in, OD = 0.038 in, $L_0 = 0.58$ in, $N_t = 38$ coils.

$D = \text{OD} - d = 0.038 - 0.007 = 0.031$ in

Eq. (10-1): $C = D/d = 0.031/0.007 = 4.429$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(4.429\right)+2}{4\left(4.429\right)-3} = 1.340$

Table (10-1): $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 12.0$ Mpsi

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.007^4\left(12.0\right)10^6}{8\left(0.031^3\right)36} = 3.358$ lbf/in

Table (10-1): $L_s = dN_t = 0.007(38) = 0.266$ in

$y_s = L_0 - L_s = 0.58 - 0.266 = 0.314$ in

$F_s = ky_s = 3.358(0.314) = 1.054$ lbf

Eq. (10-7): $\tau_s = K_B\dfrac{8F_sD}{\pi d^3} = 1.340\dfrac{8\left(1.054\right)0.031}{\pi\left(0.007^3\right)} = 325.1\left(10^3\right) \text{ psi}$ (1)

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{201}{0.007^{0.145}} = 412.7$ kpsi

Table 10-6: $S_{sy} = 0.45\ S_{ut} = 0.45(412.7) = 185.7$ kpsi

$\tau_s > S_{sy}$, that is, 325.1 > 185.7 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[185.7\left(10^3\right)/1.2\right]\pi\left(0.007^3\right)}{8\left(1.340\right)3.358\left(0.031\right)} = 0.149 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.266 + 0.149 = 0.415 \text{ in} \qquad Ans.$$

This only addresses the solid-safe criteria. There are additional problems.

10-10 Given: B159 phosphor-bronze, sq. and grd. ends, $d = 0.014$ in, OD = 0.128 in, $L_0 = 0.50$ in, $N_t = 16$ coils.

$D = \text{OD} - d = 0.128 - 0.014 = 0.114$ in

Eq. (10-1): $C = D/d = 0.114/0.014 = 8.143$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4\left(8.143\right)+2}{4\left(8.143\right)-3} = 1.169$

Table (10-1): $N_a = N_t - 2 = 16 - 2 = 14$ coils

Table 10-5: $G = 6$ Mpsi

Eq. (10-9): $k = \dfrac{d^4 G}{8D^3 N_a} = \dfrac{0.014^4\left(6\right)10^6}{8\left(0.114^3\right)14} = 1.389$ lbf/in

Table (10-1): $L_s = dN_t = 0.014(16) = 0.224$ in

$y_s = L_0 - L_s = 0.50 - 0.224 = 0.276$ in

$F_s = ky_s = 1.389(0.276) = 0.3834$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.169\dfrac{8\left(0.3834\right)0.114}{\pi\left(0.014^3\right)} = 47.42\left(10^3\right)$ psi (1)

Table 10-4: $A = 145$ kpsi·in^m, $m = 0$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{145}{0.014^0} = 145$ kpsi

Table 10-6: $S_{sy} = 0.35\ S_{ut} = 0.35(135) = 47.25$ kpsi

$\tau_s > S_{sy}$, that is, 47.42 > 47.25 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_B kD} = \frac{\left[47.25\left(10^3\right)/1.2\right]\pi\left(0.014^3\right)}{8\left(1.169\right)1.389\left(0.114\right)} = 0.229 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 0.224 + 0.229 = 0.453 \text{ in} \qquad \textit{Ans.}$$

10-11 Given: A313 stainless steel, sq. and grd. ends, $d = 0.050$ in, OD = 0.250 in, $L_0 = 0.68$ in, $N_t = 11.2$ coils.

$$D = \text{OD} - d = 0.250 - 0.050 = 0.200 \text{ in}$$

Eq. (10-1): $C = D/d = 0.200/0.050 = 4$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4)+2}{4(4)-3} = 1.385$$

Table (10-1): $N_a = N_t - 2 = 11.2 - 2 = 9.2$ coils

Table 10-5: $G = 10$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.050^4 (10)10^6}{8(0.2^3)9.2} = 106.1 \text{ lbf/in}$$

Table (10-1): $L_s = dN_t = 0.050(11.2) = 0.56$ in

$y_s = L_0 - L_s = 0.68 - 0.56 = 0.12$ in

$F_s = ky_s = 106.1(0.12) = 12.73$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.385 \frac{8(12.73)0.2}{\pi(0.050^3)} = 71.8(10^3) \text{ psi}$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.050^{0.146}} = 261.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(261.7) = 91.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{91.6}{71.8} = 1.28 \quad \text{Spring is solid-safe } (n_s > 1.2) \qquad \textit{Ans.}$$

10-12 Given: A227 hard-drawn wire, sq. and grd. ends, $d = 0.148$ in, OD = 2.12 in, $L_0 = 2.5$ in, $N_t = 5.75$ coils.

$$D = \text{OD} - d = 2.12 - 0.148 = 1.972 \text{ in}$$

Eq. (10-1): $C = D/d = 1.972/0.148 = 13.32$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.32)+2}{4(13.32)-3} = 1.099$$

Table (10-1): $N_a = N_t - 2 = 5.75 - 2 = 3.75$ coils

Table 10-5: $G = 11.4$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.148^4 (11.4)10^6}{8(1.972^3)3.75} = 23.77 \text{ lbf/in}$$

Table (10-1): $L_s = dN_t = 0.148(5.75) = 0.851$ in

$y_s = L_0 - L_s = 2.5 - 0.851 = 1.649$ in

$F_s = ky_s = 23.77(1.649) = 39.20$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.099 \dfrac{8(39.20)1.972}{\pi\left(0.148^3\right)} = 66.7\left(10^3\right)$ psi

Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{140}{0.148^{0.190}} = 201.3$ kpsi

Table 10-6: $S_{sy} = 0.35\ S_{ut} = 0.45(201.3) = 90.6$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{90.6}{66.7} = 1.36 \quad \text{Spring is solid-safe } (n_s > 1.2) \qquad \textit{Ans.}$$

10-13 Given: A229 OQ&T steel, sq. and grd. ends, $d = 0.138$ in, OD = 0.92 in, $L_0 = 2.86$ in, $N_t = 12$ coils.

$D = \text{OD} - d = 0.92 - 0.138 = 0.782$ in

Eq. (10-1): $C = D/d = 0.782/0.138 = 5.667$

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(5.667)+2}{4(5.667)-3} = 1.254$

Table (10-1): $N_a = N_t - 2 = 12 - 2 = 10$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 11.5$ Mpsi.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{0.138^4(11.5)10^6}{8\left(0.782^3\right)10} = 109.0$ lbf/in

Table (10-1): $L_s = dN_t = 0.138(12) = 1.656$ in

$y_s = L_0 - L_s = 2.86 - 1.656 = 1.204$ in

$F_s = ky_s = 109.0(1.204) = 131.2$ lbf

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.254 \dfrac{8(131.2)0.782}{\pi\left(0.138^3\right)} = 124.7\left(10^3\right)$ psi (1)

Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{147}{0.138^{0.187}} = 212.9$ kpsi

Table 10-6: $S_{sy} = 0.50\ S_{ut} = 0.50(212.9) = 106.5$ kpsi

$\tau_s > S_{sy}$, that is, 124.7 > 106.5 kpsi, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{\left(S_{sy}/n_s\right)\pi d^3}{8K_BkD} = \frac{\left[106.5\left(10^3\right)/1.2\right]\pi\left(0.138^3\right)}{8(1.254)109.0(0.782)} = 0.857 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 1.656 + 0.857 = 2.51 \text{ in} \qquad \textit{Ans.}$$

10-14 Given: A232 chrome-vanadium steel, sq. and grd. ends, $d = 0.185$ in, OD = 2.75 in, $L_0 = 7.5$ in, $N_t = 8$ coils.

$$D = \text{OD} - d = 2.75 - 0.185 = 2.565 \text{ in}$$

Eq. (10-1): $C = D/d = 2.565/0.185 = 13.86$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(13.86)+2}{4(13.86)-3} = 1.095$$

Table (10-1): $N_a = N_t - 2 = 8 - 2 = 6$ coils

Table 10-5: $G = 11.2$ Mpsi.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.185^4 (11.2)10^6}{8(2.565^3)6} = 16.20 \text{ lbf/in}$$

Table (10-1): $L_s = dN_t = 0.185(8) = 1.48$ in

$y_s = L_0 - L_s = 7.5 - 1.48 = 6.02$ in

$F_s = ky_s = 16.20(6.02) = 97.5$ lbf

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.095 \frac{8(97.5)2.565}{\pi(0.185^3)} = 110.1(10^3) \text{ psi} \qquad (1)$$

Table 10-4: $A = 169$ kpsi·in^m, $m = 0.168$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{169}{0.185^{0.168}} = 224.4 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(224.4) = 112.2$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{112.2}{110.1} = 1.02$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B k D} = \frac{[112.2(10^3)/1.2]\pi(0.185^3)}{8(1.095)16.20(2.565)} = 5.109 \text{ in}$$

The free length should be wound to

$$L_0 = L_s + y_s = 1.48 + 5.109 = 6.59 \text{ in} \qquad \textit{Ans.}$$

10-15 Given: A313 stainless steel, sq. and grd. ends, $d = 0.25$ mm, OD = 0.95 mm, $L_0 = 12.1$ mm, $N_t = 38$ coils.

$$D = \text{OD} - d = 0.95 - 0.25 = 0.7 \text{ mm}$$

Eq. (10-1): $C = D/d = 0.7/0.25 = 2.8$ (low)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(2.8)+2}{4(2.8)-3} = 1.610$$

Table (10-1): $N_a = N_t - 2 = 38 - 2 = 36$ coils (high)

Table 10-5: $G = 69.0(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.25^4 (69.0)10^3}{8(0.7^3)36} = 2.728 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 0.25(38) = 9.5$ mm
$y_s = L_0 - L_s = 12.1 - 9.5 = 2.6$ mm
$F_s = ky_s = 2.728(2.6) = 7.093$ N

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.610 \frac{8(7.093)0.7}{\pi(0.25^3)} = 1303 \text{ MPa} \qquad (1)$$

Table 10-4 (dia. less than table): $A = 1867$ MPa·mm^m, $m = 0.146$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{1867}{0.25^{0.146}} = 2286 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(2286) = 734$ MPa

$\tau_s > S_{sy}$, that is, 1303 > 734 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(734/1.2)\pi(0.25^3)}{8(1.610)2.728(0.7)} = 1.22 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 9.5 + 1.22 = 10.72 \text{ mm} \qquad Ans.$$

This only addresses the solid-safe criteria. There are additional problems.

10-16 Given: A228 music wire, sq. and grd. ends, $d = 1.2$ mm, OD = 6.5 mm, $L_0 = 15.7$ mm, $N_t = 10.2$ coils.

$D = \text{OD} - d = 6.5 - 1.2 = 5.3$ mm

Eq. (10-1): $C = D/d = 5.3/1.2 = 4.417$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(4.417)+2}{4(4.417)-3} = 1.368$$

Table (10-1): $N_a = N_t - 2 = 10.2 - 2 = 8.2$ coils

Table 10-5 ($d = 1.2/25.4 = 0.0472$ in): $G = 81.7(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{1.2^4 (81.7)10^3}{8(5.3^3)8.2} = 17.35 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 1.2(10.2) = 12.24$ mm
$y_s = L_0 - L_s = 15.7 - 12.24 = 3.46$ mm
$F_s = ky_s = 17.35(3.46) = 60.03$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.368\dfrac{8(60.03)5.3}{\pi(1.2^3)} = 641.4 \text{ MPa}$ (1)

Table 10-4: $A = 2211$ MPa·mm^m, $m = 0.145$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2211}{1.2^{0.145}} = 2153 \text{ MPa}$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45(2153) = 969$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{969}{641.4} = 1.51$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-17 Given: A229 OQ&T steel, sq. and grd. ends, $d = 3.5$ mm, OD = 50.6 mm, $L_0 = 75.5$ mm, $N_t = 5.5$ coils.

$D = \text{OD} - d = 50.6 - 3.5 = 47.1$ mm

Eq. (10-1): $C = D/d = 47.1/3.5 = 13.46$ (high)

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(13.46)+2}{4(13.46)-3} = 1.098$

Table (10-1): $N_a = N_t - 2 = 5.5 - 2 = 3.5$ coils

A229 OQ&T steel is not given in Table 10-5. From Table A-5, for carbon steels, $G = 79.3(10^3)$ MPa.

Eq. (10-9): $k = \dfrac{d^4G}{8D^3N_a} = \dfrac{3.5^4(79.3)10^3}{8(47.1^3)3.5} = 4.067$ N/mm

Table (10-1): $L_s = dN_t = 3.5(5.5) = 19.25$ mm

$y_s = L_0 - L_s = 75.5 - 19.25 = 56.25$ mm

$F_s = ky_s = 4.067(56.25) = 228.8$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.098\dfrac{8(228.8)47.1}{\pi(3.5^3)} = 702.8 \text{ MPa}$ (1)

Table 10-4: $A = 1855$ MPa·mm^m, $m = 0.187$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1855}{3.5^{0.187}} = 1468 \text{ MPa}$

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1468) = 734$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{734}{702.8} = 1.04$$ Spring is not solid-safe ($n_s < 1.2$)

Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(734/1.2)\pi(3.5^3)}{8(1.098)4.067(47.1)} = 48.96 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 19.25 + 48.96 = 68.2 \text{ mm} \qquad Ans.$$

10-18 Given: B159 phosphor-bronze, sq. and grd. ends, d = 3.8 mm, OD = 31.4 mm, L_0 = 71.4 mm, N_t = 12.8 coils.

$$D = \text{OD} - d = 31.4 - 3.8 = 27.6 \text{ mm}$$

Eq. (10-1): $C = D/d = 27.6/3.8 = 7.263$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(7.263)+2}{4(7.263)-3} = 1.192$$

Table (10-1): $N_a = N_t - 2 = 12.8 - 2 = 10.8$ coils

Table 10-5: $G = 41.4(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{3.8^4 (41.4)10^3}{8(27.6^3)10.8} = 4.752 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 3.8(12.8) = 48.64$ mm

$y_s = L_0 - L_s = 71.4 - 48.64 = 22.76$ mm

$F_s = ky_s = 4.752(22.76) = 108.2$ N

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.192 \frac{8(108.2)27.6}{\pi(3.8^3)} = 165.2 \text{ MPa} \qquad (1)$$

Table 10-4 (d = 3.8/25.4 = 0.150 in): A = 932 MPa·mm^m, m = 0.064

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{932}{3.8^{0.064}} = 855.7 \text{ MPa}$$

Table 10-6: $S_{sy} = 0.35\, S_{ut} = 0.35(855.7) = 299.5$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{299.5}{165.2} = 1.81$$ Spring is solid-safe ($n_s > 1.2$) *Ans.*

10-19 Given: A232 chrome-vanadium steel, sq. and grd. ends, d = 4.5 mm, OD = 69.2 mm, L_0 = 215.6 mm, N_t = 8.2 coils.

$$D = \text{OD} - d = 69.2 - 4.5 = 64.7 \text{ mm}$$

Eq. (10-1): $C = D/d = 64.7/4.5 = 14.38$ (high)

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(14.38)+2}{4(14.38)-3} = 1.092$$

Table (10-1): $N_a = N_t - 2 = 8.2 - 2 = 6.2$ coils

Table 10-5: $G = 77.2(10^3)$ MPa.

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{4.5^4 (77.2)10^3}{8(64.7^3)6.2} = 2.357 \text{ N/mm}$$

Table (10-1): $L_s = dN_t = 4.5(8.2) = 36.9$ mm

$y_s = L_0 - L_s = 215.6 - 36.9 = 178.7$ mm

$F_s = ky_s = 2.357(178.7) = 421.2$ N

Eq. (10-7): $\tau_s = K_B \dfrac{8F_sD}{\pi d^3} = 1.092 \dfrac{8(421.2)64.7}{\pi(4.5^3)} = 832$ MPa (1)

Table 10-4: $A = 2005$ MPa·mm^m, $m = 0.168$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{2005}{4.5^{0.168}} = 1557$ MPa

Table 10-6: $S_{sy} = 0.50\, S_{ut} = 0.50(1557) = 779$ MPa

$\tau_s > S_{sy}$, that is, 832 > 779 MPa, the spring is not solid-safe. Return to Eq. (1) with $F_s = ky_s$ and $\tau_s = S_{sy}/n_s$, and solve for y_s, giving

$$y_s = \frac{(S_{sy}/n_s)\pi d^3}{8K_B kD} = \frac{(779/1.2)\pi(4.5^3)}{8(1.092)2.357(64.7)} = 139.5 \text{ mm}$$

The free length should be wound to

$$L_0 = L_s + y_s = 36.9 + 139.5 = 176.4 \text{ mm} \qquad \textit{Ans.}$$

This only addresses the solid-safe criteria. There are additional problems.

10-20 Given: A227 HD steel.

From the figure: $L_0 = 4.75$ in, OD = 2 in, and $d = 0.135$ in. Thus

$$D = \text{OD} - d = 2 - 0.135 = 1.865 \text{ in}$$

(a) By counting, $N_t = 12.5$ coils. Since the ends are squared along 1/4 turn on each end,

$$N_a = 12.5 - 0.5 = 12 \text{ turns} \qquad \textit{Ans.}$$
$$p = 4.75 / 12 = 0.396 \text{ in} \qquad \textit{Ans.}$$

The solid stack is 13 wire diameters

$$L_s = 13(0.135) = 1.755 \text{ in} \qquad \textit{Ans.}$$

(b) From Table 10-5, $G = 11.4$ Mpsi

$$k = \frac{d^4G}{8D^3N_a} = \frac{0.135^4(11.4)(10^6)}{8(1.865^3)(12)} = 6.08 \text{ lbf/in} \qquad \textit{Ans.}$$

(c) $F_s = k(L_0 - L_s) = 6.08(4.75 - 1.755)(10^{-3}) = 18.2$ lbf *Ans.*

(d) $C = D/d = 1.865/0.135 = 13.81$

$$K_B = \frac{4(13.81) + 2}{4(13.81) - 3} = 1.096$$

$$\tau_s = K_B \frac{8F_sD}{\pi d^3} = 1.096 \frac{8(18.2)(1.865)}{\pi\left(0.135^3\right)} = 38.5\left(10^3\right) \text{ psi} = 38.5 \text{ kpsi} \quad \textit{Ans.}$$

10-21 For the wire diameter analyzed, $G = 11.75$ Mpsi per Table 10-5. Use squared and ground ends. The following is a spread-sheet study using Fig. 10-3 for parts (*a*) and (*b*). For N_a, $k = F_{\max}/y = 20/2 = 10$ lbf/in. For τ_s, $F = F_s = 20(1 + \xi) = 20(1 + 0.15) = 23$ lbf.

(a) Spring over a Rod					(b) Spring in a Hole				
Source		Parameter	Values		Source		Parameter	Values	
	d	0.075	0.080	0.085		d	0.075	0.080	0.085
	ID	0.800	0.800	0.800		OD	0.950	0.950	0.950
	D	0.875	0.880	0.885		D	0.875	0.870	0.865
Eq. (10-1)	C	11.667	11.000	10.412	Eq. (10-1)	C	11.667	10.875	10.176
Eq. (10-9)	N_a	6.937	8.828	11.061	Eq. (10-9)	N_a	6.937	9.136	11.846
Table 10-1	N_t	8.937	10.828	13.061	Table 10-1	N_t	8.937	11.136	13.846
Table 10-1	L_s	0.670	0.866	1.110	Table 10-1	L_s	0.670	0.891	1.177
$1.15y + L_s$	L_0	2.970	3.166	3.410	$1.15y + L_s$	L_0	2.970	3.191	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.603	4.629	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.603	4.576	4.550
Table 10-4	A	201.000	201.000	201.000	Table 10-4	A	201.000	201.000	201.000
Table 10-4	m	0.145	0.145	0.145	Table 10-4	m	0.145	0.145	0.145
Eq. (10-14)	S_{ut}	292.626	289.900	287.363	Eq. (10-14)	S_{ut}	292.626	289.900	287.363
Table 10-6	S_{sy}	131.681	130.455	129.313	Table 10-6	S_{sy}	131.681	130.455	129.313
Eq. (10-5)	K_B	1.115	1.122	1.129	Eq. (10-5)	K_B	1.115	1.123	1.133
Eq. (10-7)	τ_s	135.335	112.948	95.293	Eq. (10-7)	τ_s	135.335	111.787	93.434
Eq. (10-3)	n_s	0.973	1.155	1.357	Eq. (10-3)	n_s	0.973	1.167	1.384
Eq. (10-22)	fom	–0.282	–0.391	–0.536	Eq. (10-22)	fom	–0.282	–0.398	–0.555

For $n_s \geq 1.2$, the optimal size is $d = 0.085$ in for both cases.

10-22 In Prob. 10-21, there is an advantage of first selecting d as one can select from the available sizes (Table A-28). Selecting C first, requires a calculation of d where then a size must be selected from Table A-28.

Consider part (a) of the problem. It is required that

$$\text{ID} = D - d = 0.800 \text{ in.} \qquad (1)$$

From Eq. (10-1), $D = Cd$. Substituting this into the first equation yields

$$d = \frac{0.800}{C - 1} \qquad (2)$$

Starting with $C = 10$, from Eq. (2) we find that $d = 0.089$ in. From Table A-28, the closest diameter is $d = 0.090$ in. Substituting this back into Eq. (1) gives $D = 0.890$ in, with $C = 0.890/0.090 = 9.889$, which are acceptable. From this point the solution is the same as Prob. 10-21. For part (b), use

$$\text{OD} = D + d = 0.950 \text{ in.} \qquad (3)$$

and,

$$d = \frac{0.800}{C - 1} \qquad (4)$$

(a) Spring over a rod				(b) Spring in a Hole		
Source		Parameter	Values	Source	Parameter	Values
	C	10.000	10.5		C	10.000
Eq. (2)	d	0.089	0.084	Eq. (4)	d	0.086
Table A-28	d	0.090	0.085	Table A-28	d	0.085
Eq. (1)	D	0.890	0.885	Eq. (3)	D	0.865
Eq. (10-1)	C	9.889	10.412	Eq. (10-1)	C	10.176
Eq. (10-9)	N_a	13.669	11.061	Eq. (10-9)	N_a	11.846
Table 10-1	N_t	15.669	13.061	Table 10-1	N_t	13.846
Table 10-1	L_s	1.410	1.110	Table 10-1	L_s	1.177
$1.15y + L_s$	L_0	3.710	3.410	$1.15y + L_s$	L_0	3.477
Eq. (10-13)	$(L_0)_{cr}$	4.681	4.655	Eq. (10-13)	$(L_0)_{cr}$	4.550
Table 10-4	A	201.000	201.000	Table 10-4	A	201.000
Table 10-4	m	0.145	0.145	Table 10-4	m	0.145
Eq. (10-14)	S_{ut}	284.991	287.363	Eq. (10-14)	S_{ut}	287.363
Table 10-6	S_{sy}	128.246	129.313	Table 10-6	S_{sy}	129.313
Eq. (10-5)	K_B	1.135	1.128	Eq. (10-5)	K_B	1.135
Eq. (10-7)	τ_s	81.167	95.223	Eq. (10-7)	τ_s	93.643
$n_s = S_{sy}/\tau_s$	n_s	1.580	1.358	$n_s = S_{sy}/\tau_s$	n_s	1.381
Eq. (10-22)	fom	-0.725	-0.536	Eq. (10-22)	fom	-0.555

Again, for $n_s \geq 1.2$, the optimal size is = 0.085 in.

Although this approach used less iterations than in Prob. 10-21, this was due to the initial values picked and not the approach.

10-23 One approach is to select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm. Starting with $d = 2$ mm,

$$D = \text{ID} + d = 11.25 + 2 = 13.25 \text{ mm}$$

$$C = D/d = 13.25/2 = 6.625 \quad \text{(acceptable)}$$

Table 10-5 ($d = 2/25.4 = 0.0787$ in): $G = 79.3$ GPa

Eq. (10-9): $N_a = \dfrac{d^4 G}{8kD^3} = \dfrac{2^4(79.3)10^3}{8(4.286)13.25^3} = 15.9 \text{ coils}$

Assume squared and closed.

Table 10-1: $N_t = N_a + 2 = 15.9 + 2 = 17.9$ coils

$L_s = dN_t = 2(17.9) = 35.8$ mm

$y_s = L_0 - L_s = 48 - 35.8 = 12.2$ mm

$F_s = ky_s = 4.286(12.2) = 52.29$ N

Eq. (10-5): $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(6.625)+2}{4(6.625)-3} = 1.213$

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = 1.213\left[\dfrac{8(52.29)13.25}{\pi(2^3)}\right] = 267.5$ MPa

Table 10-4: $A = 1783$ MPa $\cdot$ mm^m, $m = 0.190$

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1783}{2^{0.190}} = 1563$ MPa

Table 10-6: $S_{sy} = 0.45 S_{ut} = 0.45(1563) = 703.3$ MPa

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{703.3}{267.5} = 2.63 > 1.2 \qquad O.K.$$

No other diameters in the given range work. So specify

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans*.

10-24 Select A227 HD steel for its low cost. Try $L_0 = 48$ mm, then for $y = 48 - 37.5 = 10.5$ mm when $F = 45$ N. The spring rate is $k = F/y = 45/10.5 = 4.286$ N/mm.

For a clearance of 1.25 mm with screw, ID = 10 + 1.25 = 11.25 mm.

$$D - d = 11.25 \qquad (1)$$

and, $$D = Cd \qquad (2)$$

Starting with $C = 8$, gives $D = 8d$. Substitute into Eq. (1) resulting in $d = 1.607$ mm. Selecting the nearest diameter in the given range, $d = 1.6$ mm. From this point, the calculations are shown in the third column of the spreadsheet output shown. We see that for $d = 1.6$ mm, the spring is not solid safe. Iterating on C we find that $C = 6.5$ provides acceptable results with the specifications

A227-47 HD steel, $d = 2$ mm, $D = 13.25$ mm, ID = 11.25 mm, OD = 15.25 mm, squared

and closed, $N_t = 17.9$ coils, $N_a = 15.9$ coils, $k = 4.286$ N/mm, $L_s = 35.8$ mm, and $L_0 = 48$ mm. *Ans*.

Source		Parameter Values		
	C	8.000	7	6.500
Eq. (2)	d	1.607	1.875	2.045
Table A-28	d	1.600	1.800	2.000
Eq. (1)	D	12.850	13.050	13.250
Eq. (10-1)	C	8.031	7.250	6.625
Eq. (10-9)	N_a	7.206	10.924	15.908
Table 10-1	N_t	9.206	12.924	17.908
Table 10-1	L_s	14.730	23.264	35.815
$L_0 - L_s$	y_s	33.270	24.736	12.185
$F_s = ky_s$	F_s	142.594	106.020	52.224
Table 10-4	A	1783.000	1783.000	1783.000
Table 10-4	m	0.190	0.190	0.190
Eq. (10-14)	S_{ut}	1630.679	1594.592	1562.988
Table 10-6	S_{sy}	733.806	717.566	703.345
Eq. (10-5)	K_B	1.172	1.200	1.217
Eq. (10-7)	τ_s	1335.568	724.943	268.145
$n_s = S_{sy}/\tau_s$	n_s	0.549	0.990	2.623

The only difference between selecting C first rather than d as was done in Prob. 10-23, is that once d is calculated, the closest wire size must be selected. Iterating on d uses available wire sizes from the beginning.

10-25 A stock spring catalog may have over two hundred pages of compression springs with up to 80 springs per page listed.

- Students should be made aware that such catalogs exist.
- Many springs are selected from catalogs rather than designed.
- The wire size you want may not be listed.
- Catalogs may also be available on disk or the web through search routines. For example, disks are available from Century Spring at

  1 - (800) - 237 - 5225

  www.centuryspring.com
- It is better to familiarize yourself with vendor resources rather than invent them yourself.
- Sample catalog pages can be given to students for study.

10-26 Given: ID = 0.6 in, $C = 10$, $L_0 = 5$ in, $L_s = 5 - 3 = 2$ in, sq. & grd ends, unpeened, HD A227 wire.

(**a**) With ID $= D - d = 0.6$ in and $C = D/d = 10 \Rightarrow 10\,d - d = 0.6 \Rightarrow d = 0.0667$ in *Ans.*, and $D = 0.667$ in.

(**b**) Table 10-1: $L_s = dN_t = 2$ in $\Rightarrow N_t = 2/0.0667$ 30 coils *Ans.*

(**c**) Table 10-1: $N_a = N_t - 2 = 30 - 2 = 28$ coils

Table 10-5: $G = 11.5$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.0667^4 (11.5)10^6}{8\left(0.667^3\right)28} = 3.424 \text{ lbf/in} \quad \textit{Ans.}$$

(**d**) Table 10-4: $A = 140$ kpsi·in^m, $m = 0.190$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{140}{0.0667^{0.190}} = 234.2 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\, S_{ut} = 0.45\,(234.2) = 105.4$ kpsi

$$F_s = ky_s = 3.424(3) = 10.27 \text{ lbf}$$

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4(10)+2}{4(10)-3} = 1.135$$

Eq. (10-7):

$$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.135 \frac{8(10.27)0.667}{\pi\left(0.0667^3\right)}$$

$$= 66.72\left(10^3\right) \text{ psi} = 66.72 \text{ kpsi}$$

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{105.4}{66.72} = 1.58 \quad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = 0.5\,\tau_s = 0.5(66.72) = 33.36$ kpsi, $r = \tau_a / \tau_m = 1$. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\, S_{ut} = 0.67(234.2) = 156.9$ kpsi

The Gerber ordinate intercept for the Zimmerli data is

$$S_e = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/156.9\right)^2} = 39.9 \text{ kpsi}$$

Table 6-7, p. 307,

$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$= \frac{1^2\left(156.9^2\right)}{2(39.9)}\left\{-1+\sqrt{1+\left[\frac{2(39.9)}{1(156.9)}\right]^2}\right\} = 37.61 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{37.61}{33.36} = 1.13 \quad \textit{Ans.}$$

10-27 Given: OD ≤ 0.9 in, $C = 8$, $L_0 = 3$ in, $L_s = 1$ in, $y_s = 3 - 1 = 2$ in, sq. ends, unpeened, music wire.

(**a**) Try OD = $D + d = 0.9$ in, $C = D/d = 8 \Rightarrow D = 8d \Rightarrow 9d = 0.9 \Rightarrow d = 0.1$ *Ans.*

$D = 8(0.1) = 0.8$ in

(**b**) Table 10-1: $L_s = d\,(N_t + 1) \Rightarrow N_t = L_s / d - 1 = 1/0.1 - 1 = 9$ coils *Ans.*

Table 10-1: $N_a = N_t - 2 = 9 - 2 = 7$ coils

(**c**) Table 10-5: $G = 11.75$ Mpsi

Eq. (10-9): $$k = \frac{d^4 G}{8D^3 N_a} = \frac{0.1^4\left(11.75\right)10^6}{8\left(0.8^3\right)7} = 40.98 \text{ lbf/in} \qquad \textit{Ans.}$$

(**d**) $F_s = ky_s = 40.98(2) = 81.96$ lbf

Eq. (10-5): $$K_B = \frac{4C+2}{4C-3} = \frac{4\left(8\right)+2}{4\left(8\right)-3} = 1.172$$

Eq. (10-7): $$\tau_s = K_B \frac{8F_s D}{\pi d^3} = 1.172\frac{8\left(81.96\right)0.8}{\pi\left(0.1^3\right)} = 195.7\left(10^3\right) \text{ psi} = 195.7 \text{ kpsi}$$

Table 10-4: $A = 201$ kpsi·in^m, $m = 0.145$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{201}{0.1^{0.145}} = 280.7 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.45\,S_{ut} = 0.45(280.7) = 126.3$ kpsi

$$n_s = \frac{S_{sy}}{\tau_s} = \frac{126.3}{195.7} = 0.645 \qquad \textit{Ans.}$$

(**e**) $\tau_a = \tau_m = \tau_s\,/2 = 195.7/2 = 97.85$ kpsi. Using the Gerber fatigue failure criterion with Zimmerli data,

Eq. (10-30): $S_{su} = 0.67\,S_{ut} = 0.67(280.7) = 188.1$ kpsi

The Gerber ordinate intercept for the Zimmerli data is

$$S_e = \frac{S_{sa}}{1-\left(S_{sm}/S_{su}\right)^2} = \frac{35}{1-\left(55/188.1\right)^2} = 36.83 \text{ kpsi}$$

Table 6-7, p. 307,

$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1+\sqrt{1+\left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$= \frac{1^2\left(188.1^2\right)}{2\left(38.3\right)}\left\{-1+\sqrt{1+\left[\frac{2\left(38.3\right)}{1\left(188.1\right)}\right]^2}\right\} = 36.83 \text{ kpsi}$$

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{36.83}{97.85} = 0.376 \quad Ans.$$

Obviously, the spring is severely under designed and will fail statically and in fatigue. Increasing C would improve matters. Try $C = 12$. This yields $n_s = 1.83$ and $n_f = 1.00$.

10-28 **Note to the Instructor**: In the first printing of the text, the wire material was incorrectly identified as music wire instead of oil-tempered wire. This will be corrected in subsequent printings. We are sorry for any inconvenience.

Given: $F_{\max} = 300$ lbf, $F_{\min} = 150$ lbf, $\Delta y = 1$ in, OD = 2.1 – 0.2 = 1.9 in, $C = 7$, unpeened, sq. & grd., oil-tempered wire.

(**a**) $D = \text{OD} - d = 1.9 - d$ (1)

$C = D/d = 7 \Rightarrow D = 7d$ (2)

Substitute Eq. (2) into (1)

$7d = 1.9 - d \Rightarrow d = 1.9/8 = 0.2375$ in *Ans.*

(**b**) From Eq. (2): $D = 7d = 7(0.2375) = 1.663$ in *Ans.*

(**c**) $$k = \frac{\Delta F}{\Delta y} = \frac{300 - 150}{1} = 150 \text{ lbf/in} \quad Ans.$$

(**d**) Table 10-5: $G = 11.6$ Mpsi

Eq. (10-9): $$N_a = \frac{d^4 G}{8D^3 k} = \frac{0.2375^4 (11.6)10^6}{8(1.663^3)150} = 6.69 \text{ coils}$$

Table 10-1: $N_t = N_a + 2 = 8.69$ coils *Ans.*

(**e**) Table 10-4: $A = 147$ kpsi·in^m, $m = 0.187$

Eq. (10-14): $$S_{ut} = \frac{A}{d^m} = \frac{147}{0.2375^{0.187}} = 192.3 \text{ kpsi}$$

Table 10-6: $S_{sy} = 0.5\, S_{ut} = 0.5(192.3) = 96.15$ kpsi

Eq. (10-5): $$K_B = \frac{4C + 2}{4C - 3} = \frac{4(7) + 2}{4(7) - 3} = 1.2$$

Eq. (10-7): $\tau_s = K_B \dfrac{8F_s D}{\pi d^3} = S_{sy}$

$$F_s = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi\left(0.2375^3\right)96.15\left(10^3\right)}{8\left(1.2\right)1.663} = 253.5 \text{ lbf}$$

$$y_s = F_s / k = 253.5/150 = 1.69 \text{ in}$$

Table 10-1: $L_s = N_t\, d = 8.46(0.2375) = 2.01$ in

$$L_0 = L_s + y_s = 2.01 + 1.69 = 3.70 \text{ in} \qquad \textit{Ans.}$$

10-29 For a coil radius given by:

$$R = R_1 + \frac{R_2 - R_1}{2\pi N}\theta$$

The torsion of a section is $T = PR$ where $dL = R\, d\theta$

$$\begin{aligned}
\delta_P &= \frac{\partial U}{\partial P} = \frac{1}{GJ}\int T\frac{\partial T}{\partial P}dL = \frac{1}{GJ}\int_0^{2\pi N} PR^3\, d\theta \\
&= \frac{P}{GJ}\int_0^{2\pi N}\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^3 d\theta \\
&= \frac{P}{GJ}\left(\frac{1}{4}\right)\left(\frac{2\pi N}{R_2 - R_1}\right)\left[\left(R_1 + \frac{R_2 - R_1}{2\pi N}\theta\right)^4\right]\Bigg|_0^{2\pi N} \\
&= \frac{\pi PN}{2GJ(R_2 - R_1)}\left(R_2^4 - R_1^4\right) = \frac{\pi PN}{2GJ}(R_1 + R_2)\left(R_1^2 + R_2^2\right)
\end{aligned}$$

$$J = \frac{\pi}{32}d^4 \quad \therefore \quad \delta_p = \frac{16PN}{Gd^4}(R_1 + R_2)\left(R_1^2 + R_2^2\right)$$

$$k = \frac{P}{\delta_P} = \frac{d^4 G}{16N(R_1 + R_2)\left(R_1^2 + R_2^2\right)} \qquad \textit{Ans.}$$

10-30 Given: $F_{\min} = 4$ lbf, $F_{\max} = 18$ lbf, $k = 9.5$ lbf/in, OD ≤ 2.5 in, $n_f = 1.5$.

For a food service machinery application select A313 Stainless wire.

Table 10-5: $G = 10(10^6)$ psi

Note that for $0.013 \le d \le 0.10$ in $\quad A = 169, \quad m = 0.146$

$0.10 < d \le 0.20$ in $\quad A = 128, \quad m = 0.263$

$$F_a = \frac{18 - 4}{2} = 7 \text{ lbf}, \quad F_m = \frac{18 + 4}{2} = 11 \text{ lbf}, \quad r = 7 / 11$$

Try, $d = 0.080$ in, $S_{ut} = \dfrac{169}{(0.08)^{0.146}} = 244.4$ kpsi

$S_{su} = 0.67S_{ut} = 163.7$ kpsi, $S_{sy} = 0.35S_{ut} = 85.5$ kpsi

Try unpeened using Zimmerli's endurance data: $S_{sa} = 35$ kpsi, $S_{sm} = 55$ kpsi

Gerber: $S_{se} = \dfrac{S_{sa}}{1 - (S_{sm} / S_{su})^2} = \dfrac{35}{1 - (55 / 163.7)^2} = 39.5$ kpsi

$$S_{sa} = \frac{(7 / 11)^2(163.7)^2}{2(39.5)}\left\{-1 + \sqrt{1 + \left[\frac{2(39.5)}{(7 / 11)(163.7)}\right]^2}\right\} = 35.0 \text{ kpsi}$$

$$\alpha = S_{sa} / n_f = 35.0 / 1.5 = 23.3 \text{ kpsi}$$

$$\beta = \frac{8F_a}{\pi d^2}(10^{-3}) = \left[\frac{8(7)}{\pi(0.08^2)}\right](10^{-3}) = 2.785 \text{ kpsi}$$

$$C = \frac{2(23.3) - 2.785}{4(2.785)} + \sqrt{\left[\frac{2(23.3) - 2.785}{4(2.785)}\right]^2 - \frac{3(23.3)}{4(2.785)}} = 6.97$$

$$D = Cd = 6.97(0.08) = 0.558 \text{ in}$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(6.97) + 2}{4(6.97) - 3} = 1.201$$

$$\tau_a = K_B\left(\frac{8F_aD}{\pi d^3}\right) = 1.201\left[\frac{8(7)(0.558)}{\pi(0.08^3)}(10^{-3})\right] = 23.3 \text{ kpsi}$$

$n_f = 35 / 23.3 = 1.50$ checks

$$N_a = \frac{Gd^4}{8kD^3} = \frac{10(10^6)(0.08)^4}{8(9.5)(0.558)^3} = 31.02 \text{ coils}$$

$N_t = 31.02 + 2 = 33$ coils, $L_s = dN_t = 0.08(33) = 2.64$ in

$y_{\max} = F_{\max} / k = 18 / 9.5 = 1.895$ in

$y_s = (1 + \xi)y_{\max} = (1 + 0.15)(1.895) = 2.179$ in

$L_0 = 2.64 + 2.179 = 4.819$ in

$$(L_0)_{cr} = 2.63\frac{D}{\alpha} = \frac{2.63(0.558)}{0.5} = 2.935 \text{ in}$$

$\tau_s = 1.15(18 / 7)\tau_a = 1.15(18 / 7)(23.3) = 68.9$ kpsi

$n_s = S_{sy} / \tau_s = 85.5 / 68.9 = 1.24$

$$f = \sqrt{\frac{kg}{\pi^2 d^2 D N_a \gamma}} = \sqrt{\frac{9.5(386)}{\pi^2(0.08^2)(0.558)(31.02)(0.283)}} = 109 \text{ Hz}$$

These steps are easily implemented on a spreadsheet, as shown below, for different diameters.

	d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263
A	169.000	169.000	128	128
S_{ut}	244.363	239.618	231.257	223.311
S_{su}	163.723	160.544	154.942	149.618
S_{sy}	85.527	83.866	80.940	78.159
S_{se}	39.452	39.654	40.046	40.469
S_{sa}	35.000	35.000	35.000	35.000
α	23.333	23.333	23.333	23.333
β	2.785	2.129	1.602	1.228
C	6.977	9.603	13.244	17.702
D	0.558	0.879	1.397	2.133
K_B	1.201	1.141	1.100	1.074
τ_a	23.333	23.333	23.333	23.333
n_f	1.500	1.500	1.500	1.500
N_a	30.993	13.594	5.975	2.858
N_t	32.993	15.594	7.975	4.858
L_S	2.639	1.427	0.841	0.585
y_s	2.179	2.179	2.179	2.179
L_0	4.818	3.606	3.020	2.764
$(L_0)_{\text{cr}}$	2.936	4.622	7.350	11.220
τ_s	69.000	69.000	69.000	69.000
n_s	1.240	1.215	1.173	1.133
f,(Hz)	108.895	114.578	118.863	121.775

The shaded areas depict conditions outside the recommended design conditions. Thus, one spring is satisfactory. The specifications are: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.59$ turns *Ans*.

10-31 The steps are the same as in Prob. 10-23 except that the Gerber-Zimmerli criterion is replaced with Goodman-Zimmerli:

$$S_{se} = \frac{S_{sa}}{1 - \left(S_{sm}/S_{su}\right)}$$

The problem then proceeds as in Prob. 10-23. The results for the wire sizes are shown below (see solution to Prob. 10-23 for additional details).

Iteration of d for the first trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.151	1.108	1.078	1.058
A	169.000	169.000	128.000	128.000	τ_a	29.008	29.040	29.090	29.127
S_{ut}	244.363	239.618	231.257	223.311	n_f	1.500	1.500	1.500	1.500
S_{su}	163.723	160.544	154.942	149.618	N_a	14.191	6.456	2.899	1.404
S_{sy}	85.527	83.866	80.940	78.159	N_t	16.191	8.456	4.899	3.404
S_{se}	52.706	53.239	54.261	55.345	L_s	1.295	0.774	0.517	0.410
S_{sa}	43.513	43.560	43.634	43.691	$y_{\max}$	2.875	2.875	2.875	2.875
α	29.008	29.040	29.090	29.127	L_0	4.170	3.649	3.392	3.285
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	3.809	5.924	9.354	14.219
C	9.052	12.309	16.856	22.433	τ_s	85.782	85.876	86.022	86.133
D	0.724	1.126	1.778	2.703	n_s	0.997	0.977	0.941	0.907
					f (Hz)	140.040	145.559	149.938	152.966

Without checking all of the design conditions, it is obvious that none of the wire sizes satisfy $n_s \geq 1.2$. Also, the Gerber line is closer to the yield line than the Goodman. Setting $n_f = 1.5$ for Goodman makes it impossible to reach the yield line ($n_s < 1$) . The table below uses $n_f = 2$.

Iteration of d for the second trial									
	d_1	d_2	d_3	d_4		d_1	d_2	d_3	d_4
d	0.080	0.0915	0.1055	0.1205	d	0.080	0.0915	0.1055	0.1205
m	0.146	0.146	0.263	0.263	K_B	1.221	1.154	1.108	1.079
A	169.000	169.000	128.000	128.000	τ_a	21.756	21.780	21.817	21.845
S_{ut}	244.363	239.618	231.257	223.311	n_f	2.000	2.000	2.000	2.000
S_{su}	163.723	160.544	154.942	149.618	N_a	40.243	17.286	7.475	3.539
S_{sy}	85.527	83.866	80.940	78.159	N_t	42.243	19.286	9.475	5.539
S_{se}	52.706	53.239	54.261	55.345	L_s	3.379	1.765	1.000	0.667
S_{sa}	43.513	43.560	43.634	43.691	$y_{\max}$	2.875	2.875	2.875	2.875
α	21.756	21.780	21.817	21.845	L_0	6.254	4.640	3.875	3.542
β	2.785	2.129	1.602	1.228	$(L_0)_{\text{cr}}$	2.691	4.266	6.821	10.449
C	6.395	8.864	12.292	16.485	τ_s	64.336	64.407	64.517	64.600
D	0.512	0.811	1.297	1.986	n_s	1.329	1.302	1.255	1.210
					f (Hz)	98.936	104.827	109.340	112.409

The satisfactory spring has design specifications of: A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.811 + 0.092 = 0.903 in, $L_0 = 4.266$ in, and .$N_t = 19.6$ turns. *Ans*.

10-32 This is the same as Prob. 10-30 since $S_{sa} = 35$ kpsi. Therefore, the specifications are:

A313 stainless wire, unpeened, squared and ground, $d = 0.0915$ in, OD = 0.879 + 0.092 = 0.971 in, $L_0 = 3.606$ in, and $N_t = 15.84$ turns *Ans*.

10-33 For the Gerber fatigue-failure criterion, $S_{su} = 0.67S_{ut}$,

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})^2} , \qquad S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}} \right)^2} \right]$$

The equation for S_{sa} is the basic difference. The last 2 columns of diameters of Ex. 10-5 are presented below with additional calculations.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	8.915	6.190
S_{su}	186.723	184.984	L_s	1.146	0.917
S_{se}	38.325	38.394	L_0	3.446	3.217
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.630	8.160
S_{sa}	34.658	34.652	K_B	1.111	1.095
α	23.105	23.101	τ_a	23.105	23.101
β	1.732	1.523	n_f	1.500	1.500
C	12.004	13.851	τ_s	70.855	70.844
D	1.260	1.551	n_s	1.770	1.754
ID	1.155	1.439	f_n	105.433	106.922
OD	1.365	1.663	fom	–0.973	–1.022

There are only slight changes in the results.

10-34 As in Prob. 10-35, the basic change is S_{sa}.

For Goodman, $$S_{se} = \frac{S_{sa}}{1 - (S_{sm} / S_{su})}$$

Recalculate S_{sa} with

$$S_{sa} = \frac{rS_{se}S_{su}}{rS_{su} + S_{se}}$$

Calculations for the last 2 diameters of Ex. 10-5 are given below.

d	0.105	0.112	d	0.105	0.112
S_{ut}	278.691	276.096	N_a	9.153	6.353
S_{su}	186.723	184.984	L_s	1.171	0.936
S_{se}	49.614	49.810	L_0	3.471	3.236
S_{sy}	125.411	124.243	$(L_0)_{cr}$	6.572	8.090
S_{sa}	34.386	34.380	K_B	1.112	1.096
α	22.924	22.920	τ_a	22.924	22.920
β	1.732	1.523	n_f	1.500	1.500
C	11.899	13.732	τ_s	70.301	70.289
D	1.249	1.538	n_s	1.784	1.768
ID	1.144	1.426	f_n	104.509	106.000
OD	1.354	1.650	fom	–0.986	–1.034

There are only slight differences in the results.

10-35 Use: $E = 28.6$ Mpsi, $G = 11.5$ Mpsi, $A = 140$ kpsi · in^m , $m = 0.190$, rel cost = 1.

Try $\quad d = 0.067\ in, \quad S_{ut} = \dfrac{140}{(0.067)^{0.190}} = 234.0$ kpsi

Table 10-6: $\quad S_{sy} = 0.45 S_{ut} = 105.3$ kpsi

Table 10-7: $\quad S_y = 0.75 S_{ut} = 175.5$ kpsi

Eq. (10-34) with $D/d = C$ and $C_1 = C$

$$\sigma_A = \frac{F_{\max}}{\pi d^2}[(K)_A(16C) + 4] = \frac{S_y}{n_y}$$

$$\frac{4C^2 - C - 1}{4C(C-1)}(16C) + 4 = \frac{\pi d^2 S_y}{n_y F_{\max}}$$

$$4C^2 - C - 1 = (C-1)\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)$$

$$C^2 - \frac{1}{4}\left(1 + \frac{\pi d^2 S_y}{4 n_y F_{\max}} - 1\right)C + \frac{1}{4}\left(\frac{\pi d^2 S_y}{4 n_y F_{\max}} - 2\right) = 0$$

$$C = \frac{1}{2}\left[\frac{\pi d^2 S_y}{16 n_y F_{\max}} \pm \sqrt{\left(\frac{\pi d^2 S_y}{16 n_y F_{\max}}\right)^2 - \frac{\pi d^2 S_y}{4 n_y F_{\max}} + 2}\right] \quad \text{take positive root}$$

$$= \frac{1}{2}\left\{\frac{\pi(0.067^2)(175.5)(10^3)}{16(1.5)(18)} + \sqrt{\left[\frac{\pi(0.067)^2(175.5)(10^3)}{16(1.5)(18)}\right]^2 - \frac{\pi(0.067)^2(175.5)(10^3)}{4(1.5)(18)} + 2}\right\} = 4.590$$

$$D = Cd = 4.59(0.067) = 0.3075 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\,500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range. This results in the best fom.

$$F_i = \frac{\pi(0.067)^3}{8(0.3075)}\left\{\frac{33\,500}{\exp[0.105(4.590)]} - 1000\left(4 - \frac{4.590-3}{6.5}\right)\right\} = 6.505 \text{ lbf}$$

For simplicity, we will round up to the next integer or half integer. Therefore, use $F_i = 7$ lbf

$$k = \frac{18-7}{0.5} = 22 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.067)^4(11.5)(10^6)}{8(22)(0.3075)^3} = 45.28 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 45.28 - \frac{11.5}{28.6} = 44.88 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.590) - 1 + 44.88](0.067) = 3.555 \text{ in}$$

$$L_{18\text{ lbf}} = 3.555 + 0.5 = 4.055 \text{ in}$$

Body: $K_B = \dfrac{4C+2}{4C-3} = \dfrac{4(4.590)+2}{4(4.590)-3} = 1.326$

$$\tau_{\max} = \frac{8K_B F_{\max} D}{\pi d^3} = \frac{8(1.326)(18)(0.3075)}{\pi(0.067)^3}(10^{-3}) = 62.1 \text{ kpsi}$$

$$(n_y)_{\text{body}} = \frac{S_{sy}}{\tau_{\max}} = \frac{105.3}{62.1} = 1.70$$

$$r_2 = 2d = 2(0.067) = 0.134 \text{ in}, \quad C_2 = \frac{2r_2}{d} = \frac{2(0.134)}{0.067} = 4$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4)-1}{4(4)-4} = 1.25$$

$$\tau_B = (K)_B\left[\frac{8F_{\max}D}{\pi d^3}\right] = 1.25\left[\frac{8(18)(0.3075)}{\pi(0.067)^3}\right](10^{-3}) = 58.58 \text{ kpsi}$$

$$(n_y)_B = \frac{S_{sy}}{\tau_B} = \frac{105.3}{58.58} = 1.80$$

$$\text{fom} = -(1)\frac{\pi^2 d^2 (N_b + 2)D}{4} = -\frac{\pi^2(0.067)^2(44.88+2)(0.3075)}{4} = -0.160$$

Several diameters, evaluated using a spreadsheet, are shown below.

d	0.067	0.072	0.076	0.081	0.085	0.09	0.095	0.104
S_{ut}	233.977	230.799	228.441	225.692	223.634	221.219	218.958	215.224
S_{sy}	105.290	103.860	102.798	101.561	100.635	99.548	98.531	96.851
S_y	175.483	173.100	171.331	169.269	167.726	165.914	164.218	161.418
C	4.589	5.412	6.099	6.993	7.738	8.708	9.721	11.650
D	0.307	0.390	0.463	0.566	0.658	0.784	0.923	1.212
F_i (calc)	6.505	5.773	5.257	4.675	4.251	3.764	3.320	2.621
F_i (rd)	7.0	6.0	5.5	5.0	4.5	4.0	3.5	3.0
k	22.000	24.000	25.000	26.000	27.000	28.000	29.000	30.000
N_a	45.29	27.20	19.27	13.10	9.77	7.00	5.13	3.15
N_b	44.89	26.80	18.86	12.69	9.36	6.59	4.72	2.75
L_0	3.556	2.637	2.285	2.080	2.026	2.071	2.201	2.605
$L_{18\ \text{lbf}}$	4.056	3.137	2.785	2.580	2.526	2.571	2.701	3.105
K_B	1.326	1.268	1.234	1.200	1.179	1.157	1.139	1.115
$\tau_{\max}$	62.118	60.686	59.707	58.636	57.875	57.019	56.249	55.031
$(n_y)_{\text{body}}$	1.695	1.711	1.722	1.732	1.739	1.746	1.752	1.760
τ_B	58.576	59.820	60.495	61.067	61.367	61.598	61.712	61.712
$(n_y)_B$	1.797	1.736	1.699	1.663	1.640	1.616	1.597	1.569
$(n_y)_A$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
fom	–0.160	–0.144	-0.138	–0.135	–0.133	–0.135	–0.138	–0.154

Except for the 0.067 in wire, all springs satisfy the requirements of length and number of coils. The 0.085 in wire has the highest fom.

10-36 Given: $N_b = 84$ coils, $F_i = 16$ lbf, OQ&T steel, OD = 1.5 in, $d = 0.162$ in.

$$D = \text{OD} - d = 1.5 - 0.162 = 1.338 \text{ in}$$

(a) Eq. (10-39):

$$L_0 = 2(D - d) + (N_b + 1)d$$
$$= 2(1.338 - 0.162) + (84 + 1)(0.162) = 16.12 \text{ in} \quad \textit{Ans.}$$

or $$2d + L_0 = 2(0.162) + 16.12 = 16.45 \text{ in overall}$$

(b)
$$C = \frac{D}{d} = \frac{1.338}{0.162} = 8.26$$

$$K_B = \frac{4C + 2}{4C - 3} = \frac{4(8.26) + 2}{4(8.26) - 3} = 1.166$$

$$\tau_i = K_B \left[\frac{8F_i D}{\pi d^3} \right] = 1.166 \frac{8(16)(1.338)}{\pi (0.162)^3} = 14\,950 \text{ psi} \quad \textit{Ans.}$$

(c) From Table 10-5 use: $G = 11.4(10^6)$ psi and $E = 28.5(10^6)$ psi

$$N_a = N_b + \frac{G}{E} = 84 + \frac{11.4}{28.5} = 84.4 \text{ turns}$$

$$k = \frac{d^4 G}{8D^3 N_a} = \frac{(0.162)^4(11.4)(10^6)}{8(1.338)^3(84.4)} = 4.855 \text{ lbf/in} \quad \textit{Ans.}$$

(d) Table 10-4: $A = 147 \text{ psi} \cdot \text{in}^m$, $m = 0.187$

$$S_{ut} = \frac{147}{(0.162)^{0.187}} = 207.1 \text{ kpsi}$$

$$S_y = 0.75(207.1) = 155.3 \text{ kpsi}$$

$$S_{sy} = 0.50(207.1) = 103.5 \text{ kpsi}$$

Body

$$F = \frac{\pi d^3 S_{sy}}{\pi K_B D}$$

$$= \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.166)(1.338)} = 110.8 \text{ lbf}$$

Torsional stress on hook point B

$$C_2 = \frac{2r_2}{d} = \frac{2(0.25 + 0.162/2)}{0.162} = 4.086$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} = \frac{4(4.086) - 1}{4(4.086) - 4} = 1.243$$

$$F = \frac{\pi(0.162)^3(103.5)(10^3)}{8(1.243)(1.338)} = 103.9 \text{ lbf}$$

Normal stress on hook point A

$$C_1 = \frac{2r_1}{d} = \frac{1.338}{0.162} = 8.26$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)} = \frac{4(8.26)^2 - 8.26 - 1}{4(8.26)(8.26 - 1)} = 1.099$$

$$S_{yt} = \sigma = F\left[\frac{16(K)_A D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$F = \frac{155.3(10^3)}{\left[16(1.099)(1.338)\right] / \left[\pi(0.162)^3\right] + \left\{4 / \left[\pi(0.162)^2\right]\right\}} = 85.8 \text{ lbf}$$

$$= \min(110.8,\ 103.9,\ 85.8) = 85.8 \text{ lbf} \quad \textit{Ans.}$$

(e) Eq. (10-48):

$$y = \frac{F - F_i}{k} = \frac{85.8 - 16}{4.855} = 14.4 \text{ in} \quad \textit{Ans.}$$

10-37 $F_{\min} = 9$ lbf, $F_{\max} = 18$ lbf

$$F_a = \frac{18-9}{2} = 4.5 \text{ lbf}, \quad F_m = \frac{18+9}{2} = 13.5 \text{ lbf}$$

A313 stainless: $0.013 \le d \le 0.1$ $A = 169$ kpsi $\cdot$ in^m, $m = 0.146$

$0.1 \le d \le 0.2$ $A = 128$ kpsi $\cdot$ in^m, $m = 0.263$

$E = 28$ Mpsi, $G = 10$ Gpsi

Try $d = 0.081$ in and refer to the discussion following Ex. 10-7

$$S_{ut} = \frac{169}{(0.081)^{0.146}} = 243.9 \text{ kpsi}$$

$$S_{su} = 0.67 S_{ut} = 163.4 \text{ kpsi}$$

$$S_{sy} = 0.35 S_{ut} = 85.4 \text{ kpsi}$$

$$S_y = 0.55 S_{ut} = 134.2 \text{ kpsi}$$

Table 10-8: $S_r = 0.45 S_{ut} = 109.8$ kpsi

$$S_e = \frac{S_r / 2}{1 - [S_r / (2S_{ut})]^2} = \frac{109.8 / 2}{1 - [(109.8 / 2) / 243.9]^2} = 57.8 \text{ kpsi}$$

$$r = F_a / F_m = 4.5 / 13.5 = 0.333$$

Table 7-10: $$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$$

$$S_a = \frac{(0.333)^2(243.9^2)}{2(57.8)}\left[-1 + \sqrt{1 + \left[\frac{2(57.8)}{0.333(243.9)}\right]^2}\right] = 42.2 \text{ kpsi}$$

Hook bending

$$(\sigma_a)_A = F_a\left[(K)_A \frac{16C}{\pi d^2} + \frac{4}{\pi d^2}\right] = \frac{S_a}{(n_f)_A} = \frac{S_a}{2}$$

$$\frac{4.5}{\pi d^2}\left[\frac{(4C^2 - C - 1)16C}{4C(C-1)} + 4\right] = \frac{S_a}{2}$$

This equation reduces to a quadratic in C (see Prob. 10-35). The useable root for C is

$$C = 0.5\left[\frac{\pi d^2 S_a}{144} + \sqrt{\left(\frac{\pi d^2 S_a}{144}\right)^2 - \frac{\pi d^2 S_a}{36} + 2}\right]$$

$$= 0.5\left\{\frac{\pi(0.081)^2(42.2)(10^3)}{144} + \sqrt{\left[\frac{\pi(0.081)^2(42.2)(10^3)}{144}\right]^2 - \frac{\pi(0.081)^2(42.2)(10^3)}{36} + 2}\right\}$$

$$= 4.91$$

$$D = Cd = 0.398 \text{ in}$$

$$F_i = \frac{\pi d^3 \tau_i}{8D} = \frac{\pi d^3}{8D}\left[\frac{33\ 500}{\exp(0.105C)} \pm 1000\left(4 - \frac{C-3}{6.5}\right)\right]$$

Use the lowest F_i in the preferred range.

$$F_i = \frac{\pi(0.081)^3}{8(0.398)}\left[\frac{33\ 500}{\exp[0.105(4.91)]} - 1000\left(4 - \frac{4.91-3}{6.5}\right)\right]$$

$$= 8.55 \text{ lbf}$$

For simplicity we will round up to next 1/4 integer.

$$F_i = 8.75 \text{ lbf}$$

$$k = \frac{18-9}{0.25} = 36 \text{ lbf/in}$$

$$N_a = \frac{d^4 G}{8kD^3} = \frac{(0.081)^4(10)(10^6)}{8(36)(0.398)^3} = 23.7 \text{ turns}$$

$$N_b = N_a - \frac{G}{E} = 23.7 - \frac{10}{28} = 23.3 \text{ turns}$$

$$L_0 = (2C - 1 + N_b)d = [2(4.91) - 1 + 23.3](0.081) = 2.602 \text{ in}$$

$$L_{\max} = L_0 + (F_{\max} - F_i) / k = 2.602 + (18 - 8.75) / 36 = 2.859 \text{ in}$$

$$(\sigma_a)_A = \frac{4.5(4)}{\pi d^2}\left(\frac{4C^2 - C - 1}{C - 1} + 1\right)$$

$$= \frac{18(10^{-3})}{\pi(0.081^2)}\left[\frac{4(4.91^2) - 4.91 - 1}{4.91 - 1} + 1\right] = 21.1 \text{ kpsi}$$

$$(n_f)_A = \frac{S_a}{(\sigma_a)_A} = \frac{42.2}{21.1} = 2 \text{ checks}$$

Body: $$K_B = \frac{4C + 2}{4C - 3} = \frac{4(4.91) + 2}{4(4.91) - 3} = 1.300$$

$$\tau_a = \frac{8(1.300)(4.5)(0.398)}{\pi(0.081)^3}(10^{-3}) = 11.16 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{13.5}{4.5}(11.16) = 33.47 \text{ kpsi}$$

The repeating allowable stress from Table 7-8 is

$$S_{sr} = 0.30S_{ut} = 0.30(243.9) = 73.17 \text{ kpsi}$$

The Gerber intercept is

$$S_{se} = \frac{73.17 / 2}{1 - [(73.17 / 2) / 163.4]^2} = 38.5 \text{ kpsi}$$

From Table 6-7,

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{163.4}{33.47}\right)^2\left(\frac{11.16}{38.5}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(33.47)(38.5)}{163.4(11.16)}\right]^2}\right\} = 2.53$$

Let $r_2 = 2d = 2(0.081) = 0.162$

$$C_2 = \frac{2r_2}{d} = 4, \quad (K)_B = \frac{4(4) - 1}{4(4) - 4} = 1.25$$

$$(\tau_a)_B = \frac{(K)_B}{K_B}\tau_a = \frac{1.25}{1.30}(11.16) = 10.73 \text{ kpsi}$$

$$(\tau_m)_B = \frac{(K)_B}{K_B}\tau_m = \frac{1.25}{1.30}(33.47) = 32.18 \text{ kpsi}$$

Table 10-8: $(S_{sr})_B = 0.28S_{ut} = 0.28(243.9) = 68.3$ kpsi

$$(S_{se})_B = \frac{68.3 / 2}{1 - [(68.3 / 2) / 163.4]^2} = 35.7 \text{ kpsi}$$

$$(n_f)_B = \frac{1}{2}\left(\frac{163.4}{32.18}\right)^2\left(\frac{10.73}{35.7}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(32.18)(35.7)}{163.4(10.73)}\right]^2}\right\} = 2.51$$

Yield

Bending:

$$(\sigma_A)_{\max} = \frac{4F_{\max}}{\pi d^2}\left[\frac{(4C^2 - C - 1)}{C - 1} + 1\right]$$

$$= \frac{4(18)}{\pi(0.081^2)}\left[\frac{4(4.91)^2 - 4.91 - 1}{4.91 - 1} + 1\right](10^{-3}) = 84.4 \text{ kpsi}$$

$$(n_y)_A = \frac{134.2}{84.4} = 1.59$$

Body:

$$\tau_i = (F_i / F_a)\tau_a = (8.75 / 4.5)(11.16) = 21.7 \text{ kpsi}$$
$$r = \tau_a/(\tau_m - \tau_i) = 11.16 / (33.47 - 21.7) = 0.948$$
$$(S_{sa})_y = \frac{r}{r+1}(S_{sy} - \tau_i) = \frac{0.948}{0.948+1}(85.4 - 21.7) = 31.0 \text{ kpsi}$$
$$(n_y)_{\text{body}} = \frac{(S_{sa})_y}{\tau_a} = \frac{31.0}{11.16} = 2.78$$

Hook shear:

$$S_{sy} = 0.3S_{ut} = 0.3(243.9) = 73.2 \text{ kpsi}$$
$$\tau_{\max} = (\tau_a)_B + (\tau_m)_B = 10.73 + 32.18 = 42.9 \text{ kpsi}$$
$$(n_y)_B = \frac{73.2}{42.9} = 1.71$$
$$\text{fom} = -\frac{7.6\pi^2 d^2 (N_b + 2)D}{4} = -\frac{7.6\pi^2 (0.081)^2 (23.3 + 2)(0.398)}{4} = -1.239$$

A tabulation of several wire sizes follow

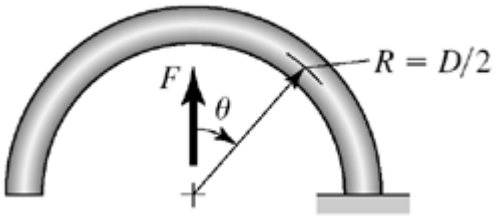
d	0.081	0.085	0.092	0.098	0.105	0.12
S_{ut}	243.920	242.210	239.427	237.229	234.851	230.317
S_{su}	163.427	162.281	160.416	158.943	157.350	154.312
S_r	109.764	108.994	107.742	106.753	105.683	103.643
S_e	57.809	57.403	56.744	56.223	55.659	54.585
S_a	42.136	41.841	41.360	40.980	40.570	39.786
C	4.903	5.484	6.547	7.510	8.693	11.451
D	0.397	0.466	0.602	0.736	0.913	1.374
OD	0.478	0.551	0.694	0.834	1.018	1.494
F_i (calc)	8.572	7.874	6.798	5.987	5.141	3.637
F_i (rd)	8.75	9.75	10.75	11.75	12.75	13.75
k	36.000	36.000	36.000	36.000	36.000	36.000
N_a	23.86	17.90	11.38	8.03	5.55	2.77
N_b	23.50	17.54	11.02	7.68	5.19	2.42
L_0	2.617	2.338	2.127	2.126	2.266	2.918
$L_{18 \text{ lbf}}$	2.874	2.567	2.328	2.300	2.412	3.036
$(\sigma_a)_A$	21.068	20.920	20.680	20.490	20.285	19.893
$(n_f)_A$	2.000	2.000	2.000	2.000	2.000	2.000
K_B	1.301	1.264	1.216	1.185	1.157	1.117
$(\tau_a)_{\text{body}}$	11.141	10.994	10.775	10.617	10.457	10.177
$(\tau_m)_{\text{body}}$	33.424	32.982	32.326	31.852	31.372	30.532
S_{sr}	73.176	72.663	71.828	71.169	70.455	69.095
S_{se}	38.519	38.249	37.809	37.462	37.087	36.371
$(n_f)_{\text{body}}$	2.531	2.547	2.569	2.583	2.596	2.616
$(K)_B$	1.250	1.250	1.250	1.250	1.250	1.250
$(\tau_a)_B$	10.705	10.872	11.080	11.200	11.294	11.391
$(\tau_m)_B$	32.114	32.615	33.240	33.601	33.883	34.173
$(S_{sr})_B$	68.298	67.819	67.040	66.424	65.758	64.489
$(S_{se})_B$	35.708	35.458	35.050	34.728	34.380	33.717

$(n_f)_B$	2.519	2.463	2.388	2.341	2.298	2.235
S_y	134.156	133.215	131.685	130.476	129.168	126.674
$(\sigma_A)_{\max}$	84.273	83.682	82.720	81.961	81.139	79.573
$(n_y)_A$	1.592	1.592	1.592	1.592	1.592	1.592
τ_i	21.663	23.820	25.741	27.723	29.629	31.097
r	0.945	1.157	1.444	1.942	2.906	4.703
$(S_{sy})_{\text{body}}$	85.372	84.773	83.800	83.030	82.198	80.611
$(S_{sa})_y$	30.958	32.688	34.302	36.507	39.109	40.832
$(n_y)_{\text{body}}$	2.779	2.973	3.183	3.438	3.740	4.012
$(S_{sy})_B$	73.176	72.663	71.828	71.169	70.455	69.095
$(\tau_B)_{\max}$	42.819	43.486	44.321	44.801	45.177	45.564
$(n_y)_B$	1.709	1.671	1.621	1.589	1.560	1.516
fom	–1.246	–1.234	–1.245	–1.283	–1.357	–1.639

optimal fom

The shaded areas show the conditions not satisfied.

10-38 For the hook,



$$M = FR\sin\theta, \quad \partial M/\partial F = R\sin\theta$$

$$\delta_F = \frac{1}{EI}\int_0^{\pi/2} F\left(R\sin\theta\right)^2 R\, d\theta = \frac{\pi}{2}\frac{FR^3}{EI}$$

The total deflection of the body and the two hooks

$$\delta = \frac{8FD^3N_b}{d^4G} + 2\left(\frac{\pi}{2}\frac{FR^3}{EI}\right) = \frac{8FD^3N_b}{d^4G} + \frac{\pi F(D/2)^3}{E(\pi/64)(d^4)}$$

$$= \frac{8FD^3}{d^4G}\left(N_b + \frac{G}{E}\right) = \frac{8FD^3N_a}{d^4G}$$

$$\therefore\ N_a = N_b + \frac{G}{E} \qquad \text{Q.E.D.}$$

10-39 Table 10-5 (d = 4 mm = 0.1575 in): E = 196.5 GPa

Table 10-4 for A227:

A = 1783 MPa · mm^m, m = 0.190

Eq. (10-14): $S_{ut} = \dfrac{A}{d^m} = \dfrac{1783}{4^{0.190}}$ = 1370 MPa

Eq. (10-57): $S_y = \sigma_{\text{all}} = 0.78\, S_{ut} = 0.78(1370) = 1069$ MPa

$$D = \text{OD} - d = 32 - 4 = 28 \text{ mm}$$

$$C = D/d = 28/4 = 7$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(7^2\right) - 7 - 1}{4(7)(7-1)} = 1.119$$

Eq. (10-44): $$\sigma = K_i \frac{32Fr}{\pi d^3}$$

At yield, $Fr = M_y$, $\sigma = S_y$. Thus,

$$M_y = \frac{\pi d^3 S_y}{32K_i} = \frac{\pi\left(4^3\right)1069\left(10^{-3}\right)}{32(1.119)} = 6.00 \text{ N} \cdot \text{m}$$

Count the turns when $M = 0$

$$N = 2.5 - \frac{M_y}{k}$$

where from Eq. (10-51): $$k = \frac{d^4 E}{10.8DN}$$

Thus,

$$N = 2.5 - \frac{M_y}{d^4 E \,/\, (10.8DN)}$$

Solving for N gives

$$N = \frac{2.5}{1 + [10.8DM_y \,/\, (d^4 E)]}$$

$$= \frac{2.5}{1 + \left\{\left[10.8(28)(6.00)\right] / \left[4^4(196.5)\right]\right\}} = 2.413 \text{ turns}$$

This means (2.5 - 2.413)(360°) or 31.3° from closed. *Ans.*

Treating the hand force as in the middle of the grip,

$$r = 112.5 - 87.5 + \frac{87.5}{2} = 68.75 \text{ mm}$$

$$F_{\max} = \frac{M_y}{r} = \frac{6.00\left(10^3\right)}{68.75} = 87.3 \text{ N} \quad \textit{Ans.}$$

10-40 The spring material and condition are unknown. Given $d = 0.081$ in and OD = 0.500,

(a) $D = 0.500 - 0.081 = 0.419$ in

Using $E = 28.6$ Mpsi for an estimate

$$k' = \frac{d^4E}{10.8DN} = \frac{(0.081)^4(28.6)(10^6)}{10.8(0.419)(11)} = 24.7 \text{ lbf} \cdot \text{in/turn}$$

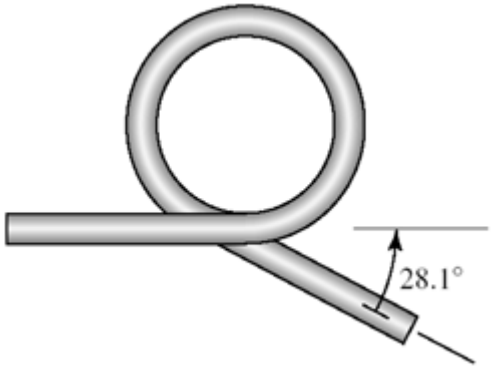
for each spring. The moment corresponding to a force of 8 lbf

$$Fr = (8/2)(3.3125) = 13.25 \text{ lbf} \cdot \text{in/spring}$$

The fraction windup turn is

$$n = \frac{Fr}{k'} = \frac{13.25}{24.7} = 0.536 \text{ turns}$$

The arm swings through an arc of slightly less than 180°, say 165°. This uses up 165/360 or 0.458 turns. So $n = 0.536 - 0.458 = 0.078$ turns are left (or $0.078(360°) = 28.1°$). The original configuration of the spring was



Ans.

(b)

$$C = \frac{D}{d} = \frac{0.419}{0.081} = 5.17$$

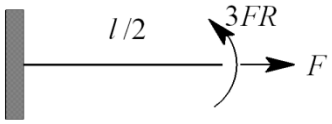
$$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4(5.17)^2 - 5.17 - 1}{4(5.17)(5.17-1)} = 1.168$$

$$\sigma = K_i \frac{32M}{\pi d^3} = 1.168\left[\frac{32(13.25)}{\pi(0.081)^3}\right] = 297\left(10^3\right) \text{ psi} = 297 \text{ kpsi} \qquad \textit{Ans.}$$

To achieve this stress level, the spring had to have set removed.

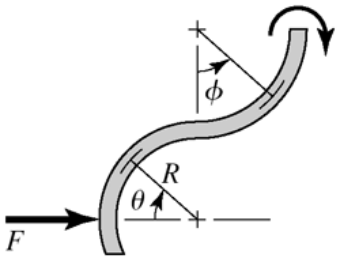
10-41 (**a**) Consider half and double results

Straight section:



$M = 3FR, \quad \dfrac{\partial M}{\partial P} = 3R$

Upper 180° section:



$$M = F[R + R(1 - \cos\phi)]$$
$$= FR(2 - \cos\phi), \quad \frac{\partial M}{\partial F} = R(2 - \cos\phi)$$

Lower section: $M = FR\sin\theta, \quad \dfrac{\partial M}{\partial F} = R\sin\theta$

Considering bending only:

$$\delta = \frac{\partial U}{\partial F} = \frac{2}{EI}\left[\int_0^{l/2} 9FR^2\,dx + \int_0^{\pi} FR^2(2 - \cos\phi)^2 R\,d\phi + \int_0^{\pi/2} F(R\sin\theta)^2 R\,d\theta\right]$$
$$= \frac{2F}{EI}\left[\frac{9}{2}R^2 l + R^3\left(4\pi - 4\sin\phi\Big|_0^{\pi} + \frac{\pi}{2}\right) + R^3\left(\frac{\pi}{4}\right)\right]$$
$$= \frac{2FR^2}{EI}\left(\frac{19\pi}{4}R + \frac{9}{2}l\right) = \frac{FR^2}{2EI}(19\pi R + 18l)$$

The spring rate is

$$k = \frac{F}{\delta} = \frac{2EI}{R^2(19\pi R + 18l)} \qquad \textit{Ans.}$$

(**b**) Given: A227 HD wire, $d = 2$ mm, $R = 6$ mm, and $l = 25$ mm.

Table 10-5 ($d = 2$ mm = 0.0787 in): $E = 197.2$ MPa

$$k = \frac{2(197.2)10^9\pi\left(0.002^4\right)/(64)}{0.006^2\left[19\pi(0.006) + 18(0.025)\right]} = 10.65\left(10^3\right)\text{ N/m} = 10.65\text{ N/mm} \qquad \textit{Ans.}$$

(**c**) The maximum stress will occur at the bottom of the top hook where the bending-moment is $3FR$ and the axial fore is F. Using curved beam theory for bending,

Eq. (3-65), p. 119: $\sigma_i = \dfrac{Mc_i}{Aer_i} = \dfrac{3FRc_i}{\left(\pi d^2/4\right)e\left(R - d/2\right)}$

Axial: $\sigma_a = \dfrac{F}{A} = \dfrac{F}{\pi d^2/4}$

Combining, $$\sigma_{\max} = \sigma_i + \sigma_a = \frac{4F}{\pi d^2}\left[\frac{3Rc_i}{e(R-d/2)}+1\right] = S_y$$

$$F = \frac{\pi d^2 S_y}{4\left[\dfrac{3Rc_i}{e(R-d/2)}+1\right]} \qquad (1) \qquad \textit{Ans.}$$

For the clip in part (b),

Eq. (10-14) and Table 10-4: $S_{ut} = A/d^m = 1783/2^{0.190} = 1563$ MPa

Eq. (10-57): $S_y = 0.78\, S_{ut} = 0.78(1563) = 1219$ MPa

Table 3-4, p. 121:

$$r_n = \frac{1^2}{2\left(6-\sqrt{6^2-1^2}\right)} = 5.95804 \text{ mm}$$

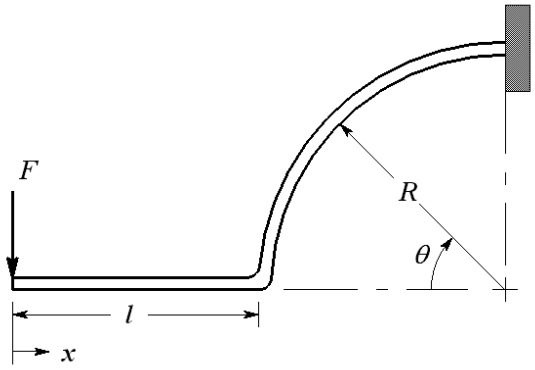
$$e = r_c - r_n = 6 - 5.95804 = 0.04196 \text{ mm}$$

$$c_i = r_n - (R - d/2) = 5.95804 - (6 - 2/2) = 0.95804 \text{ mm}$$

Eq. (1):

$$F = \frac{\pi\left(0.002^2\right)1219\left(10^6\right)}{4\left[\dfrac{3(6)0.95804}{0.04196(6-1)}+1\right]} = 46.0 \text{ N} \qquad \textit{Ans.}$$

10-42 (**a**)



$$M = -Fx, \quad \frac{\partial M}{\partial F} = -x \quad 0 \le x \le l$$

$$M = Fl + FR(1-\cos\theta), \quad \frac{\partial M}{\partial F} = l + R(1-\cos\theta) \quad 0 \le \theta \le l$$

$$\delta_F = \frac{1}{EI}\int_0^l -Fx(-x)dx + \int_0^{\pi/2} F\left[l + R(1-\cos\theta)\right]^2 Rd\theta$$

$$= \frac{F}{12EI}\left\{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]\right\}$$

The spring rate is

$$k = \frac{F}{\delta_F} = \frac{12EI}{4l^3 + 3R\left[2\pi l^2 + 4(\pi-2)lR + (3\pi-8)R^2\right]} \qquad Ans.$$

(**b**) Given: A313 stainless wire, $d = 0.063$ in, $R = 0.625$ in, and $l = 0.5$ in.

