

Mechanical Behaviour of Materials

SOLID MECHANICS AND ITS APPLICATIONS

Volume 191

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Department of Civil Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1

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Dominique François • André Pineau • André Zaoui

Mechanical Behaviour of Materials

Volume II: Fracture Mechanics and Damage



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Prof. Dr. Dominique François
École Centrale de Paris
Paris
France

Prof. André Zaoui
French Académie des Sciences
Paris
France
Academy of Engineering
Paris
France

André Pineau
École des Mines de Paris
Paris Tech
Centre des Matériaux UMR CNRS
Évry Cedex
France
Academy of Engineering
Paris
France

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Foreword

Without repeating all the considerations included in the foreword of Volume I of *Mechanical Behaviour of Materials*, we simply want to stress, in view of the problems still present in the design of components and structures, the importance of a good link between material science and mechanics, more precisely between deformation mechanisms and constitutive equations. We insisted on the need to teach these aspects to graduate students, possibly even to undergraduates and to make engineers aware of this subject. It is with the same objective in mind that we have undertaken writing the present volume.

Volume I of *Mechanical Behaviour of Materials* has given the tools to calculate the stress and strain fields in loaded bodies with different constitutive behaviours and to understand the physical mechanisms at work in each case. This should help students and engineers to reach sound results and to find ways for improving materials. With Volume II we now wish to do the same for the avoidance of failures. We think that this requires a thorough understanding of damage mechanisms. We thus tackle in six chapters brittle and ductile fracture and the transition between the two, fatigue and creep damages, environmental assisted cracking. As this is mostly rooted in the case of metallic materials, we include a seventh chapter to treat cases of non-metallic materials: ceramics, glasses, concrete, polymers, woods, composites. A complete outline on fracture mechanics precedes these seven chapters, because this constitutes an unavoidable tool to study damages and also to predict the life of components and structures. This is then developed for the various kinds of failures in the corresponding chapters. A special case of damage is that due to friction and wear. We felt that we could not avoid dealing with these very detrimental phenomena. A rather different kind of presentation is adopted in the chapter dealing with them, as it was needed first to give results of contact mechanics and we could not expand too much on the mechanisms. The extent of the various chapters is not the same according to the subjects. For instance, in the case of fatigue, object of so many researches and books, we stick to the essential. The chapters on brittle and ductile fracture and on creep are more elaborate, as updating knowledge is required.

As for Volume I, this re-edition is rooted on the Klüwer edition of *Mechanical Behaviour of Materials*, which owes much to the translation of “Comportement

mécanique des matériaux” by Jack Howlett whose work was essential. We have now added a great deal of up-to-date material and we hope that our six-hand playing, notwithstanding our inability to play the piano (there are six-hand scores), has electronically produced (a manuscript would have required three hands only) a readable and original compendium of materials science and mechanics. Hopefully, let students, professors and engineers enjoy it. This volume could not have been completed without the help and precious contributions of Jacques Verdu, Lucien Laiarinandrasana, Michel Boussuge, Marc Blétry and Henry Proudhon. We wish to thank them very warmly. We are also very grateful to Eva Héripré and to René Billardon for beautiful pictures. Contributions received over the years from our colleagues and from students have helped us greatly and gave us the incentive to carry through our project. We are grateful to Prof. Graham Gladwell from University of Waterloo, Canada, for including this volume as the last one in the series of which he is responsible.

We tried to illustrate the book with enough figures. Many are adapted from publications and the authors are acknowledged. We hope nobody was forgotten; if not so, we would welcome any request.

In the previous edition of *Mechanical Behaviour of Materials* (Klüwer 1998) exercises were included. They have disappeared in this revised version, which has expanded. We advise disappointed readers that we are now writing a volume, which will be entirely devoted to exercises. They will illustrate all chapters of both volumes. Furthermore, we would like to produce another book including case studies and what we called long exercises, that is elaborate studies of various problems. We dream to initiate some interactive production. Thus, we would encourage and welcome contributions of any kind.

Publishing a book like this one requires a lot of careful and tedious auxiliary work. We pay tribute in this respect to Joëlle Pineau and to Odile Adam. We also like to thank the staff of Springer who took good care of our work and answered our questions, especially Nathalie Jacobs and Anneke Pot. Readers will appreciate the lay out and we are thankful too for the scrupulous work of the editor.

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Notations

$\tilde{\pi}$	Angular variation of the elastic energy (HRR field)
$\tilde{\varepsilon}$	Angular variation of the strain in the HRR field
$\tilde{\sigma}$	Angular variation of the stress in the HRR field
$\bar{\varepsilon}$	Average strain
$\bar{\sigma}$	Average stress
$\dot{u}_i$	Displacement rate
$\dot{\Delta}$	Displacement rate
$\tilde{\varepsilon}$	Equivalent fracture strain
$\dot{\varepsilon}_{eq}$	Equivalent strain rate
$\dot{D}_c^I$	Intergranular damage per cycle
$\dot{N}$	Nucleation rate of cavities
$\dot{W}$	Rate of energy dissipation
$\dot{\varepsilon}_s$	Secondary creep rate
$\dot{\varepsilon}_{ij}$	Strain rate
ε	Strain tensor
σ	Stress tensor
T_{KIA}	Temperature at which the crack arrest toughness is equal to 100 MPa m ^{1/2}
∂A	Increment of crack area
A	Crack area
a	Crack length
a	Major axis of elliptical hole or ellipsoid
a	Notch depth
a_c	Critical crack length
a_{cor}	Correction to the crack length
a_f	Abscissa of a point on the crack
a_{fat}	Length of the fatigue crack
a_n	Notch length

A_n	Parameter
b	Distance to the centre of a load applied on the crack edge
b	Fatigue strength exponent
b	Interatomic distance
b	Minor axis of elliptical hole or oblate ellipsoid
B	Thickness, crack front length
b	Uncracked ligament length
BCC	Body centred cubic
BDWTT	Battelle drop weight tear test
BM	Base metal
B_n	Parameter
B_N	Remaining thickness of grooved specimen
C	Carbide size
C	Cleavage crack length
C	Compliance
c	Concentration
c	Creep distance around a cavity
c	Depth of a semi-circular crack
c	Distance to the crack tip ($a - b$)
c	Fatigue ductility exponent
C	Slope of the DBT
C^*	Critical carbide size
C^*	Secondary creep parameter (equivalent to J)
C_h^*	Primary creep parameter
C_0	Cementite platelet thickness
CAT	Crack arrest temperature curve
CCP	Centered crack panel
CDD	Continuous distribution of dislocations
CDM	Continuous damage mechanics
CGHAZ	Coarse grain heat affected zone
CMC	Ceramic matrix composites
C_{MG}	Monkman-Grant constant
COD	Crack opening displacement
C_r	Ratio of crystalline surface (in BDWTT)
CSZ	Cubic stabilized zirconia
CT	Compact tension specimen
$CTOD$	Crack tip opening displacement
C_V	Fracture energy
c_v	Limit solubility
C_{VUS}	Upper shelf energy
d	Boundary thickness
D	Damage parameter
d	Density
d	Diameter of a test piece
D	Diffusion coefficient

d	Distance from the crack tip over which are applied distributed forces
d	Grain size
D	Mesh size
D	Overall diameter of a test piece
D^*	Critical grain size
DBT	Ductile-brittle transition
D_c	Creep damage
D_c	Damage per cycle
DCB	Double cantilever beam
DCG	Ductile crack growth correction
D_f	Fatigue damage
D_{GB}	Grain boundary diffusion coefficient
D^i	Intergranular damage parameter
D^i_c	Critical value of the intergranular damage parameter
D_S	Surface diffusion coefficient
D_v	Coefficient of self diffusion
E	Remote strain
E	Young modulus
$E(k)$	Elliptical integral
E_{core}	Core energy of a dislocation
E_d^0	Remote applied strain
E_e	Elastic energy
E_{ledge}	Energy of a ledge
EPFM:	Elasto plastic fracture mechanics
f	Auxiliary force
$\bar{F}$	Force
f	Volume fraction
FA	Fracture appearance
F_a	Load at crack arrest ($= F_{iu}$)
FAD	Fracture analysis diagram
FATT	Fracture-appearance transition temperature
F_b	Intercepted boundary coefficient of a cavity
f_b	Surface fraction of cavities on grain boundary
f_b^c	Critical surface fraction of cavities on grain boundary
f_c	Critical void volume fraction
f'_c	Strength of concrete in compression
FEM	Finite elements method
F_m	Maximum load
FMFL	Frequency modified fatigue life
F_s	Surface coefficient of a cavity
f'_t	Strength of concrete in traction
FTP	Temperature below which the fracture changes from totally ductile to substantially brittle
F_v	Volume coefficient of a cavity
G	Free enthalpy

G	Strain energy release rate
$\underline{g}$	Unit vector perpendicular to the crack face
g	Vacancy potential
G^c	Critical value of the strain energy release rate
G_R	Value of G during crack propagation (R curve)
GTN	Gurson-Tvergaard-Needelman model
H	Height of brittle fracture (in BDWTT)
h	Dimension of damaged zone at crack tip
h	Dimension of transformed zone at crack tip
h	Thickness of an arm of a DCB specimen
HAZ	Heat affected zone
HCP	Hexagonal close packed
HRR	Asymptotic field (from Hutchinson, Rice and Rosengren)
HSLA	High strength low alloy (steel)
ICCGHAZ	Intercritically reheated coarse grain heat affected zone
I_N	Normalizing parameter
J	Rice-Cherepanov contour integral
J	Vacancy flux
J_i	Critical value of J
J_R	Value of J during crack extension
K'	Cyclic stress coefficient
k	Eccentricity parameter of an ellipse ($k^2 = 1 - a^2/b^2$)
k	Notch plastic reinforcement parameter
k	Thermal conductivity
K	Stress intensity factor (SIF)
k	Yield strength in shear
$K(x)$	Weight function
K_{ap}	Crack opening SIF
KC_V	Resilience (J/cm^2)
K^I	Stress concentration factor at inclusion interface
K_I, K_{II}, K_{III}	Stress intensity factors in modes I, II and III
K_{Ia}	Fracture toughness at crack arrest
$K_{Ia}^{c/f}$	Critical SIF for carbide-ferrite barriers
$K_{Ia}^{f/f}$	Critical SIF for ferrite grain boundaries
K_{Ic}	Fracture toughness
K_{Id}	Dynamic fracture toughness
K^P	Stress concentration factor inside inclusion
K_T	Stress concentration factor
K_t	Stress intensity factor due to phase transformation
k'_y	Factor for the cleavage stress ($\sigma_f = k'_y d^{-1/2}$)
k_y	Petch factor
L	% of surface occupied by shear lips (in BDWTT)
L	Distance between the centers of cavities
l	Effective length of the damaged zone
L	Size of dislocation pile-up

L	Span length
l_c	Characteristic distance
L_c	Test section length
LCF	Load-cool-fracture (warm prestress effect)
LCIKF	Load-cool-increasing K -fracture (warm prestress effect)
l_D	Dimension of damaged zone
LEFM:	Linear elastic fracture mechanics
L_f	Cavitated length along grain boundaries per unit area
LMIE	Liquid metal induced embrittlement
LSY	Large scale yielding
L_t	Total length of grain boundaries per unit area
LUCF	Load-unload-cool-fracture (warm prestress effect)
M	Eccentricity parameter of an ellipse
m	Eccentricity parameter of elliptical hole
m	Exponent
m	Shape factor of the Weibull law
M.A	Martensite-austenite
MB	Multiple barriers
M_L	Limit moment
n'	Cyclic strain hardening exponent
n	Exponent
$\underline{n}$	Normal vector
N	Number of crack initiation sites
N	Number of cycles
n	Number of dislocations in a pile-up
N	Strain hardening exponent $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$
n	Work hardening exponent $n = 1/N$ ($\sigma = \sigma_0 \varepsilon^n$)
N^*	Life in creep-fatigue
n_1	Primary creep exponent
n_2	Secondary creep exponent
NDT	Nil ductility temperature
N_f	Fatigue life
N_f	Number of cycles to failure
N_f^{CF}	Lifespan in creep-fatigue
N_i	Number of cycles to crack initiation
N_{ic}	Number of cycles to creep crack initiation
N_{if}	Number of cycles to fatigue crack initiation
N_p	Number of cycles of propagation
N_p^{CF}	Relative reduction of propagation life in creep-fatigue
N_p^{PF}	Propagation life in pure fatigue
$\underline{P}$	Applied force
p	Cumulated plastic strain
p	Distributed force per unit length
p	Particle size
p	Pressure

$p(a)$	Size distribution of cracks
$P(\sigma)$	Failure probability of an volume element
P_1, P_2, P_3	Components of an applied force
P_{GY}	Load at general yield
P_L	Limit load
P_{nucl}	Proportion of broken carbides
P_{prop}	Probability of crack propagation
P_R	Failure probability
PSZ	Partially stabilized zirconia
P_{void}	Probability for a particle to form a void
PWR	Pressurized water reactor
PZ	Plastic zone
q	Distributed force per unit area
Q	Q stress: Correction linked with the non singular stresses at the tip of a crack in EPFM
r	Distance to the crack tip
r	Distance to the head of a dislocation pile-up
R	Plastic zone size
R	Radius
R	Radius in the complex plane
r	Radius of a circle
R	Radius of curvature of a notch
r	Radius of lenticular cavity
R	Radius of void
R	Ratio P_{min}/P_{max} in cyclic loading
R	Reduction in lifespan
R^*	Critical cavity radius
R'_m	Cyclic strength
r_0	Cut-off distance to the crack tip
R6	Method of fracture assessment in EPFM
R_{CL}	Transition parameter for intergranular fracture
R_{cyc}	Cyclic plastic zone size
RKR	Ritchie, Knott and Rice model
R_m	Ultimate tensile strength (UTS)
R'_m	Ultimate tensile strength (UTS) after cyclic hardening
R'_p	Strain hardened flow strength
R_p	Yield strength
RR	Riedel and Rice asymptotic fields
RT	Model of Rice and Tracey
r_T	Stress triaxiality ratio
R_{vp}	Size of the viscoplastic zone
R_ε	Strain R ratio
R_σ	Stress R ratio
S	Area
S	Axial stress

S	Span
s	Standard deviation
SIF	Stress intensity factor
SMIE	Solid metal induced embrittlement
SRP	Strain range partitioning
SSY	Small scale yielding
SZ	Segregated zone
T	“ T stress”: second term in the development of the stresses at the crack tip
T	Stress triaxiality ratio
$\underline{t}$	Stress vector
T	Temperature
t	Thickness
t	Time
T	Torque
T	Transverse stress
T_0	Temperature in the middle of the DBT
t_1	Transition time between small scale and large scale primary creep
t_2	Transition time between small scale and large scale secondary creep
TBC	Thermal barrier coating
TEM	Transmission electron microscope
t_f	Time to fracture
T_{Fa4KN}	Temperature at which the crack arrest load is equal to 4KN
T_g	Glass transition temperature
T_{GY}	Temperature below which the fracture is macroscopically (or <i>mechanically</i>) brittle
T_i	Temperature above which the fracture is 100 percent ductile
T_K	Temperature at a given level on the DBT
t_{LL}	Ligament failure time
t_N	Nucleation time of cavities
TPZ	Tetragonal zirconia polycrystal
t_{tr}	Transition time between J dominated and C^* dominated fields in creeping materials
U	Biharmonic Airy’s stress function
U	Bonding energy
$\underline{u}$	Displacement vector
U	Potential energy of applied force
U	Ratio $\Delta K_{eff}/\Delta K$
U^*_L	Energy of nucleation of a dislocation loop
U_L	Energy of a dislocation loop
U_p	Plastic deformation energy
USE	Upper shelf energy
UTS	Ultimate tensile strength
$\underline{v}$	Displacement
V	Total volume fraction of cavities

V	Volume
v	Volume of a cavity
V_0	Reference volume (Weibull law)
V_p	Plastic part of the displacement measured at a distance z from the cracked face
V_t	Volume fraction of transformed phase
w	Specimen width
W	Width of test piece
W	Work of crack closure
$w(z)$	Analytical function
WAXS	Wide angle X-ray scattering
W_k	Kinetic energy
WPS	Warm prestressing
X_c	Critical distance for cleavage initiation
Y_N	Shape parameter
z	Complex variable ($z = x + iy = \rho e^{i\theta}$)
ZTC	Zirconia toughened ceramic
α	Angle of slip line in a Charpy V pest piece
β	Brittleness number
Γ	Contour of integration of J integral
Γ_b	Atomic concentration of impurities in the grain boundary
γ_b	Grain boundary energy
γ^{bas}	Strain on the basal plane of HCP
γ_c	Fracture energy
Γ_s	Atomic concentration of impurities on the surface
γ_s	Surface energy
γ_s^{int}	Intergranular energy
γ_{us}	Peierls energy
δ	Crack tip opening displacement (= $CTOD$)
Δ	Ratio $D_{GB}\delta_{GB}/D_s\delta_s$
ΔE_f	Formation energy of a vacancy
ΔG^*	Activation energy for the nucleation of a cavity
ΔG_b	Gibbs free energy of segregation to a grain boundary
δ_{GB}	Grain boundary diffusion thickness
δ_s	Surface diffusion thickness
$\Delta \varepsilon_p$	Plastic strain range
$\Delta \varepsilon_{tot}$	Total strain range
$\Delta \sigma_\phi$	Increase of the yield strength by irradiation
ε_0	Parameter of the constitutive law $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$
ε_{eq}	Equivalent strain
ε_f	Fatigue ductility coefficient
ε_f	Fracture strain (ductility)
ε_{ij}	Strain tensor
ε_{p1}	Plastic strain in the direction of the maximum principal stress
ε_{eq}^p	Equivalent plastic strain

ζ	Complex variable
η	Shape parameter
θ	Notch flank angle
θ	Polar angle at the crack tip
θ	Temperature (in °C)
θ_b	Fraction of boundary occupied by foreign atoms
κ	Stress state parameter
λ	Aspect ratio of ellipsoid
λ	Deflection of a beam
λ	Emission rate of vacancies
Λ	Vacancy emission distance around a cavity
Λ_0	Void distribution parameter
λ_n	Exponent
μ	Chemical potential (Gibbs free energy per atom)
μ	Shear modulus
ν	Frequency
ν	Poisson ratio
π	Elastic energy
Π	Stored potential energy
ρ	Distance from the center of a cavity
ρ	Notch root radius
ρ	Polar coordinate
ρ	Radius of curvature of the lines of principal stress
ρ	Ratio of the principal strains
ρ_c	Radius of curvature at the extremity of a notch
Σ	Macroscopic stress
σ	Uni-axial stress
σ_0	Parameter of the constitutive law $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$: yield strength
σ_1	Maximum principal stress
σ_a	Applied stress
σ_c	Cleavage stress
σ_c	Critical stress
σ_c	Theoretical cleavage stress
σ_{CI}	Critical stress for intergranular cracking
σ_d	Critical stress for particle cracking
σ_e	Effective stress
σ_{eq}	Equivalent stress
σ_f	Cleavage stress
σ_F	Fatigue limit
σ_f	Fatigue strength coefficient
σ_G	Griffith stress
σ_{ij}	Stress tensor
σ_m	Hydrostatic stress (mean value of the principal stresses)
σ_m	Mean stress in cyclic loading
σ_{max}	Maximum stress in cyclic loading

$\sigma_{\min}$	Minimum stress in cyclic loading
σ_N	Fatigue strength at N cycles
Σ_N	Nominal stress at fracture
σ_n	Normal stress
σ_{nom}	Nominal stress
σ_p	Flow stress
σ_{ref}	Reference stress
σ_{relax}	Relaxed stress
σ_t	Transformation stress
σ_u	Mean stress of the Weibull law
σ_w	Weibull stress
σ_{ww}	Modified Weibull stress
τ	Shear stress
τ_l	Friction stress
Φ	Angular parameter of an ellipse
ϕ	Diameter of the minimum section of a notched specimen
ϕ	Fluence
Ψ	Equilibrium angle of a cavity
Ω	Solid angle
Ω	Volume of a vacancy

Chapter 1

Various Types of Damage

Abstract This chapter is an introduction in which the various kinds of damage are shortly described and classified.

Any component in a structure is subjected to various loadings. They can be forces or deformations, aggressive environment, high or low temperatures. These can be permanent or variable. They can damage components and eventually destroy them. Such events can be costly or even catastrophic. It is then of great importance to be able to predict the occurrence of damages and of their evolution and so to understand their mechanism. Indeed, failures of components and structures are always linked with the presence of defects at various scales. It is interesting to note that this was understood even before the developments of mechanics from the eighteenth century on. Montaigne,¹ in chapter 14 of the second volume of the *Essays*, “As Our Mind Refrains Itself”, writes in 1588: “In the same way who will presuppose an evenly strong string everywhere, it is impossible of all impossibilities that it breaks; for where do you want that the fault starts? And to fracture everywhere together, it is not in nature.”² In this volume, we will but illustrate this statement.

Aggressive environments can produce *corrosions* of various sorts. The ensuing destructions are extremely large and detrimental. However, when acting without being combined with mechanical loadings, they are out of the scope of this volume.

Mechanical loadings can lead to *excessive deformations* of components. They can result from *buckling* or *plastic instability*. These phenomena happen when

¹Michel Eyquem de Montaigne (1533–1592) was a French writer and philosopher.

²Comme notre esprit s’empesche soy-mesmes: “... Pareillement qui présupposera une fisselle également forte partout, il est impossibl  de toute impossibilit  qu’elle rompe; car par o  voulez-vous que la fauc e commence? Et de rompre par tout ensemble, il n’est pas en nature”. A less literal translation could be: “Let us imagine an evenly strong string; it is absolutely impossible that it breaks, because where do you want a defect to be initiated? And breaking everywhere at the same time is not a natural thing.”

Table 1.1 Various types of damage

Fracture type	Volume damage	Mixed damage	Surface damage
Sudden	Cleavage Cavities (Trans or intergranular)		Liquid metal embrittlement (Hg, Cd, Ga)
Delayed	Creep Irradiation embrittlement Impurities embrittlement Hydrogen embrittlement	Creep fatigue	Fatigue Wear Stress corrosion Corrosion fatigue Wear – Fretting-Fatigue

an increasing deformation takes place without an increase of the applied forces. *Ratcheting* is another way to produce excessive deformations. However important for the stability of structures, we will not deal with these types of collapses. Nevertheless, plastic instabilities at various scales need to be considered in the occurrence of damages, so that we will have to account for them in several sections of this book. The basic treatment of plastic instability can be found in chapter 3 of the first volume.

Damage is due to initiation and development of surfaces, *cracks or cavities* at various scales. Their origin lies in microscopic defects of various kinds. They can be distributed within the volume of the material or on its surface only. They can grow and coalesce. In this way *macroscopic cracks* are created. These can propagate more or less slowly and eventually in a catastrophic fashion. The various types of damage, which bring up more or less precise notions in the mind of the reader, are listed and classified in Table 1.1.

The multiplication or the growth of these defects lead to the development of cracks and finally to fracture. We must be able to calculate the loads needed for the various steps in the evolution of damage and cracking, so as to predict when a failure would take place.

The first step, very often the most critical one, is the *initiation of damage*. This requires the production of surface energy, that is the breaking of atomic bonds. This needs a high local stress. In most cases, homogeneous loading is insufficient. Local stress raisers must be present. Thus, we need to relate the local stress and strain fields at their level to the macroscopic mechanical loadings. Chapter 2 of the first volume gives treatments of this kind of problems.

Damage initiations at various locations leads to a distribution of defects within the bulk or on the surface, depending on the mechanism. Lazar' Katchanov and Yurii Rabotnov³ proposed, for the treatment of creep rupture, a method to deal with small, distributed defects within the volume of the material. This is *damage mechanics*.

³Lazar' Markovich Katchanov (1914–1993) and Yurii Nikolaievich Rabotnov (1914–1985) were Russian professors in solid mechanics.

It considers that, owing to the presence of these defects, the effective stress is higher than the applied stress so that the elastic moduli of a damaged material are lowered. The effective stress is then introduced in the usual constitutive equations. Some care is needed in the use of damage mechanics, restricted to the case of distributed defects within the bulk, avoiding its extension outside of the limits of its basic hypotheses. We will give some notions about damage mechanics in dealing with creep rupture and with the fracture of concrete.

The coalescence of distributed defects, or the development of a single one, can lead to the formation of a crack. When cracks are present, strain concentrations exist near their tips. The *propagation of cracks* depends of course on these concentrations. Their calculation is made possible by the theory of *fracture mechanics*. It allows determining of the critical size of cracks resulting in fracture. The next chapter will be devoted to this most important subject.

The *local approach to fracture mechanics* consists in relating the macroscopic critical condition for crack propagation to the microscopic critical conditions near the crack tip for microscopic defects (cleavages or cavities) to nucleate, grow and coalesce. This synthesis of fracture mechanics and microscopic treatments allows better predictions of fracture and understanding of the influence of the microstructures of materials.

The elements of microstructure are not uniformly distributed. Precipitates and inclusions are more or less dispersed within grains, the sizes and orientations of which are not uniform. For this reason *statistical analysis* will be needed in many cases for a sound treatment of damage and fracture. One of the main tools, which we will use, is based on the *weakest link model*. It leads to the statistics of Weibull,⁴ which plays an important role in fracture theories.

To better understand Table 1.1, it is necessary to recall here elements, which are tackled in chapter 1 of the first volume.

Damage of a material results from the development of new surfaces.

On the atomic scale three basic types of damage can be envisaged (Fig. 1.1): cleavage, slip with formation of surface steps, and creation of cavities resulting from diffusion of vacancies.

The term *cleavage* is used only in connection with crystalline materials, but an analogous mechanism, the breaking of bonds normal to the plane of the crack (called mode I in fracture mechanics), is responsible for the *crazing* of *polymers*, and for the fracture of *concrete* and *glass*. This is also the case for *intergranular fractures*, which occur in grain boundaries nearly perpendicular to the crack opening displacement.

Slip is responsible for plastic deformation as was studied in Chap. 3 of Volume I. This can lead to the structural instability of *necking* or *plastic collapse* as mentioned above. Plastic deformation can also lead to the formation of small internal *cavities* in the material, which can grow, coalesce and finally cause fracture. Again, that can take place within the grains *or at the grain boundaries*.

⁴Ernst Hjalmar Wallodi Weibull (1887–1979) was a Swedish engineer.

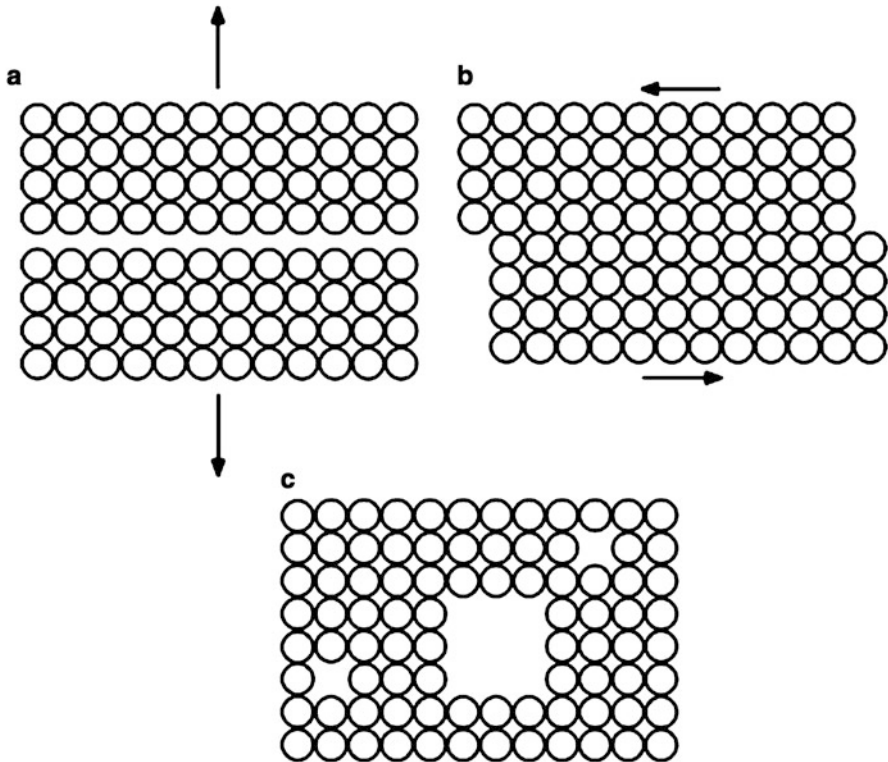


Fig. 1.1 Atomic scale damage: (a) cleavage; (b) slip; (c) appearance of cavities

Under cyclic loading slip is not perfectly reversible: after a time it can cause deteriorations, which are most often taking place at the surface, ending with *fatigue* failure.

Corrosion can interact with slip mechanisms of damage either under static loading, leading to *stress-corrosion*, or cyclic loading, producing *corrosion fatigue*.

A particular kind of aggressive environment is made of liquid metals (Hg, Cd, Ga), which affect the surface energy and can lead to sudden fractures from *liquid metal embrittlement*.

Wear results from mechanical loadings as well in many instances of chemical aggressions.

The last mechanism at the atomic scale shown in Fig. 1.1 can occur only when the temperature is high enough for vacancies to diffuse; it is the dominant effect in *creep* at high temperatures.

The creation of vacancies occurs also when *neutron irradiation* creates a large excess of vacancies over the equilibrium concentration. But *irradiation embrittlement* results also from the number of other defects created in the atomic structure.

The diffusion of impurities, in particular to grain boundaries, reducing their fracture energy, or creating precipitates, can produce *impurities embrittlement*. This results in *intergranular cracking*.

The fast diffusion of hydrogen, faster than the diffusion of other elements, and also the fact that the coalescence of hydrogen atoms produces gas bubbles and not solid precipitates, confer specific characteristics to *hydrogen embrittlement*.

Chapter 2 of this volume will be devoted to fracture mechanics, introducing notions such as the strain energy release rate, stress intensity factor, the crack opening displacement, the energy rate (or J integral) of Rice and Cherepanov.

Chapter 3 will deal with cleavage fracture including a study of the mechanisms at the microscopic scale as well as the ensuing macroscopic fracture conditions. It will also include various embrittlement mechanisms: impurities, hydrogen and irradiation embrittlements.

Chapter 4 will treat ductile fracture mechanisms of cavities nucleation, growth and coalescence.

The preceding developments will allow us, in Chap. 5, to consider the brittle ductile transition, an important aspect of the fracture of carbon steels in particular.

Chapter 6 will be devoted to fatigue, once again envisaged both at the microscopic and at the macroscopic levels. It will treat the initiation of fatigue and the propagation of cracks under cyclic loadings.

Chapter 7 will deal with environment assisted damage. It will include stress corrosion, liquid metal embrittlement and corrosion fatigue.

Creep-fatigue-oxidation phenomena will be studied in Chap. 8.

Chapter 9 will cover friction and wear. Contact mechanics will first be developed in order to understand the phenomena taking place at the interface between rubbing materials

Finally, fracture of non-metallic materials such as glass, ceramics, concrete, polymers and composites will be tackled in Chap. 10.

Chapter 2

Fracture Mechanics

Abstract After an overlook about the aim of fracture mechanics, the strain energy release rate is defined in linear as well as in nonlinear elasticity. Perturbation to the stress field produced by the presence of an elliptical hole allows introducing the notion of stress intensity factor. Stress and strain fields at the tip of a crack are calculated. The relation between the strain energy release rate and the stress intensity factor is demonstrated. The presence of a crack produces displacements, which can be calculated. Various methods exist to determine the stress intensity factor: experimental, superposition, relation with the stress concentration factor, compliance, numerical methods. Consideration is given on three-dimensional cracks.

The plastic zones at the tip of cracks are different in plane stress and in plane strain. Various approximations give their dimensions as well as the crack opening displacement, in small scale yielding and also in large scale yielding. The way to measure fracture toughness is described. The “R” curve effect explains some of the standard procedures. In elastoplastic fracture mechanics, the “R6” method provides an assessment of the safety of a structure, while the measurement of the critical value of J or of the CTOD is explained. Fracture mechanics of creeping solids follows the analysis of Riedel and Rice. It is shown how to determine the time to fracture.

The local approach to fracture mechanics is introduced. It leads to the use of cylindrical notched specimens. The ways to experiment and to analyse the results are presented.