Table 10-5: $E = 28$ Mpsi

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\left(0.063^4\right) = 7.733\left(10^{-7}\right) \text{ in}^4$$

$$k = \frac{12(28)10^6(7.733)10^{-7}}{4\left(0.5^3\right) + 3(0.625)\left[2\pi\left(0.5^2\right) + 4(\pi-2)0.5(0.625) + (3\pi-8)\left(0.625^2\right)\right]}$$

$$= 36.3 \text{ lbf/in} \qquad Ans.$$

(**c**) Table 10-4: $A = 169$ kpsi·in^m, $m = 0.146$

Eq. (10-14): $S_{ut} = A/d^m = 169/0.063^{0.146} = 253.0$ kpsi

Eq. (10-57): $S_y = 0.61\, S_{ut} = 0.61(253.0) = 154.4$ kpsi

One can use curved beam theory as in the solution for Prob. 10-41. However, the equations developed in Sec. 10-12 are equally valid.

$$C = D/d = 2(0.625 + 0.063/2)/0.063 = 20.8$$

Eq. (10-43): $$K_i = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{4\left(20.8^2\right) - 20.8 - 1}{4(20.8)(20.8-1)} = 1.037$$

Eq. (10-44), setting $\sigma = S_y$:

$$K_i \frac{32Fr}{\pi d^3} = S_y \quad \Rightarrow \quad 1.037 \frac{32F\left(0.5+0.625\right)}{\pi\left(0.063^3\right)} = 154.4\left(10^3\right)$$

Solving for F yields $F = 3.25$ lbf *Ans.*

Try solving part (c) of this problem using curved beam theory. You should obtain the same answer.

10-43 (**a**) $M = -Fx$

$$\sigma = \left|\frac{M}{I/c}\right| = \frac{Fx}{I/c} = \frac{Fx}{bh^2/6}$$

Constant stress,

$$\frac{bh^2}{6} = \frac{Fx}{\sigma} \quad \Rightarrow \quad h = \sqrt{\frac{6Fx}{b\sigma}} \qquad (1) \qquad \textit{Ans.}$$

At $x = l$,

$$h_o = \sqrt{\frac{6Fl}{b\sigma}} \qquad \Rightarrow \qquad h = h_o\sqrt{x/l} \qquad \textit{Ans.}$$

(**b**) $M = -Fx, \quad \partial M/\partial F = -x$

$$y = \int_0^l \frac{M\left(\partial M/\partial F\right)}{EI}dx = \frac{1}{E}\int_0^l \frac{-Fx(-x)}{\frac{1}{12}bh_o^3\left(x/l\right)^{3/2}}dx = \frac{12Fl^{3/2}}{bh_o^3E}\int_0^l x^{1/2}dx$$

$$= \frac{2}{3}\frac{12Fl^{3/2}}{bh_o^3E}l^{3/2} = \frac{8Fl^3}{bh_o^3E}$$

$$k = \frac{F}{y} = \frac{bh_o^3E}{8l^3} \qquad \textit{Ans.}$$

10-44 Computer programs will vary.

10-45 Computer programs will vary.

Chapter 11

11-1 For the deep-groove 02-series ball bearing with $R = 0.90$, the design life x_D, in multiples of rating life, is

$$x_D = \frac{L_D}{L_R} = \frac{60\mathcal{L}_D n_D}{L_{10}} = \frac{60(25\,000)350}{10^6} = 525 \qquad \textit{Ans.}$$

The design radial load is

$$F_D = 1.2(2.5) = 3.0 \text{ kN}$$

Eq. (11-6): $$C_{10} = 3.0\left\{\frac{525}{0.02 + (4.459 - 0.02)\left[\ln(1/0.9)\right]^{1/1.483}}\right\}^{1/3}$$

$$C_{10} = 24.3 \text{ kN} \qquad \textit{Ans.}$$

Table 11-2: Choose an 02-35 mm bearing with $C_{10} = 25.5$ kN. *Ans.*

Eq. (11-18): $$R = \exp\left\{-\left[\frac{525(3/25.5)^3 - 0.02}{4.459 - 0.02}\right]^{1.483}\right\} = 0.920 \qquad \textit{Ans.}$$

11-2 For the angular-contact 02-series ball bearing as described, the rating life multiple is

$$x_D = \frac{L_D}{L_R} = \frac{60\mathcal{L}_D n_D}{L_{10}} = \frac{60(40\,000)520}{10^6} = 1248$$

The design radial load is

$$F_D = 1.4(725) = 1015 \text{ lbf} = 4.52 \text{ kN}$$

Eq. (11-6):

$$C_{10} = 1015\left\{\frac{1248}{0.02 + (4.459 - 0.02)\left[\ln(1/0.9)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 10\,930 \text{ lbf} = 48.6 \text{ kN}$$

Table 11-2: Select an 02-60 mm bearing with $C_{10} = 55.9$ kN. *Ans.*

Eq. (11-18): $$R = \exp\left\{-\left[\frac{1248(4.52/55.9)^3 - 0.02}{4.439}\right]^{1.483}\right\} = 0.945 \qquad \textit{Ans.}$$

11-3 For the straight-roller 03-series bearing selection, x_D = 1248 rating lives from Prob. 11-2 solution.

$$F_D = 1.4(2235) = 3129 \text{ lbf} = 13.92 \text{ kN}$$

$$C_{10} = 13.92\left(\frac{1248}{1}\right)^{3/10} = 118 \text{ kN}$$

Table 11-3: Select an 03-60 mm bearing with C_{10} = 123 kN. *Ans.*

Eq. (11-18): $$R = \exp\left\{-\left[\frac{1248(13.92/123)^{10/3} - 0.02}{4.459 - 0.02}\right]^{1.483}\right\} = 0.917 \quad \textit{Ans.}$$

11-4 The combined reliability of the two bearings selected in Probs. 11-2 and 11-3 is

$$R = (0.945)(0.917) = 0.867 \quad \textit{Ans.}$$

We can choose a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. We make the selections, find the existing reliabilities, multiply them together, and observe that the reliability goal is exceeded due to the roundup of capacity upon table entry.

Another possibility is to use the reliability of one bearing, say R_1. Then set the reliability goal of the second as

$$R_2 = \frac{0.90}{R_1}$$

or vice versa. This gives three pairs of selections to compare in terms of cost, geometry implications, etc.

11-5 Establish a reliability goal of $\sqrt{0.90} = 0.95$ for each bearing. For an 02-series angular contact ball bearing,

$$C_{10} = 1015\left\{\frac{1248}{0.02 + 4.439\left[\ln(1/0.95)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 12822 \text{ lbf} = 57.1 \text{ kN}$$

Select an 02-65 mm angular-contact bearing with C_{10} = 63.7 kN.

$$R_A = \exp\left\{-\left[\frac{1248(4.52/63.7)^3 - 0.02}{4.439}\right]^{1.483}\right\} = 0.962$$

For an 03-series straight roller bearing,

$$C_{10} = 13.92\left\{\frac{1248}{0.02 + 4.439\left[\ln\left(1/0.95\right)\right]^{1/1.483}}\right\}^{3/10} = 136.5 \text{ kN}$$

Select an 03-65 mm straight-roller bearing with C_{10} = 138 kN.

$$R_B = \exp\left\{-\left[\frac{1248\left(13.92/138\right)^{10/3} - 0.02}{4.439}\right]^{1.483}\right\} = 0.953$$

The overall reliability is R = (0.962)(0.953) = 0.917, which exceeds the goal.

11-6 For the straight cylindrical roller bearing specified with a service factor of 1, R = 0.95 and F_R = 20 kN.

$$x_D = \frac{L_D}{L_R} = \frac{60\mathcal{L}_D n_D}{L_{10}} = \frac{60\left(8000\right)950}{10^6} = 456$$

$$C_{10} = 20\left\{\frac{456}{0.02 + 4.439\left[\ln\left(1/0.95\right)\right]^{1/1.483}}\right\}^{3/10} = 145 \text{ kN} \qquad \textit{Ans.}$$

11-7 Both bearings need to be rated in terms of the same catalog rating system in order to compare them. Using a rating life of one million revolutions, both bearings can be rated in terms of a Basic Load Rating.

Eq. (11-3): $$C_A = F_A\left(\frac{L_A}{L_R}\right)^{1/a} = F_A\left(\frac{\mathcal{L}_A n_A 60}{L_R}\right)^{1/a} = 2.0\left[\frac{\left(3000\right)\left(500\right)\left(60\right)}{10^6}\right]^{1/3}$$
$$= 8.96 \text{ kN}$$

Bearing B already is rated at one million revolutions, so C_B = 7.0 kN. Since $C_A > C_B$, bearing A can carry the larger load. *Ans.*

11-8 F_D = 2 kN, $L_D = 10^9$ rev, R = 0.90

Eq. (11-3): $$C_{10} = F_D\left(\frac{L_D}{L_R}\right)^{1/a} = 2\left(\frac{10^9}{10^6}\right)^{1/3} = 20 \text{ kN} \qquad \textit{Ans.}$$

11-9 F_D = 800 lbf, $\mathcal{L}_D$ = 12 000 hours, n_D = 350 rev/min, R = 0.90

Eq. (11-3): $$C_{10} = F_D\left(\frac{\mathcal{L}_D n_D 60}{L_R}\right)^{1/a} = 800\left(\frac{12\,000(350)(60)}{10^6}\right)^{1/3} = 5050 \text{ lbf} \quad \textit{Ans}$$

11-10 F_D = 4 kN, $\mathcal{L}_D$ = 8 000 hours, n_D = 500 rev/min, R = 0.90

Eq. (11-3): $$C_{10} = F_D\left(\frac{\mathcal{L}_D n_D 60}{L_R}\right)^{1/a} = 4\left(\frac{8\,000(500)(60)}{10^6}\right)^{1/3} = 24.9 \text{ kN} \quad \textit{Ans}$$

11-11 F_D = 650 lbf, n_D = 400 rev/min, R = 0.95

$\mathcal{L}_D$ = (5 years)(40 h/week)(52 week/year) = 10 400 hours

Assume an application factor of one. The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{(10\,400)(400)(60)}{10^6} = 249.6$$

Eq. (11-6): $$C_{10} = (1)(650)\left\{\frac{249.6}{0.02 + 4.439\left[\ln(1/0.95)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 4800 \text{ lbf} \quad \textit{Ans}.$$

11-12 F_D = 9 kN, $L_D = 10^8$ rev, R = 0.99

Assume an application factor of one. The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{10^8}{10^6} = 100$$

Eq. (11-6): $$C_{10} = (1)(9)\left\{\frac{100}{0.02 + 4.439\left[\ln(1/0.99)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 69.2 \text{ kN} \quad \textit{Ans}.$$

11-13 F_D = 11 kips, $\mathcal{L}_D$ = 20 000 hours, n_D = 200 rev/min, R = 0.99

Assume an application factor of one. Use the Weibull parameters for Manufacturer 2 on p. 608.

The multiple of rating life is

$$x_D = \frac{L_D}{L_R} = \frac{(20\,000)(200)(60)}{10^6} = 240$$

Eq. (11-6): $$C_{10} = (1)(11)\left\{\frac{240}{0.02 + 4.439\left[\ln(1/0.99)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 113 \text{ kips} \quad \textit{Ans.}$$

11-14 From the solution to Prob. 3-68, the ground reaction force carried by the bearing at *C* is $R_C = F_D = 178$ lbf. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{15\,000(1200)(60)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$C_{10} = 1.2(178)\left[\frac{1080}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}}\right]^{1/3}$$

$$= 2590 \text{ lbf} \quad \textit{Ans.}$$

11-15 From the solution to Prob. 3-69, the ground reaction force carried by the bearing at *C* is $R_C = F_D = 1.794$ kN. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{15\,000(1200)(60)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$C_{10} = 1.2(1.794)\left[\frac{1080}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}}\right]^{1/3}$$

$$= 26.1 \text{ kN} \quad \textit{Ans.}$$

11-16 From the solution to Prob. 3-70, $R_{Cz} = -327.99$ lbf, $R_{Cy} = -127.27$ lbf

$$R_C = F_D = \left[(-327.99)^2 + (-127.27)^2\right]^{1/2} = 351.8 \text{ lbf}$$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{15\,000(1200)(60)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_o + (\theta - x_o)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = 1.2(351.8) \left[\frac{1080}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}} \right]^{1/3}$$

$$= 5110 \text{ lbf} \quad \textit{Ans.}$$

11-17 From the solution to Prob. 3-71, $R_{Cz} = -150.7$ N, $R_{Cy} = -86.10$ N

$$R_C = F_D = \left[(-150.7)^2 + (-86.10)^2 \right]^{1/2} = 173.6 \text{ N}$$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{15\,000(1200)(60)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = 1.2(173.6) \left[\frac{1080}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}} \right]^{1/3}$$

$$= 2520 \text{ N} \quad \textit{Ans.}$$

11-18 From the solution to Prob. 3-77, $R_{Az} = 444$ N, $R_{Ay} = 2384$ N

$$R_A = F_D = \left(444^2 + 2384^2 \right)^{1/2} = 2425 \text{ N} = 2.425 \text{ kN}$$

Use the Weibull parameters for Manufacturer 2 on p. 608. The design speed is equal to the speed of shaft *AD*,

$$n_D = \frac{d_F}{d_C} n_i = \frac{125}{250}(191) = 95.5 \text{ rev/min}$$

$$x_D = \frac{L_D}{L_R} = \frac{12\,000(95.5)(60)}{10^6} = 68.76$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = (1)(2.425)\left[\frac{68.76}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}}\right]^{1/3}$$

$$= 11.7 \text{ kN} \quad \textit{Ans.}$$

11-19 From the solution to Prob. 3-79, R_{Az} = 54.0 lbf, R_{Ay} = 140 lbf

$$R_A = F_D = \left(54.0^2 + 140^2\right)^{1/2} = 150.1 \text{ lbf}$$

Use the Weibull parameters for Manufacturer 2 on p. 608. The design speed is equal to the speed of shaft *AD*,

$$n_D = \frac{d_F}{d_C} n_i = \frac{10}{5}(280) = 560 \text{ rev/min}$$

$$x_D = \frac{L_D}{L_R} = \frac{14\,000(560)(60)}{10^6} = 470.4$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}}\right]^{1/a}$$

$$C_{10} = (1)(150.1)\left[\frac{470.4}{0.02 + (4.459 - 0.02)(1 - 0.98)^{1/1.483}}\right]^{3/10}$$

$$= 1320 \text{ lbf} \quad \textit{Ans.}$$

11-20 **(a)** $F_a = 3$ kN, $F_r = 7$ kN, $n_D = 500$ rev/min, $V = 1.2$

From Table 11-2, with a 65 mm bore, C_0 = 34.0 kN.

$$F_a / C_0 = 3 / 34 = 0.088$$

From Table 11-1, $0.28 \le e \le 3.0$.

$$\frac{F_a}{VF_r} = \frac{3}{(1.2)(7)} = 0.357$$

Since this is greater than *e*, interpolating Table 11-1 with $F_a / C_0 = 0.088$, we obtain $X_2 = 0.56$ and $Y_2 = 1.53$.

Eq. (11-9): $$F_e = X_i V F_r + Y_i F_a = (0.56)(1.2)(7) + (1.53)(3) = 9.29 \text{ kN} \quad \textit{Ans.}$$

$F_e > F_r$ so use F_e.

(b) Use Eq. (11-7) to determine the necessary rated load the bearing should have to carry the equivalent radial load for the desired life and reliability. Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{10\,000(500)(60)}{10^6} = 300$$

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = (1)(9.29)\left[\frac{300}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}} \right]^{1/3}$$

$$= 73.4 \text{ kN}$$

From Table 11-2, the 65 mm bearing is rated for 55.9 kN, which is less than the necessary rating to meet the specifications. This bearing should not be expected to meet the load, life, and reliability goals. *Ans.*

11-21 **(a)** $F_a = 2$ kN, $F_r = 5$ kN, $n_D = 400$ rev/min, $V = 1$

From Table 11-2, 30 mm bore, C_{10} = 19.5 kN, C_0 = 10.0 kN

$$F_a / C_0 = 2 / 10 = 0.2$$

From Table 11-1, $0.34 \le e \le 0.38$.

$$\frac{F_a}{VF_r} = \frac{2}{(1)(5)} = 0.4$$

Since this is greater than e, interpolating Table 11-1, with $F_a / C_0 = 0.2$, we obtain $X_2 = 0.56$ and $Y_2 = 1.27$.

Eq. (11-9): $$F_e = X_i V F_r + Y_i F_a = (0.56)(1)(5) + (1.27)(2) = 5.34 \text{ kN}$$ *Ans.*

$F_e > F_r$ so use F_e.

(b) Solve Eq. (11-7) for x_D.

$$x_D = \left(\frac{C_{10}}{a_f F_D} \right)^a \left[x_0 + (\theta - x_0)(1 - R_D)^{1/b} \right]$$

$$x_D = \left(\frac{19.5}{(1)(5.34)} \right)^3 \left[0.02 + (4.459 - 0.02)(1 - 0.99)^{1/1.483} \right]$$

$$x_D = 10.66$$

$$x_D = \frac{L_D}{L_R} = \frac{\mathcal{L}_D n_D (60)}{10^6}$$

$$\mathcal{L}_D = \frac{x_D\left(10^6\right)}{n_D\left(60\right)} = \frac{10.66\left(10^6\right)}{\left(400\right)\left(60\right)} = 444 \text{ h} \quad \textit{Ans.}$$

11-22 $F_r = 8$ kN, $R = 0.9$, $L_D = 10^9$ rev

Eq. (11-3): $$C_{10} = F_D\left(\frac{L_D}{L_R}\right)^{1/a} = 8\left(\frac{10^9}{10^6}\right)^{1/3} = 80 \text{ kN}$$

From Table 11-2, select the 85 mm bore. *Ans.*

11-23 $F_r = 8$ kN, $F_a = 2$ kN, $V = 1$, $R = 0.99$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{10\,000\left(400\right)\left(60\right)}{10^6} = 240$$

First guess: Choose from middle of Table 11-1, $X = 0.56$, $Y = 1.63$

Eq. (11-9): $$F_e = 0.56\left(1\right)\left(8\right) + 1.63\left(2\right) = 7.74 \text{ kN}$$

$F_e < F_r$, so just use F_r as the design load.

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_o + \left(\theta - x_o\right)\left(1 - R_D\right)^{1/b}}\right]^{1/a}$$

$$C_{10} = \left(1\right)\left(8\right)\left[\frac{240}{0.02 + \left(4.459 - 0.02\right)\left(1 - 0.99\right)^{1/1.483}}\right]^{1/3} = 82.5 \text{ kN}$$

From Table 11-2, try 85 mm bore with $C_{10} = 83.2$ kN, $C_0 = 53.0$ kN
Iterate the previous process:

$F_a / C_0 = 2 / 53 = 0.038$

Table 11-1: $0.22 \le e \le 0.24$

$$\frac{F_a}{VF_r} = \frac{2}{1\left(8\right)} = 0.25 > e$$

Interpolate Table 11-1 with $F_a / C_0 = 0.038$ to obtain $X_2 = 0.56$ and $Y_2 = 1.89$.

Eq. (11-9): $$F_e = 0.56(1)8 + 1.89(2) = 8.26 > F_r$$

Eq. (11-7): $$C_{10} = \left(1\right)\left(8.26\right)\left[\frac{240}{0.02 + \left(4.459 - 0.02\right)\left(1 - 0.99\right)^{1/1.483}}\right]^{1/3} = 85.2 \text{ kN}$$

Table 11-2: Move up to the 90 mm bore with C_{10} = 95.6 kN, C_0 = 62.0 kN.
Iterate again:

$$F_a / C_0 = 2 / 62 = 0.032$$

Table 11-1: Again, $0.22 \le e \le 0.24$

$$\frac{F_a}{VF_r} = \frac{2}{1(8)} = 0.25 > e$$

Interpolate Table 11-1 with $F_a / C_0 = 0.032$ to obtain $X_2 = 0.56$ and $Y_2 = 1.95$.

Eq. (11-9): $$F_e = 0.56(1)8 + 1.95(2) = 8.38 > F_r$$

Eq. (11-7): $$C_{10} = (1)(8.38)\left[\frac{240}{0.02 + (4.459 - 0.02)(1 - 0.99)^{1/1.483}}\right]^{1/3} = 86.4 \text{ kN}$$

The 90 mm bore is acceptable. *Ans.*

11-24 $F_r = 8$ kN, $F_a = 3$ kN, $V = 1.2, R = 0.9, L_D = 10^8$ rev

First guess: Choose from middle of Table 11-1, X = 0.56, Y = 1.63

Eq. (11-9): $$F_e = 0.56(1.2)(8) + 1.63(3) = 10.3 \text{ kN}$$

$$F_e > F_r$$

Eq. (11-3): $$C_{10} = F_e\left(\frac{L_D}{L_R}\right)^{1/a} = 10.3\left(\frac{10^8}{10^6}\right)^{1/3} = 47.8 \text{ kN}$$

From Table 11-2, try 60 mm with C_{10} = 47.5 kN, C_0 = 28.0 kN
Iterate the previous process:

$$F_a / C_0 = 3 / 28 = 0.107$$

Table 11-1: $0.28 \le e \le 0.30$

$$\frac{F_a}{VF_r} = \frac{3}{1.2(8)} = 0.313 > e$$

Interpolate Table 11-1 with F_a / C_0 = 0.107 to obtain $X_2 = 0.56$ and $Y_2 = 1.46$

Eq. (11-9): $$F_e = 0.56(1.2)(8) + 1.46(3) = 9.76 \text{ kN} > F_r$$

Eq. (11-3): $$C_{10} = 9.76\left(\frac{10^8}{10^6}\right)^{1/3} = 45.3 \text{ kN}$$

From Table 11-2, we have converged on the 60 mm bearing. *Ans.*

11-25 $F_r = 10$ kN, $F_a = 5$ kN, $V = 1$, $R = 0.95$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{12\,000(300)(60)}{10^6} = 216$$

First guess: Choose from middle of Table 11-1, $X = 0.56$, $Y = 1.63$

Eq. (11-9): $F_e = 0.56(1)(10) + 1.63(5) = 13.75$ kN

$F_e > F_r$, so use F_e as the design load.

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = (1)(13.75) \left[\frac{216}{0.02 + (4.459 - 0.02)(1 - 0.95)^{1/1.483}} \right]^{1/3} = 97.4 \text{ kN}$$

From Table 11-2, try 95 mm bore with $C_{10} = 108$ kN, $C_0 = 69.5$ kN
Iterate the previous process:

$F_a / C_0 = 5 / 69.5 = 0.072$

Table 11-1: $0.27 \le e \le 0.28$

$$\frac{F_a}{VF_r} = \frac{5}{1(10)} = 0.5 > e$$

Interpolate Table 11-1 with $F_a / C_0 = 0.072$ to obtain $X_2 = 0.56$ and $Y_2 = 1.62 \doteq 1.63$

Since this is where we started, we will converge back to the same bearing. The 95 mm bore meets the requirements. *Ans.*

11-26 **Note to the Instructor.** In the first printing of the 9th edition, the design life was incorrectly given to be 10^9 rev and will be corrected to 10^8 rev in subsequent printings. We apologize for the inconvenience.

$F_r = 9$ kN, $F_a = 3$ kN, $V = 1.2$, $R = 0.99$

Use the Weibull parameters for Manufacturer 2 on p. 608.

$$x_D = \frac{L_D}{L_R} = \frac{10^8}{10^6} = 100$$

First guess: Choose from middle of Table 11-1, $X = 0.56$, $Y = 1.63$

Eq. (11-9): $F_e = 0.56(1.2)(9) + 1.63(3) = 10.9$ kN

$F_e > F_r$, so use F_e as the design load.

Eq. (11-7): $$C_{10} = a_f F_D \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a}$$

$$C_{10} = (1)(10.9) \left[\frac{100}{0.02 + (4.459 - 0.02)(1 - 0.99)^{1/1.483}} \right]^{1/3} = 83.9 \text{ kN}$$

From Table 11-2, try 90 mm bore with C_{10} = 95.6 kN, C_0 = 62.0 kN. Try this bearing. Iterate the previous process:

$$F_a / C_0 = 3 / 62 = 0.048$$

Table 11-1: $0.24 \le e \le 0.26$

$$\frac{F_a}{VF_r} = \frac{3}{1.2(9)} = 0.278 > e$$

Interpolate Table 11-1 with $F_a / C_0 = 0.048$ to obtain $X_2 = 0.56$ and $Y_2 = 1.79$

Eq. (11-9): $F_e = 0.56(1.2)(9) + 1.79(3) = 11.4 \text{ kN} > F_r$

$$C_{10} = \frac{11.4}{10.9} 83.9 = 87.7 \text{ kN}$$

From Table 11-2, this converges back to the same bearing. The 90 mm bore meets the requirements. *Ans.*

11-27 **(a)** $n_D = 1200$ rev/min, $L_D = 15$ kh, $R = 0.95$, $a_f = 1.2$

From Prob. 3-72, R_{Cy} = 183.1 lbf, R_{Cz} = –861.5 lbf.

$$R_C = F_D = \left[183.1^2 + (-861.5)^2 \right]^{1/2} = 881 \text{ lbf}$$

$$x_D = \frac{L_D}{L_R} = \frac{15\,000(1200)(60)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = 1.2(881) \left[\frac{1080}{0.02 + 4.439(1 - 0.95)^{1/1.483}} \right]^{1/3}$$

$$= 12\,800 \text{ lbf} = 12.8 \text{ kips} \quad \textit{Ans.}$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-28 **(a)** $n_D = 1200$ rev/min, $L_D = 15$ kh, $R = 0.95$, $a_f = 1.2$

From Prob. 3-72, R_{Oy} = –208.5 lbf, R_{Oz} = 259.3 lbf.

$$R_C = F_D = \left[259.3^2 + \left(-208.5\right)^2 \right]^{1/2} = 333 \text{ lbf}$$

$$x_D = \frac{L_D}{L_R} = \frac{15\,000\left(1200\right)\left(60\right)}{10^6} = 1080$$

Eq. (11-7): $$C_{10} = 1.2\left(333\right)\left[\frac{1080}{0.02 + 4.439\left(1 - 0.95\right)^{1/1.483}} \right]^{1/3}$$

$$= 4837 \text{ lbf} = 4.84 \text{ kips} \quad \textit{Ans.}$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-29 **(a)** $n_D = 900$ rev/min, $L_D = 12$ kh, $R = 0.98$, $a_f = 1.2$

From Prob. 3-73, R_{Cy} = 8.319 kN, R_{Cz} = –10.830 kN.

$$R_C = F_D = \left[8.319^2 + \left(-10.830\right)^2 \right]^{1/2} = 13.7 \text{ kN}$$

$$x_D = \frac{L_D}{L_R} = \frac{12\,000\left(900\right)\left(60\right)}{10^6} = 648$$

Eq. (11-7): $$C_{10} = 1.2\left(13.7\right)\left[\frac{648}{0.02 + 4.439\left(1 - 0.98\right)^{1/1.483}} \right]^{1/3} = 204 \text{ kN} \quad \textit{Ans.}$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-30 **(a)** $n_D = 900$ rev/min, $L_D = 12$ kh, $R = 0.98$, $a_f = 1.2$

From Prob. 3-73, R_{Oy} = 5083 N, R_{Oz} = 494 N.

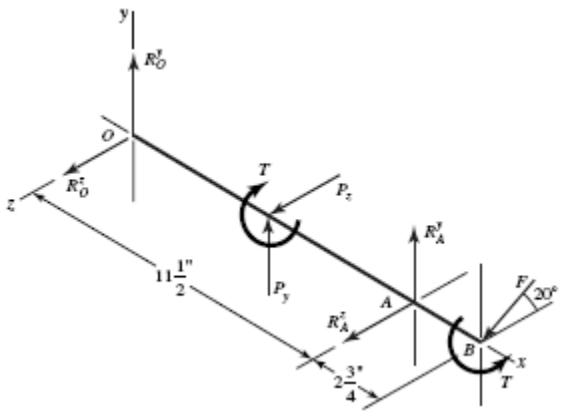
$$R_C = F_D = \left(5083^2 + 494^2 \right)^{1/2} = 5106 \text{ N} = 5.1 \text{ kN}$$

$$x_D = \frac{L_D}{L_R} = \frac{12\,000\left(900\right)\left(60\right)}{10^6} = 648$$

Eq. (11-7): $$C_{10} = 1.2\left(5.1\right)\left[\frac{648}{0.02 + 4.439\left(1 - 0.98\right)^{1/1.483}} \right]^{1/3} = 76.1 \text{ kN} \quad \textit{Ans.}$$

(b) Results will vary depending on the specific bearing manufacturer selected. A general engineering components search site such as www.globalspec.com might be useful as a starting point.

11-31 Assume concentrated forces as shown.



$$P_z = 8(28) = 224 \text{ lbf}$$
$$P_y = 8(35) = 280 \text{ lbf}$$
$$T = 224(2) = 448 \text{ lbf} \cdot \text{in}$$
$$\Sigma T^x = -448 + 1.5F\cos 20^\circ = 0$$
$$F = \frac{448}{1.5(0.940)} = 318 \text{ lbf}$$

$$\Sigma M_O^z = 5.75P_y + 11.5R_A^y - 14.25F\sin 20^\circ = 0$$
$$5.75(280) + 11.5R_A^y - 14.25(318)(0.342) = 0$$
$$R_A^y = -5.24 \text{ lbf}$$
$$\Sigma M_O^y = -5.75P_z - 11.5R_A^z - 14.25F\cos 20^\circ = 0$$
$$-5.75(224) - 11.5R_A^z - 14.25(318)(0.940) = 0$$
$$R_A^z = -482 \text{ lbf}; \quad R_A = \left[(-482)^2 + (-5.24)^2\right]^{1/2} = 482 \text{ lbf}$$
$$\Sigma F^z = R_O^z + P_z + R_A^z + F\cos 20^\circ = 0$$
$$R_O^z + 224 - 482 + 318(0.940) = 0$$
$$R_O^z = -40.9 \text{ lbf}$$
$$\Sigma F^y = R_O^y + P_y + R_A^y - F\sin 20^\circ = 0$$
$$R_O^y + 280 - 5.24 - 318(0.342) = 0$$
$$R_O^y = -166 \text{ lbf}$$
$$R_O = \left[(-40.9)^2 + (-166)^2\right]^{1/2} = 171 \text{ lbf}$$

So the reaction at A governs.
Reliability Goal: $\sqrt{0.92} = 0.96$

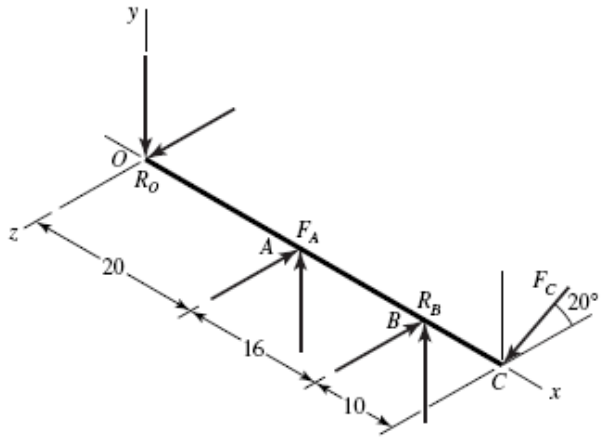
$$F_D = 1.2(482) = 578 \text{ lbf}$$
$$x_D = 35\,000(350)(60)/10^6 = 735$$
$$C_{10} = 578\left\{\frac{735}{0.02 + (4.459 - 0.02)\left[\ln(1/0.96)\right]^{1/1.483}}\right\}^{1/3}$$
$$= 6431 \text{ lbf} = 28.6 \text{ kN}$$

From Table 11-2, a 40 mm bore angular contact bearing is sufficient with a rating of

31.9 kN. *Ans.*

11-32 For a combined reliability goal of 0.95, use $\sqrt{0.95} = 0.975$ for the individual bearings.

$$x_D = \frac{40\,000(420)(60)}{10^6} = 1008$$



The resultant of the given forces are

$$R_O = [(-387)^2 + 467^2]^{1/2} = 607 \text{ lbf}$$
$$R_B = [316^2 + (-1615)^2]^{1/2} = 1646 \text{ lbf}$$

At *O*:

$$\text{Eq. (11-6):} \quad C_{10} = 1.2(607)\left\{\frac{1008}{0.02 + (4.459 - 0.02)\left[\ln(1/0.975)\right]^{1/1.483}}\right\}^{1/3}$$
$$= 9978 \text{ lbf} = 44.4 \text{ kN}$$

From Table 11-2, select an 02-55 mm angular-contact ball bearing with a basic load rating of 46.2 kN. *Ans.*

At *B*:

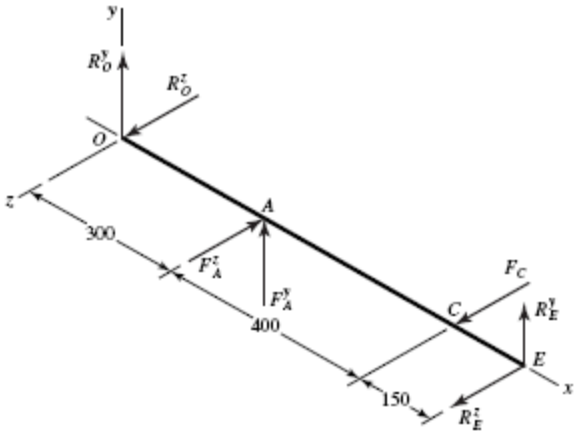
$$\text{Eq. (11-6):} \quad C_{10} = 1.2(1646)\left\{\frac{1008}{0.02 + (4.459 - 0.02)\left[\ln(1/0.975)\right]^{1/1.483}}\right\}^{3/10}$$
$$= 20827 \text{ lbf} = 92.7 \text{ kN}$$

From Table 11-3, select an 02-75 mm or 03-55 mm cylindrical roller. *Ans.*

11-33 The reliability of the individual bearings is $R = \sqrt{0.98} = 0.9899$

From statics,

$T = (270 - 50) = (P_1 - P_2)125$
$= (P_1 - 0.15\,P_1)125$
$P_1 = 310.6$ N,
$P_2 = 0.15\,(310.6) = 46.6$ N
$P_1 + P_2 = 357.2$ N
$F_A^y = 357.2 \sin 45^\circ = 252.6 \text{ N} = F_A^z$



$$\sum M_O^z = 850R_E^y + 300(252.6) = 0 \Rightarrow R_E^y = -89.2 \text{ N}$$
$$\sum F^y = 252.6 - 89.2 + R_O^y = 0 \Rightarrow R_O^y = -163.4 \text{ N}$$
$$\sum M_O^y = -850R_E^z - 700(320) + 300(252.6) = 0 \Rightarrow R_E^z = -174.4 \text{ N}$$
$$\sum F^z = -174.4 + 320 - 252.6 + R_O^z = 0 \Rightarrow R_O^z = 107 \text{ N}$$

$$R_O = \sqrt{(-163.4)^2 + 107^2} = 195 \text{ N}$$
$$R_E = \sqrt{(-89.2)^2 + (-174.4)^2} = 196 \text{ N}$$

The radial loads are nearly the same at *O* and *E*. We can use the same bearing at both locations.

$$x_D = \frac{60\,000(1500)(60)}{10^6} = 5400$$

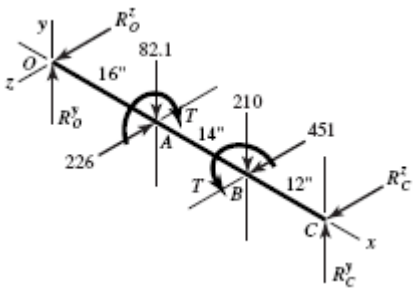
Eq. (11-6): $$C_{10} = 1(0.196)\left\{\frac{5400}{0.02 + 4.439\left[\ln(1/0.9899)\right]^{1/1.483}}\right\}^{1/3} = 5.7 \text{ kN}$$

From Table 11-2, select an 02-12 mm deep-groove ball bearing with a basic load rating of 6.89 kN. *Ans.*

11-34 $R = \sqrt{0.96} = 0.980$

$T = 12(240 \cos 20^\circ) = 2706 \text{ lbf} \cdot \text{in}$

$F = \dfrac{2706}{6 \cos 25^\circ} = 498 \text{ lbf}$



In *xy*-plane:

$$\Sigma M_O^z = -16(82.1) - 30(210) + 42R_C^y = 0$$

$$R_C^y = 181 \text{ lbf}$$

$$R_O^y = 82.1 + 210 - 181 = 111.1 \text{ lbf}$$

In *xz*-plane:

$$\Sigma M_O^y = 16(226) - 30(451) - 42R_C^z = 0$$

$$R_C^z = -236 \text{ lbf}$$

$$R_O^z = 226 - 451 + 236 = 11 \text{ lbf}$$

$$R_O = \left(111.1^2 + 11^2\right)^{1/2} = 112 \text{ lbf} \quad \textit{Ans.}$$

$$R_C = \left(181^2 + 236^2\right)^{1/2} = 297 \text{ lbf} \quad \textit{Ans.}$$

$$x_D = \frac{50\,000(300)(60)}{10^6} = 900$$

$$\left(C_{10}\right)_O = 1.2(112)\left\{\frac{900}{0.02 + 4.439\left[\ln\left(1/0.980\right)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 1860 \text{ lbf} = 8.28 \text{ kN}$$

$$\left(C_{10}\right)_C = 1.2(297)\left\{\frac{900}{0.02 + 4.439\left[\ln\left(1/0.980\right)\right]^{1/1.483}}\right\}^{1/3}$$

$$= 4932 \text{ lbf} = 21.9 \text{ kN}$$

Bearing at *O*: Choose a deep-groove 02-17 mm. *Ans*.
Bearing at *C*: Choose a deep-groove 02-35 mm. *Ans*.

11-35 Shafts subjected to thrust can be constrained by bearings, one of which supports the thrust. The shaft floats within the endplay of the second (roller) bearing. Since the thrust force here is larger than any radial load, the bearing absorbing the thrust (bearing *A*) is heavily loaded compared to bearing *B*. Bearing *B* is thus likely to be oversized and may not contribute measurably to the chance of failure. If this is the case, we may be able to obtain the desired combined reliability with bearing *A* having a reliability near 0.99 and bearing *B* having a reliability near 1. This would allow for bearing *A* to have a lower capacity than if it needed to achieve a reliability of $\sqrt{0.99}$. To determine if this is the case, we will start with bearing *B*.

Bearing B (straight roller bearing)

$$x_D = \frac{30\,000(500)(60)}{10^6} = 900$$

$$F_r = \left(36^2 + 67^2\right)^{1/2} = 76.1 \text{ lbf} = 0.339 \text{ kN}$$

Try a reliability of 1 to see if it is readily obtainable with the available bearings.

Eq. (11-6): $$C_{10} = 1.2(0.339)\left\{\frac{900}{0.02 + 4.439\left[\ln(1/1.0)\right]^{1/1.483}}\right\}^{3/10} = 10.1 \text{ kN}$$

The smallest capacity bearing from Table 11-3 has a rated capacity of 16.8 kN. Therefore, we select the 02-25 mm straight cylindrical roller bearing. *Ans.*

Bearing at A (angular-contact ball)
With a reliability of 1 for bearing *B*, we can achieve the combined reliability goal of 0.99 if bearing *A* has a reliability of 0.99.

$$F_r = \left(36^2 + 212^2\right)^{1/2} = 215 \text{ lbf} = 0.957 \text{ kN}$$
$$F_a = 555 \text{ lbf} = 2.47 \text{ kN}$$

Trial #1:
Tentatively select an 02-85 mm angular-contact with C_{10} = 90.4 kN and C_0 = 63.0 kN.

$$\frac{F_a}{C_0} = \frac{2.47}{63.0} = 0.0392$$
$$x_D = \frac{30\,000(500)(60)}{10^6} = 900$$

Table 11-1: Interpolating, $X_2 = 0.56$, $Y_2 = 1.88$

Eq. (11-9): $$F_e = 0.56(0.957) + 1.88(2.47) = 5.18 \text{ kN}$$

Eq. (11-6): $$C_{10} = 1.2(5.18)\left\{\frac{900}{0.02 + 4.439\left[\ln(1/0.99)\right]^{1/1.483}}\right\}^{1/3}$$
$$= 99.54 \text{ kN} > 90.4 \text{ kN}$$

Trial #2:
Tentatively select a 02-90 mm angular-contact ball with C_{10} = 106 kN and C_0 = 73.5 kN.

$$\frac{F_a}{C_0} = \frac{2.47}{73.5} = 0.0336$$

Table 11-1: Interpolating, $X_2 = 0.56$, $Y_2 = 1.93$

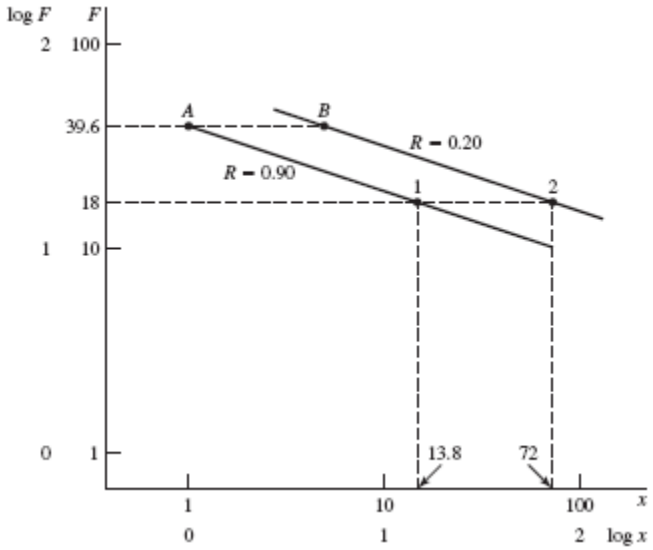
$$F_e = 0.56(0.957) + 1.93(2.47) = 5.30 \text{ kN}$$
$$C_{10} = 1.2(5.30)\left\{\frac{900}{0.02 + 4.439\left[\ln(1/0.99)\right]^{1/1.483}}\right\}^{1/3} = 102 \text{ kN} < 106 \text{ kN} \quad \text{O.K.}$$

Select an 02-90 mm angular-contact ball bearing. *Ans.*

11-36 We have some data. Let's estimate parameters b and θ from it. In Fig. 11-5, we will use line AB. In this case, B is to the right of A.

For F = 18 kN, $$(x)_1 = \frac{115(2000)(60)}{10^6} = 13.8$$

This establishes point 1 on the R = 0.90 line.



The R = 0.20 locus is above and parallel to the R = 0.90 locus. For the two-parameter Weibull distribution, $x_0 = 0$ and points A and B are related by [see Eq. (20-25)]:

$$x_A = \theta\left[\ln(1/0.90)\right]^{1/b} \qquad (1)$$
$$x_B = \theta\left[\ln(1/0.20)\right]^{1/b}$$

and x_B/x_A is in the same ratio as 600/115. Eliminating θ,

$$b = \frac{\ln\left[\ln(1/0.20)/\ln(1/0.90)\right]}{\ln(600/115)} = 1.65 \quad \textit{Ans.}$$

Solving for θ in Eq. (1),

$$\theta = \frac{x_A}{\left[\ln(1/R_A)\right]^{1/1.65}} = \frac{1}{\left[\ln(1/0.90)\right]^{1/1.65}} = 3.91 \quad \textit{Ans.}$$

Therefore, for the data at hand,

$$R = \exp\left[-\left(\frac{x}{3.91}\right)^{1.65}\right]$$

Check R at point B: $x_B = (600/115) = 5.217$

$$R = \exp\left[-\left(\frac{5.217}{3.91}\right)^{1.65}\right] = 0.20$$

Note also, for point 2 on the $R = 0.20$ line,

$$\log(5.217) - \log(1) = \log(x_m)_2 - \log(13.8)$$
$$(x_m)_2 = 72$$

11-37 This problem is rich in useful variations. Here is one.

Decision: Make straight roller bearings identical on a given shaft. Use a reliability goal of $(0.99)^{1/6} = 0.9983$.

Shaft a

$$F_A^r = \left(239^2 + 111^2\right)^{1/2} = 264 \text{ lbf} = 1.175 \text{ kN}$$
$$F_B^r = \left(502^2 + 1075^2\right)^{1/2} = 1186 \text{ lbf} = 5.28 \text{ kN}$$

Thus the bearing at B controls.

$$x_D = \frac{10\,000(1200)(60)}{10^6} = 720$$
$$0.02 + 4.439\left[\ln(1/0.9983)\right]^{1/1.483} = 0.080\,26$$
$$C_{10} = 1.2(5.28)\left(\frac{720}{0.080\,26}\right)^{0.3} = 97.2 \text{ kN}$$

Select either an 02-80 mm with $C_{10} = 106$ kN or an 03-55 mm with $C_{10} = 102$ kN. *Ans.*

Shaft b

$$F_C^r = \left(874^2 + 2274^2\right)^{1/2} = 2436 \text{ lbf} \quad \text{or} \quad 10.84 \text{ kN}$$
$$F_D^r = \left(393^2 + 657^2\right)^{1/2} = 766 \text{ lbf} \quad \text{or} \quad 3.41 \text{ kN}$$

The bearing at C controls.

$$x_D = \frac{10\,000(240)(60)}{10^6} = 144$$

$$C_{10} = 1.2(10.84)\left(\frac{144}{0.080\ 26}\right)^{0.3} = 123 \text{ kN}$$

Select either an 02-90 mm with C_{10} = 142 kN or an 03-60 mm with C_{10} = 123 kN. *Ans.*

Shaft c

$$F_E^r = \left(1113^2 + 2385^2\right)^{1/2} = 2632 \text{ lbf} \quad \text{or} \quad 11.71 \text{ kN}$$

$$F_F^r = \left(417^2 + 895^2\right)^{1/2} = 987 \text{ lbf} \quad \text{or} \quad 4.39 \text{ kN}$$

The bearing at *E* controls.

$$x_D = \frac{10\,000(80)(60)}{10^6} = 48$$

$$C_{10} = 1.2(11.71)\left(\frac{48}{0.080\,26}\right)^{0.3} = 95.7 \text{ kN}$$

Select an 02-80 mm with C_{10} = 106 kN or an 03-60 mm with C_{10} = 123 kN. *Ans.*

11-38 Express Eq. (11-1) as

$$F_1^a L_1 = C_{10}^a L_{10} = K$$

For a ball bearing, a = 3 and for an 02-30 mm angular contact bearing, C_{10} = 20.3 kN.

$$K = (20.3)^3\left(10^6\right) = 8.365\left(10^9\right)$$

At a load of 18 kN, life L_1 is given by:

$$L_1 = \frac{K}{F_1^a} = \frac{8.365\left(10^9\right)}{18^3} = 1.434\left(10^6\right) \text{ rev}$$

For a load of 30 kN, life L_2 is:

$$L_2 = \frac{8.365\left(10^9\right)}{30^3} = 0.310\left(10^6\right) \text{ rev}$$

In this case, Eq. (6-57) – the Palmgren-Miner cycle-ratio summation rule – can be expressed as

$$\frac{l_1}{L_1}+\frac{l_2}{L_2}=1$$

Substituting,

$$\frac{200\,000}{1.434\left(10^6\right)}+\frac{l_2}{0.310\left(10^6\right)}=1$$

$$l_2=0.267\left(10^6\right)\text{ rev}\quad \textit{Ans.}$$

11-39 *Total life in revolutions*

Let:
l = total turns
f_1 = fraction of turns at F_1
f_2 = fraction of turns at F_2

From the solution of Prob. 11-38, $L_1 = 1.434(10^6)$ rev and $L_2 = 0.310(10^6)$ rev.

Palmgren-Miner rule:

$$\frac{l_1}{L_1}+\frac{l_2}{L_2}=\frac{f_1 l}{L_1}+\frac{f_2 l}{L_2}=1$$

from which

$$l=\frac{1}{f_1/L_1+f_2/L_2}$$

$$l=\frac{1}{\left\{0.40/\left[1.434\left(10^6\right)\right]\right\}+\left\{0.60/\left[0.310\left(10^6\right)\right]\right\}}$$

$$=451\,585\text{ rev}\quad \textit{Ans.}$$

Total life in loading cycles

4 min at 2000 rev/min = 8000 rev/cycle

6 min at 2000 rev/min = 12 000 rev/cycle

Total rev/cycle = 8000 + 12 000 = 20 000

$$\frac{451\,585\,\text{rev}}{20\,000\text{ rev/cycle}}=22.58\text{ cycles}\quad \textit{Ans.}$$

Total life in hours

$$\left(10\frac{\text{min}}{\text{cycle}}\right)\left(\frac{22.58\text{ cycles}}{60\text{ min/h}}\right)=3.76\text{ h}\qquad \textit{Ans.}$$

11-40

$$F_{rA}=560\text{ lbf}$$
$$F_{rB}=1095\text{ lbf}$$
$$F_{ae}=200\text{ lbf}$$
$$x_D=\frac{L_D}{L_R}=\frac{40\,000(400)(60)}{90(10^6)}=10.67$$
$$R=\sqrt{0.90}=0.949$$

Eq. (11-15): $$F_{iA}=\frac{0.47F_{rA}}{K_A}=\frac{0.47(560)}{1.5}=175.5\text{ lbf}$$

Eq. (11-15): $$F_{iB}=\frac{0.47F_{rB}}{K_B}=\frac{0.47(1095)}{1.5}=343.1\text{ lbf}$$

$$F_{iA}\le ?\ge (F_{iB}+F_{ae})$$

$175.5\text{ lbf}\le(343.1+200)=543.1\text{ lbf}$, so Eq. (11-16) applies.

We will size bearing *B* first since its induced load will affect bearing *A*, but is not itself affected by the induced load from bearing *A* [see Eq. (11-16)].

From Eq. (11-16*b*), $F_{eB}=F_{rB}=1095$ lbf.

Eq. (11-7): $$F_{RB}=1.4(1095)\left(\frac{10.67}{4.48(1-0.949)^{1/1.5}}\right)^{3/10}=3607\text{ lbf}\qquad \textit{Ans.}$$

Select cone 32305, cup 32305, with 0.9843 in bore, and rated at 3910 lbf with $K = 1.95$. *Ans.*

With bearing *B* selected, we re-evaluate the induced load from bearing *B* using the actual value for *K*.

Eq. (11-15): $$F_{iB}=\frac{0.47F_{rB}}{K_B}=\frac{0.47(1095)}{1.95}=263.9\text{ lbf}$$

Find the equivalent radial load for bearing *A* from Eq. (11-16), which still applies.

Eq. (11-16*a*): $$F_{eA}=0.4F_{rA}+K_A(F_{iB}+F_{ae})$$
$$F_{eA}=0.4(560)+1.5(263.9+200)=920\text{ lbf}$$

$$F_{eA} > F_{rA}$$

Eq. (11-7): $$F_{RA} = 1.4(920)\left(\frac{10.67}{4.48(1-0.949)^{1/1.5}}\right)^{3/10} = 3030 \text{ lbf}$$

Tentatively select cone M86643, cup M86610, with 1 in bore, and rated at 3250 lbf with $K = 1.07$. Iterating with the new value for K, we get $F_{eA} = 702$ lbf and $F_{rA} = 2312$ lbf. *Ans.*

By using a bearing with a lower K, the rated load decreased significantly, providing a higher than requested reliability. Further examination with different combinations of bearing choices could yield additional acceptable solutions.

11-41 The thrust load on shaft *CD* is from the axial component of the force transmitted through the bevel gear, and is directed toward bearing *C*. By observation of Fig. 11-14, direct mounted bearings would allow bearing *C* to carry the thrust load. *Ans.*

From the solution to Prob. 3-74, the axial thrust load is $F_{ae} = 362.8$ lbf, and the bearing radial forces are $F_{Cx} = 287.2$ lbf, $F_{Cz} = 500.9$ lbf, $F_{Dx} = 194.4$ lbf, and $F_{Dz} = 307.1$ lbf. Thus, the radial forces are

$$F_{rC} = \sqrt{287.2^2 + 500.9^2} = 577 \text{ lbf}$$
$$F_{rD} = \sqrt{194.4^2 + 307.1^2} = 363 \text{ lbf}$$

The induced loads are

Eq. (11-15): $$F_{iC} = \frac{0.47F_{rC}}{K_C} = \frac{0.47(577)}{1.5} = 181 \text{ lbf}$$

Eq. (11-15): $$F_{iD} = \frac{0.47F_{rD}}{K_D} = \frac{0.47(363)}{1.5} = 114 \text{ lbf}$$

Check the condition on whether to apply Eq. (11-16) or Eq. (11-17), where bearings *C* and *D* are substituted, respectively, for labels *A* and *B* in the equations.

$$F_{iC} \le ? \ge F_{iD} + F_{ae}$$
$$181 \text{ lbf} < 114 + 362.8 = 476.8 \text{ lbf, so Eq.(11-16) applies}$$

Eq. (11-16*a*): $$F_{eC} = 0.4F_{rC} + K_C\left(F_{iD} + F_{ae}\right)$$
$$= 0.4(577) + 1.5(114 + 362.8) = 946 \text{ lbf} > F_{rC,} \text{ so use } F_{eC}$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.

$$x_D = \frac{L_D}{L_R} = \frac{10^8}{90\left(10^6\right)} = 1.11$$

$$R = \sqrt{0.90} = 0.949$$

Eq. (11-7): $$F_{RC} = 1(946)\left(\frac{1.11}{4.48(1-0.949)^{1/1.5}}\right)^{3/10} = 1130 \text{ lbf} \quad \textit{Ans.}$$

Eq. (11-16*b*): $$F_{eD} = F_{rD} = 363 \text{ lbf}$$

Eq. (11-7): $$F_{RD} = 1(363)\left(\frac{1.11}{4.48(1-0.949)^{1/1.5}}\right)^{3/10} = 433 \text{ lbf} \quad \textit{Ans.}$$

11-42 The thrust load on shaft *AB* is from the axial component of the force transmitted through the bevel gear, and is directed to the right. By observation of Fig. 11-14, indirect mounted bearings would allow bearing *A* to carry the thrust load. *Ans.*

From the solution to Prob. 3-76, the axial thrust load is F_{ae} = 92.8 lbf, and the bearing radial forces are F_{Ay} = 639.4 lbf, F_{Az} = 1513.7 lbf, F_{By} = 276.6 lbf, and F_{Bz} = 705.7 lbf. Thus, the radial forces are

$$F_{rA} = \sqrt{639.4^2 + 1513.7^2} = 1643 \text{ lbf}$$

$$F_{rB} = \sqrt{276.6^2 + 705.7^2} = 758 \text{ lbf}$$

The induced loads are

Eq. (11-15): $$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(1643)}{1.5} = 515 \text{ lbf}$$

Eq. (11-15): $$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(758)}{1.5} = 238 \text{ lbf}$$

Check the condition on whether to apply Eq. (11-16) or Eq. (11-17).

$$F_{iA} \le ? \ge F_{iB} + F_{ae}$$

$$515 \text{ lbf} > 238 + 92.8 = 330.8 \text{ lbf, so Eq.(11-17) applies}$$

Notice that the induced load from bearing *A* is sufficiently large to cause a net axial force to the left, which must be supported by bearing *B*.

Eq. (11-17*a*): $$F_{eB} = 0.4F_{rB} + K_B\left(F_{iA} - F_{ae}\right)$$

$$= 0.4(758) + 1.5(515 - 92.8) = 937 \text{ lbf} > F_{rB,} \text{ so use } F_{eB}$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.

$$x_D = \frac{L_D}{L_R} = \frac{500\left(10^6\right)}{90\left(10^6\right)} = 5.56$$

$$R = \sqrt{0.90} = 0.949$$

Eq. (11-7): $$F_{RB} = 1(937)\left(\frac{5.56}{4.48(1-0.949)^{1/1.5}}\right)^{3/10} = 1810 \text{ lbf} \quad \textit{Ans.}$$

Eq. (11-16*b*): $$F_{eA} = F_{rA} = 1643 \text{ lbf}$$

Eq. (11-7): $$F_{RA} = 1(1643)\left(\frac{5.56}{4.48(1-0.949)^{1/1.5}}\right)^{3/10} = 3180 \text{ lbf} \quad \textit{Ans.}$$

11-43 The lower bearing is compressed by the axial load, so it is designated as bearing *A*.

$$F_{rA} = 25 \text{ kN}$$
$$F_{rB} = 12 \text{ kN}$$
$$F_{ae} = 5 \text{ kN}$$

Eq. (11-15): $$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(25)}{1.5} = 7.83 \text{ kN}$$

Eq. (11-15): $$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(12)}{1.5} = 3.76 \text{ kN}$$

Check the condition on whether to apply Eq. (11-16) or Eq. (11-17)

$$F_{iA} \le ? \ge F_{iB} + F_{ae}$$
$$7.83 \text{ kN} < 3.76 + 5 = 8.76 \text{ kN, so Eq.(11-16) applies}$$

Eq. (11-16*a*): $$F_{eA} = 0.4F_{rA} + K_A\left(F_{iB} + F_{ae}\right)$$
$$= 0.4(25) + 1.5(3.76 + 5) = 23.1 \text{ kN} < F_{rA,} \text{ so use } F_{rA}$$
$$L_D = (250 \text{ rev/min})\left(\frac{60 \text{ min}}{\text{hr}}\right)\left(\frac{8 \text{ hr}}{\text{day}}\right)\left(\frac{5 \text{ day}}{\text{week}}\right)\left(\frac{52 \text{ weeks}}{\text{yr}}\right)(5 \text{ yrs})$$
$$= 156\left(10^6\right) \text{ rev}$$

Assume for tapered roller bearings that the specifications for Manufacturer 1 on p. 608 are applicable.

Eq. (11-3): $$F_{RA} = a_f F_D\left[\frac{L_D}{L_R}\right]^{3/10} = 1.2(25)\left[\frac{156\left(10^6\right)}{90\left(10^6\right)}\right]^{3/10} = 35.4 \text{ kN} \quad \textit{Ans.}$$

Eq. (11-16*b*): $$F_{eB} = F_{rB} = 12 \text{ kN}$$

Eq. (11-3): $$F_{RB} = 1.2(12)\left[\frac{156}{90}\right]^{3/10} = 17.0 \text{ kN} \quad \textit{Ans.}$$

11-44 The left bearing is compressed by the axial load, so it is properly designated as bearing *A*.

$$F_{rA} = 875 \text{ lbf}$$
$$F_{rB} = 625 \text{ lbf}$$
$$F_{ae} = 250 \text{ lbf}$$

Assume $K = 1.5$ for each bearing for the first iteration. Obtain the induced loads.

Eq. (11-15): $$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(875)}{1.5} = 274 \text{ lbf}$$

Eq. (11-15): $$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(625)}{1.5} = 196 \text{ lbf}$$

Check the condition on whether to apply Eq. (11-16) or Eq. (11-17).

$$F_{iA} \le ? \ge F_{iB} + F_{ae}$$
$$274 \text{ lbf} < 196 + 250 \text{ lbf, so Eq.(11-16) applies}$$

We will size bearing *B* first since its induced load will affect bearing *A*, but it is not affected by the induced load from bearing *A* [see Eq. (11-16)].

From Eq. (11-16*b*), $F_{eB} = F_{rB} = 625$ lbf.

Eq. (11-3): $$F_{RB} = a_f F_D \left[\frac{L_D}{L_R}\right]^{3/10} = 1(625)\left[\frac{90\,000(150)(60)}{90(10^6)}\right]^{3/10}$$
$$F_{RB} = 1208 \text{ lbf}$$

Select cone 07100, cup 07196, with 1 in bore, and rated at 1570 lbf with $K = 1.45$. *Ans.*

With bearing *B* selected, we re-evaluate the induced load from bearing *B* using the actual value for *K*.

Eq. (11-15): $$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(625)}{1.45} = 203 \text{ lbf}$$

Find the equivalent radial load for bearing *A* from Eq. (11-16), which still applies.

Eq. (11-16*a*): $F_{eA} = 0.4F_{rA} + K_A\left(F_{iB} + F_{ae}\right)$

$= 0.4\left(875\right) + 1.5\left(203 + 250\right) = 1030 \text{ lbf}$

$F_{eA} > F_{rA}$

Eq. (11-3): $F_{RA} = a_f F_D \left[\frac{L_D}{L_R}\right]^{3/10} = 1\left(1030\right)\left[\frac{90\,000\left(150\right)\left(60\right)}{90\left(10^6\right)}\right]^{3/10}$

$F_{RA} = 1990 \text{ lbf}$

Any of the bearings with 1-1/8 in bore are more than adequate. Select cone 15590, cup 15520, rated at 2480 lbf with K = 1.69. Iterating with the new value for K, we get F_{eA} = 1120 lbf and F_{rA} = 2160 lbf. The selected bearing is still adequate. *Ans.*

Chapter 12

12-1 Given: $d_{max} = 25$ mm, $b_{min} = 25.03$ mm, $l/d = 1/2$, $W = 1.2$ kN, $\mu = 55$ mPa·s, and $N = 1100$ rev/min.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{25.03 - 25}{2} = 0.015 \text{ mm}$$

$r \doteq 25/2 = 12.5$ mm

$r/c = 12.5/0.015 = 833.3$

$N = 1100/60 = 18.33$ rev/s

$P = W/(ld) = 1200/[12.5(25)] = 3.84$ N/mm^2 = 3.84 MPa

Eq. (12-7): $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 833.3^2 \left[\frac{55\left(10^{-3}\right)18.33}{3.84\left(10^{6}\right)}\right] = 0.182$$

Fig. 12-16: $h_0/c = 0.3 \quad \Rightarrow \quad h_0 = 0.3(0.015) = 0.0045$ mm *Ans.*

Fig. 12-18: $f r/c = 5.4 \quad \Rightarrow \quad f = 5.4/833.3 = 0.006\,48$

$T = f Wr = 0.006\,48(1200)12.5(10^{-3}) = 0.0972$ N·m

$H_{loss} = 2\pi TN = 2\pi(0.0972)18.33 = 11.2$ W *Ans.*

Fig. 12-19: $Q/(rcNl) = 5.1 \quad \Rightarrow \quad Q = 5.1(12.5)0.015(18.33)12.5 = 219$ mm^3/s

Fig. 12-20: $Q_s/Q = 0.81 \quad \Rightarrow \quad Q_s = 0.81(219) = 177$ mm^3/s *Ans.*

12-2 Given: $d_{max} = 32$ mm, $b_{min} = 32.05$ mm, $l = 64$ mm, $W = 1.75$ kN, $\mu = 55$ mPa·s, and $N = 900$ rev/min.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{32.05 - 32}{2} = 0.025 \text{ mm}$$

$r \doteq 32/2 = 16$ mm

$r/c = 16/0.025 = 640$

$N = 900/60 = 15$ rev/s

$P = W/(ld) = 1750/[32(64)] = 0.854$ MPa
$l/d = 64/32 = 2$

Eq. (12-7): $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 640^2 \left[\frac{55\left(10^{-3}\right)15}{0.854}\right] = 0.797$$

Eq. (12-16), Figs. 12-16, 12-19, and 12-21

	l/d	y_∞	y_1	$y_{1/2}$	$y_{1/4}$	$y_{l/d}$
h_0/c	2	0.98	0.83	0.61	0.36	0.92
P/p_{max}	2	0.84	0.54	0.45	0.31	0.65
$Q/rcNl$	2	3.1	3.45	4.2	5.08	3.20

$h_0 = 0.92\,c = 0.92(0.025) = 0.023$ mm *Ans.*

$p_{max} = P / 0.065 = 0.854/0.65 = 1.31$ MPa *Ans.*

$Q = 3.20\,rcNl = 3.20(16)0.025(15)64 = 1.23\,(10^3)$ mm^3/s *Ans.*

12-3 Given: $d_{max} = 3.000$ in, $b_{min} = 3.005$ in, $l = 1.5$ in, $W = 800$ lbf, $N = 600$ rev/min, and SAE 10 and SAE 40 at 150°F.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{3.005 - 3.000}{2} = 0.0025 \text{ in}$$
$r \doteq 3.000 / 2 = 1.500$ in
$l / d = 1.5 / 3 = 0.5$
$r / c = 1.5 / 0.0025 = 600$
$N = 600 / 60 = 10$ rev/s
$$P = \frac{W}{ld} = \frac{800}{1.5(3)} = 177.78 \text{ psi}$$

Fig. 12-12: SAE 10 at 150°F, $\mu' = 1.75\,\mu$reyn

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 600^2 \left[\frac{1.75(10^{-6})(10)}{177.78}\right] = 0.0354$$

Figs. 12-16 and 12-21: $h_0/c = 0.11$ and $P/p_{max} = 0.21$
$h_0 = 0.11(0.0025) = 0.000\,275$ in *Ans.*
$p_{max} = 177.78 / 0.21 = 847$ psi *Ans.*

Fig. 12-12: SAE 40 at 150°F, $\mu' = 4.5\,\mu$reyn

$$S = 0.0354\left(\frac{4.5}{1.75}\right) = 0.0910$$

$h_0 / c = 0.19, \quad P / p_{\max} = 0.275$

$h_0 = 0.19(0.0025) = 0.000\,475$ in *Ans.*

$p_{\max} = 177.78 / 0.275 = 646$ psi *Ans.*

12-4 Given: $d_{\max} = 3.250$ in, $b_{\min} = 3.256$ in, $l = 3.25$ in, $W = 800$ lbf, and $N = 1000$ rev/min.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{3.256 - 3.250}{2} = 0.003$$

$r \doteq 3.250 / 2 = 1.625$ in

$l / d = 3 / 3.250 = 0.923$

$r / c = 1.625 / 0.003 = 542$

$N = 1000 / 60 = 16.67$ rev/s

$$P = \frac{W}{ld} = \frac{800}{3(3.25)} = 82.05 \text{ psi}$$

Fig. 12-14: SAE 20W at 150°F, $\mu' = 2.85\ \mu$ reyn

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 542^2\left[\frac{2.85(10^{-6})(16.67)}{82.05}\right] = 0.1701$$

From Eq. (12-16), and Figs. 12-16 and 12-21:

	l/d	y_∞	y_1	$y_{1/2}$	$y_{1/4}$	$y_{l/d}$
h_o/c	0.923	0.85	0.48	0.28	0.15	0.46
$P/p_{\max}$	0.923	0.83	0.45	0.32	0.22	0.43

$h_o = 0.46c = 0.46(0.003) = 0.001\,38$ in *Ans.*

$$p_{\max} = \frac{P}{0.43} = \frac{82.05}{0.43} = 191 \text{ psi} \quad \textit{Ans.}$$

Fig. 12-14: SAE 20W-40 at 150°F, $\mu' = 4.4\ \mu$ reyn

$$S = 542^2\,\frac{4.4(10^{-6})(16.67)}{82.05} = 0.263$$

From Eq. (12-16), and Figs. 12-16 and 12-21:

	l/d	y_∞	y_1	$y_{1/2}$	$y_{1/4}$	$y_{l/d}$
h_o/c	0.923	0.91	0.6	0.38	0.2	0.58
$P/p_{\max}$	0.923	0.83	0.48	0.35	0.24	0.46

$$h_0 = 0.58c = 0.58(0.003) = 0.001\,74 \text{ in} \quad \textit{Ans.}$$

$$p_{\max} = \frac{8205}{0.46} = \frac{82.05}{0.46} = 178 \text{ psi} \quad \textit{Ans.}$$

12-5 Given: $d_{\max} = 2.000$ in, $b_{\min} = 2.0024$ in, $l = 1$ in, $W = 600$ lbf, $N = 800$ rev/min, and SAE 20 at 130°F.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{2.0024 - 2}{2} = 0.0012 \text{ in}$$

$$r \doteq \frac{d}{2} = \frac{2}{2} = 1 \text{ in}, \quad l / d = 1 / 2 = 0.50$$

$$r / c = 1 / 0.0012 = 833$$

$$N = 800 / 60 = 13.33 \text{ rev/s}$$

$$P = \frac{W}{ld} = \frac{600}{2(1)} = 300 \text{ psi}$$

Fig. 12-12: SAE 20 at 130°F, $\mu' = 3.75\,\mu$reyn

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 833^2 \left[\frac{3.75(10^{-6})(13.3)}{300}\right] = 0.115$$

From Figs. 12-16, 12-18 and 12-19:

$$h_0 / c = 0.23, \quad r f / c = 3.8, \quad Q / (rcNl) = 5.3$$

$$h_0 = 0.23(0.0012) = 0.000\,276 \text{ in} \quad \textit{Ans.}$$

$$f = \frac{3.8}{833} = 0.004\,56$$

The power loss due to friction is

$$H = \frac{2\pi f\, WrN}{778(12)} = \frac{2\pi(0.004\,56)(600)(1)(13.33)}{778(12)}$$

$$= 0.0245 \text{ Btu/s} \quad \textit{Ans.}$$

$$Q = 5.3rcNl$$

$$= 5.3(1)(0.0012)(13.33)(1)$$

$$= 0.0848 \text{ in}^3/\text{s} \quad \textit{Ans.}$$

12-6 Given: $d_{\max} = 25$ mm, $b_{\min} = 25.04$ mm, $l/d = 1$, $W = 1.25$ kN, $\mu = 50$ mPa·s, and $N = 1200$ rev/min.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{25.04 - 25}{2} = 0.02 \text{ mm}$$

$$r \doteq d / 2 = 25 / 2 = 12.5 \text{ mm}, \quad l / d = 1$$

$$r / c = 12.5 / 0.02 = 625$$

$$N = 1200 / 60 = 20 \text{ rev/s}$$

$$P = \frac{W}{ld} = \frac{1250}{25^2} = 2 \text{ MPa}$$

For $\mu = 50$ MPa · s, $\quad S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 625^2 \left[\frac{50(10^{-3})(20)}{2(10^6)}\right] = 0.195$

From Figs. 12-16, 12-18 and 12-20:

$$h_0 / c = 0.52, \quad f\, r / c = 4.5, \quad Q_s / Q = 0.57$$

$$h_0 = 0.52(0.02) = 0.0104 \text{ mm} \quad \textit{Ans.}$$

$$f = \frac{4.5}{625} = 0.0072$$

$$T = f\, Wr = 0.0072(1.25)(12.5) = 0.1125 \text{ N} \cdot \text{m}$$

The power loss due to friction is

$$H = 2\pi T N = 2\pi\,(0.1125)(20) = 14.14 \text{ W} \quad \textit{Ans.}$$

$Q_s = 0.57Q$ The side flow is 57% of Q *Ans.*

12-7 Given: $d_{\max} = 1.25$ in, $b_{\min} = 1.252$ in, $l = 2$ in, $W = 620$ lbf, $\mu' = 8.5\ \mu$ reyn, and $N = 1120$ rev/min.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{1.252 - 1.25}{2} = 0.001 \text{ in}$$

$$r = d / 2 = 1.25 / 2 = 0.625 \text{ in}$$

$$r / c = 0.625 / 0.001 = 625$$

$$N = 1120 / 60 = 18.67 \text{ rev/s}$$

$$P = \frac{W}{ld} = \frac{620}{1.25(2)} = 248 \text{ psi}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 625^2 \left[\frac{8.5(10^{-6})(18.67)}{248}\right] = 0.250$$

$$l / d = 2 / 1.25 = 1.6$$

From Eq. (12-16), and Figs. 12-16, 12-18, and 12-19

	l/d	y_∞	y_1	$y_{1/2}$	$y_{1/4}$	$y_{l/d}$
h_0/c	1.6	0.9	0.58	0.36	0.185	0.69
fr/c	1.6	4.5	5.3	6.5	8	4.92
$Q/rcNl$	1.6	3	3.98	4.97	5.6	3.59

$h_0 = 0.69\,c = 0.69(0.001) = 0.000\,69$ in *Ans.*

$f = 4.92/(r/c) = 4.92/625 = 0.007\,87$ *Ans.*

$Q = 1.6\,rcNl = 1.6(0.625)\,0.001(18.57)\,2 = 0.0833$ in^3/s *Ans.*

12-8 Given: $d_{max} = 75.00$ mm, $b_{min} = 75.10$ mm, $l = 36$ mm, $W = 2$ kN, $N = 720$ rev/min, and SAE 20 and SAE 40 at 60°C.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{75.10 - 75}{2} = 0.05 \text{ mm}$$
$$l\,/\,d = 36\,/\,75 = 0.48 \doteq 0.5 \quad \text{(close enough)}$$
$$r = d\,/\,2 = 75\,/\,2 = 37.5 \text{ mm}$$
$$r\,/\,c = 37.5\,/\,0.05 = 750$$
$$N = 720\,/\,60 = 12 \text{ rev/s}$$
$$P = \frac{W}{ld} = \frac{2000}{75(36)} = 0.741 \text{ MPa}$$

Fig. 12-13: SAE 20 at 60°C, $\mu = 18.5$ MPa · s

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 750^2 \left[\frac{18.5(10^{-3})(12)}{0.741(10^6)}\right] = 0.169$$

From Figures 12-16, 12-18 and 12-21:

$$h_0\,/\,c = 0.29, \quad f\,r\,/\,c = 5.1, \quad P\,/\,p_{max} = 0.315$$
$$h_0 = 0.29(0.05) = 0.0145 \text{ mm} \quad \textit{Ans.}$$
$$f = 5.1\,/\,750 = 0.0068$$
$$T = f\,Wr = 0.0068(2)(37.5) = 0.51 \text{ N} \cdot \text{m}$$

The heat loss rate equals the rate of work on the film

$$H_{loss} = 2\pi T\,N = 2\pi(0.51)(12) = 38.5 \text{ W} \quad \textit{Ans.}$$
$$p_{max} = 0.741/0.315 = 2.35 \text{ MPa} \quad \textit{Ans.}$$

Fig. 12-13: SAE 40 at 60°C, $\mu = 37$ MPa · s

$S = 0.169(37)/18.5 = 0.338$

From Figures 12-16, 12-18 and 12-21:

$h_0 / c = 0.42, \quad f r / c = 8.5, \quad P / p_{max} = 0.38$
$h_0 = 0.42(0.05) = 0.021 \text{ mm} \quad Ans.$
$f = 8.5 / 750 = 0.0113$
$T = f Wr = 0.0113(2)(37.5) = 0.85 \text{ N} \cdot \text{m}$
$H_{loss} = 2\pi TN = 2\pi(0.85)(12) = 64 \text{ W} \quad Ans.$
$p_{max} = 0.741 / 0.38 = 1.95 \text{ MPa} \quad Ans.$

12-9 Given: $d_{max} = 56.00$ mm, $b_{min} = 56.05$ mm, $l = 28$ mm, $W = 2.4$ kN, $N = 900$ rev/min, and SAE 40 at 65°C.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{56.05 - 56}{2} = 0.025 \text{ mm}$$
$r = d / 2 = 56 / 2 = 28 \text{ mm}$
$r / c = 28 / 0.025 = 1120$
$l / d = 28 / 56 = 0.5, \quad N = 900 / 60 = 15 \text{ rev/s}$
$$P = \frac{2400}{28(56)} = 1.53 \text{ MPa}$$

Fig. 12-13: SAE 40 at 65°C, $\mu = 30 \text{ MPa} \cdot \text{s}$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 1120^2 \left[\frac{30(10^{-3})(15)}{1.53(10^6)}\right] = 0.369$$

From Figures 12-16, 12-18, 12-19 and 12-20:

$h_0 / c = 0.44, \quad f r / c = 8.5, \quad Q_s / Q = 0.71, \quad Q / (rcNl) = 4.85$
$h_0 = 0.44(0.025) = 0.011 \text{ mm} \quad Ans.$
$f = 8.5 / 1000 = 0.007\,59$
$T = f Wr = 0.007\,59(2.4)(28) = 0.51 \text{ N} \cdot \text{m}$
$H = 2\pi TN = 2\pi(0.51)(15) = 48.1 \text{ W} \quad Ans.$
$Q = 4.85 rcNl = 4.85(28)(0.025)(15)(28) = 1426 \text{ mm}^3/\text{s}$
$Q_s = 0.71(1426) = 1012 \text{ mm}^3/\text{s} \quad Ans.$

12-10 Consider the bearings as specified by

minimum f:	$d^{+0}_{-t_d}$,	$b^{+t_b}_{-0}$
maximum W:	$d^{+0}_{-t_d}$,	$b^{+t_b}_{-0}$

and differing only in d and d'.

Preliminaries:

$$l / d = 1$$
$$P = W / (ld) = 700 / (1.25^2) = 448 \text{ psi}$$
$$N = 3600 / 60 = 60 \text{ rev/s}$$

Fig. 12-16:

minimum f: S ≐ 0.08

maximum W: S ≐ 0.20

Fig. 12-12: $\mu = 1.38(10^{-6})$ reyn

$$\mu N/P = 1.38(10^{-6})(60/448) = 0.185(10^{-6})$$

Eq. (12-7):

$$\frac{r}{c} = \sqrt{\frac{S}{\mu N / P}}$$

For minimum f:

$$\frac{r}{c} = \sqrt{\frac{0.08}{0.185(10^{-6})}} = 658$$
$$c = 0.625 / 658 = 0.000\,950 \doteq 0.001 \text{ in}$$

If this is $c_{\min}$,

$$b - d = 2(0.001) = 0.002 \text{ in}$$

The median clearance is

$$\overline{c} = c_{\min} + \frac{t_d + t_b}{2} = 0.001 + \frac{t_d + t_b}{2}$$

and the clearance range for *this* bearing is

$$\Delta c = \frac{t_d + t_b}{2}$$

which is a function only of the tolerances.

For maximum W:

$$\frac{r}{c} = \sqrt{\frac{0.2}{0.185(10^{-6})}} = 1040$$
$$c = 0.625 / 1040 = 0.000\,600 \doteq 0.0005 \text{ in}$$

If this is $c_{\min}$

$$b - d' = 2c_{\min} = 2(0.0005) = 0.001 \text{ in}$$
$$\bar{c} = c_{\min} + \frac{t_d + t_b}{2} = 0.0005 + \frac{t_d + t_b}{2}$$
$$\Delta c = \frac{t_d + t_b}{2}$$

The difference (mean) in clearance between the *two* clearance ranges, c_{range}, is

$$c_{\text{range}} = 0.001 + \frac{t_d + t_b}{2} - \left(0.0005 + \frac{t_d + t_b}{2}\right)$$
$$= 0.0005 \text{ in}$$

For the minimum f bearing

$$b - d = 0.002 \text{ in}$$

or

$$d = b - 0.002 \text{ in}$$

For the maximum W bearing

$$d' = b - 0.001 \text{ in}$$

For the same b, t_b and t_d, we need to change the journal diameter by 0.001 in.

$$d' - d = b - 0.001 - (b - 0.002)$$
$$= 0.001 \text{ in}$$

Increasing d of the minimum friction bearing by 0.001 in, defines d' of the maximum load bearing. Thus, the clearance range provides for bearing dimensions which are attainable in manufacturing. *Ans.*

12-11 Given: SAE 40, $N = 10$ rev/s, $T_s = 140°\text{F}$, $l/d = 1$, $d = 3.000$ in, $b = 3.003$ in, $W = 675$ lbf.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{3.003 - 3}{2} = 0.0015 \text{ in}$$
$$r = d / 2 = 3 / 2 = 1.5 \text{ in}$$
$$r / c = 1.5 / 0.0015 = 1000$$
$$P = \frac{W}{ld} = \frac{675}{3(3)} = 75 \text{ psi}$$

Trial #1: From Figure 12-12 for $T = 160°\text{F}$, $\mu = 3.5\ \mu$ reyn,

$$\Delta T = 2(160 - 140) = 40°F$$
$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 1000^2 \left[\frac{3.5(10^{-6})(10)}{75}\right] = 0.4667$$

From Fig. 12-24,

$$\frac{9.70\Delta T}{P} = 0.349\,109 + 6.009\,40(0.4667) + 0.047\,467(0.4667)^2 = 3.16$$
$$\Delta T = 3.16\frac{P}{9.70} = 3.16\frac{75}{9.70} = 24.4°\text{F}$$

Discrepancy = 40 – 24.4 = 15.6°F

Trial #2: $T = 150°\text{F}$, $\mu = 4.5\ \mu$ reyn,

$$\Delta T = 2(150 - 140) = 20°F$$
$$S = 1000^2\left[\frac{4.5\left(10^{-6}\right)10}{75}\right] = 0.6$$

From Fig. 12-24,

$$\frac{9.70\Delta T}{P} = 0.349\,109 + 6.009\,40(0.6) + 0.047\,467(0.6)^2 = 3.97$$
$$\Delta T = 3.97\frac{P}{9.70} = 3.97\frac{75}{9.70} = 30.7°\text{F}$$

Discrepancy = 20 – 30.7 = – 10.7°F

Trial #3: $T = 154°\text{F}$, $\mu = 4\ \mu$ reyn,

$$\Delta T = 2(154 - 140) = 28°F$$
$$S = 1000^2\left[\frac{4\left(10^{-6}\right)10}{75}\right] = 0.533$$

From Fig. 12-24,

$$\frac{9.70\Delta T}{P} = 0.349\,109 + 6.009\,40(0.533) + 0.047\,467(0.533)^2 = 3.57$$
$$\Delta T = 3.57\frac{P}{9.70} = 3.57\frac{75}{9.70} = 27.6°\text{F}$$

Discrepancy = 28 – 27.6 = 0.4°F *O.K.*

$T_{av} = 140 + 28/2 = 154°\text{F}$ *Ans.*
$T_1 = T_{av} - \Delta T\ /\ 2 = 154 - (28\ /\ 2) = 140°F$
$T_2 = T_{av} + \Delta T\ /\ 2 = 154 + (28\ /\ 2) = 168°F$
$S = 0.4$

From Figures 12-16, 12-18, to 12-20:

$$\frac{h_0}{c} = 0.75, \quad \frac{f\,r}{c} = 11, \quad \frac{Q}{rcN\,l} = 3.6, \quad \frac{Q_s}{Q} = 0.33$$

$$h_0 = 0.75(0.0015) = 0.001\,13 \text{ in} \quad \textit{Ans.}$$

$$f = \frac{11}{1000} = 0.011$$

$$T = f\,Wr = 0.0075(3)(40) = 0.9 \text{ N} \cdot \text{m}$$

$$H_{\text{loss}} = \frac{2\pi f\,WrN}{778(12)} = \frac{2\pi(0.011)675(1.5)10}{778(12)} = 0.075 \text{ Btu/s} \quad \textit{Ans.}$$

$$Q = 3.6rcN\,l = 3.6(1.5)0.0015(10)3 = 0.243 \text{ in}^3\text{/s} \quad \textit{Ans.}$$

$$Q_s = 0.33(0.243) = 0.0802 \text{ in}^3\text{/s} \quad \textit{Ans.}$$

12-12 Given: $d = 2.5$ in, $b = 2.504$ in, $c_{\min} = 0.002$ in, $W = 1200$ lbf, SAE = 20, $T_s = 110°\text{F}$, $N = 1120$ rev/min, and $l = 2.5$ in.

$$P = W/(ld) = 1200/(2.5)^2 = 192 \text{ psi}, \qquad N = 1120/60 = 18.67 \text{ rev/s}$$

For a trial film temperature, let $T_f = 150°\text{F}$

Table 12-1: $\mu' = 0.0136 \exp[1271.6/(150 + 95)] = 2.441\ \mu\text{reyn}$

Eq. (12-7): $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{2.5/2}{0.002}\right)^2 \frac{2.441\left(10^{-6}\right)18.67}{192} = 0.927$$

Fig. 12-24:

$$\Delta T = \frac{192}{9.70}\left[0.349\,109 + 6.009\,40(0.0927) + 0.047\,467\left(0.0927^2\right)\right]$$
$$= 17.9°\text{F}$$

$$T_{\text{av}} = T_s + \frac{\Delta T}{2} = 110 + \frac{17.9}{2} = 119.0°\text{F}$$

$$T_f - T_{\text{av}} = 150 - 119.0 = 31.0°\text{F}$$

which is not 0.1 or less, therefore try averaging for the new trial film temperature, let

$$(T_f)_{\text{new}} = \frac{150 + 119.0}{2} = 134.5°\text{F}$$

Proceed with additional trials using a spreadsheet (table also shows the first trial)

Trial T_f	μ'	S	ΔT	T_{av}	$T_f - T_{av}$	New T_f
150.0	2.441	0.0927	17.9	119.0	31.0	134.5
134.5	3.466	0.1317	22.6	121.3	13.2	127.9
127.9	4.084	0.1551	25.4	122.7	5.2	125.3
125.3	4.369	0.1659	26.7	123.3	2.0	124.3
124.3	4.485	0.1704	27.2	123.6	0.7	124.0
124.0	4.521	0.1717	27.4	123.7	0.3	123.8
123.8	4.545	0.1726	27.5	123.7	0.1	123.8

Note that the convergence begins rapidly. There are ways to speed this, but at this point they would only add complexity.

(a) $\mu' = 4.545(10^{-6}), \quad S = 0.1726$

From Fig. 12-16: $\dfrac{h_0}{c} = 0.482, \quad h_0 = 0.482(0.002) = 0.000\,964$ in

From Fig. 12-17: $\phi = 56°$ *Ans.*

(b) $e = c - h_0 = 0.002 - 0.000\,964 = 0.001\,04$ in *Ans.*

(c) From Fig. 12-18: $\dfrac{f\,r}{c} = 4.10, \quad f = 4.10(0.002/1.25) = 0.006\,56$ *Ans.*

(d) $T = f\,W r = 0.006\,56(1200)(1.25) = 9.84$ lbf · in

$$H = \frac{2\pi T N}{778(12)} = \frac{2\pi(9.84)(1120\ /\ 60)}{778(12)} = 0.124 \text{ Btu/s} \quad \textit{Ans.}$$

(e) From Fig. 12-19: $\dfrac{Q}{rcNl} = 4.16$

$$Q = 4.16(1.25)(0.002)\left(\frac{1120}{60}\right)(2.5) = 0.485 \text{ in}^3/\text{s} \quad \textit{Ans.}$$

From Fig. 12-20: $\dfrac{Q_s}{Q} = 0.6, \quad Q_s = 0.6(0.485) = 0.291 \text{ in}^3/\text{s}$ *Ans.*

(f) From Fig. 12-21: $\dfrac{P}{p_{\max}} = 0.45, \quad p_{\max} = \dfrac{W\ /\ (ld)}{0.45} = \dfrac{1200\ /\ 2.5^2}{0.45} = 427$ psi *Ans.*

From Fig. 12-22: $\phi_{p_{\max}} = 16°$ *Ans.*

(g) From Fig. 12-22: $\phi_{p_0} = 82°$ *Ans.*

(h) From the trial table, $T_f = 123.8°\text{F}$ *Ans.*

(i) With $\Delta T = 27.5°\text{F}$ from the trial table, $T_s + \Delta T = 110 + 27.5 = 137.5°\text{F}$ *Ans.*

12-13 Given: $d = 1.250$ in, $t_d = 0.001$ in, $b = 1.252$ in, $t_b = 0.003$ in, $l = 1.25$ in, $W = 250$ lbf, $N = 1750$ rev/min, SAE 10 lubricant, sump temperature $T_s = 120°\text{F}$.

$$P = W/(ld) = 250/1.25^2 = 160 \text{ psi}, \quad N = 1750/60 = 29.17 \text{ rev/s}$$

For the clearance, $c = 0.002 \pm 0.001$ in. Thus, $c_{\min} = 0.001$ in, $c_{\text{median}} = 0.002$ in, and $c_{\max} = 0.003$ in.

For $c_{\min} = 0.001$ in, start with a trial film temperature of $T_f = 135°\text{F}$

Table 12-1: $\mu' = 0.0158 \exp[1157.5/(135 + 95)] = 2.423$ μreyn

Eq. (12-7): $$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1.25/2}{0.001}\right)^2 \frac{2.423\left(10^{-6}\right)29.17}{160} = 0.1725$$

Fig. 12-24:

$$\Delta T = \frac{160}{9.70}\left[0.349\,109 + 6.009\,40\left(0.1725\right) + 0.047\,467\left(0.1725^2\right)\right]$$
$$= 22.9°\text{F}$$

$$T_{\text{av}} = T_s + \frac{\Delta T}{2} = 120 + \frac{22.9}{2} = 131.4°\text{F}$$
$$T_f - T_{\text{av}} = 135 - 131.4 = 3.6°\text{F}$$

which is not 0.1 or less, therefore try averaging for the new trial film temperature, let

$$(T_f)_{\text{new}} = \frac{135 + 131.4}{2} = 133.2°\text{F}$$

Proceed with additional trials using a spreadsheet (table also shows the first trial)

Trial T_f	μ'	S	ΔT	T_{av}	$T_f - T_{\text{av}}$	New T_f
135.0	2.423	0.1725	22.9	131.4	3.6	133.2
133.2	2.521	0.1795	23.6	131.8	1.4	132.5
132.5	2.560	0.1823	23.9	131.9	0.6	132.2
132.2	2.578	0.1836	24.0	132.0	0.2	132.1
132.1	2.583	0.1840	24.0	132.0	0.1	132.1

With T_f = 132.1°F, ΔT = 24.0°F, μ' = 2.583 μreyn, S = 0.1840,

$$T_{max} = T_s + \Delta T = 120 + 24.0 = 144.0°F$$

Fig. 12-16: $h_0/c = 0.50$, $h_0 = 0.50(0.001) = 0.000\ 50$ in

$$\epsilon = 1 - h_0/c = 1 - 0.50 = 0.05 \text{ in}$$

Fig. 12-18: $rf/c = 4.25$, $f = 4.25/(0.625/0.001) = 0.006\ 8$

Fig. 12-19: $Q/(rcNl) = 4.13$, $Q = 4.13(0.625)0.001(29.17)1.25 = 0.0941$ in^3/s

Fig. 12-20: $Q_s/Q = 0.58$, $Q_s = 0.58(0.0941) = 0.0546$ in^3/s

The above can be repeated for c_{median} = 0.002 in, and c_{max} = 0.003 in. The results are shown below.

	c_{min} 0.001 in	c_{median} 0.002 in	c_{max} 0.003 in
T_f	132.1	125.6	124.1
μ'	2.583	3.002	3.112
S	0.184	0.0534	0.0246
ΔT	24.0	11.1	8.2
T_{max}	144.0	131.1	128.2
h_0/c	0.5	0.23	0.125
h_0	0.00050	0.00069	0.00038
ϵ	0.50	0.77	0.88
fr/c	4.25	1.8	1.22
f	0.0068	0.0058	0.0059
$Q/(rcNl)$	4.13	4.55	4.7
Q	0.0941	0.207	0.321
Q_s/Q	0.58	0.82	0.90
Q_s	0.0546	0.170	0.289

12-14 Computer programs will vary.

12-15 **Note to the Instructor:** In the first printing of the 9th edition, the *l/d* ratio and the lubrication constant α were omitted. The values to use are $l/d = 1$, and $\alpha = 1$. This will be updated in the next printing. We apologize for any inconvenience this may have caused.

In a step-by-step fashion, we are building a skill for natural circulation bearings.
- Given the average film temperature, establish the bearing properties.
- Given a sump temperature, find the average film temperature, then establish the bearing properties.
- Now we acknowledge the environmental temperature's role in establishing the sump temperature. Sec. 12-9 and Ex. 12-5 address this problem.

Given: d_{max} = 2.500 in, b_{min} = 2.504 in, l/d = 1, N = 1120 rev/min, SAE 20 lubricant, W = 300 lbf, A = 60 in^2, T_∞ = 70°F, and α = 1.

600 lbf load with minimal clearance: We will start by using W = 600 lbf (n_d = 2). The task is to iteratively find the average film temperature, T_f, which makes H_{gen} and H_{loss} equal.

$$c_{min} = \frac{b_{min} - d_{max}}{2} = \frac{2.504 - 2.500}{2} = 0.002 \text{ in}$$

$$N = 1120/60 = 18.67 \text{ rev/s}$$

$$P = \frac{W}{ld} = \frac{600}{2.5^2} = 96 \text{ psi}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1.25}{0.002}\right)^2 \frac{\mu'(10^{-6})18.67}{96} = 0.0760\mu'$$

Table 12-1: $\mu' = 0.0136 \exp[1271.6/(T_f + 95)]$

$$H_{gen} = \frac{2545}{1050} WNc\left(\frac{f}{c}\right) = \frac{2545}{1050}(600)18.67(0.002)\frac{f}{c}$$
$$= 54.3\frac{f}{c}$$

$$H_{loss} = \frac{\hbar_{CR} A}{\alpha + 1}(T_f - T_\infty) = \frac{2.7(60/144)}{1+1}(T_f - 70)$$
$$= 0.5625(T_f - 70)$$

Start with trial values of T_f of 220 and 240°F.

Trial T_f	μ'	S	f r/c	H_{gen}	H_{loss}
220	0.770	0.059	1.9	103.2	84.4
240	0.605	0.046	1.7	92.3	95.6

As a linear approximation, let $H_{gen} = mT_f + b$. Substituting the two sets of values of T_f and H_{gen} we find that $H_{gen} = -0.545\ T_f + 223.1$. Setting this equal to H_{loss} and solving for T_f gives T_f = 237°F.

Trial T_f	μ'	S	$f\,r/c$	H_{gen}	H_{loss}
237	0.627	0.048	1.73	93.9	94.0

which is satisfactory.

Table 12-16: $h_0/c = 0.21$, $h_0 = 0.21\ (0.002) = 000\ 42$ in

Fig. 12-24:

$$\Delta T = \frac{96}{9.7}\left[0.349\,109 + 6.009\,4(0.048) + 0.047\,467(0.048^2)\right]$$

$$= 6.31°\text{F}$$

$$T_1 = T_s = T_f - \Delta T = 237 - 6.31/2 = 233.8°\text{F}$$

$$T_{\max} = T_1 + \Delta T = 233.8 + 6.31 = 240.1°\text{F}$$

Trumpler's design criteria:

$0.002 + 0.000\,04d = 0.002 + 0.000\,04(2.5) = 0.000\,30$ in $< h_0$ *O.K.*

$T_{\max} = 240.1°\text{F} < 250°\text{F}$ *O.K.*

$$\frac{W_{st}}{ld} = \frac{300}{2.5^2} = 48\ \text{psi} < 300\ \text{psi} \quad \textit{O.K.}$$

$n_d = 2$ (assessed at $W = 600$ lbf) *O.K.*

We see that the design passes Trumpler's criteria and is deemed acceptable.

For an operating load of $W = 300$ lbf, it can be shown that $T_f = 219.3°\text{F}$, $\mu' = 0.78$, $S = 0.118$, $f\,r/c = 3.09$, $H_{\text{gen}} = H_{\text{loss}} = 84$ Btu/h, $h_0 =$, $\Delta T = 10.5°\text{F}$, $T_1 = 224.6°\text{F}$, and $T_{\max} = 235.1°\text{F}$.

12-16 Given: $d = 3.500^{+0.000}_{-0.001}$ in, $b = 3.505^{+0.005}_{-0.000}$ in, SAE 30, $T_s = 120°\text{F}$, $p_s = 50$ psi, $N = 2000/60 = 33.33$ rev/s, $W = 4600$ lbf, bearing length = 2 in, groove width = 0.250 in, and $H_{\text{loss}} \le 5000$ Btu/hr.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{3.505 - 3.500}{2} = 0.0025\ \text{in}$$

$r = d/2 = 3.500/2 = 1.750$ in

$r/c = 1.750/0.0025 = 700$

$l' = (2 - 0.25)/2 = 0.875$ in

$l' / d = 0.875/3.500 = 0.25$

$$P = \frac{W}{4rl'} = \frac{4600}{4(1.750)0.875} = 751 \text{ psi}$$

Trial #1: Choose $(T_f)_1 = 150°\text{F}$. From Table 12-1,

$$\mu' = 0.0141 \exp[1360.0/(150 + 95)] = 3.63\ \mu \text{ reyn}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 700^2 \left[\frac{3.63(10^{-6})(33.33)}{751}\right] = 0.0789$$

From Figs. 12-16 and 12-18: $\epsilon = 0.9,\ fr/c = 3.6$

From Eq. (12-24),

$$\Delta T = \frac{0.0123(fr/c)SW^2}{\left(1 + 1.5\epsilon^2\right)p_s r^4}$$

$$= \frac{0.0123(3.6)0.0789\left(4600^2\right)}{\left[1 + 1.5(0.9)^2\right]50\left(1.750^4\right)} = 71.2°\text{F}$$

$T_{\text{av}} = T_s + \Delta T / 2 = 120 + 71.2/2 = 155.6°\text{F}$

Trial #2: Choose $(T_f)_2 = 160°\text{F}$. From Table 12-1

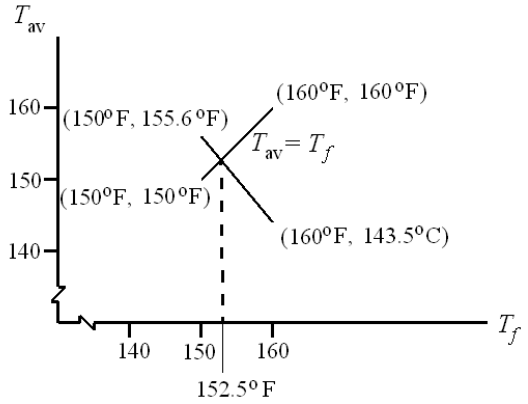
$$\mu' = 0.0141 \exp[1360.0/(160 + 95)] = 2.92\ \mu \text{ reyn}$$

$$S = 0.0789\left(\frac{2.92}{3.63}\right) = 0.0635$$

From Figs. 12-16 and 12-18: $\epsilon = 0.915,\ fr/c = 3$

$$\Delta T = \frac{0.0123(3)0.0635\left(4600^2\right)}{\left[1 + 1.5\left(0.915^2\right)\right]50\left(1.750^4\right)} = 46.9°\text{F}$$

$T_{\text{av}} = 120 + 46.9/2 = 143.5°\text{F}$



Trial #3: Thus, the plot gives $(T_f)_3 = 152.5°F$. From Table 12-1

$$\mu' = 0.0141 \exp[1360.0/(152.5 + 95)] = 3.43\ \mu \text{ reyn}$$

$$S = 0.0789\left(\frac{3.43}{3.63}\right) = 0.0746$$

From Figs. 12-16 and 12-18: $\epsilon = 0.905,\ fr/c = 3.4$

$$\Delta T = \frac{0.0123(3.4)0.0746(4600^2)}{\left[1 + 1.5(0.905^2)\right]50(1.750^4)} = 63.2°F$$

$$T_{av} = 120 + 63.2/2 = 151.6°F$$

Result is close. Choose $\bar{T}_f = \frac{152.5 + 151.6}{2} = 152.1°F$ Try 152°F

Table 12-1: $\mu' = 0.0141 \exp[1360.0/(152 + 95)] = 3.47\ \mu$ reyn

$$S = 0.0789\left(\frac{3.47}{3.63}\right) = 0.0754$$

$$\frac{fr}{c} = 3.4, \quad \epsilon = 0.902, \quad \frac{h_0}{c} = 0.098$$

$$\Delta T = \frac{0.0123(3.4)0.0754(4600^2)}{\left[1 + 1.5(0.902^2)\right]50(1.750^4)} = 64.1°F$$

$T_{av} = 120 + 64.1 / 2 = 152.1°F$ O.K.

$h_0 = 0.098(0.0025) = 0.000\ 245$ in

$T_{max} = T_s + \Delta T = 120 + 64.1 = 184.1°F$

Eq. (12-22):

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'}(1 + 1.5\epsilon^2) = \frac{\pi(50)1.750(0.0025^3)}{3(3.47)10^{-6}(0.875)}\left[1 + 1.5(0.902^2)\right]$$

$$= 1.047 \text{ in}^3/\text{s}$$

$$H_{loss} = \rho C_p Q_s \Delta T = 0.0311(0.42)1.047(64.1) = 0.877 \text{ Btu/s}$$

$$= 0.877(60^2) = 3160 \text{ Btu/h} \quad O.K.$$

Trumpler's design criteria:

$0.0002 + 0.000\ 04(3.5) = 0.000\ 34$ in $> 0.000\ 245$ *Not O.K.*

$T_{max} = 184.1°F < 250°F$ *O.K.*

$P_{st} = 751$ psi > 300 psi *Not O.K.*

$n = 1$, as done *Not O.K.*

12-17 Given: $d = 50.00^{+0.00}_{-0.05}$ mm, $b = 50.084^{+0.010}_{-0.000}$ mm, SAE 30, T_s = 55°C, p_s = 200 kPa, N = 2880/60 = 48 rev/s, W = 10 kN, bearing length = 55 mm, groove width = 5 mm, and $H_{\text{loss}} \le 300$ W.

$$c_{\min} = \frac{b_{\min} - d_{\max}}{2} = \frac{50.084 - 50}{2} = 0.042 \text{ mm}$$

$r = d/2 = 50/2 = 25$ mm

$r/c = 25/0.042 = 595$

$l' = (55 - 5)/2 = 25$ mm

$l'/d = 25/50 = 0.5$

$$P = \frac{W}{4rl'} = \frac{10(10^3)}{4(25)25} = 4 \text{ MPa}$$

Trial #1: Choose $(T_f)_1$ = 79°C. From Fig. 12-13, μ = 13 MPa · s.

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = 595^2 \left[\frac{13(10^{-3})(48)}{4(10^6)}\right] = 0.0552$$

From Figs. 12-16 and 12-18: $\epsilon = 0.85$, $fr/c = 2.3$

From Eq. (12-25),

$$\Delta T = \frac{978(10^6)}{1 + 1.5\epsilon^2} \frac{(fr/c)SW^2}{p_s r^4}$$

$$= \frac{978(10^6)}{1 + 1.5(0.85)^2} \left[\frac{2.3(0.0552)(10^2)}{200(25)^4}\right] = 76.3°\text{C}$$

$T_{\text{av}} = T_s + \Delta T/2 = 55 + 76.3/2 = 93.2°$C

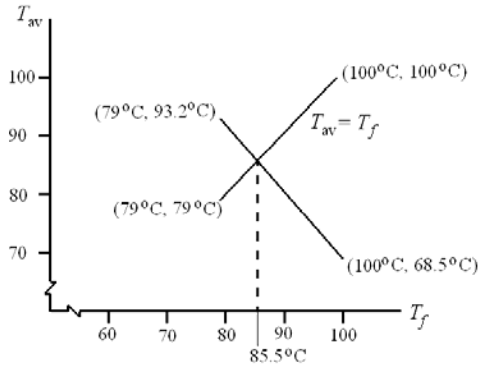
Trial #2: Choose $(T_f)_2$ = 100°C. From Fig. 12-13, μ = 7 MPa · s.

$$S = 0.0552\left(\frac{7}{13}\right) = 0.0297$$

From Figs. 12-16 and 12-18: $\epsilon = 0.90$, $fr/c = 1.6$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.9)^2} \left[\frac{1.6(0.0297)(10^2)}{200(25)^4}\right] = 26.9°\text{C}$$

$T_{\text{av}} = 55 + 26.9/2 = 68.5°$C



Trial #3: Thus, the plot gives $(T_f)_3 = 85.5°C$. From Fig. 12-13, $\mu = 10.5$ MPa · s.

$$S = 0.0552\left(\frac{10.5}{13}\right) = 0.0446$$

From Figs. 12-16 and 12-18: $\epsilon = 0.87$, $fr/c = 2.2$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.87^2)}\left[\frac{2.2(0.0457)(10^2)}{200(25)^4}\right] = 58.9°C$$

$$T_{av} = 55 + 58.9/2 = 84.5°C$$

Result is close. Choose $\overline{T}_f = \frac{85.5 + 84.5}{2} = 85°C$

Fig. 12-13: $\mu = 10.5$ MPa · s

$$S = 0.0552\left(\frac{10.5}{13}\right) = 0.0446$$

$$\epsilon = 0.87, \quad \frac{fr}{c} = 2.2, \quad \frac{h_0}{c} = 0.13$$

$$\Delta T = \frac{978(10^6)}{1 + 1.5(0.87^2)}\left[\frac{2.2(0.0457)(10^2)}{200(25^4)}\right] = 58.9°C \quad \text{or} \quad 138°F$$

$$T_{av} = 55 + 58.9 / 2 = 84.5°C \quad \text{O.K.}$$

From Eq. (12-22)

$$h_0 = 0.13(0.042) = 0.005\ 46 \text{ mm or } 0.000\ 215 \text{ in}$$

$$T_{max} = T_s + \Delta T = 55 + 58.9 = 113.9°C \quad \text{or} \quad 237°F$$

$$Q_s = (1 + 1.5\epsilon^2)\frac{\pi p_s r c^3}{3\mu l'} = \left[1 + 1.5\left(0.87^2\right)\right]\left[\frac{\pi(200)25\left(0.042^3\right)}{3(10.5)10^{-6}(25)}\right]$$

$$= 3156 \text{ mm}^3/\text{s} = 3156\left(25.4^{-3}\right) = 0.193 \text{ in}^3/\text{s}$$

$$H_{loss} = \rho C_p Q_s \Delta T = 0.0311(0.42)0.193(138) = 0.348 \text{ Btu/s}$$
$$= 1.05(0.348) = 0.365 \text{ kW} = 365 \text{ W} \quad \textit{not O.K.}$$

Trumpler's design criteria:

$0.0002 + 0.000\,04(50/25.4) = 0.000\,279$ in $> h_0$ *Not O.K.*
$T_{max} = 237°F$ *O.K.*
$P_{st} = 4000$ kPa or 581 psi > 300 psi *Not O.K.*
$n = 1$, as done *Not O.K.*

12-18 So far, we've performed elements of the design task. Now let's do it more completely.

The values of the unilateral tolerances, t_b and t_d , reflect the routine capabilities of the bushing vendor and the in-house capabilities. While the designer has to live with these, his approach should not depend on them. They can be incorporated later.

First we shall find the minimum size of the journal which satisfies Trumpler's constraint of $P_{st} \le 300$ psi.

$$P_{st} = \frac{W}{2dl'} \le 300$$

$$\frac{W}{2d^2 l'/d} \le 300 \Rightarrow d \ge \sqrt{\frac{W}{600(l'/d)}}$$

$$d_{min} = \sqrt{\frac{900}{2(300)(0.5)}} = 1.73 \text{ in}$$

In this problem we will take journal diameter as the nominal value and the bushing bore as a variable. In the next problem, we will take the bushing bore as nominal and the journal diameter as free.

To determine where the constraints are, we will set $t_b = t_d = 0$, and thereby shrink the design window to a point.

We set $d = 2.000$ in
$b = d + 2c_{min} = d + 2c$
$n_d = 2$ (This makes Trumpler's $n_d \le 2$ tight)

and construct a table.

c	b	d	$\bar{T}_f$ *	$T_{\max}$	h_o	P_{st}	$T_{\max}$	n	fom
0.0010	2.0020	2	215.50	312.0	×	√	×	√	-5.74
0.0011	2.0022	2	206.75	293.0	×	√	√	√	-6.06
0.0012	2.0024	2	198.50	277.0	×	√	√	√	-6.37
0.0013	2.0026	2	191.40	262.8	×	√	√	√	-6.66
0.0014	2.0028	2	185.23	250.4	×	√	√	√	-6.94
0.0015	2.0030	2	179.80	239.6	×	√	√	√	-7.20
0.0016	2.0032	2	175.00	230.1	×	√	√	√	-7.45
0.0017	2.0034	2	171.13	220.3	×	√	√	√	-7.65
0.0018	2.0036	2	166.92	213.9	√	√	√	√	-7.91
0.0019	2.0038	2	163.50	206.9	√	√	√	√	-8.12
0.0020	2.0040	2	160.40	200.6	√	√	√	√	-8.32

*Sample calculation for the first entry of this column.

Iteration yields: $\bar{T}_f = 215.5°\text{F}$

With $\bar{T}_f = 215.5°\text{F}$, from Table 12-1

$$\mu = 0.0136(10^{-6})\exp[1271.6 / (215.5 + 95)] = 0.817(10^{-6}) \text{ reyn}$$

$$N = 3000 / 60 = 50 \text{ rev/s}, \quad P = \frac{900}{4} = 225 \text{ psi}$$

$$S = \left(\frac{1}{0.001}\right)^2\left[\frac{0.817(10^{-6})(50)}{225}\right] = 0.182$$

From Figs. 12-16 and 12-18: $e = 0.7, \quad fr/c = 5.5$

Eq. (12–24):

$$\Delta T_F = \frac{0.0123(5.5)(0.182)(900^2)}{[1 + 1.5(0.7^2)](30)(1^4)} = 191.6°\text{F}$$

$$T_{\text{av}} = 120°\text{F} + \frac{191.6°\text{F}}{2} = 215.8°\text{F} \doteq 215.5°\text{F}$$

For the nominal 2-in bearing, the various clearances show that we have been in contact with the recurving of $(h_o)_{\min}$. The figure of merit (the parasitic friction torque plus the pumping torque negated) is best at $c = 0.0018$ in. For the nominal 2-in bearing, we will place the top of the design window at $c_{\min} = 0.002$ in, and $b = d + 2(0.002) = 2.004$ in. At this point, add the b and d unilateral tolerances:

$$d = 2.000^{+0.000}_{-0.001} \text{ in}, \quad b = 2.004^{+0.003}_{-0.000} \text{ in}$$

Now we can check the performance at $c_{\min}$, $\bar{c}$, and $c_{\max}$. Of immediate interest is the fom of the median clearance assembly, – 9.82, as compared to any other satisfactory bearing ensemble.

If a nominal 1.875 in bearing is possible, construct another table with $t_b = 0$ and $t_d = 0$.

c	b	d	$\overline{T}_f$	$T_{\max}$	h_o	P_{st}	$T_{\max}$	n	fom
0.0020	1.879	1.875	157.2	194.30	×	√	√	√	– 7.36
0.0030	1.881	1.875	138.6	157.10	√	√	√	√	– 8.64
0.0035	1.882	1.875	133.5	147.10	√	√	√	√	– 9.05
0.0040	1.883	1.875	130.0	140.10	√	√	√	√	– 9.32
0.0050	1.885	1.875	125.7	131.45	√	√	√	√	– 9.59
0.0055	1.886	1.875	124.4	128.80	√	√	√	√	– 9.63
0.0060	1.887	1.875	123.4	126.80	×	√	√	√	– 9.64

The range of clearance is $0.0030 < c < 0.0055$ in. That is enough room to fit in our design window.

$$d = 1.875^{+0.000}_{-0.001} \text{ in}, \quad b = 1.881^{+0.003}_{-0.000} \text{ in}$$

The ensemble median assembly has a fom = – 9.31.
We just had room to fit in a design window based upon the $(h_0)_{\min}$ constraint. Further reduction in nominal diameter will preclude any smaller bearings. A table constructed for a $d = 1.750$ in journal will prove this.

We choose the nominal 1.875-in bearing ensemble because it has the largest figure of merit. *Ans.*

12-19 This is the same as Prob. 12-18 but uses design variables of nominal bushing bore b and radial clearance c.

The approach is similar to that of Prob. 12-18 and the tables will change slightly. In the table for a nominal $b = 1.875$ in, note that at $c = 0.003$ in the constraints are "loose." Set

$$b = 1.875 \text{ in}$$
$$d = 1.875 - 2(0.003) = 1.869 \text{ in}$$

For the ensemble

$$b = 1.875^{+0.003}_{-0.001} \text{ in}, \quad d = 1.869^{+0.000}_{-0.001} \text{ in}$$

Analyze at $c_{\min} = 0.003$, $\overline{c} = 0.004$ in and $c_{\max} = 0.005$ in

At $c_{\min} = 0.003$ in: $\overline{T}_f = 138.4$, $\mu' = 3.160$, $S = 0.0297$, $H_{\text{loss}} = 1035$ Btu/h and the Trumpler conditions are met.

At $\overline{c} = 0.004$ in: $\overline{T}_f = 130°\text{F}$, $\mu' = 3.872$, $S = 0.0205$, $H_{\text{loss}} = 1106$ Btu/h, fom = –9.246

and the Trumpler conditions are *O.K.*
At $c_{max} = 0.005$ in: $\overline{T}_f = 125.68°\text{F}$, $\mu' = 4.325$, $S = 0.014\ 66$, $H_{loss} = 1129$ Btu/h and the Trumpler conditions are *O.K.*

The ensemble figure of merit is slightly better; this bearing is *slightly* smaller. The lubricant cooler has sufficient capacity.

12-20 Table 12-1: $\mu\,(\mu\text{reyn}) = \mu_0\,(10^6)\exp[b/(T+95)]$ b and T in °F

The conversion from μ reyn to mPa·s is given on p. 620. For a temperature of C degrees Celsius, $T = 1.8\,C + 32$. Substituting into the above equation gives

$$\mu\,(\text{mPa·s}) = 6.89\,\mu_0\,(10^6)\exp[b/(1.8\,C + 32 + 95)]$$

$$= 6.89\,\mu_0\,(10^6)\exp[b/(1.8\,C + 127)] \qquad \textit{Ans.}$$

For SAE 50 oil at 70°C, from Table 12-1, $\mu_0 = 0.0170\ (10^{-6})$ reyn, and $b = 1509.6$°F. From the equation,

$$\mu = 6.89(0.0170)\,10^{-6}(10^6)\exp\{1509.6/[1.8(70) + 127]\}$$

$$= 45.7\ \text{mPa·s} \qquad \textit{Ans.}$$

From Fig. 12-13, $\mu = 39$ mPa·s *Ans.*

The figure gives a value of about 15 % lower than the equation.

12-21 Originally

$$d = 2.000^{+0.000}_{-0.001}\ \text{in}, \quad b = 2.005^{+0.003}_{-0.000}\ \text{in}$$

Doubled,

$$d = 4.000^{+0.000}_{-0.002}\ \text{in}, \quad b = 4.010^{+0.006}_{-0.000}\ \text{in}$$

The radial load quadrupled to 3600 lbf when the analyses for parts (a) and (b) were carried out. Some of the results are:

Part	$\overline{c}$	μ'	S	T_f	fr/c	Q_s	h_0/c	e	H_{loss}	h_0	Trumpler h_0	f
(a)	0.007	3.416	0.0310	135.1	0.1612	6.56	0.1032	0.897	9898	0.000 722	0.000 360	0.005 67
(b)	0.0035	3.416	0.0310	135.1	0.1612	0.870	0.1032	0.897	1237	0.000 361	0.000 280	0.005 67

The side flow Q_s differs because there is a c^3 term and consequently an 8-fold increase. H_{loss} is related by a 9898/1237 or an 8-fold increase. The existing h_0 is related by a 2-fold increase. Trumpler's $(h_0)_{min}$ is related by a 1.286-fold increase.

12-22 Given: Oiles SP 500 alloy brass bushing, $L = 0.75$ in, $D = 0.75$ in, $T_\infty = 70°F$, $F = 400$ lbf, $N = 250$ rev/min, and $w = 0.004$ in.

Table 12-8: $K = 0.6(10^{-10})$ in^3·min/(lbf·ft·h)

$P = F/(DL) = 400/[0.75(0.75)] = 711$ psi

$V = \pi DN/12 = \pi(0.75)250/12 = 49.1$ ft/min

Tables 12-10 and 12-11: $f_1 = 1.8,\ f_2 = 1.0$

Table 12-12: $PV_{\max} = 46\,700$ psi·ft/min, $P_{\max} = 3560$ psi, $V_{\max} = 100$ ft/min

$$P_{\max} = \frac{4}{\pi}\frac{F}{DL} = \frac{4}{\pi}\frac{400}{0.75^2} = 905 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$PV = 711\ (49.1) = 34\,910$ psi·ft/min $< 46\,700$ psi·ft/min $\quad O.K.$

Eq. (12-32) can be written as

$$w = f_1 f_2 K \frac{4}{\pi}\frac{F}{DL} Vt$$

Solving for t,

$$t = \frac{\pi DLw}{4 f_1 f_2 KVF} = \frac{\pi(0.75)0.75(0.004)}{4(1.8)1.0(0.6)10^{-10}(49.1)400}$$

$$= 833.1 \text{ h} = 833.1(60) = 49\,900 \text{ min}$$

Cycles $= Nt = 250\ (49\,900) = 12.5\ (10^6)$ cycles $\quad Ans.$

12-23 Given: Oiles SP 500 alloy brass bushing, $w_{\max} = 0.002$ in for 1000 h, $N = 400$ rev/min, $F = 100$ lbf, $\hbar_{CR} = 2.7$ Btu/ (h·ft^2·°F), $T_{\max} = 300°F$, $f_s = 0.03$, and $n_d = 2$.

Estimate bushing length with $f_1 = f_2 = 1$, and $K = 0.6(10^{-10})$ in^3 · min/(lbf · ft · h)

Using Eq. (12-32) with $n_d F$ for F,

$$L = \frac{f_1 f_2 K n_d F N t}{3w} = \frac{1(1)(0.6)(10^{-10})(2)(100)(400)(1000)}{3(0.002)} = 0.80 \text{ in}$$

From Eq. (12-38), with $f_s = 0.03$ from Table 12-9 applying $n_d = 2$ to F

and $\hbar_{CR} = 2.7$ Btu/(h · ft^2 · °F)

$$L \doteq \frac{720 f_s n_d F N}{J \hbar_{CR}\left(T_f - T_\infty\right)} = \frac{720(0.03)(2)(100)(400)}{778(2.7)(300-70)} = 3.58 \text{ in}$$

$$0.80 \le L \le 3.58 \text{ in}$$

Trial 1: Let $L = 1$ in, $D = 1$ in

$$P_{\max} = \frac{4}{\pi}\frac{n_d F}{DL} = \frac{4(2)(100)}{\pi(1)(1)} = 255 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$$P = \frac{n_d F}{DL} = \frac{2(100)}{1(1)} = 200 \text{ psi}$$

$$V = \frac{\pi DN}{12} = \frac{\pi(1)(400)}{12} = 104.7 \text{ ft/min} > 100 \text{ ft/min} \quad Not\ O.K.$$

Trial 2: Try $D = 7/8$ in $= 0.875$ in, $L = 1$ in

$$P_{\max} = \frac{4(2)(100)}{\pi(0.875)(1)} = 291 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$$P = \frac{2(100)}{0.875(1)} = 229 \text{ psi}$$

$$V = \frac{\pi(0.875)(400)}{12} = 91.6 \text{ ft/min} < 100 \text{ ft/min} \quad O.K.$$

$PV = 229(91.6) = 20\,976$ psi · ft/min $< 46\,700$ psi · ft/min *O.K.*

V	f_1
33	1.3
91.6	f_1
100	1.8

$$\Rightarrow f_1 = 1.3 + (1.8 - 1.3)\left(\frac{91.6 - 33}{100 - 33}\right) = 1.74$$

$$L_{\text{new}} = f_1 L_{old} = 1.74\left(0.80\right) = 1.39 \text{ in}$$

Trial 3: Try $D = 7/8$ in $= 0.875$ in, $L = 1.5$ in

$$P_{\max} = \frac{4(2)(100)}{\pi(0.875)(1.5)} = 194 \text{ psi} < 3560 \text{ psi} \quad O.K.$$

$$P = \frac{2(100)}{0.875(1.5)} = 152 \text{ psi}, \quad V = 91.6 \text{ ft/min}$$

$PV = 152(91.6) = 13\,923$ psi · ft/min $< 46\,700$ psi · ft/min *O.K.*

$D = 7/8$ in, $L = 1.5$ in is acceptable *Ans.*

Suggestion: Try smaller sizes.

Chapter 13

13-1

$$d_P = 17/8 = 2.125 \text{ in}$$

$$d_G = \frac{N_2}{N_3} d_P = \frac{1120}{544}(2.125) = 4.375 \text{ in}$$

$$N_G = P d_G = 8(4.375) = 35 \text{ teeth} \quad \textit{Ans.}$$

$$C = (2.125 + 4.375)/2 = 3.25 \text{ in} \quad \textit{Ans.}$$

13-2

$$n_G = 1600(15/60) = 400 \text{ rev/min} \quad \textit{Ans.}$$

$$p = \pi m = 3\pi \text{ mm} \quad \textit{Ans.}$$

$$C = \left[3(15 + 60)\right]/2 = 112.5 \text{ mm} \quad \textit{Ans.}$$

13-3

$$N_G = 16(4) = 64 \text{ teeth} \quad \textit{Ans.}$$

$$d_G = N_G m = 64(6) = 384 \text{ mm} \quad \textit{Ans.}$$

$$d_P = N_P m = 16(6) = 96 \text{ mm} \quad \textit{Ans.}$$

$$C = (384 + 96)/2 = 240 \text{ mm} \quad \textit{Ans.}$$

13-4 *Mesh:*

$$a = 1/P = 1/3 = 0.3333 \text{ in} \quad \textit{Ans.}$$

$$b = 1.25/P = 1.25/3 = 0.4167 \text{ in} \quad \textit{Ans.}$$

$$c = b - a = 0.0834 \text{ in} \quad \textit{Ans.}$$

$$p = \pi/P = \pi/3 = 1.047 \text{ in} \quad \textit{Ans.}$$

$$t = p/2 = 1.047/2 = 0.523 \text{ in} \quad \textit{Ans.}$$

Pinion Base-Circle:

$$d_1 = N_1/P = 21/3 = 7 \text{ in}$$

$$d_{1b} = 7\cos 20^\circ = 6.578 \text{ in} \quad \textit{Ans.}$$

Gear Base-Circle:

$$d_2 = N_2/P = 28/3 = 9.333 \text{ in}$$

$$d_{2b} = 9.333\cos 20^\circ = 8.770 \text{ in} \quad \textit{Ans.}$$

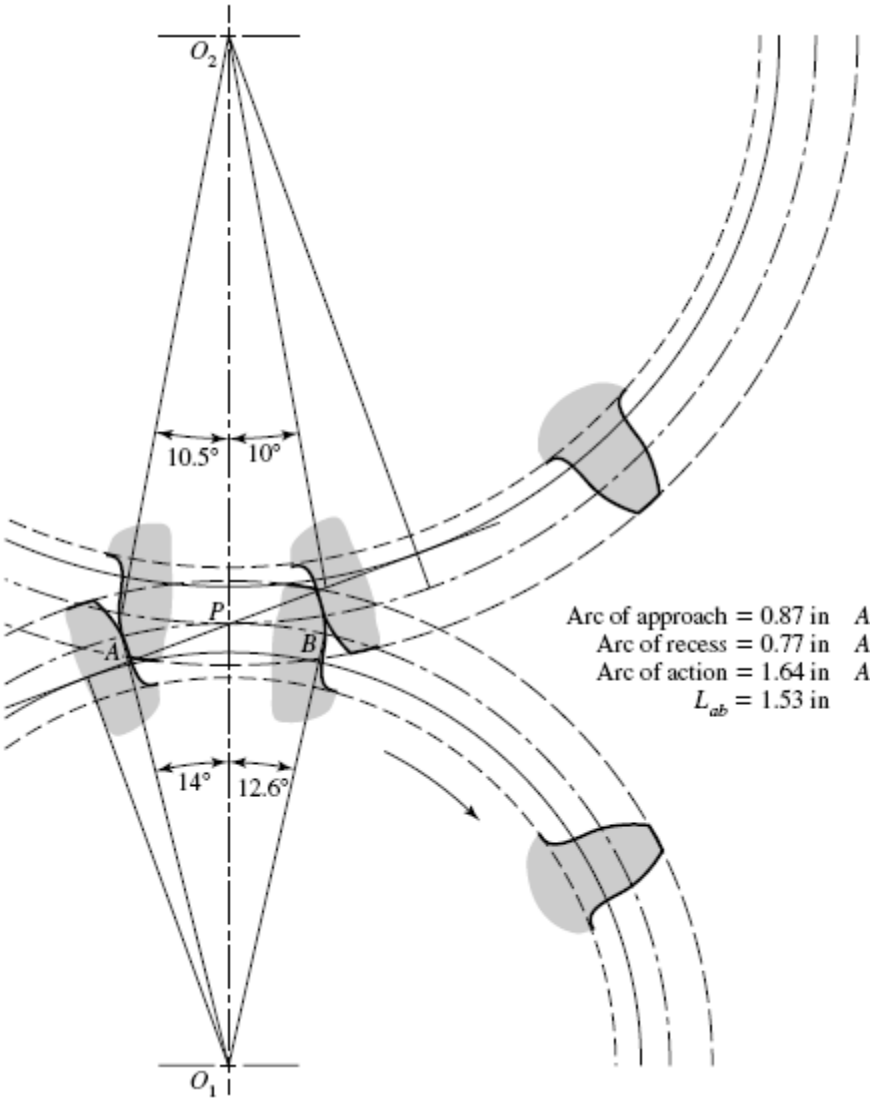
Base pitch:

$$p_b = p_c \cos\phi = (\pi/3)\cos 20^\circ = 0.984 \text{ in} \quad \textit{Ans.}$$

Contact Ratio:

$$m_c = L_{ab}/p_b = 1.53/0.984 = 1.55 \quad \textit{Ans.}$$

See the following figure for a drawing of the gears and the arc lengths.



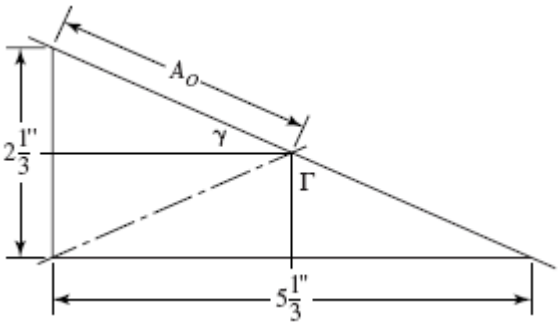
13-5

(a) $$A_0 = \left[\left(\frac{14/6}{2}\right)^2 + \left(\frac{32/6}{2}\right)^2\right]^{1/2} = 2.910 \text{ in} \quad \textit{Ans.}$$

(b)

$\gamma = \tan^{-1}(14/32) = 23.63^\circ$ *Ans.*

$\Gamma = \tan^{-1}(32/14) = 66.37^\circ$ *Ans.*



(c)

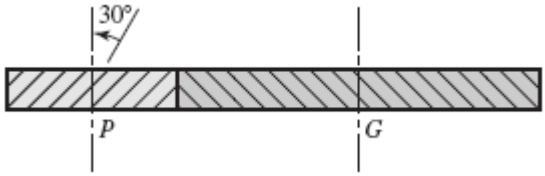
$d_P = 14/6 = 2.333$ in *Ans.*

$d_G = 32/6 = 5.333$ in *Ans.*

(d) From Table 13-3, $0.3A_0 = 0.3(2.910) = 0.873$ in and $10/P = 10/6 = 1.67$

$0.873 < 1.67 \quad \therefore\ F = 0.873$ in *Ans.*

13-6



(a) $p_n = \pi / P_n = \pi / 4 = 0.7854$ in

$p_t = p_n / \cos\psi = 0.7854 / \cos 30^\circ = 0.9069$ in

$p_x = p_t / \tan\psi = 0.9069 / \tan 30^\circ = 1.571$ in

(b) Eq. (13-7): $p_{nb} = p_n \cos\phi_n = 0.7854 \cos 25^\circ = 0.7380$ in *Ans.*

(c) $p_t = P_n \cos\psi = 4 \cos 30^\circ = 3.464$ teeth/in

$\phi_t = \tan^{-1}\left(\tan\phi_n / \cos\psi\right) = \tan^{-1}(\tan 25^\circ / \cos 30^\circ) = 28.3^\circ$ *Ans.*

(d) Table 13-4:

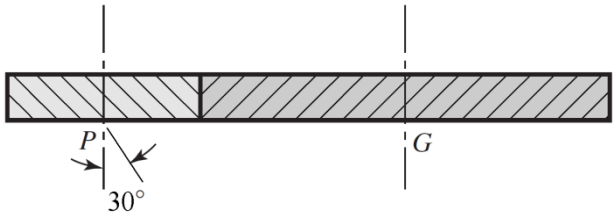
$a = 1/4 = 0.250$ in *Ans.*

$b = 1.25/4 = 0.3125$ in *Ans.*

$$d_P = \frac{20}{4\cos 30^\circ} = 5.774 \text{ in} \quad \textit{Ans.}$$

$$d_G = \frac{36}{4\cos 30^\circ} = 10.39 \text{ in} \quad \textit{Ans.}$$

13-7



$N_P = 19$ teeth, $N_G = 57$ teeth, $\phi_n = 20^\circ, m_n = 2.5$ mm

(a) $p_n = \pi m_n = \pi(2.5) = 7.854$ mm *Ans.*

$$p_t = \frac{p_n}{\cos\psi} = \frac{7.854}{\cos 30^\circ} = 9.069 \text{ mm} \quad \textit{Ans.}$$

$$p_x = \frac{p_t}{\tan\psi} = \frac{9.069}{\tan 30^\circ} = 15.71 \text{ mm} \quad \textit{Ans.}$$

(b) $$m_t = \frac{m_n}{\cos\psi} = \frac{2.5}{\cos 30^\circ} = 2.887 \text{ mm} \quad \textit{Ans.}$$

$$\phi_t = \tan^{-1}\left(\frac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.80^\circ \quad \textit{Ans.}$$

(c) $a = m_n = 2.5\,\text{mm}$ *Ans.*

$b = 1.25 m_n = 1.25(2.5) = 3.125\ \text{mm}$ *Ans.*

$$d_P = \frac{N}{P_t} = N m_t = 19(2.887) = 54.85\ \text{mm} \quad \textit{Ans.}$$

$d_G = 57(2.887) = 164.6\ \text{mm}$ *Ans.*

13-8 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20^\circ$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2) + \sqrt{(2)^2 + \left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16\ \text{teeth}$$

Round up for the minimum integer number of teeth.
$N_P = 15$ teeth *Ans.*

(b) Repeating (a) with $m = 3$, $N_P = 14.98$ teeth. Rounding up, $N_P = 15$ teeth. *Ans.*
(c) Repeating (a) with $m = 4$, $N_P = 15.44$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*
(d) Repeating (a) with $m = 5$, $N_P = 15.74$ teeth. Rounding up, $N_P = 16$ teeth. *Ans.*

Alternatively, a useful table can be generated to determine the largest gear that can mesh with a specified pinion, and thus also the maximum gear ratio with a specified pinion. The Max N_G column was generated using Eq. (13-12) with $k = 1$, $\phi = 20^\circ$, and rounding up to the next integer.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
13	16	1.23
14	26	1.86
15	45	3.00
16	101	6.31
17	1309	77.00
18	unlimited	unlimited

With this table, we can readily see that gear ratios up to 3 can be obtained with a minimum N_P of 15 teeth, and gear ratios up to 6.31 can be obtained with a minimum N_P of 16 teeth. This is consistent with the results previously obtained.

13-9 Repeating the process shown in the solution to Prob. 13-8, except with $\phi = 25°$, we obtain the following results.

(a) For $m = 2$, $N_P = 9.43$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*

(b) For $m = 3$, $N_P = 9.92$ teeth. Rounding up, $N_P = 10$ teeth. *Ans.*

(c) For $m = 4$, $N_P = 10.20$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*

(d) For $m = 5$, $N_P = 10.38$ teeth. Rounding up, $N_P = 11$ teeth. *Ans.*

For convenient reference, we will also generate the table from Eq. (13-12) for $\phi = 25°$.

Min N_P	**Max N_G**	**Max m = Max N_G / Min N_P**
9	13	1.44
10	32	3.20
11	249	22.64
12	unlimited	unlimited

13-10 **(a)** The smallest pinion tooth count that will run with itself is found from Eq. (13-10).

$$N_P \geq \frac{2k}{3\sin^2\phi}\left(1+\sqrt{1+3\sin^2\phi}\right)$$

$$\geq \frac{2(1)}{3\sin^2 20^\circ}\left(1+\sqrt{1+3\sin^2 20^\circ}\right)$$

$$\geq 12.32 \quad \rightarrow \quad 13 \text{ teeth} \quad \textit{Ans.}$$

(b) The smallest pinion that will mesh with a gear ratio of $m_G = 2.5$, from Eq. (13-11) is

$$N_P \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m+\sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

$$\geq \frac{2(1)}{\left[1+2(2.5)\right]\sin^2 20^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 20^\circ}\right\}$$

$$\geq 14.64 \quad \rightarrow \quad 15 \text{ teeth} \quad \textit{Ans.}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-12) is

$$N_G \leq \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi}$$

$$\leq \frac{15^2\sin^2 20^\circ - 4(1)^2}{4(1)-2(15)\sin^2 20^\circ}$$

$$\leq 45.49 \quad \rightarrow \quad 45 \text{ teeth} \quad \textit{Ans.}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-13),

$$N_P \geq \frac{2k}{\sin^2\phi} = \frac{2(1)}{\sin^2 20^\circ}$$
$$\geq 17.097 \quad \rightarrow \quad 18 \text{ teeth} \quad \textit{Ans.}$$

13-11 $\phi_n = 20^\circ, \psi = 30^\circ$

From Eq. (13-19), $\phi_t = \tan^{-1}\left(\tan 20^\circ / \cos 30^\circ\right) = 22.80^\circ$

(a) The smallest pinion tooth count that will run with itself, from Eq. (13-21) is

$$N_P \geq \frac{2k\cos\psi}{3\sin^2\phi_t}\left(1+\sqrt{1+3\sin^2\phi_t}\right)$$
$$\geq \frac{2(1)\cos 30^\circ}{3\sin^2 22.80^\circ}\left(1+\sqrt{1+3\sin^2 22.80^\circ}\right)$$
$$\geq 8.48 \quad \rightarrow \quad 9 \text{ teeth} \quad \textit{Ans.}$$

(b) The smallest pinion that will mesh with a gear ratio of $m = 2.5$, from Eq. (13-22) is

$$N_P \geq \frac{2(1)\cos 30^\circ}{\left[1+2(2.5)\right]\sin^2 22.80^\circ}\left\{2.5+\sqrt{2.5^2+\left[1+2(2.5)\right]\sin^2 22.80^\circ}\right\}$$
$$\geq 9.95 \quad \rightarrow \quad 10 \text{ teeth} \quad \textit{Ans.}$$

The largest gear-tooth count possible to mesh with this pinion, from Eq. (13-23) is

$$N_G \leq \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$
$$\leq \frac{10^2\sin^2 22.80^\circ - 4(1)\cos^2 30^\circ}{4(1)\cos^2 30^\circ - 2(20)\sin^2 22.80^\circ}$$
$$\leq 26.08 \quad \rightarrow \quad 26 \text{ teeth} \quad \textit{Ans.}$$

(c) The smallest pinion that will mesh with a rack, from Eq. (13-24) is

$$N_P \geq \frac{2k\cos\psi}{\sin^2\phi_t} = \frac{2(1)\cos 30^\circ}{\sin^2 22.80^\circ}$$
$$\geq 11.53 \quad \rightarrow \quad 12 \text{ teeth} \quad \textit{Ans.}$$

13-12 From Eq. (13-19), $\phi_t = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\dfrac{\tan 20^\circ}{\cos 30^\circ}\right) = 22.796^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 10$ teeth, where $N_G \le 26.01$ teeth. So $N_G = 20$ teeth will work. Higher tooth counts will work also, for example 11:22, 12:24, etc.

Use $N_P = 10$ teeth, $N_G = 20$ teeth *Ans.*

Note that the given diametral pitch (tooth size) is not relevant to the interference problem.

13-13 From Eq. (13-19), $\phi_t = \tan^{-1}\left(\dfrac{\tan\phi_n}{\cos\psi}\right) = \tan^{-1}\left(\dfrac{\tan 20^\circ}{\cos 45^\circ}\right) = 27.236^\circ$

Program Eq. (13-23) on a computer using a spreadsheet or code, and increment N_P. The first value of N_P that can be doubled is $N_P = 6$ teeth, where $N_G \le 17.6$ teeth. So $N_G = 12$ teeth will work. Higher tooth counts will work also, for example 7:14, 8:16, etc.

Use $N_P = 6$ teeth, $N_G = 12$ teeth *Ans.*

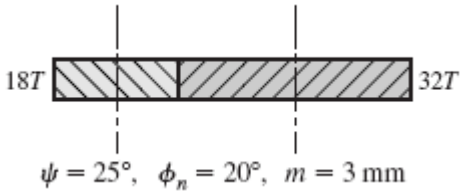
13-14 The smallest pinion that will operate with a rack without interference is given by Eq. (13-13).

$$N_P = \frac{2k}{\sin^2\phi}$$

Setting $k = 1$ for full depth teeth, $N_P = 9$ teeth, and solving for ϕ,

$$\phi = \sin^{-1}\sqrt{\frac{2k}{N_P}} = \sin^{-1}\sqrt{\frac{2(1)}{9}} = 28.126^\circ \qquad \textit{Ans.}$$

13-15



(a) Eq. (13-3): $p_n = \pi m_n = 3\pi$ mm *Ans.*

Eq. (13-16): $p_t = p_n / \cos\psi = 3\pi / \cos 25^\circ = 10.40$ mm *Ans.*

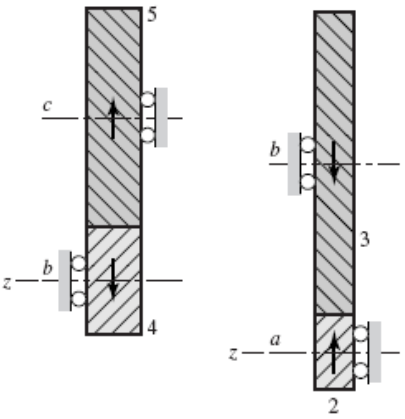
Eq. (13-17): $p_x = p_t / \tan\psi = 10.40 / \tan 25^\circ = 22.30$ mm *Ans.*

(b) Eq. (13-3): $m_t = p_t / \pi = 10.40 / \pi = 3.310$ mm *Ans.*

Eq. (13-19): $\phi_t = \tan^{-1}\dfrac{\tan\phi_n}{\cos\psi} = \tan^{-1}\dfrac{\tan 20^\circ}{\cos 25^\circ} = 21.88^\circ$ *Ans.*

(c) Eq. (13-2): $d_p = m_t N_p = 3.310\,(18) = 59.58$ mm *Ans.*
Eq. (13-2): $d_G = m_t N_G = 3.310\,(32) = 105.92$ mm *Ans.*

13-16 **(a)** Sketches of the figures are shown to determine the axial forces by inspection.



The axial force of gear 2 on shaft *a* is in the negative *z*-direction. The axial force of gear 3 on shaft *b* is in the positive *z*-direction. *Ans.*

The axial force of gear 4 on shaft *b* is in the positive *z*-direction. The axial force of gear 5 on shaft *c* is in the negative *z*-direction. *Ans.*

(b) $n_c = n_5 = \dfrac{12}{48}\left(\dfrac{16}{36}\right)(700) = +77.78$ rev/min ccw *Ans.*

(c) $d_{P2} = 12/\left(12\cos 30^\circ\right) = 1.155$ in

$d_{G3} = 48/\left(12\cos 30^\circ\right) = 4.619$ in

$C_{ab} = \dfrac{1.155 + 4.619}{2} = 2.887$ in *Ans.*

$d_{P4} = 16/\left(8\cos 25^\circ\right) = 2.207$ in

$d_{G5} = 36/\left(8\cos 25^\circ\right) = 4.965$ in

$C_{bc} = 3.586$ in *Ans.*

13-17

$$e = \frac{20}{40}\left(\frac{8}{17}\right)\left(\frac{20}{60}\right) = \frac{4}{51}$$

$$n_d = \frac{4}{51}(600) = 47.06 \text{ rev/min cw} \quad \textit{Ans.}$$

13-18

$$e = \frac{6}{10}\left(\frac{18}{38}\right)\left(\frac{20}{48}\right)\left(\frac{3}{36}\right) = \frac{3}{304}$$

$$n_9 = \frac{3}{304}(1200) = 11.84 \text{ rev/min cw} \quad \textit{Ans.}$$

13-19 **(a)** $n_c = \dfrac{12}{40} \cdot \dfrac{1}{1}(540) = 162$ rev/min cw about x. *Ans.*

(b) $d_P = 12 / \left(8 \cos 23^\circ\right) = 1.630$ in

$d_G = 40 / \left(8 \cos 23^\circ\right) = 5.432$ in

$\dfrac{d_P + d_G}{2} = 3.531$ in *Ans.*

(c) $d = \dfrac{32}{4} = 8$ in at the large end of the teeth. *Ans.*

13-20 Applying Eq. (13-30), $e = (N_2 / N_3)\,(N_4 / N_5) = 45$. For an exact ratio, we will choose to factor the train value into integers, such that

$$N_2 / N_3 = 9 \qquad (1)$$
$$N_4 / N_5 = 5 \qquad (2)$$

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$$N_2 + N_3 = N_4 + N_5 \qquad (3)$$

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 9$. From (3),

$$N_2 + N_3 = 9 + 1 = 10 = N_4 + N_5$$

Substituting $N_4 = 5\,N_5$ from (2) gives

$$10 = 5\,N_5 + N_5 = 6\,N_5$$
$$N_5 = 10 / 6 = 5 / 3$$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 3. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 17. Therefore, the smallest multiple of 3 greater than 17 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

$N_2 = 162$ teeth
$N_3 = 18$ teeth
$N_4 = 150$ teeth
$N_5 = 30$ teeth *Ans.*

13-21 The solution to Prob. 13-20 applies up to the point of determining the minimum number of teeth to avoid interference. From Eq. (13-11), with $k = 1$, $\phi = 25°$, and $m = 9$, the minimum number of teeth on the pinion to avoid interference is 11. Therefore, the smallest multiple of 3 greater than 11 is 12. Setting $N_3 = 12$ and repeating the solution of equations (1), (2), and (3) yields

$N_2 = 108$ teeth
$N_3 = 12$ teeth
$N_4 = 100$ teeth
$N_5 = 20$ teeth *Ans.*

13-22 Applying Eq. (13-30), $e = (N_2 / N_3)(N_4 / N_5) = 30$. For an exact ratio, we will choose to factor the train value into integers, such that

$$N_2 / N_3 = 6 \quad (1)$$
$$N_4 / N_5 = 5 \quad (2)$$

Assuming a constant diametral pitch in both stages, the geometry condition to satisfy the in-line requirement of the compound reverted configuration is

$$N_2 + N_3 = N_4 + N_5 \quad (3)$$

With three equations and four unknowns, one free choice is available. It is necessary that all of the unknowns be integers. We will use a normalized approach to find the minimum free choice to guarantee integers; that is, set the smallest gear of the largest stage to unity, thus $N_3 = 1$. From (1), $N_2 = 6$. From (3),

$$N_2 + N_3 = 6 + 1 = 7 = N_4 + N_5$$

Substituting $N_4 = 5\ N_5$ from (2) gives

$$7 = 5\ N_5 + N_5 = 6\ N_5$$
$$N_5 = 7 / 6$$

To eliminate this fraction, we need to multiply the original free choice by a multiple of 6. In addition, the smallest gear needs to have sufficient teeth to avoid interference. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 6$, the minimum number of teeth on the pinion to avoid interference is 16. Therefore, the smallest multiple of 3 greater than 16 is 18. Setting $N_3 = 18$ and repeating the solution of equations (1), (2), and (3) yields

$N_2 = 108$ teeth
$N_3 = 18$ teeth
$N_4 = 105$ teeth
$N_5 = 21$ teeth *Ans.*

13-23 Applying Eq. (13-30), $e = (N_2 / N_3)\,(N_4 / N_5) = 45$. For an approximate ratio, we will choose to factor the train value into two equal stages, such that

$$N_2 / N_3 = N_4 / N_5 = \sqrt{45}$$

If we choose identical pinions such that interference is avoided, both stages will be identical and the in-line geometry condition will automatically be satisfied. From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = \sqrt{45}$, the minimum number of teeth on the pinions to avoid interference is 17. Setting $N_3 = N_5 = 17$, we get

$$N_2 = N_4 = 17\sqrt{45} = 114.04 \text{ teeth}$$

Rounding to the nearest integer, we obtain

$N_2 = N_4 = 114$ teeth
$N_3 = N_5 = 17$ teeth *Ans.*

Checking, the overall train value is $e = (114 / 17)\,(114 / 17) = 44.97$.

13-24 $H = 25$ hp, $\omega_i = 2500$ rev/min
Let $\omega_o = 300$ rev/min for minimal gear ratio to minimize gear size.

$$\frac{\omega_o}{\omega_i} = \frac{300}{2500} = \frac{1}{8.333}$$

$$\frac{\omega_o}{\omega_i} = \frac{1}{8.333} = \frac{N_2}{N_3}\frac{N_4}{N_5}$$

Let $$\frac{N_2}{N_3} = \frac{N_4}{N_5} = \sqrt{\frac{1}{8.333}} = \frac{1}{2.887}$$

From Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2.887$, the minimum number of teeth on the pinions to avoid interference is 15.

Let $N_2 = N_4 = 15$ teeth
$N_3 = N_5 = 2.887(15) = 43.31$ teeth

Try $N_3 = N_5 = 43$ teeth.

$$\omega_o = \left(\frac{15}{43}\right)\left(\frac{15}{43}\right)(2500) = 304.2$$

Too big. Try $N_3 = N_5 = 44$.

$$\omega_o = \left(\frac{15}{44}\right)\left(\frac{15}{44}\right)(2500) = 290.55 \text{ rev/min}$$

$N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth *Ans.*

13-25 **(a)** The planet gears act as keys and the wheel speeds are the same as that of the ring gear. Thus,

$$n_A = n_3 = 900(16/48) = 300 \text{ rev/min} \quad \textit{Ans.}$$

(b)
$$n_F = n_5 = 0, \quad n_L = n_6, \quad e = -1$$
$$-1 = \frac{n_6 - 300}{0 - 300}$$
$$300 = n_6 - 300$$
$$n_6 = 600 \text{ rev/min} \quad \textit{Ans.}$$

(c) The wheel spins freely on icy surfaces, leaving no traction for the other wheel. The car is stalled. *Ans.*

13-26 **(a)** The motive power is divided equally among four wheels instead of two.

(b) Locking the center differential causes 50 percent of the power to be applied to the rear wheels and 50 percent to the front wheels. If one of the rear wheels rests on a slippery surface such as ice, the other rear wheel has no traction. But the front wheels still provide traction, and so you have two-wheel drive. However, if the rear differential is locked, you have 3-wheel drive because the rear-wheel power is now distributed 50-50.

13-27 Let gear 2 be first, then $n_F = n_2 = 0$. Let gear 6 be last, then $n_L = n_6 = -12$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$
$$(0 - n_A)\frac{16}{51} = -12 - n_A$$
$$n_A = \frac{-12}{35/51} = -17.49 \text{ rev/min (negative indicates cw)} \quad \textit{Ans.}$$

13-28 Let gear 2 be first, then $n_F = n_2 = 0$ rev/min. Let gear 6 be last, then $n_L = n_6 = 85$ rev/min.

$$e = \frac{20}{30}\left(\frac{16}{34}\right) = \frac{16}{51} = \frac{n_L - n_A}{n_F - n_A}$$

$$(0 - n_A)\frac{16}{51} = (85 - n_A)$$

$$-n_A\left(\frac{16}{51}\right) + n_A = 85$$

$$n_A\left(1 - \frac{16}{51}\right) = 85$$

$$n_A = \frac{85}{1 - \frac{16}{51}} = 123.9 \text{ rev/min}$$

The positive sign indicates the same direction as n_6. $\therefore n_A = 123.9$ rev/min ccw *Ans*.

13-29 The geometry condition is $d_5 / 2 = d_2 / 2 + d_3 + d_4$. Since all the gears are meshed, they will all have the same diametral pitch. Applying $d = N / P$,

$$N_5 / (2P) = N_2 / (2P) + N_3 / P + N_4 / P$$

$$N_5 = N_2 + 2N_3 + 2N_4 = 12 + 2(16) + 2(12) = 68 \text{ teeth} \quad \textit{Ans}.$$

Let gear 2 be first, $n_F = n_2 = 320$ rev/min. Let gear 5 be last, $n_L = n_5 = 0$ rev/min.

$$e = \frac{12}{16}\left(\frac{16}{12}\right)\left(\frac{12}{68}\right) = \frac{3}{17} = \frac{n_L - n_A}{n_F - n_A}$$

$$320 - n_A = \frac{17}{3}(0 - n_A)$$

$$n_A = -\frac{3}{14}(320) = -68.57 \text{ rev/min}$$

The negative sign indicates opposite of n_2. $\therefore n_A = 68.57$ rev/min cw *Ans*.

13-30 Let $n_F = n_2$, then $n_L = n_7 = 0$.

$$e = -\frac{20}{16}\left(\frac{16}{30}\right)\left(\frac{36}{46}\right) = -0.5217 = \frac{n_L - n_5}{n_F - n_5}$$

$$\frac{0 - n_5}{10 - n_5} = -0.5217$$

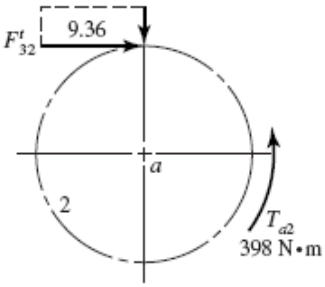
$$-0.5217\left(10-n_5\right)=-n_5$$
$$-5.217+0.5217n_5+n_5=0$$
$$n_5\left(1+0.5217\right)=5.217$$
$$n_5=\frac{5.217}{1.5217}$$
$$n_5=n_b=3.428 \text{ turns in same direction}$$

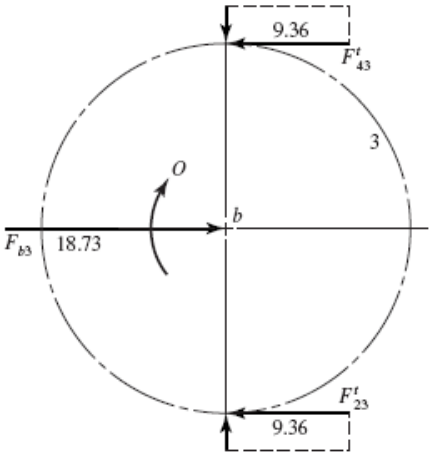
13-31 **(a)**

$$\omega=2\pi n/60$$
$$H=T\omega=2\pi Tn/60 \quad (T \text{ in N}\cdot\text{m}, H \text{ in W})$$

So
$$T=\frac{60H\left(10^3\right)}{2\pi n}$$
$$=9550\,H/n \ (H \text{ in kW}, n \text{ in rev/min})$$
$$T_a=\frac{9550\left(75\right)}{1800}=398 \text{ N}\cdot\text{m}$$
$$r_2=\frac{mN_2}{2}=\frac{5\left(17\right)}{2}=42.5 \text{ mm}$$

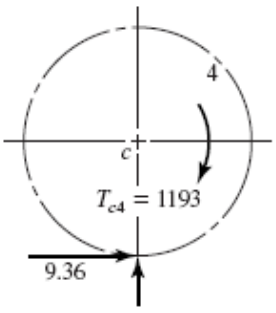
So
$$F_{32}^t=\frac{T_a}{r_2}=\frac{398}{42.5}=9.36 \text{ kN}$$





$$F_{3b}=-F_{b3}=2\left(9.36\right)=18.73 \text{ kN in the positive } x\text{-direction.} \quad \textit{Ans.}$$

(b)
$$r_4=\frac{mN_4}{2}=\frac{5\left(51\right)}{2}=127.5 \text{ mm}$$
$$T_{c4}=9.36\left(127.5\right)=1193 \text{ N}\cdot\text{m ccw}$$
$$\therefore T_{4c}=1193 \text{ N}\cdot\text{m cw} \quad \textit{Ans.}$$



Note: The solution is independent of the pressure angle.

13-32

$$d = \frac{N}{P} = \frac{N}{6}$$

$$d_2 = 4 \text{ in},\ d_4 = 4 \text{ in},\ d_5 = 6 \text{ in},\ d_6 = 24 \text{ in}$$

$$e = \left(-\frac{24}{24}\right)\left(-\frac{24}{36}\right)\left(+\frac{36}{144}\right) = 1/6$$

$$n_F = n_2 = 1000 \text{ rev/min}$$

$$n_L = n_6 = 0$$

$$e = \frac{n_L - n_A}{n_F - n_A} = \frac{0 - n_A}{1000 - n_A} = \frac{1}{6}$$

$$n_A = -200 \text{ rev/min}$$

Noting that power equals torque times angular velocity, the input torque is

$$T_2 = \frac{H}{n_2} = \frac{25 \text{ hp}}{1000 \text{ rev/min}}\left(\frac{550 \text{ lbf} \cdot \text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 1576 \text{ lbf} \cdot \text{in}$$

For 100 percent gear efficiency, the output power equals the input power, so

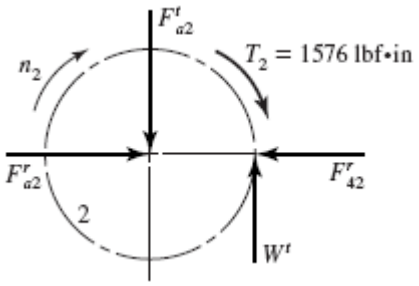
$$T_{arm} = \frac{H}{n_A} = \frac{25 \text{ hp}}{200 \text{ rev/min}}\left(\frac{550 \text{ lbf} \cdot \text{ft/s}}{\text{hp}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right) = 7878 \text{ lbf} \cdot \text{in}$$

Next, we'll confirm the output torque as we work through the force analysis and complete the free body diagrams.

Gear 2

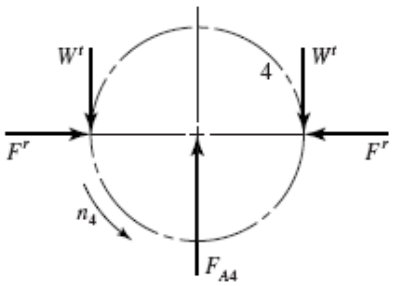
$$W^t = \frac{1576}{2} = 788 \text{ lbf}$$

$$F_{32}^r = 788 \tan 20^\circ = 287 \text{ lbf}$$

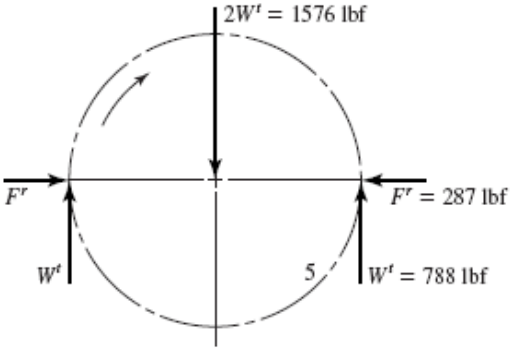


Gear 4

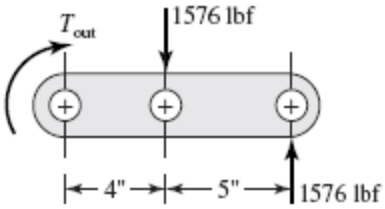
$$F_{A4} = 2W^t = 2(788) = 1576 \text{ lbf}$$



Gear 5



Arm



$$T_{\text{out}} = 1576(9) - 1576(4) = 7880 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

13-33 Given: $m = 12$ mm, $n_P = 1800$ rev/min cw,
$N_2 = 18T$, $N_3 = 32T$, $N_4 = 18T$, $N_5 = 48T$

Pitch Diameters: $d_2 = 18(12) = 216$ mm, $d_3 = 32(12) = 384$ mm,
$d_4 = 18(12) = 216$ mm, $d_5 = 48(12) = 576$ mm

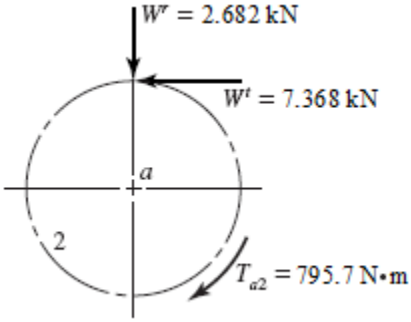
Gear 2

From Eq. (13-36),

$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(150)}{\pi(216)(1800)} = 7.368 \text{ kN}$$

$$T_{a2} = W_t\left(\frac{d_2}{2}\right) = 7.368\left(\frac{216}{2}\right) = 795.7 \text{ N} \cdot \text{m}$$

$$W^r = 7.368 \tan 20^\circ = 2.682 \text{ kN}$$



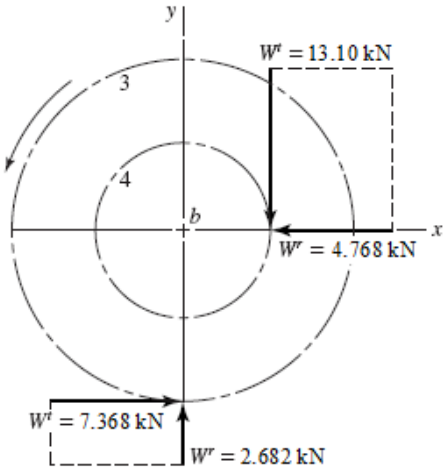
Gears 3 and 4

$$W^t\left(\frac{216}{2}\right) = 7.368\frac{(384)}{2}$$

$$W^t = 13.10 \text{ kN}$$

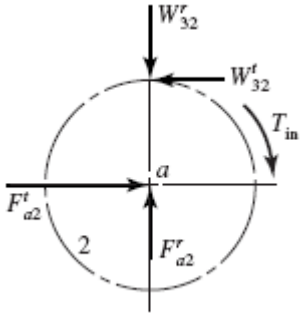
$$W^r = 13.10 \tan 20^\circ = 4.768 \text{ kN}$$

Ans.