2.1 Aims of Fracture Mechanics

After the First World War, a British engineer, Alan Griffith¹ was working in a laboratory of the Royal Aircraft Establishment on the fracture of glass. Realising that the fracture stress was very much less than the theoretical one because of the

¹Alan Arnold Griffith (1893–1963) was a British engineer.

presence of tiny cracks, he developed an energy criterion for their stability (Griffith 1921). Although he was not the first one to do so, this is considered as the beginning of fracture mechanics. However, fracture mechanics really took off in the USA, at the Naval Research Laboratory, when the large number of fractures of Liberty Ships during World War II required better fracture prevention, and when early missiles exploded during take off. So far design of steel structures was based on shock tests on notched specimens yielding the resilience of the material, which did not allow predicting quantitatively the catastrophic propagation of a crack. George Irwin² was able to develop the needed tool (Irwin 1958): fracture mechanics based on the works of mathematicians, as Muskhelishvili.³ Providing the stress and strain fields near the tip of a crack, linear elastic fracture mechanics (LEFM) enables to predict when this crack will grow.

Later, safety of nuclear pressure vessels and of gas pipelines required an extension of LEFM to situations in which plastic deformation was no longer small. Jim Rice (1968) in the USA and independently Genady Cherepanov (1974) in the Soviet Union proposed to use an integral denoted J , which is assimilated to an energy release rate in elastoplastic fracture mechanics (EPFM). In the mean time, Alan Wells in Great Britain developed the use of the crack opening displacement (COD).

In this chapter, we will study these two “global” approaches: LEFM and EPFM. We will first define the strain energy release rate G in linear elasticity and the J integral, the definition and properties of which are valid in elasticity even when nonlinear. We will define the stress intensity factor K (SIF) and explain its relation with the strain energy release rate. We will show how to determine the stress intensity factor. We will then show the corrections needed owing to the development of limited plasticity at the crack tip. This will allow explaining how to measure the fracture toughness. We will then explain extensions in EPFM. The chapter ends with description of the “local” approach to fracture mechanics, which was developed in the 1980s.

2.2 Linear Elastic Fracture Mechanics (LEFM)

2.2.1 Strain Energy Release Rate

2.2.1.1 Griffith’s Theory of Brittle Fracture

We seek a criterion for the stability of a crack inside a loaded structure. Let Π denote the potential energy stored in the structure containing a crack of area A . Let ∂A be

²George Rankin Irwin (1907–1998) was an American scientist.

³Nikoloz Muskhelishvili (1891–1976) was a Georgian mathematician.

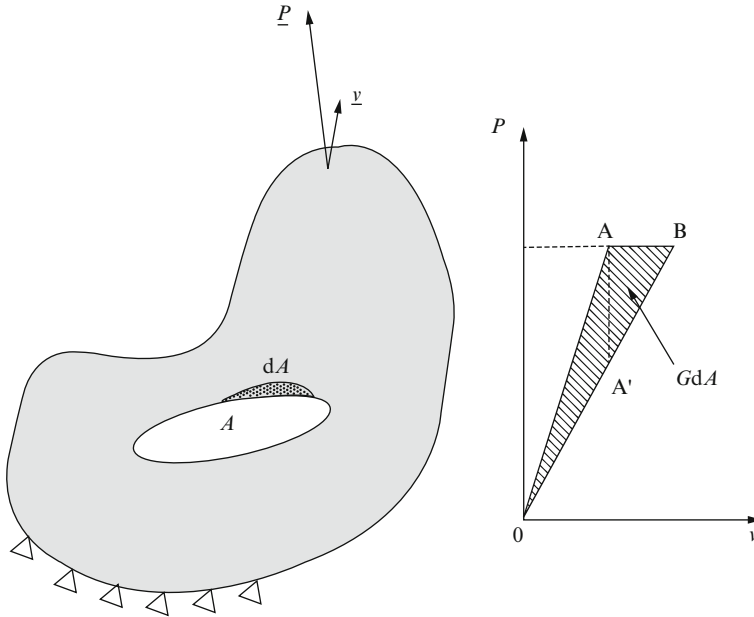


Fig. 2.1 Definition of the strain energy release rate

the increment of area of the crack and G the *crack extension force* or *rate of release of elastic energy*, defined as

$$G = -\frac{\partial \Pi}{\partial A} \quad (2.1)$$

Consider the system of Fig. 2.1. This is a closed system, exchanging no energy of any form with the exterior, the latter consisting of the material containing the crack and the forces acting on it; the material is assumed to be linear elastic.

The energy of this system, which remains constant, consists of the potential energy U of the applied forces, the stored elastic energy π , the surface energy $(S + 2A)\gamma_s$, where S is the area of the external surface and A that of the crack, which has two faces, and possibly a kinetic energy W_k . The energy conservation law gives:

$$d(U + \pi) + d(S + 2A)\gamma_s + dW_k = 0 \quad (2.2)$$

We can assume $dS = 0$, and putting $\Pi = U + \pi$, the total potential energy, we have:

$$d\Pi + 2\gamma_s dA + dW_k = 0 \quad (2.3)$$

that is:

$$G = 2\gamma_s + \frac{dW_k}{dA} \quad (2.4)$$

At equilibrium, without motion, $dW_k = 0$, and therefore:

$$G = G^c = 2\gamma_s \quad (2.5)$$

If G exceeds this critical value G^c the crack will propagate; if it is less, the crack should heal ($dA < 0$ since $dW_k > 0$). This happens sometimes, for instance with polymers if sufficient time is allowed for the migration of molecules. However in general, various irreversible processes, in particular the oxidation of the crack surfaces, prevent healing.

2.2.1.2 Relation Between the Strain Energy Release Rate and the Compliance

The response of the cracked structure is shown diagrammatically in Fig. 2.1b. If P denotes the applied force, u the displacement at the point of application and C the compliance (the inverse of the stiffness) of the structure, considering collinear force and displacement then:

$$u = CP \quad (2.6)$$

G is represented by the area of triangle OAB if a test is made at constant load, or by OAA' if at constant displacement.

Constant displacement

At constant displacement $dU = 0$ and $\pi = (1/2)Pu = (1/2)u^2/C$.

Constant load

At constant load $U = -Pu = -CP^2$ and $\pi = (1/2)Pu = (1/2)CP^2$

It is important to note that in the case of prescribed load the amount of energy stored increases as the crack propagates ($\partial\pi > 0$), but the work done by the external forces compensates for this increase.

In both cases, Eq. 2.1 with the above conditions yields the general relation:

$$G = \frac{1}{2} P^2 \frac{\partial C}{\partial A} \quad (2.7)$$

which, as we shall see later, can be useful in certain cases for determining the stress intensity factor.

2.2.2 The Contour Integral J

2.2.2.1 Definition of the J Integral

We know that in the general framework of elasticity, linear as well as nonlinear, there is an elastic potential π whose derivatives are the stresses: $\sigma_{ij} = \partial\pi/\partial\epsilon_{ij}$ (refer to Chap. 2 of Volume I). In such conditions, it is possible to calculate the amount of elastic energy released during the virtual advance of a crack, under the assumptions that the crack propagates keeping the same direction and that there are no forces acting on its sides. It can be shown that the energy change is given by the value of a certain contour integral, unaffected by the particular contour chosen. This integral, introduced by Rice (1968) and Cherepanov (1974), is:

$$J = \int_{\Gamma} \left(\pi dx_2 - \underline{t} \cdot \frac{\partial \underline{u}}{\partial x_1} ds \right) \quad (2.8)$$

where $\underline{t}$ and $\underline{u}$ are the stress and displacement vectors at a general point M on the contour Γ . The contour is oriented as in Fig. 2.2, with the normal directed outwards.

This integral has important properties demonstrated hereafter.⁴

2.2.2.2 The Integral J Is Contour Independent

To demonstrate this property, two contours Γ_1 and Γ_2 are considered, the last one being oriented in the backward direction (Fig. 2.3). The integral on the closed contours Γ_1 , Γ_2 and the portions of the crack edges between them is transformed in a surface integral. This can be done since all the quantities inside the contour are

⁴Properties demonstrated here in elasticity will be later extended in plasticity. It should be noted that this will require monotonously increasing loading with no unloading.

Fig. 2.2 Definition of the contour integral J

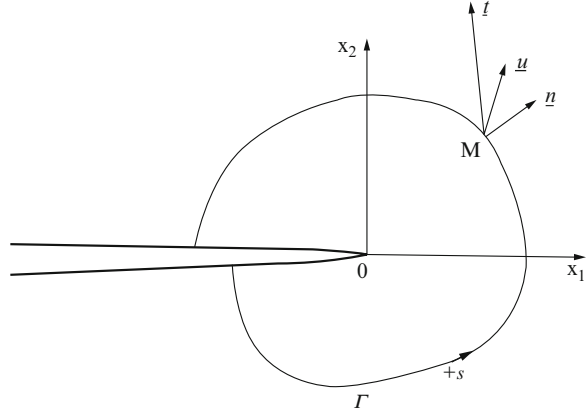
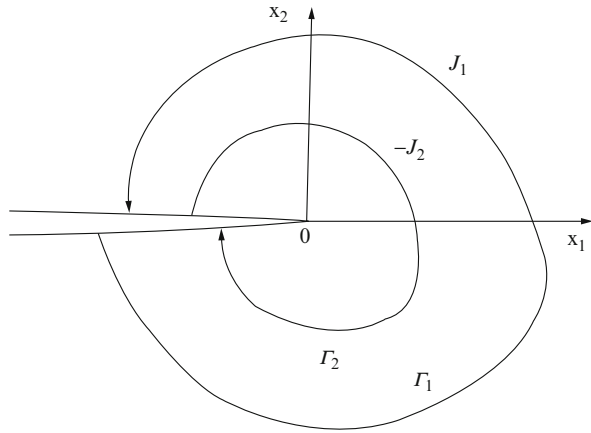


Fig. 2.3 The J integral is contour independent



continuous and can be differentiated. As the crack edges are perpendicular to the Ox_2 axis and free of stresses, the contour integral is equal to $J_1 - J_2$; and, taking account of the equilibrium equations:

$$\begin{aligned}
 J_1 - J_2 &= \oint \left(\pi n_1 - \sigma_{ij} n_j \frac{\partial u_i}{\partial x_1} \right) ds = \iint_S \left(\frac{\partial \pi}{\partial x_1} - \frac{\partial \sigma_{ij}}{\partial x_j} \frac{\partial u_i}{\partial x_1} - \sigma_{ij} \frac{\partial}{\partial x_1} \frac{\partial u_i}{\partial x_j} \right) dS \\
 &= \iint_S \left(\frac{\partial \pi}{\partial x_1} - \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial x_1} \right) dS \quad (2.9)
 \end{aligned}$$

From the definition of the elastic potential ($\pi = \int_0^{\varepsilon_{ij}} \sigma_{ij} d\varepsilon_{ij}$), this integral is equal to zero. Hence $J_1 = J_2$.

In elasticity, the J integral is contour independent.

2.2.2.3 The J Integral Is Equal to the Strain Energy Release Rate

We calculate the total energy variation ΔU when the crack length a_0 increases by Δa in a solid of unit thickness, volume V and external area S_0 . Let index 0 be related to the initial position of the crack tip. We have

$$-\Delta U = \iint_{S_0} t_i^0 \Delta u_i dS - \iiint_V \frac{\partial \pi^0}{\partial \varepsilon_{ij}} \Delta \varepsilon_{ij} dV \quad (2.10)$$

Let V_1 be the volume between the Γ contour and the external surface. We calculate the energy variation within this volume:

$$-\Delta U_1 = \iint_{S_0} t_i^0 \Delta u_i dS - \iint_{\Gamma} t_i^0 \Delta u_i dS - \iiint_{V_1} \sigma_{ij}^0 \Delta \varepsilon_{ij} dV \quad (2.11)$$

Now, the difference of the two first integrals is equal to the third, so that the energy variation within the volume V_1 is equal to zero and the calculation can be carried out on the Γ contour.

$$\begin{aligned} -\frac{dU}{da} &= \iint_{\Gamma} t_i \frac{\partial u_i}{\partial a} dS - \iint_{V-V_1} \frac{\partial \pi}{\partial a} dV \\ &= \int_{\Gamma} t_i \frac{du_i}{da} ds - \int_{\Gamma} t_i \frac{\partial u_i}{\partial x_1} ds - \iint_A \frac{d\pi}{da} dS + \iint_A \frac{\partial \pi}{\partial x_1} dS \end{aligned} \quad (2.12)$$

The surface integrals can be transformed in contour integrals:

$$\begin{aligned} \iint_A \frac{d\pi}{da} dS &= \int_{\Gamma} t_i \frac{du_i}{da} ds \\ \iint_A \frac{\partial \pi}{\partial x_1} dS &= \int_{\Gamma} \pi dx_2 \end{aligned} \quad (2.13)$$

and Eq. 2.12 shows that *the integral J is equal to the strain energy release rate*.

In linear elasticity J is equal to G .

2.2.2.4 Determination of J from the Loading Curve

The integral J being equal to the strain energy release rate $-dU/da$, it could be measured by the area between a loading curve (P, u) with a crack length a and a loading curve associated with a slightly longer crack. This is not very convenient.

However, Rice showed that it is sufficient to determine the area under the loading curve for deep enough cracks:

$$J = \frac{\eta}{Bb} \int_0^u P du - \frac{1}{Bb} \int_0^P u dP \quad (2.14)$$

where η is a shape parameter, B the thickness and b the length of the uncracked ligament. This is equivalent to the compliance equation (Eq. 2.7) for the determination of G in linear elasticity.

Equation 2.14 can be demonstrated in the following way. Dimensional analysis allows writing that the ratio of the displacement u of the loading point to a reference length, for instance b , is a function of the ratio of the load P to a reference load P_L , for instance the load needed to achieve a given displacement. But, usually P_L is taken to be the limit load in perfect plasticity (see Volume I Chap. 3). Under those conditions:

$$\begin{aligned} \frac{\partial u}{\partial b} &= f\left(\frac{P}{P_L}\right) - \frac{bP}{P_L^2} \frac{\partial P_L}{\partial b} f'\left(\frac{P}{P_L}\right) \\ \frac{\partial u}{\partial P} &= \frac{b}{P_L} f'\left(\frac{P}{P_L}\right) \end{aligned} \quad (2.15)$$

Eliminating f' between these two equations, and using the equality between J and the strain energy release rate in the form:

$$J = - \int_0^P \frac{1}{B} \frac{\partial u}{\partial b} dP \quad (2.16)$$

we get (2.14) with

$$\eta = \frac{\partial \log P_L}{\partial \log b} \quad (2.17)$$

Here are two examples (Fig. 2.4):

Single edge cracked specimen

The limit load in a single edge crack sufficiently deep (b/a small enough) is equal to BbR_p , R_p being the yield strength. Hence in that case $\eta = 1$.

In linear elasticity, Eq. 2.14 reduces to:

$$J = G = \frac{1}{2} P^2 \frac{C}{Bb} \quad (2.18)$$

Combined with (2.7), Eq. 2.18 yields $C/C_0 = b_0/b$.

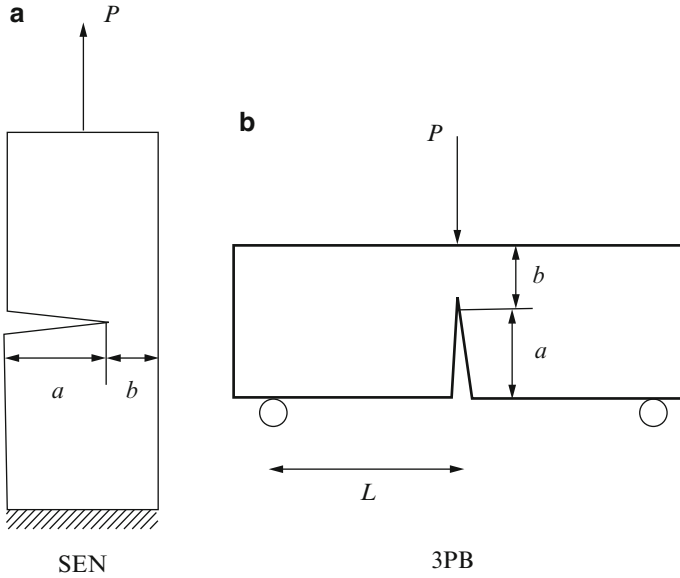


Fig. 2.4 Single edge notched (a) and three-point bend (b) specimens

Three-point bend specimen

The limit load in a cracked three-point bend specimen with a crack sufficiently deep is equal to $(Bb^2/2) (R_p/L)$, L being the span length. Thus in that case $\eta = 2$.

2.2.2.5 Strain Energy Density Singularity

At small distances, the strain energy density varies as the inverse of the distance to the crack tip.

We choose for the contour Γ a small circle of radius r . Then:

$$J = \int_{-\pi}^{+\pi} \left(\pi \cos \theta - t_i \frac{\partial u_i}{\partial x_1} \right) d\theta \quad (2.19)$$

θ being the polar angle in the x_1x_2 plane at the crack tip. For small distances, the dominant terms in the parenthesis is assumed to vary as r^q . As the integral is independent of r , q must be equal to -1 . The elastic energy can be written:

$$\pi = \frac{\tilde{\pi}(\theta)}{r} + O(r) \quad (2.20)$$

In linear elasticity, this shows that the dominant terms of the stresses and of the strains at short distances from the tip of the crack are in $1/r^{1/2}$.

2.2.2.6 Asymptotic Solutions: HRR Fields

The dominant term in the strain energy being in $1/r$ as given by (2.20), if the stress-strain curve follows the constitutive law: $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$, the dominant term of the stress must be in $r^{1/(N+1)}$ and the dominant term of the strain in $r^{N/(N+1)}$. In the case of linear elasticity $N = 1$, and we get a $r^{-1/2}$ singularity. Hutchinson (1968) and Rice and Rosengren (1968) gave the asymptotic solutions, called the HRR field, in the form:

$$\begin{aligned}\underline{\underline{\sigma}} &= \sigma_0 \left(\frac{J}{\sigma_0 \varepsilon_0 I_N r} \right)^{1/(N+1)} \underline{\underline{\tilde{\sigma}}}(\theta, N) \\ \underline{\underline{\varepsilon}} &= \varepsilon_0 \left(\frac{J}{\sigma_0 \varepsilon_0 I_N r} \right)^{N/(N+1)} \underline{\underline{\tilde{\varepsilon}}}(\theta, N)\end{aligned}\quad (2.21)$$

The functions $\tilde{\sigma}_{ij}(\theta, N)$, $\tilde{\varepsilon}_{ij}(\theta, N)$ are solutions of a differential equation and have been tabulated for various values of N (Fig. 2.5a, b); I_N is a function of N such that $(I_N/\pi)^{1/(N+1)}$ is approximately equal to 1.

In the neighbourhood of the tip of a crack – but not too close – the stress and strain fields should be asymptotically functions of a single parameter J . This property is essential for ensuring that results obtained with test pieces can be transferred to structures containing cracks. For nonlinear behaviour, however, these fields depend on the constitutive equations, which is not the case for the stress field in linear elasticity.

2.2.3 Stress Concentration Due to an Elliptical Hole

A crack is a completely flat hole. It is then appropriate to begin by the study of stress and strain fields around holes, which are known to create stress concentrations. Specifically, we consider an elliptical two-dimensional hole in a plate of infinite dimensions under a uniaxial stress σ (Fig. 2.6). This is a plane strain or plane stress problem, which can be solved by the use of biharmonic analytical functions of the complex variable $z = x + iy = \rho e^{i\theta}$.

The problem is that represented in Fig. 2.6. At infinity the stress is $\sigma_{xx} = 0$, $\sigma_{yy} = \sigma$. After a change of variable such that $z = R(\zeta + m/\zeta)$, so that $x = R(\rho + m/\rho)\cos\theta$ and $y = R(\rho - m/\rho)\sin\theta$, the perturbation that the hole creates can be calculated. The parameter m defines the eccentricity of the elliptical

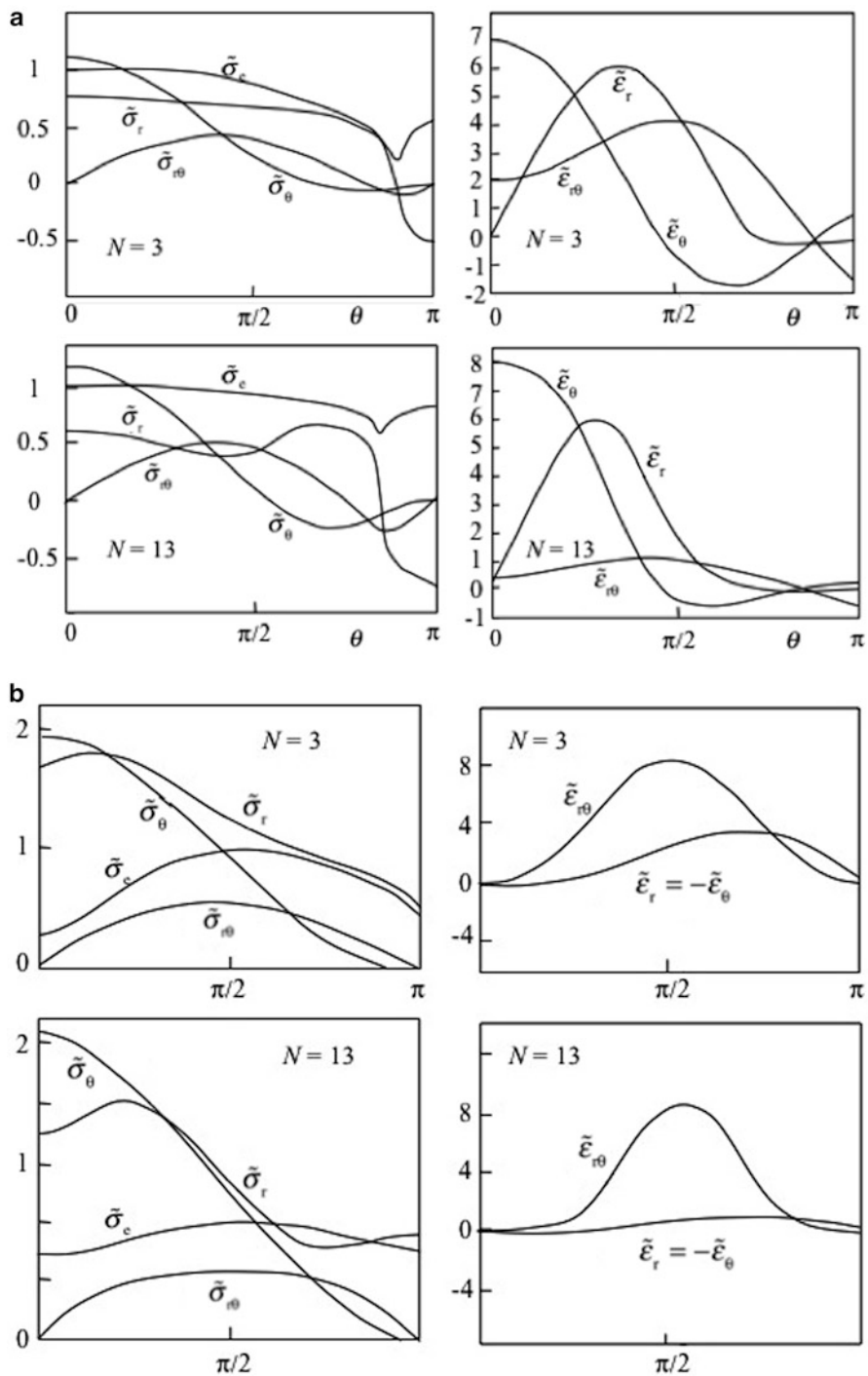


Fig. 2.5 Values of the angular variations of the stress and of the strain in the asymptotic HRR fields for $N=3$ and $N=13$; (a) plane stress; (b) plane strain

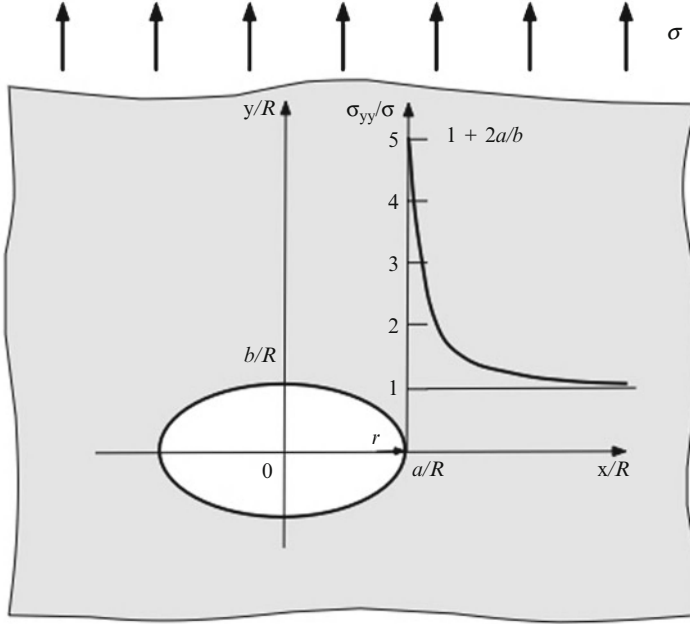


Fig. 2.6 Stress concentration at an elliptical hole in an infinite plate under uniaxial stress (In the figure $a/b = 2$)

hole of semi-axes a and b ($a = R(1 + m)$ and $b = R(1 - m)$). Along the Ox axis ($\theta = 0$), it is given by:

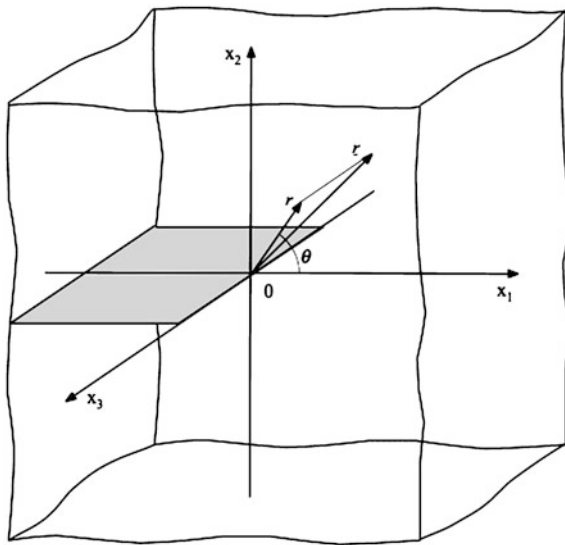
$$\begin{aligned}\sigma_{xx} + \sigma_{yy} &= 2\sigma \frac{1 + m}{\rho^2 - m} \\ \sigma_{yy} - \sigma_{xx} &= \sigma \frac{1 - m^2}{(\rho^2 - m)^2} \left[1 + \rho^2 + 2 \frac{\rho^2 (1 - \rho^2)}{\rho^2 - m} \right]\end{aligned}\quad (2.22)$$

At the extremity of the hole, for $x = a$ ($\rho = 1$), $\sigma_{xx} = 0$, as expected and the stress σ_{yy} is equal to the stress at infinity plus the perturbation given by (2.22):

$$\sigma_{yy} |_{x=a} = \sigma \left(1 + \frac{2a}{b} \right) \quad (2.23)$$

The *stress concentration factor* K_T (Peterson 1974) is thus equal to $1 + 2a/b$. This can also be expressed as $1 + 2\sqrt{a/\rho_c}$, ρ_c being the radius of curvature at the extremity of the major axis of the hole. It is equal to 3 for a circular cylindrical hole. It goes to infinity for a completely flat hole.

Fig. 2.7 Coordinate system at the *tip* of a crack



$$K_T = 1 + 2\frac{a}{b} = 1 + 2\sqrt{\frac{a}{\rho_c}} \quad \text{for an elliptical hole} \quad (2.24)$$

Note that the stress concentration is independent of the size of the hole. On the other hand, this is not the case of the stress gradient ahead of the hole. The slope of the stress versus the distance is given by:

$$\left. \frac{d(\sigma_{yy}/\sigma)}{d(x/R)} \right|_{x=a} = -\frac{1}{8} \left(\frac{2a}{b} \right)^3 \frac{a/R + 6}{(a/R)^2} \quad (2.25)$$

2.2.4 Stress Intensity Factor

2.2.4.1 The Three Elementary Fracture Modes

We consider now an ideal crack, that is a completely flat hole with a radius of curvature at the tip equal to zero. The preceding solution shows that the stress at the tip is infinite: there is a *stress singularity*. We want to study this stress singularity for any crack in any structural element in linear elasticity. We then isolate the region close to the crack tip, in which the problem becomes that of an infinite body with a crack of infinite length (Fig. 2.7). Locally, the crack front is straight. We try to determine the first term in development of the displacement $\underline{u}$ in powers of the distance r to the crack tip.

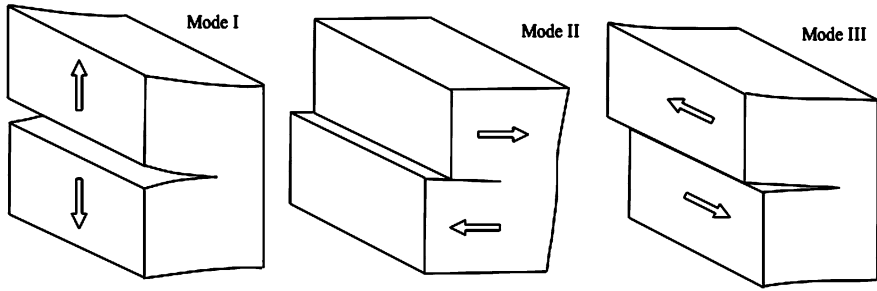


Fig. 2.8 The three modes of crack opening

In linear elasticity the problem can be decomposed in three simpler ones corresponding to the three components of the displacement of the crack faces ($x_1 < 0; x_2 = 0$) (Fig. 2.8). The opening displacement u_2 corresponds to what is denoted mode I; the sliding displacement u_1 to mode II; the sliding displacement u_3 to mode III. Modes I and II are plane strain within a thick body, plane stress in a thin plate. Mode III is antiplanar; it is the simplest as the displacement has one component only everywhere: u_3 . (This is similar to a screw dislocation. Note that Mode II is similar to the glide of an edge dislocation and mode I to its climb.)

In the following we give the displacement and stress fields in an isotropic linear elastic solid. For anisotropic materials the stress intensity factors can be found in the annex D of the handbook of Tada, Paris and Irwin (Tada et al. 2000). For cracks in inhomogeneous materials, a number of solutions can be found in section 8 of the handbook of Murakami et al. (1987) where the solutions for cracks in elastically dissimilar materials (*e.g.* a crack along the interface of two bonded dissimilar half planes) are given.

2.2.4.2 Mode III

In mode III, $u_1 = u_2 = 0$ and $\partial u_3 / \partial x_3 = 0$. Hence the only components of the stress different from 0 are σ_{13} and σ_{23} . The equilibrium equations reduce to:

$$\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} = 0 \quad (2.26)$$

or, as $\sigma_{i3} = 2\mu \varepsilon_{i3}$, μ being the shear modulus,

$$\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} = 0 \quad (2.27)$$

This allows to introduce the analytic function $w(z) = \mu (u_3^* + iu_3)$, so that

$$\sigma_{23} + i\sigma_{13} = w'(z) \quad (2.28)$$

u_3^* being an auxiliary real function.

We write the development:

$$w(z) = \sum A_n z^{\lambda_n} \quad (2.29)$$

Equation 2.28 becomes

$$\sigma_{23} + i\sigma_{13} = \sum A_n \lambda_n z^{\lambda_n - 1} \quad (2.30)$$

Now, the stress σ_{23} , the real part, is equal to 0 on the surfaces of the crack $x_2 = 0$, that is $\theta = +/\pi$, θ being the polar angle in the (Ox_1, Ox_2) plane. Hence $\lambda_n = n - 1/2$. Thus, we find the displacement

$$u_3 = \frac{A_1}{\mu} \sqrt{r} \sin \frac{\theta}{2} + O(r) \quad (2.31a)$$

$O(r)$ representing terms in powers of r at least equal to 1, which, compared with the dominant term, vanish close to the tip. For the simplification of a relation with the strain energy rate to be studied later, the constant A_1 is replaced by $2K_{III}/(2\pi)^{1/2}$ where K_{III} is called *the stress intensity factor* in mode III. It depends on the applied stress, on the crack dimensions and on the shape of the structural component.

Note that the *dimension of a stress intensity factor* is that of $\text{Newton} \times \text{meter}^{-3/2}$ or $\text{Pascal} \times \text{meter}^{1/2}$. *The convenient unit is the MPa.m^{1/2}*. The $\text{ksi.inch.}^{1/2}$ has also been used. ($1 \text{ ksi} \sqrt{\text{inch}} = 1.12 \text{ MPa} \sqrt{\text{m}}$).