13-34 Given: $P = 5$ teeth/in, $N_2 = 18T$, $N_3 = 45T$,
$\phi_n = 20^\circ$, $H = 32$ hp, $n_2 = 1800$ rev/min

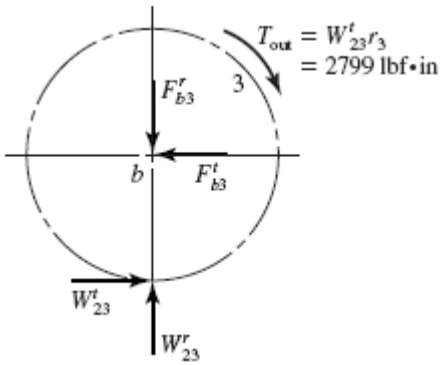
Gear 2



$$T_{\text{in}} = \frac{63\,025(32)}{1800} = 1120 \text{ lbf} \cdot \text{in}$$
$$d_P = \frac{18}{5} = 3.600 \text{ in}$$
$$d_G = \frac{45}{5} = 9.000 \text{ in}$$
$$W^t_{32} = \frac{1120}{3.6/2} = 622 \text{ lbf}$$
$$W^r_{32} = 622 \tan 20^\circ = 226 \text{ lbf}$$
$$F^t_{a2} = W^t_{32} = 622 \text{ lbf}, \quad F^r_{a2} = W^r_{32} = 226 \text{ lbf}$$
$$F_{a2} = \left(622^2 + 226^2\right)^{1/2} = 662 \text{ lbf}$$

Each bearing on shaft *a* has the same radial load of $R_A = R_B = 662/2 = 331$ lbf.

Gear 3

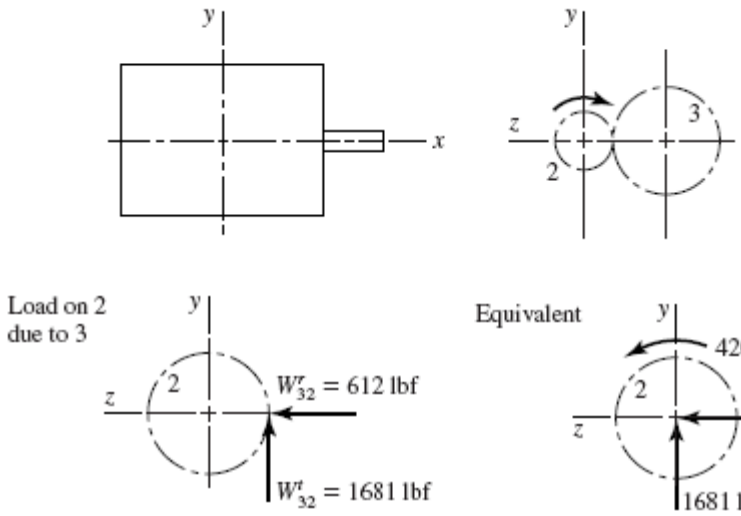


$$W^t_{23} = W^t_{32} = 622 \text{ lbf}$$
$$W^r_{23} = W^r_{32} = 226 \text{ lbf}$$
$$F_{b3} = F_{b2} = 662 \text{ lbf}$$
$$R_C = R_D = 662/2 = 331 \text{ lbf}$$

Each bearing on shaft *b* has the same radial load which is equal to the radial load of bearings *A* and *B*. Thus, all four bearings have the same radial load of 331 lbf. *Ans.*

13-35 Given: $P = 4$ teeth/in, $\phi_n = 20^\circ$, $N_P = 20T$, $n_2 = 900$ rev/min

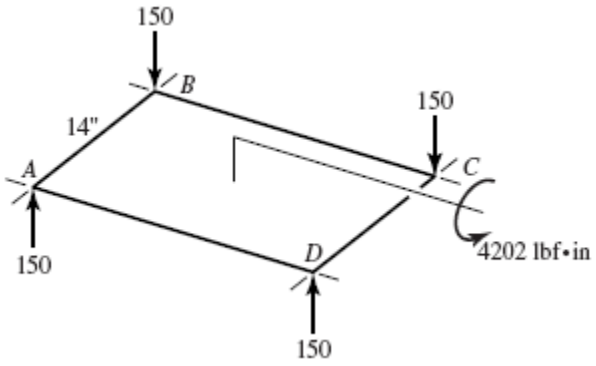
$$d_2 = \frac{N_P}{P} = \frac{20}{4} = 5.000 \text{ in}$$
$$T_{\text{in}} = \frac{63\,025(30)(2)}{900} = 4202 \text{ lbf} \cdot \text{in}$$
$$W^t_{32} = T_{\text{in}} / (d_2/2) = 4202/(5/2) = 1681 \text{ lbf}$$
$$W^r_{32} = 1681 \tan 20^\circ = 612 \text{ lbf}$$



The motor mount resists the equivalent forces and torque.
The radial force due to torque is

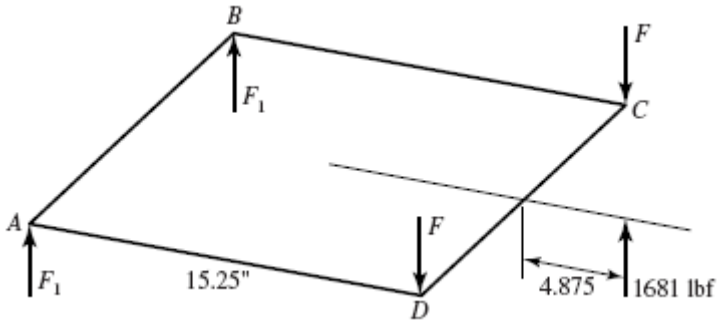
$$F^r = \frac{4202}{14(2)} = 150 \text{ lbf}$$

Forces reverse with rotational sense as torque reverses.



The compressive loads at *A* and *D* are absorbed by the base plate, not the bolts. For W_{32}^t, the tensions in *C* and *D* are

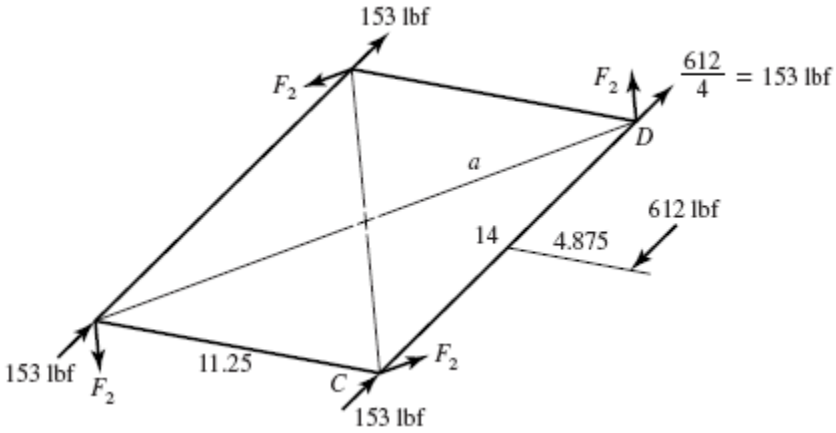
$$\Sigma M_{AB} = 0 \qquad 1681(4.875 + 15.25) - 2F(15.25) = 0 \qquad F = 1109 \text{ lbf}$$



If W_{32}^t reverses, 15.25 in changes to 13.25 in, 4.815 in changes to 2.875 in, and the forces change direction. For *A* and *B*,

$$1681(2.875) - 2F_1(13.25) = 0 \quad \Rightarrow \quad F_1 = 182.4 \text{ lbf}$$

For W_{32}^r,



$$M = 612(4.875 + 11.25/2) = 6426 \text{ lbf} \cdot \text{in}$$

$$a = \sqrt{(14/2)^2 + (11.25/2)^2} = 8.98 \text{ in}$$

$$F_2 = \frac{6426}{4(8.98)} = 179 \text{ lbf}$$

At *C* and *D*, the shear forces are:

$$F_{S1} = \sqrt{\left[153 + 179(5.625/8.98)\right]^2 + \left[179(7/8.98)\right]^2}$$

At *A* and *B*, the shear forces are:

$$F_{S2} = \sqrt{\left[153 - 179(5.625/8.98)\right]^2 + \left[179(7/8.98)\right]^2}$$
$$= 145 \text{ lbf}$$

The shear forces are independent of the rotational sense.
The bolt tensions and the shear forces for cw rotation are,

	Tension (lbf)	Shear (lbf)
A	0	145
B	0	145
C	1109	300
D	1109	300

For ccw rotation,

	Tension (lbf)	Shear (lbf)
A	182	145
B	182	145
C	0	300
D	0	300

13-36 **(a)** $N_2 = N_4 = 15$ teeth, $N_3 = N_5 = 44$ teeth

$$P = \frac{N}{d} \quad \Rightarrow \quad d = \frac{N}{P}$$

$$d_2 = d_4 = \frac{15}{6} = 2.5 \text{ in} \quad \textit{Ans.}$$

$$d_3 = d_5 = \frac{44}{6} = 7.33 \text{ in} \quad \textit{Ans.}$$

(b) $$V_i = V_2 = V_3 = \frac{\pi d_2 n_2}{12} = \frac{\pi(2.5)(2500)}{12} = 1636 \text{ ft/min} \quad \textit{Ans.}$$

$$V_o = V_4 = V_5 = \frac{\pi d_4 n_4}{12} = \frac{\pi(2.5)\left[(2500)(15/44)\right]}{12} = 558 \text{ ft/min} \quad \textit{Ans.}$$

(c) Input gears:

$$W_{ti} = 33\,000\frac{H}{V_i} = \frac{33\,000(25)}{1636} = 504.3 \text{ lbf} = 504 \text{ lbf} \quad \textit{Ans.}$$

$$W_{ri} = W_{ti}\tan\phi = 504.3\tan 20^\circ = 184 \text{ lbf} \quad \textit{Ans.}$$

$$W_i = \frac{W_{ti}}{\cos\phi} = \frac{504.3}{\cos 20^\circ} = 537 \text{ lbf} \quad \textit{Ans.}$$

Output gears:

$$W_{to} = 33\,000\frac{H}{V_o} = \frac{33\,000(25)}{558} = 1478 \text{ lbf} \quad \textit{Ans.}$$

$$W_{ro} = W_{to}\tan\phi = 1478\tan 20^\circ = 538 \text{ lbf} \quad \textit{Ans.}$$

$$W_o = \frac{W_{to}}{\cos 20^\circ} = \frac{1478}{\cos 20^\circ} = 1573 \text{ lbf} \quad \textit{Ans.}$$

(d) $$T_i = W_{ti}\left(\frac{d_2}{2}\right) = 504.3\left(\frac{2.5}{2}\right) = 630 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(e) $T_o = T_i\left(\frac{44}{15}\right)^2 = 630\left(\frac{44}{15}\right)^2 = 5420 \text{ lbf} \cdot \text{in}$ *Ans.*

13-37 $H = 35$ hp, $n_i = 1200$ rev/min, ϕ=20°

$N_2 = N_4 = 16$ teeth, $N_3 = N_5 = 48$ teeth, $P = 10$ teeth/in

(a) $n_{\text{intermediate}} = n_3 = n_4 = \frac{N_2}{N_3} n_i = \frac{16}{48}(1200) = 400 \text{ rev/min}$ *Ans.*

$n_o = \frac{N_2}{N_3}\frac{N_4}{N_5} n_i = \frac{16}{48}\left(\frac{16}{48}\right)(1200) = 133.3 \text{ rev/min}$ *Ans.*

(b) $P = \frac{N}{d} \quad \Rightarrow \quad d = \frac{N}{P}$

$d_2 = d_4 = \frac{16}{10} = 1.6 \text{ in}$ *Ans.*

$d_3 = d_5 = \frac{48}{10} = 4.8 \text{ in}$ *Ans.*

$V_i = V_2 = V_3 = \frac{\pi d_2 n_2}{12} = \frac{\pi(1.6)(1200)}{12} = 502.7 \text{ ft/min}$ *Ans.*

$V_o = V_4 = V_5 = \frac{\pi d_4 n_4}{12} = \frac{\pi(1.6)(400)}{12} = 167.6 \text{ ft/min}$ *Ans.*

(c) $W_{ti} = 33000\frac{H}{V_i} = \frac{33000(35)}{502.7} = 2298 \text{ lbf lbf}$ *Ans.*

$W_{ri} = W_{ti}\tan\phi = 2298\tan 20^\circ = 836.4 \text{ lbf}$ *Ans.*

$W_i = \frac{W_{ti}}{\cos\phi} = \frac{2298}{\cos 20^\circ} = 2445 \text{ lbf}$ *Ans.*

$W_{to} = 33000\frac{H}{V_o} = \frac{33000(35)}{167.6} = 6891 \text{ lbf}$ *Ans.*

$W_{ro} = W_{to}\tan\phi = 6891\tan 20^\circ = 2508 \text{ lbf}$ *Ans.*

$W_o = \frac{W_{to}}{\cos 20^\circ} = \frac{6891}{\cos 20^\circ} = 7333 \text{ lbf}$ *Ans.*

(d) $T_i = W_{ti}\left(\frac{d_2}{2}\right) = 2298\left(\frac{1.6}{2}\right) = 1838 \text{ lbf} \cdot \text{in}$ *Ans.*

(e) $T_o = T_i\left(\frac{48}{16}\right)^2 = 1838\left(\frac{48}{16}\right)^2 = 16\,540 \text{ lbf} \cdot \text{in}$ *Ans.*

13-38 **(a)** For $\frac{\omega_o}{\omega_i}=\frac{2}{1}$, from Eq. (13-11), with $m = 2$, $k = 1$, $\phi = 20^\circ$

$$N_P=\frac{2(1)}{\left[1+2(2)\right]\sin^2 20^\circ}\left\{2+\sqrt{2^2+\left[1+2(2)\right]\sin^2 20^\circ}\right\}=14.16$$

So $N_{P\min}=15$ *Ans.*

(b) $$P=\frac{N}{d}=\frac{15}{8}=1.875 \text{ teeth/in}$$ *Ans.*

(c) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.

For A, with $\phi = 20°$,

$$W_{tA} = F_A \cos 20° = 300 \cos 20° = 281.9 \text{ lbf}$$

For A, with $\phi = 25°$, same transmitted load,

$$F_A = W_{tA}/\cos 25° = 281.9/\cos 25° = 311.0 \text{ lbf}$$ *Ans.*

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right)=W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB}=W_{tA}\frac{(d_A/2)}{(d_B/2)}=W_{tA}\left(\frac{d_A}{d_B}\right)=(281.9)\left(\frac{20}{8}\right)=704.75 \text{ lbf}$$

$$F_B=\frac{W_{tB}}{\cos 25^\circ}=\frac{704.75}{\cos 25^\circ}=777.6 \text{ lbf}$$ *Ans.*

13-39 **(a)** For $\frac{\omega_o}{\omega_i}=\frac{5}{1}$, from Eq. (13-11), with $m = 5$, $k = 1$, $\phi = 20^\circ$

$$N_P=\frac{2(1)}{\left[1+2(5)\right]\sin^2 25^\circ}\left\{5+\sqrt{5^2+\left[1+2(5)\right]\sin^2 25^\circ}\right\}=10.4$$

So $N_{P\min}=11$ *Ans.*

(b) $$m=\frac{d}{N}=\frac{300}{11}=27.3 \text{ mm/tooth}$$ *Ans.*

(d) To transmit the same power with no change in pitch diameters, the speed and transmitted force must remain the same.

For A, with $\phi = 20°$,

$$W_{tA} = F_A \cos 20° = 11 \cos 20° = 10.33 \text{ kN}$$

For A, with $\phi = 25°$, same transmitted load,

$$F_A = W_{tA}/\cos 25° = 10.33 / \cos 25° = 11.40 \text{ kN} \quad \textit{Ans.}$$

Summing the torque about the shaft axis,

$$W_{tA}\left(\frac{d_A}{2}\right) = W_{tB}\left(\frac{d_B}{2}\right)$$

$$W_{tB} = W_{tA}\frac{(d_A/2)}{(d_B/2)} = W_{tA}\left(\frac{d_A}{d_B}\right) = (11.40)\left(\frac{600}{300}\right) = 22.80 \text{ kN}$$

$$F_B = \frac{W_{tB}}{\cos 25^\circ} = \frac{22.80}{\cos 25^\circ} = 25.16 \text{ kN} \quad \textit{Ans.}$$

13-40 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2) + \sqrt{(2)^2 + \left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.
$N_F = 15$ teeth, $N_C = 30$ teeth *Ans.*

(b) $$m = \frac{d}{N} = \frac{125}{15} = 8.33 \text{ mm/tooth} \quad \textit{Ans.}$$

(c) $$T = \frac{H}{\omega} = \frac{2 \text{ kW}}{191 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 100 \text{ N}\cdot\text{m}$$

(d) From Eq. (13-36),

$$W_t = \frac{60\,000H}{\pi dn} = \frac{60\,000(2)}{\pi(125)(191)} = 1.60 \text{ kN} = 1600 \text{ N} \quad \textit{Ans.}$$

Or, we could have obtained W_t directly from the torque and radius,

$$W_t = \frac{T}{d/2} = \frac{100}{0.125/2} = 1600 \text{ N}$$

$$W_r = W_t \tan\phi = 1600 \tan 20^\circ = 583 \text{ N} \quad \textit{Ans.}$$

$$W = \frac{W_t}{\cos\phi} = \frac{1600}{\cos 20^\circ} = 1700 \text{ N} \quad \textit{Ans.}$$

13-41 **(a)** Using Eq. (13-11) with $k = 1$, $\phi = 20°$, and $m = 2$,

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

$$= \frac{2(1)}{\left[1+2(2)\right]\sin^2(20^\circ)}\left\{(2) + \sqrt{(2)^2 + \left[1+2(2)\right]\sin^2(20^\circ)}\right\} = 14.16 \text{ teeth}$$

Round up for the minimum integer number of teeth.
$N_C = 15$ teeth, $N_F = 30$ teeth *Ans.*

(b) $$P = \frac{N}{d} = \frac{30}{10} = 3 \text{ teeth/in} \quad \textit{Ans.}$$

(c) $$T = \frac{H}{\omega} = \frac{1 \text{ hp}}{70 \text{ rev/min}}\left(\frac{550 \text{ lbf}\cdot\text{ft/s}}{\text{hp}}\right)\left(\frac{12 \text{ in}}{\text{ft}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)$$

$$T = 900 \text{ lbf}\cdot\text{in} \quad \textit{Ans.}$$

(d) From Eqs. (13-34) and (13-35),

$$V = \frac{\pi dn}{12} = \frac{\pi(10)(70)}{12} = 183.3 \text{ ft/min}$$

$$W_t = 33\,000\frac{H}{V} = \frac{33\,000(1)}{183.3} = 180 \text{ lbf} \quad \textit{Ans.}$$

$$W_r = W_t \tan\phi = 180 \tan 20^\circ = 65.5 \text{ lbf} \quad \textit{Ans.}$$

$$W = \frac{W_t}{\cos\phi} = \frac{180}{\cos 20^\circ} = 192 \text{ lbf} \quad \textit{Ans.}$$

13-42 **(a)** Eq. (13-14): $$\gamma = \tan^{-1}\left(\frac{N_P}{N_G}\right) = \tan^{-1}\left(\frac{d_P}{d_G}\right) = \tan^{-1}\left(\frac{1.30}{3.88}\right) = 18.5^\circ \quad \textit{Ans.}$$

(b) Eq. (13-34): $$V = \frac{\pi dn}{12} = \frac{\pi(2)(1.30)(600)}{12} = 408.4 \text{ ft/min} \quad \textit{Ans.}$$

(c) Eq. (13-35): $W_t = 33\,000\dfrac{H}{V} = 33\,000\left(\dfrac{10}{408.4}\right) = 808 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_r = W_t \tan\phi \cos\gamma = 808 \tan 20^\circ \cos 18.5^\circ = 279 \text{ lbf}$ *Ans.*

Eq. (13-38): $W_a = W_t \tan\phi \sin\gamma = 808 \tan 20^\circ \sin 18.5^\circ = 93.3 \text{ lbf}$ *Ans.*

The tangential and axial forces agree with Prob. 3-74, but the radial force given in Prob. 3-74 is shown here to be incorrect. *Ans.*

13-43

$T_{\text{in}} = 63\,025H/n = 63025(2.5)/240 = 656.5 \text{ lbf}\cdot\text{in}$

$W^t = T/r = 656.5/2 = 328.3 \text{ lbf}$

$\gamma = \tan^{-1}(2/4) = 26.565^\circ$

$\Gamma = \tan^{-1}(4/2) = 63.435^\circ$

$a = 2 + (1.5\cos 26.565^\circ)/2 = 2.67 \text{ in}$

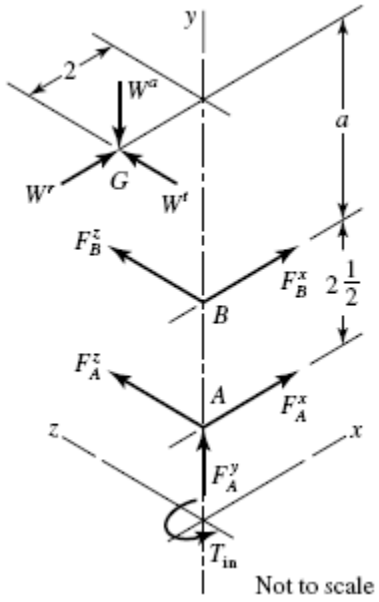
$W^r = 328.3 \tan 20^\circ \cos 26.565^\circ = 106.9 \text{ lbf}$

$W^a = 328.3 \tan 20^\circ \sin 26.565^\circ = 53.4 \text{ lbf}$

$\mathbf{W} = 106.9\mathbf{i} - 53.4\mathbf{j} + 328.3\mathbf{k} \text{ lbf}$

$\mathbf{R}_{AG} = -2\mathbf{i} + 5.17\mathbf{j}, \quad \mathbf{R}_{AB} = 2.5\mathbf{j}$

$\Sigma\mathbf{M}_A = \mathbf{R}_{AG} \times \mathbf{W} + \mathbf{R}_{AB} \times \mathbf{F}_B + \mathbf{T} = \mathbf{0}$



Solving gives

$\mathbf{R}_{AB} \times \mathbf{F}_B = 2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k}$

$\mathbf{R}_{AG} \times \mathbf{W} = 1697\mathbf{i} + 656.6\mathbf{j} - 445.9\mathbf{k}$

So

$(1697\mathbf{i} + 656.6\mathbf{j} - 445.9\mathbf{k}) + (2.5F_B^z\mathbf{i} - 2.5F_B^x\mathbf{k} + T\mathbf{j}) = \mathbf{0}$

$F_B^z = -1697/2.5 = -678.8 \text{ lbf}$

$T = -656.6 \text{ lbf}\cdot\text{in}$

$F_B^x = -445.9/2.5 = -178.4 \text{ lbf}$

So

$F_B = \left[(-678.8)^2 + (-178.4)^2\right]^{1/2} = 702 \text{ lbf}$ *Ans.*

$\mathbf{F}_A = -(\mathbf{F}_B + \mathbf{W})$
$= -(-178.8\mathbf{i} - 678.8\mathbf{k} + 106.9\mathbf{i} - 53.4\mathbf{j} + 328.3\mathbf{k})$
$= 71.5\mathbf{i} + 53.4\mathbf{j} + 350.5\mathbf{k}$

$$F_A(\text{radial}) = \left(71.5^2 + 350.5^2\right)^{1/2} = 358 \text{ lbf} \quad \textit{Ans.}$$

$$F_A(\text{thrust}) = 53.4 \text{ lbf} \quad \textit{Ans.}$$

13-44

$$d_2 = 18/10 = 1.8 \text{ in}, \quad d_3 = 30/10 = 3.0 \text{ in}$$

$$\gamma = \tan^{-1}\left(\frac{d_2/2}{d_3/2}\right) = \tan^{-1}\left(\frac{0.9}{1.5}\right) = 30.96^\circ$$

$$\Gamma = 180^\circ - \gamma = 59.04^\circ$$

$$DE = \frac{9}{16} + 0.5\cos 59.04^\circ = 0.8197 \text{ in}$$

$$W^t = 25 \text{ lbf}$$

$$W^r = 25\tan 20^\circ \cos 59.04^\circ = 4.681 \text{ lbf}$$

$$W^a = 25\tan 20^\circ \sin 59.04^\circ = 7.803 \text{ lbf}$$

$$\mathbf{W} = -4.681\mathbf{i} - 7.803\mathbf{j} + 25\mathbf{k}$$

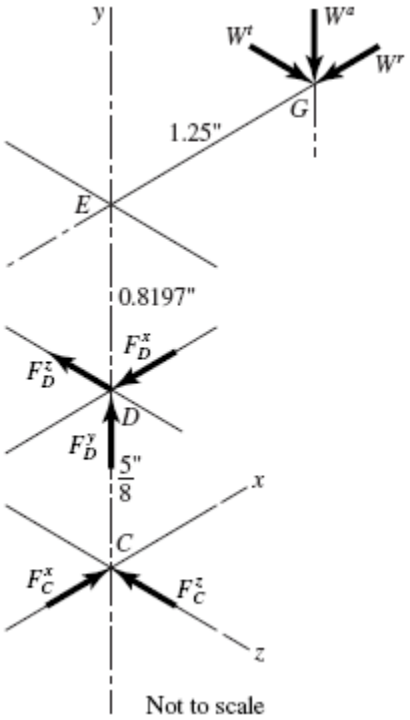
$$\mathbf{R}_{DG} = 0.8197\mathbf{j} + 1.25\mathbf{i}$$

$$\mathbf{R}_{DC} = -0.625\mathbf{j}$$

$$\Sigma\mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{DG} \times \mathbf{W} = 20.49\mathbf{i} - 31.25\mathbf{j} - 5.917\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -0.625F_C^z\mathbf{i} + 0.625F_C^x\mathbf{k}$$



$$\left(20.49\mathbf{i} - 31.25\mathbf{j} - 5.917\mathbf{k}\right) + \left(-0.625F_C^z\mathbf{i} + 0.625F_C^x\mathbf{k}\right) + T\mathbf{j} = \mathbf{0}$$

$$T = 31.25 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$\mathbf{F}_C = 9.47\mathbf{i} + 32.8\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

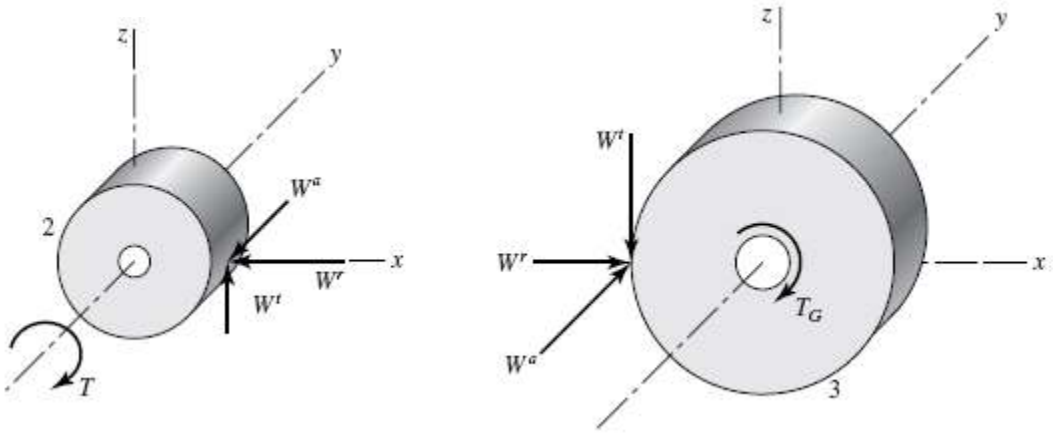
$$F_C = \left(9.47^2 + 32.8^2\right)^{1/2} = 34.1 \text{ lbf} \quad \textit{Ans.}$$

$$\Sigma\mathbf{F} = 0 \qquad \mathbf{F}_D = -4.79\mathbf{i} + 7.80\mathbf{j} - 57.8\mathbf{k} \text{ lbf}$$

$$F_D(\text{radial}) = \left[\left(-4.79\right)^2 + \left(-57.8\right)^2\right]^{1/2} = 58.0 \text{ lbf} \quad \textit{Ans.}$$

$$F_D(\text{thrust}) = W^a = 7.80 \text{ lbf} \quad \textit{Ans.}$$

13-45



$$P_t = P_n \cos\psi = 4\cos 30^\circ = 3.464 \text{ teeth/in}$$

$$\phi_t = \tan^{-1}\frac{\tan\phi_n}{\cos\psi} = \tan^{-1}\frac{\tan 20^\circ}{\cos 30^\circ} = 22.80^\circ$$

$$d_P = \frac{18}{3.464} = 5.196 \text{ in}$$

The forces on the shafts will be equal and opposite of the forces transmitted to the gears through the meshing teeth.

Pinion (Gear 2)

$$W^r = W^t \tan\phi_t = 800\tan 22.80^\circ = 336 \text{ lbf}$$

$$W^a = W^t \tan\psi = 800\tan 30^\circ = 462 \text{ lbf}$$

$$\mathbf{W} = -336\mathbf{i} - 462\mathbf{j} + 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = \left[(-336)^2 + (-462)^2 + 800^2\right]^{1/2} = 983 \text{ lbf} \quad \textit{Ans.}$$

Gear 3

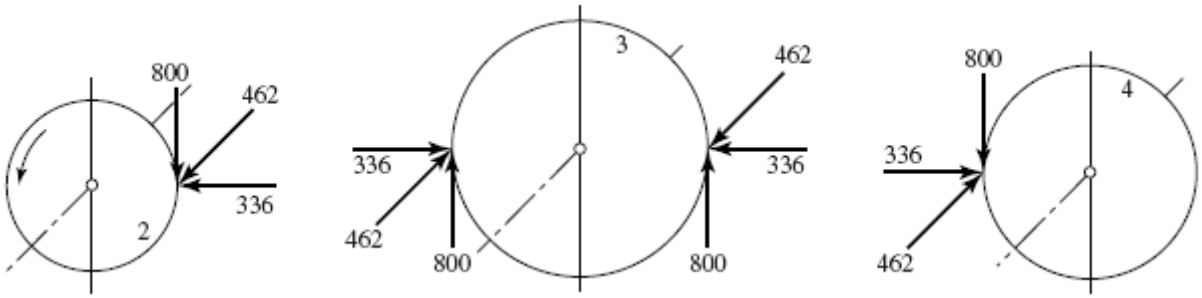
$$\mathbf{W} = 336\mathbf{i} + 462\mathbf{j} - 800\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$W = 983 \text{ lbf} \quad \textit{Ans.}$$

$$d_G = \frac{32}{3.464} = 9.238 \text{ in}$$

$$T_G = W^t r = 800(9.238) = 7390 \text{ lbf}\cdot\text{in}$$

13-46 From Prob. 13-45 solution,



Notice that the idler shaft reaction contains a couple tending to turn the shaft end-over-end. Also the idler teeth are bent both ways. Idlers are more severely loaded than other gears, belying their name. Thus, be cautious.

13-47 *Gear 3:*

$$P_t = P_n \cos\psi = 7\cos 30^\circ = 6.062 \text{ teeth/in}$$

$$\tan\phi_t = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.4203, \quad \phi_t = 22.8^\circ$$

$$d_3 = \frac{54}{6.062} = 8.908 \text{ in}$$

$$W^t = 500 \text{ lbf}$$

$$W^a = 500\tan 30^\circ = 288.7 \text{ lbf}$$

$$W^r = 500\tan 22.8^\circ = 210.2 \text{ lbf}$$

$$\mathbf{W}_3 = 210.2\mathbf{i} + 288.7\mathbf{j} - 500\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

Gear 4:

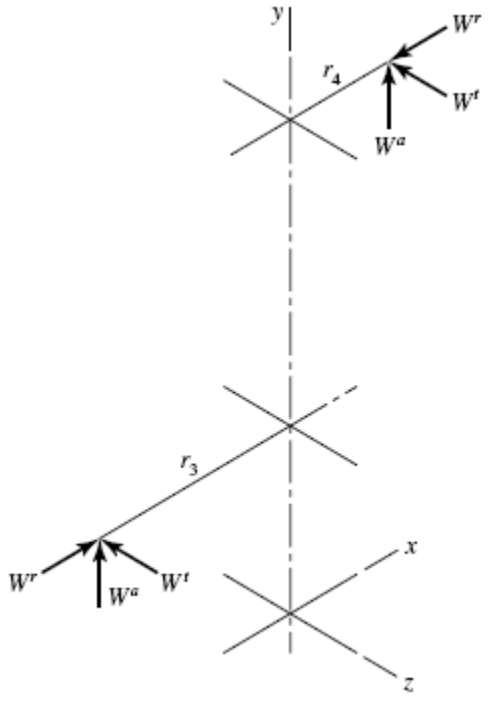
$$d_4 = \frac{14}{6.062} = 2.309 \text{ in}$$

$$W^t = 500\frac{8.908}{2.309} = 1929 \text{ lbf}$$

$$W^a = 1929\tan 30^\circ = 1114 \text{ lbf}$$

$$W^r = 1929\tan 22.8^\circ = 811 \text{ lbf}$$

$$\mathbf{W}_4 = -811\mathbf{i} + 1114\mathbf{j} - 1929\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

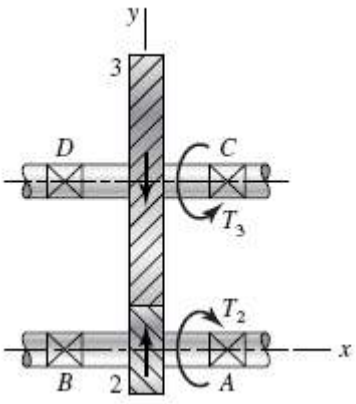


13-48

$$P_t = 6\cos 30^\circ = 5.196 \text{ teeth/in}$$

$$d_3 = \frac{42}{5.196} = 8.083 \text{ in}$$

$$\phi_t = 22.8^\circ$$



$$d_2 = \frac{16}{5.196} = 3.079 \text{ in}$$

$$T_2 = \frac{63\,025(25)}{1720} = 916 \text{ lbf} \cdot \text{in}$$

$$W^t = \frac{T}{r} = \frac{916}{3.079/2} = 595 \text{ lbf}$$

$$W^a = 595 \tan 30^\circ = 344 \text{ lbf}$$

$$W^r = 595 \tan 22.8^\circ = 250 \text{ lbf}$$

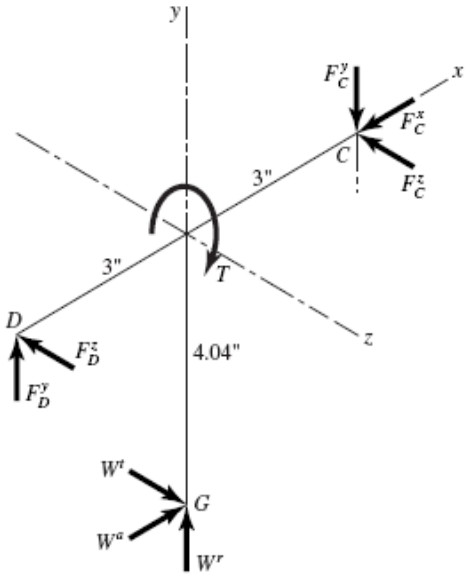
$$\mathbf{W} = 344\mathbf{i} + 250\mathbf{j} + 595\mathbf{k} \text{ lbf}$$

$$\mathbf{R}_{DC} = 6\mathbf{i}, \quad \mathbf{R}_{DG} = 3\mathbf{i} - 4.04\mathbf{j}$$

$$\Sigma \mathbf{M}_D = \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{R}_{DG} \times \mathbf{W} + \mathbf{T} = \mathbf{0} \qquad (1)$$

$$\mathbf{R}_{DG} \times \mathbf{W} = -2404\mathbf{i} - 1785\mathbf{j} + 2140\mathbf{k}$$

$$\mathbf{R}_{DC} \times \mathbf{F}_C = -6F_C^z\mathbf{j} + 6F_C^y\mathbf{k}$$



Substituting and solving Eq. (1) gives

$$\mathbf{T} = 2404\mathbf{i} \text{ lbf} \cdot \text{in}$$

$$F_C^z = -297.5 \text{ lbf}$$

$$F_C^y = -365.7 \text{ lbf}$$

$$\Sigma \mathbf{F} = \mathbf{F}_D + \mathbf{F}_C + \mathbf{W} = \mathbf{0}$$

Substituting and solving gives

$$F_C^x = -344 \text{ lbf}$$

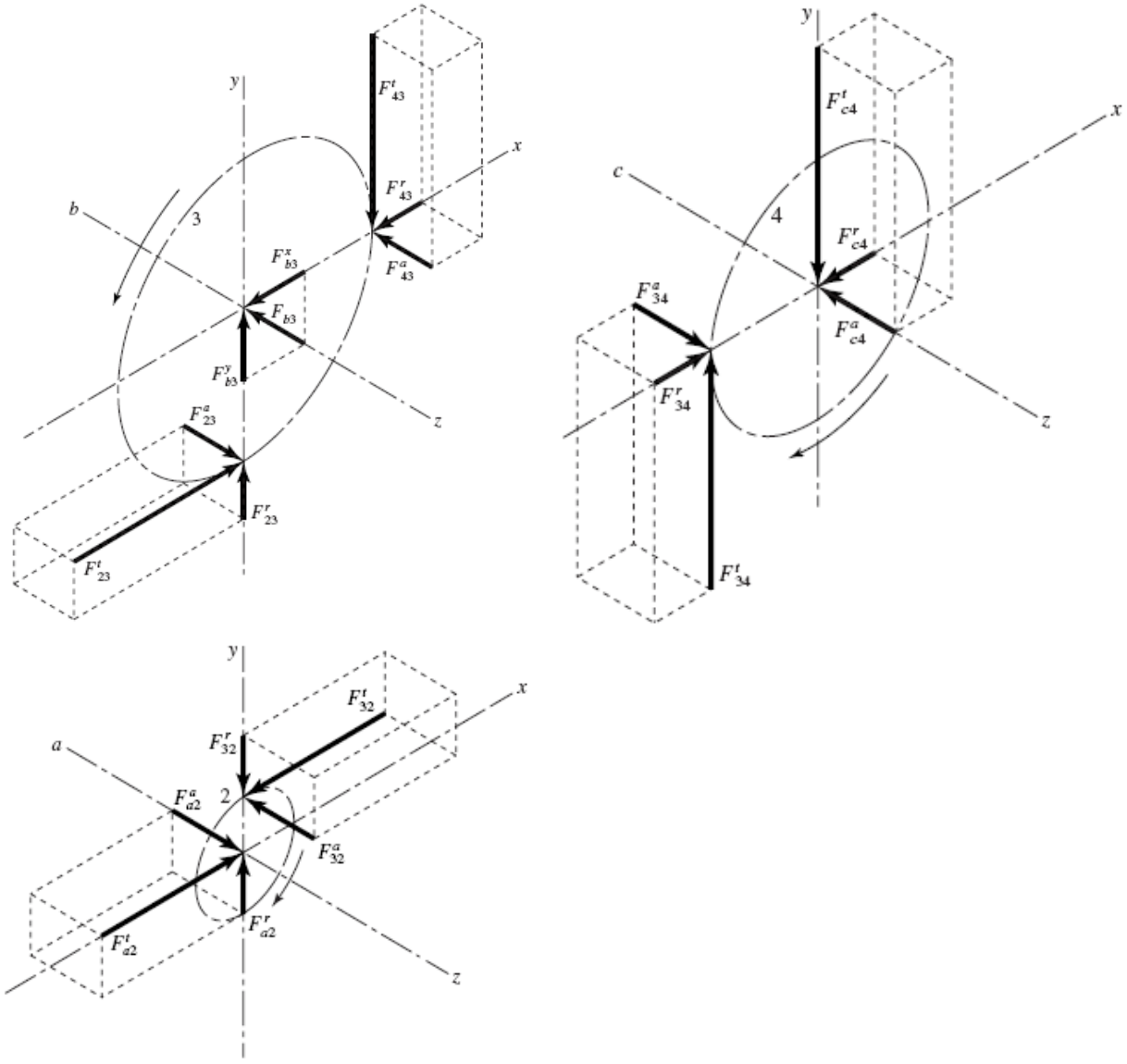
$$F_D^y = 106.7 \text{ lbf}$$

$$F_D^z = -297.5 \text{ lbf}$$

$$\mathbf{F}_C = -344\mathbf{i} - 356.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

$$\mathbf{F}_D = 106.7\mathbf{j} - 297.5\mathbf{k} \text{ lbf} \quad \textit{Ans.}$$

13-49



Since the transverse pressure angle is specified, we will assume the given module is also in terms of the transverse orientation.

$$d_2 = mN_2 = 4(16) = 64 \text{ mm}$$

$$d_3 = mN_3 = 4(36) = 144 \text{ mm}$$

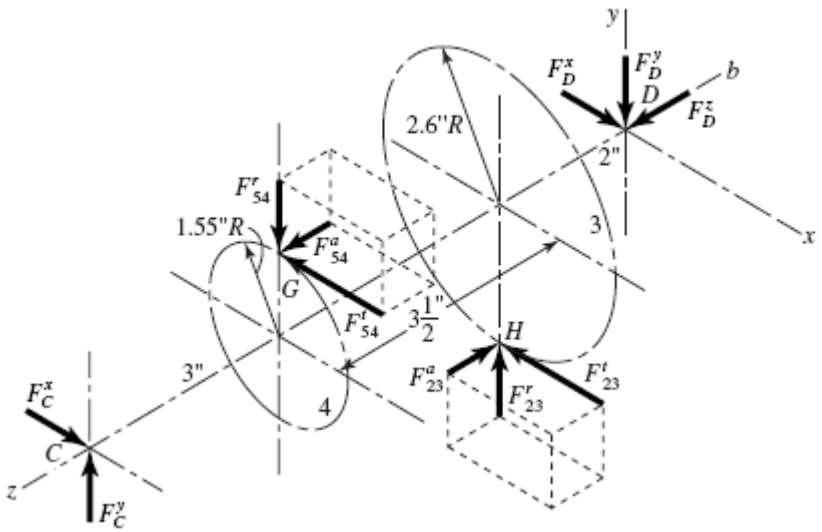
$$d_4 = mN_4 = 4(28) = 112 \text{ mm}$$

$$T = \frac{H}{\omega} = \frac{6 \text{ kW}}{1600 \text{ rev/min}}\left(\frac{1000 \text{ W}}{\text{kW}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 35.81 \text{ N}\cdot\text{m}$$

$$W^t = \frac{T}{d_2/2} = \frac{35.81}{0.064/2} = 1119 \text{ N}$$

$$W^r = W^t \tan\phi_t = 1119 \tan 20^\circ = 407.3 \text{ N}$$
$$W^a = W^t \tan\psi = 1119 \tan 15^\circ = 299.8 \text{ N}$$
$$\mathbf{F}_{2a} = -1119\mathbf{i} - 407.3\mathbf{j} - 299.8\mathbf{k} \text{ N} \quad \textit{Ans.}$$
$$\mathbf{F}_{3b} = (1119 - 407.3)\mathbf{i} - (1119 - 407.3)\mathbf{j}$$
$$= 711.7\mathbf{i} - 711.7\mathbf{j} \text{ N} \quad \textit{Ans.}$$
$$\mathbf{F}_{4c} = 407.3\mathbf{i} + 1119\mathbf{j} + 299.8\mathbf{k} \text{ N} \quad \textit{Ans.}$$

13-50



$$d_2 = \frac{N}{P_n \cos\psi} = \frac{14}{8\cos 30^\circ} = 2.021 \text{ in}, \quad d_3 = \frac{36}{8\cos 30^\circ} = 5.196 \text{ in}$$
$$d_4 = \frac{15}{5\cos 15^\circ} = 3.106 \text{ in}, \quad d_5 = \frac{45}{5\cos 15^\circ} = 9.317 \text{ in}$$

For gears 2 and 3: $\phi_t = \tan^{-1}\left(\tan\phi_n / \cos\psi\right) = \tan^{-1}\left(\tan 20^\circ / \cos 30^\circ\right) = 22.8^\circ$

For gears 4 and 5: $\phi_t = \tan^{-1}\left(\tan 20^\circ / \cos 15^\circ\right) = 20.6^\circ$

$$F_{23}^t = T_2 / r_2 = 1200 / (2.021/2) = 1188 \text{ lbf}$$
$$F_{54}^t = 1188 \frac{5.196}{3.106} = 1987 \text{ lbf}$$
$$F_{23}^r = F_{23}^t \tan\phi_t = 1188 \tan 22.8^\circ = 499 \text{ lbf}$$
$$F_{54}^r = 1986 \tan 20.6^\circ = 746 \text{ lbf}$$
$$F_{23}^a = F_{23}^t \tan\psi = 1188 \tan 30^\circ = 686 \text{ lbf}$$
$$F_{54}^a = 1986 \tan 15^\circ = 532 \text{ lbf}$$

Next, designate the points of action on gears 4 and 3, respectively, as points *G* and *H*, as shown. Position vectors are

$$\mathbf{R}_{CG} = 1.553\mathbf{j} - 3\mathbf{k}$$
$$\mathbf{R}_{CH} = -2.598\mathbf{j} - 6.5\mathbf{k}$$
$$\mathbf{R}_{CD} = -8.5\mathbf{k}$$

Force vectors are

$$\mathbf{F}_{54} = -1986\mathbf{i} - 748\mathbf{j} + 532\mathbf{k}$$
$$\mathbf{F}_{23} = -1188\mathbf{i} + 500\mathbf{j} - 686\mathbf{k}$$
$$\mathbf{F}_C = F_C^x\mathbf{i} + F_C^y\mathbf{j}$$
$$\mathbf{F}_D = F_D^x\mathbf{i} + F_D^y\mathbf{j} + F_D^z\mathbf{k}$$

Now, a summation of moments about bearing *C* gives

$$\Sigma\mathbf{M}_C = \mathbf{R}_{CG} \times \mathbf{F}_{54} + \mathbf{R}_{CH} \times \mathbf{F}_{23} + \mathbf{R}_{CD} \times \mathbf{F}_D = \mathbf{0}$$

The terms for this equation are found to be

$$\mathbf{R}_{CG} \times \mathbf{F}_{54} = -1412\mathbf{i} + 5961\mathbf{j} + 3086\mathbf{k}$$
$$\mathbf{R}_{CH} \times \mathbf{F}_{23} = 5026\mathbf{i} + 7722\mathbf{j} - 3086\mathbf{k}$$
$$\mathbf{R}_{CD} \times \mathbf{F}_D = 8.5F_D^y\mathbf{i} - 8.5F_D^x\mathbf{j}$$

When these terms are placed back into the moment equation, the **k** terms, representing the shaft torque, cancel. The **i** and **j** terms give

$$F_D^y = -\frac{3614}{8.5} = -425 \text{ lbf} \quad \textit{Ans.}$$
$$F_D^x = \frac{13\,683}{8.5} = 1610 \text{ lbf} \quad \textit{Ans.}$$

Next, we sum the forces to zero.

$$\Sigma\mathbf{F} = \mathbf{F}_C + \mathbf{F}_{54} + \mathbf{F}_{23} + \mathbf{F}_D = \mathbf{0}$$

Substituting, gives

$$\left(F_C^x\mathbf{i} + F_C^y\mathbf{j}\right) + \left(-1987\mathbf{i} - 746\mathbf{j} + 532\mathbf{k}\right) + \left(-1188\mathbf{i} + 499\mathbf{j} - 686\mathbf{k}\right) + \left(1610\mathbf{i} - 425\mathbf{j} + F_D^z\mathbf{k}\right) = \mathbf{0}$$

Solving gives

$$F_C^x = 1987 + 1188 - 1610 = 1565 \text{ lbf} \quad \textit{Ans.}$$
$$F_C^y = 746 - 499 + 425 = 672 \text{ lbf} \quad \textit{Ans.}$$
$$F_D^z = -532 + 686 = 154 \text{ lbf} \quad \textit{Ans.}$$

13-51

$$V_W = \frac{\pi d_W n_W}{60} = \frac{\pi(0.100)(600)}{60} = \pi \text{ m/s}$$

$$W_{Wt} = \frac{H}{V_W} = \frac{2000}{\pi} = 637 \text{ N}$$

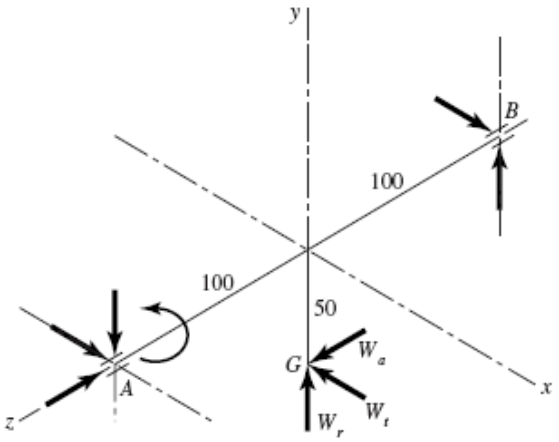
$$L = p_x N_W = 25(1) = 25 \text{ mm}$$

$$\lambda = \tan^{-1}\frac{L}{\pi d_W}$$

$$= \tan^{-1}\frac{25}{\pi(100)} = 4.550^\circ \quad \text{lead angle}$$

$$W = \frac{W_{Wt}}{\cos\phi_n \sin\lambda + f\cos\lambda}$$

$$V_S = \frac{V_W}{\cos\lambda} = \frac{\pi}{\cos 4.550^\circ} = 3.152 \text{ m/s}$$



Worm shaft diagram

In ft/min: $V_S = 3.28(3.152) = 10.33$ ft/s = 620 ft/min
Use $f = 0.043$ from curve A of Fig. 13-42. Then, from the first of Eq. (13-43)

$$W = \frac{637}{\cos 14.5^\circ\left(\sin 4.55^\circ\right) + 0.043\cos 4.55^\circ} = 5323 \text{ N}$$

$$W^y = W\sin\phi_n = 5323\sin 14.5^\circ = 1333 \text{ N}$$

$$W^z = 5323\left[\cos 14.5^\circ\left(\cos 4.55^\circ\right) - 0.043\sin 4.55^\circ\right] = 5119 \text{ N}$$

The force acting against the worm is

$$\mathbf{W} = -637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} \text{ N}$$

Thus, A is the thrust bearing. *Ans.*

$$\mathbf{R}_{AG} = -0.05\mathbf{j} - 0.10\mathbf{k}, \quad \mathbf{R}_{AB} = -0.20\mathbf{k}$$

$$\Sigma\mathbf{M}_A = \mathbf{R}_{AG}\times\mathbf{W} + \mathbf{R}_{AB}\times\mathbf{F}_B + \mathbf{T} = \mathbf{0}$$

$$\mathbf{R}_{AG}\times\mathbf{W} = -122.6\mathbf{i} + 63.7\mathbf{j} - 31.85\mathbf{k}$$

$$\mathbf{R}_{AB}\times\mathbf{F}_B = 0.2F_B^y\mathbf{i} - 0.2F_B^x\mathbf{j}$$

Substituting and solving gives

$$T = 31.85 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

$$F_B^x = 318.5 \text{ N}, \quad F_B^y = 613 \text{ N}$$

So $\mathbf{F}_B = 318.5\mathbf{i} + 613\mathbf{j}$ N *Ans.*

Or $F_B = \left[(613)^2 + (318.5)^2 \right]^{1/2} = 691 \text{ N radial}$

$\Sigma \mathbf{F} = \mathbf{F}_A + \mathbf{W} + \mathbf{R}_B = \mathbf{0}$

$\mathbf{F}_A = -(\mathbf{W} + \mathbf{F}_B) = -(-637\mathbf{i} + 1333\mathbf{j} + 5119\mathbf{k} + 318.5\mathbf{i} + 613\mathbf{j})$

$= 318.5\mathbf{i} - 1946\mathbf{j} - 5119\mathbf{k}$ *Ans.*

Radial $\mathbf{F}_A^r = 318.5\mathbf{i} - 1946\mathbf{j} \text{ N}$

$F_A^r = \left[(318.5)^2 + (-1946)^2 \right]^{1/2} = 1972 \text{ N}$

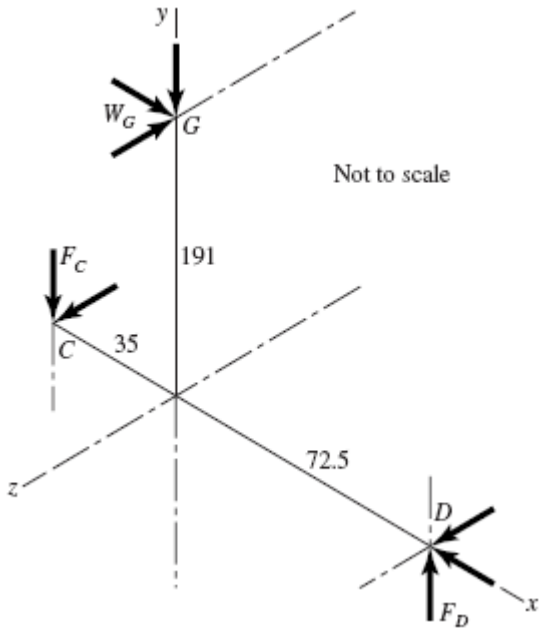
Thrust $F_A^a = -5119 \text{ N}$

13-52 From Prob. 13-51,

$\mathbf{W}_G = 637\mathbf{i} - 1333\mathbf{j} - 5119\mathbf{k} \text{ N}$

$p_t = p_x$

So $d_G = \dfrac{N_G p_x}{\pi} = \dfrac{48(25)}{\pi} = 382 \text{ mm}$



Bearing *D* takes the thrust load.

$\Sigma \mathbf{M}_D = \mathbf{R}_{DG} \times \mathbf{W}_G + \mathbf{R}_{DC} \times \mathbf{F}_C + \mathbf{T} = \mathbf{0}$

$\mathbf{R}_{DG} = -0.0725\mathbf{i} + 0.191\mathbf{j}$

$\mathbf{R}_{DC} = -0.1075\mathbf{i}$

The position vectors are in meters.

$\mathbf{R}_{DG} \times \mathbf{W}_G = -977.7\mathbf{i} - 371.1\mathbf{j} - 25.02\mathbf{k}$

$\mathbf{R}_{DC} \times \mathbf{F}_C = 0.1075F_C^z\mathbf{j} - 0.1075F_C^y\mathbf{k}$

Putting it together and solving,

$T = 977.7 \text{ N} \cdot \text{m}$ *Ans.*

$\mathbf{F}_C = -233\mathbf{j} + 3450\mathbf{k} \text{ N}, \quad F_C = 3460 \text{ N}$ *Ans.*

$\Sigma \mathbf{F} = \mathbf{F}_C + \mathbf{W}_G + \mathbf{F}_D = \mathbf{0}$

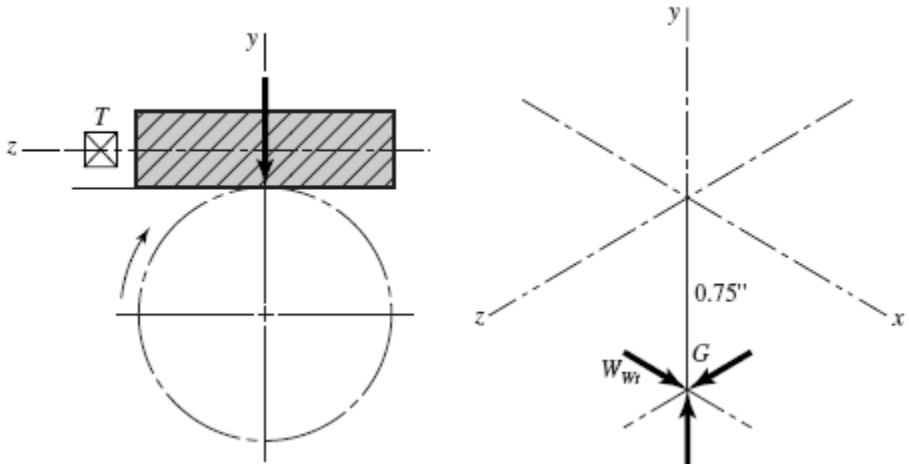
$\mathbf{F}_D = -(\mathbf{F}_C + \mathbf{W}_G) = -637\mathbf{i} + 1566\mathbf{j} + 1669\mathbf{k} \text{ N}$ *Ans.*

Radial $\mathbf{F}_D^r = 1566\mathbf{j} + 1669\mathbf{k} \text{ N}$

Or $F_D^r = \left(1566^2 + 1669^2\right)^{1/2} = 2289 \text{ N (total radial)}$

$\mathbf{F}_D^t = -637\mathbf{i} \text{ N}$ (thrust)

13-53



$$V_W = \frac{\pi(1.5)(600)}{12} = 235.7 \text{ ft/min}$$

$$W^x = W_{Wt} = \frac{33\,000(0.75)}{235.7} = 105.0 \text{ lbf}$$

$$p_t = p_x = \frac{\pi}{8} = 0.3927 \text{ in}$$

$$L = 0.3927(2) = 0.7854 \text{ in}$$

$$\lambda = \tan^{-1}\frac{0.7854}{\pi(1.5)} = 9.46^\circ$$

$$W = \frac{105.0}{\cos 20^\circ \sin 9.46^\circ + 0.05\cos 9.46^\circ} = 515.3 \text{ lbf}$$

$$W^y = 515.3 \sin 20^\circ = 176.2 \text{ lbf}$$

$$W^z = 515.3\left[\cos 20^\circ\left(\cos 9.46^\circ\right) - 0.05 \sin 9.46^\circ\right] = 473.4 \text{ lbf}$$

So $\mathbf{W} = 105\mathbf{i} + 176\mathbf{j} + 473\mathbf{k}$ lbf *Ans.*

$$T = 105(0.75) = 78.8 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

13-54 Computer programs will vary.

Chapter 14

14-1 $d = \dfrac{N}{P} = \dfrac{22}{6} = 3.667$ in

Table 14-2: $Y = 0.331$

Eq. (13-34): $V = \dfrac{\pi dn}{12} = \dfrac{\pi(3.667)(1200)}{12} = 1152$ ft/min

Eq. (14-4*b*): $K_v = \dfrac{1200 + 1152}{1200} = 1.96$

Eq. (13-35) : $W^t = 33\,000\dfrac{H}{V} = 33\,000\dfrac{15}{1152} = 429.7$ lbf

Eq. (14-7): $\sigma = \dfrac{K_v W^t P}{FY} = \dfrac{1.96(429.7)(6)}{2(0.331)} = 7633$ psi $= 7.63$ kpsi *Ans.*

14-2 $d = \dfrac{N}{P} = \dfrac{18}{10} = 1.8$ in

Table 14-2: $Y = 0.309$

Eq. (13-34): $V = \dfrac{\pi dn}{12} = \dfrac{\pi(1.8)(600)}{12} = 282.7$ ft/min

Eq. (14-4*b*): $K_v = \dfrac{1200 + 282.7}{1200} = 1.236$

Eq. (13-35) : $W^t = 33\,000\dfrac{H}{V} = 33\,000\dfrac{2}{282.7} = 233.5$ lbf

Eq. (14-7): $\sigma = \dfrac{K_v W^t P}{FY} = \dfrac{1.236(233.5)(10)}{1.0(0.309)} = 9340$ psi $= 9.34$ kpsi *Ans.*

14-3 $d = mN = 1.25(18) = 22.5$ mm

Table 14-2: $Y = 0.309$

$V = \dfrac{\pi dn}{60} = \dfrac{\pi(22.5)(10^{-3})(1800)}{60} = 2.121$ m/s

Eq. (14-6*b*): $K_v = \dfrac{6.1 + 2.121}{6.1} = 1.348$

Eq. (13-36): $W^t = \dfrac{60\,000H}{\pi dn} = \dfrac{60\,000(0.5)}{\pi(22.5)(1800)} = 0.2358$ kN $= 235.8$ N

Eq. (14-8): $\sigma = \dfrac{K_v W^t}{FmY} = \dfrac{1.348(235.8)}{12(1.25)(0.309)} = 68.6$ MPa *Ans.*

14-4 $d = mN = 8(16) = 128$ mm

Table 14-2: $Y = 0.296$

$$V = \frac{\pi dn}{60} = \frac{\pi(128)(10^{-3})(150)}{60} = 1.0053 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 1.0053}{6.1} = 1.165$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(6)}{\pi(128)(150)} = 5.968 \text{ kN} = 5968 \text{ N}$$

Eq. (14-8): $$\sigma = \frac{K_v W^t}{FmY} = \frac{1.165(5968)}{90(8)(0.296)} = 32.6 \text{ MPa}$$ *Ans.*

14-5 $d = mN = 1(16) = 16$ mm

Table 14-2: $Y = 0.296$

$$V = \frac{\pi dn}{60} = \frac{\pi(16)(10^{-3})(400)}{60} = 0.335 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 0.335}{6.1} = 1.055$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(0.15)}{\pi(16)(400)} = 0.4476 \text{ kN} = 447.6 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.055(447.6)}{150(1)(0.296)} = 10.6 \text{ mm}$$

From Table 13-2, use $F = 11$ mm or 12 mm, depending on availability. *Ans.*

14-6 $d = mN = 2(20) = 40$ mm

Table 14-2: $Y = 0.322$

$$V = \frac{\pi dn}{60} = \frac{\pi(40)(10^{-3})(200)}{60} = 0.419 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 0.419}{6.1} = 1.069$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(0.5)}{\pi(40)(200)} = 1.194 \text{ kN} = 1194 \text{ N}$$

Eq. (14-8): $$F = \frac{K_v W^t}{\sigma mY} = \frac{1.069(1194)}{75(2.0)(0.322)} = 26.4 \text{ mm}$$

From Table 13-2, use $F = 28$ mm. *Ans.*

14-7 $d = \dfrac{N}{P} = \dfrac{24}{5} = 4.8$ in

Table 14-2: $Y = 0.337$

Eq. (13-34): $V = \dfrac{\pi dn}{12} = \dfrac{\pi(4.8)(50)}{12} = 62.83$ ft/min

Eq. (14-4*b*): $K_v = \dfrac{1200 + 62.83}{1200} = 1.052$

Eq. (13-35) : $W^t = 33\,000\dfrac{H}{V} = 33\,000\dfrac{6}{62.83} = 3151$ lbf

Eq. (14-7): $F = \dfrac{K_v W^t P}{\sigma Y} = \dfrac{1.052(3151)(5)}{20(10^3)(0.337)} = 2.46$ in

Use $F = 2.5$ in *Ans.*

14-8 $d = \dfrac{N}{P} = \dfrac{16}{4} = 4.0$ in

Table 14-2: $Y = 0.296$

Eq. (13-34): $V = \dfrac{\pi dn}{12} = \dfrac{\pi(4.0)(400)}{12} = 418.9$ ft/min

Eq. (14-4*b*): $K_v = \dfrac{1200 + 418.9}{1200} = 1.349$

Eq. (13-35) : $W^t = 33\,000\dfrac{H}{V} = 33\,000\dfrac{20}{418.9} = 1575.6$ lbf

Eq. (14-7): $F = \dfrac{K_v W^t P}{\sigma Y} = \dfrac{1.349(1575.6)(4)}{12(10^3)(0.296)} = 2.39$ in

Use $F = 2.5$ in *Ans.*

14-9 Try $P = 8$ which gives $d = 18/8 = 2.25$ in and $Y = 0.309$.

Eq. (13-34): $V = \dfrac{\pi dn}{12} = \dfrac{\pi(2.25)(600)}{12} = 353.4$ ft/min

Eq. (14-4*b*): $K_v = \dfrac{1200 + 353.4}{1200} = 1.295$

Eq. (13-35): $W^t = 33\,000\dfrac{H}{V} = 33\,000\dfrac{2.5}{353.4} = 233.4$ lbf

Eq. (14-7): $F = \dfrac{K_v W^t P}{\sigma Y} = \dfrac{1.295(233.4)(8)}{10(10^3)(0.309)} = 0.783$ in

Using coarse integer pitches from Table 13-2, the following table is formed.

P	d	V	K_v	W^t	F
2	9.000	1413.717	2.178	58.356	0.082
3	6.000	942.478	1.785	87.535	0.152
4	4.500	706.858	1.589	116.713	0.240
6	3.000	471.239	1.393	175.069	0.473
8	2.250	353.429	1.295	233.426	0.782
10	1.800	282.743	1.236	291.782	1.167
12	1.500	235.619	1.196	350.139	1.627
16	1.125	176.715	1.147	466.852	2.773

Other considerations may dictate the selection. Good candidates are $P = 8$ ($F = 7/8$ in) and $P = 10$ ($F = 1.25$ in). *Ans.*

14-10 Try $m = 2$ mm which gives $d = 2(18) = 36$ mm and $Y = 0.309$.

$$V = \frac{\pi dn}{60} = \frac{\pi(36)(10^{-3})(900)}{60} = 1.696 \text{ m/s}$$

Eq. (14-6*b*): $$K_v = \frac{6.1 + 1.696}{6.1} = 1.278$$

Eq. (13-36): $$W^t = \frac{60\,000H}{\pi dn} = \frac{60\,000(1.5)}{\pi(36)(900)} = 0.884 \text{ kN} = 884 \text{ N}$$

Eq. (14-8): $$F = \frac{1.278(884)}{75(2)(0.309)} = 24.4 \text{ mm}$$

Using the preferred module sizes from Table 13-2:

m	d	V	K_v	W^t	F
1.00	18.0	0.848	1.139	1768.388	86.917
1.25	22.5	1.060	1.174	1414.711	57.324
1.50	27.0	1.272	1.209	1178.926	40.987
2.00	36.0	1.696	1.278	884.194	24.382
3.00	54.0	2.545	1.417	589.463	12.015
4.00	72.0	3.393	1.556	442.097	7.422
5.00	90.0	4.241	1.695	353.678	5.174
6.00	108.0	5.089	1.834	294.731	3.888
8.00	144.0	6.786	2.112	221.049	2.519
10.00	180.0	8.482	2.391	176.839	1.824
12.00	216.0	10.179	2.669	147.366	1.414
16.00	288.0	13.572	3.225	110.524	0.961
20.00	360.0	16.965	3.781	88.419	0.721
25.00	450.0	21.206	4.476	70.736	0.547
32.00	576.0	27.143	5.450	55.262	0.406
40.00	720.0	33.929	6.562	44.210	0.313
50.00	900.0	42.412	7.953	35.368	0.243

$$\sigma_C = -2100\left[\frac{1.204(202.6)}{F\cos 20°}\left(\frac{1}{0.228}+\frac{1}{0.684}\right)\right]^{1/2} = -100(10^3)$$

$$F = \left(\frac{2100}{100(10^3)}\right)^2\left[\frac{1.204(202.6)}{\cos 20°}\right]\left(\frac{1}{0.228}+\frac{1}{0.684}\right) = 0.669 \text{ in}$$

Use $F = 0.75$ in *Ans.*

14-13

$$d_p = 5(24) = 120 \text{ mm}, \quad d_G = 5(48) = 240 \text{ mm}$$

$$V = \frac{\pi(120)(10^{-3})(50)}{60} = 0.3142 \text{ m/s}$$

Eq. (14-6*a*): $$K_v = \frac{3.05 + 0.3142}{3.05} = 1.103$$

$$W^t = \frac{60\,000H}{\pi dn} = \frac{60(10^3)H}{\pi(120)(50)} = 3.183H$$

where H is in kW and W^t is in kN

Table 14-8: $C_p = 163\sqrt{\text{MPa}}$ [Note: Using Eq. (14-13) can result in wide variation in C_p due to wide variation in cast iron properties].

Eq. (14-12): $$r_1 = \frac{120\sin 20°}{2} = 20.52 \text{ mm}, \quad r_2 = \frac{240\sin 20°}{2} = 41.04 \text{ mm}$$

Eq. (14-14): $$-690 = -163\left[\frac{1.103(3.183)\left(10^3\right)H}{60\cos 20^o}\left(\frac{1}{20.52}+\frac{1}{41.04}\right)\right]^{1/2}$$

$$H = 3.94 \text{ kW} \quad \textit{Ans.}$$

14-14

$$d_P = 4(20) = 80 \text{ mm}, \quad d_G = 4(32) = 128 \text{ mm}$$

$$V = \frac{\pi(80)(10^{-3})(1000)}{60} = 4.189 \text{ m/s}$$

Eq. (14-6*a*): $$K_v = \frac{3.05 + 4.189}{3.05} = 2.373$$

$$W^t = \frac{60(10)(10^3)}{\pi(80)(1000)} = 2.387 \text{ kN} = 2387 \text{ N}$$

Table 14-8: $C_p = 163\sqrt{\text{MPa}}$ [Note: Using Eq. (14-13) can result in wide variation in C_p due to wide variation in cast iron properties.]

Eq. (14-12): $$r_1 = \frac{80\sin 20°}{2} = 13.68 \text{ mm}, \quad r_2 = \frac{128\sin 20°}{2} = 21.89 \text{ mm}$$

Eq. (14-14): $$\sigma_C = -163\left[\frac{2.373(2387)}{50\cos 20^\circ}\left(\frac{1}{13.68}+\frac{1}{21.89}\right)\right]^{1/2} = -617 \text{ MPa} \quad \textit{Ans.}$$

14-15 The pinion controls the design.

Bending $Y_P = 0.303, \quad Y_G = 0.359$

$$d_P = \frac{17}{12} = 1.417 \text{ in}, \quad d_G = \frac{30}{12} = 2.500 \text{ in}$$

$$V = \frac{\pi d_P n}{12} = \frac{\pi(1.417)(525)}{12} = 194.8 \text{ ft/min}$$

Eq. (14-4*b*): $$K_v = \frac{1200 + 194.8}{1200} = 1.162$$

Eq. (6-8), p. 282: $S'_e = 0.5(76) = 38.0$ kpsi

Eq. (6-19), p. 287: $k_a = 2.70(76)^{-0.265} = 0.857$

$$l = \frac{2.25}{P_d} = \frac{2.25}{12} = 0.1875 \text{ in}$$

Eq. (14-3): $$x = \frac{3Y_P}{2P} = \frac{3(0.303)}{2(12)} = 0.0379 \text{ in}$$

Eq. (b), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.1875)(0.0379)} = 0.1686$ in

Eq. (6-25), p. 289: $d_e = 0.808\sqrt{hb} = 0.808\sqrt{0.875(0.1686)} = 0.310$ in

Eq. (6-20), p. 288: $$k_b = \left(\frac{0.310}{0.3}\right)^{-0.107} = 0.996$$

$k_c = k_d = k_e = 1$

Account for one-way bending with $k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18), p. 287: $S_e = 0.857(0.996)(1)(1)(1.66)(38.0) = 53.84$ kpsi

For stress concentration, find the radius of the root fillet (See Ex. 14-2).

$$r_f = \frac{0.300}{P} = \frac{0.300}{12} = 0.025 \text{ in}$$

From Fig. A-15-6,

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.025}{0.1686} = 0.148$$

Approximate $D/d = \infty$ with $D/d = 3$; from Fig. A-15-6, $K_t = 1.68$.
From Fig. 6-20, with $S_{ut} = 76$ kpsi and $r = 0.025$ in, $q = 0.62$.

Eq. (6-32): $K_f = 1 + 0.62\ (1.68 - 1) = 1.42$

$$\sigma_{all} = \frac{S_e}{K_f n_d} = \frac{53.84}{1.42(2.25)} = 16.85 \text{ psi}$$

$$W^t = \frac{FY_P\sigma_{all}}{K_v P_d} = \frac{0.875(0.303)(16\,850)}{1.162(12)} = 320.4 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{320.4(194.8)}{33\,000} = 1.89 \text{ hp} \quad \textit{Ans.}$$

Wear

$\nu_1 = \nu_2 = 0.292, \quad E_1 = E_2 = 30(10^6)$ psi

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\frac{1-0.292^2}{30(10^6)}\right)}\right]^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{1.417}{2}\sin 20^\circ = 0.242 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{2.500}{2}\sin 20^\circ = 0.428 \text{ in}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{0.242} + \frac{1}{0.428} = 6.469 \text{ in}^{-1}$$

Eq. (6-68), p. 329: $(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi} = [0.4(149) - 10](10^3) = 49\,600$ psi

From the discussion and equation developed on the bottom of p. 329,

$$\sigma_{C,\text{all}} = -\frac{(S_C)_{10^8}}{\sqrt{n}} = \frac{-49\,600}{\sqrt{2.25}} = -33\,067 \text{ psi}$$

Eq. (14-14): $$W^t = \left(\frac{-33\,067}{2285}\right)^2\left[\frac{0.875\cos 20^\circ}{1.162(6.469)}\right] = 22.6 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{22.6(194.8)}{33\,000} = 0.133 \text{ hp} \quad \textit{Ans.}$$

Rating power (pinion controls):

$H_1 = 1.89$ hp

$H_2 = 0.133$ hp

$H_{\text{all}} = (\min 1.89, 0.133) = 0.133$ hp *Ans.*

14-16 See Prob. 14-15 solution for equation numbers.

Pinion controls: $Y_P = 0.322, \quad Y_G = 0.447$

Bending $d_P = 20/3 = 6.667$ in, $d_G = 100/3 = 33.333$ in

$V = \pi d_P n / 12 = \pi(6.667)(870) / 12 = 1519$ ft/min

$K_v = (1200 + 1519) / 1200 = 2.266$

$S'_e = 0.5(113) = 56.5$ kpsi

$k_a = 2.70(113)^{-0.265} = 0.771$

$l = 2.25 / P_d = 2.25 / 3 = 0.75$ in

$x = 3(0.322) / [2(3)] = 0.161$ in

$t = \sqrt{4(0.75)(0.161)} = 0.695$ in

$d_e = 0.808\sqrt{2.5(0.695)} = 1.065$ in

$k_b = (1.065 / 0.30)^{-0.107} = 0.873$

$k_c = k_d = k_e = 1$

$k_f = 1.66$ (See Ex. 14-2.)

$S_e = 0.771(0.873)(1)(1)(1)(1.66)(56.5) = 63.1$ kpsi

$r_f = 0.300 / 3 = 0.100$ in

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.100}{0.695} = 0.144$$

$K_t = 1.75$, $q = 0.85$, $K_f = 1.64$

$$\sigma_{all} = \frac{S_e}{K_f n_d} = \frac{63.1}{1.64(1.5)} = 25.7 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2.5(0.322)(25\,700)}{2.266(3)} = 3043 \text{ lbf}$$

$H = W^t V / 33\,000 = 3043(1519) / 33\,000 = 140$ hp *Ans.*

Wear

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\frac{1 - 0.292^2}{30\left(10^6\right)}\right)}\right]^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $r_1 = (6.667/2) \sin 20° = 1.140$ in

$r_2 = (33.333/2) \sin 20° = 5.700$ in

Eq. (6-68), p. 329: $S_C = [0.4(262) - 10](10^3) = 94\,800$ psi

$\sigma_{C,\text{all}} = -S_C / \sqrt{n_d} = -94\,800 / \sqrt{1.5} = -77\,400$ psi

$$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\left(\frac{1}{1/r_1 + 1/r_2}\right)$$

$$= \left(\frac{-77\ 400}{2285}\right)^2 \left(\frac{2.5\cos 20°}{2.266}\right)\left(\frac{1}{1/1.140 + 1/5.700}\right)$$

$$= 1130 \text{ lbf}$$

$$H = \frac{W^t V}{33\ 000} = \frac{1130(1519)}{33\ 000} = 52.0 \text{ hp} \quad \textit{Ans.}$$

For 10^8 cycles (revolutions of the pinion), the power based on wear is 52.0 hp.
Rating power (pinion controls):

$$H_1 = 140 \text{ hp}$$
$$H_2 = 52.0 \text{ hp}$$
$$H_{\text{rated}} = \min(140,\ 52.0) = 52.0 \text{ hp} \quad \textit{Ans.}$$

14-17 See Prob. 14-15 solution for equation numbers.
Given: $\phi = 20°$, $n = 1145$ rev/min, $m = 6$ mm, $F = 75$ mm, $N_P = 16$ milled teeth, $N_G = 30T$, $S_{ut} = 900$ MPa, $H_B = 260$, $n_d = 3$, $Y_P = 0.296$, and $Y_G = 0.359$.

Pinion bending

$$d_P = mN_P = 6(16) = 96 \text{ mm}$$
$$d_G = 6(30) = 180 \text{ mm}$$
$$V = \frac{\pi d_P n}{60} = \frac{\pi(96)(10^{-3})(1145)}{(60)} = 5.76 \text{ m/s}$$
$$K_v = \frac{6.1 + 5.76}{6.1} = 1.944$$
$$S'_e = 0.5(900) = 450 \text{ MPa}$$
$$k_a = 4.51(900)^{-0.265} = 0.744$$
$$l = 2.25m = 2.25(6) = 13.5 \text{ mm}$$
$$x = 3Ym/2 = 3(0.296)6/2 = 2.664 \text{ mm}$$
$$t = \sqrt{4lx} = \sqrt{4(13.5)(2.664)} = 12.0 \text{ mm}$$
$$d_e = 0.808\sqrt{75(12.0)} = 24.23 \text{ mm}$$
$$k_b = \left(\frac{24.23}{7.62}\right)^{-0.107} = 0.884$$
$$k_c = k_d = k_e = 1$$
$$k_f = 1.66 \text{ (See Ex. 14-2)}$$
$$S_e = 0.744(0.884)(1)(1)(1)(1.66)(450) = 491.3 \text{ MPa}$$
$$r_f = 0.300m = 0.300(6) = 1.8 \text{ mm}$$
$$r/d = r_f/t = 1.8/12 = 0.15,\ K_t = 1.68,\ q = 0.86,\ K_f = 1.58$$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{491.3}{1.58(1.3)} = 239.2 \text{ MPa}$$

Eq. (14-8): $$W^t = \frac{FYm\sigma_{\text{all}}}{K_v} = \frac{75(0.296)(6)(239.2)}{1.944} = 16\,390 \text{ N}$$

Eq. (13-36): $$H = \frac{W^t \pi dn}{60\,000} = \frac{16.39\pi(96)(1145)}{60\,000} = 94.3 \text{ kW} \quad \textit{Ans.}$$

Wear: Pinion and gear

Eq. (14-12): $r_1 = (96/2) \sin 20° = 16.42$ mm
$r_2 = (180/2) \sin 20° = 30.78$ mm

Eq. (14-13): $$C_p = \left[\frac{1}{2\pi\left(\frac{1 - 0.292^2}{207(10^3)}\right)}\right]^{1/2} = 190\sqrt{\text{MPa}}$$

Eq. (6-68), p. 329: $S_C = 6.89[0.4(260) - 10] = 647.7$ MPa

$$\sigma_{C,\text{all}} = -S_C / \sqrt{n_d} = \frac{-647.7}{\sqrt{1.3}} = -568 \text{ MPa}$$

Eq. (14-14): $$W^t = \left(\frac{\sigma_{C,\text{all}}}{C_p}\right)^2 \frac{F\cos\phi}{K_v}\left(\frac{1}{1/r_1 + 1/r_2}\right)$$

$$= \left(\frac{-568}{190}\right)^2 \left(\frac{75\cos 20^\circ}{1.944}\right)\left(\frac{1}{1/16.42 + 1/30.78}\right) = 3469 \text{ N}$$

Eq. (13-36): $$H = \frac{W^t \pi dn}{60\,000} = \frac{3.469\pi(96)(1145)}{60\,000} = 20.0 \text{ kW}$$

Thus, wear controls the gearset power rating; $H = 20.0$ kW. *Ans.*

14-18

$N_P = 17$ teeth, $N_G = 51$ teeth

$$d_P = \frac{N}{P} = \frac{17}{6} = 2.833 \text{ in}$$

$$d_G = \frac{51}{6} = 8.500 \text{ in}$$

$$V = \pi d_P n / 12 = \pi(2.833)(1120) / 12 = 830.7 \text{ ft/min}$$

Eq. (14-4*b*): $K_v = (1200 + 830.7)/1200 = 1.692$

$$\sigma_{\text{all}} = \frac{S_y}{n_d} = \frac{90\,000}{2} = 45\,000 \text{ psi}$$

Table 14-2: $Y_P = 0.303, \quad Y_G = 0.410$

Eq. (14-7): $W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P} = \frac{2(0.303)(45\,000)}{1.692(6)} = 2686 \text{ lbf}$

Eq. (13-35): $H = \frac{W^t V}{33\,000} = \frac{2686(830.7)}{33\,000} = 67.6 \text{ hp}$

Based on yielding in bending, the power is 67.6 hp.

(a) Pinion fatigue

Bending

Eq. (2-121), p. 41: $S_{ut} = 0.5\, H_B = 0.5(232) = 116$ kpsi

Eq. (6-8), p. 282: $S'_e = 0.5 S_{ut} = 0.5(116) = 58$ kpsi

Eq. (6-19), p. 287: $k_a = 2.70(116)^{-0.265} = 0.766$

Table 13-1, p. 696: $l = \frac{1}{P_d} + \frac{1.25}{P_d} = \frac{2.25}{P_d} = \frac{2.25}{6} = 0.375$ in

Eq. (14-3): $x = \frac{3Y_P}{2P} = \frac{3(0.303)}{2(6)} = 0.0758$ in

Eq. (*b*), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.375)(0.0758)} = 0.337$ in

Eq. (6-25), p. 289: $d_e = 0.808\sqrt{F\,t} = 0.808\sqrt{2(0.337)} = 0.663$ in

Eq. (6-20), p. 288: $k_b = \left(\frac{0.663}{0.30}\right)^{-0.107} = 0.919$

$k_c = k_d = k_e = 1$

Account for one-way bending with $k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18): $S_e = 0.766(0.919)(1)(1)(1)(1.66)(58) = 67.8$ kpsi

For stress concentration, find the radius of the root fillet (See Ex. 14-2).

$$r_f = \frac{0.300}{P} = \frac{0.300}{6} = 0.050 \text{ in}$$

Fig. A-15-6: $\frac{r}{d} = \frac{r_f}{t} = \frac{0.05}{0.338} = 0.148$

Estimate $D/d = \infty$ by setting $D/d = 3$, $K_t = 1.68$.

Fig. 6-20, p. 295: $q = 0.86$

Eq. (6-32), p. 295: $K_f = 1 + (0.86)(1.68 - 1) = 1.58$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{67.8}{1.58(2)} = 21.5 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.303)(21\,500)}{1.692(6)} = 1283 \text{ lbf}$$

$$H = \frac{W^t V}{33\,000} = \frac{1283(830.7)}{33\,000} = 32.3 \text{ hp} \quad \textit{Ans.}$$

(b) Pinion fatigue

Wear

Eq. (14-13): $$C_p = \left\{ \frac{1}{2\pi[(1 - 0.292^2)/30(10^6)]} \right\}^{1/2} = 2285\sqrt{\text{psi}}$$

Eq. (14-12): $$r_1 = \frac{d_P}{2}\sin\phi = \frac{2.833}{2}\sin 20^\circ = 0.485 \text{ in}$$

$$r_2 = \frac{d_G}{2}\sin\phi = \frac{8.500}{2}\sin 20^\circ = 1.454 \text{ in}$$

$$\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{1}{0.485} + \frac{1}{1.454} = 2.750 \text{ in}$$

Eq. (6-68): $$(S_C)_{10^8} = 0.4H_B - 10 \text{ kpsi}$$

In terms of gear notation

$$\sigma_C = [0.4(232) - 10]10^3 = 82\,800 \text{ psi}$$

We will introduce the design factor of $n_d = 2$ and because it is a contact stress apply it to the load W^t by dividing by $\sqrt{2}$. (See p. 329.)

$$\sigma_{C,\text{all}} = -\frac{\sigma_c}{\sqrt{2}} = -\frac{82\,800}{\sqrt{2}} = -58\,548 \text{ psi}$$

Solve Eq. (14-14) for W^t:

$$W^t = \left(\frac{-58\,548}{2285}\right)^2 \left[\frac{2\cos 20^\circ}{1.692(2.750)}\right] = 265 \text{ lbf}$$

$$H_{\text{all}} = \frac{W^t V}{33\,000} = \frac{265(830.7)}{33\,000} = 6.67 \text{ hp} \quad \textit{Ans.}$$

For 10^8 cycles (turns of pinion), the allowable power is 6.67 hp.

(c) Gear fatigue due to bending and wear

Bending

Eq. (14-3): $x = \dfrac{3Y_G}{2P} = \dfrac{3(0.4103)}{2(6)} = 0.1026$ in

Eq. (*b*), p. 737: $t = \sqrt{4lx} = \sqrt{4(0.375)(0.1026)} = 0.392$ in ±

Eq. (6-25): $d_e = 0.808\sqrt{F\,t} = 0.808\sqrt{2(0.392)} = 0.715$ in

Eq. (6-20): $k_b = \left(\dfrac{0.715}{0.30}\right)^{-0.107} = 0.911$

$k_c = k_d = k_e = 1$

$k_f = 1.66$. (See Ex. 14-2.)

Eq. (6-18): $S_e = 0.766(0.911)(1)(1)(1)(1.66)(58) = 67.2$ kpsi

$$\frac{r}{d} = \frac{r_f}{t} = \frac{0.050}{0.392} = 0.128$$

Approximate $D/d = \infty$ by setting $D/d = 3$ for Fig. A-15-6; $K_t = 1.80$.

Fig. 6-20: $q = 0.82$

Eq. (6-32): $K_f = 1 + (0.82)(1.80 - 1) = 1.66$

$$\sigma_{\text{all}} = \frac{S_e}{K_f n_d} = \frac{67.2}{1.66(2)} = 20.2 \text{ kpsi}$$

$$W^t = \frac{FY_P\sigma_{\text{all}}}{K_v P_d} = \frac{2(0.4103)(20\,200)}{1.692(6)} = 1633 \text{ lbf}$$

$$H_{\text{all}} = \frac{W^t V}{33\,000} = \frac{1633(830.7)}{33\,000} = 41.1 \text{ hp} \quad \textit{Ans.}$$

The gear is thus stronger than the pinion in bending.

Wear

Since the material of the pinion and the gear are the same, and the contact stresses are the same, the allowable power transmission of both is the same. Thus, $H_{\text{all}} = 6.67$ hp for 10^8 revolutions of each. As yet, we have no way to establish S_C for $10^8/3$ revolutions.

(d)

Pinion bending: $H_1 = 32.3$ hp
Pinion wear: $H_2 = 6.67$ hp
Gear bending: $H_3 = 41.1$ hp
Gear wear: $H_4 = 6.67$ hp

Power rating of the gear set is thus

$H_{\text{rated}} = \min(32.3, 6.67, 41.1, 6.67) = 6.67$ hp *Ans.*

14-19 $d_P = 16/6 = 2.667$ in, $d_G = 48/6 = 8$ in

$$V = \frac{\pi(2.667)(300)}{12} = 209.4 \text{ ft/min}$$

$$W^t = \frac{33\,000(5)}{209.4} = 787.8 \text{ lbf}$$

Assuming uniform loading, $K_o = 1$.

Eq. (14-28): $Q_v = 6, \quad B = 0.25(12 - 6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{59.77 + \sqrt{209.4}}{59.77}\right)^{0.8255} = 1.196$$

Table 14-2: $Y_P = 0.296, \quad Y_G = 0.4056$

From Eq. (*a*), Sec. 14-10 with $F = 2$ in

$$(K_s)_P = 1.192\left(\frac{2\sqrt{0.296}}{6}\right)^{0.0535} = 1.088$$

$$(K_s)_G = 1.192\left(\frac{2\sqrt{0.4056}}{6}\right)^{0.0535} = 1.097$$

From Eq. (14-30) with $C_{mc} = 1$

$$C_{pf} = \frac{2}{10(2.667)} - 0.0375 + 0.0125(2) = 0.0625$$

$C_{pm} = 1, \quad C_{ma} = 0.093$ (Fig. 14 - 11), $\quad C_e = 1$

$$K_m = 1 + 1[0.0625(1) + 0.093(1)] = 1.156$$

Assuming constant thickness of the gears → $K_B = 1$

$$m_G = N_G/N_P = 48/16 = 3$$

With N (pinion) = 10^8 cycles and N (gear) = $10^8/3$, Fig. 14-14 provides the relations:

$$(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$$

$$(Y_N)_G = 1.3558(10^8 / 3)^{-0.0178} = 0.996$$

Fig. 14-6: $J_P = 0.27, \quad J_G \doteq 0.38$

Table 14-10: $K_R = 0.85$

$K_T = C_f = 1$

Eq. (14-23): $$I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{3}{3 + 1}\right) = 0.1205$$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Strength: Grade 1 steel with $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(S_t)_P = (S_t)_G = 77.3(200) + 12\,800 = 28\,260$ psi

Fig. 14-5: $(S_c)_P = (S_c)_G = 322(200) + 29\,100 = 93\,500$ psi

Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$
$(Z_N)_G = 1.4488(10^8/3)^{-0.023} = 0.973$

Sec. 14-12: $H_{BP}/H_{BG} = 1 \quad \therefore C_H = 1$

Pinion tooth bending

Eq. (14-15): $$(\sigma)_P = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}$$
$$= 787.8(1)(1.196)(1.088)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.27}\right]$$
$$= 13\,170 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-41): $$(S_F)_P = \left[\frac{S_t Y_N \,/\, (K_T K_R)}{\sigma}\right]$$
$$= \frac{28\,260(0.977) \,/\, [(1)(0.85)]}{13\,170} = 2.47 \quad \textit{Ans.}$$

Gear tooth bending

Eq. (14-15): $$(\sigma)_G = 787.8(1)(1.196)(1.097)\left(\frac{6}{2}\right)\left[\frac{(1.156)(1)}{0.38}\right] = 9433 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-41): $$(S_F)_G = \frac{28\,260(0.996) \,/\, [(1)(0.85)]}{9433} = 3.51 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16): $$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2}$$

$$= 2300\left[787.8(1)(1.196)(1.088)\left(\frac{1.156}{2.667(2)}\right)\left(\frac{1}{0.1205}\right)\right]^{1/2}$$
$$= 98\,760 \text{ psi} \quad \textit{Ans.}$$

Eq. (14-42):
$$(S_H)_P = \left[\frac{S_c Z_N /(K_T K_R)}{\sigma_c}\right]_P = \left\{\frac{93\,500(0.948)/[(1)(0.85)]}{98\,760}\right\} = 1.06 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1.097}{1.088}\right)^{1/2}(98\ 760) = 99\ 170 \text{ psi} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{93\ 500(0.973)(1)/[(1)(0.85)]}{99\ 170} = 1.08 \quad \textit{Ans.}$$

The hardness of the pinion and the gear should be increased.

14-20

$$d_P = 2.5(20) = 50 \text{ mm}, \quad d_G = 2.5(36) = 90 \text{ mm}$$

$$V = \frac{\pi d_P n_P}{60} = \frac{\pi(50)(10^{-3})(100)}{60} = 0.2618 \text{ m/s}$$

$$W^t = \frac{60(120)}{\pi(50)(10^{-3})(100)} = 458.4 \text{ N}$$

With no specific information given to indicate otherwise, assume

$$K_B = K_o = Y_\theta = Z_R = 1$$

Eq. (14-28): $Q_v = 6$, $B = 0.25(12 - 6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left[\frac{59.77 + \sqrt{200(0.2618)}}{59.77}\right]^{0.8255} = 1.099$$

Table 14-2: $Y_P = 0.322, \quad Y_G = 0.3775$

Similar to Eq. (*a*) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$

$$(K_s)_P = 0.8433\left[2.5(18)\sqrt{0.322}\right]^{0.0535} = 1.003 \quad \text{use 1}$$

$$(K_s)_G = 0.8433\left[2.5(18)\sqrt{0.3775}\right]^{0.0535} = 1.007 \quad \text{use 1}$$

$$C_{mc} = C_e = C_{pm} = 1$$

$$F = 18 / 25.4 = 0.709 \text{ in}, \quad C_{pf} = \frac{18}{10(50)} - 0.025 = 0.011$$

$$C_{ma} = 0.247 + 0.0167(0.709) - 0.765(10^{-4})(0.709^2) = 0.259$$

$$K_H = 1 + 1[0.011(1) + 0.259(1)] = 1.27$$

Fig. 14-14: $(Y_N)_P = 1.3558(10^8)^{-0.0178} = 0.977$

$(Y_N)_G = 1.3558(10^8/1.8)^{-0.0178} = 0.987$

Fig. 14-6: $(Y_J)_P = 0.33, \quad (Y_J)_G = 0.38$

Eq. (14-38): $Y_Z = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$

Eq. (14-23): $Z_I = \dfrac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\dfrac{1.8}{1.8+1}\right) = 0.103$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Strength Grade 1 steel, $H_{BP} = H_{BG} = 200$

Fig. 14-2: $(S_t)_P = (S_t)_G = 0.533(200) + 88.3 = 194.9$ MPa
Fig. 14-5: $(S_c)_P = (S_c)_G = 2.22(200) + 200 = 644$ MPa
Fig. 14-15: $(Z_N)_P = 1.4488(10^8)^{-0.023} = 0.948$
$(Z_N)_G = 1.4488(10^8 / 1.8)^{-0.023} = 0.961$
Fig. 14-12: $H_{BP} / H_{BG} = 1 \quad \therefore \; Z_W = C_H = 1$

Pinion tooth bending

Eq. (14-15): $(\sigma)_P = \left(W^t K_o K_v K_s \dfrac{1}{bm_t}\dfrac{K_H K_B}{Y_J}\right)_P$

$$= 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.33}\right] = 43.08 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-41) for SI: $(S_F)_P = \left(\dfrac{S_t}{\sigma}\dfrac{Y_N}{Y_\theta Y_Z}\right)_P = \dfrac{194.9}{43.08}\left[\dfrac{0.977}{1(0.885)}\right] = 4.99 \quad \textit{Ans.}$

Gear tooth bending

$$(\sigma)_G = 458.4(1)(1.099)(1)\left[\frac{1}{18(2.5)}\right]\left[\frac{1.27(1)}{0.38}\right] = 37.42 \text{ MPa} \quad \textit{Ans.}$$

$$(S_F)_G = \frac{194.9}{37.42}\left[\frac{0.987}{1(0.885)}\right] = 5.81 \quad \textit{Ans.}$$

Pinion tooth wear

Eq. (14-16): $(\sigma_c)_P = \left(Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1}b}\dfrac{Z_R}{Z_I}}\right)_P$

$$= 191\sqrt{458.4(1)(1.099)(1)\left[\frac{1.27}{50(18)}\right]\left[\frac{1}{0.103}\right]} = 501.8 \text{ MPa} \quad \textit{Ans.}$$

Eq. (14-42) for SI: $(S_H)_P = \left(\dfrac{S_c}{\sigma_c}\dfrac{Z_N Z_W}{Y_\theta Y_Z}\right)_P = \dfrac{644}{501.8}\left[\dfrac{0.948(1)}{1(0.885)}\right] = 1.37 \quad \textit{Ans.}$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{(K_s)_G}{(K_s)_P}\right]^{1/2}(\sigma_c)_P = \left(\frac{1}{1}\right)^{1/2}(501.8) = 501.8 \text{ MPa} \quad \textit{Ans.}$$

$$(S_H)_G = \frac{644}{501.8}\frac{0.961(1)}{1(0.885)} = 1.39 \quad \textit{Ans.}$$

14-21

$$P_t = P_n \cos\psi = 6\cos 30° = 5.196 \text{ teeth/in}$$
$$d_P = \frac{16}{5.196} = 3.079 \text{ in}, \quad d_G = \frac{48}{16}(3.079) = 9.238 \text{ in}$$
$$V = \frac{\pi(3.079)(300)}{12} = 241.8 \text{ ft/min}$$
$$W^t = \frac{33\,000(5)}{241.8} = 682.3 \text{ lbf}, \quad K_v = \left(\frac{59.77 + \sqrt{241.8}}{59.77}\right)^{0.8255} = 1.210$$

From Prob. 14-19:

$$Y_P = 0.296, \quad Y_G = 0.4056$$
$$(K_s)_P = 1.088, \quad (K_s)_G = 1.097, \quad K_B = 1$$
$$m_G = 3, \quad (Y_N)_P = 0.977, \quad (Y_N)_G = 0.996, \quad K_R = 0.85$$
$$(S_t)_P = (S_t)_G = 28\,260 \text{ psi}, \quad C_H = 1, \quad (S_c)_P = (S_c)_G = 93\,500 \text{ psi}$$
$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973, \quad C_p = 2300\sqrt{\text{psi}}$$

The pressure angle is:

$$\phi_t = \tan^{-1}\left(\frac{\tan 20°}{\cos 30°}\right) = 22.80°$$
$$(r_b)_P = \frac{3.079}{2}\cos 22.8° = 1.419 \text{ in}, \quad (r_b)_G = 3(r_b)_P = 4.258 \text{ in}$$
$$a = 1 / P_n = 1 / 6 = 0.167 \text{ in}$$

Eq. (14-25):

$$Z = \left[\left(\frac{3.079}{2} + 0.167\right)^2 - 1.419^2\right]^{1/2} + \left[\left(\frac{9.238}{2} + 0.167\right)^2 - 4.258^2\right]^{1/2}$$
$$-\left(\frac{3.079}{2} + \frac{9.238}{2}\right)\sin 22.8°$$
$$= 0.9479 + 2.1852 - 2.3865 = 0.7466 \quad \text{Conditions } \textit{O.K.} \text{ for use}$$

$$p_N = p_n \cos\phi_n = \frac{\pi}{6}\cos 20° = 0.4920 \text{ in}$$

Eq. (14-21): $\quad m_N = \dfrac{p_N}{0.95Z} = \dfrac{0.492}{0.95(0.7466)} = 0.6937$

Eq. (14-23): $$I = \left[\frac{\sin 22.8^\circ \cos 22.8^\circ}{2(0.6937)}\right]\left(\frac{3}{3+1}\right) = 0.193$$

Fig. 14-7: $J'_P \doteq 0.45, \quad J'_G \doteq 0.54$

Fig. 14-8: Corrections are 0.94 and 0.98.

$$J_P = 0.45(0.94) = 0.423, \quad J_G = 0.54(0.98) = 0.529$$
$$C_{mc} = 1, \quad C_{pf} = \frac{2}{10(3.079)} - 0.0375 + 0.0125(2) = 0.0525$$
$$C_{pm} = 1, \quad C_{ma} = 0.093, \quad C_e = 1$$
$$K_m = 1 + (1)[0.0525(1) + 0.093(1)] = 1.146$$

Pinion tooth bending

$$(\sigma)_P = 682.3(1)(1.21)(1.088)\left(\frac{5.196}{2}\right)\frac{1.146(1)}{0.423} = 6323 \text{ psi} \quad \textit{Ans.}$$
$$(S_F)_P = \frac{28\,260(0.977)\ /\ [1(0.85)]}{6323} = 5.14 \quad \textit{Ans.}$$

Gear tooth bending

$$(\sigma)_G = 682.3(1)(1.21)(1.097)\left(\frac{5.196}{2}\right)\frac{1.146(1)}{0.529} = 5097 \text{ psi} \quad \textit{Ans.}$$
$$(S_F)_G = \frac{28\,260(0.996)\ /\ [1(0.85)]}{5097} = 6.50 \quad \textit{Ans.}$$

Pinion tooth wear

$$(\sigma_c)_P = 2300\left\{682.3(1)(1.21)(1.088)\left[\frac{1.146}{3.078(2)}\right]\left(\frac{1}{0.193}\right)\right\}^{1/2} = 67\,700 \text{ psi} \quad \textit{Ans.}$$
$$(S_H)_P = \frac{93\,500(0.948)\ /\ [(1)(0.85)]}{67\,700} = 1.54 \quad \textit{Ans.}$$

Gear tooth wear

$$(\sigma_c)_G = \left[\frac{1.097}{1.088}\right]^{1/2}(67\,700) = 67\,980 \text{ psi} \quad \textit{Ans.}$$
$$(S_H)_G = \frac{93\,500(0.973)/[(1)(0.85)]}{67\,980} = 1.57 \quad \textit{Ans.}$$

14-22 Given: $R = 0.99$ at 10^8 cycles, $H_B = 232$ through-hardening Grade 1, core and case, both gears. $N_P = 17T$, $N_G = 51T$,
Table 14-2: $Y_P = 0.303$, $Y_G = 0.4103$

Fig. 14-6: $J_P = 0.292,\ J_G = 0.396$

$d_P = N_P / P = 17 / 6 = 2.833$ in, $d_G = 51 / 6 = 8.500$ in.

Pinion bending

From Fig. 14-2:

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12\ 800$$
$$= 77.3(232) + 12\ 800 = 30\ 734 \text{ psi}$$

Fig. 14-14: $Y_N = 1.6831(10^8)^{-0.0323} = 0.928$

$$V = \pi d_P n / 12 = \pi(2.833)(1120 / 12) = 830.7 \text{ ft/min}$$
$$K_T = K_R = 1, \quad S_F = 2, \quad S_H = \sqrt{2}$$
$$\sigma_{\text{all}} = \frac{30\ 734(0.928)}{2(1)(1)} = 14\ 261 \text{ psi}$$
$$Q_v = 5, \quad B = 0.25(12 - 5)^{2/3} = 0.9148$$
$$A = 50 + 56(1 - 0.9148) = 54.77$$
$$K_v = \left(\frac{54.77 + \sqrt{830.7}}{54.77} \right)^{0.9148} = 1.472$$
$$K_s = 1.192\left(\frac{2\sqrt{0.303}}{6} \right)^{0.0535} = 1.089 \Rightarrow \text{use } 1$$
$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$
$$C_{mc} = 1$$
$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F$$
$$= \frac{2}{10(2.833)} - 0.0375 + 0.0125(2) = 0.0581$$
$$C_{pm} = 1$$
$$C_{ma} = 0.127 + 0.0158(2) - 0.093(10^{-4})(2^2) = 0.1586$$
$$C_e = 1$$
$$K_m = 1 + 1[0.0581(1) + 0.1586(1)] = 1.217$$
$$K_B = 1$$

Eq. (14-15): $$W^t = \frac{FJ_P\sigma_{\text{all}}}{K_o K_v K_s P_d K_m K_B}$$
$$= \frac{2(0.292)(14\ 261)}{1(1.472)(1)(6)(1.217)(1)} = 775 \text{ lbf}$$
$$H = \frac{W^t V}{33\ 000} = \frac{775(830.7)}{33\ 000} = 19.5 \text{ hp}$$

Pinion wear

Fig. 14-15: $Z_N = 2.466N^{-0.056} = 2.466(10^8)^{-0.056} = 0.879$

$m_G = 51 / 17 = 3$

Eq. (14-23): $I = \dfrac{\cos 20^\circ \sin 20^\circ}{2}\left(\dfrac{3}{3+1}\right) = 1.205, \quad C_H = 1$

Fig. 14-5: $_{0.99}(S_c)_{10^7} = 322H_B + 29\,100$

$= 322(232) + 29\,100 = 103\,804$ psi

$\sigma_{c,\text{all}} = \dfrac{103\,804(0.879)}{\sqrt{2}(1)(1)} = 64\,519$ psi

Eq. (14-16): $W^t = \left(\dfrac{\sigma_{c,\text{all}}}{C_p}\right)^2 \dfrac{Fd_P I}{K_o K_v K_s K_m C_f}$

$= \left(\dfrac{64\,519}{2300}\right)^2 \left[\dfrac{2(2.833)(0.1205)}{1(1.472)(1)(1.2167)(1)}\right]$

$= 300$ lbf

$H = \dfrac{W^t V}{33\,000} = \dfrac{300(830.7)}{33\,000} = 7.55$ hp

The pinion controls, therefore $H_{\text{rated}} = 7.55$ hp *Ans.*

14-23 $l = 2.25/P_d, \qquad x = 3Y/2P_d$

$$t = \sqrt{4lx} = \sqrt{4\left(\frac{2.25}{P_d}\right)\left(\frac{3Y}{2P_d}\right)} = \frac{3.674}{P_d}\sqrt{Y}$$

$$d_e = 0.808\sqrt{Ft} = 0.808\sqrt{F\left(\frac{3.674}{P_d}\right)\sqrt{Y}} = 1.5487\sqrt{\frac{F\sqrt{Y}}{P_d}}$$

$$k_b = \left(\frac{1.5487\sqrt{F\sqrt{Y}\ /\ P_d}}{0.30}\right)^{-0.107} = 0.8389\left(\frac{F\sqrt{Y}}{P_d}\right)^{-0.0535}$$

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P_d}\right)^{0.0535} \qquad \textit{Ans.}$$

14-24 $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $K_o = 1.25$. The service conditions are adequately described by K_o. Set $S_F = S_H = 1$.

$d_P = 22 / 4 = 5.500$ in

$d_G = 60 / 4 = 15.000$ in

$$V = \frac{\pi(5.5)(1145)}{12} = 1649 \text{ ft/min}$$

Pinion bending

$$_{0.99}(S_t)_{10^7} = 77.3H_B + 12\,800 = 77.3(250) + 12\,800 = 32\,125 \text{ psi}$$

$$Y_N = 1.6831[3(10^9)]^{-0.0323} = 0.832$$

Eq. (14-17): $$\left(\sigma_{\text{all}}\right)_P = \frac{32\,125(0.832)}{1(1)(1)} = 26\,728 \text{ psi}$$

$$B = 0.25(12 - 6)^{2/3} = 0.8255$$

$$A = 50 + 56(1 - 0.8255) = 59.77$$

$$K_v = \left(\frac{59.77 + \sqrt{1649}}{59.77}\right)^{0.8255} = 1.534$$

$$K_s = 1, \quad C_m = 1$$

$$C_{mc} = \frac{F}{10d} - 0.0375 + 0.0125F$$

$$= \frac{3.25}{10(5.5)} - 0.0375 + 0.0125(3.25) = 0.0622$$

$$C_{ma} = 0.127 + 0.0158(3.25) - 0.093(10^{-4})(3.25^2) = 0.178$$

$$C_e = 1$$

$$K_m = C_{mf} = 1 + (1)[0.0622(1) + 0.178(1)] = 1.240$$

$$K_B = 1, \quad K_T = 1$$

Eq. (14-15): $$W_1^t = \frac{26\,728(3.25)(0.345)}{1.25(1.534)(1)(4)(1.240)} = 3151 \text{ lbf}$$

$$H_1 = \frac{3151(1649)}{33\,000} = 157.5 \text{ hp}$$

Gear bending By similar reasoning, $W_2^t = 3861$ lbf and $H_2 = 192.9$ hp

Pinion wear

$$m_G = 60 / 22 = 2.727$$

$$I = \frac{\cos 20^\circ \sin 20^\circ}{2}\left(\frac{2.727}{1 + 2.727}\right) = 0.1176$$

$$_{0.99}(S_c)_{10^7} = 322(250) + 29\,100 = 109\,600 \text{ psi}$$

$$(Z_N)_P = 2.466[3(10^9)]^{-0.056} = 0.727$$

$$(Z_N)_G = 2.466[3(10^9) / 2.727]^{-0.056} = 0.769$$

$$(\sigma_{c,\text{all}})_P = \frac{109\,600(0.727)}{1(1)(1)} = 79\,679 \text{ psi}$$

$$W_3^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{F d_P I}{K_o K_v K_s K_m C_f}$$

$$= \left(\frac{79\ 679}{2300}\right)^2 \left[\frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)}\right] = 1061 \text{ lbf}$$

$$H_3 = \frac{1061(1649)}{33\ 000} = 53.0 \text{ hp}$$

Gear wear

Similarly, $W_4^t = 1182$ lbf, $H_4 = 59.0$ hp

Rating

$$H_{\text{rated}} = \min(H_1, H_2, H_3, H_4)$$

$$= \min(157.5, 192.9, 53, 59) = 53 \text{ hp} \quad \textit{Ans.}$$

Note differing capacities. Can these be equalized?

14-25 From Prob. 14-24:

$$W_1^t = 3151 \text{ lbf}, \quad W_2^t = 3861 \text{ lbf},$$
$$W_3^t = 1061 \text{ lbf}, \quad W_4^t = 1182 \text{ lbf}$$
$$W^t = \frac{33\ 000 K_o H}{V} = \frac{33\ 000(1.25)(40)}{1649} = 1000 \text{ lbf}$$

Pinion bending: The factor of safety, based on load and stress, is

$$(S_F)_P = \frac{W_1^t}{1000} = \frac{3151}{1000} = 3.15$$

Gear bending based on load and stress

$$(S_F)_G = \frac{W_2^t}{1000} = \frac{3861}{1000} = 3.86$$

Pinion wear

based on load: $n_3 = \dfrac{W_3^t}{1000} = \dfrac{1061}{1000} = 1.06$

based on stress: $(S_H)_P = \sqrt{1.06} = 1.03$

Gear wear

based on load: $n_4 = \dfrac{W_4^t}{1000} = \dfrac{1182}{1000} = 1.18$

based on stress: $(S_H)_G = \sqrt{1.18} = 1.09$

Factors of safety are used to assess the relative threat of loss of function 3.15, 3.86, 1.06, 1.18 where the threat is from pinion wear. By comparison, the AGMA safety factors

$$(S_F)_P,\ (S_F)_G,\ (S_H)_P,\ (S_H)_G$$

are

$$3.15,\ 3.86,\ 1.03,\ 1.09 \quad \text{or} \quad 3.15,\ 3.86,\ 1.06^{1/2},\ 1.18^{1/2}$$

and the threat is again from pinion wear. Depending on the magnitude of the numbers, using S_F and S_H as defined by AGMA, does not *necessarily* lead to the same conclusion concerning threat. Therefore be cautious.

14-26 Solution summary from Prob. 14-24: $n = 1145$ rev/min, $K_o = 1.25$, Grade 1 materials, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.345$, $J_G = 0.410$, $P_d = 4T$ /in, $F = 3.25$ in, $Q_v = 6$, $(N_c)_P = 3(10^9)$, $R = 0.99$, $K_m = 1.240$, $K_T = 1$, $K_B = 1$, $d_P = 5.500$ in, $d_G = 15.000$ in, $V = 1649$ ft/min, $K_v = 1.534$, $(K_s)_P = (K_s)_G = 1$, $(Y_N)_P = 0.832$, $(Y_N)_G = 0.859$, $K_R = 1$

Pinion H_B: 250 core, 390 case
Gear H_B: 250 core, 390 case

Bending

$$(\sigma_{\text{all}})_P = 26\ 728 \text{ psi} \qquad (S_t)_P = 32\ 125 \text{ psi}$$
$$(\sigma_{\text{all}})_G = 27\ 546 \text{ psi} \qquad (S_t)_G = 32\ 125 \text{ psi}$$
$$W_1^t = 3151 \text{ lbf}, \qquad H_1 = 157.5 \text{ hp}$$
$$W_2^t = 3861 \text{ lbf}, \qquad H_2 = 192.9 \text{ hp}$$

Wear

$$\phi = 20^\circ, \quad I = 0.1176, \quad (Z_N)_P = 0.727$$
$$(Z_N)_G = 0.769, \quad C_P = 2300\sqrt{\text{psi}}$$
$$(S_c)_P = S_c = 322(390) + 29\ 100 = 154\ 680 \text{ psi}$$
$$(\sigma_{c,\text{all}})_P = \frac{154\ 680(0.727)}{1(1)(1)} = 112\ 450 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = \frac{154\ 680(0.769)}{1(1)(1)} = 118\ 950 \text{ psi}$$

$$W_3^t = \left(\frac{112\ 450}{79\ 679}\right)^2 (1061) = 2113 \text{ lbf}, \qquad H_3 = \frac{2113(1649)}{33\ 000} = 105.6 \text{ hp}$$

$$W_4^t = \left(\frac{118\ 950}{109\ 600(0.769)}\right)^2 (1182) = 2354 \text{ lbf}, \quad H_4 = \frac{2354(1649)}{33\ 000} = 117.6 \text{ hp}$$

Rated power

$$H_{\text{rated}} = \min(157.5, 192.9, 105.6, 117.6) = 105.6 \text{ hp} \quad \textit{Ans.}$$

Prob. 14-24:

$$H_{\text{rated}} = \min(157.5, 192.9, 53.0, 59.0) = 53 \text{ hp}$$

The rated power approximately doubled.

14-27 The gear and the pinion are 9310 grade 1, carburized and case-hardened to obtain Brinell 285 core and Brinell 580–600 case.

Table 14-3: $_{0.99}(S_t)_{10^7} = 55\,000$ psi

Modification of S_t by $(Y_N)_P = 0.832$ produces

$$(\sigma_{\text{all}})_P = 45\,657 \text{ psi},$$

Similarly for $(Y_N)_G = 0.859$

$$(\sigma_{\text{all}})_G = 47\,161 \text{ psi}, \quad \text{and}$$
$$W_1^t = 4569 \text{ lbf}, \quad H_1 = 228 \text{ hp}$$
$$W_2^t = 5668 \text{ lbf}, \quad H_2 = 283 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$$_{0.99}(S_c)_{10^7} = 180\,000 \text{ psi}$$

Modification of S_c by Y_N produces

$$(\sigma_{c,\text{all}})_P = 130\,525 \text{ psi}$$
$$(\sigma_{c,\text{all}})_G = 138\,069 \text{ psi}$$

and

$$W_3^t = 2489 \text{ lbf}, \quad H_3 = 124.3 \text{ hp}$$
$$W_4^t = 2767 \text{ lbf}, \quad H_4 = 138.2 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(228, 283, 124, 138) = 124 \text{ hp} \quad \textit{Ans.}$$

14-28 Grade 2, 9310 carburized and case-hardened to 285 core and 580 case in Prob. 14-27.

Summary:

Table 14-3: $_{0.99}(S_t)_{10^7} = 65\ 000$ psi

$(\sigma_{all})_P = 53\ 959$ psi

$(\sigma_{all})_G = 55\ 736$ psi

and it follows that

$$W_1^t = 5400 \text{ lbf}, \quad H_1 = 270 \text{ hp}$$
$$W_2^t = 6699 \text{ lbf}, \quad H_2 = 335 \text{ hp}$$

From Table 14-8, $C_p = 2300\sqrt{\text{psi}}$. Also, from Table 14-6:

$$S_c = 225\ 000 \text{ psi}$$
$$(\sigma_{c,all})_P = 181\ 285 \text{ psi}$$
$$(\sigma_{c,all})_G = 191\ 762 \text{ psi}$$

Consequently,

$$W_3^t = 4801 \text{ lbf}, \quad H_3 = 240 \text{ hp}$$
$$W_4^t = 5337 \text{ lbf}, \quad H_4 = 267 \text{ hp}$$

Rating

$H_{\text{rated}} = \min(270, 335, 240, 267) = 240$ hp. *Ans.*

14-29 Given: $n = 1145$ rev/min, $K_o = 1.25$, $N_P = 22T$, $N_G = 60T$, $m_G = 2.727$, $d_P = 2.75$ in, $d_G = 7.5$ in, $Y_P = 0.331$, $Y_G = 0.422$, $J_P = 0.335$, $J_G = 0.405$, $P = 8T$ /in, $F = 1.625$ in, $H_B = 250$, case and core, both gears. $C_m = 1$, $F/d_P = 0.0591$, $C_f = 0.0419$, $C_{pm} = 1$, $C_{ma} = 0.152$, $C_e = 1$, $K_m = 1.1942$, $K_T = 1$, $K_B = 1$, $K_s = 1$, $V = 824$ ft/min, $(Y_N)_P = 0.8318$, $(Y_N)_G = 0.859$, $K_R = 1$, $I = 0.117\ 58$

$$_{0.99}(S_t)_{10^7} = 32\ 125 \text{ psi}$$
$$(\sigma_{all})_P = 26\ 668 \text{ psi}$$
$$(\sigma_{all})_G = 27\ 546 \text{ psi}$$

and it follows that

$$W_1^t = 879.3 \text{ lbf}, \quad H_1 = 21.97 \text{ hp}$$
$$W_2^t = 1098 \text{ lbf}, \quad H_2 = 27.4 \text{ hp}$$

For wear

$$W_3^t = 304 \text{ lbf}, \quad H_3 = 7.59 \text{ hp}$$
$$W_4^t = 340 \text{ lbf}, \quad H_4 = 8.50 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(21.97, 27.4, 7.59, 8.50) = 7.59 \text{ hp}$$

In Prob. 14-24, $H_{\text{rated}} = 53$ hp. Thus,

$$\frac{7.59}{53.0} = 0.1432 = \frac{1}{6.98}, \quad \text{not } \frac{1}{8} \quad \textit{Ans.}$$

The transmitted load rating is

$$W_{\text{rated}}^t = \min(879.3, 1098, 304, 340) = 304 \text{ lbf}$$

In Prob. 14-24

$$W_{\text{rated}}^t = 1061 \text{ lbf}$$

Thus

$$\frac{304}{1061} = 0.2865 = \frac{1}{3.49}, \quad \text{not } \frac{1}{4} \quad \textit{Ans.}$$

14-30 $S_P = S_H = 1, \quad P_d = 4, \quad J_P = 0.345, \quad J_G = 0.410, \quad K_o = 1.25$

Bending

Table 14-4: ${}_{0.99}(S_t)_{10^7} = 13\,000$ psi

$$(\sigma_{\text{all}})_P = (\sigma_{\text{all}})_G = \frac{13\,000(1)}{1(1)(1)} = 13\,000 \text{ psi}$$

$$W_1^t = \frac{\sigma_{\text{all}} F J_P}{K_o K_v K_s P_d K_m K_B} = \frac{13\,000(3.25)(0.345)}{1.25(1.534)(1)(4)(1.24)(1)} = 1533 \text{ lbf}$$

$$H_1 = \frac{1533(1649)}{33\,000} = 76.6 \text{ hp}$$

$$W_2^t = W_1^t J_G \,/\, J_P = 1533(0.410) \,/\, 0.345 = 1822 \text{ lbf}$$

$$H_2 = H_1 J_G \,/\, J_P = 76.6(0.410) \,/\, 0.345 = 91.0 \text{ hp}$$

Wear

Table 14-8: $C_p = 1960\sqrt{\text{psi}}$

Table 14-7: ${}_{0.99}(S_c)_{10^7} = 75\,000 \text{ psi} = (\sigma_{c,\text{all}})_P = (\sigma_{c,\text{all}})_G$

$$W_3^t = \left[\frac{(\sigma_{c,\text{all}})_P}{C_p}\right]^2 \frac{F d_p I}{K_o K_v K_s K_m C_f}$$

$$W_3^t = \left(\frac{75\ 000}{1960}\right)^2 \frac{3.25(5.5)(0.1176)}{1.25(1.534)(1)(1.24)(1)} = 1295 \text{ lbf}$$

$$W_4^t = W_3^t = 1295 \text{ lbf}$$

$$H_4 = H_3 = \frac{1295(1649)}{33\ 000} = 64.7 \text{ hp}$$

Rating

$$H_{\text{rated}} = \min(76.7, 94.4, 64.7, 64.7) = 64.7 \text{ hp} \quad \textit{Ans.}$$

Notice that the balance between bending and wear power is improved due to CI's more favorable S_c/S_t ratio. Also note that the life is 10^7 pinion revolutions which is (1/300) of $3(10^9)$. Longer life goals require power de-rating.

14-31 From Table A-24*a*, $E_{av} = 11.8(10^6)$ Mpsi
For $\phi = 14.5°$ and $H_B = 156$

$$S_C = \sqrt{\frac{1.4(81)}{2\sin 14.5° \,/\, [11.8(10^6)]}} = 51\ 693 \text{ psi}$$

For $\phi = 20°$

$$S_C = \sqrt{\frac{1.4(112)}{2\sin 20° \,/\, [11.8(10^6)]}} = 52\ 008 \text{ psi}$$

$$S_C = 0.32(156) = 49.9 \text{ kpsi}$$

The first two calculations were approximately 4 percent higher.

14-32 Programs will vary.

14-33

$$(Y_N)_P = 0.977, \quad (Y_N)_G = 0.996$$

$$(S_t)_P = (S_t)_G = 82.3(250) + 12\ 150 = 32\ 725 \text{ psi}$$

$$(\sigma_{\text{all}})_P = \frac{32\ 725(0.977)}{1(0.85)} = 37\ 615 \text{ psi}$$

$$W_1^t = \frac{37\ 615(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 1558 \text{ lbf}$$

$$H_1 = \frac{1558(925)}{33\ 000} = 43.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{32\ 725(0.996)}{1(0.85)} = 38\ 346 \text{ psi}$$

$$W_2^t = \frac{38\ 346(1.5)(0.5346)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2007 \text{ lbf}$$

$$H_2 = \frac{2007(925)}{33\ 000} = 56.3 \text{ hp}$$

$$(Z_N)_P = 0.948, \quad (Z_N)_G = 0.973$$

Table 14-6: ${}_{0.99}(S_c)_{10^7} = 150\ 000$ psi

$$(\sigma_{c,\text{allow}})_P = 150\ 000\left[\frac{0.948(1)}{1(0.85)}\right] = 167\ 294 \text{ psi}$$

$$W_3^t = \left(\frac{167\ 294}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2074 \text{ lbf}$$

$$H_3 = \frac{2074(925)}{33\ 000} = 58.1 \text{ hp}$$

$$(\sigma_{c,\text{allow}})_G = \frac{0.973}{0.948}(167\ 294) = 171\ 706 \text{ psi}$$

$$W_4^t = \left(\frac{171\ 706}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.052)}\right] = 2167 \text{ lbf}$$

$$H_4 = \frac{2167(925)}{33\ 000} = 60.7 \text{ hp}$$

$$H_{\text{rated}} = \min(43.7,\ 56.3,\ 58.1,\ 60.7) = 43.7 \text{ hp} \quad \textit{Ans.}$$

Pinion bending is controlling.

14-34

$$(Y_N)_P = 1.6831(10^8)^{-0.0323} = 0.928$$

$$(Y_N)_G = 1.6831(10^8 / 3.059)^{-0.0323} = 0.962$$

Table 14-3: $S_t = 55\ 000$ psi

$$(\sigma_{\text{all}})_P = \frac{55\ 000(0.928)}{1(0.85)} = 60\ 047 \text{ psi}$$

$$W_1^t = \frac{60\ 047(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)(1)} = 2487 \text{ lbf}$$

$$H_1 = \frac{2487(925)}{33\ 000} = 69.7 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{0.962}{0.928}(60\ 047) = 62\ 247 \text{ psi}$$

$$W_2^t = \frac{62\ 247}{60\ 047}\left(\frac{0.5346}{0.423}\right)(2487) = 3258 \text{ lbf}$$

$$H_2 = \frac{3258}{2487}(69.7) = 91.3 \text{ hp}$$

Table 14-6: $S_c = 180\ 000$ psi

$$(Z_N)_P = 2.466(10^8)^{-0.056} = 0.8790$$

$$(Z_N)_G = 2.466(10^8 / 3.059)^{-0.056} = 0.9358$$

$$(\sigma_{c,\text{all}})_P = \frac{180\ 000(0.8790)}{1(0.85)} = 186\ 141 \text{ psi}$$

$$W_3^t = \left(\frac{186\ 141}{2300}\right)^2\left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 2568 \text{ lbf}$$

$$H_3 = \frac{2568(925)}{33\ 000} = 72.0 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(186\ 141) = 198\ 169 \text{ psi}$$

$$W_4^t = \left(\frac{198\ 169}{186\ 141}\right)^2\left(\frac{1.043}{1.052}\right)(2568) = 2886 \text{ lbf}$$

$$H_4 = \frac{2886(925)}{33\ 000} = 80.9 \text{ hp}$$

$$H_{\text{rated}} = \min(69.7,\ 91.3,\ 72,\ 80.9) = 69.7 \text{ hp} \quad \textit{Ans.}$$

Pinion bending controlling

14-35 $(Y_N)_P = 0.928$, $(Y_N)_G = 0.962$ (See Prob. 14-34)

Table 14-3: $S_t = 65\ 000$ psi

$$(\sigma_{\text{all}})_P = \frac{65\ 000(0.928)}{1(0.85)} = 70\ 965 \text{ psi}$$

$$W_1^t = \frac{70\ 965(1.5)(0.423)}{1(1.404)(1.043)(8.66)(1.208)} = 2939 \text{ lbf}$$

$$H_1 = \frac{2939(925)}{33\ 000} = 82.4 \text{ hp}$$

$$(\sigma_{\text{all}})_G = \frac{65\ 000(0.962)}{1(0.85)} = 73\ 565 \text{ psi}$$

$$W_2^t = \frac{73\ 565}{70\ 965}\left(\frac{0.5346}{0.423}\right)(2939) = 3850 \text{ lbf}$$

$$H_2 = \frac{3850}{2939}(82.4) = 108 \text{ hp}$$

Table 14-6: $S_c = 225\ 000$ psi

$(Z_N)_P = 0.8790, \quad (Z_N)_G = 0.9358$

$$(\sigma_{c,\text{all}})_P = \frac{225\ 000(0.879)}{1(0.85)} = 232\ 676 \text{ psi}$$

$$W_3^t = \left(\frac{232\ 676}{2300}\right)^2 \left[\frac{1.963(1.5)(0.195)}{1(1.404)(1.043)}\right] = 4013 \text{ lbf}$$

$$H_3 = \frac{4013(925)}{33\ 000} = 112.5 \text{ hp}$$

$$(\sigma_{c,\text{all}})_G = \frac{0.9358}{0.8790}(232\ 676) = 247\ 711 \text{ psi}$$

$$W_4^t = \left(\frac{247\ 711}{232\ 676}\right)^2 \left(\frac{1.043}{1.052}\right)(4013) = 4509 \text{ lbf}$$

$$H_4 = \frac{4509(925)}{33\ 000} = 126 \text{ hp}$$

$H_{\text{rated}} = \min(82.4, 108, 112.5, 126) = 82.4$ hp *Ans.*

The bending of the pinion is the controlling factor.

14-36

$P = 2$ teeth/in, $d = 8$ in, $N = dP = 8\ (2) = 16$ teeth

$$F = 4p = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{2}\right) = 2\pi$$

$$\sum M_x = 0 = 10(300)\cos 20^\circ - 4F_B \cos 20^\circ$$

$F_B = 750$ lbf

$W^t = F_B \cos 20^\circ = 750 \cos 20^\circ = 705$ lbf

$n = 2400 / 2 = 1200$ rev/min

$$V = \frac{\pi dn}{12} = \frac{\pi(8)(1200)}{12} = 2513 \text{ ft/min}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Fig. 14-2: $S_t = 102(300) + 16\ 400 = 47\ 000$ psi

Fig. 14-5: $S_c = 349(300) + 34\ 300 = 139\ 000$ psi

Fig. 14-6: $J = 0.27$

Eq. (14-23): $$I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{2}{2+1}\right) = 0.107$$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Assume a typical quality number of 6.

Eq. (14-28): $B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$

$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B = \left(\frac{59.77 + \sqrt{2513}}{59.77} \right)^{0.8255} = 1.65$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, $Y = 0.296$. From Eq. (*a*), Sec. 14-10,

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} = 1.192 \left(\frac{2\pi\sqrt{0.296}}{2} \right)^{0.0535} = 1.23$$

The load distribution factor is applicable for straddle-mounted gears, which is not the case here since the gear is mounted outboard of the bearings. Lacking anything better, we will use the load distribution factor as a rough estimate.

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $$C_{pf} = \frac{2\pi}{10(8)} - 0.0375 + 0.0125(2\pi) = 0.1196$$

Eq. (14-33): $C_{pm} = 1.1$

Fig. 14-11: $C_{ma} = 0.23$ (commercial enclosed gear unit)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $K_m = 1 + 1[0.1196(1.1) + 0.23(1)] = 1.36$

For the stress-cycle factors, we need the desired number of load cycles.

$$N = 15\,000 \text{ h } (1200 \text{ rev/min})(60 \text{ min/h}) = 1.1\ (10^9) \text{ rev}$$

Fig. 14-14: $Y_N = 0.9$

Fig. 14-15: $Z_N = 0.8$

Eq. 14-38: $$K_R = 0.658 - 0.0759 \ln(1 - R) = 0.658 - 0.0759 \ln(1 - 0.95) = 0.885$$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = C_f = 1$

Tooth bending

Eq. (14-15): $$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

$$= 705(1)(1.65)(1.23) \left(\frac{2}{2\pi} \right) \left[\frac{(1.36)(1)}{0.27} \right] = 2294 \text{ psi}$$

Eq. (14-41): $$S_F = \left[\frac{S_t Y_N \ / \ (K_T K_R)}{\sigma} \right]$$

$$= \frac{47\,000(0.9) \ / \ [(1)(0.885)]}{2294} = 20.8 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $$\sigma_c = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)^{1/2}$$

$$= 2300\left[705(1)(1.65)(1.23)\left(\frac{1.36}{8(2\pi)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 43\,750 \text{ psi}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \frac{S_c Z_N \,/\, (K_T K_R)}{\sigma_c}$$

$$= \left\{\frac{139\,000(0.8) \,/\, [(1)(0.885)]}{43\,750}\right\} = 2.9 \quad \textit{Ans}$$

14-37

$m = 18.75$ mm/tooth, $d = 300$ mm

$N = d/m = 300 / 18.75 = 16$ teeth

$F = b = 4p = 4(\pi m) = 4\pi(18.75) = 236$ mm

$\sum M_x = 0 = 300(11)\cos 20^\circ - 150F_B \cos 25^\circ$

$F_B = 22.81$ kN

$W^t = F_B \cos 25^\circ = 22.81\cos 25^\circ = 20.67$ kN

$n = 1800 / 2 = 900$ rev/min

$$V = \frac{\pi dn}{60} = \frac{\pi(0.300)(900)}{60} = 14.14 \text{ m/s}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Fig. 14-2: $S_t = 0.703(300) + 113 = 324$ MPa

Fig. 14-5: $S_c = 2.41(300) + 237 = 960$ MPa

Fig. 14-6: $J = Y_J = 0.27$

Eq. (14-23): $$I = Z_I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{5}{5+1}\right) = 0.134$$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Assume a typical quality number of 6.

Eq. (14-28): $B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$

$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200(14.14)}}{59.77}\right)^{0.8255} = 1.69$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, $Y = 0.296$. Similar to Eq. (a) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$

$$K_s = 0.8433\left[18.75(236)\sqrt{0.296}\right]^{0.0535} = 1.28$$

Convert the diameter and facewidth to inches for use in the load-distribution factor equations. $d = 300/25.4 = 11.81$ in, $F = 236/25.4 = 9.29$ in

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $C_{pf} = \frac{9.29}{10(11.81)} - 0.0375 + 0.0125(9.29) = 0.1573$

Eq. (14-33): $C_{pm} = 1.1$

Fig. 14-11: $C_{ma} = 0.27$ (commercial enclosed gear unit)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $K_m = K_H = 1 + 1[0.1573(1.1) + 0.27(1)] = 1.44$

For the stress-cycle factors, we need the desired number of load cycles.

$N = 12\,000$ h (900 rev/min)(60 min/h) = 6.48 (10^8) rev

Fig. 14-14: $Y_N = 0.9$

Fig. 14-15: $Z_N = 0.85$

Eq. 14-38: $K_R = 0.658 - 0.0759\ln(1-R) = 0.658 - 0.0759\ln(1-0.98) = 0.955$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = Z_R = 1$.

Tooth bending

Eq. (14-15): $\sigma = W^t K_o K_v K_s \frac{1}{bm_t}\frac{K_H K_B}{Y_J}$

$$= 20\,670(1)(1.69)(1.28)\left[\frac{1}{236(18.75)}\right]\left[\frac{(1.44)(1)}{0.27}\right] = 53.9 \text{ MPa}$$

Eq. (14-41): $S_F = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma}\right]$

$$= \frac{324(0.9) / [(1)(0.955)]}{53.9} = 5.66 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $\sigma_c = Z_E\left(W^t K_o K_v K_s \frac{K_H}{d_{w1}b}\frac{Z_R}{Z_I}\right)^{1/2}$

$$= 191\left[20\,670(1)(1.69)(1.28)\left(\frac{1.44}{300(236)}\right)\left(\frac{1}{0.134}\right)\right]^{1/2}$$
$$= 498 \text{ MPa}$$

Since gear B is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \frac{S_c Z_N \,/\, (K_T K_R)}{\sigma_c}$$
$$= \frac{960(0.85) \,/\, [(1)(0.955)]}{498} = 1.72 \quad \textit{Ans}$$

14-38 From the solution to Prob. 13-40, n = 191 rev/min, W^t = 1600 N, d = 125 mm, N = 15 teeth, m = 8.33 mm/tooth.

$$F = b = 4p = 4(\pi m) = 4\pi(8.33) = 105 \text{ mm}$$
$$V = \frac{\pi dn}{60} = \frac{\pi(0.125)(191)}{60} = 1.25 \text{ m/s}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Table 14-3: S_t = 65 kpsi = 448 MPa
Table 14-6: S_c = 225 kpsi = 1550 MPa
Fig. 14-6: $J = Y_J = 0.25$

Eq. (14-23): $$I = Z_I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{2}{2+1}\right) = 0.107$$

Table 14-8: $Z_E = 191\sqrt{\text{MPa}}$

Assume a typical quality number of 6.

Eq. (14-28): $$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{200V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{200(1.25)}}{59.77}\right)^{0.8255} = 1.21$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, Y = 0.290. Similar to Eq. (*a*) of Sec. 14-10 but for SI units:

$$K_s = \frac{1}{k_b} = 0.8433\left(mF\sqrt{Y}\right)^{0.0535}$$
$$K_s = 0.8433\left[8.33(105)\sqrt{0.290}\right]^{0.0535} = 1.17$$

Convert the diameter and facewidth to inches for use in the load-distribution factor

equations. $d = 125/25.4 = 4.92$ in, $F = 105/25.4 = 4.13$ in

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $C_{pf} = \dfrac{4.13}{10(4.92)} - 0.0375 + 0.0125(4.13) = 0.0981$

Eq. (14-33): $C_{pm} = 1$

Fig. 14-11: $C_{ma} = 0.32$ (open gearing)

Eq. (14-35): $C_e = 1$

Eq. (14-30): $K_m = K_H = 1 + 1[0.0981(1) + 0.32(1)] = 1.42$

For the stress-cycle factors, we need the desired number of load cycles.

$N = 12\,000$ h (191 rev/min)(60 min/h) = 1.4 (10^8) rev

Fig. 14-14: $Y_N = 0.95$

Fig. 14-15: $Z_N = 0.88$

Eq. 14-38: $K_R = 0.658 - 0.0759\ln(1-R) = 0.658 - 0.0759\ln(1-0.95) = 0.885$

With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = Z_R = 1$.

Tooth bending

Eq. (14-15): $\sigma = W^t K_o K_v K_s \dfrac{1}{bm_t}\dfrac{K_H K_B}{Y_J}$

$$= 1600(1)(1.21)(1.17)\left[\frac{1}{105(8.33)}\right]\left[\frac{(1.42)(1)}{0.25}\right] = 14.7 \text{ MPa}$$

Since gear is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_F = \left[\frac{S_t Y_N / (K_T K_R)}{\sigma}\right]$$

$$= \frac{448(0.95) / [(1)(0.885)]}{14.7} = 32.7 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $\sigma_c = Z_E\left(W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}\right)^{1/2}$

$$= 191\left[1600(1)(1.21)(1.17)\left(\frac{1.42}{125(105)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 289 \text{ MPa}$$

Eq. (14-42): $S_H = \left[\dfrac{S_c Z_N / (K_T K_R)}{\sigma_c}\right]$

$$= \left\{\frac{1550(0.88) / [(1)(0.885)]}{289}\right\} = 5.33 \quad \textit{Ans}$$

14-39 From the solution to Prob. 13-41, $n = 2(70) = 140$ rev/min, $W^t = 180$ lbf, $d = 5$ in
$N = 15$ teeth, $P = 3$ teeth/in.

$$F = 4p = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{3}\right) = 4.2 \text{ in}$$

$$V = \frac{\pi dn}{12} = \frac{\pi(5)(140)}{12} = 183.3 \text{ ft/min}$$

We will obtain all of the needed factors, roughly in the order presented in the textbook.

Table 14-3: $S_t = 65$ kpsi
Table 14-6: $S_c = 225$ kpsi
Fig. 14-6: $J = 0.25$

Eq. (14-23): $$I = \frac{\cos 20^\circ \sin 20^\circ}{2(1)}\left(\frac{2}{2+1}\right) = 0.107$$

Table 14-8: $C_p = 2300\sqrt{\text{psi}}$

Assume a typical quality number of 6.

Eq. (14-28): $$B = 0.25(12 - Q_v)^{2/3} = 0.25(12 - 6)^{2/3} = 0.8255$$
$$A = 50 + 56(1 - B) = 50 + 56(1 - 0.8255) = 59.77$$

Eq. (14-27): $$K_v = \left(\frac{A + \sqrt{V}}{A}\right)^B = \left(\frac{59.77 + \sqrt{183.3}}{59.77}\right)^{0.8255} = 1.18$$

To estimate a size factor, get the Lewis Form Factor from Table 14-2, $Y = 0.290$.
From Eq. (*a*), Sec. 14-10,

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.192\left(\frac{4.2\sqrt{0.290}}{3}\right)^{0.0535} = 1.17$$

Eq. (14-31): $C_{mc} = 1$ (uncrowned teeth)

Eq. (14-32): $$C_{pf} = \frac{4.2}{10(5)} - 0.0375 + 0.0125(4.2) = 0.099$$

Eq. (14-33): $C_{pm} = 1$
Fig. 14-11: $C_{ma} = 0.32$ (Open gearing)
Eq. (14-35): $C_e = 1$
Eq. (14-30): $K_m = 1 + 1[0.099(1) + 0.32(1)] = 1.42$

For the stress-cycle factors, we need the desired number of load cycles.
$N = 14\,000$ h (140 rev/min)(60 min/h) = 1.2 (10^8) rev
Fig. 14-14: $Y_N = 0.95$
Fig. 14-15: $Z_N = 0.88$
Eq. 14-38: $K_R = 0.658 - 0.0759\ln(1 - R) = 0.658 - 0.0759\ln(1 - 0.98) = 0.955$
With no specific information given to indicate otherwise, assume $K_o = K_B = K_T = C_f = 1$.

Tooth bending

Eq. (14-15): $$\sigma = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_B}{J}$$

$$= 180(1)(1.18)(1.17)\left(\frac{3}{4.2}\right)\left[\frac{(1.42)(1)}{0.25}\right] = 1010 \text{ psi}$$

Eq. (14-41): $$S_F = \left[\frac{S_t Y_N \ / \ (K_T K_R)}{\sigma}\right]$$

$$= \frac{65\,000(0.95) \ / \ [(1)(0.955)]}{1010} = 64.0 \quad \textit{Ans.}$$

Tooth wear

Eq. (14-16): $$\sigma_c = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)^{1/2}$$

$$= 2300\left[180(1)(1.18)(1.17)\left(\frac{1.42}{5(4.2)}\right)\left(\frac{1}{0.107}\right)\right]^{1/2}$$

$$= 28\,800 \text{ psi}$$

Since gear *B* is a pinion, C_H is not used in Eq. (14-42) (see p. 761), where

$$S_H = \left[\frac{S_c Z_N \ / \ (K_T K_R)}{\sigma_c}\right]$$

$$= \left\{\frac{225\,000(0.88) \ / \ [(1)(0.955)]}{28\,800}\right\} = 7.28 \quad \textit{Ans}$$

Chapter 15

15-1 Given: Uncrowned, through-hardened 300 Brinell core and case, Grade 1, $N_C = 10^9$ rev of pinion at $R = 0.999$, $N_P = 20$ teeth, $N_G = 60$ teeth, $Q_v = 6$, $P_d = 6$ teeth/in, shaft angle = 90°, $n_p = 900$ rev/min, $J_P = 0.249$ and $J_G = 0.216$ (Fig. 15-7), $F = 1.25$ in, $S_F = S_H = 1$, $K_o = 1$.

Mesh $d_P = 20/6 = 3.333$ in, $d_G = 60/6 = 10.000$ in

Eq. (15-7): $v_t = \pi(3.333)(900/12) = 785.3$ ft/min

Eq. (15-6): $B = 0.25(12 - 6)^{2/3} = 0.8255$
$A = 50 + 56(1 - 0.8255) = 59.77$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{785.3}}{59.77} \right)^{0.8255} = 1.374$$

Eq. (15-8): $v_{t,\max} = [59.77 + (6 - 3)]^2 = 3940$ ft/min

Since $785.3 < 3904$, $K_v = 1.374$ is valid. The size factor for bending is:

Eq. (15-10): $K_s = 0.4867 + 0.2132 / 6 = 0.5222$

For one gear straddle-mounted, the load-distribution factor is:

Eq. (15-11): $K_m = 1.10 + 0.0036\,(1.25)^2 = 1.106$

Eq. (15-15): $(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$
$(K_L)_G = 1.6831(10^9 / 3)^{-0.0323} = 0.893$

Eq. (15-14): $(C_L)_P = 3.4822(10^9)^{-0.0602} = 1$
$(C_L)_G = 3.4822(10^9 / 3)^{-0.0602} = 1.069$

Eq. (15-19): $K_R = 0.50 - 0.25 \log(1 - 0.999) = 1.25$ (or Table 15-3)
$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$

Bending

Fig. 15-13: ${}_{0.99}S_t = s_{at} = 44(300) + 2100 = 15\,300$ psi

Eq. (15-4): $$(\sigma_{\text{all}})_P = s_{wt} = \frac{s_{at} K_L}{S_F K_T K_R} = \frac{15\,300(0.862)}{1(1)(1.25)} = 10\,551 \text{ psi}$$

Eq. (15-3): $$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$

$$= \frac{10\ 551(1.25)(1)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 690 \text{ lbf}$$

$$H_1 = \frac{690(785.3)}{33\ 000} = 16.4 \text{ hp}$$

Eq. (15-4): $$(\sigma_{\text{all}})_G = \frac{15\ 300(0.893)}{1(1)(1.25)} = 10\ 930 \text{ psi}$$

$$W_G^t = \frac{10\ 930(1.25)(1)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 620 \text{ lbf}$$

$$H_2 = \frac{620(785.3)}{33\ 000} = 14.8 \text{ hp} \quad \textit{Ans.}$$

The gear controls the bending rating.

15-2 Refer to Prob. 15-1 for the gearset specifications.

Wear

Fig. 15-12: $s_{ac} = 341(300) + 23\ 620 = 125\ 920$ psi

For the pinion, $C_H = 1$. From Prob. 15-1, $C_R = 1.118$. Thus, from Eq. (15-2):

$$(\sigma_{c,\text{all}})_P = \frac{s_{ac}(C_L)_P C_H}{S_H K_T C_R}$$

$$(\sigma_{c,\text{all}})_P = \frac{125\ 920(1)(1)}{1(1)(1.118)} = 112\ 630 \text{ psi}$$

For the gear, from Eq. (15-16),

$$B_1 = 0.008\ 98(300 / 300) - 0.008\ 29 = 0.000\ 69$$

$$C_H = 1 + 0.000\ 69(3 - 1) = 1.001\ 38$$

From Prob. 15-1, $(C_L)_G = 1.0685$. Equation (15-2) thus gives

$$(\sigma_{c,\text{all}})_G = \frac{s_{ac}(C_L)_G C_H}{S_H K_T C_R}$$

$$(\sigma_{c,\text{all}})_G = \frac{125\ 920(1.0685)(1.001\ 38)}{1(1)(1.118)} = 120\ 511 \text{ psi}$$

For steel: $C_p = 2290\sqrt{\text{psi}}$

Eq. (15-9): $C_s = 0.125(1.25) + 0.4375 = 0.593\ 75$

Fig. 15-6: $I = 0.083$

Eq. (15-12): $C_{xc} = 2$

Eq. (15-1):
$$W_P^t = \left(\frac{(\sigma_{c,\text{all}})_P}{C_p}\right)^2 \frac{Fd_P I}{K_o K_v K_m C_s C_{xc}}$$
$$= \left(\frac{112\ 630}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.5937)(2)}\right]$$
$$= 464 \text{ lbf}$$
$$H_3 = \frac{464(785.3)}{33\ 000} = 11.0 \text{ hp}$$
$$W_G^t = \left(\frac{120\ 511}{2290}\right)^2 \left[\frac{1.25(3.333)(0.083)}{1(1.374)(1.106)(0.593\ 75)(2)}\right]$$
$$= 531 \text{ lbf}$$
$$H_4 = \frac{531(785.3)}{33\ 000} = 12.6 \text{ hp}$$

The pinion controls wear: $H = 11.0$ hp *Ans.*

The power rating of the mesh, considering the power ratings found in Prob. 15-1, is

$$H = \min(16.4, 14.8, 11.0, 12.6) = 11.0 \text{ hp} \quad \textit{Ans.}$$

15-3 AGMA 2003-B97 does not fully address cast iron gears. However, approximate comparisons can be useful. This problem is similar to Prob. 15-1, but not identical. We will organize the method. A follow-up could consist of completing Probs. 15-1 and 15-2 with identical pinions, and cast iron gears.

Given: Uncrowned, straight teeth, $P_d = 6$ teeth/in, $N_P = 30$ teeth, $N_G = 60$ teeth, ASTM 30 cast iron, material Grade 1, shaft angle 90°, $F = 1.25$, $n_P = 900$ rev/min, $\phi_n = 20°$, one gear straddle-mounted, $K_o = 1$, $J_P = 0.268$, $J_G = 0.228$, $S_F = 2$, $S_H = \sqrt{2}$.

Mesh $d_P = 30/6 = 5.000$ in, $d_G = 60/6 = 10.000$ in

$$v_t = \pi(5)(900 / 12) = 1178 \text{ ft/min}$$

Set $N_L = 10^7$ cycles for the pinion. For $R = 0.99$,

Table 15-7: $s_{at} = 4500$ psi

Table 15-5: s_{ac} = 50 000 psi

Eq. (15-4): $$s_{wt} = \frac{s_{at}K_L}{S_F K_T K_R} = \frac{4500(1)}{2(1)(1)} = 2250 \text{ psi}$$

The velocity factor K_v represents stress augmentation due to mislocation of tooth profiles along the pitch surface and the resulting "falling" of teeth into engagement. Equation (5-67) shows that the induced bending moment in a cantilever (tooth) varies directly with $\sqrt{\text{E}}$ of the tooth material. If only the material varies (cast iron vs. steel) in the same geometry, I is the same. From the Lewis equation of Section 14-1,

$$\sigma = \frac{M}{I/c} = \frac{K_v W^t P}{FY}$$

We expect the ratio $\sigma_{\text{CI}}/\sigma_{\text{steel}}$ to be

$$\frac{\sigma_{CI}}{\sigma_{\text{steel}}} = \frac{(K_v)_{\text{CI}}}{(K_v)_{\text{steel}}} = \sqrt{\frac{E_{CI}}{E_{\text{steel}}}}$$

In the case of ASTM class 30, from Table A-24(*a*)

$$(E_{\text{CI}})_{\text{av}} = (13 + 16.2)/2 = 14.7 \text{ kpsi}$$

Then, $$(K_v)_{\text{CI}} = \sqrt{\frac{14.7}{30}}(K_v)_{\text{steel}} = 0.7(K_v)_{\text{steel}}$$

Our modeling is rough, but it convinces us that $(K_v)_{\text{CI}} < (K_v)_{\text{steel}}$, but we are not sure of the value of $(K_v)_{\text{CI}}$. We will use K_v for steel as a basis for a conservative rating.

Eq. (15-6): $B = 0.25(12 - 6)^{2/3} = 0.8255$

$A = 50 + 56(1 - 0.8255) = 59.77$

Eq. (15-5): $$K_v = \left(\frac{59.77 + \sqrt{1178}}{59.77}\right)^{0.8255} = 1.454$$

Pinion bending $(\sigma_{\text{all}})_P = s_{wt} = 2250$ psi

From Prob. 15-1, $K_x = 1$, $K_m = 1.106$, $K_s = 0.5222$

Eq. (15-3): $$W_P^t = \frac{(\sigma_{\text{all}})_P F K_x J_P}{P_d K_o K_v K_s K_m}$$

$$= \frac{2250(1.25)(1)(0.268)}{6(1)(1.454)(0.5222)(1.106)} = 149.6 \text{ lbf}$$

$$H_1 = \frac{149.6(1178)}{33\,000} = 5.34 \text{ hp}$$

Gear bending

$$W_G^t = W_P^t \frac{J_G}{J_P} = 149.6\left(\frac{0.228}{0.268}\right) = 127.3 \text{ lbf}$$

$$H_2 = \frac{127.3(1178)}{33\,000} = 4.54 \text{ hp}$$

The gear controls in bending fatigue. $H = 4.54$ hp *Ans.*

15-4 Continuing Prob. 15-3,

Table 15-5: $s_{ac} = 50\,000$ psi

$$s_{wt} = \sigma_{c,\text{all}} = \frac{50\,000}{\sqrt{2}} = 35\,355 \text{ psi}$$

Eq. (15-1): $$W^t = \left(\frac{\sigma_{c,\text{all}}}{C_p}\right)^2 \frac{Fd_P I}{K_o K_v K_m C_s C_{xc}}$$

Fig. 15-6: $I = 0.86$

From Probs. 15-1 and 15-2: $C_s = 0.593\,75$, $K_s = 0.5222$, $K_m = 1.106$, $C_{xc} = 2$

From Table 14-8: $C_p = 1960\sqrt{\text{psi}}$

Thus, $$W^t = \left(\frac{35\,355}{1960}\right)^2 \left[\frac{1.25(5.000)(0.086)}{1(1.454)(1.106)(0.59375)(2)}\right] = 91.6 \text{ lbf}$$

$$H_3 = H_4 = \frac{91.6(1178)}{33\,000} = 3.27 \text{ hp}$$

Rating

Based on results of Probs. 15-3 and 15-4,

$$H = \min(5.34, 4.54, 3.27, 3.27) = 3.27 \text{ hp} \quad \textit{Ans.}$$

The mesh is weakest in wear fatigue.

15-5 Uncrowned, through-hardened to 180 Brinell (core and case), Grade 1, 10^9 rev of pinion at $R = 0.999$, $N_P = z_1 = 22$ teeth, $N_G = z_2 = 24$ teeth, $Q_v = 5$, $m_{et} = 4$ mm, shaft angle 90°, $n_1 = 1800$ rev/min, $S_F = 1$, $S_H = \sqrt{S_F} = \sqrt{1}$, $J_P = Y_{J1} = 0.23$, $J_G = Y_{J2} = 0.205$, $F = b = 25$ mm, $K_o = K_A = K_T = K_\theta = 1$ and $C_p = 190\sqrt{\text{MPa}}$.

Mesh $d_P = d_{e1} = mz_1 = 4(22) = 88$ mm, $d_G = m_{et}\, z_2 = 4(24) = 96$ mm

Eq. (15-7): $v_{et} = 5.236(10^{-5})(88)(1800) = 8.29$ m/s

Eq. (15-6): $B = 0.25(12 - 5)^{2/3} = 0.9148$
$A = 50 + 56(1 - 0.9148) = 54.77$

Eq. (15-5): $$K_v = \left(\frac{54.77 + \sqrt{200(8.29)}}{54.77} \right)^{0.9148} = 1.663$$

Eq. (15-10): $K_s = Y_x = 0.4867 + 0.008\,339(4) = 0.520$

Eq. (15-11): with $K_{mb} = 1$ (both straddle-mounted),
$K_m = K_{H\beta} = 1 + 5.6(10^{-6})(25^2) = 1.0035$

From Fig. 15-8,
$(C_L)_P = (Z_{NT})_P = 3.4822(10^9)^{-0.0602} = 1.00$
$(C_L)_G = (Z_{NT})_G = 3.4822[10^9(22 / 24)]^{-0.0602} = 1.0054$

Eq. (15-12): $C_{xc} = Z_{xc} = 2$ (uncrowned)

Eq. (15-19): $K_R = Y_Z = 0.50 - 0.25 \log (1 - 0.999) = 1.25$
$C_R = Z_Z = \sqrt{Y_Z} = \sqrt{1.25} = 1.118$

From Fig. 15-10, $C_H = Z_w = 1$

Eq. (15-9): $Z_x = 0.004\,92(25) + 0.4375 = 0.560$

Wear of Pinion

Fig. 15-12: $\sigma_{H\,\lim} = 2.35H_B + 162.89$
$= 2.35(180) + 162.89 = 585.9$ MPa

Fig. 15-6: $I = Z_I = 0.066$

Eq. (15-2): $$(\sigma_H)_P = \frac{(\sigma_{H\ \lim})_P (Z_{NT})_P Z_W}{S_H K_\theta Z_Z}$$
$$= \frac{585.9(1)(1)}{\sqrt{1}(1)(1.118)} = 524.1 \text{ MPa}$$

Eq. (15-1): $$W_P^t = \left(\frac{\sigma_H}{C_p} \right)^2 \frac{bd_{e1}Z_I}{1000K_A K_v K_{H\beta} Z_x Z_{xc}}$$

The constant 1000 expresses W^t in kN.

$$W_P^t = \left(\frac{524.1}{190}\right)^2\left[\frac{25(88)(0.066)}{1000(1)(1.663)(1.0035)(0.56)(2)}\right] = 0.591 \text{ kN}$$

Eq. (13-36): $$H_3 = \frac{\pi d n_1 W^t}{60\,000} = \frac{\pi(88)(1800)(0.591)}{60\,000} = 4.90 \text{ kW}$$

Wear of Gear

$$\sigma_{H\lim} = 585.9 \text{ MPa}$$

$$(\sigma_H)_G = \frac{585.9(1.0054)}{\sqrt{1}(1)(1.118)} = 526.9 \text{ MPa}$$

$$W_G^t = W_P^t \frac{(\sigma_H)_G}{(\sigma_H)_P} = 0.591\left(\frac{526.9}{524.1}\right) = 0.594 \text{ kN}$$

$$H_4 = \frac{\pi(88)(1800)(0.594)}{60\,000} = 4.93 \text{ kW}$$

Thus in wear, the pinion controls the power rating; $H = 4.90$ kW *Ans.*

We will rate the gear set after solving Prob. 15-6.

15-6 Refer to Prob. 15-5 for terms not defined below.

Bending of Pinion

$$(K_L)_P = (Y_{NT})_P = 1.6831(10^9)^{-0.0323} = 0.862$$
$$(K_L)_G = (Y_{NT})_G = 1.6831[10^9(22/24)]^{-0.0323} = 0.864$$

Fig. 15-13: $$\sigma_{F\lim} = 0.30H_B + 14.48$$
$$= 0.30(180) + 14.48 = 68.5 \text{ MPa}$$

Eq. (15-13): $K_x = Y_\beta = 1$

From Prob. 15-5: $Y_Z = 1.25$, $v_{et} = 8.29$ m/s,
$K_A = 1$, $K_v = 1.663$, $K_\theta = 1$,
$Y_x = 0.52$, $K_{H\beta} = 1.0035$, $Y_{J1} = 0.23$

Eq. (5-4): $$(\sigma_F)_P = \frac{\sigma_{F\lim} Y_{NT}}{S_F K_\theta Y_Z} = \frac{68.5(0.862)}{1(1)(1.25)} = 47.2 \text{ MPa}$$

Eq. (5-3): $$W_P^t = \frac{(\sigma_F)_P b m_{et} Y_\beta Y_{J1}}{1000 K_A K_v Y_x K_{H\beta}}$$
$$= \frac{47.2(25)(4)(1)(0.23)}{1000(1)(1.663)(0.52)(1.0035)} = 1.25 \text{ kN}$$

$$H_1 = \frac{\pi(88)(1800)(1.25)}{60\,000} = 10.37 \text{ kW}$$

Bending of Gear

$$\sigma_{F\lim} = 68.5 \text{ MPa}$$

$$(\sigma_F)_G = \frac{68.5(0.864)}{1(1)(1.25)} = 47.3 \text{ MPa}$$

$$W_G^t = \frac{47.3(25)(4)(1)(0.205)}{1000(1)(1.663)(0.52)(1.0035)} = 1.12 \text{ kN}$$

$$H_2 = \frac{\pi(88)(1800)(1.12)}{60\,000} = 9.29 \text{ kW}$$

Rating of mesh is

$H_{\text{rating}} = \min(10.37, 9.29, 4.90, 4.93) = 4.90$ kW *Ans.*

with pinion wear controlling.