Equation 2.31a is now written:

$$u_3 = \frac{2K_{III}}{\mu} \sqrt{\frac{r}{2\pi}} \sin \frac{\theta}{2} + O(r) \quad (2.31b)$$

The stress field near the crack tip is then given by:

$$\begin{Bmatrix} \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \frac{K_{III}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin(\theta/2) \\ \cos(\theta/2) \end{Bmatrix} + O(r) \quad (2.32)$$

Note that for a given value of the stress intensity factor K_{III} the dominant term of the stress field near the crack tip is independent of the crack length and shape, of the shape of the structural component and of the material. In particular, it is the same in a test piece and in a structural component, which, assuming linear elasticity and infinitely sharp crack tip, would fail for the same critical value of K_{III} as that of a test piece. In this possibility of *transposition* of results obtained on a test piece to any structural component lies the interest of fracture mechanics. However, it must not be forgotten that further away from the tip regular terms in the expansion become increasingly important.

Note also that a real material cannot sustain an infinite stress, such as the one found at the crack tip in the preceding calculation. First of all the radius of curvature at the crack tip cannot be equal to zero as was assumed in the calculation. Secondly, the very high stresses near the crack tip produce plastic deformation and microscopic damages. The correction needed to take this into account will be studied later.

2.2.4.3 Modes I and II

Modes I and II are planar problems. The equilibrium equations then reduce to:

$$\begin{aligned}\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} &= 0 \\ \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} &= 0\end{aligned}\quad (2.33)$$

The biharmonic Airy's function⁵ $U(x_1, x_2)$ can be used, such that:

$$\sigma_{ij} = -\frac{\partial^2 U}{\partial x_i \partial x_j} + \delta_{ij} \left(\frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_2^2} \right) \quad i, j = 1, 2 \quad (2.34)$$

U can be expressed by Goursat's functions⁶ of the complex variable $z = x + iy$:

$$U = \text{Re} \left[z\phi(z) + \int \psi(z)dz \right] \quad (2.35)$$

Now:

$$\begin{aligned}\sigma_{11} + \sigma_{22} &= 4\text{Re}[\phi'(z)] \\ \sigma_{22} - \sigma_{11} + 2i\sigma_{12} &= 2[\bar{z}\phi''(z) + \psi'(z)]\end{aligned}\quad (2.36)$$

$$u_1 + iu_2 = \frac{1}{2\mu} [\kappa\phi(z) - z\bar{\phi}'(z) - \bar{\psi}(z)] \quad (2.37)$$

where μ is the shear modulus and $\kappa = 3 - 4\nu$ in plane strain and $\kappa = (3 - \nu)/(1 + \nu)$ in plane stress, ν being the Poisson ratio.

As we did in mode III, we develop the functions ϕ and ψ in powers of z :

$$\begin{cases} \phi(z) = \sum A_n z^{\lambda_n} \\ \int \psi(z)dz = \sum B_n z^{\lambda_n+1} \end{cases} \quad \lambda_n > 0. \quad (2.38)$$

⁵Sir George Biddell Airy (1801–1892) was a British mathematician.

⁶Édouard Jean-Baptiste Goursat (1858–1936) was a French mathematician.

From (2.36), shifting to polars:

$$\sigma_{\theta\theta} + i\sigma_{r\theta} = \sum \lambda_n r^{\lambda_n-1} \left\{ \begin{array}{l} \lambda_n A_n \exp[i(\lambda_n - 1)\theta] \\ + \bar{A}_n \exp[-i(\lambda_n - 1)\theta] \\ + (\lambda_n + 1) B_n \exp[i(\lambda_n + 1)\theta] \end{array} \right\} \quad (2.39)$$

As this must be equal to 0 for $\theta = +/\pi$:

$$\begin{aligned} \sum \{ \lambda_n A_n \exp[i\lambda_n \pi] + \bar{A}_n \exp[-i\lambda_n \pi] + (\lambda_n + 1) B_n \exp[i\lambda_n \pi] \} &= 0 \\ \sum \{ \lambda_n A_n \exp[-i\lambda_n \pi] + \bar{A}_n \exp[i\lambda_n \pi] + (\lambda_n + 1) B_n \exp[-i\lambda_n \pi] \} &= 0 \end{aligned} \quad (2.40)$$

Hence:

$\lambda_n = n/2$ and

$$\frac{n}{2} A_n + (-1)^n \bar{A}_n + \left(\frac{n}{2} + 1\right) B_n = 0 \quad (2.41)$$

In particular this equation yields the dominant term of the function $\phi(z)$ in $z^{1/2}$:

$$3B_1 = \frac{A_1 + \bar{A}_1}{2} - 3\frac{A_1 - \bar{A}_1}{2} \quad (2.42)$$

With

$$\begin{aligned} \frac{A_1 + \bar{A}_1}{2} &= \frac{K_I}{\sqrt{2\pi}} \\ \frac{A_1 - \bar{A}_1}{2} &= -i \frac{K_{II}}{\sqrt{2\pi}} \end{aligned} \quad (2.43)$$

the following results are found: Mode I

$$\left\{ \begin{array}{l} u_1 \\ u_2 \end{array} \right\} = \frac{K_I}{2\mu} \sqrt{\frac{r}{2\pi}} \left\{ \begin{array}{l} \cos(\theta/2) [\kappa - 1 + 2\sin^2(\theta/2)] \\ \sin(\theta/2) [\kappa + 1 - 2\cos^2(\theta/2)] \end{array} \right\} + O(r) \quad (2.44)$$

where $\kappa = 3 - 4\nu$ in plane strain and $\kappa = (3 - \nu)/(1 + \nu)$ in plane stress, ν being the Poisson ratio.

Equation 2.44 shows that in linear elasticity *the crack tip opening displacement* $2u_2$ ($\theta = \pi$) is a parabolic function of the distance to the tip. At a given distance from the tip, it is a measure of the stress intensity factor.

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2) \begin{Bmatrix} 1 - \sin(\theta/2) \sin(3\theta/2) \\ 1 + \sin(\theta/2) \sin(3\theta/2) \\ \sin(\theta/2) \cos(3\theta/2) \end{Bmatrix} + O(r) \quad (2.45)$$

In plane strain $\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) = \left(K_I / \sqrt{2\pi r}\right) \cos(\theta/2)$ while $\sigma_{33} = 0$ in plane stress.

We can make the same remarks as the ones above in mode III regarding the transposition of results, as the singular stress field depends of a single parameter, notwithstanding correction needed to take into account plasticity and damage near the crack tip. However, the second term in the development of the Goursat's functions, called the *T stress*, cannot be completely neglected (Sect. 2.2.4.4).

The principal stresses are:

$$\begin{Bmatrix} \sigma_I \\ \sigma_{II} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos(\theta/2) \begin{Bmatrix} [1 + \sin(\theta/2)] \\ [1 - \sin(\theta/2)] \end{Bmatrix} \quad (2.46)$$

The maximum principal stress is the highest for $\theta = \pi/3$ and is then equal to $3\sqrt{3}/4$.

The maximum shear stress is:

$$\tau = \frac{K_I}{\sqrt{2\pi r}} \frac{\sin \theta}{2} \quad (2.47)$$

The normal stress on a plane making an angle θ with the crack plane is:

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos^3(\theta/2) \quad (2.48)$$

while the shear stress on that plane is:

$$\sigma_{r\theta} = \frac{K_I}{\sqrt{2\pi r}} \cos^2(\theta/2) \sin(\theta/2) \quad (2.49)$$

This shear stress is the highest for $\sin \theta = 2/3$.

As the normal stress ahead of the crack as given by Eq. 2.48 is maximum for $\theta = 0$, in mode I cracks do not have a tendency to deviate.

Mode II

In mode II the results are:

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{K_{II}}{2\mu} \sqrt{\frac{r}{2\pi}} \begin{Bmatrix} \sin(\theta/2) [\kappa + 1 + 2\cos^2(\theta/2)] \\ -\cos(\theta/2) [\kappa - 1 - 2\sin^2(\theta/2)] \end{Bmatrix} + O(r) \quad (2.50)$$

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \frac{K_{II}}{\sqrt{2\pi r}} \begin{Bmatrix} -\sin(\theta/2) [2 + \cos(\theta/2) \cos(3\theta/2)] \\ \sin(\theta/2) \cos(\theta/2) \cos(3\theta/2) \\ \cos(\theta/2) [1 - \sin(\theta/2) \sin(3\theta/2)] \end{Bmatrix} + O(r) \quad (2.51)$$

$$\sigma_{\theta\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \frac{3}{2} \sin \theta \cos(\theta/2) \quad (2.52)$$

The maximum of this normal stress is found for $\cos \theta = 1/3$. This is the reason why, in mode II, cracks have a strong tendency to deviate in this direction.

2.2.4.4 Non-singular Terms, T and Q Stresses

So far we only considered the singular terms in the development of the stresses, which vary as $r^{-1/2}$. We now investigate the following terms, the variation of which is in positive powers of the r , and thus are negligible by comparison with the first close enough from the tip of the crack. Let us consider the second terms corresponding to λ_2 .

In mode III $\lambda_2 = n - 1/2 = 3/2$, so that the second term in the development of the stresses vary as $r^{1/2}$ (refer to Eq. 2.30).

In mode I, $\lambda_2 = n/2 = 1$, so that the second terms in the development of the stresses are constant (refer to Eq. 2.36). More precisely, Eq. 2.41 yields $B_2 = -2(A_2 + \bar{A}_2)$ and from (2.36) we find

$$\begin{aligned} \sigma_{11} &= 2(A_2 + \bar{A}_2) \\ \sigma_{22} &= \sigma_{12} = 0 \end{aligned} \quad (2.53)$$

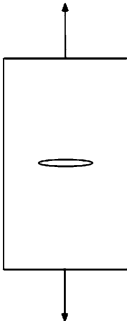
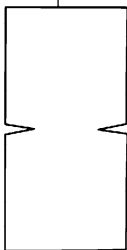
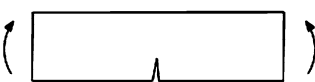
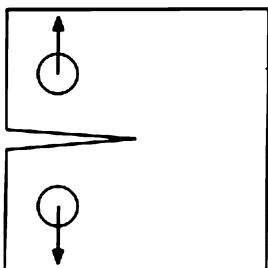
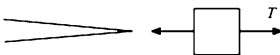
The stress σ_{11} , called the T stress, which is different from 0 at the crack tip,⁷ cannot be completely ignored. Strictly speaking, the transposition of results from a specimen to a structure would need not only the stress intensity factor to be the same but also the T stress.

It is a function of the far field conditions. Table 2.1 gives values of the T stress in term of $K_I(\pi a)^{-1/2}$, a being the crack length.

In the same way, for nonlinear behaviour, the HRR stress field (2.21), which describes the stress and strain singularity, ought to be completed by terms of higher power in the development. In that case it is called the $\underline{Q}$ stress field (Betegon and Hancock 1991; O'Dowd and Shih 1991). It must be added in Eq. 2.21. The difficulty in that case is that it depends, like the singular stress field, on the strain-hardening

⁷This could not be so if the crack tip was blunted. However, it was assumed to be perfectly sharp.

Table 2.1 Values of the parameter $\beta = T(\pi a)^{1/2}/K_I$ for different configurations

Geometry	Configuration	β
	Center notched	-0.59
	Double notched	-0.14
	Pure bending	0.03
	Compact tension CT	0.29
		

parameter. For the determination of $\underline{Q}$, finite elements calculations are needed. They require very fine meshes near the tip of the crack, in view of the high stress gradients.

We will give other developments later, when dealing with elastoplastic fracture mechanics (Sect. 2.5.2.4).

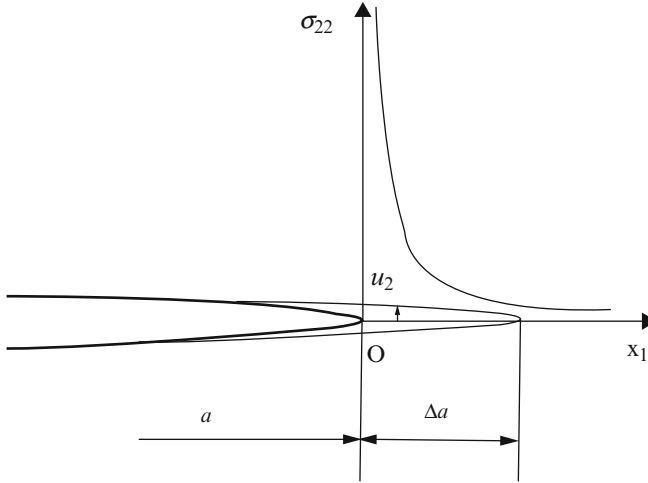


Fig. 2.9 Work done in crack closure over a distance Δa

2.2.5 Relation Between the Stress Intensity Factor and the Strain Energy Release Rate

The strain energy release rate G ($=J$ in linear elasticity) is the amount of energy released when the crack tip moves a small distance Δa , relaxing stresses within the singularity. Hence, there must exist a relation between the strain energy release rate and the stress intensity factor, on which only depends the stress singularity.

We consider the problem in mode I (Fig. 2.9). The crack propagates a small distance Δa *without deviation*. The strain energy release rate is equal to the work per unit area done in closing down the crack back to its original length. At a distance x_1 from the position O of the crack tip before propagation the stress is given by (2.45), namely $\sigma_{22}(x_1, \theta = 0)$; the crack opening displacement at this same point after propagation is $2u_2(\Delta a - x_1, \theta = \pi)$, given by Eq. 2.44. The work done at this point is $dW = u_2 \sigma_{22} dx_1$. Hence:

$$G \Delta a = \int_0^{\Delta a} \frac{K_I}{\sqrt{2\pi x_1}} \frac{K_I}{2\mu} \sqrt{\frac{\Delta a - x_1}{2\pi}} (\kappa + 1) dx_1 = \frac{(\kappa + 1)}{8\mu} K_I^2 \Delta a \quad (2.54)$$

Given the values of κ we get:

$$\text{in plane strain, } G = \frac{K_I^2}{E} (1 - \nu^2) \quad (2.55a)$$

$$\text{in plane stress, } G = \frac{K_I^2}{E} \quad (2.55b)$$

(E is the Young modulus and ν the Poisson ratio).

The simplicity of these formulae comes from the choice made for the definition of the stress intensity factor K_I as a function of $A_I + \bar{A}_I$ as given by (2.43).

The same demonstration in modes II and III yields the general relation:

$$G = \frac{1}{E} [(K_I^2 + K_{II}^2)(1 - \nu^2) + K_{III}^2(1 + \nu)] \quad (2.56)$$

provided the propagation of the crack is without deviation.

As a general rule, the extension of the crack does not remain in the original plane; we have then to treat the propagation as that of a virtual crack at an angle α and find the work done by the “cohesion forces” of the *initial* crack in the displacement field of the *branched* crack. A simple argument would assume that the stress intensity factors for the branched crack tend towards those for the initial crack as Δa tends to zero, but this would be false (See also Lawn and Wilshaw 1975).

2.2.6 Displacement in a Cracked Body

The compliance equation (Eq. 2.7) shows that the knowledge of the strain energy release rate should enable to determine the displacement of a load point. We consider a body with a crack of area A loaded by a force $\underline{P}$. We want to determine the displacement $\underline{u}_f$ of a point M in a certain direction. We apply at point M in the desired direction an auxiliary unit force $\underline{f}$. The strain energy release rate is then given by (2.56) in which the stress intensity factors are replaced by the sum of the stress intensity factors $K_{JP} + K_{Jf}$ ($J = \text{I, II or III}$) due to the load $\underline{P}$ and to the unit force $\underline{f}$, respectively.

For collinear vectors $\underline{P}$, $\underline{f}$ and $\underline{u}_f$, using the corresponding scalars P , f and u_f , Castigliano's theorem states that:

$$u_f = \frac{\partial \pi}{\partial f} \quad (2.57)$$

π being the elastic energy. We have

$$\pi = \pi_0 + \int_0^A \frac{\partial \pi}{\partial A} dA = \pi_0 + \int_0^A G dA \quad (2.58)$$

π_0 being the elastic energy in the body with no crack. Thus from (2.57), at constant P :

$$\begin{aligned} u_f(P) - u_f(P=0) &= u_{f0} + \frac{2}{E} (1 - \nu^2) \int_0^A \left(K_{IP} \frac{\partial K_{If}}{\partial f} + K_{IIP} \frac{\partial K_{IIf}}{\partial f} \right) dA \\ &\quad + \frac{2}{E} (1 + \nu) \int_0^A K_{IIIP} \frac{\partial K_{IIIf}}{\partial f} dA \end{aligned} \quad (2.59)$$

In mode I, the crack opening displacement, from (2.59) is given by:

$$\text{COD}(a_f) = \frac{4}{E'} \int_{a_f}^a K_{IP} \frac{\partial K_{If}}{\partial f} da \quad (2.60)$$

where E' is equal to $E/(1 - \nu^2)$ in plane strain and to E in plane stress, a_f is the abscissa of the point of interest and a the abscissa of the crack tip along the $\underline{x}_1$ axis.

2.2.7 Determination of the Stress Intensity Factor

We now need to know how the stress intensity factor is expressed as a function of the loading, of the size and shape of the crack and of the shape of the structural element. Experimental methods can be used. There are analytical solutions for several situations. Otherwise, finite elements calculations provide the solutions. We give now an overview of various methods of determination of the stress intensity factor (see Table 2.5 at the end of the chapter for various cases for which the stress intensity factor is given).

2.2.7.1 Experimental Methods

Compliance method

The first method uses the relation between K and G (2.55a and 2.55b): the variation of compliance with crack length is determined experimentally, and the values of K follow immediately.

Crack tip stress and displacement fields

Other methods are based on the experimental determination of the stress and strain fields in the region of the crack tip. The measured distributions are studied in as great detail as possible in the light of the facts that the stresses vary like $r^{-1/2}$ and the displacements like $r^{1/2}$, and that the constant of proportionality is in each case directly related to K . Since the singularity occurs only at the crack tip the measurements have to be very precise and allow high gradients to be studied: *photo-elasticity* has been used to achieve this. There are possibilities of using this method for three-dimensional problems.

The method of *caustics* uses the reflection of a beam of light on the surface close to the crack-tip. As the stress on this surface is zero, there is a plane stress state so that the reflected beam, or the transmitted beam in a transparent material, produce a caustic on a screen. Its dimension is proportional to the stress intensity factor. This method is well suited for the determination of the evolution of the SIF in dynamic crack propagation experiments. An illustration showing the use of caustics method to measure the displacement field associated with a propagating crack is shown in Fig. 2.10 (Theocaris and Papadopoulos 1980).

Analogies

Finally, there are experimental methods based on analogies between the equations of mechanics on the one hand and on different physical phenomena on the other, but these are little used in practice. The interested reader will find details in Bui (1978).

2.2.7.2 Applications of the Superposition Principle

Stress on the crack location

Solutions to many problems have been published in the literature (Tada et al. 2000; Murakami et al. 1987). Some of these are given at the end of this chapter in Table 2.5. The commonest practice in attacking a new problem is to attempt to reduce it to a set of problems for which the solutions are known and appeal to the *principle of superposition* to construct the solution as a combination of these known solutions: clearly, this requires the conditions to be those of linear elasticity.

Applications of the superposition principle make use of the following important theorem

The stress intensity factor in a load-controlled structure is the same as the one which would result from the application on the crack faces of the opposite stresses to the ones existing on its location before cracking.

Indeed, the problem can be considered as the superposition of the loaded uncracked structure (problem N° 1) and of the structure loaded only along the crack

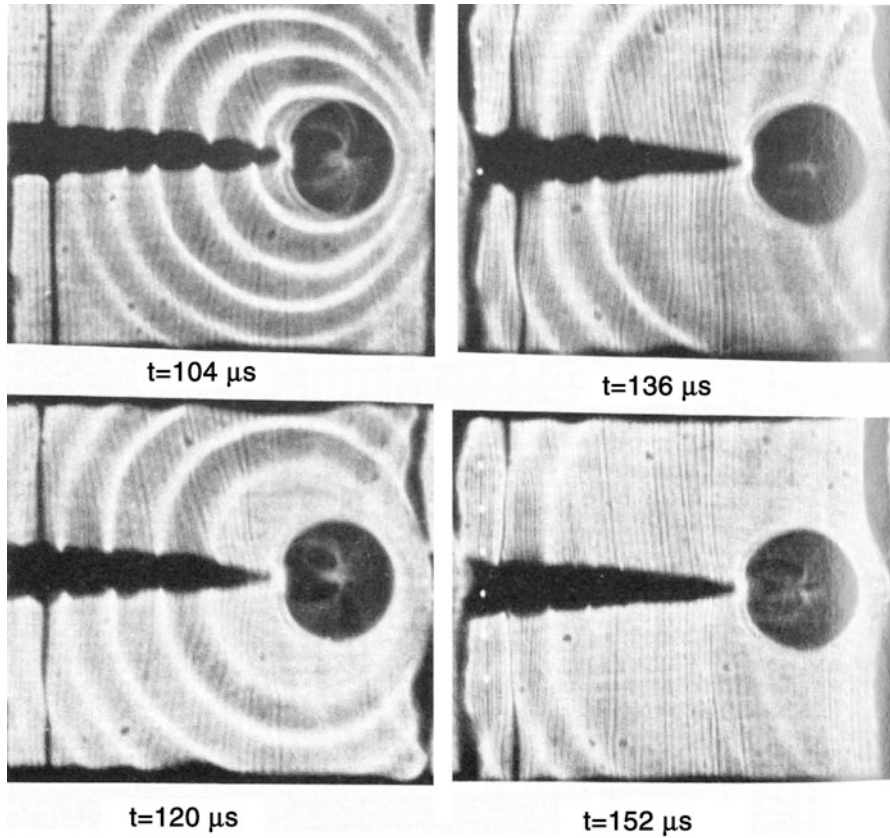


Fig. 2.10 A series of photographs of an arrested crack in a polycarbonate plate where the shear stress wave engendered at the crack tip travels along the specimen (initial crack length $a_0 = 10$ mm). The times at which the photographs were taken are indicated. The *black circle* next to the *tip* is due to the stress distribution there and its radius is linked with the SIF (Theocaris and Papadopoulos 1980)

location by distributed forces opposite to the stresses existing there, so as to cancel them (problem N° 2) (Fig. 2.11). As this second problem only includes a stress singularity, it provides the solution for the stress intensity factor.

Specifically in problem N° 1 we first try to find the solution with the same boundary conditions but with no crack in the structure. This solution will give the stress and strain distributions at all points of the solid. With a general point M on the edges of the crack we associate a pair of forces $+/-\underline{P}(M)$ such that $P_i(M) = \sigma_{ij}n_j$ where $\underline{n}$ is the normal to the plane of the crack and is directed towards this. The force $\underline{P}$ can be split into three parts:

$P_I(M)$ normal to the plane of the crack

$P_{II}(M)$ in this plane and parallel to it

$P_{III}(M)$ normal to the plane of the problem

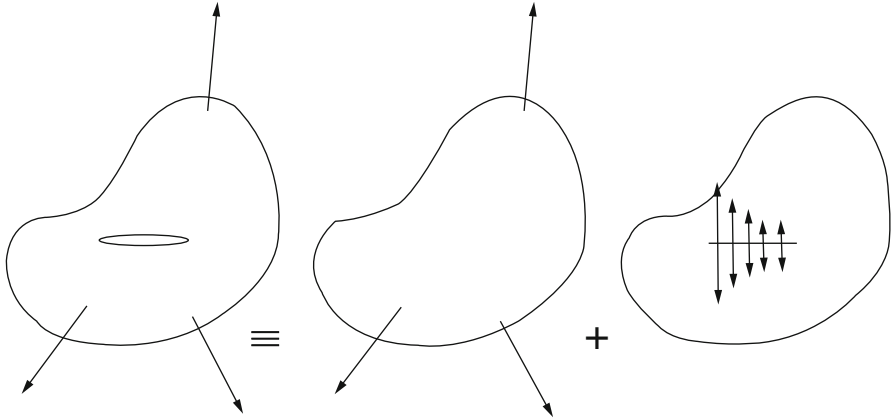


Fig. 2.11 Cracked load-controlled structure decomposed in the load-controlled uncracked structure and the structure loaded by distributed forces to cancel the stresses along the crack length

Problem N° 2 is as follows. With boundary conditions of zero forces or displacements at the external surface, find the stress intensity factor for a crack subjected to the load $\underline{P}(M)$.

Weight functions

The solution to the problem of distributed forces $\underline{P}(M)$ along the surface of the crack may be known for certain forms for $\underline{P}(M)$, for example for constant value or for linear or parabolic variation with distance; for a given direction of $\underline{P}(M)$, we can then approximate it by a polynomial $P(M) = A_0 + A_1x + A_2x^2 + \dots$, where x is the distance from M to the tip of the crack and obtain a solution in the form $A_0K_{I0} + A_1K_{I1} + A_2K_{I2} + \dots$ where K_{I0} , etc. are the solutions corresponding to the elementary distributions $P(M) = 1, x, x^2$, etc. Figure 2.12 gives the solutions for various geometries; these are taken from Buchalet and Bamford (1976) p. 385. Using this figure, to the third order, K_I is expressed as

$$K_I = \sqrt{(\pi a)} [A_0 P_1 + (2a/\pi) A_1 P_2 + (a^2/2) A_2 P_3 + (4a^3/3\pi) A_3 P_4] \quad (2.61)$$

The values of the factors P_i are given in Fig. 2.12 for each of the three geometries.

Finally, the solution to the original problem is expressed as a linear combination of the solutions to problems N° 1 and N° 2.

An alternative method for attacking the same problem is to attempt to find the value of $K_I(x)$ due to a unit force acting at a distance x from the tip of the crack; given this the value K_I corresponding to a distribution $F(x)$ is

$$K_I = \int F(x) K(x) dx \quad (2.62)$$

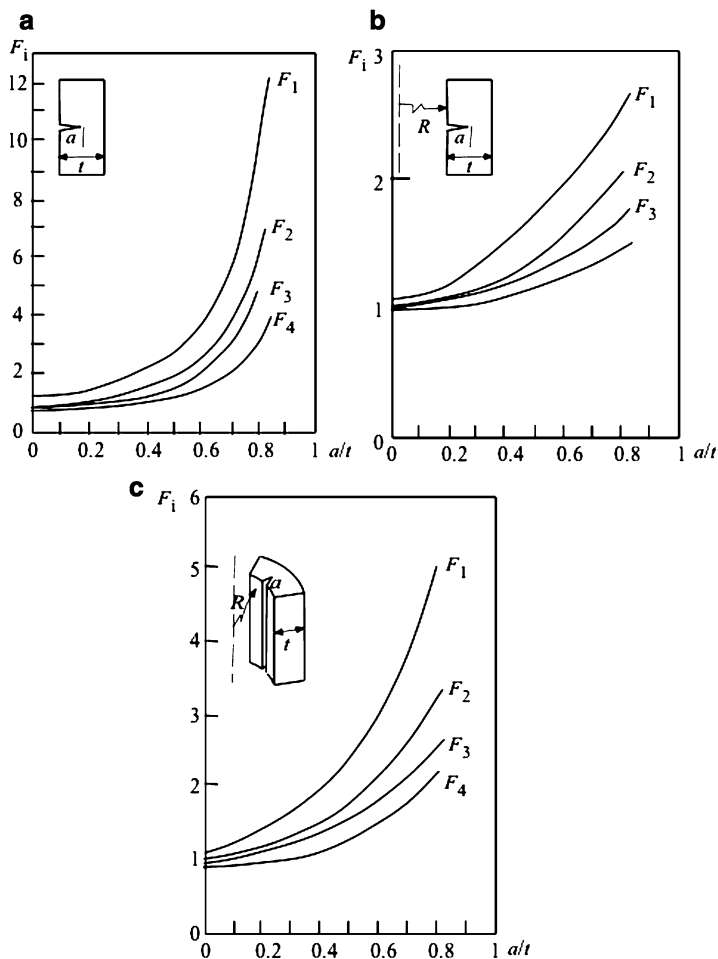


Fig. 2.12 Application of the method of superposition to determine the stress intensity factor K_I in three geometries: (a) plate with an edge crack; (b) cylinder with a circumferential crack $t/R = 0.1$; (c) cylinder with longitudinal crack $t/R = 0.1$

The function $K(x)$, called the *weight function*, has been determined for various particular cases (Labbens et al. 1976).

Notes:

- Before attempting to use these methods it is important to check that the boundary conditions are fulfilled. In any practical problem the dimensions are necessarily finite and the true solution may be very different from that obtained with the mathematically convenient but physically unrealistic assumption of infinite size. Similarly for problems of revolution, it is important to ensure that the values of

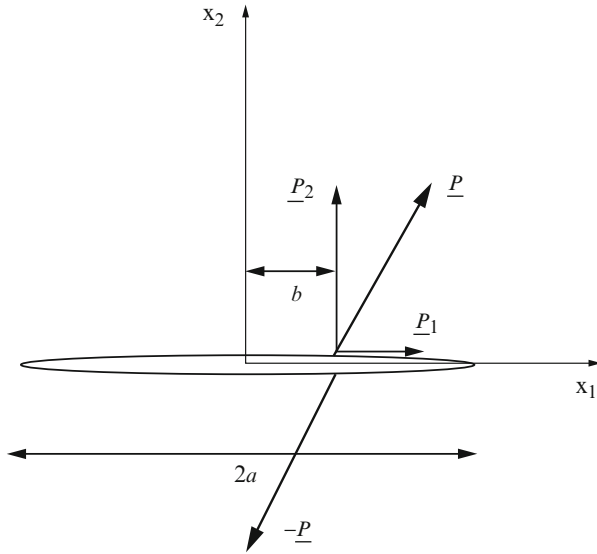


Fig. 2.13 Stress intensity factor for a pair of point forces on the crack edges

the thickness-to-radius ratio B/R for the problem are in the range for which the given solution has been tabulated.

- The influence of the boundary conditions is seen in the expression for K : the value increases with the length of the crack when the stress is prescribed and decreases when the displacement is prescribed.
- The weight function method assumes that the stress is prescribed. If what is available is the solution corresponding to these conditions but what is wanted is the solution for prescribed displacements, then displacements can be transformed into forces by use of the compliance of the component (Eq. 2.6). Further, (2.7) and (2.56) make it possible to pass from one problem to the other.

A crack in an infinite plate loaded by a pair of concentrated forces (Fig. 2.13)

The length of the crack is $2a$, and the pair of concentrated forces applied on the two opposite faces of the crack is $+/-\underline{P}$ per unit thickness at a distance b from the crack centre. Using again analytical function it can be demonstrated that, for a force P_2 in the direction Ox_2 perpendicular to the crack:

$$K_I = \frac{P_2}{\sqrt{\pi a}} \left(\frac{a+b}{a-b} \right)^{1/2} \quad (2.63)$$

K_{II} is given by the same equation, replacing P_2 by P_1 , and K_{III} by P_3 .

In particular, *if the forces are applied at the centre of the crack* $b = 0$ and:

$$K_I = \frac{P_2}{\sqrt{\pi a}} \quad (2.64)$$

Note that the stress intensity factor decreases when the crack length increases under constant load. Such a situation leads to crack arrest.

When *the forces are applied at a small distance c from the tip*, Eq. 2.63 yields:

$$K_I = P_2 \sqrt{\frac{2}{\pi c}} \quad (2.65)$$

A crack in an infinite plate loaded by distributed forces

Let the distributed normal forces be $\pm q$ per unit area applied on a crack of length $2a$ between distances from the centre b_1 and b_2 . Because in linear elasticity we are allowed to superimpose solutions, (2.63) yields:

$$\begin{aligned} K_I &= \frac{q}{\sqrt{\pi a}} \int_{b_1}^{b_2} \left(\frac{a+x}{a-x} \right)^{1/2} dx \\ &= 2q \sqrt{\frac{a}{\pi}} \left[\arcsin \left(\frac{a-b_1}{2a} \right)^{1/2} - \arcsin \left(\frac{a-b_2}{2a} \right)^{1/2} \right. \\ &\quad \left. + \frac{(a^2 - b_1^2)^{1/2}}{2a} - \frac{(a^2 - b_2^2)^{1/2}}{2a} \right] \end{aligned} \quad (2.66)$$

In particular if the forces are distributed all over the surface of the crack as if the crack was *under a pressure p* ($b_1 = -a$, $b_2 = +a$):

$$K_I = p \sqrt{\pi a} \quad (2.67a)$$

This is equivalent to a *crack in an infinite plate under a uniform applied tensile stress $\sigma = p$* at infinity:

$$K_I = \sigma \sqrt{\pi a} \quad (2.67b)$$

In that case, in contrast with the case of a concentrated force, the stress intensity factor increases with the crack length under constant stress. Such a situation leads to crack instability.