15-7

(a) $$(S_F)_P = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_P = (S_F)_G = \left(\frac{\sigma_{\text{all}}}{\sigma}\right)_G$$

$$\frac{(s_{at}K_L \,/\, K_T K_R)_P}{(W^t P_d K_o K_v K_s K_m \,/\, FK_x J)_P} = \frac{(s_{at}K_L \,/\, K_T K_R)_G}{(W^t P_d K_o K_v K_s K_m \,/\, FK_x J)_G}$$

All terms cancel except for s_{at} , K_L , and J,

$$(s_{at})_P (K_L)_P J_P = (s_{at})_G (K_L)_G J_G$$

From which

$$(s_{at})_G = \frac{(s_{at})_P (K_L)_P J_P}{(K_L)_G J_G} = (s_{at})_P \frac{J_P}{J_G} m_G^{\beta}$$

where $\beta = -0.0178$ or $\beta = -0.0323$ as appropriate. This equation is the same as Eq. (14-44). *Ans.*

(b) In bending

$$W^t = \left(\frac{\sigma_{\text{all}}}{S_F}\frac{FK_x J}{P_d K_o K_v K_s K_m}\right)_{11} = \left(\frac{s_{at}}{S_F}\frac{K_L}{K_T K_R}\frac{FK_x J}{P_d K_o K_v K_s K_m}\right)_{11} \qquad (1)$$

In wear

$$\left(\frac{s_{ac} C_L C_U}{S_H K_T C_R}\right)_{22} = C_p \left(\frac{W^t K_o K_v K_m C_s C_{xc}}{F d_P I}\right)_{22}^{1/2}$$

Squaring and solving for W^t gives

$$W^t = \left(\frac{s_{ac}^2 C_L^2 C_H^2}{S_H^2 K_T^2 C_R^2 C_P^2} \right)_{22} \left(\frac{F d_P I}{K_o K_v K_m C_s C_{xc}} \right)_{22} \tag{2}$$

Equating the right-hand sides of Eqs. (1) and (2) and canceling terms, and recognizing that $C_R = \sqrt{K_R}$ and $P_d d_P = N_P$, we obtain

$$(s_{ac})_{22} = \frac{C_p}{(C_L)_{22}} \sqrt{\frac{S_H^2}{S_F} \frac{(s_{at})_{11} (K_L)_{11} K_x J_{11} K_T C_s C_{xc}}{C_H^2 N_P K_s I}}$$

For equal W^t in bending and wear

$$\frac{S_H^2}{S_F} = \frac{\left(\sqrt{S_F}\right)^2}{S_F} = 1$$

So we get

$$(s_{ac})_G = \frac{C_p}{(C_L)_G C_H} \sqrt{\frac{(s_{at})_P (K_L)_P J_P K_x K_T C_s C_{xc}}{N_P I K_s}} \quad \textit{Ans.}$$

(c)

$$(S_H)_P = (S_H)_G = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c} \right)_P = \left(\frac{\sigma_{c,\text{all}}}{\sigma_c} \right)_G$$

Substituting in the right-hand equality gives

$$\frac{[s_{ac} C_L \,/\, (C_R K_T)]_P}{\left[C_p \sqrt{W^t K_o K_v K_m C_s C_{xc} \,/\, (F d_P I)} \right]_P} = \frac{[s_{ac} C_L C_H \,/\, (C_R K_T)]_G}{\left[C_p \sqrt{W^t K_o K_v K_m C_s C_{xc} \,/\, (F d_P I)} \right]_G}$$

Denominators cancel, leaving

$$(s_{ac})_P (C_L)_P = (s_{ac})_G (C_L)_G C_H$$

Solving for $(s_{ac})_P$ gives,

$$(s_{ac})_P = (s_{ac})_G \frac{(C_L)_G}{(C_L)_P} C_H \tag{1}$$

From Eq. (15-14), $\left(C_L\right)_P = 3.4822 N_L^{-0.0602}$ and $\left(C_L\right)_G = 3.4822 \left(N_L / m_G\right)^{-0.0602}$. Thus,

$$\left(s_{ac}\right)_P = \left(s_{ac}\right)_G \left(1/m_G\right)^{-0.0602} C_H = \left(s_{ac}\right)_G m_G^{0.0602} C_H \quad \textit{Ans.}$$

This equation is the transpose of Eq. (14-45).

15-8

	Core	Case
Pinion	$(H_B)_{11}$	$(H_B)_{12}$
Gear	$(H_B)_{21}$	$(H_B)_{22}$

Given $(H_B)_{11}$ = 300 Brinell

Eq. (15-23): $(s_{at})_P = 44(300) + 2100 = 15\,300$ psi

$$(s_{at})_G = (s_{at})_P \frac{J_P}{J_G} m_G^{-0.0323} = 15\,300\left(\frac{0.249}{0.216}\right)\left(3^{-0.0323}\right) = 17\,023 \text{ psi}$$

$$(H_B)_{21} = \frac{17\,023 - 2100}{44} = 339 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_G = \frac{2290}{1.0685(1)}\sqrt{\frac{15\,300(0.862)(0.249)(1)(0.593\,25)(2)}{20(0.086)(0.5222)}}$$

$$= 141\,160 \text{ psi}$$

$$(H_B)_{22} = \frac{141\,160 - 23\,600}{341} = 345 \text{ Brinell} \quad \textit{Ans.}$$

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H \doteq 141\,160(3^{0.0602})(1) = 150\,811 \text{ psi}$$

$$(H_B)_{12} = \frac{150\,811 - 23\,600}{341} = 373 \text{ Brinell} \quad \textit{Ans.}$$

	Core	Case
Pinion	300	373
Gear	339	345

Ans.

15-9

Pinion core

$$(s_{at})_P = 44(300) + 2100 = 15\,300 \text{ psi}$$

$$(\sigma_{all})_P = \frac{15\,300(0.862)}{1(1)(1.25)} = 10\,551 \text{ psi}$$

$$W^t = \frac{10\,551(1.25)(0.249)}{6(1)(1.374)(0.5222)(1.106)} = 689.7 \text{ lbf}$$

Gear core

$$(s_{at})_G = 44(352) + 2100 = 17\,588 \text{ psi}$$

$$(\sigma_{all})_G = \frac{17\,588(0.893)}{1(1)(1.25)} = 12\,565 \text{ psi}$$

$$W^t = \frac{12\,565(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 712.5 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(372) + 23\,620 = 150\,472 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,472(1)}{1(1)(1.118)} = 134\,590 \text{ psi}$$

$$W^t = \left(\frac{134\,590}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 685.8 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(344) + 23\,620 = 140\,924 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{140\,924(1.0685)(1)}{1(1)(1.118)} = 134\,685 \text{ psi}$$

$$W^t = \left(\frac{134\,685}{2290}\right)^2 \frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)} = 686.8 \text{ lbf}$$

The rating load would be

$$W^t_{\text{rated}} = \min(689.7,\ 712.5,\ 685.8,\ 686.8) = 685.8 \text{ lbf}$$

which is slightly less than intended.

Pinion core

$$(s_{at})_P = 15\,300 \text{ psi} \quad \text{(as before)}$$

$$(\sigma_{\text{all}})_P = 10\,551 \text{ psi} \quad \text{(as before)}$$

$$W^t = 689.7 \text{ lbf} \quad \text{(as before)}$$

Gear core

$$(s_{at})_G = 44(339) + 2100 = 17\,016 \text{ psi}$$

$$(\sigma_{\text{all}})_G = \frac{17\,016(0.893)}{1(1)(1.25)} = 12\,156 \text{ psi}$$

$$W^t = \frac{12\,156(1.25)(0.216)}{6(1)(1.374)(0.5222)(1.106)} = 689.3 \text{ lbf}$$

Pinion case

$$(s_{ac})_P = 341(373) + 23\,620 = 150\,813 \text{ psi}$$

$$(\sigma_{c,\text{all}})_P = \frac{150\,813(1)}{1(1)(1.118)} = 134\,895 \text{ psi}$$

$$W^t = \left(\frac{134\,895}{2290}\right)^2 \left[\frac{1.25(3.333)(0.086)}{1(1.374)(1.106)(0.593\,75)(2)}\right] = 689.0 \text{ lbf}$$

Gear case

$$(s_{ac})_G = 341(345) + 23\,620 = 141\,265 \text{ psi}$$

$$(\sigma_{c,\text{all}})_G = \frac{141\,265(1.0685)(1)}{1(1)(1.118)} = 135\,010 \text{ psi}$$

$$W^t = \left(\frac{135\ 010}{2290}\right)^2\left[\frac{1.25(3.333)(0.086)}{1(1.1374)(1.106)(0.593\ 75)(2)}\right] = 690.1 \text{ lbf}$$

The equations developed within Prob. 15-7 are effective.

15-10 The catalog rating is 5.2 hp at 1200 rev/min for a straight bevel gearset. Also given: N_P = 20 teeth, N_G = 40 teeth, ϕ_n = 20°, F = 0.71 in, J_P = 0.241, J_G = 0.201, P_d = 10 teeth/in, through-hardened to 300 Brinell-General Industrial Service, and Q_v = 5 uncrowned.

Mesh

$$d_P = 20 / 10 = 2.000 \text{ in}, \quad d_G = 40 / 10 = 4.000 \text{ in}$$

$$v_t = \frac{\pi d_P n_P}{12} = \frac{\pi(2)(1200)}{12} = 628.3 \text{ ft/min}$$

$$K_o = 1, \quad S_F = 1, \quad S_H = 1$$

Eq. (15-6): $B = 0.25(12 - 5)^{2/3} = 0.9148$
$A = 50 + 56(1 - 0.9148) = 54.77$

Eq. (15-5): $$K_v = \left(\frac{54.77 + \sqrt{628.3}}{54.77}\right)^{0.9148} = 1.412$$

Eq. (15-10): $K_s = 0.4867 + 0.2132/10 = 0.508$

Eq. (15-11): $K_m = 1.25 + 0.0036(0.71)^2 = 1.252$, where $K_{mb} = 1.25$

Eq. (15-15): $(K_L)_P = 1.6831(10^9)^{-0.0323} = 0.862$
$(K_L)_G = 1.6831(10^9/2)^{-0.0323} = 0.881$

Eq. (15-14): $(C_L)_P = 3.4822(10^9)^{-0.0602} = 1.000$
$(C_L)_G = 3.4822(10^9/2)^{-0.0602} = 1.043$

Analyze for 10^9 pinion cycles at 0.999 reliability.

Eq. (15-19): $K_R = 0.50 - 0.25 \log(1 - 0.999) = 1.25$
$C_R = \sqrt{K_R} = \sqrt{1.25} = 1.118$

Bending
Pinion:
Eq. (15-23): $(s_{at})_P = 44(300) + 2100 = 15\ 300$ psi

Eq. (15-4): $(s_{wt})_P = \dfrac{15\ 300(0.862)}{1(1)(1.25)} = 10\ 551$ psi

Eq. (15-3): $W^t = \dfrac{(s_{wt})_P F K_x J_P}{P_d K_o K_v K_s K_m}$

$= \dfrac{10\ 551(0.71)(1)(0.241)}{10(1)(1.412)(0.508)(1.252)} = 201$ lbf

$H_1 = \dfrac{201(628.3)}{33\ 000} = 3.8$ hp

Gear: $(s_{at})_G = 15\ 300$ psi

Eq. (15-4): $(s_{wt})_G = \dfrac{15\ 300(0.881)}{1(1)(1.25)} = 10\ 783$ psi

Eq. (15-3): $W^t = \dfrac{10\ 783(0.71)(1)(0.201)}{10(1)(1.412)(0.508)(1.252)} = 171.4$ lbf

$H_2 = \dfrac{171.4(628.3)}{33\ 000} = 3.3$ hp

Wear

Pinion:

$(C_H)_G = 1, \quad I = 0.078, \quad C_p = 2290\sqrt{\text{psi}}, \quad C_{xc} = 2$

$C_s = 0.125(0.71) + 0.4375 = 0.526\ 25$

Eq. (15-22): $(s_{ac})_P = 341(300) + 23\ 620 = 125\ 920$ psi

$(\sigma_{c,\text{all}})_P = \dfrac{125\ 920(1)(1)}{1(1)(1.118)} = 112\ 630$ psi

Eq. (15-1): $W^t = \left[\dfrac{(\sigma_{c,\text{all}})_P}{C_p}\right]^2 \dfrac{F d_P I}{K_o K_v K_m C_s C_{xc}}$

$= \left(\dfrac{112\ 630}{2290}\right)^2 \left[\dfrac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\ 25)(2)}\right]$

$= 144.0$ lbf

$H_3 = \dfrac{144(628.3)}{33\ 000} = 2.7$ hp

Gear:

$(s_{ac})_G = 125\ 920$ psi

$(\sigma_{c,\text{all}}) = \dfrac{125\ 920(1.043)(1)}{1(1)(1.118)} = 117\ 473$ psi

$W^t = \left(\dfrac{117\ 473}{2290}\right)^2 \left[\dfrac{0.71(2.000)(0.078)}{1(1.412)(1.252)(0.526\ 25)(2)}\right] = 156.6$ lbf

$$H_4 = \frac{156.6(628.3)}{33\,000} = 3.0 \text{ hp}$$

Rating:

$$H = \min(3.8, 3.3, 2.7, 3.0) = 2.7 \text{ hp}$$

Pinion wear controls the power rating. While the basis of the catalog rating is unknown, it is overly optimistic (by a factor of 1.9).

15-11 From Ex. 15-1, the core hardness of both the pinion and gear is 180 Brinell. So $(H_B)_{11}$ and $(H_B)_{21}$ are 180 Brinell and the bending stress numbers are:

$$(s_{at})_P = 44(180) + 2100 = 10\,020 \text{ psi}$$
$$(s_{at})_G = 10\,020 \text{ psi}$$

The contact strength of the gear case, based upon the equation derived in Prob. 15-7, is

$$(s_{ac})_G = \frac{C_p}{(C_L)_G C_H}\sqrt{\frac{S_H^2}{S_F}\frac{(s_{at})_P (K_L)_P K_x J_P K_T C_s C_{xc}}{N_P I K_s}}$$

Substituting $(s_{at})_P$ from above and the values of the remaining terms from Ex. 15-1,

$$(s_{ac})_G = \frac{2290}{1.32(1)}\sqrt{\frac{1.5^2}{1.5}\left(\frac{10\,020(1)(1)(0.216)(1)(0.575)(2)}{25(0.065)(0.529)}\right)}$$
$$= 114\,331 \text{ psi}$$
$$(H_B)_{22} = \frac{114\,331 - 23\,620}{341} = 266 \text{ Brinell}$$

The pinion contact strength is found using the relation from Prob. 15-7:

$$(s_{ac})_P = (s_{ac})_G m_G^{0.0602} C_H = 114\,331(1)^{0.0602}(1) = 114\,331 \text{ psi}$$
$$(H_B)_{12} = \frac{114\,331 - 23\,600}{341} = 266 \text{ Brinell}$$

	Core	Case
Pinion	180	266
Gear	180	266

Realization of hardnesses

The response of students to this part of the question would be a function of the extent to which heat-treatment procedures were covered in their materials and manufacturing prerequisites, and how quantitative it was. The most important

thing is to have the student think about it.

The instructor can comment in class when students' curiosity is heightened. Options that will surface may include:

(a) Select a through-hardening steel which will meet or exceed core hardness in the hot-rolled condition, then heat-treating to gain the additional 86 points of Brinell hardness by bath-quenching, then tempering, then generating the teeth in the blank.

(b) Flame or induction hardening are possibilities.

(c) The hardness goal for the case is sufficiently modest that carburizing and case hardening may be too costly. In this case the material selection will be different.

(d)The initial step in a nitriding process brings the core hardness to 33–38 Rockwell C-scale (about 300–350 Brinell), which is too much.

15-12 Computer programs will vary.

15-13 A design program would ask the user to make the a priori decisions, as indicated in Sec. 15-5, p. 806, of the text. The decision set can be organized as follows:

A priori decisions:
- Function: H, K_o, rpm, m_G, temp., N_L, R
- Design factor: n_d ($S_F = n_d$, $S_H = \sqrt{n_d}$)
- Tooth system: Involute, Straight Teeth, Crowning, ϕ_n
- Straddling: K_{mb}
- Tooth count: N_P ($N_G = m_G N_P$)

Design decisions:
- Pitch and Face: P_d , F
- Quality number: Q_v
- Pinion hardness: $(H_B)_1$, $(H_B)_3$
- Gear hardness: $(H_B)_2$, $(H_B)_4$

First, gather all of the equations one needs, then arrange them before coding. Find the required hardnesses, express the consequences of the chosen hardnesses, and allow for revisions as appropriate.

	Pinion Bending	Gear Bending	Pinion Wear	Gear Wear
Load-induced stress (Allowable stress)	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_P} = s_{11}$	$s_t = \dfrac{W^t P K_o K_v K_m K_s}{F K_x J_G} = s_{21}$	$\sigma_c = C_p \left(\dfrac{W^t K_o K_v C_s C_{xc}}{F d_P I} \right)^{1/2} = s_{12}$	$s_{22} = s_{12}$
Tabulated strength	$(s_{at})_P = \dfrac{s_{11} S_F K_T K_R}{(K_L)_P}$	$(s_{at})_G = \dfrac{s_{21} S_F K_T K_R}{(K_L)_G}$	$(s_{ac})_P = \dfrac{s_{12} S_H K_T C_R}{(C_L)_P (C_H)_P}$	$(s_{ac})_G = \dfrac{s_{22} S_H K_T C_R}{(C_L)_G (C_H)_G}$
Associated hardness	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_P - 2100}{44} \\ \dfrac{(s_{at})_P - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{at})_G - 2100}{44} \\ \dfrac{(s_{at})_G - 5980}{48} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_P - 23\,620}{341} \\ \dfrac{(s_{ac})_P - 29\,560}{363.6} \end{cases}$	$\text{Bhn} = \begin{cases} \dfrac{(s_{ac})_P - 23\,620}{341} \\ \dfrac{(s_{ac})_P - 29\,560}{363.6} \end{cases}$
Chosen hardness	$(H_B)_{11}$	$(H_B)_{21}$	$(H_B)_{12}$	$(H_B)_{22}$
New tabulated strength	$(s_{at1})_P = \begin{cases} 44(H_B)_{11} + 2100 \\ 48(H_B)_{11} + 5980 \end{cases}$	$(s_{at1})_G = \begin{cases} 44(H_B)_{21} + 2100 \\ 48(H_B)_{21} + 5980 \end{cases}$	$(s_{ac1})_P = \begin{cases} 341(H_B)_{12} + 23\,620 \\ 363.6(H_B)_{12} + 29\,560 \end{cases}$	$(s_{ac1})_G = \begin{cases} 341(H_B)_{22} + 23\,620 \\ 363.6(H_B)_{22} + 29\,560 \end{cases}$
Factor of safety	$n_{11} = \dfrac{\sigma_{\text{all}}}{\sigma} = \dfrac{(s_{at1})_P (K_L)_P}{s_{11} K_T K_R}$	$n_{21} = \dfrac{(s_{at1})_G (K_L)_G}{s_{21} K_T K_R}$	$n_{12} = \left[\dfrac{(s_{ac1})_P (C_L)_P (C_H)_P}{s_{12} K_T C_R} \right]^2$	$n_{22} = \left[\dfrac{(s_{ac1})_G (C_L)_G (C_H)_G}{s_{22} K_T C_R} \right]^2$

Note: $S_F = n_d, \quad S_H = \sqrt{S_F}$

15-14 $N_W = 1$, $N_G = 56$, $P_t = 8$ teeth/in, $d = 1.5$ in, $H_o = 1$hp, $\phi_n = 20°$, $t_a = 70°$F, $K_a = 1.25$, $n_d = 1$, $F_e = 2$ in, $A = 850$ in^2

(a) $m_G = N_G/N_W = 56$, $\quad d_G = N_G/P_t = 56/8 = 7.0$ in

$p_x = \pi / 8 = 0.3927$ in, $\quad C = 1.5 + 7 = 8.5$ in

Eq. (15-39): $a = p_x / \pi = 0.3927 / \pi = 0.125$ in

Eq. (15-40): $b = 0.3683\, p_x = 0.1446$ in

Eq. (15-41): $h_t = 0.6866\, p_x = 0.2696$ in

Eq. (15-42): $d_o = 1.5 + 2(0.125) = 1.75$ in

Eq. (15-43): $d_r = 3 - 2(0.1446) = 2.711$ in

Eq. (15-44): $D_t = 7 + 2(0.125) = 7.25$ in

Eq. (15-45): $D_r = 7 - 2(0.1446) = 6.711$ in

Eq. (15-46): $c = 0.1446 - 0.125 = 0.0196$ in

Eq. (15-47): $(F_W)_{\max} = 2\sqrt{2(7)(0.125)} = 2.646$ in

$$V_W = \pi(1.5)(1725/12) = 677.4 \text{ ft/min}$$

$$V_G = \frac{\pi(7)(1725/56)}{12} = 56.45 \text{ ft/min}$$

Eq. (13-27): $L = p_x N_W = 0.3927$ in

Eq. (13-28): $\lambda = \tan^{-1}\left(\dfrac{0.3927}{\pi(1.5)}\right) = 4.764^{\circ}$

$$P_n = \frac{P_t}{\cos\lambda} = \frac{8}{\cos 4.764°} = 8.028$$

$$p_n = \frac{\pi}{P_n} = 0.3913 \text{ in}$$

Eq. (15-62): $V_s = \dfrac{\pi(1.5)(1725)}{12\cos 4.764°} = 679.8$ ft/min

(b)

Eq. (15-38): $f = 0.103\exp\left[-0.110(679.8)^{0.450}\right] + 0.012 = 0.0250$

Eq. (15-54):

$$e = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} = \frac{\cos 20° - 0.0250\tan 4.764°}{\cos\ 20° + 0.0250\cot 4.764°} = 0.7563 \quad \textit{Ans.}$$

Eq. (15-58): $$W_G^t = \frac{33\,000 n_d H_o K_a}{V_G e} = \frac{33\,000(1)(1)(1.25)}{56.45(0.7563)} = 966 \text{ lbf} \quad \textit{Ans.}$$

Eq. (15-57): $$W_W^t = W_G^t \frac{\cos\phi_n \sin\lambda + f\cos\lambda}{\cos\phi_n \cos\lambda - f\sin\lambda}$$

$$= 966\left(\frac{\cos 20^\circ \sin 4.764^\circ + 0.025\cos 4.764^\circ}{\cos 20^\circ \cos 4.764^\circ - 0.025\sin 4.764^\circ}\right)$$

$$= 106.4 \text{ lbf} \quad \textit{Ans.}$$

(c)

Eq. (15-33): $C_s = 1190 - 477 \log 7.0 = 787$

Eq. (15-36): $$C_m = 0.0107\sqrt{-56^2 + 56(56) + 5145} = 0.767$$

Eq. (15-37): $$C_v = 0.659\exp[-0.0011(679.8)] = 0.312$$

Eq. (15-38): $(W^t)_{\text{all}} = 787(7)^{0.8}(2)(0.767)(0.312) = 1787$ lbf

Since $W_G^t < (W^t)_{\text{all}}$, the mesh will survive at least 25 000 h.

Eq. (15-61): $$W_f = \frac{0.025(966)}{0.025\sin 4.764^\circ - \cos 20^\circ \cos 4.764^\circ} = -29.5 \text{ lbf}$$

Eq. (15-63): $$H_f = \frac{29.5(679.8)}{33\,000} = 0.608 \text{ hp}$$

$$H_W = \frac{106.4(677.4)}{33\,000} = 2.18 \text{ hp}$$

$$H_G = \frac{966(56.45)}{33\,000} = 1.65 \text{ hp}$$

The mesh is sufficient *Ans.*

$$P_n = P_t / \cos\lambda = 8 / \cos 4.764^\circ = 8.028$$

$$p_n = \pi / 8.028 = 0.3913 \text{ in}$$

$$\sigma_G = \frac{966}{0.3913(0.5)(0.125)} = 39\,500 \text{ psi}$$

The stress is high. At the rated horsepower,

$$\sigma_G = \frac{1}{1.65} 39\,500 = 23\,940 \text{ psi} \quad \text{acceptable}$$

(d)

Eq. (15-52): $A_{\min} = 43.2(8.5)^{1.7} = 1642 \text{ in}^2 < 1700 \text{ in}^2$

Eq. (15-49): $H_{\text{loss}} = 33\,000(1 - 0.7563)(2.18) = 17\,530 \text{ ft} \cdot \text{lbf/min}$

Assuming a fan exists on the worm shaft,

Eq. (15-50): $$h_{CR} = \frac{1725}{3939} + 0.13 = 0.568 \text{ ft} \cdot \text{lbf/(min} \cdot \text{in}^2 \cdot {}^\circ\text{F)}$$

Eq. (15-51): $$t_s = 70 + \frac{17\,530}{0.568(1700)} = 88.2^\circ\text{F} \quad \textit{Ans.}$$

15-15 Problem statement values of 25 hp, 1125 rev/min, m_G = 10, K_a = 1.25, n_d = 1.1, ϕ_n = 20°, t_a = 70°F are not referenced in the table. The first four parameters listed in the table were selected as design decisions.

	15-15	15-16	15-17	15-18	15-19	15-20	15-21	15-22
p_x	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
d_W	3.60	3.60	3.60	3.60	3.60	4.10	3.60	3.60
F_G	2.40	1.68	1.43	1.69	2.40	2.25	2.4	2.4
A	2000	2000	2000	2000	2000	2000	2500	2600
							FAN	FAN
H_W	38.2	38.2	38.2	38.2	38.2	38.0	41.2	41.2
H_G	36.2	36.2	36.2	36.2	36.2	36.1	37.7	37.7
H_f	1.87	1.47	1.97	1.97	1.97	1.85	3.59	3.59
N_W	3	3	3	3	3	3	3	3
N_G	30	30	30	30	30	30	30	30
K_W				125	80	50	115	185
C_s	607	854	1000					
C_m	0.759	0.759	0.759					
C_v	0.236	0.236	0.236					
V_G	492	492	492	492	492	563	492	492
W_G^t	2430	2430	2430	2430	2430	2120	2524	2524
W_W^t	1189	1189	1189	1189	1189	1038	1284	1284
f	0.0193	0.0193	0.0193	0.0193	0.0193	0.0183	0.034	0.034
e	0.948	0.948	0.948	0.948	0.948	0.951	0.913	0.913
$(P_t)_G$	1.795	1.795	1.795	1.795	1.795	1.571	1.795	1.795
P_n	1.979	1.979	1.979	1.979	1.979	1.732	1.979	1.979
C-to-C	10.156	10.156	10.156	10.156	10.156	11.6	10.156	10.156
t_s	177	177	177	177	177	171	179.6	179.6
L	5.25	5.25	5.25	5.25	5.25	6.0	5.25	5.25
λ	24.9	24.9	24.9	24.9	24.9	24.98	24.9	24.9
σ_G	5103	7290	8565	7247	5103	4158	5301	5301
d_G	16.71	16.71	16.71	16.71	16.71	19.099	16.7	16.71

Chapter 16

16-1 Given: $r = 300/2 = 150$ mm, $a = R = 125$ mm, $b = 40$ mm, $f = 0.28$, $F = 2.2$ kN, $\theta_1 = 0°$, $\theta_2 = 120°$, and $\theta_a = 90°$. From which, $\sin\theta_a = \sin 90° = 1$.

Eq. (16-2):

$$M_f = \frac{0.28 p_a (0.040)(0.150)}{1} \int_{0°}^{120°} \sin\theta (0.150 - 0.125\cos\theta)\, d\theta$$

$$= 2.993\left(10^{-4}\right) p_a \ \text{N} \cdot \text{m}$$

Eq. (16-3): $$M_N = \frac{p_a (0.040)(0.150)(0.125)}{1} \int_{0°}^{120°} \sin^2\theta\, d\theta = 9.478\left(10^{-4}\right) p_a \ \text{N} \cdot \text{m}$$

$c = 2(0.125 \cos 30°) = 0.2165$ m

Eq. (16-4): $$F = \frac{9.478\left(10^{-4}\right) p_a - 2.993\left(10^{-4}\right) p_a}{0.2165} = 2.995\left(10^{-3}\right) p_a$$

$p_a = F/[2.995(10^{-3})] = 2200/[2.995(10^{-3})]$
$= 734.5(10^3)$ Pa for cw rotation

Eq. (16-7): $$2200 = \frac{9.478\left(10^{-4}\right) p_a + 2.993\left(10^{-4}\right) p_a}{0.2165}$$

$p_a = 381.9(10^3)$ Pa for ccw rotation

A maximum pressure of 734.5 kPa occurs on the RH shoe for cw rotation. *Ans.*

(b) *RH shoe*:

Eq. (16-6):

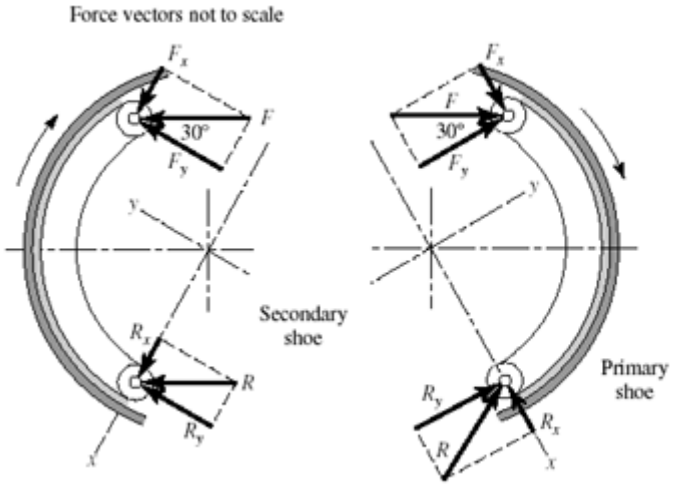
$$T_R = \frac{0.28(734.5)10^3(0.040)0.150^2(\cos 0^\circ - \cos 120^\circ)}{1} = 277.6 \ \text{N} \cdot \text{m}$$ *Ans.*

LH shoe:

$$T_L = 277.6\frac{381.9}{734.5} = 144.4 \ \text{N} \cdot \text{m}$$ *Ans.*

$T_{\text{total}} = 277.6 + 144.4 = 422 \ \text{N} \cdot \text{m}$ *Ans.*

(c)



RH shoe: $F_x = 2200 \sin 30° = 1100$ N, $F_y = 2200 \cos 30° = 1905$ N

Eqs. (16-8): $A = \left(\frac{1}{2}\sin^2\theta\right)_{0°}^{120°} = 0.375, \quad B = \left(\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right)_{0}^{2\pi/3 \text{ rad}} = 1.264$

Eqs. (16-9): $R_x = \frac{734.5\left(10^3\right)0.040(0.150)}{1}[0.375 - 0.28(1.264)] - 1100 = -1007$ N

$R_y = \frac{734.5\left(10^3\right)0.04(0.150)}{1}[1.264 + 0.28(0.375)] - 1905 = 4128$ N

$R = [(-1007)^2 + 4128^2]^{1/2} = 4249$ N *Ans.*

LH shoe: $F_x = 1100$ N, $F_y = 1905$ N

Eqs. (16-10): $R_x = \frac{381.9\left(10^3\right)0.040(0.150)}{1}[0.375 + 0.28(1.264)] - 1100 = 570$ N

$R_y = \frac{381.9\left(10^3\right)0.040(0.150)}{1}[1.264 - 0.28(0.375)] - 1905 = 751$ N

$R = \left(597^2 + 751^2\right)^{1/2} = 959$ N *Ans.*

16-2 Given: $r = 300/2 = 150$ mm, $a = R = 125$ mm, $b = 40$ mm, $f = 0.28$, $F = 2.2$ kN, $\theta_1 = 15°$, $\theta_2 = 105°$, and $\theta_a = 90°$. From which, $\sin\theta_a = \sin 90° = 1$.

Eq. (16-2):

$$M_f = \frac{0.28 p_a (0.040)(0.150)}{1} \int_{15°}^{105°} \sin\theta(0.150 - 0.125\cos\theta)\, d\theta = 2.177\left(10^{-4}\right) p_a$$

Eq. (16-3): $$M_N = \frac{p_a(0.040)(0.150)(0.125)}{1}\int_{15^\circ}^{105^\circ}\sin^2\theta\, d\theta = 7.765\left(10^{-4}\right)p_a$$

$$c = 2(0.125)\cos 30^\circ = 0.2165 \text{ m}$$

Eq. (16-4): $$F = \frac{7.765\left(10^{-4}\right)p_a - 2.177\left(10^{-4}\right)p_a}{0.2165} = 2.581\left(10^{-3}\right)p_a$$

RH shoe: $p_a = 2200/\,[2.581(10^{-3})] = 852.4\,(10^3)$ Pa
$= 852.4$ kPa on RH shoe for cw rotation *Ans.*

Eq. (16-6): $$T_R = \frac{0.28(852.4)10^3(0.040)(0.150^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 263 \text{ N}\cdot\text{m}$$

LH shoe:

$$2200 = \frac{7.765\left(10^{-4}\right)p_a + 2.177\left(10^{-4}\right)p_a}{0.2165}$$

$$p_a = 479.1\left(10^3\right) \text{ Pa} = 479.1 \text{ kPa on LH shoe for ccw rotation} \quad \textit{Ans.}$$

$$T_L = \frac{0.28(479.1)10^3(0.040)(0.150^2)(\cos 15^\circ - \cos 105^\circ)}{1} = 148 \text{ N}\cdot\text{m}$$

$$T_{\text{total}} = 263 + 148 = 411 \text{ N}\cdot\text{m} \quad \textit{Ans.}$$

Comparing this result with that of Prob. 16-1, a 2.6% reduction in torque is obtained by using 25% less braking material.

16-3 Given: $\theta_1 = 0°$, $\theta_2 = 120°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = R = 3.5$ in, $b = 1.25$ in, $f = 0.30$, $F = 225$ lbf, $r = 11/2 = 5.5$ in, counter-clockwise rotation.

LH shoe:

Eq. (16-2), with $\theta_1 = 0$:

$$M_f = \frac{f\,p_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta\left(r - a\cos\theta\right)d\theta = \frac{f\,p_a br}{\sin\theta_a}\left[r(1-\cos\theta_2) - \frac{a}{2}\sin^2\theta_2\right]$$

$$= \frac{0.30\,p_a(1.25)5.5}{1}\left[5.5(1-\cos 120^\circ) - \frac{3.5}{2}\sin^2 120^\circ\right]$$

$$= 14.31\,p_a \text{ lbf}\cdot\text{in}$$

Eq. (16-3), with $\theta_1 = 0$:

$$M_N = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta d\theta = \frac{p_a bra}{\sin\theta_a}\left[\frac{\theta_2}{2} - \frac{1}{4}\sin 2\theta_2\right]$$

$$= \frac{p_a(1.25)5.5(3.5)}{1}\left[\frac{120^\circ}{2}\left(\frac{\pi}{180^\circ}\right) - \frac{1}{4}\sin 2(120^\circ)\right]$$

$$= 30.41\,p_a \text{ lbf}\cdot\text{in}$$

$$c = 2r\cos\left(\frac{180^\circ - \theta_2}{2}\right) = 2(5.5)\cos 30^\circ = 9.526 \text{ in}$$

$$F = 225 = \frac{30.41p_a - 14.31p_a}{9.526} = 1.690\, p_a$$

$$p_a = 225 / 1.690 = 133.1 \text{ psi}$$

Eq. (16-6):

$$T_L = \frac{f\, p_a b r^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.30(133.1)1.25(5.5^2)}{1}[1 - (-0.5)]$$

$$= 2265 \text{ lbf} \cdot \text{in} = 2.265 \text{ kip} \cdot \text{in} \quad \textit{Ans.}$$

RH shoe:

$$F = 225 = \frac{30.41p_a + 14.31p_a}{9.526} = 4.694\, p_a$$

$$p_a = 225 / 4.694 = 47.93 \text{ psi}$$

$$T_R = \frac{47.93}{133.1}2265 = 816 \text{ lbf} \cdot \text{in} = 0.816 \text{ kip} \cdot \text{in}$$

$$T_{\text{total}} = 2.27 + 0.82 = 3.09 \text{ kip} \cdot \text{in} \quad \textit{Ans.}$$

16-4 **(a)** Given: $\theta_1 = 10°$, $\theta_2 = 75°$, $\theta_a = 75°$, $p_a = 10^6$ Pa, $f = 0.24$, $b = 0.075$ m (shoe width), $a = 0.150$ m, $r = 0.200$ m, $d = 0.050$ m, $c = 0.165$ m.

Some of the terms needed are evaluated here:

$$A = \left[r\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta - a\int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta \right] = r\left[-\cos\theta\right]_{\theta_1}^{\theta_2} - a\left[\frac{1}{2}\sin^2\theta\right]_{\theta_1}^{\theta_2}$$

$$= 200\left[-\cos\theta\right]_{10^\circ}^{75^\circ} - 150\left[\frac{1}{2}\sin^2\theta\right]_{10^\circ}^{75^\circ} = 77.5 \text{ mm}$$

$$B = \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta = \left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_{10\pi/180 \text{ rad}}^{75\pi/180 \text{ rad}} = 0.528$$

$$C = \int_{\theta_1}^{\theta_2} \sin\theta\cos\theta\, d\theta = 0.4514$$

Now converting to Pascals and meters, we have from Eq. (16-2),

$$M_f = \frac{f\, p_a b r}{\sin\theta_a} A = \frac{0.24\left(10^6\right)(0.075)(0.200)}{\sin 75^\circ}(0.0775) = 289 \text{ N} \cdot \text{m}$$

From Eq. (16-3),

$$M_N = \frac{p_a bra}{\sin\theta_a} B = \frac{10^6(0.075)(0.200)(0.150)}{\sin 75°}(0.528) = 1230 \text{ N} \cdot \text{m}$$

Finally, using Eq. (16-4), we have

$$F = \frac{M_N - M_f}{c} = \frac{1230 - 289}{165} = 5.70 \text{ kN} \quad \textit{Ans.}$$

(b) Use Eq. (16-6) for the primary shoe.

$$T = \frac{fp_a br^2(\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$
$$= \frac{0.24\left(10^6\right)(0.075)(0.200)^2(\cos 10° - \cos 75°)}{\sin 75°} = 541 \text{ N} \cdot \text{m}$$

For the secondary shoe, we must first find p_a. Substituting

$$M_N = \frac{1230}{10^6} p_a \text{ and } M_f = \frac{289}{10^6} p_a \quad \text{into Eq. (16 - 7),}$$
$$5.70 = \frac{(1230 / 10^6)p_a + (289 / 10^6)p_a}{165}, \quad \text{solving gives} \quad p_a = 619\left(10^3\right) \text{ Pa}$$

Then

$$T = \frac{0.24\left[619\left(10^3\right)\right]0.075\left(0.200^2\right)(\cos 10° - \cos 75°)}{\sin 75°} = 335 \text{ N} \cdot \text{m}$$

so the braking capacity is $T_{\text{total}} = 2(541) + 2(335) = 1750 \text{ N} \cdot \text{m}$ *Ans.*

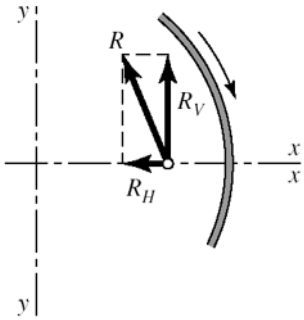
(c) *Primary shoes*:

$$R_x = \frac{p_a br}{\sin\theta_a}(C - f B) - F_x$$
$$= \frac{10^6(0.075)0.200}{\sin 75°}[0.4514 - 0.24(0.528)](10^{-3}) - 5.70 = -0.658 \text{ kN}$$
$$R_y = \frac{p_a br}{\sin\theta_a}(B + f C) - F_y$$
$$= \frac{10^6(0.075)0.200}{\sin 75°}[0.528 + 0.24(0.4514)]\left(10^{-3}\right) - 0 = 9.88 \text{ kN}$$

Secondary shoes:

$$R_x = \frac{p_a br}{\sin\theta_a}(C + f\,B) - F_x$$
$$= \frac{0.619\left(10^6\right)0.075(0.200)}{\sin 75°}[0.4514 + 0.24(0.528)]\left(10^{-3}\right) - 5.70$$
$$= -0.143 \text{ kN}$$
$$R_y = \frac{p_a br}{\sin\theta_a}(B - f\,C) - F_y$$
$$= \frac{0.619\left(10^6\right)0.075(0.200)}{\sin 75°}[0.528 - 0.24(0.4514)]\left(10^{-3}\right) - 0$$
$$= 4.03 \text{ kN}$$

Note from figure that +*y* for secondary shoe is opposite to +*y* for primary shoe.



Combining horizontal and vertical components,

$R_H = -0.658 - 0.143 = -0.801$ kN

$R_V = 9.88 - 4.03 = 5.85$ kN

$$R = \sqrt{(-0.801)^2 + 5.85^2}$$
$$= 5.90 \text{ kN} \quad \textit{Ans.}$$

16-5 Given: Face width $b = 1.25$ in, $F = 90$ lbf, $f = 0.25$.

Preliminaries: $\theta_1 = 45° - \tan^{-1}(6/8) = 8.13°$, $\theta_2 = 98.13°$, $\theta_a = 90°$, $a = (6^2 + 8^2)^{1/2} = 10$ in

Eq. (16-2):

$$M_f = \frac{f\,p_a br}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin\theta\left(r - a\cos\theta\right)d\theta = \frac{0.25p_a(1.25)6}{1}\int_{8.13°}^{98.13°}\sin\theta\left(6 - 10\cos\theta\right)d\theta$$
$$= 3.728p_a \text{ lbf}\cdot\text{in}$$

Eq. (16-3):

$$M_N = \frac{p_a bra}{\sin\theta_a}\int_{\theta_1}^{\theta_2}\sin^2\theta\,d\theta = \frac{p_a(1.25)6(10)}{1}\int_{8.13°}^{98.13°}\sin^2\theta\,d\theta$$
$$= 69.405p_a \text{ lbf}\cdot\text{in}$$

Eq. (16-4): Using $Fc = M_N - M_f$, we obtain

$$90(20) = (69.405 - 3.728)p_a \quad \Rightarrow \quad p_a = 27.4 \text{ psi} \quad \textit{Ans.}$$

Eq. (16-6):

$$T = \frac{f p_a b r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a} = \frac{0.25(27.4)1.25\left(6^2\right)(\cos 8.13° - \cos 98.13°)}{1}$$

$$= 348.7 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-6 For $+3\hat{\sigma}_f$:

$$f = \bar{f} + 3\hat{\sigma}_f = 0.25 + 3(0.025) = 0.325$$

From Prob. 16-5, with $f = 0.25$, $M_f = 3.728\, p_a$. Thus, $M_f = (0.325/0.25)\, 3.728\, p_a = 4.846\, p_a$. From Prob. 16-5, $M_N = 69.405\, p_a$.

Eq. (16-4): Using $Fc = M_N - M_f$, we obtain

$$90(20) = (69.405 - 4.846) p_a \quad \Rightarrow \quad p_a = 27.88 \text{ psi} \quad \textit{Ans.}$$

From Prob. 16-5, $p_a = 27.4$ psi and $T = 348.7$ lbf·in. Thus,

$$T = \left(\frac{0.325}{0.25}\right)\left(\frac{27.88}{27.4}\right)348.7 = 461.3 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

Similarly, for $-3\hat{\sigma}_f$:

$$f = \bar{f} - 3\hat{\sigma}_f = 0.25 - 3(0.025) = 0.175$$
$$M_f = (0.175 / 0.25)\, 3.728 p_a = 2.610 p_a$$

$$90(20) = (69.405 - 2.610)\, p_a \quad \Rightarrow \quad p_a = 26.95 \text{ psi}$$
$$T = \left(\frac{0.175}{0.25}\right)\left(\frac{26.95}{27.4}\right)348.7 = 240.1 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-7 Preliminaries: $\theta_2 = 180° - 30° - \tan^{-1}(3/12) = 136°$, $\theta_1 = 20° - \tan^{-1}(3/12) = 6°$, $\theta_a = 90°$, $\sin\theta_a = 1$, $a = (3^2 + 12^2)^{1/2} = 12.37$ in, $r = 10$ in, $f = 0.30$, $b = 2$ in, $p_a = 150$ psi.

Eq. (16-2): $$M_f = \frac{0.30(150)(2)(10)}{\sin 90°} \int_{6°}^{136°} \sin\theta(10 - 12.37\cos\theta)\, d\theta = 12\,800 \text{ lbf} \cdot \text{in}$$

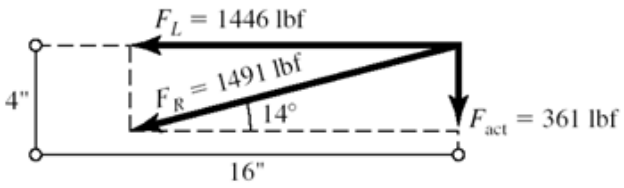
Eq. (16-3): $$M_N = \frac{150(2)(10)(12.37)}{\sin 90°} \int_{6°}^{136°} \sin^2\theta\, d\theta = 53\,300 \text{ lbf} \cdot \text{in}$$

LH shoe:

$$c_L = 12 + 12 + 4 = 28 \text{ in}$$

Now note that M_f is cw and M_N is ccw. Thus,

$$F_L = \frac{53\,300 - 12\,800}{28} = 1446 \text{ lbf}$$



Eq. (16-6): $$T_L = \frac{0.30(150)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 15\,420 \text{ lbf} \cdot \text{in}$$

RH shoe:

$$M_N = 53\,300\frac{p_a}{150} = 355.3p_a, \quad M_f = 12\,800\frac{p_a}{150} = 85.3p_a$$

On this shoe, both M_N and M_f are ccw. Also,

$$c_R = (24 - 2\tan 14°)\cos 14° = 22.8 \text{ in}$$
$$F_{act} = F_L \sin 14° = 361 \text{ lbf} \quad \textit{Ans.}$$
$$F_R = F_L / \cos 14° = 1491 \text{ lbf}$$

Thus, $$1491 = \frac{355.3 + 85.3}{22.8} p_a \;\Rightarrow\; p_a = 77.2 \text{ psi}$$

Then, $$T_R = \frac{0.30(77.2)(2)(10)^2(\cos 6° - \cos 136°)}{\sin 90°} = 7940 \text{ lbf} \cdot \text{in}$$

$$T_{\text{total}} = 15\,420 + 7940 = 23\,400 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

16-8

$$M_f = 2\int_0^{\theta_2} (f dN)(a'\cos\theta - r) \qquad \text{where } dN = pbr\, d\theta$$
$$= 2fpbr\int_0^{\theta_2} (a'\cos\theta - r)\, d\theta = 0$$

From which

$$a'\int_0^{\theta_2} \cos\theta\, d\theta = r\int_0^{\theta_2} d\theta$$
$$a' = \frac{r\theta_2}{\sin\theta_2} = \frac{r(60°)(\pi / 180)}{\sin 60°} = 1.209r \quad \textit{Ans.}$$

Eq. (16-15):

$$a = \frac{4r\sin 60°}{2(60)(\pi / 180) + \sin[2(60)]} = 1.170r \qquad \textit{Ans.}$$

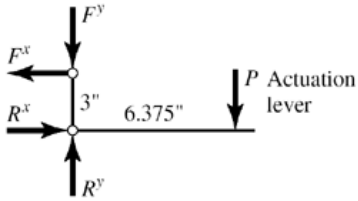
a differs with a' by $100(1.170 - 1.209)/1.209 = -3.23\ \%$ *Ans.*

16-9 **(a)** Counter-clockwise rotation, $\theta_2 = \pi/4$ rad, $r = 13.5/2 = 6.75$ in
Eq. (16-15):

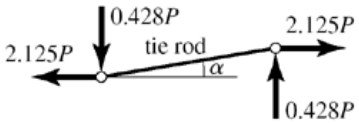
$$a = \frac{4r\sin\theta_2}{2\theta_2 + \sin 2\theta_2} = \frac{4(6.75)\sin(\pi / 4)}{2\pi / 4 + \sin(2\pi / 4)} = 7.426 \text{ in}$$

$$e = 2a = 2(7.426) = 14.85 \text{ in} \qquad \textit{Ans.}$$

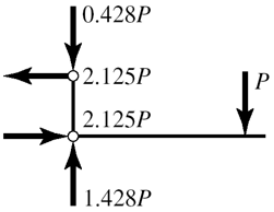
(b)



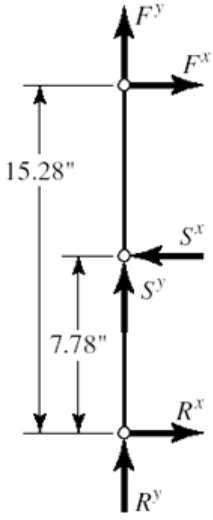
$$\alpha = \tan^{-1}(3/14.85) = 11.4°$$



$$\sum M_R = 0 = 3F^x - 6.375P \quad \Rightarrow \quad F^x = 2.125P$$
$$\sum F_x = 0 = -F^x + R^x \quad \Rightarrow \quad R^x = F^x = 2.125P$$

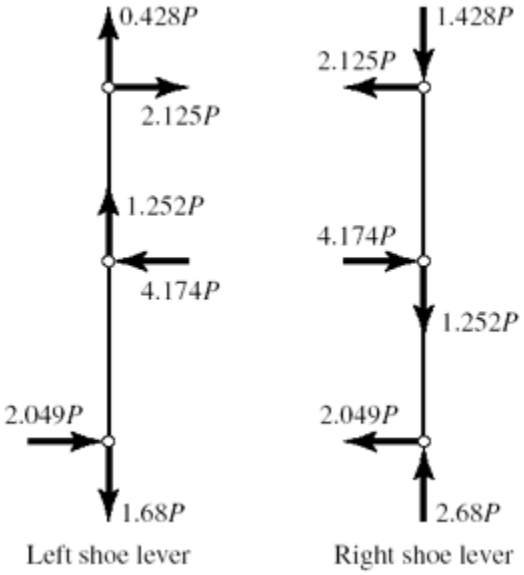


$$F^y = F^x \tan 11.4° = 0.428P$$
$$\sum F_y = -P - F^y + R^y$$
$$R^y = P + 0.428P = 1.428P$$



Left shoe lever.

$$\sum M_R = 0 = 7.78S^x - 15.28F^x$$
$$S^x = \frac{15.28}{7.78}(2.125P) = 4.174P$$
$$S^y = f S^x = 0.30(4.174P) = 1.252P$$
$$\sum F_y = 0 = R^y + S^y + F^y$$
$$R^y = -F^y - S^y = -0.428P - 1.252P = -1.68P$$
$$\sum F_x = 0 = R^x - S^x + F^x$$
$$R^x = S^x - F^x = 4.174P - 2.125P = 2.049P$$



Ans.

(c) The direction of brake pulley rotation affects the sense of S^y, which has no effect on the brake shoe lever moment and hence, no effect on S^x or the brake torque.

The brake shoe levers carry identical bending moments but the left lever carries a tension while the right carries compression (column loading). The right lever is designed and used as a left lever, producing interchangeable levers (identical levers). But do not infer from these identical loadings.

16-10 $r = 13.5/2 = 6.75$ in, $b = 6$ in, $\theta_2 = 45° = \pi / 4$ rad.

From Table 16-3 for a rigid, molded non-asbestos lining use a conservative estimate of $p_a = 100$ psi, $f = 0.33$.

Equation (16-16) gives the horizontal brake hinge pin reaction which corresponds to S^x in Prob. 16-9. Thus,

$$N = S^x = \frac{p_a b r}{2}\left(2\theta_2 + \sin 2\theta_2\right) = \frac{100(6)6.75}{2}\left\{2\left(\pi / 4\right) + \sin\left[2\left(45°\right)\right]\right\}$$
$$= 5206 \text{ lbf}$$

which, from Prob. 6-9 is 4.174 P. Therefore,

$$4.174\,P = 5206 \quad \Rightarrow \quad P = 1250 \text{ lbf} = 1.25 \text{ kip} \qquad \textit{Ans.}$$

Applying Eq. (16-18) for two shoes, where from Prob. 16-9, $a = 7.426$ in

$$T = 2a\,f\,N = 2(7.426)0.33(5206)$$
$$= 25\,520 \text{ lbf} \cdot \text{in} = 25.52 \text{ kip} \cdot \text{in} \qquad \textit{Ans.}$$

16-11 Given: $D = 350$ mm, $b = 100$ mm, $p_a = 620$ kPa, $f = 0.30$, $\phi = 270°$.

Eq. (16-22):

$$P_1 = \frac{p_a b D}{2} = \frac{620(0.100)0.350}{2} = 10.85 \text{ kN} \quad \textit{Ans.}$$

$$f\phi = 0.30(270°)(\pi / 180°) = 1.414$$

Eq. (16-19): $P_2 = P_1 \exp(-f\phi) = 10.85 \exp(-1.414) = 2.64 \text{ kN}$ *Ans.*

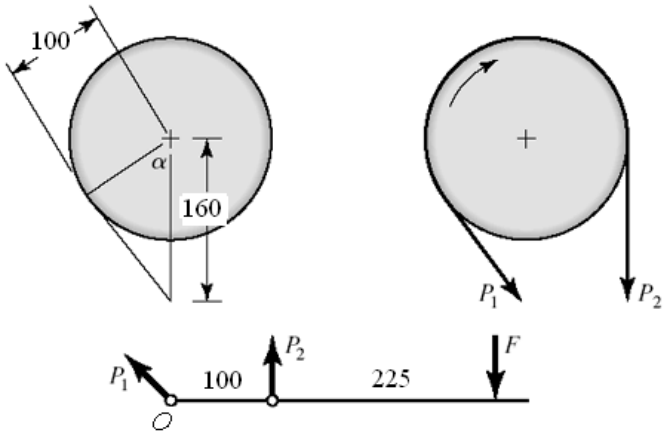
$$T = (P_1 - P_2)(D / 2) = (10.85 - 2.64)(0.350 / 2) = 1.437 \text{ kN} \cdot \text{m} \quad \textit{Ans.}$$

16-12 Given: $D = 12$ in, $f = 0.28$, $b = 3.25$ in, $\phi = 270°$, $P_1 = 1800$ lbf.

Eq. (16-22): $p_a = \dfrac{2P_1}{bD} = \dfrac{2(1800)}{3.25(12)} = 92.3 \text{ psi}$ *Ans.*

$$f\phi = 0.28(270^\circ)(\pi / 180^\circ) = 1.319$$
$$P_2 = P_1 \exp(-f\phi) = 1800 \exp(-1.319) = 481 \text{ lbf}$$
$$T = (P_1 - P_2)(D / 2) = (1800 - 481)(12 / 2)$$
$$= 7910 \text{ lbf} \cdot \text{in} = 7.91 \text{ kip} \cdot \text{in} \quad \textit{Ans.}$$

16-13



$$\Sigma M_O = 0 = 100 P_2 - 325 F \quad \Rightarrow \quad P_2 = 325(300)/100 = 975 \text{ N} \quad \textit{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{100}{160}\right) = 51.32°$$

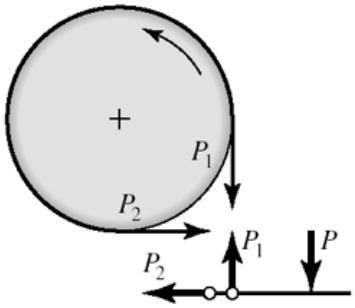
$$\phi = 270° - 51.32° = 218.7°$$

$$f\phi = 0.30(218.7)\left(\pi / 180°\right) = 1.145$$

$$P_1 = P_2 \exp(f\phi) = 975 \exp(1.145) = 3064 \text{ N} \quad \textit{Ans.}$$

$$T = \left(P_1 - P_2\right)(D / 2) = (3064 - 975)(200 / 2)$$
$$= 209\left(10^3\right) \text{ N} \cdot \text{mm} = 209 \text{ N} \cdot \text{m} \quad \textit{Ans.}$$

16-14 **(a)** $D = 16$ in, $b = 3$ in
$n = 200$ rev/min
$f = 0.20$, $p_a = 70$ psi



Eq. (16-22):

$$P_1 = \frac{p_a b D}{2} = \frac{70(3)(16)}{2} = 1680 \text{ lbf}$$

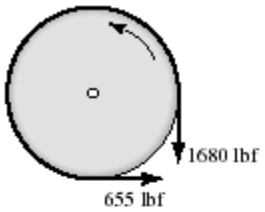
$$f\phi = 0.20(3\pi/2) = 0.942$$

Eq. (16-14): $P_2 = P_1 \exp(-f\phi) = 1680 \exp(-0.942) = 655$ lbf

$$T = (P_1 - P_2)\frac{D}{2} = (1680 - 655)\frac{16}{2}$$
$$= 8200 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$H = \frac{Tn}{63\,025} = \frac{8200(200)}{63\,025} = 26.0 \text{ hp} \quad \textit{Ans.}$$

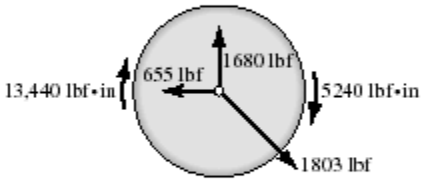
$$P = \frac{3P_1}{10} = \frac{3(1680)}{10} = 504 \text{ lbf} \quad \textit{Ans.}$$



(b) Force of belt on the drum:

$$R = (1680^2 + 655^2)^{1/2} = 1803 \text{ lbf}$$

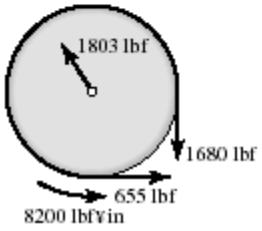
Force of shaft on the drum: 1680 and 655 lbf



$$T_{P_1} = 1680(8) = 13\,440 \text{ lbf} \cdot \text{in}$$
$$T_{P_2} = 655(8) = 5240 \text{ lbf} \cdot \text{in}$$

Net torque on drum due to brake band:

$$T = T_{P_1} - T_{P_2}$$
$$= 13\,440 - 5240$$
$$= 8200 \text{ lbf} \cdot \text{in}$$



The radial load on the bearing pair is 1803 lbf. If the bearing is straddle mounted with the drum at center span, the bearing radial load is 1803/2 = 901 lbf.

(c) Eq. (16-21):

$$p = \frac{2P}{bD}$$

$$p\big|_{\theta=0^\circ} = \frac{2P_1}{3(16)} = \frac{2(1680)}{3(16)} = 70 \text{ psi} \quad \textit{Ans.}$$

$$p\big|_{\theta=270^\circ} = \frac{2P_2}{3(16)} = \frac{2(655)}{3(16)} = 27.3 \text{ psi} \quad \textit{Ans.}$$

16-15 Given: $\phi = 270°$, $b = 2.125$ in, $f = 0.20$, $T = 150$ lbf · ft, $D = 8.25$ in, $c_2 = 2.25$ in (see figure). Notice that the pivoting rocker is not located on the vertical centerline of the drum.

(a) To have the band tighten for ccw rotation, it is necessary to have $c_1 < c_2$. When friction is fully developed,

$$P_1 / P_2 = \exp(f\phi) = \exp[0.2(3\pi / 2)] = 2.566$$

If friction is not fully developed,

$$P_1/P_2 \le \exp(f\phi)$$

To help visualize what is going on let's add a force W parallel to P_1, at a lever arm of c_3. Now sum moments about the rocker pivot.

$$\sum M = 0 = c_3 W + c_1 P_1 - c_2 P_2$$

From which

$$W = \frac{c_2 P_2 - c_1 P_1}{c_3}$$

The device is self locking for ccw rotation if W is no longer needed, that is, $W \le 0$. It follows from the equation above

$$\frac{P_1}{P_2} \ge \frac{c_2}{c_1}$$

When friction is fully developed

$$2.566 = 2.25 / c_1$$

$$c_1 = \frac{2.25}{2.566} = 0.877 \text{ in}$$

When P_1/P_2 is less than 2.566, friction is not fully developed. Suppose $P_1/P_2 = 2.25$,

then

$$c_1 = \frac{2.25}{2.25} = 1 \text{ in}$$

We don't want to be at the point of slip, and we need the band to tighten.

$$\frac{c_2}{P_1 / P_2} \le c_1 \le c_2$$

When the developed friction is very small, $P_1/P_2 \to 1$ and $c_1 \to c_2$ *Ans.*

(b) Rocker has $c_1 = 1$ in

$$\frac{P_1}{P_2} = \frac{c_2}{c_1} = \frac{2.25}{1} = 2.25$$
$$f = \frac{\ln(P_1 / P_2)}{\phi} = \frac{\ln\ 2.25}{3\pi/2} = 0.172$$

Friction is not fully developed, no slip.

$$T = (P_1 - P_2)\frac{D}{2} = P_2\left(\frac{P_1}{P_2} - 1\right)\frac{D}{2}$$

Solve for P_2

$$P_2 = \frac{2T}{[(P_1 / P_2) - 1]D} = \frac{2(150)(12)}{(2.25 - 1)(8.25)} = 349 \text{ lbf}$$
$$P_1 = 2.25P_2 = 2.25(349) = 785 \text{ lbf}$$
$$p = \frac{2P_1}{bD} = \frac{2(785)}{2.125(8.25)} = 89.6 \text{ psi} \quad \textit{Ans.}$$

(c) The torque ratio is 150(12)/100 or 18-fold.

$$P_2 = \frac{349}{18} = 19.4 \text{ lbf}$$
$$P_1 = 2.25P_2 = 2.25(19.4) = 43.6 \text{ lbf}$$
$$p = \frac{89.6}{18} = 4.98 \text{ psi} \quad \textit{Ans.}$$

Comment:
As the torque opposed by the locked brake increases, P_2 and P_1 increase (although ratio is still 2.25), then p follows. The brake can self-destruct. Protection could be provided by a shear key.

16-16 Given: OD = 250 mm, ID = 175 mm, $f = 0.30$, $F = 4$ kN.

(a) From Eq. (16-23),

$$p_a = \frac{2F}{\pi d(D-d)} = \frac{2(4000)}{\pi(175)(250-175)} = 0.194 \text{ N/mm}^2 = 194 \text{ kPa} \qquad \textit{Ans.}$$

Eq. (16-25):

$$T = \frac{Ff}{4}(D+d) = \frac{4000(0.30)}{4}(250+175)10^{-3} = 127.5 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

(b) From Eq. (16-26),

$$p_a = \frac{4F}{\pi(D^2-d^2)} = \frac{4(4000)}{\pi(250^2-175^2)} = 0.159 \text{ N/mm}^2 = 159 \text{ kPa} \qquad \textit{Ans.}$$

Eq. (16-27):

$$T = \frac{\pi}{12} f\, p_a(D^3-d^3) = \frac{\pi}{12}(0.30)159\left(10^3\right)\left(250^3-175^3\right)\left(10^{-3}\right)^3$$
$$= 128 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

16-17 Given: OD = 6.5 in, ID = 4 in, $f = 0.24$, $p_a = 120$ psi.

(a) Eq. (16-23):

$$F = \frac{\pi p_a d}{2}(D-d) = \frac{\pi(120)(4)}{2}(6.5-4) = 1885 \text{ lbf} \qquad \textit{Ans.}$$

Eq. (16-24) with N sliding planes:

$$T = \frac{\pi f p_a d}{8}(D^2-d^2)N = \frac{\pi(0.24)(120)(4)}{8}(6.5^2-4^2)(6)$$
$$= 7125 \text{ lbf} \cdot \text{in} \qquad \textit{Ans.}$$

(b) $$T = \frac{\pi(0.24)(120d)}{8}(6.5^2-d^2)(6)$$

d, in	T, lbf · in	
2	5191	
3	6769	
4	7125	*Ans.*
5	5853	
6	2545	

(c) The torque-diameter curve exhibits a stationary point maximum in the range of diameter d. The clutch has nearly optimal proportions.

16-18 **(a)** Eq. (16-24) with N sliding planes:

$$T = \frac{\pi f\, p_a d(D^2 - d^2)N}{8} = \frac{\pi f\, p_a N}{8}\left(D^2 d - d^3\right)$$

Differentiating with respect to d and equating to zero gives

$$\frac{dT}{dd} = \frac{\pi f\, p_a N}{8}\left(D^2 - 3d^2\right) = 0$$

$$d^* = \frac{D}{\sqrt{3}} \quad \textit{Ans.}$$

$$\frac{d^2T}{dd^2} = -6\frac{\pi f\, p_a N}{8}d = -\frac{3\pi f\, p_a N}{4}d$$

which is negative for all positive d. We have a stationary point *maximum*.

(b) $$d^* = \frac{6.5}{\sqrt{3}} = 3.75 \text{ in} \quad \textit{Ans.}$$

Eq. (16-24):

$$T^* = \frac{\pi(0.24)(120)\left(6.5 / \sqrt{3}\right)}{8}\left[6.5^2 - \left(6.5 / \sqrt{3}\right)^2\right](6) = 7173 \text{ lbf} \cdot \text{in}$$

(c) The table indicates a maximum within the range: $3 \le d \le 5$ in

(d) Consider: $$0.45 = \frac{d}{D} = 0.80$$

Multiply through by D,

$$0.45D \le d \le 0.80D$$

$$0.45(6.5) \le d \le 0.80(6.5)$$

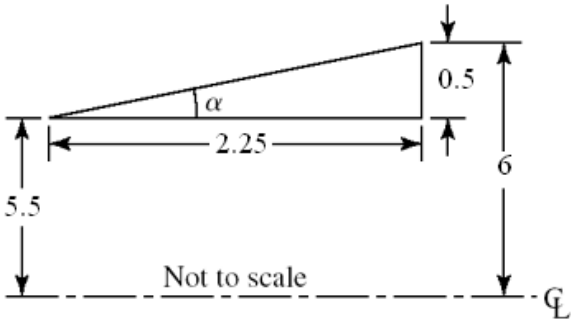
$$2.925 \le d \le 5.2 \text{ in}$$

$$\left(\frac{d}{D}\right)^* = d^*/D = \frac{1}{\sqrt{3}} = 0.577$$

which lies within the common range of clutches.

Yes. *Ans.*

16-19 Given: $d = 11$ in, $l = 2.25$ in, $T = 1800$ lbf · in, $D = 12$ in, $f = 0.28$.



$$\alpha = \tan^{-1}\left(\frac{0.5}{2.25}\right) = 12.53°$$

Uniform wear

Eq. (16-45):

$$T = \frac{\pi f\ p_a d}{8\sin\alpha}\left(D^2 - d^2\right)$$

$$1800 = \frac{\pi(0.28)p_a(11)}{8\sin 12.53°}\left(12^2 - 11^2\right) = 128.2\,p_a$$

$$p_a = \frac{1800}{128.2} = 14.04 \text{ psi} \quad \textit{Ans.}$$

Eq. (16-44):

$$F = \frac{\pi p_a d}{2}(D - d) = \frac{\pi(14.04)11}{2}(12 - 11) = 243 \text{ lbf} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-48):

$$T = \frac{\pi f\ p_a}{12\sin\alpha}\left(D^3 - d^3\right)$$

$$1800 = \frac{\pi(0.28)p_a}{12\sin 12.53°}\left(12^3 - 11^3\right) = 134.1\,p_a$$

$$p_a = \frac{1800}{134.1} = 13.42 \text{ psi} \quad \textit{Ans.}$$

Eq. (16-47):

$$F = \frac{\pi p_a}{4}(D^2 - d^2) = \frac{\pi(13.42)}{4}\left(12^2 - 11^2\right) = 242 \text{ lbf} \quad \textit{Ans.}$$

16-20 *Uniform wear*

Eq. (16-34): $T = \frac{1}{2}(\theta_2 - \theta_1) f\ p_a r_i \left(r_o^2 - r_i^2\right)$

Eq. (16-33): $F = (\theta_2 - \theta_1)\, p_a r_i\, (r_o - r_i)$

Thus,

$$\frac{T}{f\ FD} = \frac{(1\ /\ 2)(\theta_2 - \theta_1) f\ p_a r_i \left(r_o^2 - r_i^2\right)}{f(\theta_2 - \theta_1) p_a r_i (r_o - r_i)(D)}$$

$$= \frac{r_o + r_i}{2D} = \frac{D\ /\ 2 + d\ /\ 2}{2D} = \frac{1}{4}\left(1 + \frac{d}{D}\right) \quad \textit{O.K.} \quad \textit{Ans.}$$

Uniform pressure

Eq. (16-38): $T = \frac{1}{3}(\theta_2 - \theta_1) f\ p_a \left(r_o^3 - r_i^3\right)$

Eq. (16-37): $$F = \frac{1}{2}(\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right)$$

Thus,

$$\frac{T}{f\ FD} = \frac{(1/3)(\theta_2 - \theta_1)\, f\ p_a \left(r_o^3 - r_i^3\right)}{(1/2) f (\theta_2 - \theta_1) p_a \left(r_o^2 - r_i^2\right) D} = \frac{2}{3}\left\{\frac{(D/2)^3 - (d/2)^3}{\left[(D/2)^2 - (d/2)^2 D\right]}\right\}$$

$$= \frac{2(D/2)^3\left[1 - (d/D)^3\right]}{3(D/2)^2\left[1 - (d/D)^2\right]D} = \frac{1}{3}\left[\frac{1 - (d/D)^3}{1 - (d/D)^2}\right] \quad O.K. \quad Ans.$$

16-21

$$\omega = 2\pi n / 60 = 2\pi\ 500 / 60 = 52.4 \text{ rad/s}$$

$$T = \frac{H}{\omega} = \frac{2(10^3)}{52.4} = 38.2 \text{ N} \cdot \text{m}$$

Key:

$$F = \frac{T}{r} = \frac{38.2}{12} = 3.18 \text{ kN}$$

Average shear stress in key is

$$\tau = \frac{3.18(10^3)}{6(40)} = 13.2 \text{ MPa} \quad Ans.$$

Average bearing stress is

$$\sigma_b = -\frac{F}{A_b} = -\frac{3.18(10^3)}{3(40)} = -26.5 \text{ MPa} \quad Ans.$$

Let one jaw carry the entire load.

$$r_{av} = \frac{1}{2}\left(\frac{26}{2} + \frac{45}{2}\right) = 17.75 \text{ mm}$$

$$F = \frac{T}{r_{av}} = \frac{38.2}{17.75} = 2.15 \text{ kN}$$

The bearing and shear stress estimates are

$$\sigma_b = \frac{-2.15\left(10^3\right)}{10(22.5 - 13)} = -22.6 \text{ MPa} \quad Ans.$$

$$\tau = \frac{2.15(10^3)}{10\left[0.25\pi(17.75)^2\right]} = 0.869 \text{ MPa} \quad Ans.$$

16-22

$$\omega_1 = 2\pi n / 60 = 2\pi(1600) / 60 = 167.6 \text{ rad/s}$$
$$\omega_2 = 0$$

From Eq. (16-51),

$$\frac{I_1 I_2}{I_1 + I_2} = \frac{T t_1}{\omega_1 - \omega_2} = \frac{2800(8)}{167.6 - 0} = 133.7 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

Eq. (16-52):

$$E = \frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2 = \frac{133.7}{2}(167.6 - 0)^2 = 1.877(10^6) \text{ lbf} \cdot \text{in}$$

In Btu, Eq. (16-53): $H = E / 9336 = 1.877(10^6) / 9336 = 201$ Btu

Eq. (16-54):

$$\Delta T = \frac{H}{C_p W} = \frac{201}{0.12(40)} = 41.9°\text{F} \quad \textit{Ans.}$$

16-23

$$n = \frac{n_1 + n_2}{2} = \frac{260 + 240}{2} = 250 \text{ rev/min}$$

Eq. (16-62): $C_s = (\omega_2 - \omega_1) / \omega = (n_2 - n_1) / n = (260 - 240) / 250 = 0.08$ *Ans.*

$$\omega = 2\pi(250) / 60 = 26.18 \text{ rad/s}$$

From Eq. (16-64):

$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{6.75(10^3)}{0.08(26.18)^2} = 123.1 \text{ N} \cdot \text{m} \cdot \text{s}^2$$

$$I = \frac{m}{8}(d_o^2 + d_i^2) \quad \Rightarrow \quad m = \frac{8I}{d_o^2 + d_i^2} = \frac{8(123.1)}{1.5^2 + 1.4^2} = 233.9 \text{ kg}$$

Table A-5, cast iron unit weight = 70.6 kN/m^3 $\Rightarrow \rho = 70.6(10^3) / 9.81 = 7197$ kg / m^3.

Volume: $V = m / \rho = 233.9 / 7197 = 0.0325$ m^3

$$V = \pi t(d_o^2 - d_i^2) / 4 = \pi t(1.5^2 - 1.4^2) / 4 = 0.2278t$$

Equating the expressions for volume and solving for t,

$$t = \frac{0.0325}{0.2278} = 0.143 \text{ m} = 143 \text{ mm} \quad \textit{Ans.}$$

16-24 **(a)** The useful work performed in one revolution of the crank shaft is

$$U = 320\ (10^3)\ 200\ (10^{-3})\ 0.15 = 9.6\ (10^3)\ \text{J}$$

Accounting for friction, the total work done in one revolution is

$$U = 9.6(10^3) / (1 - 0.20) = 12.0(10^3)\ \text{J}$$

Since 15% of the crank shaft stroke accounts for 7.5% of a crank shaft revolution, the energy fluctuation is

$$E_2 - E_1 = 9.6(10^3) - 12.0(10^3)(0.075) = 8.70(10^3)\ \text{J} \quad \textit{Ans.}$$

(b) For the flywheel,

$$n = 6(90) = 540\ \text{rev/min}$$
$$\omega = \frac{2\pi n}{60} = \frac{2\pi(540)}{60} = 56.5\ \text{rad/s}$$

Since $C_s = 0.10$

Eq. (16-64): $$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{8.70(10^3)}{0.10(56.5)^2} = 27.25\ \text{N} \cdot \text{m} \cdot \text{s}^2$$

Assuming all the mass is concentrated at the effective diameter, d,

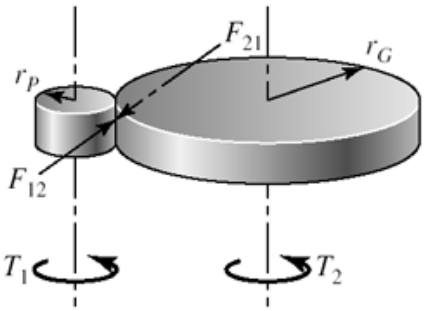
$$I = mr^2 = \frac{md^2}{4}$$
$$m = \frac{4I}{d^2} = \frac{4(27.25)}{1.2^2} = 75.7\ \text{kg} \quad \textit{Ans.}$$

16-25 Use Ex. 16-6 and Table 16-6 data for one cylinder of a 3-cylinder engine.

$$C_s = 0.30$$
$$n = 2400\ \text{rev/min} \quad \text{or} \quad 251\ \text{rad/s}$$
$$T_m = \frac{3(3368)}{4\pi} = 804\ \text{lbf} \cdot \text{in} \quad \textit{Ans.}$$
$$E_2 - E_1 = 3(3531) = 10\ 590\ \text{in} \cdot \text{lbf}$$
$$I = \frac{E_2 - E_1}{C_s \omega^2} = \frac{10\ 590}{0.30(251^2)} = 0.560\ \text{in} \cdot \text{lbf} \cdot \text{s}^2 \quad \textit{Ans.}$$

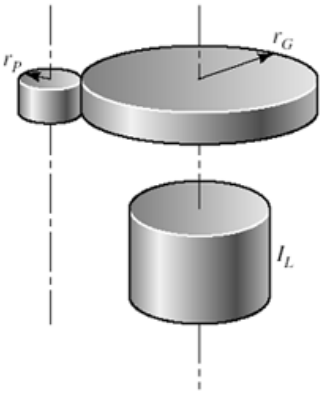
16-26 **(a)**

(1)



$$(T_2)_1 = -F_{21} r_P = -\frac{T_2}{r_G} r_P = \frac{T_2}{-n} \quad \textit{Ans.}$$

(2)



Equivalent energy

$$(1/2) I_2 \omega_2^2 = (1/2)(I_2)_1 \left(\omega_1^2\right)$$

$$(I_2)_1 = \frac{\omega_2^2}{\omega_1^2} I_2 = \frac{I_2}{n^2} \quad \textit{Ans.}$$

(3)

$$\left(\frac{r_G}{r_P}\right)^2 \left(\frac{r_G}{r_P}\right)^2 = n^4$$

From (2) $$(I_2)_1 = \frac{I_G}{n^2} = \frac{n^4 I_P}{n^2} = n^2 I_P \quad \textit{Ans.}$$

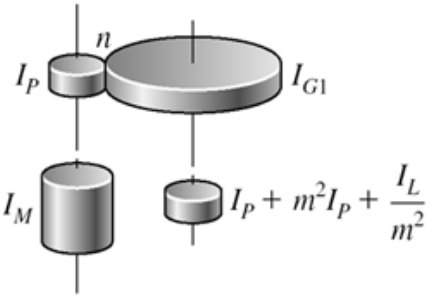
(b) $I_e = I_M + I_P + n^2 I_P + \dfrac{I_L}{n^2}$ *Ans.*

(c) $I_e = 10 + 1 + 10^2(1) + \dfrac{100}{10^2} = 112$

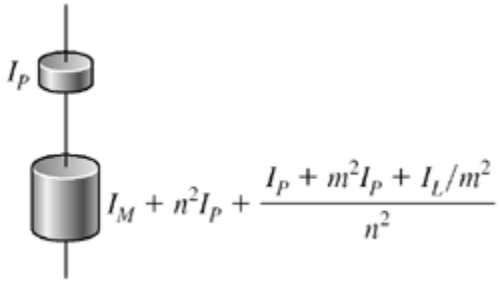
- armature inertia
- pinion inertia
- reflected gear inertia
- reflected load inertia

Ans.