For a very long crack loaded by distributed normal forces q per unit area over a small distance d from the tip:

$$K_I = q \sqrt{\frac{2}{\pi}} \int_0^d \frac{dx}{\sqrt{x}} = \sqrt{\frac{8d}{\pi}} q \quad (2.68)$$

If the forces are distributed over a distance d from the tip on both sides

$$K_I = 4p \sqrt{\frac{a}{\pi}} \arcsin \sqrt{\frac{d}{2a}} \quad (2.69)$$

Penny-shaped crack

For a penny-shaped crack of radius a in an infinite medium loaded on its edges by distributed forces perpendicular to the plane of the crack $+/-q$ per unit length along two opposite circles of radius b :

$$K_I = \frac{2q}{\sqrt{\pi a}} \frac{b}{a} \left[1 - \left(\frac{b}{a} \right)^2 \right]^{-1/2} \quad (2.70)$$

This yields for a penny-shaped crack loaded by a pressure p along a zone of radii b_1 and b_2 :

$$K_I = \frac{1}{\sqrt{\pi a}} \frac{p}{a} \int_{b_1}^{b_2} \frac{2x dx}{[1 - (x/a)^2]^{1/2}} = \frac{2}{\pi} p \sqrt{\pi a} \left[\sqrt{1 - \left(\frac{b_1}{a} \right)^2} - \sqrt{1 - \left(\frac{b_2}{a} \right)^2} \right] \quad (2.71)$$

Thus, the stress intensity factor for a *penny-shaped crack of radius a under pressure p* in an infinite medium is:

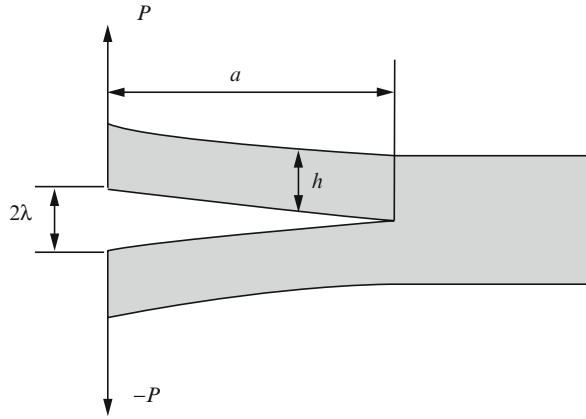
$$K_I = \frac{2}{\pi} p \sqrt{\pi a} \quad (2.72)$$

If the penny-shaped crack is in an infinite medium loaded by a tensile stress σ at infinity the stress intensity factor is given by (2.72) replacing p by σ .

The stress intensity factor of a *penny-shaped crack in an infinite medium loaded by concentrated forces $+/-F$ perpendicular to the plane of the crack and applied on its centre*, from (2.71), with $b_1 = 0$ and b_2/a going to 0, is given by:

$$K_I = \frac{F}{(\pi a)^{3/2}} \quad (2.73)$$

The same remarks can be made concerning the evolution of the stress intensity factor with the crack length as the ones above for two-dimensional cracks.

Fig. 2.14 DCB specimen

2.2.7.3 Relation Between the Stress Intensity Factor and the Stress Concentration Factor

There is a fundamental difference between the stress intensity factor K and the stress concentration factor K_T ; however, the argument used to establish the presence of the singularity suggests that the two are related, and Irwin has shown that K_I can be expressed as a limit, thus:

$$K_I = \lim_{\rho_c \rightarrow 0} \frac{\sqrt{\pi}}{2} \sigma_{\max} \sqrt{\rho_c} \quad (2.74)$$

where ρ_c is the radius of curvature of the extremity of the notch (Fig. 2.4). For the geometry of this figure this gives (2.67b), for a crack of length $2a$ in an infinite plate.

2.2.7.4 Compliance Methods Based on G and J

DCB specimen

There are cases for which the compliance can be calculated. That is the case of the double cantilever beam specimen (DCB) sketched in Fig. 2.14. It can be assimilated to two cantilever beams of length a and width B loaded by forces perpendicular to the plane of the crack $+/-P$ at their ends. Under the condition that their height h is small in front of a , the beam theory yields the deflection of one beam:

$$\lambda = \frac{4Pa^3}{EBh^3} \quad (2.75)$$

in which case the compliance is $8a^3/EBh^3$ and:

$$\frac{\partial C}{\partial A} = \frac{24}{E} \left(\frac{a}{Bh^{3/2}} \right)^2 \quad (2.76)$$

so that, from (2.7) and (2.55a and 2.55b):

$$G = \frac{12}{E} \left(\frac{aP}{Bh^{3/2}} \right)^2 = \frac{K_I^2}{E} (1 - \nu^2) \quad (2.77)$$

The stress intensity factor increases with the crack length a .

In case of imposed opening displacement 2λ , in (2.77) P must be replaced by its value given by (2.75):

$$G = \frac{3}{4} E \left(\frac{h^{3/2} \lambda}{a^2 B} \right)^2 \quad (2.78)$$

The stress intensity factor then decreases as the crack length increases. The DCB specimen is well suited to determine crack arrest conditions and thresholds of critical SIF.

The two beams heights can be designed so as to obtain a constant stress intensity factor whatever the crack length. This is convenient in crack propagation studies.

Spot weld

Using the same method, it can be demonstrated that the stress intensity factor of two spot-welded thin disks of thickness h and radius R , loaded along their periphery by distributed normal forces p per unit length, is given by:

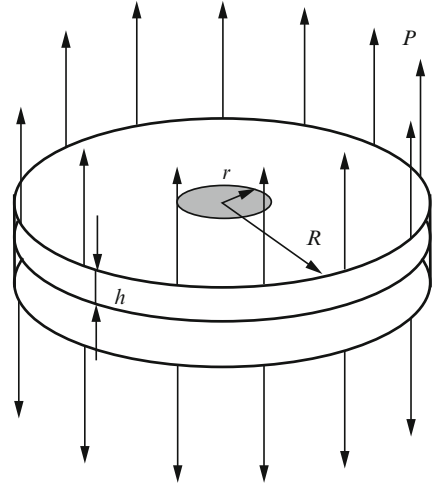
$$K_I (1 - \nu^2)^{1/2} = \frac{2\sqrt{3}Rp}{h^{3/2}} \left(\frac{R}{r} - 1 \right) \quad (2.79)$$

r being the radius of the spot-weld (Fig. 2.15).

Due to the technical importance of resistance spot (RS) welding in particular in car industry (a modern car contains between 3,000 and 4,000 RS welds!), this geometry has been largely investigated to determine the stress intensity factor under various loading conditions. Several types of specimens are used to test RS welds. The most largely used are the lap-shear (LS) specimen (Fig. 2.16a) and the U-shape cross tensile (USCT) specimen (Fig. 2.16b). It is only recently that closed-form solutions of the stress intensity factors have been proposed for these geometries (Lin and Pan 2008; Lin and Wang 2010).

The LS geometry develops not only mode I, but also modes II and III. Moreover in this specimen the stress intensity factors K_I and K_{II} are not constant along the

Fig. 2.15 Spot weld of radius r loaded by distributed forces p per unit length along a circle of radius R



periphery of the RS weld of diameter $2r$. At the critical points A and B (Fig. 2.16a) the stress intensity factors for modes I and II are given by:

$$\begin{aligned} K_I &= \frac{\sqrt{3}F}{4B\sqrt{h}XY} [B^2X + 2YB^4(r^4 + B^4)] \\ K_{II} &= \frac{F}{\pi r^{3/2}} \end{aligned} \quad (2.80)$$

where

$$\begin{aligned} X &= (1 - \nu)(r^4 + B^4)^2 + 4(1 + \nu)r^2B^6 \\ Y &= (1 - \nu)r^2 + (1 + \nu)B^2 \end{aligned} \quad (2.81)$$

The USCT geometry develops essentially mode I opening and K_I is almost constant all along the periphery of the RS weld. This is the reason why this geometry should be preferred to the LS specimen for testing the fracture toughness of RS welds. The stress intensity factor, with a geometry such that $2L = B$, is given by:

$$K_I = \frac{\sqrt{3}}{2\pi h^{3/2}} \left\{ \begin{aligned} &\frac{0.837B^2[r^2 - 4B^2/\pi + 2r^2 \log(2B/\sqrt{\pi}r)]}{[(1 - \nu)r^2 + (1 + \nu)B^2](r^2 - 4B^2/\pi)} \\ &- \frac{(r^2 - 4B^2/\pi)(1 - \nu) - (8B^2/\pi) \log(2B/\sqrt{\pi}r)}{(1 - \nu)r^2 + (1 + \nu)4B^2/\pi} \end{aligned} \right\} \quad (2.82)$$

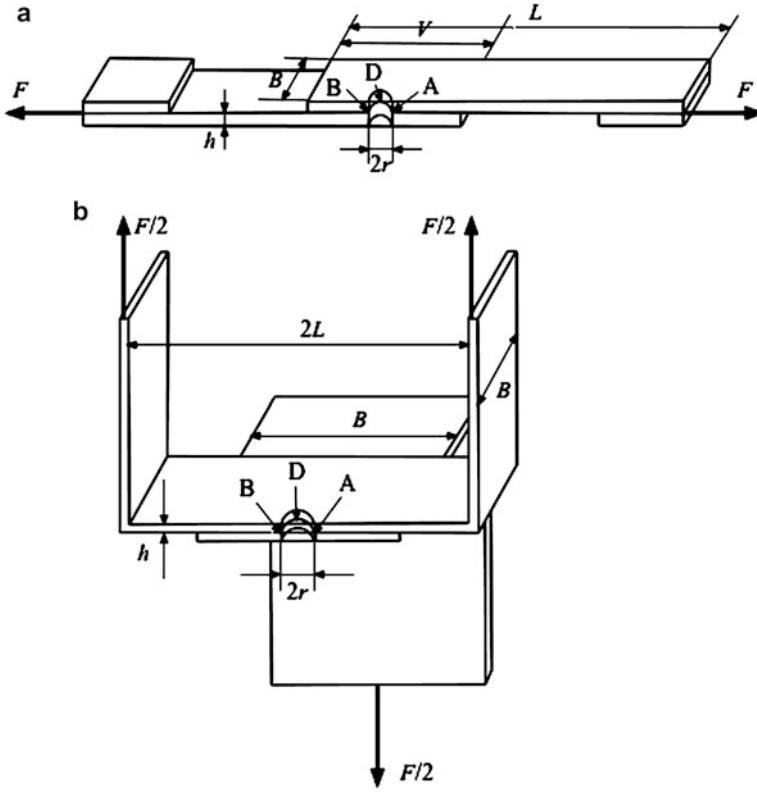


Fig. 2.16 Two types of half spot weld specimens with applied forces indicated by *bold arrows*: (a) lap-shear (LS) specimen; (b) U-shaped cross tensile (USCT) specimen (Lin and Pan 2008)

Adhesive joint

To calculate the stress intensity factor of an adhesive joint of thickness t , containing a crack of a certain length not too long with respect to its length, loaded by constant displacement u_2 , we can make use of the integral J . We chose the Γ contour to follow the specimen edges (Fig. 2.17). Then:

$$J = \int_{\text{ABCDEF}} \left(\pi dx_2 - t_i \frac{\partial u_i}{\partial x_1} ds \right) = \int_{\text{CD}} \pi dx_2 = \frac{E u_2^2}{2t} = G = \frac{K_I^2}{E} (1 - \nu^2) \quad (2.83)$$

because π and $\underline{t}$ are equal to zero along AB and EF, dx_2 and the derivative of u_2 are equal to zero along BC and DE.

The stress intensity factor does not depend on the crack length.

It can be noted that this calculation is easily extended to nonlinear behaviour.

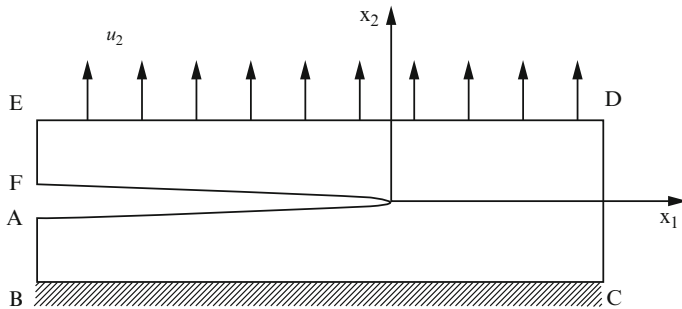


Fig. 2.17 Adhesive joint

2.2.7.5 Numerical Methods

Numerical methods are now the ones most commonly used

Analytic functions

The first we shall describe is based on the known form of the solutions of the equations of elasticity, involving one or more unknown functions of a complex variable. Singularities are accounted for by including a term in $\zeta^{1/2}$ and it is assumed that all other terms are regular and can be expressed as power series in ζ , with coefficients determined by the boundary conditions.

More precisely, we know (Sect. 2.2.4.3) that in plane stress and plane strain the solution involves two functions $\phi(\zeta)$ and $\psi(\zeta)$. In order to satisfy the boundary conditions on the faces of the crack these are put in the form:

$$\begin{aligned}\phi(\zeta) &= \zeta^{1/2} F(\zeta) \\ \psi(\zeta) &= \zeta^{1/2} \bar{F}(\zeta) - \zeta \phi'(\zeta)\end{aligned}\tag{2.84}$$

with $\bar{F}(\zeta) = F(\bar{\zeta})$.

We recall that:

$$\begin{aligned}\sigma_{11} + \sigma_{22} &= 4\text{Re}[\phi'(\zeta)] \\ \sigma_{22} - \sigma_{11} + 2i\sigma_{12} &= 2[\bar{\zeta}\phi''(\zeta) + \psi'(\zeta)]\end{aligned}\tag{2.36bis}$$

We can check that these forms for ϕ and ψ give a singularity $\zeta^{1/2}$ in the stress, as required; consequently we have to determine the function $F(\zeta)$ so that the other boundary conditions for the problem are satisfied. For this we can express it either as:

$$F(\zeta) = a_0 + a_1\zeta + a_2\zeta^2 + \dots \quad (2.85a)$$

or as:

$$F(\zeta) = a_0 + a_1\zeta + a_2\zeta^2 + \dots + b_1/(\zeta - C_1) + b_2/(\zeta - C_2) \quad (2.85b)$$

The second expression enables certain geometries to be handled better, with the proviso that the points C_i all lie outside the external boundary.

The process reduces the problem to that of minimising, in the least-squares sense, the difference between the actual boundary values and those given by the approximation chosen for $F(\zeta)$.

Continuously distributed dislocations (CDD) method

The theory of continuous dislocation distributions has been presented in Volume I (Chap. 3 Sect. 3.5.1.2c). This theory can also be used to calculate the stress intensity factors (see *e.g.* the book by Hills et al. 1996). Note that the CDD method involves the followings steps: (i) modelling the discontinuity across the crack surface by using a continuous distribution of edge dislocations, $b(s)$, where s measures length along the crack line; (ii) formulating the corresponding stress field, (iii) calculating the tractions leading to equilibrium integral equations, and (iv) solving the resulting system of integral equations for the unknown, $b(s)$, often, numerically by a collocation method. Note that the above steps are based on the superposition principle.

The crack-surface boundary conditions can alternatively be formulated by considering the *resultant forces on the crack surfaces* (in contrast to *crack-surface tractions*). The primary unknown in these methods can be the dislocation density or the crack opening displacement (COD); each may be formulated using either the crack-surface tractions or the crack-surface resultant-forces. Thus the CDD method may fall into four categories. Note that the CDD method is also referred to as integral transforms/continuous dislocations method (see also Azhdari et al. 2000).

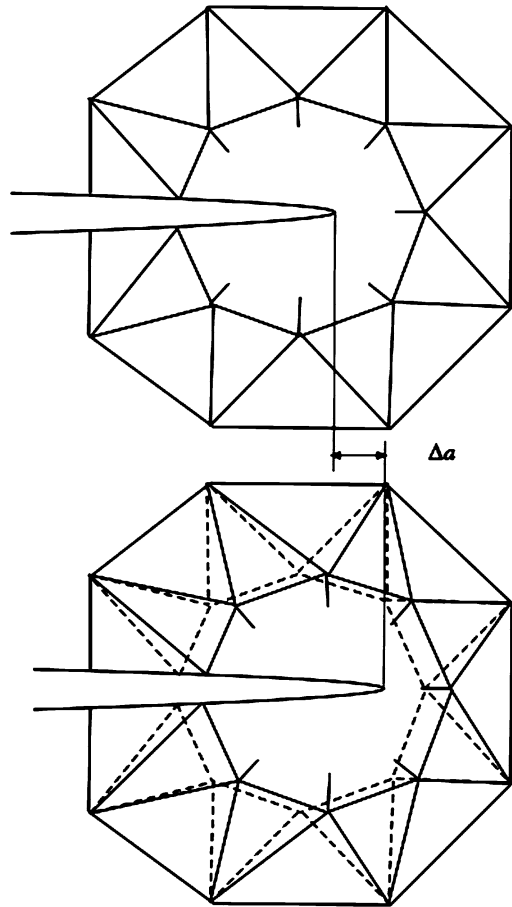
Finite elements method

The second method, the *finite elements method*, is the one most widely used and is the only practical method for any but simple geometries. There are several forms.

Extrapolation of the stress and strain fields. This is related to the experimental method described in Sect. 2.2.7.1. It requires a knowledge of the stress and strain fields, for the determination of which very fine meshes must be used so the high gradients at the crack tip are represented accurately. Most often special elements are used, which give the $r^{1/2}$ singularities automatically.

Energy method. This also is based on the known relation (2.55a and 2.55b) between K and G and the determination of the compliance (Eq. 2.7). Requiring two meshes it is relatively costly; the meshes must have comparable finenesses,

Fig. 2.18 Park's method:
advancing crack by
displacing nodes in the
immediate vicinity of the tip
(Parks 1974)



otherwise numerical errors will be introduced. A further disadvantage is that what the method gives is the value of G and that in the case of a mixture of modes I and II the separate values of K_I and K_{II} cannot be determined directly.

Perturbation method. This method, proposed by Parks (1974) involves calculating the change in energy when the node representing the crack is moved. No special elements are needed at the tip of the crack, and the method is less costly than the energy method since it requires only the solution corresponding to a given crack length. Further, it lends itself reasonably well to generalisation to three-dimensional problems.

Figure 2.18 illustrates the method. The mesh in full lines is that for the initial position of the crack and that in dotted lines is for the displaced position. The contour Γ_1 remains unchanged while Γ_0 , in the immediate neighborhood of the crack, moves when the crack advances a distance Δa .

Parks (1974) showed that if $[K]$ is the stiffness matrix the rate of release of energy can be determined with the aid of the expression

$$G = -\frac{1}{2} \underline{u}^T \cdot \frac{d[K]}{dl} \cdot \underline{u} \quad (2.86)$$

where $\underline{u}^T$ is the transpose of the displacement vector $\underline{u}$; $[K] = \sum_{i=1}^{N_c} [k_i^c]$.

$[k_i^c]$ is the stiffness matrix for an element between the contours Γ_0 and Γ_1 and N_c is the number of elements between these contours. Thus in contrast to the previous methods this requires only a few nodes of the stiffness matrix to be modified.

This method has advantages, but being based on the calculation of G it suffers from the same disadvantages as the previous method in the case of mixed-mode opening of the crack.

Method of the integral J .

It was shown that in linear elasticity $J = G$. From this it becomes possible to calculate G and therefore K_I for mode I.

2.2.7.6 Three-Dimensional Cracks

General considerations

In three dimensions, at all points along the tip of a crack there are three stress intensity factors, $K_I(s)$, $K_{II}(s)$, $K_{III}(s)$ (Fig. 2.19).

Theory shows that the form of the stress field in the immediate neighbourhood of a point on the crack tip is the same as in plane strain; in contrast, the points at which the crack reaches the surface are special in the sense that the singularity there is weaker than $r^{1/2}$. Figure 2.20 shows this edge effect for a crack in the form of a circular arc of radius a , emerging at a free surface; note that values of K_I for θ close to zero are not given when c , the crack depth, is much greater than a .

In two dimensions, writing the fracture conditions in the form of $G = G_c$ or of $K_I = K_{Ic}$ comes to the same thing, but this is not so in three dimensions since then K_I varies along the crack tip. Some authors prefer to use an energy criterion, and therefore the form $G = G_c$, whilst others take the view that fracture results when $\max_S K_I(s) = K_{Ic}$ along the crack front.

Finally, local approach to brittle fracture in steels by cleavage (see Chap. 3) leads one to think that the fracture condition would be better expressed as

$$\int_S K_I^4 ds = K_{Ic}^4 B \quad (2.87)$$

where B is the thickness of a plate for which an experiment on fracture with a two-dimensional crack has given the value K_{Ic} .

Fig. 2.19 Three dimensional cracks: (a) imbedded crack (b) corner crack

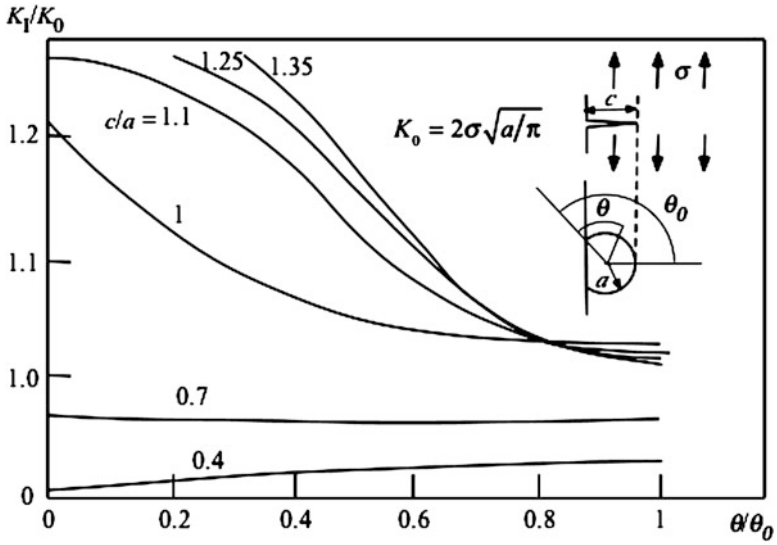
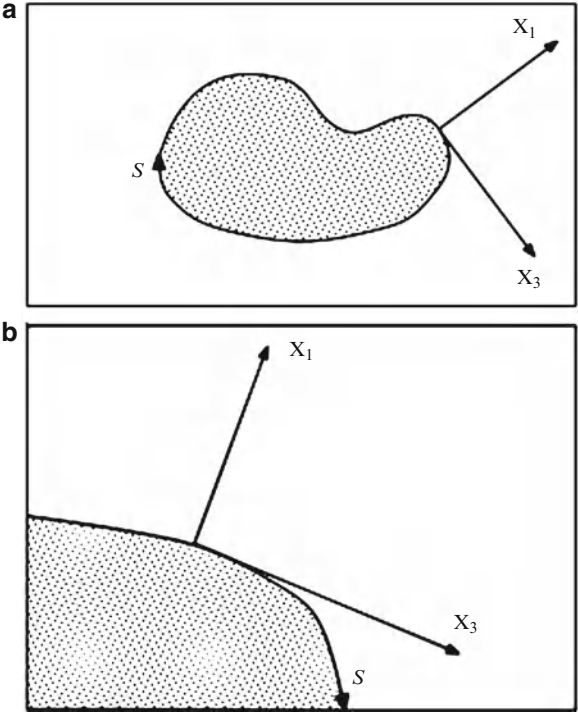


Fig. 2.20 Circular crack emanating from a semi-infinite body under uniform tensile stress. K_0 is the stress intensity factor of a penny-shaped crack

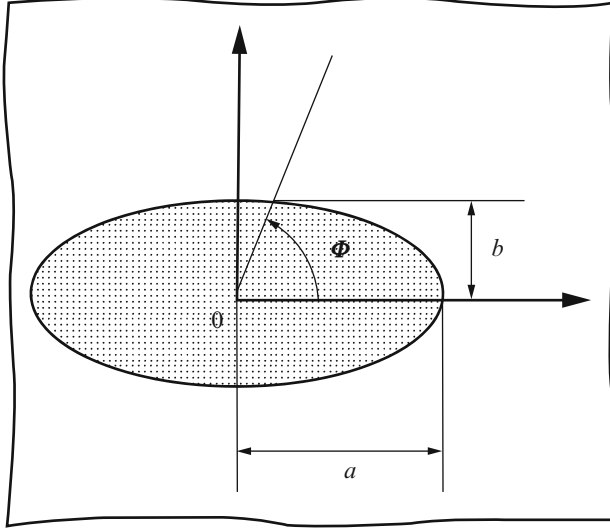


Fig. 2.21 Ellipsoidal crack in an infinite body under homogeneous uniaxial stress (mode I)

More information on 3D cracks is given in Labbens (1980), Bui (1978) and in Murakami et al. (1987).

Ellipsoidal crack

A generalisation of the penny-shaped crack discussed before in Sect. 2.2.7.1 is the ellipsoidal crack. We consider (Fig. 2.21) an ellipsoidal crack of semi-axes a and b in an infinite medium loaded by an homogenous uni-axial stress σ in mode I. The stress intensity factor, which is not constant along the tip, is given by:

$$K_I(\Phi) = \frac{\sigma \sqrt{\pi a}}{E(k)} \left(\sin^2 \Phi + \frac{a^2}{b^2} \cos^2 \Phi \right)^{1/4} \quad (2.88)$$

where $E(k)$ is the elliptic integral $E(k) = \int_0^{\pi/2} (1 - k^2 \cos^2 \Phi)^{1/2} d\Phi$ and where $k^2 = 1 - a^2/b^2$. Φ is the angle parameter of the ellipse.

K_I is maximum at the end of the minor axis ($\Phi = \pi/2$), where it has the value

$$K_{I\max} = \frac{\sigma \sqrt{\pi a}}{E(k)} \quad (2.89)$$

It increases as the eccentricity of the ellipse becomes greater. If $a = b$, the crack is circular (penny-shaped) and (2.88) reduces to (2.73).

2.2.7.7 Recapitulation

Table 2.5 at the end of the chapter provides solutions for some configurations, while Table 2.3 (see at the end of Sect. 2.5.4) contains the solutions for CT and 3 PB specimens. It is interesting to compare the loads per unit thickness needed to obtain, for a given value of the a/w ratio, say 0.5, a given value of the SIF for these specimens, with the load per unit thickness needed to obtain the same SIF for a central cracked panel of width W : $K_I W/(\pi a)^{1/2}$. These ratios are:

$$\begin{aligned}\frac{P_{CT}}{P_{CCP}} &= \frac{\sqrt{\pi}}{g_2(a/w)} \frac{w}{W} \sqrt{\frac{a}{w}} \\ \frac{P_{3PB}}{P_{CCP}} &= \frac{\sqrt{\pi}}{g_1(a/w)} \frac{w^2}{WS} \sqrt{\frac{a}{w}}\end{aligned}\quad (2.90)$$

Assuming w/W and w^2/WS to be equal to 1, those ratios are equal to 0.3 for $a/w = 0.5$. This shows that the loads needed in tests are smaller for specimens in which there is a bending component than in purely tensile cases.

It must be remembered also that the T stress is different for these various configurations.

2.3 Plastic Zones at the Crack Tip

2.3.1 Introduction

It is clear that the solution based on the assumption that conditions are elastic is not physically realistic, especially in the case of metals; qualitatively, one can see that peaks in the stresses at the tip of the crack will be smoothed out by plasticity. Two important questions arise. In the first place, to what extent does plasticity affect the elasticity solution? In the second place, the meaning of the energy relations for the fracture derived from Griffith's energy balance (Sect. 2.2.1.1) is less certain here and, in fact, since plasticity is by definition irreversible, the meaning of energy release rate is less clear.

In the study of perturbations due to plasticity the J integral will be essential. We defined and discussed the properties of this integral in Sect. 2.2.2 in elasticity. We now extend these to plasticity. This can be done inasmuch as the loading remains proportional and increasing monotonously and the crack does not grow. For, in that case, the behaviour is not different from nonlinear elasticity. So under those conditions, now again, the integral is contour independent and is equal to the strain energy release rate. It is equal to G in the limit of linear elasticity. The stress and strain fields ahead of the crack are those of Hutchinson (1968) and Rice and Rosengren (1968) (HRR fields in Sect. 2.2.2.6)

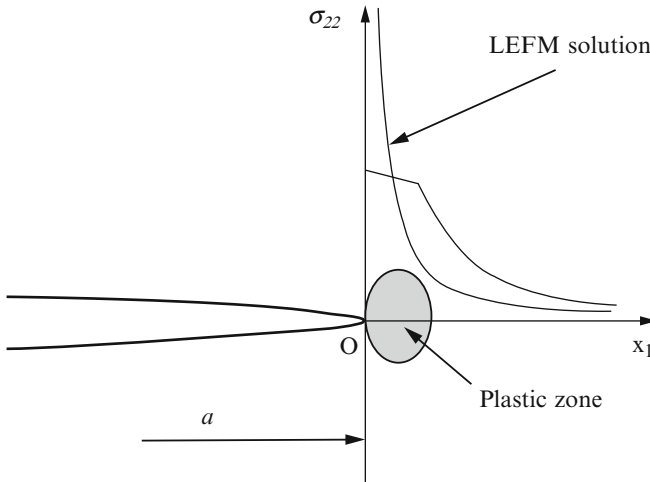


Fig. 2.22 Effect of plasticity in reducing the stresses around the *tip* of a crack

In this section we study the case in which the plastic region is confined to the immediate neighbourhood of the tip of the crack, as illustrated in the diagram of Fig. 2.22. The assumption is that the extent of this region is small compared with the rest of the structure, that is, with the length a of the crack and the width b of material ahead of the crack. We shall show that under certain conditions concerning the dimensions of the sample the problem can still be treated within the framework of linear fracture mechanics.

We will also discuss models describing more extensive plasticity.

As shown in the next section, it will be important to distinguish between plane stress and plane strain situations.

2.3.2 *Qualitative Description of Plastic Flow at the Crack Tip in Plane Stress and in Plane Strain*

There is a large difference in plastic flow at the tip of cracks according to plane stress, in a thin sheet or at a free surface, or plane strain in the interior of a thick body. We now give a qualitative description of these two situations limited to mode I.

2.3.2.1 Plane Stress

In plane stress (Fig. 2.23), as σ_{i3} are equal to zero, Ox_3 is a principal direction. In mode I, for reason of symmetry, Ox_1 is a principal direction. It follows that so is Ox_2 . The Mohr circle (Fig. 2.23c) representing the stress state in this zone has

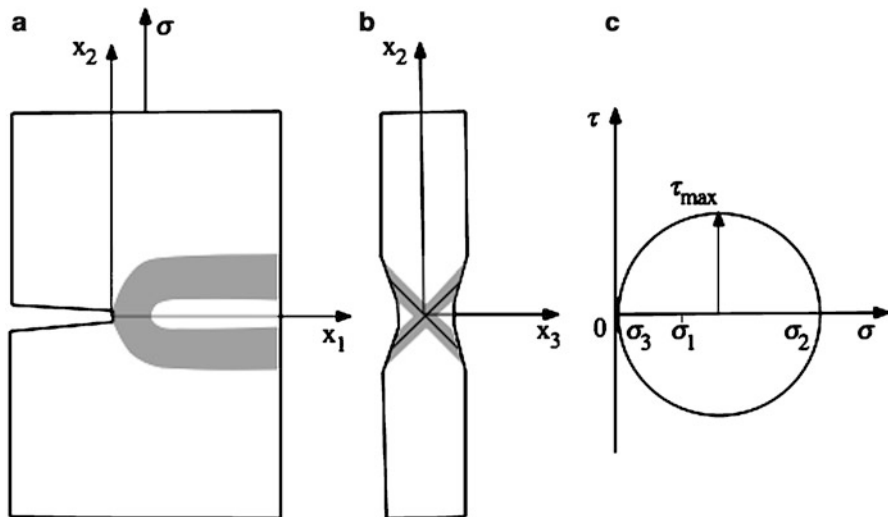


Fig. 2.23 Plastic deformation in plane stress (a) view in the plane of the plate (b) section of the plate (c) Mohr circle

radius k , the yield strength of the material in shear, and passes through the origin since $\sigma_3 = 0$. Since the maximum principal stress is σ_2 , the maximum shear acts on planes at angles of $\pm \pi/4$ to the directions Ox_2 , Ox_3 . As shown in Fig. 2.23, the plastic deformation results from slip along these directions (a), producing local necking (b) of an extent depending on the thickness of the plate. (a) shows the trace of the slip lines on the side surface of the plate.

With Tresca criterion, $k = R_p/2$, so that the maximum principal stress $\sigma_2 = R_p$.

2.3.2.2 Plane Strain

As in plane stress, in mode I again, the principal stresses are in the directions of the axes (Fig. 2.24). It is easily seen that $\sigma_1 < \sigma_2$ because of the blunting of the crack, however small this effect may be. In plane strain and in plasticity σ_3 is half the sum of σ_1 and σ_2 : this relation derives from the fact that, in plane strain, the Tresca and von Mises criteria coincide (as well as all the isotropic criteria independent of the mean stress σ_m) and from the Levy – von Mises equation $\epsilon_{33}^p = 0$ (see Volume I, Chap. 3, Sect. 3.5.2.4) associated with the vanishing of ϵ_{33}^p . Thus the intermediate stress is now σ_3 instead of σ_1 as in plane stress. The slip planes contain Ox_3 and are at angles $\pm \pi/4$ with Ox_1 and Ox_2 , as shown in (b). Slip takes place ahead of the crack in these directions. With the plasticity confined to a small volume the plastic zone has the shape of butterfly wings (a). Coming near to the surface, where flow occurs more easily, approaching that of the plane stress case, the slip directions change to $\pm \pi/4$ of $Ox_2 - Ox_3$ and the plastic zone is more extended. Far from

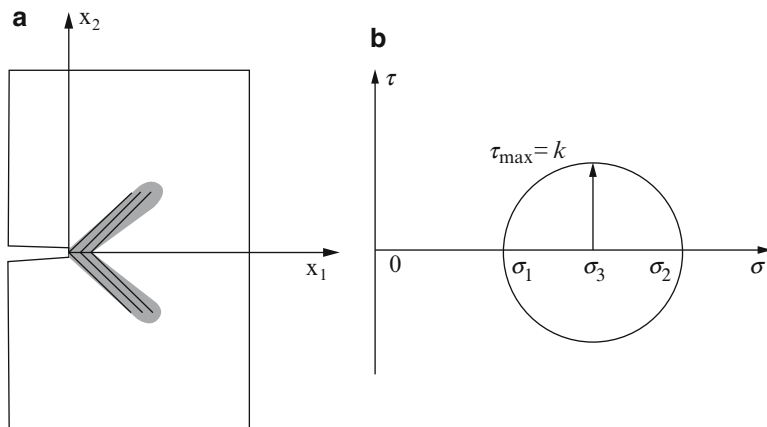


Fig. 2.24 Plastic deformation in plane strain (a) section in the *middle* of the thick structure (b) Mohr circle

the surface, the Mohr circle (b) is displaced on the tensile axis because, at the edge of the plastic zone along Ox_1 , the stress σ_1 must be equal to its value in the elastic singularity, which is positive (Eq. 2.34). Thus in the plastic zone there is a *high level of stress triaxiality*, characterised by the ratio $\sigma_m/2k$ where $\sigma_m = \sigma_{kk}/3$.

2.3.3 Plane Stress Yielding Solutions

2.3.3.1 Small Scale Yielding in Plane Stress

Small scale yielding refers to a plastic zone at the crack tip completely embedded in the elastic singularity in $r^{-1/2}$. In the plastic zone, for a material with no work-hardening, the stress remains constant, so that the strain, the product of which with the stress must vary as J , that is as r^{-1} , is proportional to r^{-1} . Outside of the plastic zone the stresses and the strains are given by Eq. (2.45), except for a small displacement of the origin, as we shall see. For a strain-hardening material the stress and strain fields are the HRR fields given in Sect. 2.2.2.6.

We give two solutions, one proposed by Irwin (1958) and the other based on the Dugdale-Barenblatt model; both are important for understanding the effect of plasticity.

Irwin's model

Irwin's model is applied with conditions of plane stress (Fig. 2.25). The material is assumed to be perfectly plastic and to fulfill Tresca's criterion $2k = R_p$, where R_p is the yield strength. As we have just seen, the stress σ_2 on the plane prolonging that

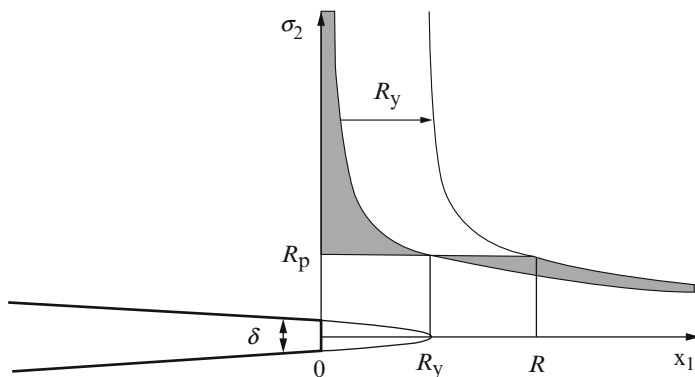


Fig. 2.25 Calculation of the extent of the plastic zone at the crack tip, using Irwin's model in plane stress

of the crack is a principal stress and in the plastic zone it is constant with the value R_p . With the stresses previously calculated for elastic conditions, in the plane $\theta = 0$, we get $\sigma_2 = K_I / \sqrt{(2\pi r)}$ and therefore σ_2 reaches the limit R_p at a distance R_y given by (see Eq. 2.45):

$$R_y = \frac{1}{2\pi} \left(\frac{K_I}{R_p} \right)^2 \quad (2.91)$$

In reality, the plastically deformed zone is more extended, because the equilibrium conditions for the stresses have to be satisfied. The need is to compensate for the truncation of the elastic distribution for $x_1 \leq R_y$; with this as aim a simple assumption is made, that the elastic distribution of σ_2 is shifted a distance X determined by equating the areas of the shaded regions in Fig. 2.25. Thus

$$\int_0^{R_y} \left(\frac{K_I}{\sqrt{2\pi x_1}} - R_p \right) dx_1 = R_p X \quad (2.92)$$

and hence the size of the plastic zone R in plane stress:

$$R = R_y + X = 2R_y = \frac{1}{\pi} \left(\frac{K_I}{R_p} \right)^2 \quad (2.93)$$

This result is clearly only approximate, since all that the calculation ensures is that the stresses are in equilibrium in the prolongation of the plane of the crack only. We shall see that the same result is given by a more rigorous calculation in mode III. It is valid only so long as the extent of the plastic zone remains small – the assumption of small scale yielding.

In small scale yielding the crack behaves as a longer fictitious crack in linear elasticity, the tip of which is in the middle of the plastic zone. The length increase $r_Y = R_Y$ is known as *the correction of Irwin*.

For a strain-hardening material, the constitutive equation of which is $\varepsilon/\varepsilon_0 = \alpha(\sigma/\sigma_0)^N$, where ε_0 is the deformation at the elastic limit and $\sigma_0 = E\varepsilon_0$, an approximate size of the plastic zone in the plane $\theta = 0$ is:

$$R = \frac{1}{\pi} \frac{N-1}{N+1} \left(\frac{K_I}{\sigma_0} \right)^2 \quad (2.94)$$

Since for most metallic materials N is large, between 5 and 20, we can see that work-hardening has only a small effect on the result (2.91). An approximation for the size of the plastic zone is often used, in which the yield strength R_p in (2.91) is replaced by a flow stress equal to the mean of R_p and the ultimate strength R_m .

The model of Fig. 2.25 shows also that the crack is blunted by plasticity. At the tip, which is at a distance R_Y from the tip of the fictitious elastic crack, the faces are separated by a quantity δ known as the *Crack Tip Opening Displacement* (CTOD), whose approximate value, using (2.44), is

$$\delta = 2u_2 \big|_{\theta=\pi, r=R_Y} = 2 \frac{4}{1+\nu} \frac{K_I}{2\mu} \sqrt{\frac{R_Y}{2\pi}} = \frac{4}{\pi} \frac{K_I^2}{E R_p} \quad (2.95)$$

(E is Young's modulus and μ the shear modulus).

CTOD from the J integral

The J integral allows calculation of the CTOD. The Γ contour is chosen to follow the limits of the plastic zone. As in a thin plate of thickness B , the plastic zone has a constant small height equal to B , corresponding to the side way slips, dx_2 can be considered as equal to zero, and:

$$\begin{aligned} J &= \int_{\Gamma} \left(\pi dx_2 - t_i \frac{\partial u_i}{\partial x_1} ds \right) = 2 \int_{\Gamma} \sigma_2 \frac{\partial u_2}{\partial x_1} dx_1 = 2 \int_0^{\delta/2} \sigma_2 du_2 \\ &= \int_0^{\delta/B} \sigma(\varepsilon) d\varepsilon \end{aligned} \quad (2.96)$$

With the constitutive behaviour just given, this equation yields:

$$J = G = \frac{K_I^2}{E} = \frac{N}{N+1} \frac{\sigma_0}{(B\alpha\sigma_0)^{1/N}} \delta^{\frac{N+1}{N}} \quad (2.97)$$

hence the CTOD. For a non work-hardening material, with $1/N = 0$, the CTOD is equal to K_I^2/ER_p not very different from that given by Eq. 2.95.

Various more rigorous calculations have been made, giving in all cases

$$CTOD = \delta \approx K_I^2 / E R_p \quad (2.98)$$

This result is easily obtained by recalling that K_I^2/E is the strain energy release rate and equating this to the work required in deforming, by a distance δ , a virtual tensile test piece of yield strength R_p situated at the tip of the crack.

Reasoning by analogy can lift the restriction to plane stress. In this a fictitious elastic yield strength is introduced so as to take account *in plane strain* of the triaxiality, as noted previously (Sect. 2.3.2.2). The calculation, which is not completely rigorous, gives the result:

$$R_{PS} \simeq \frac{1}{3\pi} \left(\frac{K_I}{R_p} \right)^2 \quad (2.99a)$$

In plane strain the correction of Irwin r_Y is $(1/6\pi)(K_I/R_p)^2$.

Detailed numerical calculations in plane strain yielding were made by Mc Meeking (1977). He proposed the following relation for a material with the constitutive equation $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$:

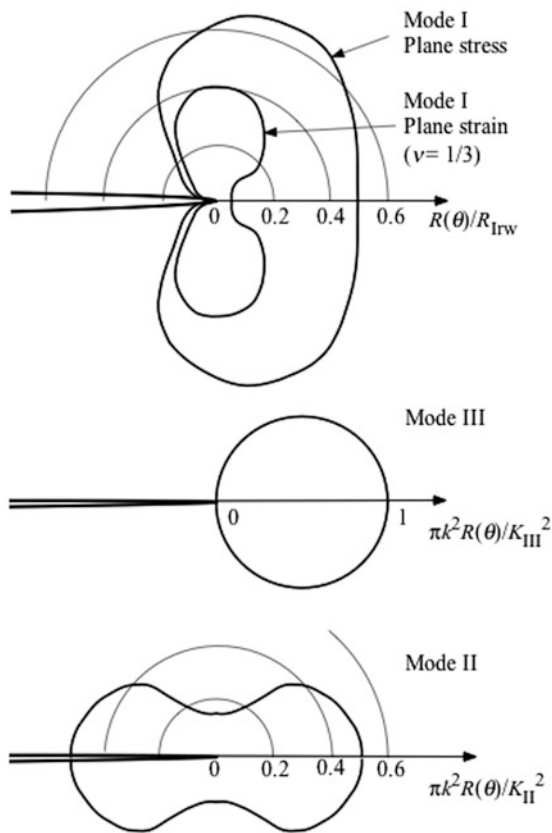
$$\delta = 0.55 \frac{J}{\sigma_0} \left[\frac{\left(2/\sqrt{3} \right) (1+\nu) (N+1) \sigma_0}{E} \right]^N \quad (2.99b)$$

Notes:

1. The model just described does not claim to represent the plastic zone correctly over the whole of the plane. More elaborate analytical and numerical calculations have been made, some of the results of which are given in Fig. 2.26 together with references sources of further information. The estimates of size given by Irwin's simple model are always of the right order of magnitude, but those of the *shape* depend strongly on the problem treated, as we should expect from the discussion on the slip planes in the previous section.
2. In *mode III*, Rice (1971) obtained a rigorous solution for a non-hardening material. The plastic zone is circular of radius $(1/2\pi) (K_{III}/k)^2$, the crack behaving as a fictitious crack in linear elasticity, the tip of which is at the centre of the plastic zone. The displacement at the crack tip is $2K_{III}^2/\pi\mu k$.
3. Some *experimental investigations* have been made of the extent of the plastic zone, and in some cases of the form also; most of these have concerned fatigue cracks. Various techniques have been used, including
 - measurements of micro-hardness
 - examination of surface relief, for example by holography
 - metallographic attack

Figure 2.27 gives an example.

Fig. 2.26 Results of calculations of plastic zone shapes in small scale yielding



2.3.3.2 Dugdale (1960), Barenblatt (1962), and Bilby-Cottrell-Swinden (1963) Models

These models enable us to relax requirement that the plastic zone is confined to a small region in the immediate neighbourhood of a singularity; they provide an illustration of the extension of the principle of superposition of solutions.

Figure 2.28 gives the principle. We consider a crack of length $2a$ in an infinite plate acted on by a uniform stress σ at infinity; the model is in plane stress and mode I. The plastic zone, which we have shown to be of small height equal to the plate thickness B (Sect. 2.3.2.1), is represented by constant forces equal to the yield strength of the non work-hardening material. These are exerted over lengths R in the material just before each extremity of the crack, where R is the size of the plastic zone, the quantity to be determined.

The solution can be obtained by applying the weight function method (Sect. 2.2.7.1) to a fictitious crack of length $2(a + R)$, which includes the plastic

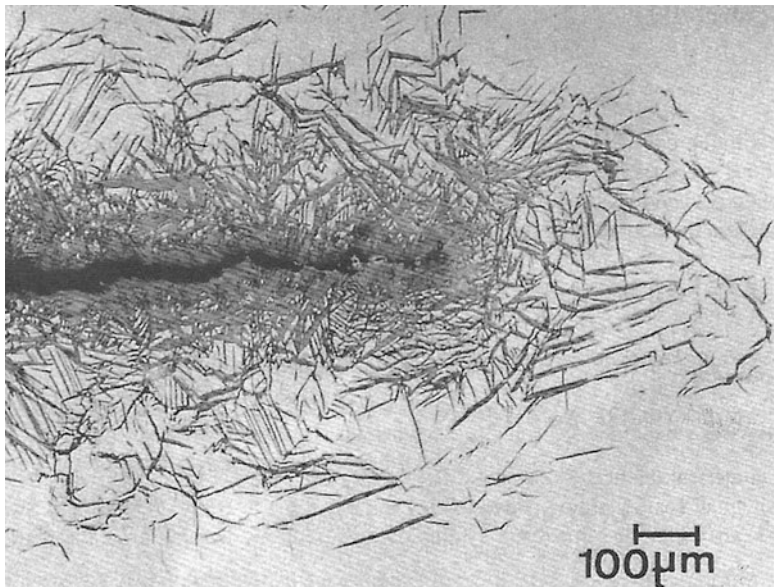
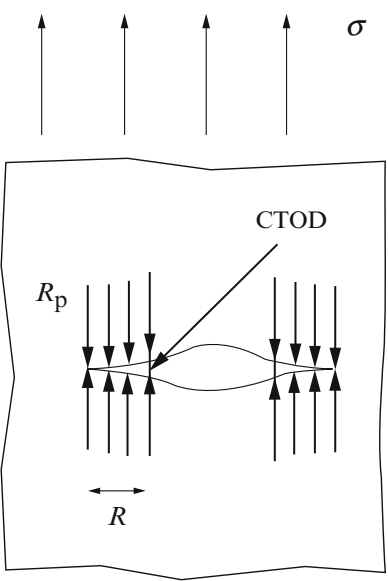


Fig. 2.27 Plastic wake surrounding a fatigue crack in an unstable stainless steel. The martensitic transformation $\gamma \rightarrow \alpha'$ and mechanical twinning display the plastic zone

Fig. 2.28 The Dugdale-Barenblatt model. There is no stress singularity at the fictitious crack tip and the radius of curvature there is zero



zones, and at the tip of which there is no stress singularity so as to give finite stresses on the boundary of the plastic zone. Applying (2.66) and (2.67b), we find:

$$\sigma \sqrt{\pi a} - \left[\frac{\pi}{2} - \arcsin\left(\frac{a+R}{2a}\right)^{1/2} + \arcsin\left(\frac{a-R}{2a}\right)^{1/2} \right] = 0 \quad (2.100)$$

Hence:

$$R = a[\sec(\pi\sigma/2R_p) - 1] \quad (2.101)$$

If $\sigma \ll R_p$ the power-series expansion of this can be truncated to give

$$R \approx (\pi/8)(K_I/R_p)^2 \quad (2.102)$$

The numerical factor $\pi/8$ does not differ greatly from the $1/\pi$ given by Irwin's model in which small-scale yielding is assumed. The model due to Bilby et al. (1963) arrived at the same result by employing a fictitious distribution of dislocations.

These two models correspond quite well with reality in the case of thin plates. Further, they enable the CTOD δ to be determined.

The separation at the tips of the real crack is given by

$$\delta/a = (8R_p/\pi E)\log\sec(\pi\sigma/2R_p) \quad (2.103)$$

This equation, often referred to as the “logsec formula”, was the basis of the CTOD design method, which was developed by Wells and the British Welding Institute (Standard ISO 12135:2002).

It is interesting that experimental measurements made with a polymer confirm this result quite well: the “crazing” (see Sect. 9.1) found in such material represents a system that is doubtless well adapted to the cohesion forces associated with plasticity.

The Dugdale-Barenblatt model also enables sharper bounds to be given for the validity of the small-scale yielding solutions. For this, the size R of the plastic zone and the CTOD δ , calculated from (2.93) and (2.95), respectively, are represented as non-dimensional numbers and these are compared with the values found from (2.101) to (2.102), plane stress being assumed in all cases. Figure 2.29 gives this comparison, showing that the previous solutions cease to be reasonably valid when the applied stress exceeds one-third, perhaps one-half, of the yield strength.

Equations 2.101 and 2.102 give a plastic zone size and a CTOD, which go to infinity when the applied stress reaches the yield strength. The crack then loses its sharpness completely. This is of course what is expected.

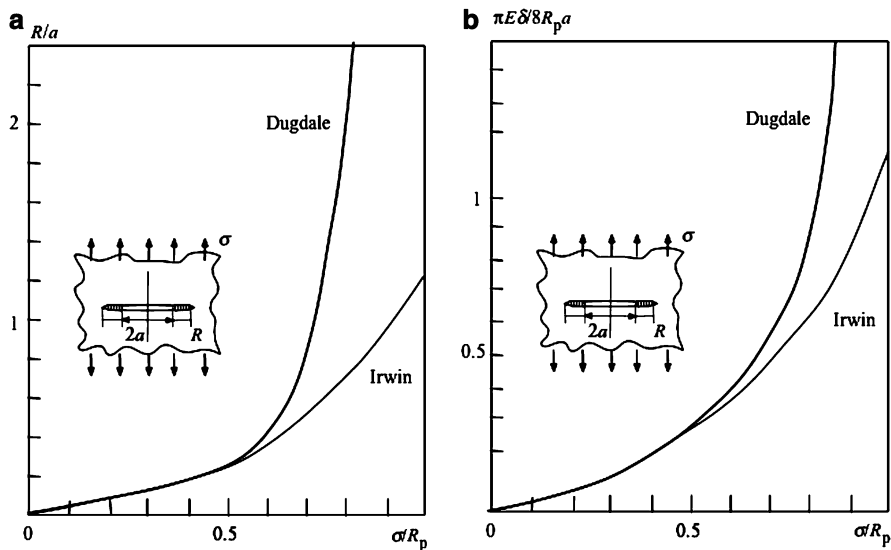


Fig. 2.29 Comparison of the Dugdale-Barenblatt and Irwin models for plane stress in mode I. (a) size of the plastic zone; (b) CTOD δ

2.3.3.3 Plastic Zone at the Surface Where the Crack Emerges

At the surface where the crack emerges, the plastic zone is not of small height as in a thin plate notwithstanding that the condition is plane stress. The plastic zone is again the place where the slips inclined at 45 degrees reach the surface, but they originate deep inside the body from the region in plane strain (Fig. 2.30).

An approximate shape of the plastic zone can be obtained by the locus in plane stress of the von Mises yield strength deduced from (2.45):

$$R(\theta) = \frac{R_{\text{Irwin}}}{2} \cos^2 \frac{\theta}{2} \left(1 + 3 \sin^2 \frac{\theta}{2} \right) \quad (2.104)$$

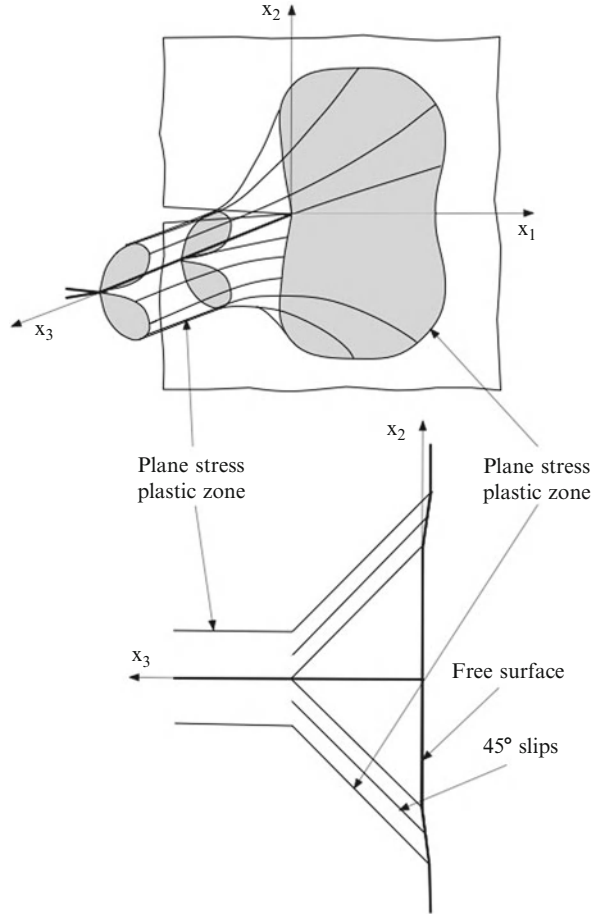
Thus $R(0) = 0.5 R_{\text{Irwin}}$ and the maximum distance $R_{\text{max}} = 0.67 R_{\text{Irwin}}$.

The shape of the plastic zone given by this equation is shown in Fig. 2.26.

The T stress (see Sect. 2.2.4.4) modifies the value of the von Mises yield strength and thus of the shape and extent of the plastic zone. In plane stress, the von Mises equivalent stress along the Ox_1 axis ahead of the crack, taking account of the T stress, is given by:

$$\sigma_{\text{eq}}^2 = \frac{K_I^2}{2\pi x_1} + T \frac{K_I}{\sqrt{2\pi x_1}} + T^2 \quad (2.105)$$

Fig. 2.30 Plane strain and plane stress *plastic zones* in a thick plate, showing the way the plane stress slips emerge at 45° to the surface. This creates a *dip* at the surface



The ratio of the extent r_p of the plastic zone along this Ox_1 axis to its dimension r_Y for $T = 0$ (without the Irwin correction) can then be easily calculated:

$$\frac{r_p}{r_Y} = \left[1 - \frac{1}{2} \left(\frac{T}{R_p} \right)^2 - \frac{T}{R_p} \sqrt{1 - \frac{3}{4} \left(\frac{T}{R_p} \right)^2} \right]^{-1} \quad (2.106)$$

It goes through a minimum for $T/R_p = -\sqrt{3}/3$ and keeps increasing with T/R_p above this value, and tends to infinity when the T stress approaches the yield strength. Thus, the extent of the plastic zone is smaller in a CT specimen (positive T stress) than in a CCP one (negative T stress) (see Table 2.1).

2.3.3.4 Validity of the Small Scale Yielding Approximation

The small scale yielding approximation assumes that the external surface of the body is at infinity. Thus, for this approximation to be valid, the ligament of length b ahead of the crack must be large enough. Now, the stress intensity factor can generally be written:

$$K_I = F\left(\frac{a}{b}\right) \sigma \sqrt{\pi a} \quad (2.107)$$

The distance r^* at which the stress in the elastic singularity becomes equal to the applied stress σ is given by:

$$r^* = \frac{a}{2} F^2\left(\frac{a}{b}\right) = \frac{1}{2\pi} \left(\frac{K_I}{R_p}\right)^2 \left(\frac{R_p}{\sigma}\right)^2 \quad (2.108)$$

Assuming that at a stress level equal to half of the yield strength, the plastic zone size R must remain less than a quarter of the ligament length b , the following condition follows:

$$b \geq 2.5 \left(\frac{K_I}{R_p}\right)^2 \quad (2.109)$$

This is the condition of validity of the small scale yielding approximation, and thus of linear elastic fracture mechanics, which is recommended in standards.

2.3.4 Plane Strain Solutions

2.3.4.1 von Mises Locus of the Plastic Zone Limit

From Eq. 2.45 the distance $R(\theta)$ of the plastic zone limit to the crack tip is given by:

$$R(\theta) = \frac{R_{Irwin}}{2} \cos^2 \frac{\theta}{2} \left[3 \sin^2 \frac{\theta}{2} + (1 - 2\nu)^2 \right] \quad (2.110)$$

The largest distance is such that:

$$R_{\max} = \frac{R_{Irwin}}{24} \left[3 + (1 - 2\nu)^2 \right]^2 \quad (2.111)$$

If the Poisson ratio ν is equal to $1/3$, $R(0) = 0.056 R_{Irwin}$ and $R_{\max} = 0.4 R_{Irwin}$.

If it is equal to $1/2$, $R(0) = 0$ and $R_{\max} = 0.375 R_{Irwin}$.

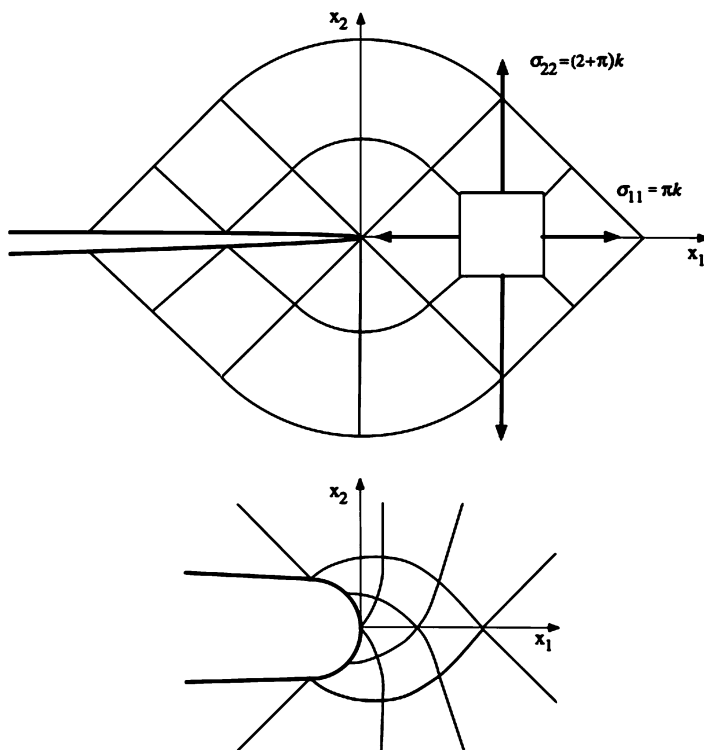


Fig. 2.31 The Prandtl slip lines field at the *tip* of a crack. The *lower figure* shows the logarithmic spirals at the *blunted tip*

These values are to be compared with the ones found above in plane stress (Sect. 2.3.3.3).

The shape is that shown in Fig. 2.26.

In plane strain the calculation of the extension of the plastic zone under the influence of the T stress is manageable but more complicated than in plane stress. It can be surmised that its influence would be similar.

2.3.4.2 Perfect Plasticity; The Prandtl⁸ Slip Lines Field

We consider here a mode I crack in a perfectly plastic material in conditions of plane strain. The stress distribution ahead of the crack can be obtained from the equilibrium equations and the plasticity criterion, for which we can use the slip-lines technique (see Volume I, Sect. 3.4). This enables to show, analogously to the case of a punch (*cf.* Volume I, Sect. 3.4.3.5), that the stresses are as shown in Fig. 2.31.

⁸Ludwig Prandtl (1875–1953) was a German scientist.

In this figure, in the first “diamond” region $0 < \theta < \pi/4$

$$\sigma_{11} = \pi k, \quad \sigma_{12} = 0, \quad \sigma_{22} = (2 + \pi)k$$

In the “fan” region $\pi/4 < \theta < 3\pi/4$

$$\sigma_{\theta\theta} = (1 + 3\pi/2 - 2\theta)k, \quad \sigma_{r\theta} = k, \quad \sigma_{rr} = (1 + 3\pi/2 - 2\theta)k$$

In the second “diamond” region $3\pi/4 < \theta < \pi$

$$\sigma_{11} = k, \quad \sigma_{12} = 0, \quad \sigma_{22} = 0.$$

where k , the yield strength in shear, is $R_p/\sqrt{3}$ for a material to which the von Mises criterion applies and $R_p/2$ for the Tresca criterion.

It follows that there is no singularity at the tip of the crack and that the stresses near there depend only on θ . (Note that the slip lines extend to the plastic zone limit, the shape of which is shown in Figs. 2.26 and 2.30.). As discussed before (Sect. 2.3.2.2), the Mohr circles are displaced to the tensile axis and there is a high stress triaxiality ahead of the crack.

At the tip of the crack the stress σ_{11} should be equal to zero. This is the case if we take account of the blunting of the crack. Assuming that the crack profile at the tip is circular, there is a zone of slip lines, which are logarithmic spirals, as shown in the bottom part of Fig. 2.31. In this zone the stresses increase logarithmically to reach the preceding solution at a distance $r/\delta = (1/2) \exp(\pi/2) = 2.4$, δ being the CTOD. It can be shown that all the strains are regular with the exception of the $\varepsilon_{r\theta}$ component in the fan, which has the form $(k/2\mu r)R(\theta)$, $R(\theta)$ being the distance of the plastic zone limit. This strain field thus has a strong singularity and deformation is concentrated in the two fans on either sides of the crack.

2.3.4.3 Mixed Modes

The limits of the plastic zone in small scale yielding were calculated in mixed mode for various values of the strain hardening exponent N , the hardening behaviour being represented by $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$ (Hult and Mc Clintock 1956; Rice 1971; Shih 1976). The size of the plastic zone keeps being proportional to $(K/\sigma_0)^2$. Figure 2.32 shows the results for various values of the parameter $M^c = (2/\pi)\arctan(K_I/K_{II})$.

2.4 Fracture Toughness Measurements (LEFM)

In LEFM crack propagation takes place when the stress intensity factor K (or the strain energy release rate G) reaches a critical value, the fracture toughness. However, in many situations the extent of plasticity is such that the linear elastic

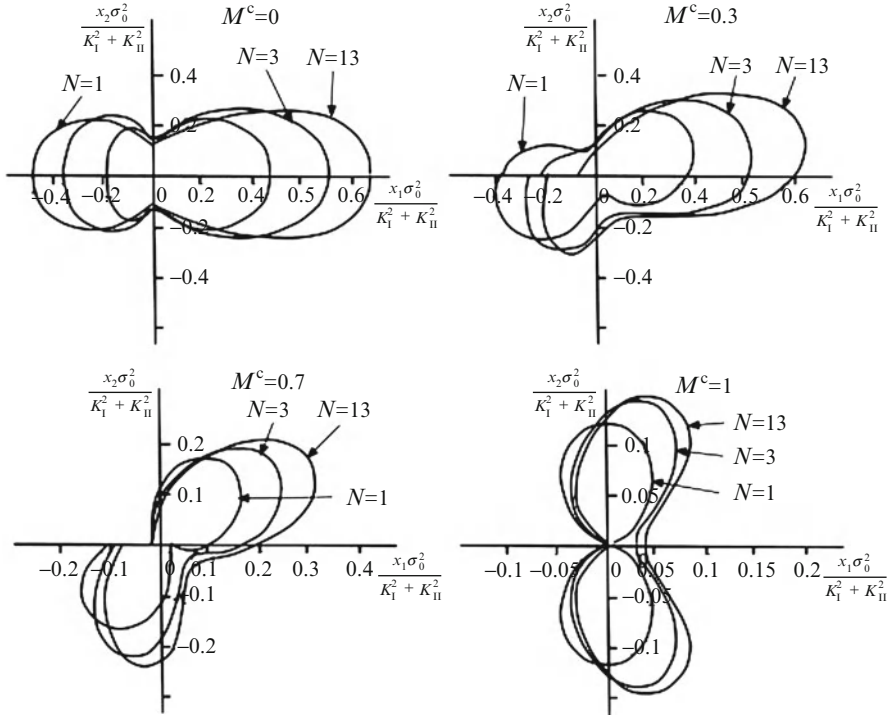


Fig. 2.32 Plastic zone for various values of the mixed mode parameter: $M^c = (2/\pi)\arctan(K_I/K_{II})$ and for various strain hardening exponents N

approximation is no longer valid. In that case the behaviour of structures containing cracks is approached by methods of elastoplastic fracture mechanics (EPFM), which are based on the CTOD on the one hand and on the J integral on the other. It is considered that crack propagation begins when one of these reaches a critical value.