16-27 **(a)** Reflect I_L, I_{G2} to the center shaft



Reflect the center shaft to the motor shaft



$$I_e = I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{m^2}{n^2} I_P + \frac{I_L}{m^2 n^2} \qquad Ans.$$

(b) For R = constant = nm, $I_e = I_M + I_P + n^2 I_P + \dfrac{I_P}{n^2} + \dfrac{R^2 I_P}{n^4} + \dfrac{I_L}{R^2}$ *Ans.*

(c) For $R = 10$, $\dfrac{\partial I_e}{\partial n} = 0 + 0 + 2n(1) - \dfrac{2(1)}{n^3} - \dfrac{4(10^2)(1)}{n^5} + 0 = 0$

$$n^6 - n^2 - 200 = 0$$

From which

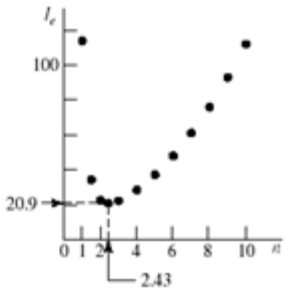
$$n^* = 2.430 \qquad Ans.$$
$$m^* = \frac{10}{2.430} = 4.115 \qquad Ans.$$

Notice that $n^*$and m^* are independent of I_L.

16-28 From Prob. 16-27,

$$\begin{aligned} I_e &= I_M + I_P + n^2 I_P + \frac{I_P}{n^2} + \frac{R^2 I_P}{n^4} + \frac{I_L}{R^2} \\ &= 10 + 1 + n^2(1) + \frac{1}{n^2} + \frac{100(1)}{n^4} + \frac{100}{10^2} \\ &= 12 + n^2 + \frac{1}{n^2} + \frac{100}{n^4} \end{aligned}$$

n	I_e
1.00	114.00
1.50	34.40
2.00	22.50
2.43	20.90
3.00	22.30
4.00	28.50
5.00	37.20
6.00	48.10
7.00	61.10
8.00	76.00
9.00	93.00
10.00	112.02



Optimizing the partitioning of a double reduction lowered the gear-train inertia to 20.9/112 = 0.187, or to 19% of that of a single reduction. This includes the two additional gears.

16-29 Figure 16-29 applies,

$$t_2 = 10\ s, \quad t_1 = 0.5\ s$$
$$\frac{t_2 - t_1}{t_1} = \frac{10 - 0.5}{0.5} = 19$$

The load torque, as seen by the motor shaft (Rule 1, Prob. 16-26), is

$$T_L = \left|\frac{1300(12)}{10}\right| = 1560\ \text{lbf} \cdot \text{in}$$

The rated motor torque T_r is

$$T_r = \frac{63\ 025(3)}{1125} = 168.07\ \text{lbf} \cdot \text{in}$$

For Eqs. (16-65):

$$\omega_r = \frac{2\pi}{60}(1125) = 117.81\ \text{rad/s}$$
$$\omega_s = \frac{2\pi}{60}(1200) = 125.66\ \text{rad/s}$$
$$a = \frac{-T_r}{\omega_s - \omega_r} = -\frac{168.07}{125.66 - 117.81} = -21.41\ \text{lbf} \cdot \text{in} \cdot \text{s/rad}$$
$$b = \frac{T_r \omega_s}{\omega_s - \omega_r} = \frac{168.07(125.66)}{125.66 - 117.81} = 2690.4\ \text{lbf} \cdot \text{in}$$

The linear portion of the squirrel-cage motor characteristic can now be expressed as

$$T_M = -21.41\omega + 2690.4 \text{ lbf} \cdot \text{in}$$

Eq. (16-68):

$$T_2 = 168.07\left(\frac{1560 - 168.07}{1560 - T_2}\right)^{19}$$

One root is 168.07 which is for infinite time. The root for 10 s is desired. Use a successive substitution method

T_2	New T_2
0.00	19.30
19.30	24.40
24.40	26.00
26.00	26.50
26.50	26.67

Continue until convergence to

$$T_2 = 26.771 \text{ lbf} \cdot \text{in}$$

Eq. (16-69):

$$I = \frac{a\left(t_2 - t_1\right)}{\ln\left(T_2 / T_r\right)} = \frac{-21.41(10 - 0.5)}{\ln(26.771 / 168.07)} = 110.72 \text{ lbf} \cdot \text{in} \cdot \text{s}^2$$

$$\omega = \frac{T - b}{a}$$

$$\omega_{\max} = \frac{T_2 - b}{a} = \frac{26.771 - 2690.4}{-21.41} = 124.41 \text{ rad/s} \quad \textit{Ans.}$$

$$\omega_{\min} = 117.81 \text{ rad/s} \quad \textit{Ans.}$$

$$\bar{\omega} = \frac{124.41 + 117.81}{2} = 121.11 \text{ rad/s}$$

$$C_s = \frac{\omega_{\max} - \omega_{\min}}{(\omega_{\max} + \omega_{\min}) / 2} = \frac{124.41 - 117.81}{(124.41 + 117.81) / 2} = 0.0545 \quad \textit{Ans.}$$

$$E_1 = \frac{1}{2} I \omega_r^2 = \frac{1}{2}(110.72)(117.81)^2 = 768\,352 \text{ in} \cdot \text{lbf}$$

$$E_2 = \frac{1}{2} I \omega_2^2 = \frac{1}{2}(110.72)(124.41)^2 = 856\,854 \text{ in} \cdot \text{lbf}$$

$$\Delta E = E_2 - E_1 = 856\,854 - 768\,352 = 88\,502 \text{ in} \cdot \text{lbf}$$

Eq. (16-64):

$$\Delta E = C_s I \bar{\omega}^2 = 0.0545(110.72)(121.11)^2$$
$$= 88\,508 \text{ in} \cdot \text{lbf}, \quad \text{close enough} \quad \textit{Ans.}$$

During the punch

$$T = \frac{63\,025H}{n}$$
$$H = \frac{T_L\bar{\omega}(60/2\pi)}{63\,025} = \frac{1560(121.11)(60/2\pi)}{63\,025} = 28.6 \text{ hp}$$

The gear train has to be sized for 28.6 hp under shock conditions since the flywheel is on the motor shaft. From Table A-18,

$$I = \frac{m}{8}\left(d_o^2 + d_i^2\right) = \frac{W}{8g}\left(d_o^2 + d_i^2\right)$$
$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(110.72)}{d_o^2 + d_i^2}$$

If a mean diameter of the flywheel rim of 30 in is acceptable, try a rim thickness of 4 in

$$d_i = 30 - (4 / 2) = 28 \text{ in}$$
$$d_o = 30 + (4 / 2) = 32 \text{ in}$$
$$W = \frac{8(386)(110.72)}{32^2 + 28^2} = 189.1 \text{ lbf}$$

Rim volume V is given by

$$V = \frac{\pi l}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi l}{4}(32^2 - 28^2) = 188.5l$$

where l is the rim width as shown in Table A-18. The specific weight of cast iron is $\gamma = 0.260$ lbf / in^3, therefore the volume of cast iron is

$$V = \frac{W}{\gamma} = \frac{189.1}{0.260} = 727.3 \text{ in}^3$$

Equating the volumes,

$$188.5\, l = 727.3$$
$$l = \frac{727.3}{188.5} = 3.86 \text{ in wide}$$

Proportions can be varied.

16-30 Prob. 16-29 solution has I for the motor shaft flywheel as

$$I = 110.72 \ \text{lbf} \cdot \text{in} \cdot \text{s}^2$$

A flywheel located on the crank shaft needs an inertia of $10^2\, I$ (Prob. 16-26, rule 2)

$$I = 10^2(110.72) = 11\,072 \ \text{lbf} \cdot \text{in} \cdot \text{s}^2$$

A 100-fold inertia increase. On the other hand, the gear train has to transmit 3 hp under shock conditions.

Stating the problem is most of the solution. Satisfy yourself that on the crankshaft:

$$T_L = 1300(12) = 15\,600 \ \text{lbf} \cdot \text{in}$$
$$T_r = 10(168.07) = 1680.7 \ \text{lbf} \cdot \text{in}$$
$$\omega_r = 117.81 / 10 = 11.781 \ \text{rad/s}$$
$$\omega_s = 125.66 / 10 = 12.566 \ \text{rad/s}$$
$$a = -21.41(100) = -2141 \ \text{lbf} \cdot \text{in} \cdot \text{s/rad}$$
$$b = 2690.35(10) = 26903.5 \ \text{lbf} \cdot \text{in}$$
$$T_M = -2141\omega_c + 26\,903.5 \ \text{lbf} \cdot \text{in}$$
$$T_2 = 1680.6\left(\frac{15\,600 - 1680.5}{15\,600 - T_2}\right)^{19}$$

The root is $10(26.67) = 266.7$ lbf · in

$$\bar{\omega} = 121.11 / 10 = 12.111 \ \text{rad/s}$$
$$C_s = 0.0549 \ \text{(same)}$$
$$\omega_{\max} = 121.11 / 10 = 12.111 \ \text{rad/s} \ \textit{Ans.}$$
$$\omega_{\min} = 117.81 / 10 = 11.781 \ \text{rad/s} \ \textit{Ans.}$$

E_1, E_2, ΔE and peak power are the same. From Table A-18

$$W = \frac{8gI}{d_o^2 + d_i^2} = \frac{8(386)(11\,072)}{d_o^2 + d_i^2} = \frac{34.19\left(10^6\right)}{d_o^2 + d_i^2}$$

Scaling will affect d_o and d_i , but the gear ratio changed I. Scale up the flywheel in the Prob. 16-29 solution by a factor of 2.5. Thickness becomes $4(2.5) = 10$ in.

$$\bar{d} = 30(2.5) = 75 \ \text{in}$$
$$d_o = 75 + (10 / 2) = 80 \ \text{in}$$
$$d_i = 75 - (10 / 2) = 70 \ \text{in}$$

$$W = \frac{34.19\left(10^6\right)}{80^2 + 70^2} = 3026 \text{ lbf}$$

$$V = \frac{W}{\gamma} = \frac{3026}{0.260} = 11\,638 \text{ in}^3$$

$$V = \frac{\pi}{4} l(80^2 - 70^2) = 1178\, l$$

$$l = \frac{11\,638}{1178} = 9.88 \text{ in}$$

Proportions can be varied. The weight has increased 3026/189.1 or about 16-fold while the moment of inertia I increased 100-fold. The gear train transmits a steady 3 hp. But the motor armature has its inertia magnified 100-fold, and during the punch there are deceleration stresses in the train. With no motor armature information, we cannot comment.

16-31 This can be the basis for a class discussion.

Chapter 17

17-1 Given: F-1 Polyamide, $b = 6$ in, $d = 2$ in with $n = 1750$ rev/min, $H_{\text{nom}} = 2$ hp, $C = 9(12) = 108$ in, velocity ratio = 0.5, $K_s = 1.25$, $n_d = 1$

$$V = \pi d \text{ n} / 12 = \pi(2)(1750) / 12 = 916.3 \text{ ft/min}$$

$$D = d / \text{vel ratio} = 2 / 0.5 = 4 \text{ in}$$

Eq. (17-1): $$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C} = \pi - 2\sin^{-1}\left[\frac{4-2}{2(108)}\right] = 3.123 \text{ rad}$$

Table 17-2: $t = 0.05$ in, $d_{\min} = 1.0$ in, $F_a = 35$ lbf/in, $\gamma = 0.035$ lbf/in^3, $f = 0.5$

$$w = 12\,\gamma bt = 12(0.035)6(0.05) = 0.126 \text{ lbf/ft}$$

(a) Eq. (*e*), p. 885: $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.126}{32.17}\left(\frac{916.3}{60}\right)^2 = 0.913 \text{ lbf} \quad \textit{Ans.}$$

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(2)(1.25)(1)}{1750} = 90.0 \text{ lbf} \cdot \text{in}$$

$$\Delta F = \left(F_1\right)_a - F_2 = \frac{2T}{d} = \frac{2(90.0)}{2} = 90.0 \text{ lbf}$$

Table 17-4: $C_p = 0.70$

Eq. (17-12): $(F_1)_a = bF_aC_pC_v = 6(35)(0.70)(1) = 147$ lbf *Ans.*

$F_2 = (F_1)_a - [(F_1)_a - F_2] = 147 - 90 = 57$ lbf *Ans.*

Do not use Eq. (17-9) because we do not yet know f'

Eq. (*i*), p. 886: $$F_i = \frac{\left(F_1\right)_a + F_2}{2} - F_c = \frac{147 + 57}{2} - 0.913 = 101.1 \text{ lbf} \quad \textit{Ans.}$$

Using Eq. (17-7) solved for f' (see step 8, p.888),

$$f' = \frac{1}{\theta_d}\ln\left[\frac{(F_1)_a - F_c}{F_2 - F_c}\right] = \frac{1}{3.123}\ln\left(\frac{147 - 0.913}{57 - 0.913}\right) = 0.307$$

The friction is thus underdeveloped.

(b) The transmitted horsepower is, with $\Delta F = (F_1)_a - F_2 = 90$ lbf,

Eq. (j), p. 887: $$H = \frac{(\Delta F)V}{33\,000} = \frac{90(916.3)}{33\,000} = 2.5 \text{ hp} \quad \textit{Ans.}$$

$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{2.5}{2(1.25)} = 1$$

Eq. (17-1): $$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C} = \pi + 2\sin^{-1}\left[\frac{4-2}{2(108)}\right] = 3.160 \text{ rad}$$

Eq. (17-2): $$L = [4C^2 - (D-d)^2]^{1/2} + (D\theta_D + d\theta_d)/2$$

$$= [4(108)^2 - (4-2)^2]^{1/2} + [4(3.160) + 2(3.123)]/2 = 225.4 \text{ in} \quad \textit{Ans.}$$

(c) Eq. (17-13): $$\text{dip} = \frac{3C^2 w}{2F_i} = \frac{3(108/12)^2(0.126)}{2(101.1)} = 0.151 \text{ in} \quad \textit{Ans.}$$

Comment: The solution of the problem is finished; however, a note concerning the design is presented here.

The friction is under-developed. Narrowing the belt width to 5 in (if size is available) will increase f'. The limit of narrowing is $b_{\min} = 4.680$ in, whence

$w = 0.0983$ lbf/ft
$F_c = 0.713$ lbf
$T = 90$ lbf · in (same)
$\Delta F = (F_1)_a - F_2 = 90$ lbf
$F_i = 68.9$ lbf
$(F_1)_a = 114.7$ lbf
$F_2 = 24.7$ lbf
$f' = f = 0.50$
dip $= 0.173$ in

Longer life can be obtained with a 6-inch wide belt by reducing F_i to attain $f' = 0.50$. Prob. 17-8 develops an equation we can use here

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$F_2 = F_1 - \Delta F$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right)$$

$$\text{dip} = \frac{3C^2 w}{2F_i}$$

which in this case, $\theta_d = 3.123$ rad, $\exp(f\theta) = \exp[0.5(3.123)] = 4.766$, $w = 0.126$ lbf/ft, $\Delta F = 90.0$ lbf, $F_c = 0.913$ lbf, and gives

$$F_1 = \frac{(0.913 + 90)4.766 - 0.913}{4.766 - 1} = 114.8 \text{ lbf}$$

$$F_2 = 114.8 - 90 = 24.8 \text{ lbf}$$

$$F_i = (114.8 + 24.8)/\,2 - 0.913 = 68.9 \text{ lbf}$$

$$f' = \frac{1}{3.123}\ln\left(\frac{114.8 - 0.913}{24.8 - 0.913}\right) = 0.50$$

$$\text{dip} = \frac{3(108 / 12)^2\, 0.126}{2(68.9)} = 0.222 \text{ in}$$

So, reducing F_i from 101.1 lbf to 68.9 lbf will bring the undeveloped friction up to 0.50, with a corresponding dip of 0.222 in. Having reduced F_1 and F_2, the endurance of the belt is improved. Power, service factor and design factor have remained intact.

17-2 Double the dimensions of Prob. 17-1.

In Prob. 17-1, F-1 Polyamide was used with a thickness of 0.05 in. With what is available in Table 17-2 we will select the Polyamide A-2 belt with a thickness of 0.11 in. Also, let $b = 12$ in, $d = 4$ in with $n = 1750$ rev/min, $H_{\text{nom}} = 2$ hp, $C = 18(12) = 216$ in, velocity ratio = 0.5, $K_s = 1.25$, $n_d = 1$.

$$V = \pi d \text{ n} / 12 = \pi(4)(1750) / 12 = 1833 \text{ ft/min}$$

$$D = d / \text{ vel ratio} = 4 / 0.5 = 8 \text{ in}$$

Eq. (17-1): $$\theta_d = \pi - 2\sin^{-1}\frac{D - d}{2C} = \pi - 2\sin^{-1}\left[\frac{8 - 4}{2(216)}\right] = 3.123 \text{ rad}$$

Table 17-2: $t = 0.11$ in, $d_{\min} = 2.4$ in, $F_a = 60$ lbf/in, $\gamma = 0.037$ lbf/in^3, $f = 0.8$

$$w = 12\,\gamma bt = 12(0.037)12(0.11) = 0.586 \text{ lbf/ft}$$

(a) Eq. (*e*), p. 885: $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.586}{32.17}\left(\frac{1833}{60}\right)^2 = 17.0 \text{ lbf} \quad \textit{Ans.}$$

$$T = \frac{63\,025 H_{nom} K_s n_d}{n} = \frac{63\,025(2)(1.25)(1)}{1750} = 90.0 \text{ lbf} \cdot \text{in}$$

$$\Delta F = (F_1)_a - F_2 = \frac{2T}{d} = \frac{2(90.0)}{4} = 45.0 \text{ lbf}$$

Table 17-4: $C_p = 0.73$

Eq. (17-12): $(F_1)_a = bF_aC_pC_v = 12(60)(0.73)(1) = 525.6$ lbf *Ans.*

$F_2 = (F_1)_a - [(F_1)_a - F_2] = 525.6 - 45 = 480.6$ lbf *Ans.*

Eq. (*i*), p. 886: $F_i = \dfrac{(F_1)_a + F_2}{2} - F_c = \dfrac{525.6 + 480.6}{2} - 17.0 = 486.1$ lbf *Ans.*

Eq. (17-9):

$$f' = \frac{1}{\theta_d}\ln\left[\frac{(F_1)_a - F_c}{F_2 - F_c}\right] = \frac{1}{3.123}\ln\left(\frac{525.6 - 17.0}{480.6 - 17.0}\right) = 0.0297$$

The friction is thus underdeveloped.

(b) The transmitted horsepower is, with $\Delta F = (F_1)_a - F_2 = 45$ lbf,

$$H = \frac{(\Delta F)V}{33\,000} = \frac{45(1833)}{33\,000} = 2.5 \text{ hp} \quad \textit{Ans.}$$

$$n_{fs} = \frac{H}{H_{\text{nom}}K_s} = \frac{2.5}{2(1.25)} = 1$$

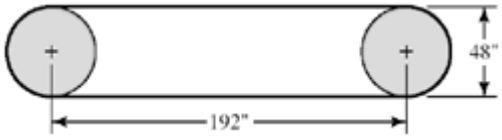
Eq. (17-1): $\theta_D = \pi + 2\sin^{-1}\dfrac{D - d}{2C} = \pi + 2\sin^{-1}\left[\dfrac{8 - 4}{2(216)}\right] = 3.160$ rad

Eq. (17-2): $L = [4C^2 - (D - d)^2]^{1/2} + (D\theta_D + d\theta_d)/2$

$= [4(216)^2 - (8 - 4)^2]^{1/2} + [8(3.160) + 4(3.123)]/2 = 450.9$ in *Ans.*

(c) Eq. (17-13): $\text{dip} = \dfrac{3C^2w}{2F_i} = \dfrac{3(216 / 12)^2(0.586)}{2(486.1)} = 0.586$ in *Ans.*

17-3



As a design task, the decision set on p. 893 is useful.
A priori decisions:
• Function: $H_{\text{nom}} = 60$ hp, $n = 380$ rev/min, $C = 192$ in, $K_s = 1.1$
• Design factor: $n_d = 1$
• Initial tension: Catenary
• Belt material. Table 17-2: Polyamide A-3, $F_a = 100$ lbf/in, $\gamma = 0.042$ lbf/in^3, $f = 0.8$
• Drive geometry: $d = D = 48$ in
• Belt thickness: $t = 0.13$ in

Design variable: Belt width.
Use a method of trials. Initially, choose $b = 6$ in

$$V = \frac{\pi dn}{12} = \frac{\pi(48)(380)}{12} = 4775 \text{ ft/min}$$
$$w = 12\gamma bt = 12(0.042)(6)(0.13) = 0.393 \text{ lbf/ft}$$
$$F_c = \frac{wV^2}{g} = \frac{0.393(4775 / 60)^2}{32.17} = 77.4 \text{ lbf}$$
$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(60)(1.1)(1)}{380} = 10\,946 \text{ lbf} \cdot \text{in}$$
$$\Delta F = \frac{2T}{d} = \frac{2(10\,946)}{48} = 456.1 \text{ lbf}$$
$$F_1 = (F_1)_a = bF_aC_pC_v = 6(100)(1)(1) = 600 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 600 - 456.1 = 143.9 \text{ lbf}$$

Transmitted power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{456.1(4775)}{33\,000} = 66 \text{ hp}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{600 + 143.9}{2} - 77.4 = 294.6 \text{ lbf}$$
$$f' = \frac{1}{\theta_d} \ln \frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi} \ln \left(\frac{600 - 77.4}{143.9 - 77.4} \right) = 0.656$$

Eq. (17-2): $L = [4(192)^2 - (48 - 48)^2]^{1/2} + [48(\pi) + 48(\pi)] / 2 = 534.8$ in

Friction is not fully developed, so b_{min} is just a little smaller than 6 in (5.7 in). Not having a figure of merit, we choose the most narrow belt available (6 in). We can improve the design by reducing the initial tension, which reduces F_1 and F_2, thereby increasing belt life (see the result of Prob. 17-8). This will bring f' to 0.80

$$F_1 = \frac{(\Delta F + F_c)\exp(f\theta) - F_c}{\exp(f\theta) - 1}$$

$$\exp(f\theta) = \exp(0.80\pi) = 12.345$$

Therefore

$$F_1 = \frac{(456.1 + 77.4)(12.345) - 77.4}{12.345 - 1} = 573.7 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 573.7 - 456.1 = 117.6 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{573.7 + 117.6}{2} - 77.4 = 268.3 \text{ lbf}$$

These are small reductions since f' is close to f, but improvements nevertheless.

$$f' = \frac{1}{\theta_d}\ln\frac{F_1 - F_c}{F_2 - F_c} = \frac{1}{\pi}\ln\left(\frac{573.7 - 77.4}{117.6 - 77.4}\right) = 0.80$$

$$\text{dip} = \frac{3C^2 w}{2F_i} = \frac{3(192\ /\ 12)^2(0.393)}{2(268.3)} = 0.562 \text{ in}$$

17-4 From the last equation given in the problem statement,

$$\exp(f\phi) = \frac{1}{1 - \{2T\ /\ [d(a_0 - a_2)b]\}}$$

$$\left[1 - \frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = 1$$

$$\left[\frac{2T}{d(a_0 - a_2)b}\right]\exp(f\phi) = \exp(f\phi) - 1$$

$$b = \frac{1}{a_0 - a_2}\left(\frac{2T}{d}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right]$$

But $2T/d = 33\,000H_d/V$. Thus,

$$b = \frac{1}{a_0 - a_2}\left(\frac{33\,000H_d}{V}\right)\left[\frac{\exp(f\phi)}{\exp(f\phi) - 1}\right] \quad Q.E.D.$$

17-5 Refer to Ex. 17-1 on p. 890 for the values used below.
(a) The maximum torque prior to slip is,

$$T = \frac{63\,025H_{\text{nom}}K_s n_d}{n} = \frac{63\,025(15)(1.25)(1.1)}{1750} = 742.8 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

The corresponding initial tension, from Eq. (17-9), is,

$$F_i = \frac{T}{d}\left(\frac{\exp(f\theta) + 1}{\exp(f\theta) - 1}\right) = \frac{742.8}{6}\left(\frac{11.17 + 1}{11.17 - 1}\right) = 148.1 \text{ lbf} \quad \textit{Ans.}$$

(b) See Prob. 17-4 statement. The final relation can be written

$$b_{\min} = \frac{1}{F_a C_p C_v - (12\gamma t / 32.174)(V / 60)^2}\left\{\frac{33\,000 H_a \exp(f\theta)}{V[\exp(f\theta) - 1]}\right\}$$

$$= \frac{1}{100(0.7)(1) - \{[12(0.042)(0.13)] / 32.174\}(2749 / 60)^2}\left[\frac{33\,000(20.6)(11.17)}{2749(11.17 - 1)}\right]$$

$= 4.13$ in *Ans.*

This is the minimum belt width since the belt is at the point of slip. The design must round up to an available width.

Eq. (17-1):

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D - d}{2C}\right) = \pi - 2\sin^{-1}\left[\frac{18 - 6}{2(96)}\right]$$
$$= 3.016\,511 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D - d}{2C}\right) = \pi + 2\sin^{-1}\left[\frac{18 - 6}{2(96)}\right]$$
$$= 3.266\,674 \text{ rad}$$

Eq. (17-2):

$$L = [4(96)^2 - (18 - 6)^2]^{1/2} + \frac{1}{2}[18(3.266\,674) + 6(3.016\,511)]$$
$$= 230.074 \text{ in} \quad \textit{Ans.}$$

(c)

$$\Delta F = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$
$$(F_1)_a = bF_a C_p C_v = F_1 = 4.13(100)(0.70)(1) = 289.1 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 289.1 - 247.6 = 41.5 \text{ lbf}$$
$$w = 12\gamma bt = 12(0.042)4.13(0.130) = 0.271 \text{ lbf/ft}$$
$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.271}{32.17}\left(\frac{2749}{60}\right)^2 = 17.7 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{289.1 + 41.5}{2} - 17.7 = 147.6 \text{ lbf}$$

Transmitted belt power H

$$H = \frac{\Delta F(V)}{33\,000} = \frac{247.6(2749)}{33\,000} = 20.6 \text{ hp}$$
$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1$$

Dip: $$dip = \frac{3C^2 w}{2F_i} = \frac{3(96/12)^2(0.271)}{2(147.6)} = 0.176 \text{ in}$$

(**d**) If you only change the belt width, the parameters in the following table change as shown.

	Ex. 17-1	This Problem
b	6.00	4.13
w	0.393	0.271
F_c	25.6	17.7
$(F_1)_a$	420	289
F_2	172.4	41.5
F_i	270.6	147.6
f'	0.33*	0.80**
dip	0.139	0.176

*Friction underdeveloped
**Friction fully developed

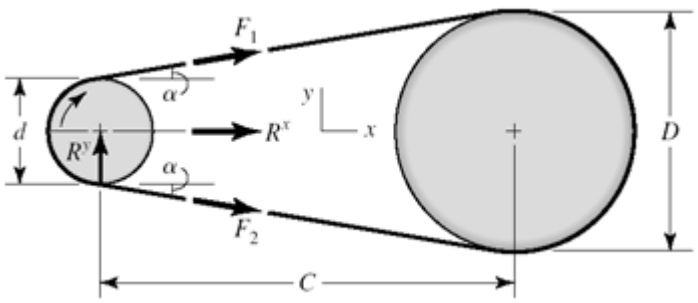
17-6 The transmitted power is the same.

	$b = 6$ in	$b = 12$ in	n-Fold Change
F_c	25.65	51.3	2
F_i	270.35	664.9	2.46
$(F_1)_a$	420	840	2
F_2	172.4	592.4	3.44
H_a	20.62	20.62	1
n_{fs}	1.1	1.1	1
f'	0.139	0.125	0.90
dip	0.328	0.114	0.34

If we relax F_i to develop full friction ($f = 0.80$) and obtain longer life, then

	$b = 6$ in	$b = 12$ in	n-Fold Change
F_c	25.6	51.3	2
F_i	148.1	148.1	1
F_1	297.6	323.2	1.09
F_2	50	75.6	1.51
f'	0.80	0.80	1
dip	0.255	0.503	2

17-7



Find the resultant of F_1 and F_2:

$$\alpha = \sin^{-1}\frac{D-d}{2C}$$

$$\sin\alpha = \frac{D-d}{2C}$$

$$\cos\alpha \doteq 1 - \frac{1}{2}\left(\frac{D-d}{2C}\right)^2$$

$$R^x = F_1\cos\alpha + F_2\cos\alpha = (F_1 + F_2)\left[1 - \frac{1}{2}\left(\frac{D-d}{2C}\right)^2\right] \quad \textit{Ans.}$$

$$R^y = F_1\sin\alpha - F_2\sin\alpha = (F_1 - F_2)\frac{D-d}{2C} \quad \textit{Ans.}$$

From Ex. 17-2, $d = 16$ in, $D = 36$ in, $C = 16(12) = 192$ in, $F_1 = 940$ lbf, $F_2 = 276$ lbf

$$\alpha = \sin^{-1}\left[\frac{36-16}{2(192)}\right] = 2.9855^\circ$$

$$R^x = (940 + 276)\left[1 - \frac{1}{2}\left(\frac{36-16}{2(192)}\right)^2\right] = 1214.4 \text{ lbf}$$

$$R^y = (940 - 276)\left[\frac{36-16}{2(192)}\right] = 34.6 \text{ lbf}$$

$$T = (F_1 - F_2)\left(\frac{d}{2}\right) = (940 - 276)\left(\frac{16}{2}\right) = 5312 \text{ lbf}\cdot\text{in}$$

17-8 Begin with Eq. (17-10),

$$F_1 = F_c + F_i\frac{2\exp(f\theta)}{\exp(f\theta) - 1}$$

Introduce Eq. (17-9):

$$F_1 = F_c + d\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right]\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = F_c + \frac{2T}{d}\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$F_1 = F_c + \Delta F\,\frac{\exp(f\theta)}{\exp(f\theta)-1}$$

Now add and subtract $F_c\left[\dfrac{\exp(f\theta)}{\exp(f\theta)-1}\right]$

$$F_1 = F_c + F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + \Delta F\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$= (F_c + \Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] + F_c - F_c\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right]$$

$$= (F_c + \Delta F)\left[\frac{\exp(f\theta)}{\exp(f\theta)-1}\right] - \frac{F_c}{\exp(f\theta)-1}$$

$$= \frac{(F_c + \Delta F)\exp(f\theta) - F_c}{\exp(f\theta)-1} \quad \textit{Q.E.D.}$$

From Ex. 17-2: $\theta_d = 3.037$ rad, $\Delta F = 664$ lbf, $\exp(f\theta) = \exp[0.80(3.037)] = 11.35$, and $F_c = 73.4$ lbf.

$$F_1 = \frac{(73.4 + 664)11.35 - 73.4}{(11.35 - 1)} = 802 \text{ lbf}$$

$$F_2 = F_1 - \Delta F = 802 - 664 = 138 \text{ lbf}$$

$$F_i = \frac{802 + 138}{2} - 73.4 = 396.6 \text{ lbf}$$

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1 - F_c}{F_2 - F_c}\right) = \frac{1}{3.037}\ln\left(\frac{802 - 73.4}{138 - 73.4}\right) = 0.80 \quad \textit{Ans.}$$

17-9 This is a good class project. Form four groups, each with a belt to design. Once each group agrees internally, all four should report their designs including the forces and torques on the line shaft. If you give them the pulley locations, they could design the line shaft.

17-10 If you have the students implement a computer program, the design problem selections may differ, and the students will be able to explore them. For $K_s = 1.25$, $n_d = 1.1$, $d = 14$ in and $D = 28$ in, a polyamide A-5 belt, 8 inches wide, will do ($b_{\min} = 6.58$ in)

17-11 An efficiency of less than unity lowers the output for a given input. Since the object of

the drive is the output, the efficiency must be incorporated such that the belt's capacity is increased. The design power would thus be expressed as

$$H_d = \frac{H_{\text{nom}} K_s n_d}{\text{eff}} \qquad \textit{Ans.}$$

17-12 Some perspective on the size of F_c can be obtained from

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{12\gamma bt}{g}\left(\frac{V}{60}\right)^2$$

An approximate comparison of non-metal and metal belts is presented in the table below.

	Non-metal	Metal
γ, lbf/in^3	0.04	0.280
b, in	5.00	1.000
t, in	0.20	0.005

The ratio w / w_m is

$$\frac{w}{w_m} = \frac{12(0.04)(5)(0.2)}{12(0.28)(1)(0.005)} \doteq 29$$

The second contribution to F_c is the belt peripheral velocity which tends to be low in metal belts used in instrument, printer, plotter and similar drives. The velocity ratio squared influences any $F_c / (F_c)_m$ ratio.

It is common for engineers to treat F_c as negligible compared to other tensions in the belting problem. However, when developing a computer code, one should include F_c.

17-13 Eq. (17-8):

$$\Delta F = F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\theta) - 1}{\exp(f\theta)} \doteq F_1 \frac{\exp(f\theta) - 1}{\exp(f\theta)}$$

Assuming negligible centrifugal force and setting $F_1 = ab$ from step 3, p. 897,

$$b_{\min} = \frac{\Delta F}{a}\frac{\exp(f\theta)}{\exp(f\theta) - 1} \qquad (1)$$

Also,

$$H_d = H_{\text{nom}} K_s n_d = \frac{(\Delta F)V}{33\,000}$$

$$\Delta F = \frac{33\,000 H_{\text{nom}} K_s n_d}{V}$$

Substituting into Eq. (1), $$b_{\min} = \frac{1}{a}\left(\frac{33\,000H_d}{V}\right)\frac{\exp(f\theta)}{\exp(f\theta)-1} \quad \textit{Ans.}$$

17-14 The decision set for the friction metal flat-belt drive is:

A priori decisions

- Function: $H_{\text{nom}} = 1$ hp, $n = 1750$ rev/min, $VR = 2$, $C \doteq 15$ in, $K_s = 1.2$, $N_p = 10^6$ belt passes.
- Design factor: $n_d = 1.05$
- Belt material and properties: 301/302 stainless steel
  Table 17-8: $S_y = 175$ kpsi, $E = 28$ Mpsi, $\nu = 0.285$
- Drive geometry: $d = 2$ in, $D = 4$ in
- Belt thickness: $t = 0.003$ in

Design variables:

- Belt width, b
- Belt loop periphery

Preliminaries

$$H_d = H_{\text{nom}}K_s n_d = 1(1.2)(1.05) = 1.26 \text{ hp}$$
$$T = \frac{63\,025(1.26)}{1750} = 45.38 \text{ lbf} \cdot \text{in}$$

A 15 in center-to-center distance corresponds to a belt loop periphery of 39.5 in. The 40 in loop available corresponds to a 15.254 in center distance.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{4-2}{2(15.254)}\right] = 3.010 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left[\frac{4-2}{2(15.274)}\right] = 3.273 \text{ rad}$$

For full friction development

$$\exp(f\theta_d) = \exp[0.35(3.010)] = 2.868$$
$$V = \frac{\pi dn}{12} = \frac{\pi(2)(1750)}{12} = 916.3 \text{ ft/s}$$
$$S_y = 175 \text{ kpsi}$$

Eq. (17-15):

$$S_y = 14.17\left(10^6\right)N_p^{-0.407} = 14.17\left(10^6\right)\left(10^6\right)^{-0.407} = 51.212\left(10^3\right) \text{ psi}$$

From selection step 3, p. 897,

$$a = \left[S_f - \frac{Et}{(1-\nu^2)d} \right] t = \left[51.212(10^3) - \frac{28(10^6)(0.003)}{(1-0.285^2)(2)} \right] (0.003)$$
$$= 16.50 \text{ lbf/in} \quad \text{of belt width}$$

$$(F_1)_a = ab = 16.50b$$

For full friction development, from Prob. 17-13,

$$b_{\min} = \frac{\Delta F}{a} \frac{\exp(f\theta_d)}{\exp(f\theta_d) - 1}$$

$$\Delta F = \frac{2T}{d} = \frac{2(45.38)}{2} = 45.38 \text{ lbf}$$

So

$$b_{\min} = \frac{45.38}{16.50} \left(\frac{2.868}{2.868 - 1} \right) = 4.23 \text{ in}$$

Decision #1: $b = 4.5$ in

$$F_1 = (F_1)_a = ab = 16.5(4.5) = 74.25 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 74.25 - 45.38 = 28.87 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} = \frac{74.25 + 28.87}{2} = 51.56 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d} \ln \left(\frac{F_1}{F_2} \right) = \frac{1}{3.010} \ln \left(\frac{74.25}{28.87} \right) = 0.314$$
$$H_t = \frac{(\Delta F)V}{33\,000} = \frac{45.38(916.3)}{33\,000} = 1.26 \text{ hp}$$
$$n_{fs} = \frac{H_t}{H_{\text{nom}} K_s} = \frac{1.26}{1(1.2)} = 1.05$$

This is a non-trivial point. The methodology preserved the factor of safety corresponding to $n_d = 1.1$ *even* as we rounded $b_{\min}$ up to b.

Decision #2 was taken care of with the adjustment of the center-to-center distance to accommodate the belt loop. Use Eq. (17-2) as is and solve for C to assist in this. Remember to subsequently recalculate θ_d and θ_D .

17-15 Decision set:

A priori decisions
• Function: $H_{\text{nom}} = 5$ hp, $N = 1125$ rev/min, $VR = 3$, $C \doteq 20$ in, $K_s = 1.25$, $N_p = 10^6$ belt passes
• Design factor: $n_d = 1.1$
• Belt material: BeCu, $S_y = 170$ kpsi, $E = 17$ Mpsi, $\nu = 0.220$
• Belt geometry: $d = 3$ in, $D = 9$ in
• Belt thickness: $t = 0.003$ in

Design decisions
• Belt loop periphery
• Belt width b

Preliminaries:

$$H_d = H_{\text{nom}} K_s n_d = 5(1.25)(1.1) = 6.875 \text{ hp}$$
$$T = \frac{63\,025(6.875)}{1125} = 385.2 \text{ lbf} \cdot \text{in}$$

Decision #1: Choose a 60-in belt loop with a center-to-center distance of 20.3 in.

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 2.845 \text{ rad}$$
$$\theta_D = \pi + 2\sin^{-1}\left[\frac{9-3}{2(20.3)}\right] = 3.438 \text{ rad}$$

For full friction development:

$$\exp(f\theta_d) = \exp[0.32(2.845)] = 2.485$$
$$V = \frac{\pi dn}{12} = \frac{\pi(3)(1125)}{12} = 883.6 \text{ ft/min}$$
$$S_f = 56.67 \text{ kpsi}$$

From selection step 3, p. 897,

$$a = \left[S_f - \frac{E_t}{(1-\nu^2)d} \right] t = \left[56.67(10^3) - \frac{17(10^6)(0.003)}{(1-0.22^2)(3)} \right](0.003) = 116.4 \text{ lbf/in}$$

$$\Delta F = \frac{2T}{d} = \frac{2(385.2)}{3} = 256.8 \text{ lbf}$$

$$b_{\min} = \frac{\Delta F}{a}\left[\frac{\exp(f\theta_d)}{\exp(f\theta_d)-1} \right] = \frac{256.8}{116.4}\left(\frac{2.485}{2.485-1} \right) = 3.69 \text{ in}$$

Decision #2: $b = 4$ in

$$F_1 = (F_1)_a = ab = 116.4(4) = 465.6 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 465.6 - 256.8 = 208.8 \text{ lbf}$$
$$F_i = \frac{F_1 + F_2}{2} = \frac{465.6 + 208.8}{2} = 337.3 \text{ lbf}$$

Existing friction

$$f' = \frac{1}{\theta_d}\ln\left(\frac{F_1}{F_2} \right) = \frac{1}{2.845}\ln\left(\frac{465.6}{208.8} \right) = 0.282$$

$$H = \frac{(\Delta F)V}{33\,000} = \frac{256.8(883.6)}{33\,000} = 6.88 \text{ hp}$$

$$n_{fs} = \frac{H}{5(1.25)} = \frac{6.88}{5(1.25)} = 1.1$$

F_i can be reduced only to the point at which $f' = f = 0.32$. From Eq. (17-9)

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1} \right] = \frac{385.2}{3}\left(\frac{2.485+1}{2.485-1} \right) = 301.3 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1} \right] = 301.3\left[\frac{2(2.485)}{2.485+1} \right] = 429.7 \text{ lbf}$$
$$F_2 = F_1 - \Delta F = 429.7 - 256.8 = 172.9 \text{ lbf}$$

and $f' = f = 0.32$

17-16 This solution is the result of a series of five design tasks involving different belt thicknesses. The results are to be compared as a matter of perspective. These design tasks are accomplished in the same manner as in Probs. 17-14 and 17-15 solutions.

The details will not be presented here, but the table is provided as a means of learning. Five groups of students could each be assigned a belt thickness. You can form a table

from their results or use the table given here.

	t, in				
	0.002	0.003	0.005	0.008	0.010
b	4.000	3.500	4.000	1.500	1.500
CD	20.300	20.300	20.300	18.700	20.200
a	109.700	131.900	110.900	194.900	221.800
d	3.000	3.000	3.000	5.000	6.000
D	9.000	9.000	9.000	15.000	18.000
F_i	310.600	333.300	315.200	215.300	268.500
F_1	439.000	461.700	443.600	292.300	332.700
F_2	182.200	209.000	186.800	138.200	204.300
n_{fs}	1.100	1.100	1.100	1.100	1.100
L	60.000	60.000	60.000	70.000	80.000
f'	0.309	0.285	0.304	0.288	0.192
F_i	301.200	301.200	301.200	195.700	166.600
F_1	429.600	429.600	429.600	272.700	230.800
F_2	172.800	172.800	172.800	118.700	102.400
f	0.320	0.320	0.320	0.320	0.320

The first three thicknesses result in the same adjusted F_i, F_1 and F_2 (why?). We have no figure of merit, but the costs of the belt and pulleys are about the same for these three thicknesses. Since the same power is transmitted and the belts are widening, belt forces are lessening.

17-17 This is a design task. The decision variables would be belt length and belt section, which could be combined into one, such as B90. The number of belts is not an issue.

We have no figure of merit, which is not practical in a text for this application. It is suggested that you gather sheave dimensions and costs and V-belt costs from a principal vendor and construct a figure of merit based on the costs. Here is one trial.

Preliminaries: For a single V-belt drive with $H_{\text{nom}} = 3$ hp, $n = 3100$ rev/min, $D = 12$ in, and $d = 6.2$ in, choose a B90 belt, $K_s = 1.3$ and $n_d = 1$. From Table 17-10, select a circumference of 90 in. From Table 17-11, add 1.8 in giving

$$L_p = 90 + 1.8 = 91.8 \text{ in}$$

Eq. (17-16*b*):

$$C = 0.25\left\{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right] + \sqrt{\left[91.8 - \frac{\pi}{2}(12 + 6.2)\right]^2 - 2(12 - 6.2)^2}\right\}$$

$$= 31.47 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\left[\frac{12-6.2}{2(31.47)}\right] = 2.9570 \text{ rad}$$

$$\exp(f\theta_d) = \exp[0.5123(2.9570)] = 4.5489$$

$$V = \frac{\pi dn}{12} = \frac{\pi(6.2)(3100)}{12} = 5031.8 \text{ ft/min}$$

Table 17-13:

$$\text{Angle}\theta = \theta_d \frac{180°}{\pi} = (2.957 \text{ rad})\left(\frac{180°}{\pi}\right) = 169.42°$$

The footnote regression equation of Table 17-13 gives K_1 without interpolation:

$$K_1 = 0.143\ 543 + 0.007\ 468(169.42°) - 0.000\ 015\ 052(169.42°)^2 = 0.9767$$

The design power is

$$H_d = H_{\text{nom}} K_s n_d = 3(1.3)(1) = 3.9 \text{ hp}$$

From Table 17-14 for B90, $K_2 = 1$. From Table 17-12 take a marginal entry of $H_{\text{tab}} = 4$, although extrapolation would give a slightly lower H_{tab}.

Eq. (17-17): $H_a = K_1 K_2 H_{\text{tab}} = 0.9767(1)(4) = 3.91 \text{ hp}$

The allowable ΔF_a is given by

$$\Delta F_a = \frac{63\ 025 H_a}{n(d/2)} = \frac{63\ 025(3.91)}{3100(6.2/2)} = 25.6 \text{ lbf}$$

The allowable torque T_a is

$$T_a = \frac{\Delta F_a d}{2} = \frac{25.6(6.2)}{2} = 79.4 \text{ lbf} \cdot \text{in}$$

From Table 17-16, $K_c = 0.965$. Thus, Eq. (17-21) gives,

$$F_c = K_c\left(\frac{V}{1000}\right)^2 = 0.965\left(\frac{5031.8}{1000}\right)^2 = 24.4 \text{ lbf}$$

At incipient slip, Eq. (17-9) provides:

$$F_i = \left(\frac{T}{d}\right)\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right] = \left(\frac{79.4}{6.2}\right)\left(\frac{4.5489+1}{4.5489-1}\right) = 20.0 \text{ lbf}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 24.4 + 20\left[\frac{2(4.5489)}{4.5489+1}\right] = 57.2 \text{ lbf}$$

Thus, $F_2 = F_1 - \Delta F_a = 57.2 - 25.6 = 31.6$ lbf

Eq. (17-26): $$n_{fs} = \frac{H_a N_b}{H_d} = \frac{(3.91)(1)}{3.9} = 1.003 \quad \textit{Ans.}$$

If we had extrapolated for H_{tab}, the factor of safety would have been slightly less than one.

Life Use Table 17-16 to find equivalent tensions T_1 and T_2 .

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d} = 57.2 + \frac{576}{6.2} = 150.1 \text{ lbf}$$
$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D} = 57.2 + \frac{576}{12} = 105.2 \text{ lbf}$$

From Table 17-17, $K = 1193$, $b = 10.926$, and from Eq. (17-27), the number of belt passes is:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1}$$
$$= \left[\left(\frac{1193}{150.1}\right)^{-10.926} + \left(\frac{1193}{105.2}\right)^{-10.926}\right]^{-1} = 6.72(10^9) \text{ passes}$$

From Eq. (17-28) for $N_P > 10^9$,

$$t = \frac{N_P L_p}{720V} > \frac{10^9(91.8)}{720(5031.8)}$$
$$t > 25\,340 \text{ h} \quad \textit{Ans.}$$

Suppose n_{fs} was too small. Compare these results with a 2-belt solution.

$$H_{\text{tab}} = 4 \text{ hp/belt}, \quad T_a = 39.6 \text{ lbf} \cdot \text{in/belt},$$
$$\Delta F_a = 12.8 \text{ lbf/belt}, \quad H_a = 3.91 \text{ hp/belt}$$
$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{N_b H_a}{H_{\text{nom}} K_s} = \frac{2(3.91)}{3(1.3)} = 2.0$$

Also, $F_1 = 40.8$ lbf/belt, $F_2 = 28.0$ lbf/belt

$F_i = 9.99$ lbf/belt, $F_c = 24.4$ lbf/belt
$(F_b)_1 = 92.9$ lbf/belt, $(F_b)_2 = 48$ lbf/belt
$T_1 = 133.7$ lbf/belt, $T_2 = 88.8$ lbf/belt
$N_P = 2.39(10^{10})$ passes, $t > 605\ 600$ h

Initial tension of the drive:

$$(F_i)_{\text{drive}} = N_b F_i = 2(9.99) = 20 \text{ lbf}$$

17-18 Given: two B85 V-belts with $d = 5.4$ in, $D = 16$ in, $n = 1200$ rev/min, and $K_s = 1.25$

Table 17-11: $L_p = 85 + 1.8 = 86.8$ in

Eq. (17-17*b*):

$$C = 0.25\left\{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right] + \sqrt{\left[86.8 - \frac{\pi}{2}(16 + 5.4)\right]^2 - 2(16 - 5.4)^2}\right\}$$

$$= 26.05 \text{ in} \quad \textit{Ans.}$$

Eq. (17-1):

$$\theta_d = 180° - 2\sin^{-1}\left[\frac{16 - 5.4}{2(26.05)}\right] = 156.5°$$

From table 17-13 footnote:

$$K_1 = 0.143\ 543 + 0.007\ 468(156.5°) - 0.000\ 015\ 052(156.5°)^2 = 0.944$$

Table 17-14: $K_2 = 1$

Belt speed: $$V = \frac{\pi(5.4)(1200)}{12} = 1696 \text{ ft/min}$$

Use Table 17-12 to interpolate for H_{tab}.

$$H_{\text{tab}} = 1.59 + \left(\frac{2.62 - 1.59}{2000 - 1000}\right)(1696 - 1000) = 2.31 \text{ hp/belt}$$

Eq. (17-17) for two belts: $H_a = K_1 K_2 N_b H_{\text{tab}} = 0.944(1)(2)(2.31) = 4.36$ hp

Assuming $n_d = 1$,

$$H_d = K_s H_{\text{nom}} n_d = 1.25(1) H_{\text{nom}}$$

For a factor of safety of one,

$$H_a = H_d$$
$$4.36 = 1.25H_{\text{nom}}$$
$$H_{\text{nom}} = \frac{4.36}{1.25} = 3.49 \text{ hp} \quad \textit{Ans.}$$

17-19 Given: $H_{\text{nom}} = 60$ hp, $n = 400$ rev/min, $K_s = 1.4$, $d = D = 26$ in on 12 ft centers.

Design task: specify V-belt and number of strands (belts). *Tentative decision*: Use D360 belts.

Table 17-11: $L_p = 360 + 3.3 = 363.3$ in

Eq. (17-16*b*):

$$C = 0.25\left\{\left[363.3 - \frac{\pi}{2}(26 + 26)\right] + \sqrt{\left[363.3 - \frac{\pi}{2}(26 + 26)\right]^2 - 2(26 - 26)^2}\right\}$$
$$= 140.8 \text{ in (nearly 144 in)}$$

$$\theta_d = \pi, \quad \theta_D = \pi, \quad \exp[0.5123\pi] = 5.0,$$
$$V = \frac{\pi dn}{12} = \frac{\pi(26)(400)}{12} = 2722.7 \text{ ft/min}$$

Table 17-13: For $\theta = 180°$, $K_1 = 1$

Table 17-14: For D360, $K_2 = 1.10$

Table 17-12: $H_{\text{tab}} = 16.94$ hp by interpolation

Thus, $H_a = K_1 K_2 H_{\text{tab}} = 1(1.1)(16.94) = 18.63$ hp / belt

Eq. (17-19): $H_d = H_{\text{nom}} K_s n_d = 60(1.4)(1) = 84$ hp

Number of belts, N_b

$$N_b = \frac{H_d}{H_a} = \frac{84}{18.63} = 4.51$$

Round up to five belts. It is left to the reader to repeat the above for belts such as C360 and E360.

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(18.63)}{400(26/2)} = 225.8 \text{ lbf/belt}$$

$$T_a = \frac{(\Delta F_a)d}{2} = \frac{225.8(26)}{2} = 2935 \text{ lbf} \cdot \text{in/belt}$$

Eq. (17-21):

$$F_c = 3.498\left(\frac{V}{1000}\right)^2 = 3.498\left(\frac{2722.7}{1000}\right)^2 = 25.9 \text{ lbf/belt}$$

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta)+1}{\exp(f\theta)-1}\right] = \frac{2935}{26}\left(\frac{5+1}{5-1}\right) = 169.3 \text{ lbf/belt}$$

Eq. (17-10): $$F_1 = F_c + F_i\left[\frac{2\exp(f\theta)}{\exp(f\theta)+1}\right] = 25.9 + 169.3\left[\frac{2(5)}{5+1}\right] = 308.1 \text{ lbf/belt}$$

$$F_2 = F_1 - \Delta F_a = 308.1 - 225.8 = 82.3 \text{ lbf/belt}$$

$$n_{fs} = \frac{H_a N_b}{H_d} = \frac{18.63(5)}{84} = 1.109 \quad \textit{Ans.}$$

Life From Table 17-16,

$$T_1 = T_2 = F_1 + \frac{K_b}{d} = 308.1 + \frac{5\,680}{26} = 526.6 \text{ lbf}$$

Eq. (17-27):

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} = 5.28\left(10^{-9}\right) \text{ passes}$$

Thus, $N_P > 10^{-9}$ passes *Ans.*

Eq. (17-28): $$t = \frac{N_P L_p}{720V} > \frac{10^9(363.3)}{720(2722.7)}$$

Thus, $t > 185\,320$ h *Ans.*

17-20 Preliminaries: $D \doteq 60$ in, 14-in wide rim, $H_{\text{nom}} = 50$ hp, $n = 875$ rev/min, $K_s = 1.2$, $n_d = 1.1$, $m_G = 875/170 = 5.147$, $d \doteq 60/5.147 = 11.65$ in

(a) From Table 17-9, an 11-in sheave exceeds C-section minimum diameter and precludes D- and E-section V-belts.

Decision: Use $d = 11$ in, C270 belts

Table 17-11: $L_p = 270 + 2.9 = 272.9$ in

Eq. (17-16*b*):

$$C = 0.25\left\{\left[272.9 - \frac{\pi}{2}(60+11)\right] + \sqrt{\left[272.9 - \frac{\pi}{2}(60+11)\right]^2 - 2(60-11)^2}\right\}$$
$$= 76.78 \text{ in}$$

This fits in the range

$$D < C < 3(D+d) \;\Rightarrow\; 60 < C < 3(60+11) \;\Rightarrow\; 60 \text{ in} < C < 213 \text{ in}$$

$$\theta_d = \pi - 2\sin^{-1}\frac{60-11}{2(76.78)} = 2.492 \text{ rad} = 142.8°$$

$$\theta_D = \pi + 2\sin^{-1}\frac{60-11}{2(76.78)} = 3.791 \text{ rad}$$

$$\exp(f\,\theta_d) = \exp[0.5123(2.492)] = 3.5846$$

For the flat on flywheel, $f = 0.13$ (see p. 900), $\exp(f\,\theta_D) = \exp[0.13(3.791)] = 1.637$.

The belt speed is

$$V = \frac{\pi dn}{12} = \frac{\pi(11)(875)}{12} = 2520 \text{ ft/min}$$

Table 17-13:

$$K_1 = 0.143\,543 + 0.007\,468(142.8°) - 0.000\,015\,052(142.8°)^2 = 0.903$$

Table 17-14: $K_2 = 1.15$

For interpolation of Table 17-12, let x be entry for $d = 11.65$ in and $n = 2000$ ft/min, and y be entry for $d = 11.65$ in and $n = 3000$ ft/min. Then,

$$\frac{x - 6.74}{11.65 - 11} = \frac{7.17 - 6.74}{12 - 11} \;\Rightarrow\; x = 7.01 \text{ hp at } 2000 \text{ ft/min}$$

and

$$\frac{8.11 - y}{11.65 - 11} = \frac{8.84 - 8.11}{12 - 11} \;\Rightarrow\; y = 8.58 \text{ hp at } 3000 \text{ ft/min}$$

Interpolating these for 2520 ft/min gives

$$\frac{8.58 - H_{\text{tab}}}{8.58 - 7.01} = \frac{3000 - 2520}{3000 - 2000} \;\Rightarrow\; H_{\text{tab}} = 7.83 \text{ hp/belt}$$

Eq. (17-17): $H_a = K_1 K_2 H_{\text{tab}} = 0.903(1.15)(7.83) = 8.13$ hp

Eq. (17-19): $H_d = H_{\text{nom}} K_s n_d = 50(1.2)(1.1) = 66$ hp

Eq. (17-20): $N_b = \dfrac{H_d}{H_a} = \dfrac{66}{8.13} = 8.1$ belts

Decision: Use 9 belts. On a per belt basis,

$$\Delta F_a = \frac{63\,025 H_a}{n(d/2)} = \frac{63\,025(8.13)}{875(11/2)} = 106.5 \text{ lbf/belt}$$

$$T_a = \frac{\Delta F_a d}{2} = \frac{106.5(11)}{2} = 586.8 \text{ lbf} \cdot \text{in per belt}$$

Table 17-16: $K_c = 1.716$

Eq. (17-21): $F_c = 1.716\left(\dfrac{V}{1000}\right)^2 = 1.716\left(\dfrac{2520}{1000}\right)^2 = 10.9$ lbf/belt

At fully developed friction, Eq. (17-9) gives

$$F_i = \frac{T}{d}\left[\frac{\exp(f\theta_d)+1}{\exp(f\theta_d)-1}\right] = \frac{586.9}{11}\left[\frac{3.5846+1}{3.5846-1}\right] = 94.6 \text{ lbf/belt}$$

Eq. (17-10):

$$F_1 = F_c + F_i\left[\frac{2\exp(f\theta_d)}{\exp(f\theta_d)+1}\right] = 10.9 + 94.6\left[\frac{2(3.5846)}{3.5846+1}\right] = 158.8 \text{ lbf/belt}$$

$$F_2 = F_1 - \Delta F_a = 158.8 - 106.7 = 52.1 \text{ lbf/belt}$$

$$n_{fs} = \frac{N_b H_a}{H_d} = \frac{9(8.13)}{66} = 1.11 \quad O.K. \quad \textit{Ans.}$$

Durability:

$$(F_b)_1 = K_b / d = 1600/11 = 145.5 \text{ lbf/belt}$$

$$(F_b)_2 = K_b / D = 1600/60 = 26.7 \text{ lbf/belt}$$

$$T_1 = F_1 + (F_b)_1 = 158.8 + 145.5 = 304.3 \text{ lbf/belt}$$

$$T_2 = F_1 + (F_b)_2 = 158.8 + 26.7 = 185.5 \text{ lbf/belt}$$

Eq. (17-27) with Table 17-17:

$$N_P = \left[\left(\frac{K}{T_1}\right)^{-b} + \left(\frac{K}{T_2}\right)^{-b}\right]^{-1} = \left[\left(\frac{2038}{304.3}\right)^{-11.173} + \left(\frac{2038}{185.5}\right)^{-11.173}\right]^{-1}$$

$$= 1.68\left(10^9\right) \text{ passes} \quad > 10^9 \text{ passes} \qquad \textit{Ans.}$$

Since N_P is greater than 10^9 passes and is out of the range of Table 17-17, life from Eq. (17-27) is

$$t = \frac{N_P L_p}{720V} > \frac{10^9(272.9)}{720(2520)} = 150\left(10^3\right) \text{ h}$$

Remember: $(F_i)_{\text{drive}} = 9(94.6) = 851.4$ lbf

Table 17-9: C-section belts are 7/8 in wide. Check sheave groove spacing to see if 14 in width is accommodating.

(b) The fully developed friction torque on the flywheel using the flats of the V-belts, from Eq. (17-9), is

$$T_{\text{flat}} = F_i D\left[\frac{\exp(f\theta) - 1}{\exp(f\theta) + 1}\right] = 94.6(60)\left(\frac{1.637 - 1}{1.637 + 1}\right) = 1371 \text{ lbf} \cdot \text{in per belt}$$

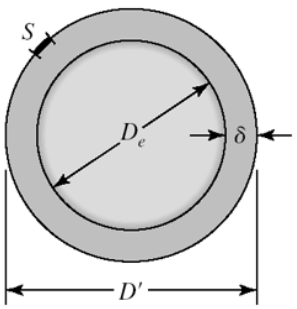
The flywheel torque should be

$$T_{\text{fly}} = m_G T_a = 5.147(586.9) = 3021 \text{ lbf} \cdot \text{in per belt}$$

but it is not. There are applications, however, in which it will work. For example, make the flywheel controlling. Yes. *Ans.*

17-21

(a)



S is the spliced-in string segment length
D_e is the equatorial diameter
D' is the spliced string diameter
δ is the radial clearance
$S + \pi D_e = \pi D' = \pi(D_e + 2\delta) = \pi D_e + 2\pi\delta$

From which

$$\delta = \frac{S}{2\pi}$$

The radial clearance is thus *independent* of D_e.

$$\delta = \frac{12(6)}{2\pi} = 11.5 \text{ in} \quad \textit{Ans.}$$

This is true whether the sphere is the earth, the moon or a marble. Thinking in terms of a radial or diametral increment removes the basic size from the problem.

(b) and **(c)**

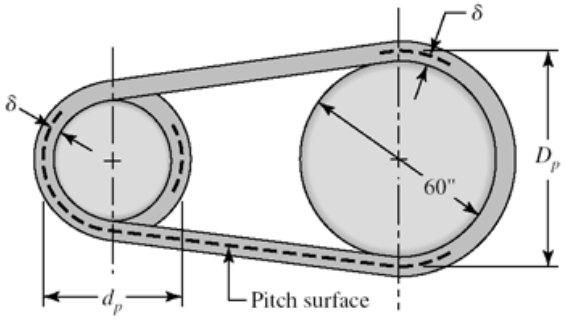
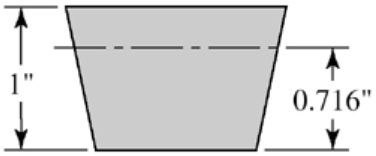


Table 17-9: For an E210 belt, the thickness is 1 in.



$$d_P - d_i = \frac{210 + 4.5}{\pi} - \frac{210}{\pi} = \frac{4.5}{\pi}$$

$$2\delta = \frac{4.5}{\pi}$$

$$\delta = \frac{4.5}{2\pi} = 0.716 \text{ in}$$

The pitch diameter of the flywheel is

$$D_P - 2\delta = D \quad \Rightarrow \quad D_P = D + 2\delta = 60 + 2(0.716) = 61.43 \text{ in}$$

We could make a table:

Diametral Growth	Section				
	A	*B*	*C*	*D*	*E*
2δ	$\frac{1.3}{\pi}$	$\frac{1.8}{\pi}$	$\frac{2.9}{\pi}$	$\frac{3.3}{\pi}$	$\frac{4.5}{\pi}$

The velocity ratio for the D-section belt of Prob. 17-20 is

$$m'_G = \frac{D + 2\delta}{d} = \frac{60 + 3.3/\pi}{11} = 5.55 \quad \textit{Ans.}$$

for the V-flat drive as compared to $m_a = 60/11 = 5.455$ for the VV drive.
The pitch diameter of the pulley is still $d = 11$ in, so the new angle of wrap, θ_d, is

$$\theta_d = \pi - 2\sin^{-1}\frac{D + 2\delta - d}{2C} \quad \textit{Ans.}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D + 2\delta - d}{2C} \quad \textit{Ans.}$$

Equations (17-16*a*) and (17-16*b*) are modified as follows

$$L_p = 2C + \frac{\pi}{2}(D + 2\delta + d) + \frac{(D + \delta - d)^2}{4C} \quad \textit{Ans.}$$

$$C_p = 0.25\left\{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D + 2\delta + d)\right]^2 - 2(D + 2\delta - d)^2}\right\} \quad \textit{Ans.}$$

The changes are small, but if you are writing a computer code for a V-flat drive, remember that θ_d and θ_D changes are exponential.

17-22 This design task involves specifying a drive to couple an electric motor running at 1720 rev/min to a blower running at 240 rev/min, transmitting two horsepower with a center distance of at least 22 inches. Instead of focusing on the steps, we will display two different designs side-by-side for study. Parameters are in a "per belt" basis with per drive quantities shown along side, where helpful.

Parameter	Four A-90 Belts	Two A-120 Belts
m_G	7.33	7.142
K_s	1.1	1.1
n_d	1.1	1.1
K_1	0.877	0.869
K_2	1.05	1.15
d, in	3.0	4.2
D, in	22	30
θ_d, rad	2.333	2.287
V, ft/min	1350.9	1891
$\exp(f\theta_d)$	3.304	3.2266
L_p, in	91.3	101.3
C, in	24.1	31
H_{tab}, uncorr.	0.783	1.662
$N_b H_{\text{tab}}$, uncorr.	3.13	3.326
T_a, lbf · in	26.45(105.8)	60.87(121.7)
ΔF_a, lbf	17.6(70.4)	29.0(58)
H_a, hp	0.721(2.88)	1.667(3.33)
n_{fs}	1.192	1.372
F_1, lbf	26.28(105.2)	44(88)
F_2, lbf	8.67(34.7)	15(30)
$(F_b)_1$, lbf	73.3(293.2)	52.4(109.8)
$(F_b)_2$, lbf	10(40)	7.33(14.7)
F_c, lbf	1.024	2.0
F_i, lbf	16.45(65.8)	27.5(55)
T_1, lbf · in	99.2	96.4

T_2, lbf · in	36.3	57.4
N', passes	$1.61(10^9)$	$2.3(10^9)$
$t > h$	93 869	89 080

Conclusions:
- Smaller sheaves lead to more belts.
- Larger sheaves lead to larger D and larger V.
- Larger sheaves lead to larger tabulated power.
- The discrete numbers of belts obscures some of the variation. The factors of safety exceed the design factor by differing amounts.

17-23 In Ex. 17-5 the selected chain was 140-3, making the pitch of this 140 chain14/8 = 1.75 in. Table 17-19 confirms.

17-24 **(a)** Eq. (17-32): $H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)}$

Eq. (17-33): $$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}}$$

Equating and solving for n_1 gives

$$n_1 = \left[\frac{0.25(10^6) K_r N_1^{0.42}}{p^{(2.2-0.07p)}} \right]^{1/2.4} \qquad \textit{Ans.}$$

(b) For a No. 60 chain, $p = 6/8 = 0.75$ in, $N_1 = 17$, $K_r = 17$

$$n_1 = \left\{ \frac{0.25(10^6)(17)(17)^{0.42}}{0.75^{[2.2-0.07(0.75)]}} \right\}^{1/2.4} = 1227 \text{ rev/min} \qquad \textit{Ans.}$$

Table 17-20 confirms that this point occurs at 1200 ± 200 rev/min.

(c) Life predictions using Eq. (17-40) are possible at speeds greater than 1227 rev/min. *Ans.*

17-25 Given: a double strand No. 60 roller chain with $p = 0.75$ in, $N_1 = 13$ teeth at 300 rev/min, $N_2 = 52$ teeth.

(a) Table 17-20: $H_{\text{tab}} = 6.20$ hp
Table 17-22: $K_1 = 0.75$
Table 17-23: $K_2 = 1.7$
Use $K_s = 1$
Eq. (17-37):

$$H_a = K_1 K_2 H_{\text{tab}} = 0.75(1.7)(6.20) = 7.91 \text{ hp} \qquad \textit{Ans.}$$

(b) Eqs. (17-35) and (17-36) with $L/p = 82$

$$A = \frac{13 + 52}{2} - 82 = -49.5$$

$$C = \frac{p}{4}\left[49.5 + \sqrt{49.5^2 - 8\left(\frac{52 - 13}{2\pi}\right)^2}\right] = 23.95p$$

$$C = 23.95(0.75) = 17.96 \text{ in}, \quad \text{round up to } 18 \text{ in} \quad \textit{Ans.}$$

(c) For 30 percent less power transmission,

$$H = 0.7(7.91) = 5.54 \text{ hp}$$

$$T = \frac{63\,025(5.54)}{300} = 1164 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

Eq. (17-29):

$$D = \frac{0.75}{\sin(180^\circ / 13)} = 3.13 \text{ in}$$

$$F = \frac{T}{r} = \frac{1164}{3.13/2} = 744 \text{ lbf} \quad \textit{Ans.}$$

17-26 Given: No. 40-4 chain, $N_1 = 21$ teeth for $n = 2000$ rev/min, $N_2 = 84$ teeth, $h = 20\,000$ hours.

(a) Chain pitch is $p = 4/8 = 0.500$ in and $C \doteq 20$ in.

Eq. (17-34):

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_1 - N_2)^2}{4\pi^2 C / p}$$

$$= \frac{2(20)}{0.5} + \frac{21 + 84}{2} + \frac{(84 - 21)^2}{4\pi^2(20 / 0.5)} = 135 \text{ pitches} \quad \text{(or links)}$$

$$L = 135(0.500) = 67.5 \text{ in} \quad \textit{Ans.}$$

(b) Table 17-20: $H_{\text{tab}} = 7.72$ hp (post-extreme power)

Eq. (17-40): Since K_1 is required, the $N_1^{3.75}$ term is omitted (see p. 914).

$$\text{constant} = \frac{\left(7.72^{2.5}\right)(15\,000)}{135} = 18\,399$$

$$H'_{\text{tab}} = \left[\frac{18\,399(135)}{20\,000}\right]^{1/2.5} = 6.88 \text{ hp} \quad \textit{Ans.}$$

(c) Table 17-22:

$$K_1 = \left(\frac{21}{17}\right)^{1.5} = 1.37$$

Table 17-23: $K_2 = 3.3$

$$H_a = K_1 K_2 H'_{\text{tab}} = 1.37(3.3)(6.88) = 31.1 \text{ hp} \quad \textit{Ans.}$$

(d)

$$V = \frac{N_1 pn}{12} = \frac{21(0.5)(2000)}{12} = 1750 \text{ ft/min}$$

$$F_1 = \frac{33\,000(31.1)}{1750} = 586 \text{ lbf} \quad \textit{Ans.}$$

17-27 This is our first design/selection task for chain drives. A possible decision set:

A priori decisions
• Function: H_{nom}, n_1, space, life, K_s
• Design factor: n_d
• Sprockets: Tooth counts N_1 and N_2, factors K_1 and K_2

Decision variables
• Chain number
• Strand count
• Lubrication type
• Chain length in pitches

Function: Motor with $H_{\text{nom}} = 25$ hp at $n = 700$ rev/min; pump at $n = 140$ rev/min;
$m_G = 700/140 = 5$
Design Factor: $n_d = 1.1$
Sprockets: Tooth count $N_2 = m_G N_1 = 5(17) = 85$ teeth–odd and unavailable. Choose 84 teeth. *Decision*: $N_1 = 17$, $N_2 = 84$

Evaluate K_1 and K_2
Eq. (17-38): $H_d = H_{\text{nom}} K_s n_d$
Eq. (17-37): $H_a = K_1 K_2 H_{\text{tab}}$

Equate H_d to H_a and solve for H_{tab} :

$$H_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2}$$

Table 17-22: $K_1 = 1$

Table 17-23: $K_2 = 1, 1.7, 2.5, 3.3$ for 1 through 4 strands

$$H'_{\text{tab}} = \frac{1.5(1.1)(25)}{(1)K_2} = \frac{41.25}{K_2}$$

Prepare a table to help with the design decisions:

Strands	K_2	H'_{tab}	Chain No.	H_{tab}	n_{fs}	Lub. Type
1	1.0	41.3	100	59.4	1.58	B
2	1.7	24.3	80	31.0	1.40	B
3	2.5	16.5	80	31.0	2.07	B
4	3.3	12.5	60	13.3	1.17	B

Design Decisions
We need a figure of merit to help with the choice. If the best was 4 strands of No. 60 chain, then

Decision #1 and #2: Choose four strand No. 60 roller chain with $n_{fs} = 1.17$.

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1(3.3)(13.3)}{1.5(25)} = 1.17$$

Decision #3: Choose Type B lubrication
Analysis:
Table 17-20: $H_{\text{tab}} = 13.3$ hp
Table 17-19: $p = 0.75$ in

Try $C = 30$ in in Eq. (17-34):

$$\begin{aligned}\frac{L}{p} &\doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C / p} \\ &= 2(30 / 0.75) + \frac{17 + 84}{2} + \frac{(84 - 17)^2}{4\pi^2 (30 / 0.75)} \\ &= 133.3\end{aligned}$$

$$L = 0.75(133.3) = 100 \text{ in (no need to round)}$$

Eq. (17-36) with $p = 0.75$ in: $A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{17 + 84}{2} - \frac{100}{0.75} = -82.83$

Eq. (17-35):

$$\begin{aligned}C &= \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right] \\ &= \frac{0.75}{4}\left[-(-82.83) + \sqrt{(-82.83)^2 - 8\left(\frac{84 - 17}{2\pi}\right)^2}\right] = 30.0 \text{ in}\end{aligned}$$

Decision #4: Choose C = 30.0 in.

17-28 Follow the decision set outlined in Prob. 17-27 solution. We will form two tables, the first for a 15 000 h life goal, and a second for a 50 000 h life goal. The comparison is useful.

Function: H_{nom} = 50 hp at n = 1800 rev/min, n_{pump} = 900 rev/min, m_G = 1800/900 = 2,
K_s = 1.2, life = 15 000 h, then repeat with life = 50 000 h
Design factor: n_d = 1.1
Sprockets: N_1 = 19 teeth, N_2 = 38 teeth
Table 17-22 (post extreme):

$$K_1 = \left(\frac{N_1}{17}\right)^{1.5} = \left(\frac{19}{17}\right)^{1.5} = 1.18$$

Table 17-23: K_2 = 1, 1.7, 2.5, 3.3, 3.9, 4.6, 6.0

Decision variables for 15 000 h life goal:

$$H'_{\text{tab}} = \frac{K_s n_d H_{\text{nom}}}{K_1 K_2} = \frac{1.2(1.1)(50)}{1.18K_2} = \frac{55.9}{K_2} \tag{1}$$

$$n_{fs} = \frac{K_1 K_2 H_{\text{tab}}}{K_s H_{\text{nom}}} = \frac{1.18K_2 H_{\text{tab}}}{1.2(50)} = 0.0197K_2 H_{\text{tab}}$$

Form a table for a 15 000 h life goal using these equations.

K_2	H'_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	55.90	120	21.6	0.423	C'
1.7	32.90	120	21.6	0.923	C'
2.5	22.40	120	21.6	1.064	C'
3.3	16.90	120	21.6	1.404	C'
3.9	14.30	80	15.6	1.106	C'
4.6	12.20	60	12.4	1.126	C'
6	9.32	60	12.4	1.416	C'

There are 4 possibilities where $n_{fs} \geq 1.1$

Decision variables for 50 000 h life goal
From Eq. (17-40), the power-life tradeoff is:

$$(H'_{\text{tab}})^{2.5}15\ 000 = (H''_{\text{tab}})^{2.5}50\ 000$$

$$H''_{\text{tab}} = \left[\frac{15\ 000}{50\ 000}(H'_{\text{tab}})^{2.5}\right]^{1/2.5} = 0.618\,H'_{\text{tab}}$$

Substituting from (1),

$$H''_{\text{tab}} = 0.618\left(\frac{55.9}{K_2}\right) = \frac{34.5}{K_2}$$

The H'' notation is only necessary because we constructed the first table, which we normally would not do.

$$n_{fs} = \frac{K_1K_2H''_{\text{tab}}}{K_sH_{\text{nom}}} = \frac{K_1K_2(0.618H'_{\text{tab}})}{K_sH_{\text{nom}}} = 0.618[(0.0197)K_2H_{\text{tab}}]$$
$$= 0.0122K_2H_{\text{tab}}$$

Form a table for a 50 000 h life goal.

K_2	H''_{tab}	Chain #	H_{tab}	n_{fs}	Lub
1	34.50	120	21.6	0.264	C'
1.7	20.30	120	21.6	0.448	C'
2.5	13.80	120	21.6	0.656	C'
3.3	10.50	120	21.6	0.870	C'
3.9	8.85	120	21.6	1.028	C'
4.6	7.60	120	21.6	1.210	C'
6	5.80	80	15.6	1.140	C'

There are two possibilities in the second table with $n_{fs} \geq 1.1$. (The tables allow for the identification of a longer life of the outcomes.) We need a figure of merit to help with the choice; costs of sprockets and chains are thus needed, but is more information than we have.

Decision #1: #80 Chain (smaller installation) *Ans.*
$n_{fs} = 0.0122K_2H_{\text{tab}} = 0.0122(8.0)(15.6) = 1.14$ *O.K.*
Decision #2: 8-Strand, No. 80 *Ans.*
Decision #3: Type C′ Lubrication *Ans.*
Decision #4: $p = 1.0$ in, C is in midrange of 40 pitches

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C / p}$$

$$= 2(40) + \frac{19 + 38}{2} + \frac{(38 - 19)^2}{4\pi^2(40)}$$

$$= 108.7 \Rightarrow 110 \text{ even integer} \quad \textit{Ans.}$$

Eq. (17-36):

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{19 + 38}{2} - \frac{110}{1} = -81.5$$

Eq. (17-35): $$\frac{C}{p} = \frac{1}{4}\left[-(-81.5) + \sqrt{(-81.5)^2 - 8\left(\frac{38 - 19}{2\pi}\right)^2}\right] = 40.64$$

$$C = p(C/p) = 1.0(40.64/1.0) = 40.64 \text{ in (for reference)} \quad \textit{Ans.}$$

17-29 The objective of the problem is to explore factors of safety in wire rope. We will express strengths as tensions.

(a) Monitor steel 2-in 6 × 19 rope, 480 ft long.

Table 17-2: Minimum diameter of a sheave is $30d = 30(2) = 60$ in, preferably $45(2) = 90$ in. The hoist abuses the wire when it is bent around a sheave. Table 17-24 gives the nominal tensile strength as 106 kpsi. The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\left[\frac{\pi(2)^2}{4}\right] = 333 \text{ kip} \quad \textit{Ans.}$$

The tensile loading of the wire is given by Eq. (17-46)

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right)$$

$$W = 4(2) = 8 \text{ kip}, \quad m = 1$$

Table (17-24):

$$wl = 1.60d^2 l = 1.60(2^2)(480) = 3072 \text{ lbf} = 3.072 \text{ kip}$$

Therefore,

$$F_t = (8 + 3.072)\left(1 + \frac{2}{32.2}\right) = 11.76 \text{ kip} \quad \textit{Ans.}$$

Eq. (17-48):

$$F_b = \frac{E_r d_w A_m}{D}$$

and for the 72-in drum

$$F_b = \frac{12(10^6)(2 / 13)(0.38)(2^2)(10^{-3})}{72} = 39 \text{ kip} \quad \textit{Ans.}$$

For use in Eq. (17-44), from Fig. 17-21

$$(p / S_u) = 0.0014$$
$$S_u = 240 \text{ kpsi}, \quad \text{p. 920}$$
$$F_f = \frac{0.0014(240)(2)(72)}{2} = 24.2 \text{ kip} \quad \textit{Ans.}$$

(b) Factors of safety

Static, no bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3 \quad \textit{Ans.}$$

Static, with bending:

Eq. (17-49): $$n_s = \frac{F_u - F_b}{F_t} = \frac{333 - 39}{11.76} = 25.0 \quad \textit{Ans.}$$

Fatigue without bending:

$$n_f = \frac{F_f}{F_t} = \frac{24.2}{11.76} = 2.06 \quad \textit{Ans.}$$

Fatigue, with bending: For a life of $0.1(10^6)$ cycles, from Fig. 17-21

$$(p / S_u) = 4 / 1000 = 0.004$$
$$F_f = \frac{0.004(240)(2)(72)}{2} = 69.1 \text{ kip}$$

Eq. (17-50): $$n_f = \frac{69.1 - 39}{11.76} = 2.56 \quad \textit{Ans.}$$

If we were to use the endurance strength at 10^6 cycles ($F_f = 24.2$ kip) the factor of safety would be less than 1 indicating 10^6 cycle life impossible.