But in the first place we will discuss initiation of crack propagation and propagation itself up to instability using the concept of the “R curve”. We will exclude for the time being stable propagation phenomena associated with fatigue, stress-corrosion or creep, which will be studied in specific chapters further on.

This will allow to explain how is measured the fracture toughness. Afterwards, we will discuss the limit loads in plasticity. Finally, we will give some indications about methods to insure the integrity of structures.

2.4.1 Stable and Unstable Crack Propagation

Experiment shows that in many cases, a crack in a structure begins to propagate for a certain critical condition and then grows only if the applied load keeps increasing,

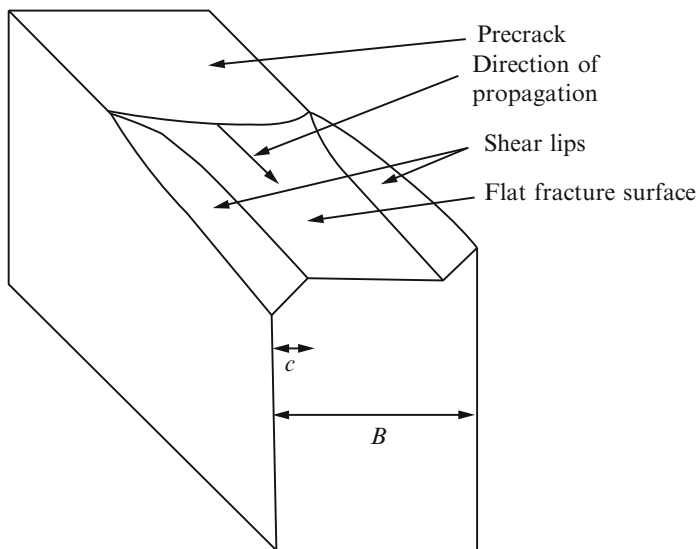


Fig. 2.33 Propagation of a pre-crack across a plate, showing the flat fracture surface in the *centre* (plane strain) and the shear lips on both sides (plane stress)

until instability occurs and instant fracture takes place. This behaviour is associated with the interaction between the evolution of the loading parameter K in LEFM or J in EPFM, and the energy needed for growth, the fracture toughness, G_c or J_c .

Equations 2.64 and 2.67b correspond to classical situations of stable and unstable crack propagation. If the propagation takes place when $G = G_c$, Eq. 2.64 shows that a crack loaded by point forces propagates only if they increase as G is a decreasing function of the crack length a . This is not the case in the loading situation corresponding to Eq. 2.67b in which G is an increasing function of a .

However, stable propagation can also occur in this last situation if the fracture toughness G_c increases as the crack grows. We call this the *R-curve* effect.

2.4.2 *R-Curve*

2.4.2.1 Shear Lips

As seen in Fig. 2.33 a pre-crack, most often created by fatigue in a test piece or in a component, starts to propagate in the centre where the conditions are plane strain. Indeed this is where the stress triaxiality is the highest, which favors the various modes of fracture. There the fracture occurs in mode I and the fracture surface is flat. But, over the sides where the conditions are those of plane stress, fracture occurs not in pure mode I but by shear, as indicated in Fig. 2.33. The shear lips created do

not have immediately their full extent, corresponding to the size of the plane stress plastic zone. Their development is progressive. The shear lips can be oriented as well in the same direction (as shown in Fig. 2.33) or in opposite directions.

2.4.2.2 Energy for Crack Propagation

The energy dissipated in the central plane strain region is much less than the one dissipated in the shear lips. This is due to the higher stress triaxiality in this zone, which favors the micro-mechanisms of damage. These occur for a low strain to which corresponds a low density of plastic deformation energy. Moreover, the volume of the plastic zone being limited, the total plastic deformation work is reduced. The energy needed for crack propagation in the plane strain zone in mode I is denoted G_{Ic} .

In contrast, the plastic deformation work in the shear lips, denoted G_c , is large for inverse reasons: the strain needed for damage to occur there is large and so is the plastic zone size.

The energy dissipated by unit area of crack propagation G_R is the average of the energy dissipated in plane strain fracture and of the one dissipated in plane stress in the shear lips. Crack propagation begins in the plate centre when $G_R = G_{Ic}$. It then increases as the shear lips become wider; this is the *R-curve* of the plate. When they reach their full extent with a width equal to c the fracture toughness is given by:

$$G_{Rmax} = G^c = \frac{2c}{B}G_c + \frac{B-2c}{B}G_{Ic} \quad (2.112)$$

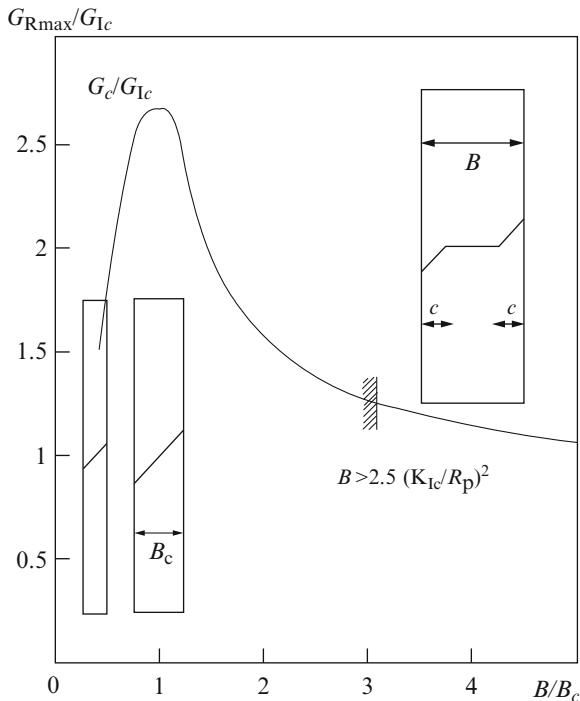
where B is the plate thickness. In view of (2.93), which gives the plastic zone size in plane stress, the shear lip width is given by:

$$c = \frac{1}{2\pi} \left(\frac{K_c}{R_p} \right)^2 = \frac{1}{2\pi} \frac{EG_c}{R_p^2} \quad (2.113)$$

hence the evolution of the fracture toughness G_c as a function of the plate thickness B shown in Fig. 2.34. When the thickness becomes very large, the shear lips size is negligible, and the fracture toughness is that corresponding to plane strain denoted G_{Ic} . For a width smaller than twice the full shear lip size c , the fracture toughness decreases as the plate thickness becomes smaller, because the shear lips, which occupy the whole thickness, decrease in size themselves.

There is another reason for the increase of the strain energy with crack propagation. It is a modification of the stress singularity, which becomes weaker for the moving crack (Rice and Sorensen 1978). Hence, a higher stress intensity factor is needed to reach the same damage in the plastic zone. Moreover in ductile fracture the mechanisms of crack initiation and crack propagation are different (Lautridou and Pineau 1981), as explained in Chap. 4.

Fig. 2.34 Evolution of the fracture toughness G_{Rmax} as a function of the plate thickness B . The critical thickness B_c , corresponding to the disappearance of the flat fracture surface, is equal to twice the shear lip size c given by (2.113). The figure includes also the size limit for plane strain measurement



2.4.2.3 Loading Curve

In Fig. 2.35 the evolution of the crack driving force G is shown, for two different loading conditions, as a function of crack length a for constant values of applied stress σ and applied load F respectively. In the first case the variation is linear, whereas in the second it is hyperbolic. The R curve $G_c(a)$, a property of the material and of the plate thickness, relates to an initial length a_0 . Crack propagation requires $G > G_c$. If $\partial G/\partial a > dG_c/da$: it occurs only if the stress or the load is increased, that is, there is stable crack growth. But if $\partial G/\partial a < dG_c/da$ then keeping the stress or the load constant will provide more energy than is needed and consequently the growth becomes unstable.

In Fig. 2.35 the first case represented is that of a large plate loaded at infinity by distributed constant forces. The stress intensity factor is given by Eq. (2.67b), so that

$$G = \frac{\pi \sigma^2 a}{E} \quad (2.114)$$

in plane stress, E being the Young modulus. For constant stress, G is a linear function of a . As long as σ remains smaller than the value corresponding to

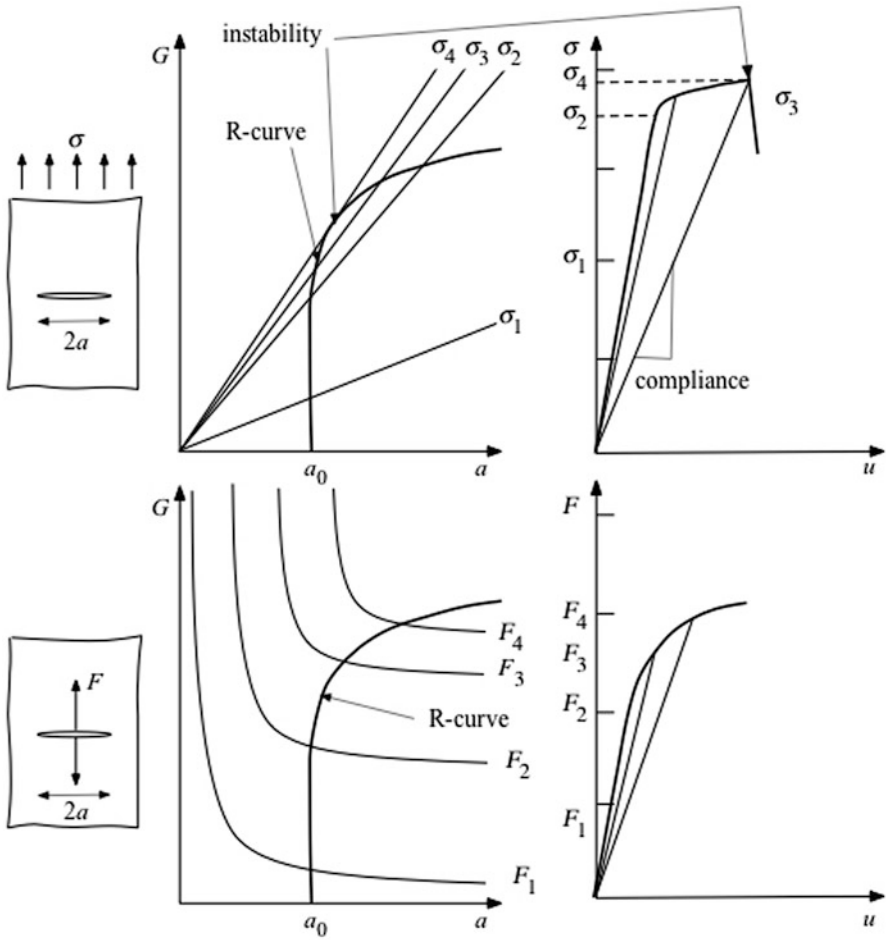


Fig. 2.35 Stable crack propagation and instability. Crack propagation starts when $G = G_{Ic}$. In a large plate loaded by distributed forces $G(a)$ is an increasing function; when it tangents the R curve, the propagation becomes unstable. In a large plate loaded by point forces $G(a)$ is a decreasing function; there is no instability

$G = G_{Ic}$, the crack opens, but does not propagate; the crack tip blunts owing to plastic deformation (Fig. 2.36b). When this condition is fulfilled, crack growth begins in the central part of the plate, in plane strain condition; it is the initiation of tearing. Simultaneously, the crack tip opening displacement reaches a critical value δ_c (Fig. 2.36c). For propagation to continue, an increase of σ is needed. The propagation is stable. It becomes unstable, when the stress reaches a value such that the $G(a)$ is tangent to the R curve.

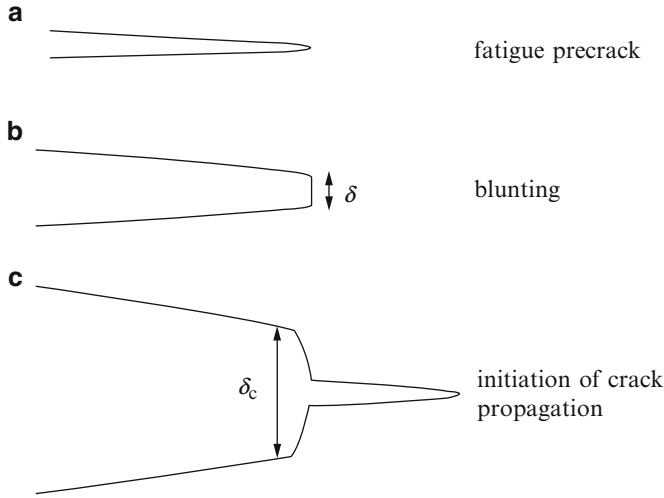


Fig. 2.36 Stage of crack tip evolution: fatigue pre-crack; crack tip blunting; initiation of tearing

The second case represented in Fig. 2.35 is that of a large plate loaded by point forces applied on the middle of the crack. The stress intensity factor is given by (2.64), so that

$$G = \frac{F^2}{\pi E a} \quad (2.115)$$

In that case $G(a)$ is a decreasing function; there is no instability.

Figure 2.37 shows also the loading curve σ as a function of the displacement u of the applied forces. As the crack growth occurs in stable condition, the compliance of the plate increases. This could be measured by the unloading slope. This decrease produces a deviation from linearity of the loading curve.

Figure 2.37 shows the R curves for two different initial crack lengths, a_0 and a_1 . It is assumed that the R curve is simply translated. The construction of the condition of instability shows that it does not take place for the same values of G . The amount of stable crack propagation is also different. Therefore, the fracture toughness measured at crack instability is not an intrinsic property. Similarly, for other specimen geometries, for other structural members, the set of $G(a)$ curves would differ, and the point of instability would be different in each case. In order to predict stable crack propagation and instability a full knowledge of the R curve would be required. This is not a simple requirement, because the R curve is thickness dependent. However, the initiation of crack propagation in the central part of the specimens in plane strain condition always occurs for the same value of toughness: G_{Ic} .

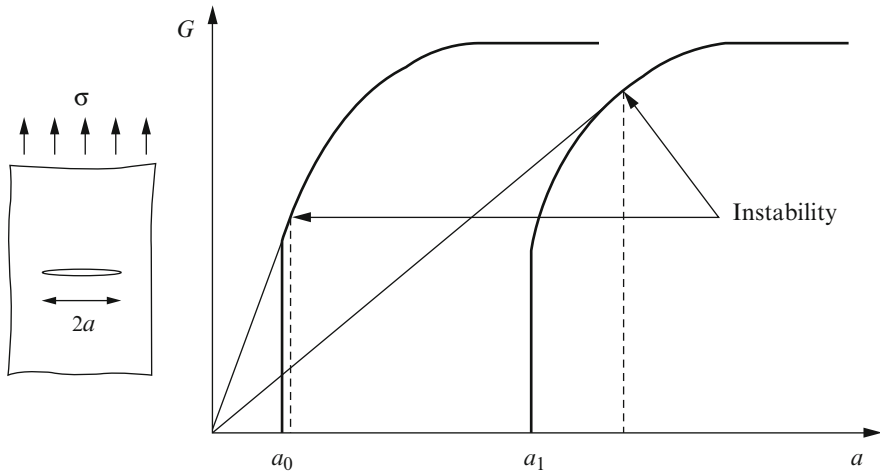


Fig. 2.37 The condition of crack propagation instability depends on the initial crack length

2.4.2.4 K_{Ic} Measurements

We wish to find a value for the fracture energy, and therefore for the fracture toughness, that is independent of thickness and is an intrinsic property of the material. We need the condition of small scale yielding to be fulfilled in order to be able to use LEFM. This value of the fracture toughness is denoted K_{Ic} , the mode I fracture toughness in plane strain. So we must ensure that the thickness of the test piece is great enough. Further, we should wish the plastic zone not to emerge on the front or on the back surfaces. In Sect. 2.3.3.4 we gave already the condition for the ligament size b (see Eq. 2.109). It holds also for the crack depth a . Thus the practical conditions are:

$$a, B, b > 2.5(K_{Ic}/R_p)^2 \quad (2.116)$$

When the asymptotic value G_{Ic} is reached the toughness K_{Ic} is given by:

$$K_{Ic} = \left(\frac{EG_{Ic}}{1 - \nu^2} \right)^{1/2} \quad (2.117)$$

The *specimens* used to measure the fracture toughness can be of different shapes, as far as the stress intensity factor is known as a function of the crack length. Standards propose various possibilities (ISO 12135:2002). The most widely used specimens are the *three (or four) points bend* (3PB) specimen and the *compact*

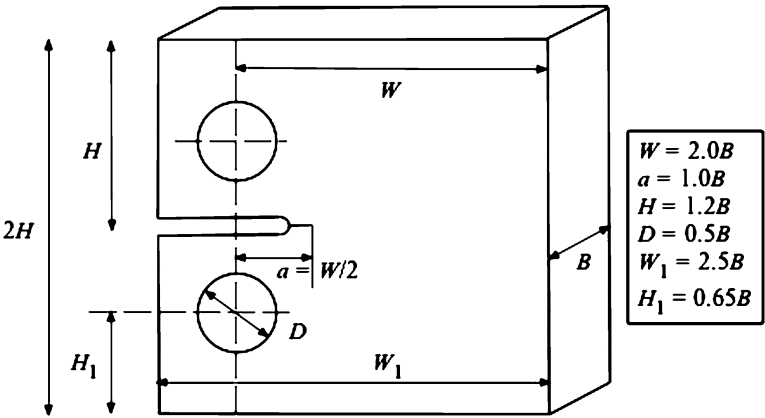


Fig. 2.38 The CT specimen

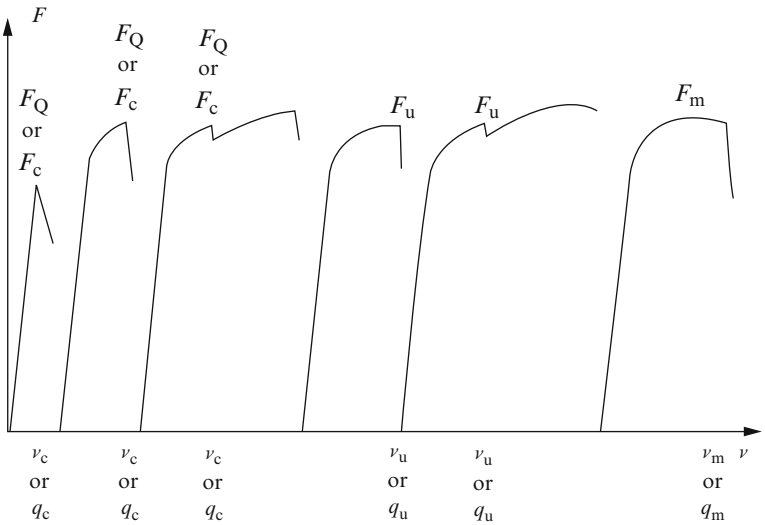


Fig. 2.39 Various shapes of load/displacement recordings and standard denominations

tension (CT) specimen (Fig. 2.38). This specimen shape has the advantage of a limited volume of material. The dimensions are homothetic, once the thickness for instance has been chosen.

The deformation of the specimens must be followed in the course of the test so as to insure there is no deviation from linear elasticity. There is no need to measure the displacement of the point of application of the load. We can simply measure the crack opening displacement on the back face of the specimen. This is usually done with a clip-gauge. If the material is very brittle, the loading curve remains completely linear up to sudden fracture (Fig. 2.39). Working with metallic

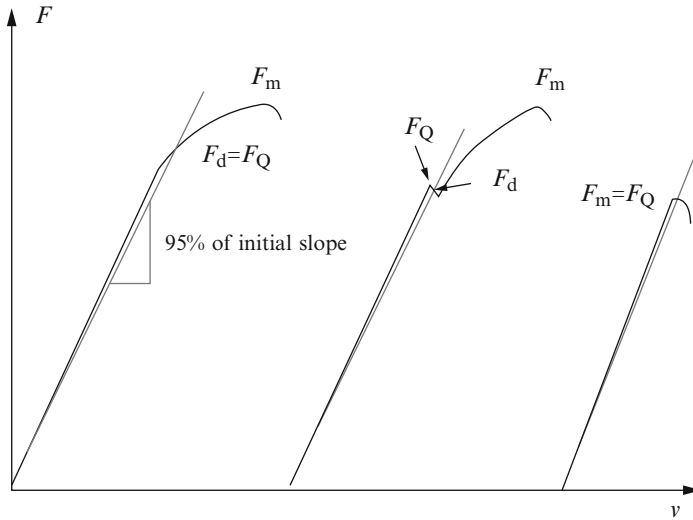


Fig. 2.40 Determination of the conventional load F_Q

materials however, a deviation from linearity is most often observed. This is due to the development of plasticity at the crack tip and most of all to some stable crack propagation, as we have seen in Sect. 2.4.2.3. This deviation should not be too great in order to remain in the conditions of LEFM. The standards require the maximum load not to exceed 10% of a conventional load denoted F_Q , which we now define. Figure 2.39 shows various kinds of recording that can be obtained, depending on the tested materials. It gives the standard denominations of the characteristic points in these recordings. The corresponding values of the crack opening displacement (and as we shall see later of J) bear the same index.

The ideal would be to determine the very initiation of crack tearing inducing a deviation from linearity. This is difficult to measure precisely (as it is to determine the yield strength in a tensile test). The standards require determining the load F_Q as defined in Fig. 2.40. In case of departure from linearity (with no pop-in), F_Q corresponds to an increase of the compliance such that the unloading curve would have a slope 5% smaller than the initial slope of the loading curve (Fig. 2.40).

In summary, the determination of the fracture toughness K_{Ic} includes the following main steps (omitting some details which are described in the standards):

1. Choose a suitable specimen shape and dimensions according to the thickness B .
2. Pre-crack the specimen (most often by fatigue) in conditions such that the plastic zone at the crack tip remains sufficiently small. This is achieved by following the crack propagation velocity, which must remain below a specified value.

Table 2.2 Fracture toughness K_{Ic} and validity criterion of LEFM for some materials

Material	R_p (MPa)	K_{Ic} (MPa m ^{1/2})	$2.5 (K_{Ic}/R_p)^2$ (mm)
Very high resistance steel (e.g. 35NCD16)	1,500	60	4
Medium resistance steel at low temperature	1,000	40	4
15MND6	500	200	400
High resistance aluminium alloy (e.g. 7075)	500	30	9
High resistance titanium alloy (e.g. TA6V)	800	80	25

- Put a clip-gauge on the specimen and perform the test while recording the load displacement curve, up to fracture.
- On the recording determine the conventional load F_Q .
- Check that $F_m/F_Q < 1.1$, F_m being the maximum load.
- Calculate K_Q as a function of F_Q and of the pre-crack length, which can be measured on the fracture surface. The standards include tables and formulae for this calculation. (The formulae for the 3PB and for the CT specimens are given in Table 2.3).
- Check that $a, b, B > 2.5 (K_Q/R_p)^2$.
- If these conditions are fulfilled $K_Q = K_{Ic}$. If not, the test must be repeated with a larger thickness, or, when this is not possible, the interpretation of the test must be achieved by the method of elastoplastic fracture mechanics (EPFM).

Table 2.2 gives values of the fracture toughness of some materials and the dimensions of the specimens for valid measurements.

2.5 Elastoplastic Fracture Mechanics (EPFM)

2.5.1 Limit Load and R-6 Method

If a component made from a perfectly plastic material is acted on by a steadily increasing load, a stage is reached at which there is full-scale plastic yielding, as shown in Fig. 2.41. Since there is no work-hardening the load cannot then be increased further, and this is called the *limit load*.

The limit load concept applies to any component; a crack always reduces its value below that for the uncracked component. Calculation of the value involves using tools developed in the theory of plasticity, in particular the extremal theorems of Sect. 3.5.3 in Volume I. Thus in the simple case corresponding to Fig. 2.41 it can be shown that:

$$P_L = WB(1 - 2a/W) R_p \quad (2.118)$$

where B , W are the thickness and width of the plate respectively.

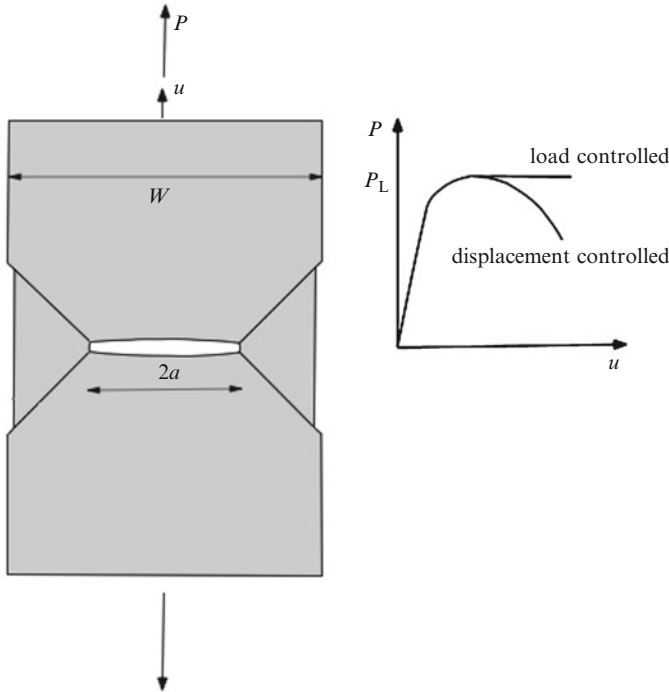


Fig. 2.41 Limit load of a plate with a central crack. When the limit load is reached, flow occurs along slip bands emanating at 45 degrees from the crack tips

The limit load represents a distinctive mode of failure when the structure is subjected to a steadily increasing load; but this is not necessarily the case when the displacement is prescribed. Further, when the crack propagates stably the progressive tearing entails a decreasing load.

A very brittle material breaks much before the limit load is reached. LEFM is fully applicable and fracture occurs when $K_I/K_{Ic} = 1$. On the contrary, a very ductile material fails when $P/P_L = 1$. In Fig. 2.42 the limit load and brittle failure criteria are shown for a plate with a central crack as represented in Fig. 2.41. It can be seen that the limit load is reached before brittle fracture can occur for short and long cracks. Otherwise, the brittle failure criterion is reached first. There are intermediary cases between fully plastic and fully brittle cases the condition of failure of which might be represented by a line in a K_I/K_{Ic} versus P/P_L diagram (Fig. 2.43). This is the root of the so-called *R-6 method* (See also Shih and Hutchinson 1976). In order to trace this failure line a failure criterion is needed; it could be for instance a critical value of the CTOD produced by plastic blunting of the crack tips and the failure line could be calculated using (2.103) giving δ as a function of the applied stress. We now study the possible fracture criteria in EPFM.

Fig. 2.42 Limit load and brittle fracture criterion as a function of the crack length

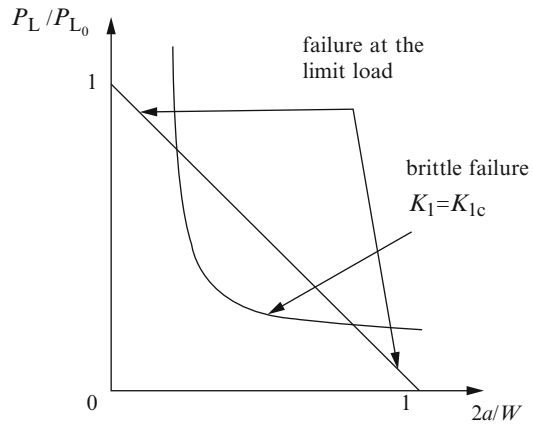
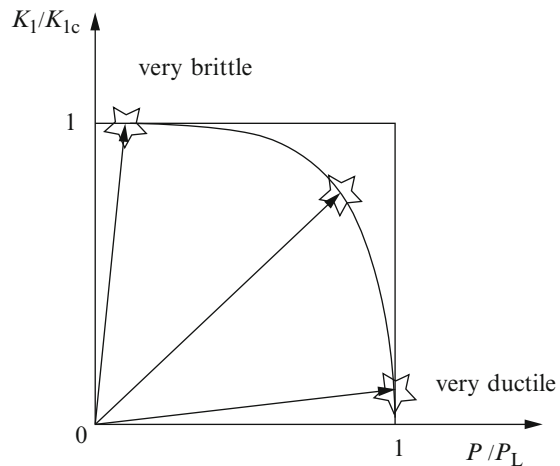


Fig. 2.43 The R-6 diagram of the ratio K_I/K_{Ic} versus P/P_L . The straight lines through the origin represent the loading trajectories. According to the fracture toughness of the material, the failure can be fully brittle, fully ductile or intermediary



2.5.2 Fracture Toughness in Term of the Critical Value of J

2.5.2.1 Extension of the J Integral in EPFM

In Sect. 2.2.2 we introduced the J integral and we discussed its fundamental properties in elasticity. We now extend these properties to plasticity. Inasmuch that *loading remains proportional* (the principal stresses remain in a constant ratio) and that there is *no unloading*, the behaviour of a plastic material does not differ from that of a nonlinear elastic material. Then, the demonstrations in Sect. 2.2.2 remain valid. It means in particular that we can determine the value of J from the area under the loading curve and that the HRR asymptotic fields describe the state of stress and

strain near the crack tip (Eq. 2.21). An important consequence is that, in elastoplasticity, J plays the same role as does K in linear elasticity. It characterises the stress intensity whatever the crack surrounding. The propagation of the crack depends then on some critical material dependent value of J , which we might denote J_c for the time being. And this can be transposed from a specimen to a structural member.

In Sect. 2.4.2.3 we saw that at some critical value of crack blunting, corresponding to the plane strain fracture toughness G_{Ic} , there was initiation of tearing. As $J = G$ in linear elasticity, the corresponding critical value of J is denoted $J_i = G_{Ic}$. We have seen that the very beginning of tearing is difficult to determine on the loading curve. We need to be able to determine the evolution of J as a function of crack propagation Δa .

In principle, as crack propagation induces some local unloading, we should not continue to use J as a characterising parameter during crack propagation. However, if the unloading concerns a small enough zone, the surrounding remains an HRR singularity and J continues to be a characterising parameter. The condition for this to be valid can be written:

$$\omega = \frac{b}{J_R} \frac{dJ_R}{da} \gg 1 \quad (2.119)$$

2.5.2.2 Measurements of the Crack Extension Δa

There are several experimental methods to determine the evolution of J as a function of Δa . The determination of J relies on the measurement of the area under the loading curve, provided the displacement is that of the point of application of the load, and on the use of (2.14) with the value of η according to the specimen used.

To determine Δa it is possible to perform a series of *interrupted tests* and to complete the fracture of the specimens in a different condition, at low temperature for instance, to be able to distinguish the various zones on the fractography.

It is simpler to perform *periodic unloadings* during the test, so as to determine the evolution of the elastic compliance, which is a function of the crack length.

It is also possible to measure the evolution of the *electrical resistance* of the specimen, which is a function of the crack length.

2.5.2.3 The Critical Values of J

Whatever the method, a plot of J as a function of Δa is finally obtained (Fig. 2.44). Crack blunting produces a small amount of apparent crack propagation, so that a correction needs to be introduced to take this phenomenon into account. This is the *blunting line*. Standards describe it by the equation:

$$J = 3.75 R_m \Delta a \quad (2.120)$$

R_m being the ultimate tensile stress of the material.