Comments:

- There are a number of factors of safety used in wire rope analysis. They are different, with different meanings. There is no substitute for knowing exactly which factor of safety is written or spoken.
- Static performance of a rope in tension is impressive.
- In this problem, at the drum, we have a finite life.
- The remedy for fatigue is the use of smaller diameter ropes, with multiple ropes

supporting the load. See Ex. 17-6 for the effectiveness of this approach. It will also be used in Prob. 17-30.

- Remind students that wire ropes do not fail suddenly due to fatigue. The outer wires gradually show wear and breaks; such ropes should be retired. Periodic inspections prevent fatigue failures by parting of the rope.

17-30 Since this is a design task, a decision set is useful.

A priori decisions

- Function: load, height, acceleration, velocity, life goal
- Design Factor: n_d
- Material: IPS, PS, MPS or other
- Rope: Lay, number of strands, number of wires per strand

Decision variables:
- Nominal wire size: d
- Number of load-supporting wires: m

From experience with Prob. 17-29, a 1-in diameter rope is not likely to have much of a life, so approach the problem with the d and m decisions open.

Function: 5000 lbf load, 90 foot lift, acceleration = 4 ft/s^2, velocity = 2 ft/s, life goal = 10^5 cycles
Design Factor: $n_d = 2$
Material: IPS
Rope: Regular lay, 1-in plow-steel 6 × 19 hoisting

Design variables
Choose 30-in $D_{\min}$. Table 17-27: $w = 1.60d^2$ lbf/ft

$$wl = 1.60d^2 l = 1.60d^2(90) = 144d^2 \text{ lbf, each}$$

Eq. (17-46):

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left(\frac{5000}{m} + 144d^2\right)\left(1 + \frac{4}{32.2}\right)$$

$$= \frac{5620}{m} + 162d^2 \text{ lbf, each wire}$$

Eq. (17-47):

$$F_f = \frac{(p / S_u)S_u D d}{2}$$

From Fig. 17-21 for 10^5 cycles, $p/S_u = 0.004$. From p. 920, $S_u = 240$ kpsi, based on metal area.

$$F_f = \frac{0.004(240\,000)(30d)}{2} = 14\,400d \text{ lbf each wire}$$

Eq. (17-48) and Table 17-27:

$$F_b = \frac{E_w d_w A_m}{D} = \frac{12\left(10^6\right)0.067d\left(0.4d^2\right)}{30} = 10\ 720d^3 \text{ lbf, each wire}$$

Eq. (17-45):

$$n_f = \frac{F_f - F_b}{F_t} = \frac{14\ 400d - 10\ 720d^3}{(5620\ /\ m) + 162d^2}$$

We could use a computer program to build a table similar to that of Ex. 17-6. Alternatively, we could recognize that $162\ d^2$ is small compared to $5620\ /\ m$, and therefore eliminate the $162d^2$ term.

$$n_f \doteq \frac{14\ 400d - 10\ 720d^3}{5620\ /\ m} = \frac{m}{5620}(14\ 400d - 10\ 720d^3)$$

Maximize n_f,

$$\frac{\partial n_f}{\partial d} = 0 = \frac{m}{5620}[14\ 400 - 3(10\ 720)d^2]$$

From which

$$d^* = \sqrt{\frac{14\ 400}{3(10\ 720)}} = 0.669 \text{ in}$$

Back-substituting

$$n_f = \frac{m}{5620}[14\ 400(0.669) - 10\ 720(0.669^3)] = 1.14 \text{ m}$$

Thus $n_f = 1.14, 2.28, 3.42, 4.56$ for m=1, 2, 3, 4 respectively. If we choose $d = 0.50$ in, then $m = 2$.

$$n_f = \frac{14\ 400(0.5) - 10\ 720(0.5^3)}{(5620\ /\ 2) + 162(0.5)^2} = 2.06$$

This exceeds $n_d = 2$

Decision #1: $d = 1/2$ in

Decision #2: $m = 2$ ropes supporting load. Rope should be inspected weekly for any signs of fatigue (broken outer wires).

Comment: Table 17-25 gives n for freight elevators in terms of velocity.

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\ 000\left(\frac{\pi d^2}{4}\right) = 83\ 252d^2 \text{ lbf, each wire}$$

$$n = \frac{F_u}{F_t} = \frac{83\ 452(0.5)^2}{(5620/2) + 162(0.5)^2} = 7.32$$

By comparison, interpolation for 120 ft/min gives 7.08 - close. The category of construction hoists is not addressed in Table 17-25. We should investigate this before proceeding further.

17-31 Given: 2000 ft lift, 72 in drum, 6 × 19 MS rope, cage and load 8000 lbf, accel. = 2 ft/s^2.

(a) Table 17-24: $(S_u)_{\text{nom}} = 106$ kpsi; $S_u = 240$ kpsi (p. 920); Fig. 17-21: $(p/S_u)10^6 = 0.0014$

Eq. (17-44):

$$F_f = \frac{(p\ /\ S_u)S_u dD}{2} = \frac{0.0014(240)d(72)}{2} = 12.1d \text{ kip}$$

Table 17-24: $wl = 1.6d^2\ 2000(10^{-3}) = 3.2d^2$ kip

Eq. (17-46):

$$F_t = (W + wl)\left(1 + \frac{a}{g}\right)$$

$$= (8 + 3.2d^2)\left(1 + \frac{2}{32.2}\right)$$

$$= 8.5 + 3.4d^2 \text{ kip}$$

Note that bending is not included.

$$n = \frac{F_f}{F_t} = \frac{12.1d}{8.5 + 3.4d^2}$$

d, in	n	
0.500	0.650	
1.000	1.020	
1.500	1.124	
1.625	1.125	← maximum n *Ans.*
1.750	1.120	
2.000	1.095	

(b) Try $m = 4$ strands

$$F_t = \left(\frac{8}{4} + 3.2d^2\right)\left(1 + \frac{2}{32.2}\right)$$
$$= 2.12 + 3.4d^2 \text{ kip}$$
$$F_f = 12.1d \text{ kip}$$
$$n = \frac{12.1d}{2.12 + 3.4d^2}$$

d, in	n	
0.5000	2.037	
0.5625	2.130	
0.6520	2.193	
0.7500	2.250	← maximum n *Ans.*
0.8750	2.242	
1.0000	2.192	

Comparing tables, multiple ropes supporting the load increases the factor of safety, and reduces the corresponding wire rope diameter, a useful perspective.

17-32

$$n = \frac{ad}{b/m + cd^2}$$

$$\frac{dn}{dd} = \frac{(b/m + cd^2)a - ad(2cd)}{(b/m + cd^2)^2} = 0$$

From which

$$d^* = \sqrt{\frac{b}{mc}} \quad Ans.$$

$$n^* = \frac{a\sqrt{b/(mc)}}{(b/m) + c[b/(mc)]} = \frac{a}{2}\sqrt{\frac{m}{bc}} \quad Ans.$$

These results agree closely with the Prob. 17-31 solution. The small differences are due to rounding in Prob. 17-31.

17-33 From Prob. 17-32 solution:

$$n_1 = \frac{ad}{b/m + cd^2}$$

Solve the above equation for m

$$m = \frac{b}{ad/n_1 - cd^2} \qquad (1)$$

$$\frac{dm}{dd} = 0 = \frac{\left[\left(ad/n_1\right) - ad^2\right](0) - b\left[\left(a/n_1\right) - 2cd\right]}{\left[\left(ad/n_1\right) - cd^2\right]^2}$$

From which $d^* = \dfrac{a}{2cn_1}$ *Ans.*

Substituting this result for d into Eq. (1) gives

$$m^* = \frac{4bcn_1}{a^2} \quad Ans.$$

17-34 **Note to the Instructor**. In the first printing of the ninth edition, the wording of this problem is incorrect. It should read " For Prob. 17-29 estimate the elongation of the rope if a 7000 lbf loaded mine cart is placed in the cage which weighs 1000 lbf. The results of Prob. 4-7 may be useful". This will be corrected in subsequent printings. We apologize for any inconvenience encountered.

Table 17-27:

$$A_m = 0.40d^2 = 0.40(2^2) = 1.6 \text{ in}^2$$

$$E_r = 12 \text{ Mpsi}, \quad w = 1.6d^2 = 1.6(2^2) = 6.4 \text{ lbf/ft}$$

$$wl = 6.4(480) = 3072 \text{ lbf}$$

$$\gamma \doteq wl / \left(A_m l\right) = 3072 / \left[1.6(480)12\right] = 0.333 \text{ lbf/in}^3$$

Treat the rest of the system as rigid, so that all of the stretch is due to the load of 7000 lbf, the cage weighing 1000 lbf, and the wire's weight. From the solution of Prob. 4-7,

$$\delta_1 = \frac{Wl}{AE} + \frac{\gamma l^2}{2E}$$

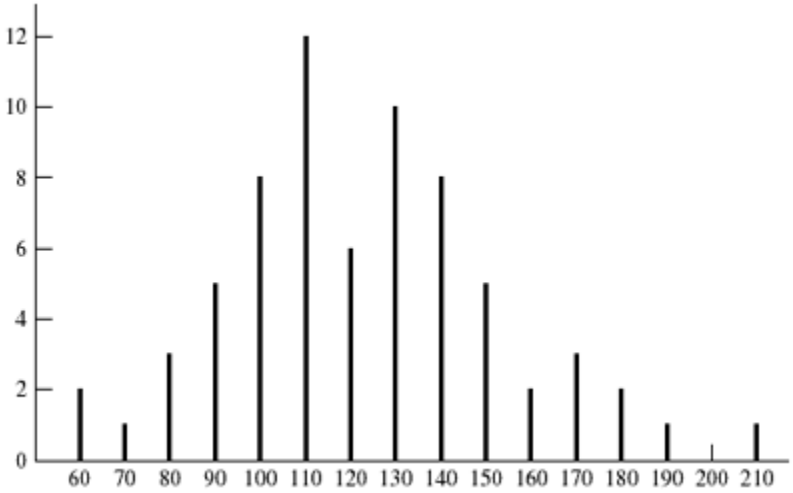
$$= \frac{(1000 + 7000)(480)(12)}{1.6(12)(10^6)} + \frac{0.333(480^2)12^2}{2(12)(10^6)}$$

$$= 2.4 + 0.460 = 2.860 \text{ in} \qquad \textit{Ans.}$$

17-35 to 17-38 Computer programs will vary.

Chapter 20

20-1 (**a**)



(**b**) $f / (N\Delta x) = f / [69(10)] = f / 690$

x	f	fx	fx^2	$f / (N\Delta x)$
60	2	120	7200	0.0029
70	1	70	4900	0.0015
80	3	240	19200	0.0043
90	5	450	40500	0.0072
100	8	800	80000	0.0116
110	12	1320	145200	0.0174
120	6	720	86400	0.0087
130	10	1300	169000	0.0145
140	8	1120	156800	0.0116
150	5	750	112500	0.0174
160	2	320	51200	0.0029
170	3	510	86700	0.0043
180	2	360	64800	0.0029
190	1	130	36100	0.0015
200	0	0	0	0
210	1	210	44100	0.0015
Σ	69	8480	1 104 600	

Eq. (20-9): $\bar{x} = \dfrac{8480}{69} = 122.9 \text{ kcycles}$

Eq. (20-10): $s_x = \left[\dfrac{1\,104\,600 - 8480^2 / 69}{69 - 1} \right]^{1/2} = 30.3 \text{ kcycles}$ *Ans.*

20-2 Data represents a 7-class histogram with $N = 197$.

x	f	fx	fx^2
174	6	1044	181 656
182	9	1638	298 116
190	44	8360	1 588 400
198	67	13 266	2 626 688
206	53	10 918	2 249 108
214	12	2568	549 552
220	6	1320	290 400
Σ	197	39 114	7 789 900

$$\bar{x} = \frac{39\,114}{197} = 198.55 \text{ kpsi} \quad \textit{Ans.}$$

$$s = \left[\frac{7\,783\,900 - 39\,114^2 / 197}{197 - 1} \right]^{1/2} = 9.55 \text{ kpsi} \quad \textit{Ans.}$$

20-3 Form a Table:

x	f	fx	fx^2
64	2	128	8192
68	6	408	27 744
72	6	432	31 104
76	9	684	51 984
80	19	1520	121 600
84	10	840	70 560
88	4	352	30 976
92	2	184	16 928
Σ	58	4548	359 088

$$\bar{x} = \frac{4548}{58} = 78.4 \text{ kpsi} \qquad \textit{Ans.}$$

$$s_x = \left[\frac{359\ 088 - 4548^2 / 58}{58 - 1} \right]^{1/2} = 6.57 \text{ kpsi} \qquad \textit{Ans.}$$

From Eq. 20-14

$$f(x) = \frac{1}{6.57\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 78.4}{6.57} \right)^2 \right] \qquad \textit{Ans.}$$

20-4 (a)

x	f	fy	fy^2	y	$f/(Nw)$	$f(y)$	$g(y)$
5.625	1	5.625	31.64063	5.625	0.072 727	0.001 262	0.000 295
5.875	0	0	0	5.875	0	0.008 586	0.004 088
6.125	0	0	0	6.125	0	0.042 038	0.031 194
6.375	3	19.125	121.9219	6.375	0.218 182	0.148 106	0.140 262
6.625	3	19.875	131.6719	6.625	0.218 182	0.375 493	0.393 667
6.875	6	41.25	283.5938	6.875	0.436 364	0.685 057	0.725 002
7.125	14	99.75	710.7188	7.125	1.018 182	0.899 389	0.915 128
7.375	15	110.625	815.8594	7.375	1.090 909	0.849 697	0.822 462
7.625	10	76.25	581.4063	7.625	0.727 273	0.577 665	0.544 251
7.875	2	15.75	124.0313	7.875	0.145 455	0.282 608	0.273 138
8.125	1	8.125	66.015 63	8.125	0.072 727	0.099 492	0.106 720
Σ	55	396.375	2866.859				

For a normal distribution,

$$\bar{y} = 396.375/55 = 7.207, \qquad s_y = \left(\frac{2866.859 - \left(396.375^2/55\right)}{55 - 1} \right)^{1/2} = 0.4358$$

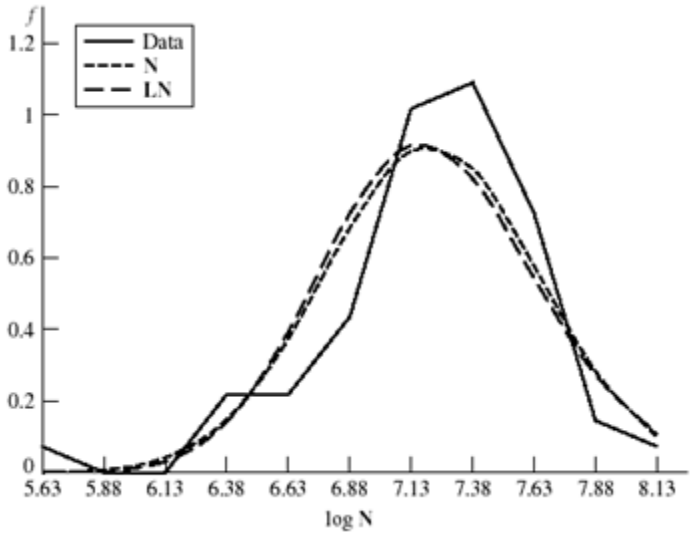
$$f(y) = \frac{1}{0.4358\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 7.207}{0.4358} \right)^2 \right]$$

For a lognormal distribution,

$$\bar{x} = \ln 7.206\ 818 - \ln\sqrt{1 + 0.060\ 474^2} = 1.9732, \qquad s_x = \ln\sqrt{1 + 0.060\ 474^2} = 0.0604$$

$$g(y) = \frac{1}{x(0.0604)\left(\sqrt{2\pi}\right)} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 1.9732}{0.0604} \right)^2 \right]$$

(b) Histogram



20-5 Distribution is uniform in interval 0.5000 to 0.5008 in, range numbers are a = 0.5000 in, b = 0.5008 in.

(a) Eq. (20-22) $$\mu_x = \frac{a+b}{2} = \frac{0.5000 + 0.5008}{2} = 0.5004$$

Eq. (20-23) $$\sigma_x = \frac{b-a}{2} = \frac{0.5008 - 0.5000}{2\sqrt{3}} = 0.000\ 231$$

(b) PDF, Eq. (20-20)

$$f(x) = \begin{cases} 1250 & 0.5000 \le \text{x} \le 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

(c) CDF, Eq. (20-21)

$$F(x) = \begin{cases} 0 & x < 0.5000 \text{ in} \\ (x - 0.5) / 0.0008 & 0.5000 \le x \le 0.5008 \text{ in} \\ 1 & x > 0.5008 \text{ in} \end{cases}$$

If all smaller diameters are removed by inspection, a = 0.5002 in, b = 0.5008 in,

$$\mu_x = \frac{0.5002 + 0.5008}{2} = 0.5005 \text{ in}$$

$$\hat{\sigma}_x = \frac{0.5008 - 0.5002}{2\sqrt{3}} = 0.000\ 173 \text{ in}$$

$$f(x) = \begin{cases} 1666.7 & 0.5002 \le x \le 0.5008 \text{ in} \\ 0 & \text{otherwise} \end{cases}$$

$$F(x)=\begin{cases}0 & x<0.5002 \text{ in}\\ 1666.7(x-0.5002) & 0.5002\le x\le 0.5008 \text{ in}\\ 1 & x>0.5008 \text{ in}\end{cases}$$

20-6 Dimensions produced are due to tool dulling and wear. When parts are mixed, the distribution is uniform. From Eqs. (20-22) and (20-23),

$$a=\mu_x-\sqrt{3}s=0.6241-\sqrt{3}\left(0.000\ 581\right)=0.6231 \text{ in}$$
$$b=\mu_x+\sqrt{3}s=0.6241+\sqrt{3}\left(0.000\ 581\right)=0.6251 \text{ in}$$

We suspect the dimension was $\dfrac{0.623}{0.625}$ in *Ans.*

20-7 $F(x) = 0.555x - 33$ mm.

(a) Since $F(x)$ is linear, distribution is uniform at $x = a$

$$F(a) = 0 = 0.555(a) - 33$$

$\therefore$ $a = 59.46$ mm. Therefore at $x = b$

$$F(b) = 1 = 0.555b - 33$$

$\therefore$ $b = 61.26$ mm. Therefore,

$$F(x)=\begin{cases}0 & x<59.46 \text{ mm}\\ 0.555x-33 & 59.46\le x\le 61.26 \text{ mm}\\ 1 & x>61.26 \text{ mm}\end{cases}$$

The PDF is dF/dx, thus the range numbers are:

$$f(x)=\begin{cases}0.555 & 59.46\le x\le 61.26 \text{ mm}\\ 0 & \text{otherwise}\end{cases}$$ *Ans.*

From the range numbers,

$$\mu_x=\frac{59.46+61.26}{2}=60.36 \text{ mm}$$ *Ans.*

$$\hat{\sigma}_x=\frac{61.26-59.46}{2\sqrt{3}}=0.520 \text{ mm}$$ *Ans.*

(*b*) σ is an uncorrelated quotient $\bar{F} = 3600$ lbf, $\bar{A} = 0.112$ in^2

$$C_F = 300/3600 = 0.083\ 33,\ C_A = 0.001/0.112 = 0.008\ 929$$

From Table 20-6, For σ

$$\bar{\sigma} = \frac{\mu_F}{\mu_A} = \frac{3600}{0.112} = 32\ 143 \text{ psi} \quad \textit{Ans.}$$

$$\hat{\sigma}_\sigma = 32\ 143\left[\frac{\left(0.08333^2 + 0.008929^2\right)}{\left(1 + 0.008929^2\right)}\right]^{1/2} = 2694 \text{ psi} \quad \textit{Ans.}$$

$$C_\sigma = 2694 / 32\ 143 = 0.0838 \quad \textit{Ans.}$$

Since **F** and **A** are lognormal, division is closed and $\boldsymbol{\sigma}$ is lognormal too.

$$\boldsymbol{\sigma} = \mathbf{LN}(32\ 143,\ 2694) \text{ psi} \quad \textit{Ans.}$$

20-8 Cramer's rule

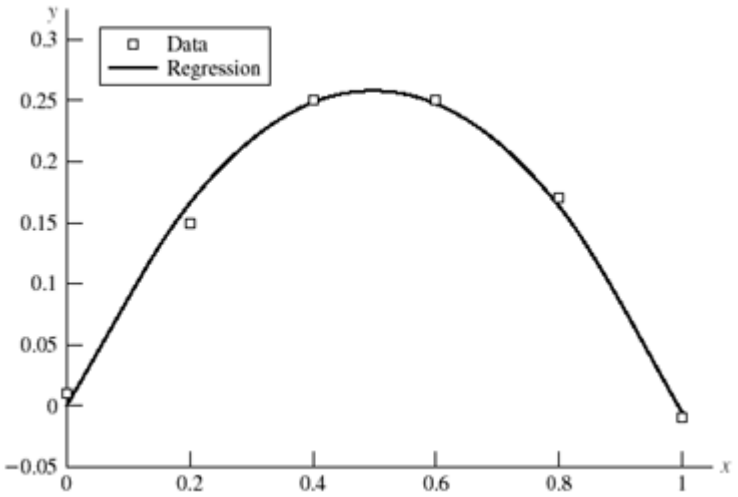
$$a_1 = \frac{\begin{vmatrix} \sum y & \sum x^2 \\ \sum xy & \sum x^3 \end{vmatrix}}{\begin{vmatrix} \sum x & \sum x^2 \\ \sum x^2 & \sum x^3 \end{vmatrix}} = \frac{\sum y \sum x^3 - \sum xy \sum x^2}{\sum x \sum x^3 - \left(\sum x^2\right)^2} \quad \textit{Ans.}$$

$$a_2 = \frac{\begin{vmatrix} \sum x & \sum y \\ \sum x^2 & \sum xy \end{vmatrix}}{\begin{vmatrix} \sum x & \sum x^2 \\ \sum x^2 & \sum x^3 \end{vmatrix}} = \frac{\sum y \sum xy - \sum y \sum x^2}{\sum x \sum x^3 - \left(\sum x^2\right)^2} \quad \textit{Ans.}$$

	x	y	x^2	x^3	xy
	0	0.01	0	0	0
	0	0.15	0.04	0.008	0.030
	0	0.25	0.16	0.064	0.100
	1	0.25	0.36	0.216	0.150
	1	0.17	0.64	0.512	0.136
	1	–0.01	1.00	1.000	–0.010
Σ	3	0.82	2.20	1.800	0.406

$a_1 =$ 1.040 714 $\quad a_2 = -1.046\ 43$ *Ans.*

	Data	Regression
x	y	y
0	0.01	0
0.2	0.15	0.166 286
0.4	0.25	0.248 857
0.6	0.25	0.247 714
0.8	0.17	0.162 857
1.0	-0.01	-0.005 710

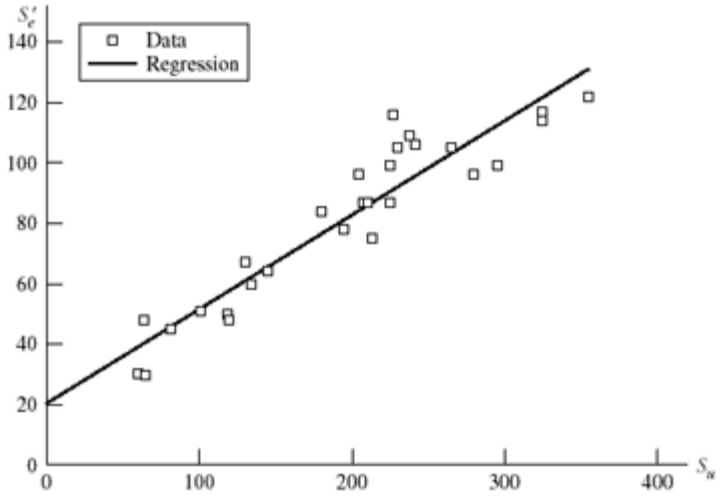


20-9

	Data	Regression		
S_u	S_e'	S_e'	S_u^2	$S_u S_e'$
0		20.356 75		
60	30	39.080 78	3 600	1 800
64	48	40.329 05	4 096	3 072
65	29.5	40.641 12	4 225	1 917.5
82	45	45.946 26	6 724	3 690
101	51	51.875 54	10 201	5 151
119	50	57.492 75	14 161	5 950
120	48	57.804 81	14 400	5 760
130	67	60.925 48	16 900	8 710
134	60	62.173 75	17 956	8 040
145	64	65.606 49	21 025	9 280
180	84	76.528 84	32 400	15 120

	195	78	81.209 85	38 025	15 210
	205	96	84.330 52	42 025	19 680
	207	87	84.954 66	42 849	18 009
	210	87	85.890 86	44 100	18 270
	213	75	86.827 06	45 369	15 975
	225	99	90.571 87	50 625	22 275
	225	87	90.571 87	50 625	19 575
	227	116	91.196 00	51 529	26 332
	230	105	92.132 20	52 900	24 150
	238	109	94.628 74	56 644	25 942
	242	106	95.877 01	58 564	25 652
	265	105	103.054 60	70 225	27 825
	280	96	107.735 60	78 400	26 880
	295	99	112.416 60	87 025	29 205
	325	114	121.778 60	105 625	37 050
	325	117	121.778 60	105 625	38 025
	355	122	131.140 60	126 025	43 310
Σ	5462	2274.5		1 251 868	501 855.5

$m = 0.312\,067$, $b = 20.356\,75$ *Ans.*



20-10

$$\varepsilon = \sum \left(y - a_0 - a_2 x^2 \right)^2$$

$$\frac{\partial \varepsilon}{\partial a_o} = -2 \sum \left(y - a_0 - a_2 x^2 \right) = 0$$

$$\sum y - na_0 - a_2\sum x^2 = 0 \quad \Rightarrow \quad \sum y = na_0 + a_2\sum x^2$$

$$\frac{\partial \varepsilon}{\partial a_2} = 2\sum\left(y - a_0 - a_2x^2\right)(2x) = 0 \quad \Rightarrow \quad \sum xy = a_0\sum x + a_2\sum x^3$$

Ans.

Cramer's rule

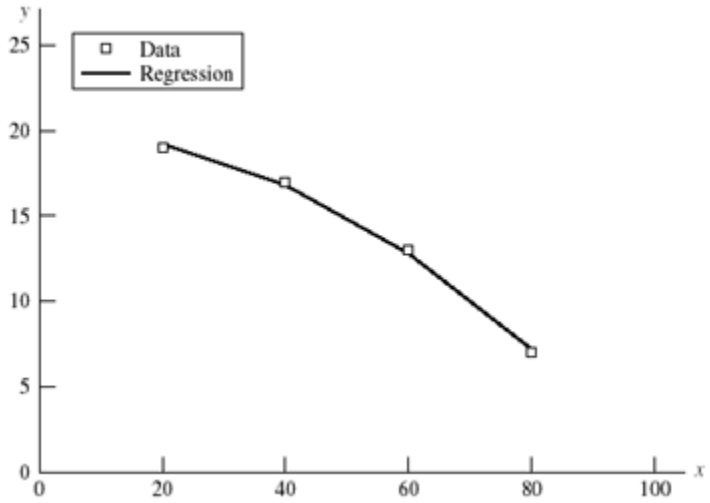
$$a_o = \frac{\begin{vmatrix} \sum y & \sum x^2 \\ \sum xy & \sum x^3 \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{\sum x^3 \sum y - \sum x^2 \sum xy}{n\sum x^3 - \sum x \sum x^2}$$

$$a_1 = \frac{\begin{vmatrix} n & \sum y \\ \sum x & \sum xy \end{vmatrix}}{\begin{vmatrix} n & \sum x^2 \\ \sum x & \sum x^3 \end{vmatrix}} = \frac{n\sum xy - \sum x \sum y}{n\sum x^3 - \sum x \sum x^2}$$

		Data	Regression			
	x	y	y	x^2	x^3	xy
	20	19	19.2	400	8 000	380
	40	17	16.8	1600	64 000	680
	60	13	12.8	3600	216 000	780
	80	7	7.2	6400	512 000	560
Σ	200	56		12 000	800 000	2400

$$a_0 = \frac{800\ 000(56) - 12\ 000(2400)}{4(800\ 000) - 200(12\ 000)} = 20$$

$$a_1 = \frac{4(2400) - 200(56)}{4(800\ 000) - 200(12\ 000)} = -0.002$$

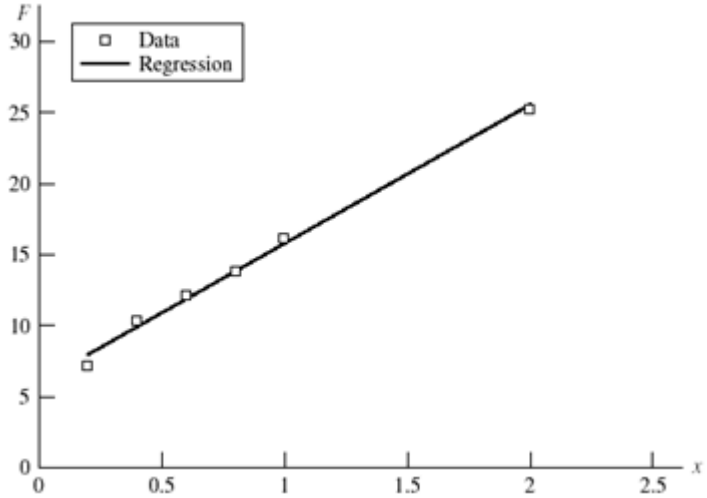


20-11

	Data		Regression					
	x	y	y	x^2	y^2	xy	$x-\bar{x}$	$(x-\bar{x})^2$
	0.2	7.1	7.931 803	0.04	50.41	1.42	-0.633 333	0.401 111 111
	0.4	10.3	9.884 918	0.16	106.09	4.12	-0.433 333	0.187 777 778
	0.6	12.1	11.838 032	0.36	146.41	7.26	-0.233 333	0.054 444 444
	0.8	13.8	13.791 147	0.64	190.44	11.04	-0.033 333	0.001 111 111
	1	16.2	15.744 262	1	262.44	16.2	0.166 666	0.027 777 778
	2	25.2	25.509 836	4	635.04	50.4	1.166 666	1.361 111 111
Σ	5	84.7		6.2	1390.83	90.44	0	2.033 333 333

$$\hat{m} = \bar{k} = \frac{6(90.44) - 5(84.7)}{6(6.2) - (5)^2} = 9.7656$$

$$\hat{b} = \bar{F}_i = \frac{84.7 - 9.7656(5)}{6} = 5.9787$$



(a) $\bar{x} = \frac{5}{6}; \quad \bar{y} = \frac{84.7}{6} = 14.117$

Eq. (20-37):

$$s_{yx} = \sqrt{\frac{1390.83 - 5.9787(84.7) - 9.7656(90.44)}{6-2}}$$

$$= 0.556$$

Eq. (20-36):

$$s_{\hat{b}} = 0.556\sqrt{\frac{1}{6} + \frac{(5/6)^2}{2.0333}} = 0.3964 \text{ lbf}$$

$$F_i = (5.9787, 0.3964) \text{ lbf} \quad \textit{Ans.}$$

(b) Eq. (20-35):

$$s_{\hat{m}} = \frac{0.556}{\sqrt{2.0333}} = 0.3899 \text{ lbf/in}$$

$$k = (9.7656, 0.3899) \text{ lbf/in} \quad \textit{Ans.}$$

20-12 The expression $\boldsymbol{\epsilon} = \boldsymbol{\delta} / \mathbf{l}$ is of the form $\mathbf{x} / \mathbf{y}$. Now $\boldsymbol{\delta}$ = (0.0015, 0.000 092) in, unspecified distribution; and $\mathbf{l}$ = (2,000, 0.008 1) in, unspecified distribution;

$$C_x = 0.000\ 092 / 0.0015 = 0.0613$$
$$C_y = 0.0081 / 2.000 = 0.004\ 05$$

Table 20-6: $\bar{\epsilon} = 0.0015 / 2.000 = 0.000\ 75$

$$\hat{\sigma}_\epsilon = 0.000\ 75\left[\frac{0.0613^2 + 0.004\ 05^2}{1 + 0.004\ 05^2}\right]^{1/2}$$
$$= 4.607\left(10^{-5}\right) = 0.000\ 046$$

We can predict $\bar{\epsilon}$ and $\hat{\sigma}_\epsilon$ but not the distribution of $\boldsymbol{\epsilon}$.

20-13 $\boldsymbol{\sigma} = \boldsymbol{\epsilon}\mathbf{E}$

$\boldsymbol{\epsilon}$ = (0.0005, 0.000 034), distribution unspecified; $\mathbf{E}$ = (29.5, 0.885) Mpsi, distribution unspecified;

$$C_x = 0.000\ 034 / 0.0005 = 0.068$$
$$C_y = 0.0885 / 29.5 = 0.03$$

$\boldsymbol{\sigma}$ is of the form $\mathbf{xy}$

Table 20-6: $\bar{\sigma} = \bar{\epsilon}\,\bar{E} = 0.0005(29.5)10^6 = 14\ 750$ psi

$$\hat{\sigma}_\sigma = 14\ 750\left[0.068^2 + 0.030^2 + 0.068^2\left(0.030^2\right)\right]^{1/2}$$
$$= 1096.7 \text{ psi}$$
$$C_\sigma = 1096.7 / 14\ 750 = 0.074\ 35$$

20-14

$$\boldsymbol{\delta} = \frac{\mathbf{Fl}}{\mathbf{AE}}$$

where $\mathbf{F}$ = (14.7, 1.3) kip, $\mathbf{A}$ = (0.226, 0.003) in^2, $\mathbf{l}$ = (1.5, 0.004) in, and $\mathbf{E}$ = (29.5, 0.885) Mpsi, distributions unspecified.

C_F = 1.3 / 14.7 = 0.0884; C_A = 0.003 / 0.226 = 0.0133; C_l = 0.004 / 1.5 = 0.00267; C_E =0.885 / 29.5 = 0.03

$$\boldsymbol{\delta} = \frac{\mathbf{Fl}}{\mathbf{AE}} = \mathbf{Fl}\left(\frac{1}{\mathbf{A}}\right)\left(\frac{1}{\mathbf{E}}\right)$$

Table 20-6:

$$\bar{\delta} = \bar{F}\,\bar{l}\,\overline{(1/A)}\;\overline{(1/E)} \doteq \bar{F}\,\bar{l}\,(1/\bar{A})(1/\bar{E})$$

$$= 14\ 700(1.5)\left(\frac{1}{0.226}\right)\left[\frac{1}{29.5(10^6)}\right] = 0.003\ 31 \text{ in.} \quad \textit{Ans.}$$

For the standard deviation, using the first-order terms in Table 20-6,

$$\hat{\sigma}_\delta \doteq \frac{\bar{F}\,\bar{l}}{\bar{A}\bar{E}}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2} = \bar{\delta}\left(C_F^2 + C_l^2 + C_A^2 + C_E^2\right)^{1/2}$$

$$\hat{\sigma}_\delta = 0.003\ 31\left(0.0844^2 + 0.002\ 67^2 + 0.0133^2 + 0.03^2\right)^{1/2}$$

$$= 0.000\ 313 \text{ in} \quad \textit{Ans.}$$

COV: $C_\delta = \hat{\sigma}_\delta / \bar{\delta} = 0.000\ 313 / 0.003\ 31 = 0.0945$ *Ans.*

Force COV dominates. There is no distributional information on δ.

20-15 **M** = (15 000, 1350) lbf · in, distribution unspecified; **d** = (2.00, 0.005) in, distribution unspecified.

$$\boldsymbol{\sigma} = \frac{32\mathbf{M}}{\pi\mathbf{d}^3}$$

C_M = 1350 / 15 000 = 0.09, C_d = 0.005 / 2.00 = 0.0025

σ is of the form **x/y**^{3}, Table 20-6.

Mean: $\bar{M} = 15\ 000$ lbf · in

$$\overline{\left(\frac{1}{d^3}\right)} = \frac{1}{\bar{d}^3}\left(1 + 6C_x^2\right) = \frac{1}{2^3}\left[1 + 6\left(0.0025^2\right)\right] = 0.125 \text{ in}^3 \ *$$

* Note: $\overline{\left(\frac{1}{d^3}\right)} \doteq \frac{1}{\bar{d}^3}$

$$\bar{\sigma} = \frac{32\bar{M}}{\pi \bar{d}^3} = \frac{32(15\ 000)}{\pi}(0.125)$$
$$= 19\ 099 \text{ psi} \quad \textit{Ans.}$$

Standard Deviation:

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + C_{d^3}^2\right)/\left(1 + C_{d^3}^2\right)\right]^{1/2}$$

Table 20-6:

$$C_{d^3} \doteq 3C_d = 3(0.0025) = 0.0075$$

$$\hat{\sigma}_\sigma = \bar{\sigma}\left[\left(C_M^2 + \left(3C_d\right)^2\right)\Big/\left(1 + \left(3C_d\right)^2\right)\right]^{1/2}$$

$$= 19\ 099\left[\left(0.09^2 + 0.0075^2\right)\Big/\left(1 + 0.0075^2\right)\right]^{1/2}$$

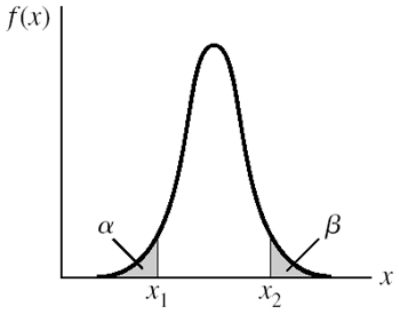
$$= 1725 \text{ psi} \quad \textit{Ans.}$$

COV:

$$C_\sigma = \frac{1725}{19\ 099} = 0.0903 \quad \textit{Ans.}$$

Stress COV dominates. No information of distribution of σ.

20-16



Fraction discarded is $\alpha + \beta$. The area under the PDF was unity. Having discarded $\alpha + \beta$ fraction, the ordinates to the truncated PDF are multiplied by a.

$$a = \frac{1}{1 - (\alpha + \beta)}$$

New PDF, $g(x)$, is given by

$$g(x) = \begin{cases} f(x)\Big/\left[1 - (\alpha + \beta)\right] & x_1 \le x \le x_2 \\ 0 & \text{otherwise} \end{cases}$$

A more formal proof: $g(x)$ has the property

$$1 = \int_{x_1}^{x_2} g(x)\,dx = a\int_{x_1}^{x_2} f(x)\,dx$$

$$1 = a\left[\int_{-\infty}^{\infty} f(x)\,dx - \int_{0}^{x_1} f(x)\,dx - \int_{x_2}^{\infty} f(x)\,dx\right]$$

$$1 = a\left\{1 - F(x_1) - \left[1 - F(x_2)\right]\right\}$$

$$a = \frac{1}{F(x_2) - F(x_1)} = \frac{1}{(1-\beta) - \alpha} = \frac{1}{1-(\alpha+\beta)}$$

20-17 (**a**) **d** = **U**(0.748, 0.751)

$$\mu_d = \frac{0.751 + 0.748}{2} = 0.7495 \text{ in}$$

$$\hat{\sigma}_d = \frac{0.751 - 0.748}{2\sqrt{3}} = 0.000\ 866 \text{ in}$$

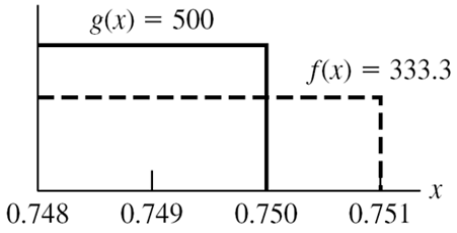
$$f(x) = \frac{1}{b-a} = \frac{1}{0.751 - 0.748} = 333.3 \text{ in}^{-1}$$

$$F(x) = \frac{x - 0.748}{0.751 - 0.748} = 333.3(x - 0.748)$$

(**b**) $F(x_1) = F(0.748) = 0$

$F(x_2) = (0.750 - 0.748)\ 333.3 = 0.6667$

If $g(x)$ is truncated, PDF becomes



$$g(x) = \frac{f(x)}{F(x_2) - F(x_1)} = \frac{333.3}{0.6667 - 0} = 500 \text{ in}^{-1}$$

$$\mu_x = \frac{a' + b'}{2} = \frac{0.748 + 0.750}{2} = 0.749 \text{ in}$$

$$\hat{\sigma}_x = \frac{b' - a'}{2\sqrt{3}} = \frac{0.750 - 0.748}{2\sqrt{3}} = 0.000\ 577 \text{ in}$$

20-18 From Table A-10, 8.1% corresponds to $z_1 = -1.4$ and 5.5% corresponds to $z_2 = +1.6$.

$$k_1 = \mu + z_1\hat{\sigma}$$

$$k_2 = \mu + z_2\hat{\sigma}$$

From which

$$\mu = \frac{z_2 k_1 - z_1 k_2}{z_2 - z_1} = \frac{1.6(9) - (-1.4)11}{1.6 - (-1.4)}$$

$$= 9.933$$

$$\hat{\sigma} = \frac{k_2 - k_1}{z_2 - z_1} = \frac{11 - 9}{1.6 - (-1.4)} = 0.6667$$

The original density function is

$$f(k) = \frac{1}{0.6667\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{k - 9.933}{0.6667} \right)^2 \right] \quad \textit{Ans.}$$

20-19 From Prob. 20-1, $\mu = 122.9$ kcycles and $\hat{\sigma} = 30.3$ kcycles.

$$z_{10} = \frac{x_{10} - \mu}{\hat{\sigma}} = \frac{x_{10} - 122.9}{30.3}$$

$$x_{10} = 122.9 + 30.3 z_{10}$$

From Table A-10, for 10 percent failure, $z_{10} = -1.282$

$$x_{10} = 122.9 + 30.3(-1.282)$$
$$= 84.1 \text{ kcycles} \quad \textit{Ans.}$$

20-20

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
60	2	120	7200	0.002899	0.000399
70	1	70	4900	0.001449	0.001206
80	3	240	19200	0.004348	0.003009
90	5	450	40500	0.007246	0.006204
100	8	800	80000	0.011594	0.010567
110	12	1320	145200	0.017391	0.014871
120	6	720	86400	0.008696	0.017292
130	10	1300	169000	0.014493	0.016612
140	8	1120	156800	0.011594	0.013185
150	5	750	112500	0.007246	0.008647
160	2	320	51200	0.002899	0.004685
170	3	510	86700	0.004348	0.002097
180	2	360	64800	0.002899	0.000776
190	1	190	36100	0.001449	0.000237
200	0	0	0	0	5.98E-05
210	1	210	44100	0.001449	1.25E-05
Σ	69	8480			

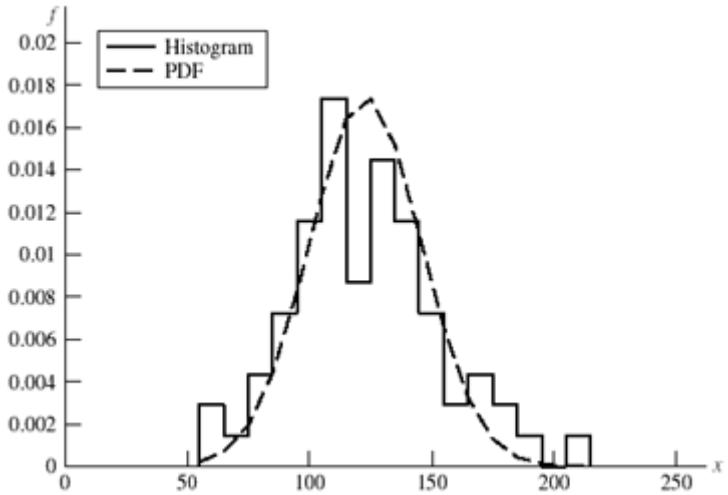
$\bar{x} = 122.8986$ $\quad$ $s_x = 22.887\ 19$

Eq. (20-14):

$$f(x) = \frac{1}{\hat{\sigma}_x\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\hat{\sigma}_x}\right)^2\right]$$

$$= \frac{1}{22.88719\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-122.8986}{22.88719}\right)^2\right]$$

x	f / (Nw)	$f(x)$	x	f / (Nw)	$f(x)$
55	0	0.000 214	145	0.011 594	0.010 935
55	0.002 899	0.000 214	145	0.007 246	0.010 935
65	0.002 899	0.000 711	155	0.007 246	0.006 518
65	0.001 449	0.000 711	155	0.002 899	0.006 518
75	0.001 449	0.001 951	165	0.002 899	0.002 21
75	0.004 348	0.001 951	165	0.004 348	0.003 21
85	0.004 348	0.004 425	175	0.004 348	0.001 306
85	0.007 246	0.004 425	175	0.002 899	0.001 306
95	0.007 246	0.008 292	185	0.002 899	0.000 439
95	0.011 594	0.008 292	185	0.001 449	0.000 439
105	0.011 594	0.012 839	195	0.001 449	0.000 122
105	0.017 391	0.012 839	195	0	0.000 122
115	0.017 391	0.016 423	205	0	2.8E-05
115	0.008 696	0.016 423	205	0.001 499	2.8E-05
125	0.008 696	0.017 357	215	0.001 499	5.31E-06
125	0.014 493	0.017 357	215	0	5.31E-06
135	0.014 493	0.015 157			
135	0.011 594	0.015 157			

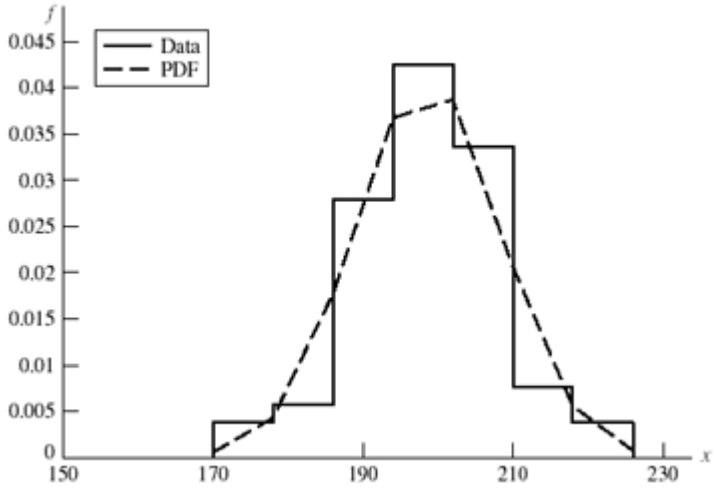


20-21

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
174	6	1044	181656	0.003807	0.001642
182	9	1638	298116	0.005711	0.009485
190	44	8360	1588400	0.027919	0.027742
198	67	13266	2626668	0.042513	0.041068
206	53	10918	2249108	0.033629	0.030773
214	12	2568	549552	0.007614	0.011671
222	6	1332	295704	0.003807	0.002241
1386	197	39126	7789204		

$\bar{x}$ = 198.6091 s_x = 9.695 071

x	$f/(Nw)$	$f(x)$
170	0	0.000529
170	0.003807	0.000529
178	0.003807	0.004297
178	0.005711	0.004297
186	0.005711	0.017663
186	0.027919	0.017663
194	0.027919	0.036752
194	0.042513	0.036752
202	0.042513	0.038708
202	0.033629	0.038708
210	0.033629	0.020635
210	0.007614	0.020635
218	0.007614	0.005568
218	0.003807	0.005568
226	0.003807	0.00076
226	0	0.00076

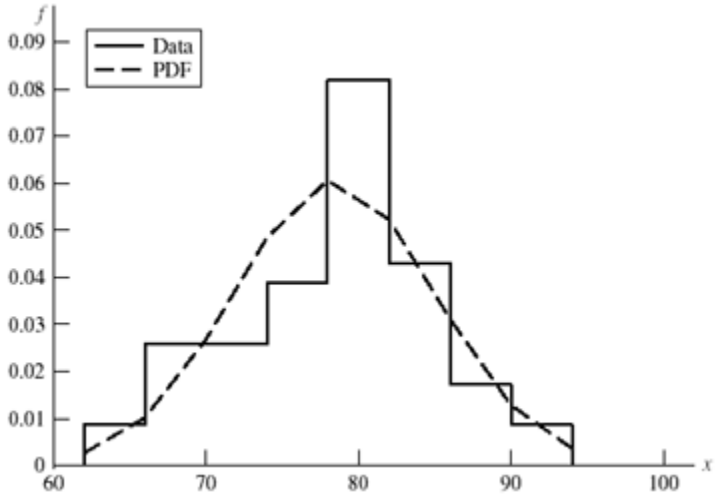


20-22

x	f	fx	fx^2	$f/(Nw)$	$f(x)$
64	2	128	8192	0.008621	0.00548
68	6	408	27744	0.025862	0.017299
72	6	432	31104	0.025862	0.037705
76	9	684	51984	0.038793	0.056742
80	19	1520	121600	0.081897	0.058959
84	10	840	70560	0.043103	0.042298
88	4	352	30976	0.017241	0.020952
92	2	184	16928	0.008621	0.007165
624	58	4548	359088		

$\bar{x} = 78.041379$ $s_x = 6.572\ 229$

x	$f/(Nw)$	$f(x)$
62	0	0.002684
62	0.008621	0.002684
66	0.008621	0.010197
66	0.025862	0.010197
70	0.025862	0.026749
70	0.025862	0.026749
74	0.025862	0.048446
74	0.038793	0.048446
78	0.038793	0.060581
78	0.0381897	0.060581
82	0.081897	0.052305
82	0.043103	0.052305
86	0.043103	0.03118
86	0.017241	0.03118
90	0.017241	0.012833
90	0.008621	0.012833
94	0.008621	0.003647
94	0	0.003647



20-23

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(40)}{\pi\left(1^2\right)} = 50.93 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(8.5)}{\pi\left(1^2\right)} = 10.82 \text{ kpsi}$$

$$\hat{\sigma}_{S_y} = 5.9 \text{ kpsi}$$

For no yield, $m = S_y - \sigma \geq 0$

$$z = \frac{m - \mu_m}{\hat{\sigma}_m} = \frac{0 - \mu_m}{\hat{\sigma}_m} = -\frac{\mu_m}{\hat{\sigma}_m}$$

$$\mu_m = \bar{S}_y - \bar{\sigma} = 78.4 - 50.93 = 27.47 \text{ kpsi}$$

$$\hat{\sigma}_m = \left(\hat{\sigma}_\sigma^2 + \sigma_{S_y}^2\right)^{1/2} = \left(10.82^2 + 5.9^2\right)^2 = 12.32 \text{ kpsi}$$

$$z = -\frac{\mu_m}{\hat{\sigma}_m} = -\frac{27.47}{12.32} = -2.230$$

Table A-10, $p_f = 0.0129$

$R = 1 - p_f = 1 - 0.0129 = 0.987$ *Ans.*

20-24 For a lognormal distribution,

$$\text{Eq. (20-18)} \quad \mu_y = \ln \mu_x - \ln \sqrt{1 + C_x^2}$$

$$\text{Eq. (20-19)} \quad \hat{\sigma}_y = \sqrt{\ln\left(1 + C_x^2\right)}$$

From Prob. (20-23)

$$\mu_m = \bar{S}_y - \bar{\sigma} = \mu_x$$

$$\mu_y = \left(\ln \bar{S}_y - \ln \sqrt{1 + C_{S_y}^2}\right) - \left(\ln \bar{\sigma} - \ln \sqrt{1 + C_\sigma^2}\right)$$

$$= \ln \left[\frac{\bar{S}_y}{\bar{\sigma}} \sqrt{\frac{1 + C_\sigma^2}{1 + C_{S_y}^2}} \right]$$

$$\hat{\sigma}_y = \left[\ln\left(1 + C_{S_y}^2\right) + \ln\left(1 + C_\sigma^2\right) \right]^{1/2}$$

$$= \sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}$$

$$z = -\frac{\mu}{\hat{\sigma}} = -\frac{\ln\left(\dfrac{\bar{S}_y}{\bar{\sigma}} \sqrt{\dfrac{1 + C_\sigma^2}{1 + C_{S_y}^2}} \right)}{\sqrt{\ln\left[\left(1 + C_{S_y}^2\right)\left(1 + C_\sigma^2\right)\right]}}$$

$$\bar{\sigma} = \frac{4\bar{P}}{\pi d^2} = \frac{4(30)}{\pi\left(1^2\right)} = 38.197 \text{ kpsi}$$

$$\hat{\sigma}_\sigma = \frac{4\hat{\sigma}_P}{\pi d^2} = \frac{4(5.1)}{\pi\left(1^2\right)} = 6.494 \text{ kpsi}$$

$$C_\sigma = \frac{6.494}{38.197} = 0.1700$$

$$C_{S_y} = \frac{3.81}{49.6} = 0.076\ 81$$

$$z = -\frac{\ln\left[\dfrac{49.6}{38.197} \sqrt{\dfrac{1 + 0.170^2}{1 + 0.07681^2}} \right]}{\sqrt{\ln\left[\left(1 + 0.076\ 81^2\right)\left(1 + 0.170^2\right)\right]}} = -1.470$$

Table A-10

$$p_f = 0.0708$$

$$R = 1 - p_f = 0.929 \quad \textit{Ans.}$$

20-25

x	n	nx	nx^2
93	19	1767	164 311
95	25	2375	225 625
97	38	3686	357 542
99	17	1683	166 617
101	12	1212	122 412
103	10	1030	106 090
105	5	525	55 125
107	4	428	45 796
109	4	436	47 524
111	2	222	24 642
	136	13 364	1 315 704

$$\bar{x} = 13\ 364/136 = 98.26 \text{ kpsi}$$

$$s_x = \left(\frac{1\ 315\ 704 - 13\ 364^2\ /136}{136-1} \right)^{1/2} = 4.30 \text{ kpsi}$$

Under normal hypothesis,

$$z_{0.01} = \left(x_{0.01} - 98.26 \right) / 4.30$$

$$x_{0.01} = 98.26 + 4.30 z_{0.01}$$

$$= 98.26 + 4.30\left(-2.3267\right)$$

$$= 88.26 \doteq 88.3 \text{ kpsi} \quad \textit{Ans.}$$

20-26 From Prob. 20.25, $\mu_x = 98.26$ kpsi, and $\hat{\sigma}_x = 4.30$ kpsi.

$$C_x = \hat{\sigma}_x / \mu_x = 4.30 / 98.26 = 0.043\ 76$$

From Eqs. (20-18) and (20-19),

$$\mu_y = \ln\left(98.26\right) - 0.043\ 76^2 / 2 = 4.587$$

$$\hat{\sigma}_y = \sqrt{\ln\left(1 + 0.043\ 76^2\right)} = 0.043\ 74$$

For a yield strength exceeded by 99% of the population,

$$z_{0.01} = \left(\ln x_{0.01} - \mu_y \right) / \hat{\sigma}_y \quad \Rightarrow \quad \ln x_{0.01} = \mu_y + \hat{\sigma}_y z_{0.01}$$

From Table A-10, for 1% failure, $z_{0.01} = -2.326$. Thus,

$$\ln x_{0.01} = 4.587 + 0.043\ 74(-2.326) = 4.485$$

$$x_{0.01} = 88.7 \text{ kpsi} \quad \textit{Ans.}$$

The normal PDF is given by Eq. (20-14) as

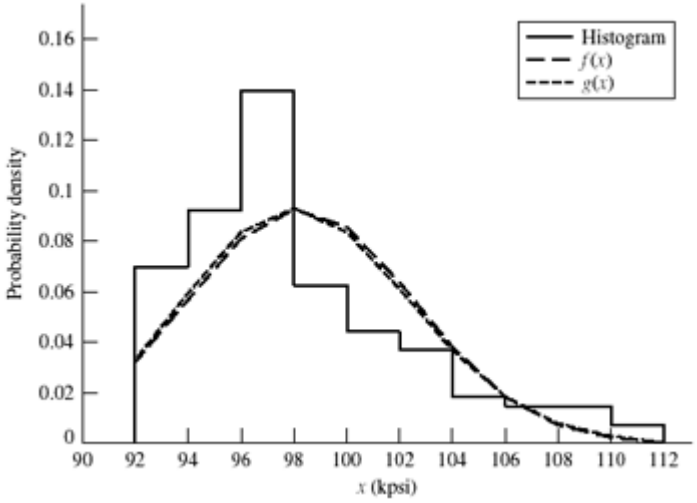
$$f(x) = \frac{1}{4.30\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - 98.26}{4.30} \right)^2 \right]$$

For the lognormal distribution, from Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.043\ 74)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 4.587}{0.043\ 74} \right)^2 \right]$$

x(kpsi)	$f/(Nw)$	$f(x)$	$g(x)$	x(kpsi)	$f/(Nw)$	$f(x)$	$g(x)$
92	0.000 00	0.032 15	0.032 63	102	0.036 76	0.063 56	0.061 34
92	0.069 85	0.032 15	0.032 63	104	0.036 76	0.038 06	0.037 08
94	0.069 85	0.056 80	0.058 90	104	0.018 38	0.038 06	0.037 08
94	0.091 91	0.056 80	0.058 90	106	0.018 38	0.018 36	0.018 69
96	0.091 91	0.080 81	0.083 08	106	0.014 71	0.018 36	0.018 69
96	0.139 71	0.080 81	0.083 08	108	0.014 71	0.007 13	0.007 93
98	0.139 71	0.092 61	0.092 97	108	0.014 71	0.007 13	0.007 93
98	0.062 50	0.092 61	0.092 97	110	0.014 71	0.002 23	0.002 86
100	0.062 50	0.085 48	0.083 67	110	0.007 35	0.002 23	0.002 86
100	0.044 12	0.085 48	0.083 67	112	0.007 35	0.000 56	0.000 89
102	0.044 12	0.063 56	0.061 34	112	0.000 00	0.000 56	0.000 89

Note: rows are repeated to draw histogram



The normal and lognormal are almost the same. However, the data is quite skewed and perhaps a Weibull distribution should be explored. For a method of establishing the

Weibull parameters see Shigley, J. E., and C. R. Mishke, *Mechanical Engineering Design*, McGraw-Hill, 5th ed., 1989, Sec. 4-12.

20-27 $\mathbf{x} = \left(\mathbf{S}'_{fe}\right)_{10^4}$ $x_0 = 79$ kpsi, $\theta = 86.2$ kpsi, $b = 2.6$

Eq. (20-28):

$$\begin{aligned}\overline{x} &= x_0 + (\theta - x_0)\Gamma(1 + 1/b) \\ &= 79 + (86.2 - 79)\Gamma(1 + 1/2.6) \\ &= 79 + 7.2\Gamma(1.38)\end{aligned}$$

From Table A-34, $\Gamma(1.38) = 0.888\ 54$

$$\overline{x} = 79 + 7.2(0.888\ 54) = 85.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-29)

$$\begin{aligned}\hat{\sigma}_x &= (\theta - x_0)\left[\Gamma(1 + 2/b) - \Gamma^2(1 + 1/b)\right]^{1/2} \\ &= (86.2 - 79)\left[\Gamma(1 + 2/2.6) - \Gamma^2(1 + 1/2.6)\right]^{1/2} \\ &= 7.2\left[0.923\ 76 - 0.888\ 54^2\right]^{1/2} \\ &= 2.64 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$C_x = \frac{\hat{\sigma}_x}{\overline{x}} = \frac{2.64}{85.4} = 0.031 \quad \textit{Ans.}$$

20-28 $\mathbf{x} = \mathbf{S}_{ut}$ $x_0 = 27.7$ kpsi, $\theta = 46.2$ kpsi, $b = 4.38$

$$\begin{aligned}\mu_x &= 27.7 + (46.2 - 27.7)\Gamma(1 + 1/4.38) \\ &= 27.7 + 18.5\Gamma(1.23) \\ &= 27.7 + 18.5(0.910\ 75) \\ &= 44.55 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$\begin{aligned}\hat{\sigma}_x &= (46.2 - 27.7)\left[\Gamma(1 + 2/4.38) - \Gamma^2(1 + 1/4.38)\right]^{1/2} \\ &= 18.5\left[\Gamma(1.46) - \Gamma^2(1.23)\right]^{1/2} \\ &= 18.5\left[0.8856 - 0.920\ 75^2\right]^{1/2} \\ &= 4.38 \text{ kpsi} \quad \textit{Ans.}\end{aligned}$$

$$C_x = \frac{4.38}{44.55} = 0.098 \quad \textit{Ans.}$$

From the Weibull survival equation

$$R = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right] = 1 - p$$

$$R_{40} = \exp\left[-\left(\frac{x_{40} - x_0}{\theta - x_0}\right)^b\right] = 1 - p_{40}$$

$$= \exp\left[-\left(\frac{40 - 27.7}{46.2 - 27.7}\right)^{4.38}\right] = 0.846$$

$$p_{40} = 1 - R_{40} = 1 - 0.846 = 0.154 = 15.4\% \quad \textit{Ans.}$$

20-29 $\mathbf{x} = \mathbf{S}_{ut}$, $x_0 = 151.9$ kpsi, $\theta = 193.6$ kpsi, $b = 8$

$$\mu_x = 151.9 + (193.6 - 151.9)\Gamma(1 + 1/8)$$

$$= 151.9 + 41.7\Gamma(1.125)$$

$$= 151.9 + 41.7(0.941\ 76)$$

$$= 191.2 \text{ kpsi} \quad \textit{Ans.}$$

$$\hat{\sigma}_x = (193.6 - 151.9)\left[\Gamma(1 + 2/8) - \Gamma^2(1 + 1/8)\right]^{1/2}$$

$$= 41.7\left[\Gamma(1.25) - \Gamma^2(1.125)\right]^{1/2}$$

$$= 41.7\left[0.906\ 40 - 0.941\ 76^2\right]^{1/2}$$

$$= 5.82 \text{ kpsi} \quad \textit{Ans.}$$

$$C_x = \frac{5.82}{191.2} = 0.030$$

20-30 $\mathbf{x} = \mathbf{S}_{ut}$, $x_0 = 47.6$ kpsi, $\theta = 125.6$ kpsi, $b = 11.4$

$$\overline{x} = 47.6 + (125.6 - 47.6)\Gamma(1 + 1/11.84)$$

$$= 47.6 + 78\Gamma(1.08)$$

$$= 47.6 + 78(0.959\ 73) = 122.5 \text{ kpsi}$$

$$\hat{\sigma}_x = (125.6 - 47.6)\left[\Gamma(1 + 2/11.84) - \Gamma^2(1 + 1/11.84)\right]^{1/2}$$

$$= 78\left[\Gamma(1.08) - \Gamma^2(1.17)\right]^{1/2}$$

$$= 78\left[0.959\ 73 - 0.936\ 70^2\right]^{1/2} = 22.4 \text{ kpsi}$$

From Prob. 20-28,

$$p = 1 - \exp\left[-\left(\frac{x - x_0}{\theta - \theta_0}\right)^b\right] = 1 - \exp\left[-\left(\frac{100 - 47.6}{125.6 - 47.6}\right)^{11.84}\right] = 0.0090 \quad \textit{Ans.}$$

$\mathbf{y} = \mathbf{S}_y$, $y_0 = 64.1$ kpsi, $\theta = 81.0$ kpsi, $b = 3.77$

$$\bar{y} = 64.1 + (81.0 - 64.1)\Gamma\left(1 + 1/3.77\right)$$
$$= 64.1 + 16.9\,\Gamma(1.27)$$
$$= 64.1 + 16.9(0.902\ 50) = 79.35 \text{ kpsi}$$
$$\sigma_y = (81 - 64.1)\left[\Gamma(1 + 2/3.77) - \Gamma\left(1 + 1/3.77\right)\right]^{1/2}$$
$$= 16.9\left[\left(0.887\ 57\right) - 0.902\ 50^2\right]^{1/2} = 4.57 \text{ kpsi}$$
$$p = 1 - \exp\left[-\left(\frac{y - y_0}{\theta - y_0}\right)^{3.77}\right]$$
$$= 1 - \exp\left[-\left(\frac{70 - 64.1}{81 - 64.1}\right)^{3.77}\right] = 0.019 \quad \textit{Ans.}$$

20-31 $\mathbf{x} = \mathbf{S}_{ut} = \mathbf{W}$[122.3, 134.6, 3.64] kpsi, $p(x > 120) = 1 = 100\%$ since $x_0 > 120$ kpsi

$$p(x > 133) = \exp\left[-\left(\frac{133 - 122.3}{134.6 - 122.3}\right)^{3.64}\right]$$
$$= 0.548 = 54.8\% \quad \textit{Ans.}$$

20-32 Using Eqs. (20-28) and (20-29) and Table A-34,

$$\mu_n = n_0 + \left(\theta - n_0\right)\Gamma\left(1 + 1/b\right) = 36.9 + \left(133.6 - 36.9\right)\Gamma\left(1 + 1/2.66\right)$$
$$= 122.85 \text{ kcycles}$$
$$\hat{\sigma}_n = \left(\theta - n_0\right)\left[\Gamma\left(1 + 2/b\right) - \Gamma^2\left(1 + 1/b\right)\right] = 34.79 \text{ kcycles}$$

For the Weibull density function, Eq. (20-27),

$$f_W(n) = \frac{2.66}{133.6 - 36.9}\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66-1} \exp\left[-\left(\frac{n - 36.9}{133.6 - 36.9}\right)^{2.66}\right]$$

For the lognormal distribution, Eqs. (20-18) and (20-19) give,

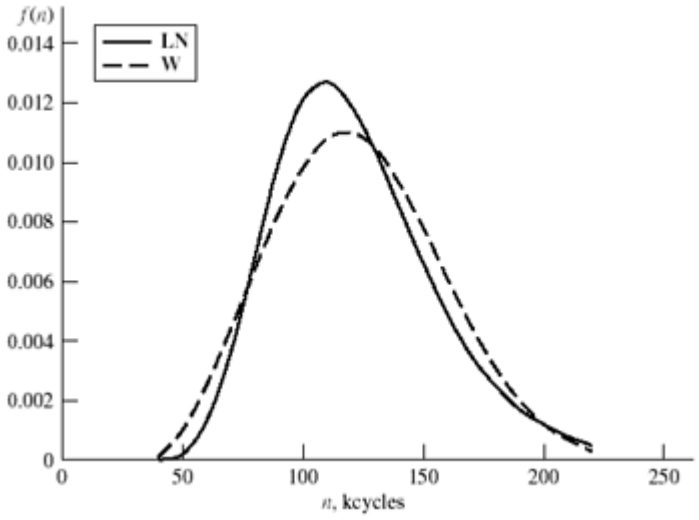
$$\mu_y = \ln\left(122.85\right) - \left(34.79/122.85\right)^2/2 = 4.771$$
$$\hat{\sigma}_y = \sqrt{\left[1 + \left(34.79/122.85\right)^2\right]} = 0.2778$$

From Eq. (20-17), the lognormal PDF is

$$f_{\text{LN}}(n) = \frac{1}{0.2778 n\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln n - 4.771}{0.2778} \right)^2 \right]$$

We form a table of densities $f_{\text{W}}(n)$ and $f_{\text{LN}}(n)$ and plot.

n(kcycles)	$f_{\text{W}}(n)$	$f_{\text{LN}}(n)$
40	9.1E-05	1.82E-05
50	0.000 991	0.000 241
60	0.002 498	0.001 233
70	0.004 380	0.003 501
80	0.006 401	0.006 739
90	0.008 301	0.009 913
100	0.009 822	0.012 022
110	0.010 750	0.012 644
120	0.010 965	0.011 947
130	0.010 459	0.010 399
140	0.009 346	0.008 492
150	0.007 827	0.006 597
160	0.006 139	0.004 926
170	0.004 507	0.003 564
180	0.003 092	0.002 515
190	0.001 979	0.001 739
200	0.001 180	0.001 184
210	0.000 654	0.000 795
220	0.000 336	0.000 529



The Weibull L10 life comes from Eq. (20-26) with reliability of $R = 0.90$. Thus,

$$n_{0.10} = 36.9 + (133.6 - 36.9)\left[\ln(1/0.90)\right]^{1/2.66} = 78.4 \text{ kcycles} \quad \textit{Ans.}$$

The lognormal L10 life comes from the definition of the z variable. That is,

$$\ln n_0 = \mu_y + \hat{\sigma}_y z \quad \text{or} \quad n_0 = \exp\left(\mu_y + \hat{\sigma}_y z\right)$$

From Table A-10, for $R = 0.90$, $z = -1.282$. Thus,

$$n_0 = \exp\left[4.771 + 0.2778\left(-1.282\right)\right] = 82.7 \text{ kcycles} \quad \textit{Ans.}$$

20-33 Form a table

i	x $(10^{-5})L$	f_i	$f_i\,x\cdot(10^{-5})$	$f_i\,x^2\cdot(10^{-10})$	$g(x)\cdot(10^5)$
1	3.05	3	9.15	27.9075	0.0557
2	3.55	7	24.85	88.2175	0.1474
3	4.05	11	44.55	180.4275	0.2514
4	4.55	16	72.80	331.24	0.3168
5	5.05	21	106.05	535.5525	0.3216
6	5.55	13	72.15	400.4325	0.2789
7	6.05	13	78.65	475.8325	0.2151
8	6.55	6	39.30	257.415	0.1517
9	7.05	2	14.10	99.405	0.1000
10	7.55	0	0	0	0.0625
11	8.05	4	32.20	259.21	0.0375
12	8.55	3	25.65	219.3075	0.0218
13	9.05	0	0	0	0.0124
14	9.55	0	0	0	0.0069
15	10.05	1	10.05	101.0025	0.0038
		100	529.50	2975.95	

$$\bar{x} = 529.5\left(10^5\right)/100 = 5.295\left(10^5\right) \text{ cycles} \quad \textit{Ans.}$$

$$s_x = \left[\frac{2975.95\left(10^{10}\right) - \left[529.5\left(10^5\right)\right]^2/100}{100-1}\right]^{1/2}$$

$$= 1.319\left(10^5\right) \text{ cycles} \quad \textit{Ans.}$$

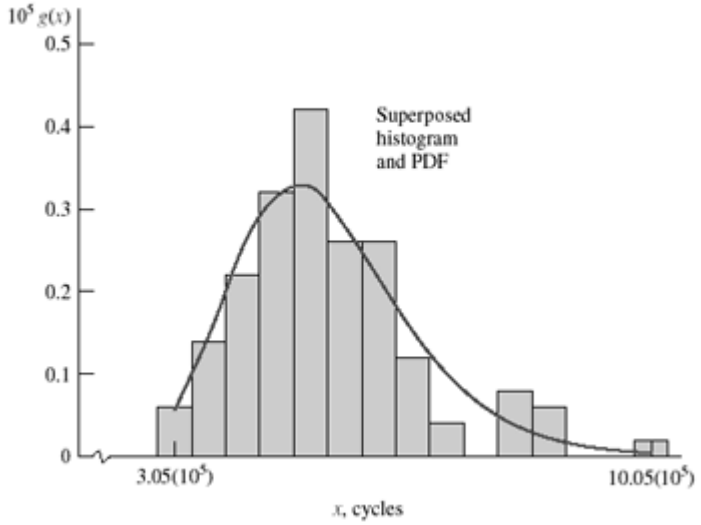
$$C_x = s/\bar{x} = 1.319/5.295 = 0.249$$

$$\mu_y = \ln 5.295\left(10^5\right) - 0.249^2/2 = 13.149$$

$$\hat{\sigma}_y = \sqrt{\ln\left(1+0.249^2\right)} = 0.245$$

$$g(x) = \frac{1}{x\hat{\sigma}_y\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu_y}{\hat{\sigma}_y}\right)^2\right]$$

$$= \frac{1.628}{x}\exp\left[-\frac{1}{2}\left(\frac{\ln x - 13.149}{0.245}\right)^2\right]$$



20-34 $\mathbf{X} = \mathbf{S}_u = \mathbf{W}[70.3, 84.4, 2.01]$

$$\mu_x = 70.3 + \left(84.4 - 70.3\right)\Gamma\left(1 + 1/2.01\right)$$

Eq. (2-28): $$= 70.3 + (84.4 - 70.3)\Gamma\left(1.498\right)$$

$$= 82.8\,\text{kpsi} \quad \textit{Ans.}$$

Eq. (2-29):

$$\hat{\sigma}_x = (84.4 - 70.3)\left[\Gamma(1+2/2.01) - \Gamma^2(1+1/2.01)\right]^{1/2}$$

$$\hat{\sigma}_x = 14.1\left[0.997\ 91 - 0.886\ 17^2\right]^{1/2}$$

$$= 6.502 \text{ kpsi}$$

$$C_x = \frac{6.502}{82.8} = 0.079 \quad \textit{Ans.}$$

20-35 Take the Weibull equation for the standard deviation

$$\hat{\sigma}_x = (\theta - x_0)\left[\Gamma(1+2/b) - \Gamma^2(1+1/b)\right]^{1/2}$$

and the mean equation solved for $\bar{x} - x_0$

$$\bar{x} - x_0 = (\theta - x_0)\Gamma(1+1/b)$$

and divide the first by the second,

$$\frac{\hat{\sigma}_x}{\bar{x} - x_0} = \frac{\left[\Gamma(1+2/b) - \Gamma^2(1+1/b)\right]^{1/2}}{\Gamma(1+1/b)}$$

$$\frac{4.2}{49 - 33.8} = \sqrt{\frac{\Gamma(1+2/b)}{\Gamma^2(1+1/b)} - 1} = \sqrt{R} = 0.2763$$

Make a table and solve for *b* iteratively

b	$1 + 2/b$	$1 + 1/b$	$\Gamma(1+2/b)$	$\Gamma(1+1/b)$	$\sqrt{R}$
3	1.67	1.33	0.903 30	0.893 38	0.363
4	1.5	1.25	0.886 23	0.906 40	0.280
4.1	1.49	1.24	0.885 95	0.908 52	0.271

$b \doteq 4.068$ Using MathCad *Ans.*

$$\theta = x_0 + \frac{\bar{x} - x_0}{\Gamma(1+1/b)} = 33.8 + \frac{49 - 33.8}{\Gamma(1+1/4.068)}$$

$$= 49.8 \text{ kpsi} \quad \textit{Ans.}$$

20-36 $\mathbf{x} = \mathbf{S}_y = \mathbf{W}[34.7, 39, 2.93]$ kpsi

$$\bar{x} = 34.7 + (39 - 34.7)\Gamma(1 + 1/2.93) = 34.7 + 4.3\Gamma(1.34)$$
$$= 34.7 + 4.3(0.892\ 22) = 38.5 \text{ kpsi}$$
$$\hat{\sigma}_x = (39 - 34.7)\left[\Gamma(1 + 2/2.93) - \Gamma^2(1 + 1/2.93)\right]^{1/2}$$
$$= 4.3\left[\Gamma(1.68) - \Gamma^2(1.34)\right]^{1/2}$$
$$= 4.3\left[0.905\ 00 - 0.892\ 22^2\right]^{1/2} = 1.42 \text{ kpsi} \quad \textit{Ans.}$$
$$C_x = 1.42/38.5 = 0.037 \quad \textit{Ans.}$$

20-37

	x (Mrev)	f	fx	fx^2
	1	11	11	11
	2	22	44	88
	3	38	114	342
	4	57	228	912
	5	31	155	775
	6	19	114	684
	7	15	105	735
	8	12	96	768
	9	11	99	891
	10	9	90	900
	11	7	77	847
	12	5	60	720
Sum	78	237	1193	7673

$$\mu_x = 1193(10^6)/237 = 5.034(10^6) \text{ cycles}$$
$$\hat{\sigma}_x = \sqrt{\frac{7673(10^{12}) - \left[1193(10^6)\right]^2/237}{237 - 1}} = 2.658(10^6) \text{ cycles}$$
$$C_x = 2.658/5.034 = 0.528$$

From Eqs. (20-18) and (20-19),

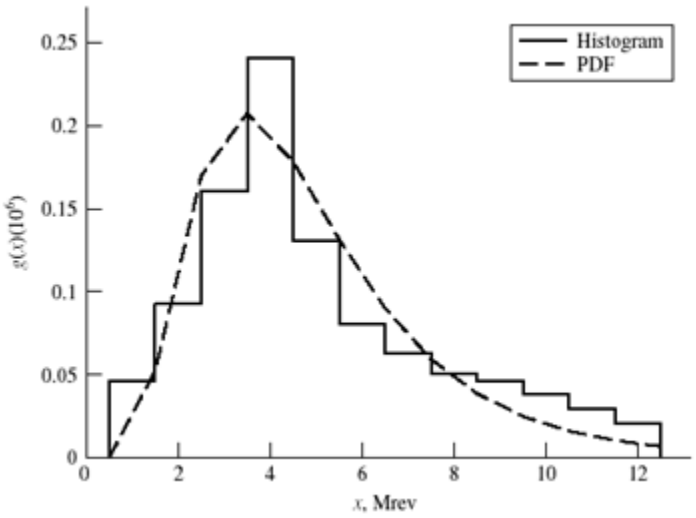
$$\mu_y = \ln\left[5.034(10^6)\right] - 0.528^2/2 = 15.292$$
$$\hat{\sigma}_y = \sqrt{\ln(1 + 0.528^2)} = 0.496$$

From Eq. (20-17), defining $g(x)$,

$$g(x) = \frac{1}{x(0.496)\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - 15.292}{0.496}\right)^2\right]$$

x (Mrev)	$f/(Nw)$	$g(x)\cdot(10^6)$
0.5	0.000 00	0.000 11

0.5	0.046414	0.000 11
1.5	0.046414	0.052 03
1.5	0.092827	0.052 03
2.5	0.092827	0.169 92
2.5	0.160338	0.169 92
3.5	0.160338	0.207 54
3.5	0.240506	0.207 54
4.5	0.240506	0.178 47
4.5	0.130802	0.178 47
5.5	0.130802	0.131 58
5.5	0.080 17	0.13158
6.5	0.080 17	0.090 11
6.5	0.063 29	0.090 11
7.5	0.063 29	0.059 53
7.5	0.050 63	0.059 53
8.5	0.050 63	0.038 69
8.5	0.046 41	0.038 69
9.5	0.046 41	0.025 01
9.5	0.037 97	0.025 01
10.5	0.037 97	0.016 18
10.5	0.029 54	0.016 18
11.5	0.029 54	0.010 51
11.5	0.021 10	0.010 51
12.5	0.021 10	0.006 87
12.5	0.000 00	0.006 87



$$z = \frac{\ln x - \mu_y}{\hat{\sigma}_y} \quad \Rightarrow \quad \ln x = \mu_y + \hat{\sigma}_y z = 15.292 + 0.496z$$

L_{10} life, where 10% of bearings fail, from Table A-10, $z = -1.282$. Thus,

$$\ln x = 15.292 + 0.496(-1.282) = 14.66$$

$$\therefore \quad x = 2.33\ (10^6)\ \text{rev} \qquad \textit{Ans.}$$