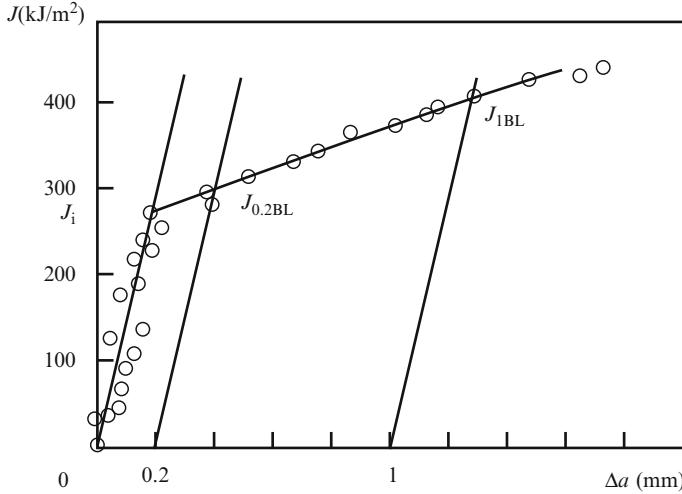


Fig. 2.44 $J - \Delta a$ curve for aged austeno-ferritic steel CFM8M obtained with a CTJ12.5 specimen at 350°C. $J_i = 271 \text{ kJ/m}^2$; $J_{0.2BL} = 302 \text{ kJ/m}^2$; $J_{1BL} = 403 \text{ kJ/m}^2$. ($J_{BL} = 3.75 R_m \Delta a$) (Courtesy of P. Nicolino)

It is then possible, on the $J(\Delta a)$ curve, to determine J_i at the initiation of tearing; $J_{c(B)}$ at crack propagation instability if $\Delta a < 0.2 \text{ mm}$, B being the specimen thickness; $J_{u(B)}$ at instability if $\Delta a > 0.2 \text{ mm}$; $J_{0.2BL}$ at 0.2 mm from the blunting line. The fracture toughness at crack instability is denoted $J_{uc(B)}$ if Δa cannot be determined. We have seen that the value at instability is not really an intrinsic property. This is the reason why the fracture toughness $J_{0.2BL}$ is to be preferred.

LEFM toughness measurements require small scale yielding conditions to be fulfilled (Eq. 2.116). In the same way, in EPFM the HRR field in a region close to the crack tip should not be perturbed by boundary conditions around the specimen. It means that the extent of the HRR field, of the order of the CTOD, should remain small with respect to the ligament size b . This is written for bend type specimens:

$$b \geq 30 \frac{J}{R_p} \quad (2.121)$$

It is recommended to use a value larger than 30 (about 100) for tensile specimens such as single edge notched (SEN) specimens. The J-integral characterisation of fracture toughness is described in ASTM standard E 1737 (1996).

2.5.2.4 Influence of the T and Q Stresses

We have seen that in LEFM the existence of the T stress could not be completely neglected with respect to the elastic singularity when determining the shape and

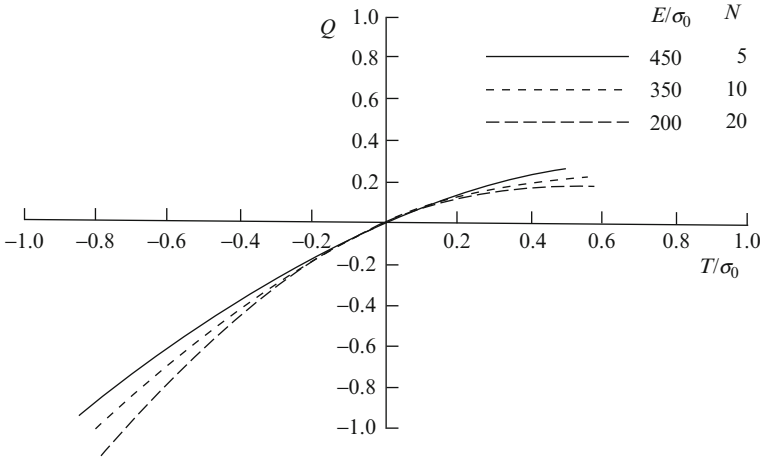


Fig. 2.45 Relationship between T/σ_0 and Q for three values of E/σ_0 and N (Sherry et al. 2005)

extend of the small scale yielding plastic zone, and therefore the fracture toughness. In the same way, in EPFM the HRR field, even if it is confined within the ligament, is perturbed by second terms in the development of the Q stress. Finite elements calculations allow introducing corrective terms to take into account the boundary conditions on the periphery of the specimen. They require very fine meshes near the crack tip.

Several authors have attempted to relate T and Q parameters. 2D plane strain finite element analyses have been made for the purpose to simulate a crack in an infinite body using a semi-circular mesh of initial radius, R , containing a radial crack (see *e.g.* Gao and Dodds 2001; Sherry et al. 2005). This type of model is often termed a “modified boundary layer” (MBL) model. The boundary conditions applied to the model are related to the simulation of a K_I and T -stress field at the crack tip. This is achieved by specifying displacements to the outer nodes of the model, u_1 and u_2 , according to the displacement due to the K field and T stress effect.

Elastoplastic calculations with a power law constitutive equation $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$ have been made to determine the Q parameter. The value of this quantity is extremely sensitive to the mesh size. It is thus possible to relate the T stress and the Q parameter, as shown in Fig. 2.45 (Sherry et al. 2005). This figure shows that for small values of T/σ_0 , *i.e.*, $-0.2 < T/\sigma_0 < 0.10$, the relationship is close to a 1:1 correlation for all values of E/σ_0 and N . However, with increasing positive or negative T/σ_0 the relationship deviates from a 1:1 correlation. For positive values of T/σ_0 , the value of Q falls below T/σ_0 , whilst for negative T/σ_0 the value of Q is more negative. This deviation is more significant the lower the value of E/σ_0 and the higher the value of N . Correlations between Q and the T stress have also been proposed such as:

$$Q = A_N \left(\frac{T}{\sigma_0} \right) + B_N \left(\frac{T}{\sigma_0} \right)^2 + C_N \left(\frac{T}{\sigma_0} \right)^3 \quad (2.122)$$

where the coefficients A_N , B_N and C_N depend on the work-hardening coefficient, N (see *e.g.* O'Dowd and Shih 1991, 1992).

2.5.3 Fracture Toughness in Term of the Critical CTOD

In view of the physical mechanisms taking place near the crack tip, which are responsible for crack propagation, it is justified to use a critical value of the CTOD as a criterion. The difficulty is to be able to determine this critical value, as direct access to deformation at the crack tip is not easy at all. However there is a relation between the CTOD and J , as evidenced by the equations of the HRR field (2.21) (see also Eqs. 2.97 and 2.99b). It is of the form:

$$J = m\sigma_0\delta \quad (2.123)$$

m being a parameter which is a function of the specimen geometry and which varies between 1 and 2; δ is the CTOD and σ_0 the yield parameter of the law $\varepsilon/\varepsilon_0 = (\sigma/\sigma_0)^N$. Approximate solutions for J -integral have been given by Zahoor (1989).

In view of this relation there is no difference between the determination of the fracture toughness in term of J or in term of δ .

However, the CTOD can be determined from the displacement V measured at a distance z from the cracked face of the specimen. The formulae for CT and 3PB specimens are given in Table 2.3. It is based on the analysis of the deformation as a rotation of the two halves of the specimen around a centre located ahead of the crack tip.

The determination of the critical values then follows the same procedure as the one described for J in Sect. 2.5.2. The same limitations hold as the ones insuring the validity of J .

2.5.4 Conclusion

In EPFM uncertainties concerning the exact stress and strain fields in the vicinity of the crack tip require detailed calculations by finite elements to be as rigorous as possible. They require mesh refining near the crack tip. Under those conditions, as far as the stress and strain fields are calculated in the vicinity of the crack tip, why not confront them with local fracture criteria linked with the mechanisms operating at the microscopic level? This is the local approach to fracture mechanics.

Table 2.3 Formulae for the determination of K , J and δ (ISO 12135:2002)

Compact tension specimen CT

$$\begin{aligned}
K & K_I = \frac{F}{(B B_N w)^{0.5}} g_2 \left(\frac{a}{w} \right) \\
g_2 &= \frac{(2 + \frac{a}{w}) \left[0.886 + 4.64 \frac{a}{w} - 13.32 \left(\frac{a}{w} \right)^2 + 14.72 \left(\frac{a}{w} \right)^3 - 5.6 \left(\frac{a}{w} \right)^4 \right]}{\left(1 - \frac{a}{w} \right)^{1.5}} \\
J & J = \frac{1 - \nu^2}{E} K_I^2 + \frac{\eta_p U_p}{B_N (w - a_0)} \left[1 - \frac{(0.75 \eta_p - 1) \Delta a}{(w - a_0)} \right] \\
& \eta_p = 2 + 0.522 \left(1 - \frac{a_0}{w} \right) \\
\delta & \delta = \frac{1 - \nu^2}{E} \frac{K_I^2}{2R_{p0.2}} + \frac{0.54 \Delta a + 0.46 (w - a_0) V_p}{0.54 (a_0 + \Delta a) + 0.46 w + z}
\end{aligned}$$

Three-point bend specimen 3PB

$$\begin{aligned}
K & K_I = \frac{S}{w} \frac{F}{(B B_N w)^{0.5}} g_1 \left(\frac{a}{w} \right) \\
g_1 &= \frac{3 \left(\frac{a}{w} \right)^{0.5} \left[1.99 - \frac{a}{w} \left(1 - \frac{a}{w} \right) \left(2.15 - 3.93 \frac{a}{w} + 2.7 \frac{a^2}{w^2} \right) \right]}{2 \left(1 + 2 \frac{a}{w} \right) \left(1 - \frac{a}{w} \right)^{1.5}} \\
J & J = \frac{1 - \nu^2}{E} K_I^2 + \frac{2U_p}{B_N (w - a_0)} \left[1 + \frac{\Delta a}{2 (w - a_0)} \right] \\
\delta & \delta = \frac{1 - \nu^2}{E} \frac{K_I^2}{2R_{p0.2}} + \frac{0.6 \Delta a + 0.4 (w - a_0) V_p}{0.6 (a_0 + \Delta a) + 0.4 w + z}
\end{aligned}$$

B specimen thickness, B_N remaining thickness in case of lateral grooves, a crack length, w specimen width, U_p plastic deformation energy, V_p plastic part of the displacement measured at a distance z from the cracked face, S span (distance between the two extreme loading points)

Furthermore, the application of global LEFM and EPFM methodologies to fracture toughness measurement (ISO 12135:2002) can have strong limitations. These approaches assume that fracture can be determined with only one parameter (K_{Ic} , K_{Jc} , $CTOD$) or two parameters ($K - T$, $J - Q$). These parameters are assumed to be intrinsic and to be specific to a given material provided that the limitations in terms of specimen size are satisfied. It has been shown by many researchers that this assumption is not verified when testing ferritic steels in the lower shelf and in the ductile-to-brittle transition regime, as detailed in Chap. 3. Many experiments (see *e.g.* the largest European experimental program “European fracture toughness data set”, Heerens and Hellmann 2002) have shown that, even when the specimen size requirements are fulfilled, the fracture toughness is intrinsically scattered and is a decreasing function of specimen size. This is one of the main reasons why the local approach to fracture was introduced in the early 1980s (Pineau 1981; Beremin 1983). This local approach based on a detailed analysis of the failure micro-mechanisms (see Chap. 3) has led to standards. The size effect and the

intrinsic scatter can be taken into account using the “Master Curve” approach (ASTM E 1921-02). The transferability of the results obtained on small laboratory test specimens to larger structures with different constraint conditions can also be made using the recent ISO standard (ISO – 27306: 2009). More details are given in Chaps. 3, 4, and 5.

We devote Sect. 2.7 to explain how local criteria, which depend much on the stress triaxiality, can be determined.

The next section (Sect. 2.6) extends the previous concepts of fracture mechanics to time-dependent situations, those met with creeping solids.

2.6 Fracture Mechanics of Creeping Solids

2.6.1 Introduction

The stress-strain field around the tip of a sharp crack in an infinite creeping solid (for the definition of creep and viscoplasticity, see Volume I, Chap. 4) can be calculated in an approximate way as a function of time and distance from the crack tip (Riedel 1981, 1987). The total strain is assumed to be equal to the sum of instantaneous time-independent elastic and plastic strain and of time-dependent creep strain. It is thus possible to define asymptotic fields, similar to the HRR J field in elastoplasticity (Sect. 2.2.2.6) which apply to some distance of the crack tip but not too close from it because they do not take into account crack tip blunting effects. Moreover these theoretical asymptotic fields may also have some limitations when applied to finite size specimens.

2.6.2 RR (Riedel and Rice) Creep Stress Fields

2.6.2.1 Secondary Creep

Let us consider first a material which creeps by secondary power law only, *i.e.*:

$$\dot{\epsilon}_S = B\sigma^{n_2} \quad (2.124)$$

The stresses in the viscoplastic zone of a stationary crack are given for $r \rightarrow 0$ by the time-dependent RR singularity:

$$\sigma_{ij} = \left[\frac{C_S(t)}{B_2 I_{n_2} r} \right]^{\frac{1}{n_2+1}} \tilde{\sigma}_{ij}(\theta, n_2) \quad (2.125)$$

This singularity is similar to that of a plastic material (Eq. 2.21). The loading parameter $C_S(t)$ is given by:

$$C_S(t) = \frac{J}{(n_2 + 1)t} \quad \text{for } t \ll t_{tr} \quad (2.126a)$$

$$C_S(t) = C^* \quad \text{for } t \gg t_{tr} \quad (2.126b)$$

where C^* is the equivalent of the J integral, except for the units, and the transition time t_{tr} is given by:

$$t_{tr} = \frac{J}{(n_2 + 1)C^*} \quad (2.127)$$

The transient load-geometry parameter between short and long times can be determined by a simple interpolation as:

$$C_S(t) = C^* \left[1 + \frac{J}{(n_2 + 1)C^*t} \right] \quad (2.128)$$

The size of the viscoplastic (or creep) zone, R_{vp} in which the creep strains are dominant, increases for short times $t \ll t_{tr}$ as:

$$R_{vp} = \frac{J}{E} (B_2 E n_2 t)^{\frac{2}{n_2 - 1}} \quad (2.129)$$

The creep zone grows from dimensions which are negligible with respect to characteristic specimen dimensions for short times ($t \ll t_{tr}$) to large dimensions for long times ($t \gg t_{tr}$).

It is important to note that the C^* -integral, which can be theoretically measured far from the crack tip, unequivocally determines the crack-tip stress field. For a two-dimensional planar crack, C^* is defined similarly to the J -integral, as the contour integral:

$$C^* = \int_{\Gamma} \left(\dot{W} dx_2 - \sigma_{ij} n_i \frac{\partial \dot{u}_j}{\partial x_1} ds \right) \quad (2.130)$$

with $\dot{W} = \int \sigma_{ij} d\dot{\epsilon}_{ij}$. For the definition of the symbols, see Fig. 2.2. In principle, C^* can be measured by testing a pair of specimens having infinitesimally different crack lengths, a and $a + da$, and which are otherwise identical. From the measured

load-displacement rate records of the two specimens, C^* can be determined by differentiating the area under the load (P) – displacement rate ($\dot{\Delta}$) curve with respect to crack length:

$$C^* = -\frac{\partial}{\partial a} \int P d\dot{\Delta} \quad (2.131)$$

Here, P is the load per unit specimen thickness and $\dot{\Delta}$ is the displacement rate of the load point.

This expression is similar to Eq. 2.16 used to determine the J -integral.

2.6.2.2 Primary Creep

For materials in which primary creep strains cannot be neglected, a primary creep zone develops first and grows around the crack tip. Riedel (1981) showed that the crack tip stresses in the primary creep zone are given for $r \rightarrow 0$ by:

$$\sigma_{ij}(r, \theta) = \left[\frac{C_p(t)}{B'_1 I_{n_1} r} \right]^{\frac{1}{n_1+1}} \tilde{\sigma}_{ij}(\theta, n'_1) \quad (2.132)$$

where the primary creep law is expressed as:

$$\dot{\varepsilon} = B_1 \sigma^{n_1} \varepsilon^{-p_1} \quad (2.133)$$

and

$$B'_1 = B_1 (1 + p_1)^{\frac{1}{1+p_1}} \quad (2.134a)$$

$$n'_1 = \frac{n_1}{1 + p_1} \quad (2.134b)$$

$$p'_1 = \frac{1}{1 + p_1} \quad (2.134c)$$

The load geometry parameter $C_p(t)$ is given by:

$$C_p(t) = \frac{J}{(n'_1 + 1)^{p'_1} t^{p'_1}} \quad \text{for } t \ll t_1 \quad (2.135a)$$

$$C_p(t) = C_h^* \quad \text{for } t \gg t_1 \quad (2.135b)$$

with

$$C_h^* = \frac{J}{t^{p_1}} \quad (2.136)$$

The transition time, t_1 , between small-scale and large-scale primary creep conditions is given by:

$$t_1 = \frac{1}{n'_1} \left(\frac{J}{C_h^*} \right)^{1+p_1} \quad (2.137)$$

The stresses in the secondary creep zone which develops inside the primary creep zone for short times and eventually overgrows it for long times, have also been calculated as a function of J , C_h^* and C^* . These expressions can largely be simplified by assuming that crack-tip stress relaxation by primary creep occurs rapidly, and that large-scale primary creep conditions prevail at $t \rightarrow 0$. The crack-tip stresses are then equal to the RR singularities (Eq. 2.125) where:

$$C_s(t) = \frac{n_2 p_1 + 1}{n_2 + 1} \frac{C_h^*}{t^{1-p'_1}} \quad \text{for } t \ll t_2 \quad (2.138a)$$

$$C_s(t) = C^* \quad \text{for } t \gg t_2 \quad (2.138b)$$

where t_2 corresponds to the transition time between small-scale and large-scale secondary creep and is defined by:

$$t_2 = \left(\frac{n_2 p'_1 + 1}{n_2 + 1} \frac{C_h^*}{C^*} \right)^{\frac{1}{1-p'_1}} \quad (2.139)$$

These stress fields apply to stationary cracks, as indicated earlier. The situation of propagating cracks has also been studied by Hui and Riedel (1981) but is not considered here.

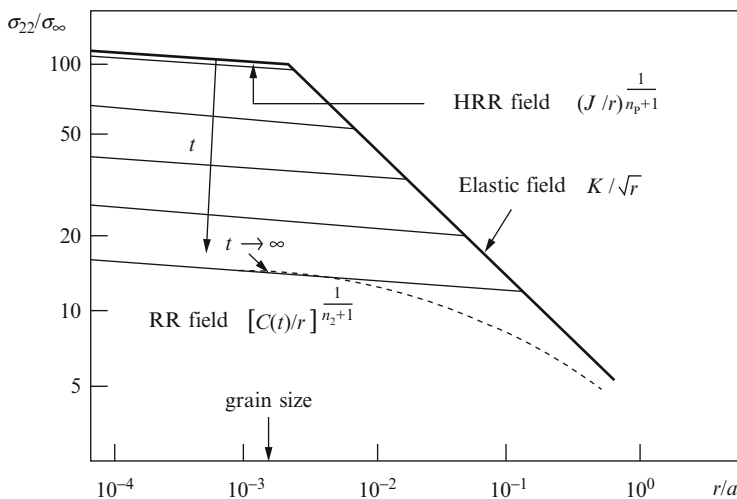
To summarise, the singular asymptotic stresses in the plastic, small and large primary and secondary creep zones can be written as:

$$\sigma_{ij}(r, \theta) = \left(\frac{A}{B_n I_n} \right)^{\frac{1}{n+1}} t^{-\eta} r^{-\rho} \tilde{\sigma}_{ij}(\theta, n) \quad (2.140)$$

where n and B_n are respectively equal to N and $\sigma_0 / \varepsilon_0^{1/N}$ (plastic law $\varepsilon / \varepsilon_0 = (\sigma / \sigma_0)^N$), n'_1 and B'_1 or n_2 and B_2 . The load geometry parameter A in (2.140) is proportional to J , C_h^* or C^* . The expressions for A , ρ and η are given in Table 2.4 for each regime. The evolution of the tensile stress normalised by the applied stress at infinity ahead of the crack tip (a = crack length, r = distance from the crack-tip) is shown in Fig. 2.46. The comparison between these theoretical fields with those numerically calculated was made (Pineau 1992a, b). This comparison showed a good agreement for the spatial and time singularities except very close to the crack tip. This is due to the fact that the HRR and the RR fields do not account for the crack blunting effect.

Table 2.4 Theoretical values for the parameters n , B_n , A , ρ and η in (2.140)

	Plasticity	Small-scale primary creep	Large-scale primary creep	Small-scale secondary creep	Large scale secondary creep
n	N	n'_1	n'_1	n_2	n_2
B_n	ε_0/σ_0^N	B'_1	B'_1	B_2	B_2
A	J	$\frac{J}{(n'_1+1)p'_1}$	C_h^*	$\frac{n_2 p'_1 + 1}{n_2 + 1} C_h^*$	C^*
ρ	$1/(N+1)$	$1/(n'_1+1)$	$1/(n'_1+1)$	$1/(n'_2+1)$	$1/(n'_2+1)$
η	0	$p'_1/(n'_1+1)$	0	$(1-p'_1)/(n_2+1)$	0

**Fig. 2.46** Sketch showing the various stress-strain fields ahead of a stationary creep crack in plastic-viscoplastic material

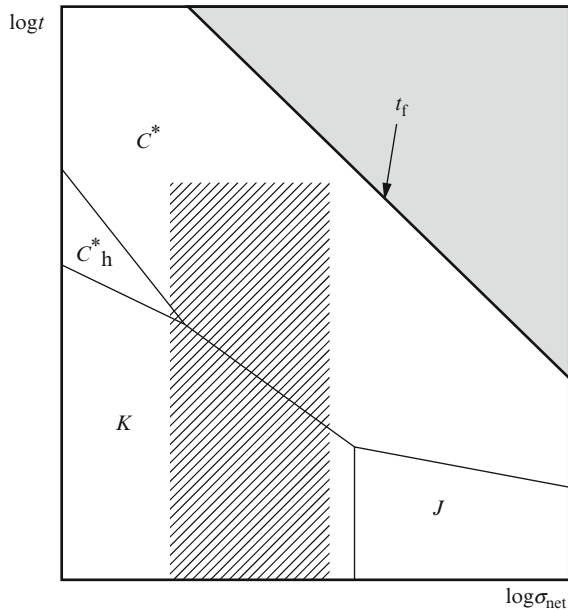
2.6.3 Characteristic Load-Geometry Parameters

Assuming that stress relaxation ahead of the crack tip occurs rapidly by primary creep and, in particular that $t_1 \ll t_2$, the stress, strain and strain-rate crack-tip fields are theoretically fully characterised for $r \rightarrow 0$ by the following load-geometry parameters:

- (a) J in the small-scale primary creep zone and the surrounding instantaneous plastic zone for short times ($t \ll t_1$). It should be kept in mind that J reduces to K for small scale yielding conditions and, in particular that:

$$J \simeq \frac{K^2}{E} \quad (2.141)$$

Fig. 2.47 Load-geometry parameters as a function of time and applied stress σ_{net} ; t_f is the time to failure or the time to initiate creep crack growth. The shaded area corresponds to the situation found in many materials



- (b) C^*_h in the small-scale secondary creep zone and the surrounding primary creep zone for short times ($t_1 < t < t_2$) and loading conditions leading to large-scale primary creep;
- (c) C^* in the large-scale secondary creep for long times ($t \gg t_2$).

Slightly different conclusions are obtained when neglecting primary creep. Under stationary creep, only the J and C^* dominated regimes remain. The situation is summarised in Fig. 2.47 where the various load-geometry parameters are indicated as a function of time and applied stress, σ_{net} (logarithmic scales).

It is usually assumed that the C^* parameter applies to *creep ductile materials*, such as austenitic stainless steels or Cr-Mo-V ferritic steels used in thermal power plants, while K applies to *creep brittle materials*, such as a number of high strength Al alloys, and Ni base superalloys. One simple way to distinguish these two extreme cases is to calculate the transition time, t_{tr} , (Eq. 2.127) and to compare t_{tr} with the time spent during the experiments, t_{exp} . In *creep ductile materials*, it is observed that $t_{\text{exp}} \gg t_{\text{tr}}$ while the opposite situation takes place in *creep brittle materials*.

2.6.4 Simplified Methods to Calculate the C^* Parameter

The values of the K factor for a large number of specimen geometries can be found in the literature or using the methods presented in Sect. 2.2.7. The methods used to determine the J parameter have also been described (Sect. 2.5.2 and Table 2.2).

The analogy between power-law plasticity and stationary power-law creep is obvious. It is therefore clear that the expressions developed to calculate J in simple geometries can also be used to determine the experimental value of C^* , which is called C_{exp}^* . This experimental value is directly calculated from the load-line displacement rate, $\dot{\Delta}_{\text{exp}}$, instead of Δ_{exp} which would be the factor used to calculate J (see the expressions for CT specimen and 3 PB specimens in Table 2.3). For instance for CT type specimens C_{exp}^* is expressed as (Landes and Begley 1976; Harper and Ellison 1977):

$$C_{\text{exp}}^* = \frac{n_2}{n_2 + 1} A(a/w, n_2) \frac{P \dot{\Delta}_{\text{exp}}}{B(w-a)} \simeq 2 \frac{P \dot{\Delta}_{\text{exp}}}{B(w-a)} \quad (2.142)$$

where P is the applied load, B the specimen thickness, $(w-a)$ the ligament length and $A \approx 2$.

In the presence of predominant primary creep strains, it was shown that C_h^* could be expressed as (Bensussan et al. 1989):

$$C_h^* = (p_1 + 1) t \frac{p_1}{1 + p_1} C_{\text{exp}}^* \quad (2.143)$$

where C_{exp}^* is given by (2.142).

For more complex geometries such as those found in components, the analogy between J and C^* can be used to calculate C^* by the finite element method. A number of solutions have been published (Kumar 1980).

Simplified methods have been developed mainly in the United Kingdom and in France at Ecole des Mines. These methods are based on the reference stress and reference length concepts (Pineau 1996).

2.6.5 Time to Initiate Creep Crack Growth and Crack Growth Rate

Let us consider a defect or a fatigue crack in a specimen or in a component, which are submitted to a constant creep load at elevated temperature. It is usually observed that a certain time, called the time to creep crack initiation (CCI), t_i , is needed before this defect or this crack grows over a small distance, Δa . The definition of t_i depends on the length chosen for Δa . Material scientists define Δa as a metallurgical distance of the order of the grain size ($\sim 50 \mu\text{m}$ in many structural materials). This prescribed crack extension can be measured using appropriate techniques (optical microscope observations, potential drop technique, etc.). It is observed that t_i is directly related to K in *creep brittle materials* or to C^* in *creep ductile materials* by power laws:

$$t_i K^\alpha = \text{constant} \quad (2.144a)$$

$$t_i(C^*)^\beta = \text{constant} \quad (2.144b)$$

where α and β are constants which are eventually dependent on test temperature.

In *creep brittle materials*, the creep crack growth rates (CCGR) are theoretically correlated with the stress intensity factor K , by a power-law, such as:

$$da/dt = AK^\beta \quad (2.145a)$$

where A is constant (which depends also on temperature). The growth time, t_G , is simply calculated by integrating (2.145a):

$$t_G = \int_{a_0}^{a_f} \frac{da}{AK^\beta} \quad (2.145b)$$

where a_0 and a_f correspond to the initial crack length and the crack length at failure, respectively.

In *creep ductile materials* many authors have suggested to correlate da/dt with the C^* parameter using also a power-law:

$$da/dt = B(C^*)^\gamma \quad (2.145c)$$

where B is a constant (which is temperature dependent) and γ is an exponent close to 1.

Details about the procedures to determine CCI and CCGR properties are given in Standard ISO/TTA 2007.

2.7 Introduction to Local Approach to Fracture Mechanics

2.7.1 Introduction

Crack propagation results from local damage near the tip of the crack. The fracture mechanisms at the microscopic level will be studied in further chapters of this book. One important aspect is their sensibility to the stress triaxiality. The higher the hydrostatic stress the easier their initiation. Another important consideration is that the development of damage requires plastic deformation.

In the study of the plastic zone in plane strain we saw (Sect. 2.3.4.2) that, for a non-hardening material, the stress σ_{22} increases from the blunted crack tip to some maximum at the extremity of the zone of logarithmic spirals of the slip line field.

It is tempting to consider that crack propagation initiates when this maximum stress at a critical distance ρ_c ahead of the crack tip reaches a critical value σ_f . This kind of approach, which was that of Alan Tetelman (Wilshaw et al. 1968), introduces two fundamental parameters: σ_f and ρ_c . The latter must be related to the average distance between microscopic fracture sites.

The same idea was later developed by Ritchie, Knott and Rice (RKR model) who used the HRR stress field instead of the slip line field (Ritchie et al. 1973).

However, these crude methods are but approximate and, with the development of computing facilities, much more precise crack tip stress fields could be determined and better tests were introduced for the measurements of local fracture parameters.

The local approach to fracture (LAF) was introduced in the early 1980s (Beremin⁹1980a, b, 1981, 1983; Pineau 1981, 1992a, b; Berdin et al. 2004). This approach is largely based on a detailed study of the micromechanisms of failure, contrarily to the conventional “global” LEFM and EPFM. These global approaches are extremely useful and absolutely necessary, but they have also a number of serious limitations, in particular when large-scale yielding conditions are met or when dealing with non-isothermal loadings. This is illustrated in Fig. 2.48 where the ductile-to-brittle transition curve giving the variation of fracture toughness (K_{Ic} or K_{Jc}) or the variation of Charpy V-notch toughness (see Chap. 3), is obtained under isothermal conditions with specimens tested at each temperature. Let us consider non-isothermal loadings, as indicated in Fig. 2.48 where two trajectories are shown: (i) load-unload-cool-fracture (LUCF) and (ii) load-cool-fracture (LCF). It is now firmly established that the effective fracture toughness measured at low temperature is much larger than the fracture toughness determined at this low temperature without preloading. Otherwise stated, the material can cross the transition curve obtained under isothermal conditions without fracture. This “warm-prestress” effect improves the fracture toughness of the material, which is no longer an intrinsic property but is dependent on the loading history. This illustrates one of the limitations of the “global” approach to fracture when this methodology is followed too abruptly. In Chap. 3, an attempt is made to show how the beneficial effect of warm prestressing can be explained using the LAF methodology.

Another limitation of the “global” approaches to fracture is the “size effect” which is observed when ferritic steels are tested in the lower shelf or in the ductile-to-brittle transition regime. It is now well established that fracture toughness is dependent on specimen thickness even when LEFM conditions are satisfied, as stated previously (Sect. 2.5.2). Moreover the intrinsic scatter in fracture toughness measurements, which is directly linked to this size effect, as shown in Chap. 3, must also be taken into account.

These limitations raise the important problem of the transferability of laboratory fracture toughness test results to large components.

⁹F. M. Beremin is the name of a French research group founded in the early 1980s including researchers and engineers from Ecole des Mines de Paris (Centre des Matériaux), Centre de Calcul de la Division des Fabrications de Framatome (Saint-Marcel) and Bureau de Contrôle de la Construction Nucléaire (Dijon).

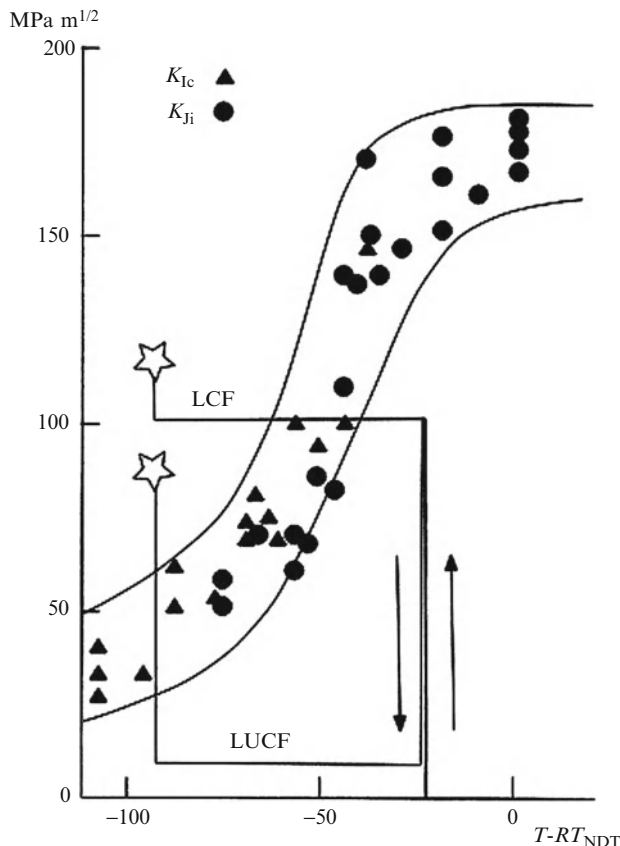


Fig. 2.48 Results of fracture toughness measurements for a pressure vessel steel and sketch showing two warm pre-stressing loading cycles

The LAFs attempt to circumvent these difficulties. This explains why there are now considerable researches devoted to these approaches. Two types of approaches must be distinguished: (i) the uncoupled models in which the damage has no effect on the constitutive equation (see *e.g.* brittle cleavage fracture in Chap. 3) and (ii) the coupled models in which the constitutive equation is coupled with the damage which produces a softening effect (see *e.g.* ductile fracture in Chap. 3).

The LAFs introduce new parameters, which have to be identified from experiments. This raises the problem of the strategy which has to be used to determine these parameters. For instance, the seminal work of Beremin on cleavage fracture (Beremin 1983) introduces the Weibull stress, σ_w , as a probabilistic fracture parameter. When implemented in a finite element code, the Beremin model predicts the evolution of the Weibull stress with applied (macroscopic) load to define conditions leading to (local) material failure. The developments for transferability models of EPFM toughness values rely on the notion of a Weibull stress as a

crack-tip driving force (see *e.g.* Ruggieri and Dodds 1996; Ruggieri et al. 1998; Standard ISO 2009). The control feature in this methodology adopts the simple axiom that unstable cleavage crack propagation occurs at a critical value of the Weibull stress. In this context the Weibull modulus m plays a major role in the process to correlate effects of constraint loss for varying crack configurations and loading modes (tension, bending). Therefore robust schemes of Weibull parameters calibration become a key element in fracture assessment procedures based upon Weibull stress.

The strategy used to determine these parameters was initially based upon tests on volume elements, in particular notched specimens with various geometries leading to different stress triaxiality ratios r_T . An active effort has been made to standardise these tests (ESIS PE-98 1998). Another strategy relies upon a direct calibration of the fracture parameters on the results of fracture toughness measurements obtained on cracked specimens, which are calculated using finite element method. This second procedure has also been discussed in some detail (Ruggieri et al. 2000).

2.7.2 Specimens and Testing

Notched axi-symmetric specimens are used since their cylindrical symmetry does not require three-dimensional analyses. Figure 2.49 shows typical specimens and the notations. The initial notch radius is denoted R , the initial diameter of the smallest cross-section ϕ_0 (current value ϕ), the maximum diameter of the specimen ϕ_m . The various specimens are labelled $AE\chi$, according to the ratio $\chi = 10 R/\phi_0$.

During testing the stress in the smallest cross-section is not constant. Its mean value is denoted $\bar{\sigma}$. It is equal to F , the applied load, divided by the current cross-section area S . The von Mises equivalent stress is denoted σ_e and the mean (hydrostatic) stress σ_m , so that the stress triaxiality ratio is $r_T = \sigma_m/\sigma_e$.

The mean value of the strain in the smallest cross-section is denoted $\bar{\epsilon}$

$$\bar{\epsilon} = \log \left(\frac{S_0}{S} \right) = 2 \log \left(\frac{\phi_0}{\phi} \right) \quad (2.146)$$

The dimensions of the typical specimens shown in Fig. 2.49 can be modified proportionally. They afford different levels of stress triaxiality ratios, increasing as the radius of curvature R decreases.

During testing, the strain is concentrated in the smallest cross-section. It is therefore needed to measure the deformation there, the usual axial measurement in tensile tests being insufficient. The radial displacement at the root of the notch is determined using an extensometer, which records the variations of the diameter ϕ . Of course, the load is recorded at the same time. These tests can be carried out at various temperatures.

When the material is not isotropic, the cross-section can become elliptic. This requires then measuring the diameter variations in two perpendicular directions.

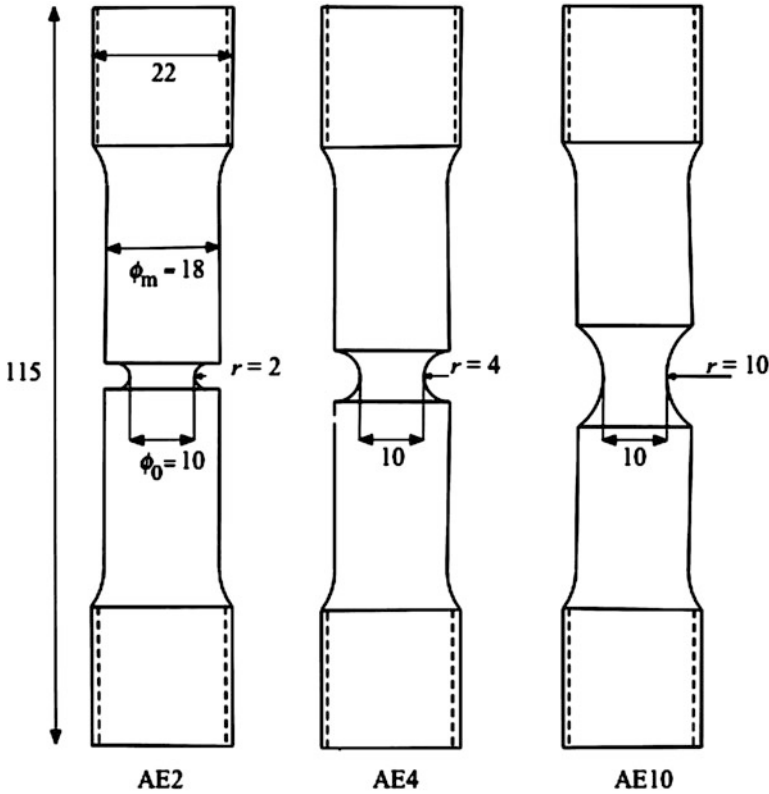


Fig. 2.49 Typical notched axi-symmetric specimens AE2, AE4, AE10 (Dimensions in mm). The radius of the notch is denoted R in the text

In the course of the test damage develops in the minimum cross-section, where the stress triaxiality ratio is the highest. The load drops, when the damage reaches a critical value. These critical conditions of mean stress and strain are thus recorded. The use of an extensometer and strain control conditions allow interrupting the test before failure.

Observations of an axial section through the tested specimen allow characterising the nature of damage.

2.7.3 Analysis of the Results

As just explained, the results are the evolution of the diameter of the minimum cross-section, and thus of the average strain there, as a function of the applied load, and thus of the average axial stress in this cross-section. The critical values at fracture are also recorded. We want now to relate this information to the local critical parameters (Bridgman 1952; Kachanov 1974; Beremin 1980a; Besson 2008).

We need to be able to calculate the stress and strain distributions in the notched area. One way to do it is to use finite elements calculations, knowing the constitutive behaviour from tests on un-notched specimens. We also have an approximate closed form solution, that given by Bridgman. We already gave the results in Volume I, Sect.1.3.3.3, for necking. As they do not differ too much from the results obtained by finite elements calculations, we detail now the *analysis of Bridgman*, with the notations adopted here for the case of notched specimens.

2.7.3.1 Conventional Bridgman Analysis

The Bridgman analysis assumes uniform strain in the minimum cross-section. It is rather well verified for not too small notch radii. If that is the case, the radial displacement u_r is proportional to the radius r in the minimum cross-section. Hence, the radial and hoop strains are equal and constant. Thus, the stresses σ_{rr} and $\sigma_{\theta\theta}$ are equal. The equation of equilibrium for the coordinate $z = 0$ is then written:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} = 0 \quad (2.147)$$

Owing to symmetry $\sigma_{rz} = 0$ for $z = 0$. As the deformation is plastic in the minimum cross-section

$$\sigma_{zz} - \sigma_{rr} = R_p \quad (2.148)$$

As the strain is assumed constant in the minimum cross-section, so is the yield strength. No assumption is needed about strain-hardening.

Figure 2.50 shows the lines of the principal stresses close to the minimum cross-section, ψ being the small angle between the line of the first principal direction with the z axis, $\sigma_{rz} \approx \psi(\sigma_1 - \sigma_2) = \psi R_p$. Close to the minimum cross-section we can then write:

$$\left(\frac{\partial \sigma_{rz}}{\partial z} \right)_{z=0} = R_p \left(\frac{\partial \psi}{\partial z} \right)_{z=0} = \frac{R_p}{\rho} \quad (2.149)$$

ρ being the radius of curvature of the line of first principal stress.

The equilibrium equation (Eq. 2.147, with use of Eq. 2.148) becomes:

$$\frac{\partial \sigma_{zz}}{\partial r} + \frac{R_p}{\rho} = 0 \quad (2.150)$$

The solution of this differential equation depends on the variation of the radius of curvature ρ as a function of the radius r . We assume that the lines of principal stresses are circles. Referring to Fig. 2.50, this allows writing:

$$CT^2 = CM \cdot CN = OC^2 - OM^2 \quad (2.151)$$

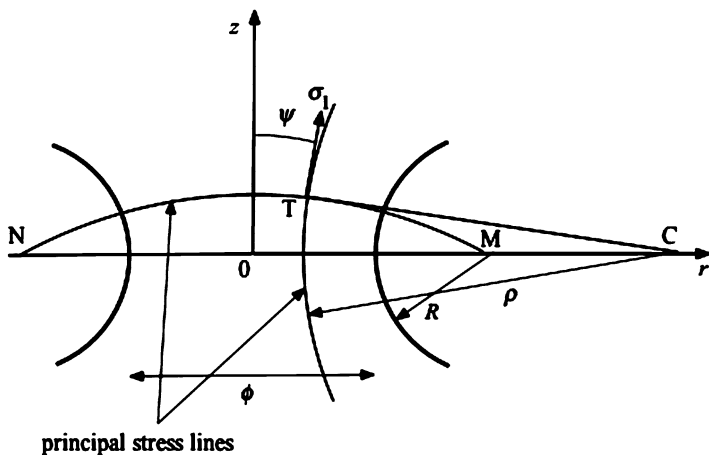


Fig. 2.50 Bridgman analysis

Hence:

$$r^2 + 2\rho r = \text{Cte} = \left(\frac{\phi}{2}\right)^2 + \phi R \quad (2.152)$$

Equation 2.152 provides an expression for the radius of curvature as a function of the radius r , so that Eq. 2.150 can be integrated. The boundary condition is $\sigma_{zz}(\phi/2) = R_p$.

The axial stress σ_{zz} in the minimum cross-section is then given by:

$$\sigma_{zz} = R_p \left[1 + \log \left(1 + \frac{\phi^2/4 - r^2}{\phi R} \right) \right] \quad (2.153)$$

This enables the calculation of the average axial stress:

$$\bar{\sigma} = R_p \left(1 + \frac{4R}{\phi} \right) \log \left(1 + \frac{\phi}{4R} \right) \quad (2.154)$$

The stress triaxiality ratio in the centre of the specimen is then:

$$r_T = \frac{1}{3} + \log \left(1 + \frac{\phi}{4R} \right) \quad (2.155)$$

The stress triaxiality ratios according to this formula are 0.43, 0.54 and 0.68 for specimens AE10, AE4 and AE2, respectively. They are yet well below the stress triaxiality ratio which is predicted ahead of a sharp crack according to the Prandtl slip lines field, *i.e.* $(1 + \pi)/3$. Nevertheless, this does not weaken the interest of these

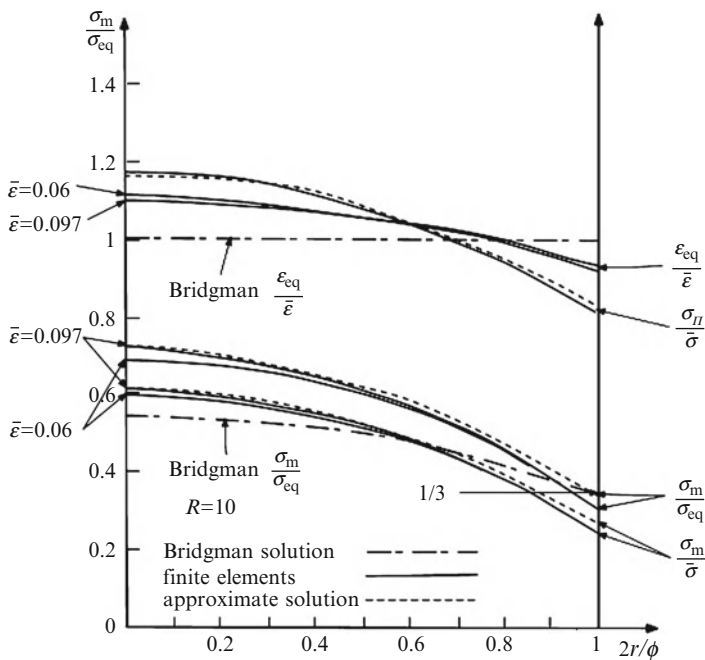


Fig. 2.51 Variations of the stresses and the strains in the minimum section of AE10 specimen. Comparison between finite elements calculations and approximate solution (Eqs. 2.157a, 2.157b, and 2.157c)

notched axi-symmetric specimens for the determination of local fracture parameters. Applications will be discussed when we study various kinds of damages.

2.7.3.2 Extension of the Bridgman Analysis

The results obtained from the simplified Bridgman analysis have been compared to those obtained from finite element calculations (Beremin 1980a). These numerical calculations were performed on a low strength ferritic steel. The comparison for slightly notched AE10 specimens is shown in Fig. 2.51 where the values of the normalised strain, $\varepsilon_{eq}/\bar{\varepsilon}$, that of the normalised maximum principal stress, $\sigma_{zz}/\bar{\sigma}$ and that of the stress triaxiality ratio $r_T = \sigma_m/\sigma_{eq}$ in the minimum section ($z=0$) are shown. It is observed that the Bridgman solution gives a reasonable approximation for the strains and stresses in the minimum section. The situation is quite different for sharper notches AE4 (Fig. 2.52) and AE2 (Fig. 2.53) where finite element calculations show that the strain is not constant through the minimum section and is maximum at the edge of the notch ($2r/\phi = 1$). This clearly illustrates the advantage of using notched specimens to investigate whether fracture is essentially strain or stress controlled.

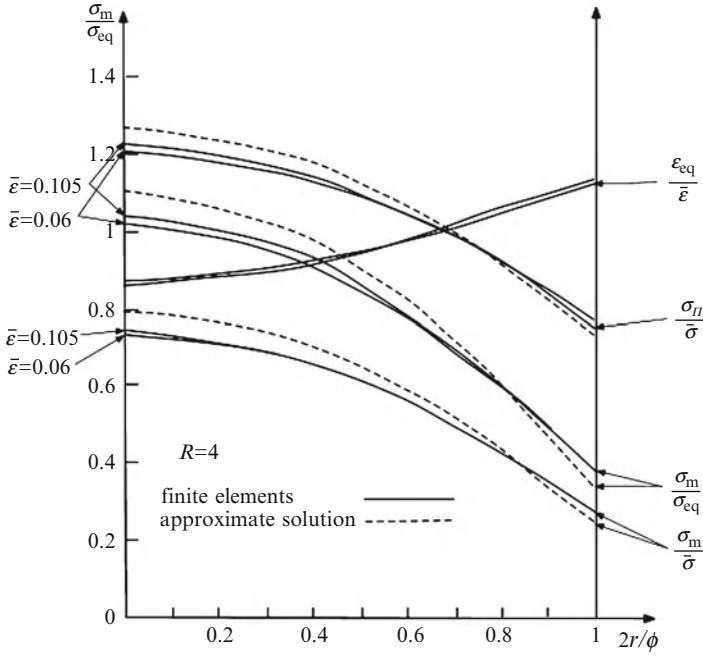


Fig. 2.52 Variations of the stresses and the strains in the minimum section of AE4 specimen. Comparison between finite elements calculations and approximate solution (Eqs. 2.157a, 2.157b, and 2.157c)

The variation of the maximum principal stress or that of the hydrostatic stress, σ_m , through the minimum section is given by (2.155) when using Bridgman analysis. This variation can also be expressed by a parabolic law:

$$\frac{\bar{\sigma}}{\sigma_{eq}(\bar{\varepsilon})} = 1 + k \quad (2.156)$$

where k is thus a simple measurement of the plastic reinforcement due to the presence of a notch.

A simplified analysis has been proposed by Mudry 1982 (see also Beremin 1980a) to calculate the distribution of strains and stresses not only in the minimum section of the specimen but also along the longitudinal axis z . This analysis leads to:

$$\underline{\underline{\varepsilon}}(r, z) \simeq \bar{\varepsilon} \exp\left(-\frac{12kz^2}{R^2}\right) \begin{pmatrix} 1/2 & 0 & 6krz/R^2 \\ 0 & -1/2 & 0 \\ 6krz/R^2 & 0 & 1 \end{pmatrix} \quad (2.157a)$$

$$\varepsilon_{eq}(r, z) = \bar{\varepsilon} \exp\left(-\frac{12kz^2}{R^2}\right) \quad (2.157b)$$

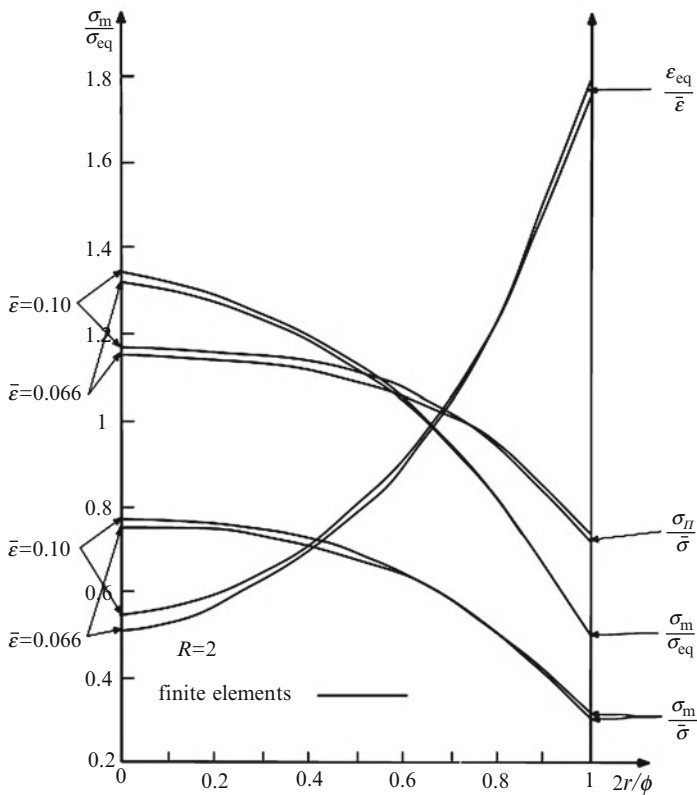


Fig. 2.53 Variations of the stresses and the strains in the minimum section of AE2 specimen. Results of finite elements calculations

$$\frac{\underline{\underline{\sigma}}(r, z)}{\sigma_{eq}(r, z)} = \left(\frac{1}{3} + 2k - 4k \frac{z^2}{R^2} \right) \underline{\underline{\delta}} + \begin{pmatrix} -1/3 & 0 & 4krz/R^2 \\ 0 & -1/3 & 0 \\ 4krz/R^2 & 0 & 2/3 \end{pmatrix} \quad (2.157c)$$

These solutions are represented with dotted lines for AE10, AE4 and AE specimens in Figs. 2.51, 2.52, and 2.53, respectively. They provide acceptable approximations of the finite element calculations close to the minimum section provided that $4k(rz/R^2) \ll 1$ and that the strain through the minimum section ($z=0$) can be considered as homogeneously distributed, which is not too far from the reality in AE10 and AE4 geometries.

The variations of $\varepsilon_{eq}/\bar{\varepsilon}$, $\sigma_m/\bar{\sigma}$, σ_m/σ_{eq} , and $\sigma_{zz}/\bar{\sigma}$ along the z axis in AE4 specimens as given by (2.157a, 2.157b, and 2.157c) are compared to the results of finite element calculations in Fig. 2.54 where it is observed that the predictions are satisfactory, especially close to the symmetry axis of the specimens.

Table 2.5 gives some values of the stress intensity factors for various geometries.

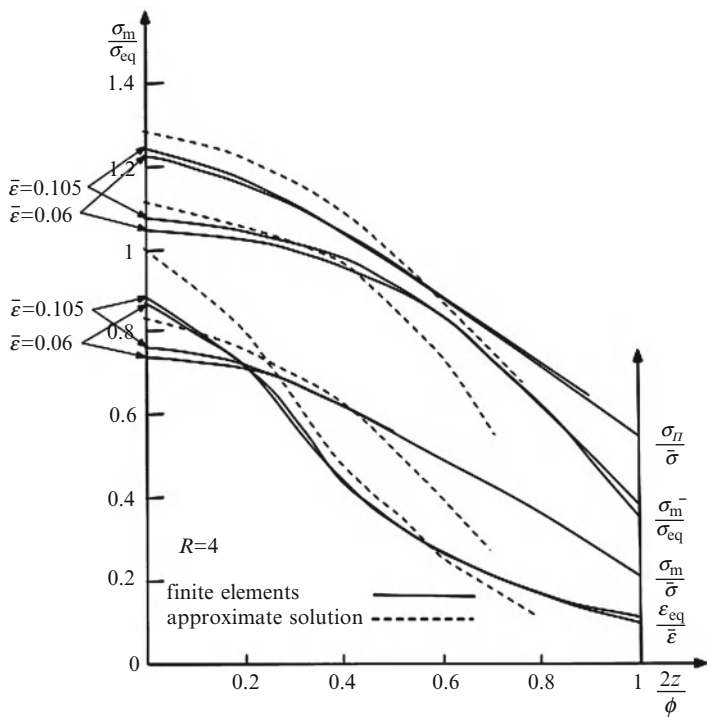


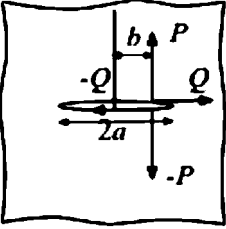
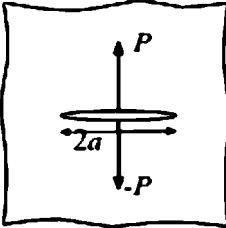
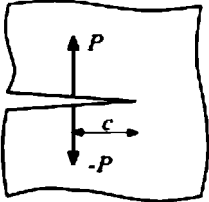
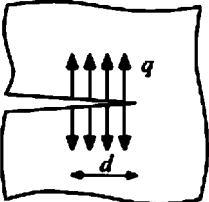
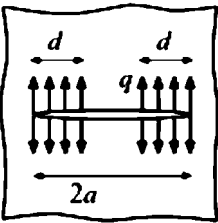
Fig. 2.54 Variations of the stresses and strains along the longitudinal axis ($2z/\phi$) of AE4 specimen. Comparison between finite elements calculations and approximate solution (Eqs. 2.157a, 2.157b, and 2.157c)

Table 2.5 Some formulae for stress intensity factors

Sketch	Description	Stress intensity factor
	Crack of length $2a$ in an infinite plate under uniaxial stress σ	$K_I = \sigma \sqrt{\pi a}$
	Inclined crack of length $2a$ in an infinite plate under uniaxial stress σ	$K_I = \sigma \sqrt{\pi a} \cos^2 \beta$ $K_{II} = \sigma \sqrt{\pi a} \cos \beta \sin \beta$

(continued)

Table 2.5 (continued)

Sketch	Description	Stress intensity factor
	Crack of length $2a$ in an infinite plate under concentrated loads per unit thickness applied at a distance b from the centre, in directions Ox_2 ($+/-P$), Ox_1 ($+/-Q$), Ox_3 ($+/-R$)	$K_I = \frac{P}{\sqrt{\pi a}} \left(\frac{a+b}{a-b} \right)^{1/2}$ $K_{II} = \frac{Q}{\sqrt{\pi a}} \left(\frac{a+b}{a-b} \right)^{1/2}$ $K_{III} = \frac{R}{\sqrt{\pi a}} \left(\frac{a+b}{a-b} \right)^{1/2}$
	Crack of length $2a$ in an infinite plate under concentrated loads $+/-P$ per unit thickness applied at the centre	$K_I = \frac{P}{\sqrt{\pi a}}$
	Semi-infinite crack in an infinite plate loaded by two opposite concentrated loads $+/-P$ per unit thickness at a distance c from the tip	$K_I = P \sqrt{\frac{2}{\pi c}}$
	Semi-infinite crack in an infinite plate loaded by distributed forces $+/-q$ per unit area over a distance d at the tip	$K_I = q \sqrt{\frac{8d}{\pi}}$
	Crack of length $2a$ in an infinite plate loaded over a distance d at the two extremities by distributed forces q per unit area	$K_I = 4q \sqrt{\frac{a}{\pi}} \arcsin \sqrt{\frac{d}{2a}}$

(continued)

Table 2.5 (continued)

Sketch	Description	Stress intensity factor
	Crack of length $2a$ in the centre of a plate of width w under uniaxial stress σ	$K_I = \sigma \sqrt{\pi a} \left(\cos \pi \frac{a}{w} \right)^{1/2}$
	Same case under shear stress in the direction Ox_3	$K_{III} = \tau \sqrt{\pi a} \left(\frac{w}{\pi a} \tan \frac{\pi a}{w} \right)^{1/2}$
	Double-edge cracks of length a in a plate of width w under uniaxial stress σ	$K_I = \sigma \sqrt{\pi a} \left(\frac{w}{\pi a} \tan \frac{\pi a}{w} \right)^{1/2}$
	Single-edge crack of length a in a semi-infinite plate under uniaxial stress σ	$K_I = 1.122 \sigma \sqrt{\pi a} = 2 \sigma \sqrt{a}$
	Cracks of length $2a$ distant of b in an infinite plate under uniaxial stress σ	$K_I = \sigma \sqrt{\pi a} \left(\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right)^{1/2}$
	Semi-infinite crack in a strip of height h under prescribed displacement u	$K_I = \frac{E}{\sqrt{1-\nu^2}} \frac{u}{\sqrt{2h}}$ (plane strain) (in plane stress let $\nu = 0$)
	Double cantilever beam (DCB) specimen of height h with a crack of length a under loads per unit thickness $\pm P$ applied on the extremities	$K_I = 4\sqrt{6} \left(0.32 + \frac{a}{h} \right) \frac{P}{\sqrt{h(1-\nu^2)}}$ (plane strain) (in plane stress let $\nu = 0$)

(continued)

Table 2.5 (continued)

Sketch	Description	Stress intensity factor
	Penny shaped crack of radius a in an infinite medium loaded by distributed forces p per unit length over a radius b	$K_I = \frac{2p}{\sqrt{\pi a}} \frac{b}{a} \left[1 - \left(\frac{b}{a} \right)^2 \right]^{-1/2}$
	Penny shaped crack of radius a in an infinite medium under uniaxial stress σ	$K_I = \frac{2}{\pi} \sigma \sqrt{\pi a}$
	Penny shaped crack of radius a in an infinite medium under concentrated loads $+/-P$ applied at the centre	$K_I = \frac{P}{(\pi a)^{3/2}}$
	Spot weld of radius r of two plates of thickness h loaded by distributed forces per unit length p over a radius R	$K_I = \frac{2\sqrt{3}p}{\sqrt{1-\nu^2}} \frac{R}{h^{3/2}} \left(\frac{R}{r} - 1 \right)$
	Elliptical crack of semi-axes a and b in an infinite medium under uniaxial stress σ	$K_I = \frac{\sigma \sqrt{\pi a}}{E(k)} \left[\sin^2 \phi + \left(\frac{a}{b} \right)^2 \cos^2 \phi \right]^{1/4}$ $E(k) = \int_0^{\pi/2} (1 - k^2 \cos^2 \phi)^{1/2} d\phi$ $k^2 = 1 - \left(\frac{a}{b} \right)^2$

E is the Young modulus, ν the Poisson ratio

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Chapter 3

Brittle Fracture

Abstract This chapter deals with brittle fracture especially by cleavage and embrittlement by segregation of impurities at grain boundaries or by irradiation.

Cleavage follows specific crystallographic planes in metals. A criterion gives the tendency for cleavage versus intergranular fracture. The conditions for crack tip blunting allow writing another criterion indicating the tendency for intrinsic brittleness. Owing to the very high value of the theoretical fracture stress, local stress raisers are needed to explain cleavage initiation: they are various defects in ceramics, dislocation pile-ups and inclusions in metals. Other obstacles, such as grain boundaries must be overcome for cleavage crack propagation. The Beremin model based on the analysis of Weibull accounts for the statistical nature of cleavage. The theory of Batdorf introduces the effect of the crack orientations. Care is needed to insure the validity of the models. They are extended to fracture toughness, in small scale yielding as well as in large scale yielding. The way to obtain the parameters of the models is described. In applications to steels, the presence of multiple barriers needs to be introduced in the modelling. It is necessary to take account of dynamic and of stress triaxiality effects. The Beremin model was applied to bainitic steels, and adapted to include plasticity. The fracture toughness of steels is statistically distributed and shows size effects. Loading rate and prestraining effects, warm prestraining effect, effect of inhomogeneities, especially in welds, are described and modelling discussed. Description of cleavage in other BCC metals as well as in HCP zinc and magnesium are given.

Temper embrittlement of steels results from the segregation of impurities to the grain boundaries. The thermodynamics and kinetics models of this segregation are discussed. The embrittlement is explained by the modifications of the boundary and surface energies due to this segregation. It influences also the conditions for intrinsic brittleness as opposed to crack blunting. Examples of temper embrittlement of steels are given. Overheating of steels is another phenomenon producing intergranular fracture.

Irradiation hardens metals and produces a shift of the DBT temperature. Impurities play a major role in this phenomenon. It is possible to model this shift.

3.1 Introduction

Among the various damage mechanisms described in Chap. 1, we begin by studying those associated with brittle fracture: cleavage, intergranular fracture and irradiation embrittlement in particular.

The *first objective* of this chapter and Chaps. 4 and 5 is to present an overview of the approaches which are based on the investigation of the micromechanisms at a local scale and, through a multiscale approach, on the transfer of this local information to the macroscale. Indeed, nowadays, the final goal is to develop predictive approaches, which can be used in finite element (FEM) codes for structural analysis or for simulation of forming operations. Several excellent reviews and books have already been written on the micromechanisms of failure in metals (see *e.g.* Knott 1973; Besson 2004; François et al. 1998) but very few of them have attempted to provide a comprehensive synthesis of the state of the art. This is one of the goals of this chapter and Chaps. 4 and 5. In particular, a special effort is made to incorporate the most recent developments in the theoretical and numerical modelling of both brittle and ductile fracture.

The *second objective* of these chapters is to introduce the four tools available for microstructural/micromechanical investigations and to show their complementarity. These tools include: (i) macro-mechanical tests under various loading conditions (iso- and non-isothermal tests, multi-axial tests, etc.), (ii) advanced characterisation methods (scanning and transmission electron microscopy, electron back scattering diffraction, X-ray tomography, with an emphasis on *in-situ* testing), (iii) computational unit cell calculations used to investigate the mechanical response of elementary volume elements or to simulate numerically crack initiation and crack growth, (iv) theoretical models which remain essential tools in structural analysis.

The *third objective* is to investigate the main mechanisms of fracture based on the use of these four tools. This chapter only deals with brittle (cleavage and intergranular) fracture. These two modes of failure are analysed separately. Irradiation embrittlement is also presented. Ductile fracture by nucleation, growth and coalescence of voids and by plastic localisation are addressed in Chap. 4. Then the ductile-to-brittle transition (DBT) in ferritic steels is reviewed in Chap. 5.

A *fourth objective* of these chapters is to cover a wide range of metals and metallic alloys, including steels and other BCC materials (Mo, Nb), HCP metals (Zn, Mg), aluminium alloys, titanium alloys, etc. In steels, both ferritic and austenitic microstructures are considered. Multiphased and quenched-tempered steels are also included. Specific aspects encountered with welds are underlined. Many examples dealing with cast and wrought aluminium alloys are also given in particular in Chap. 4.

These four objectives are those of what is now called the *local approach to fracture* (see Sect. 2.7 in this volume).

The influence of crack tip constraint and stress triaxiality on ductile and brittle fracture is of major importance for the assessment of structural integrity. This assessment is usually made by means of linear and nonlinear fracture mechanics

concepts (see Chap. 2 in this volume). Compared with conventional fracture mechanics concepts, micromechanical models developed in the frame of the local approach to fracture have the advantage that the corresponding material parameters for fracture can be transferred in a more general way between different specimen geometries. It is not even necessary to use specimens with an initial crack; of course, initially uncracked structures will also break if the local degradation conditions of the material have exceeded some critical state. In the Gurson model (Gurson 1977) initiation and propagation of a crack are a natural result of the local softening of the material due to the void coalescence which starts when a critical void volume fraction, f_c , is reached over a characteristic distance l_c . The parameter f_c can therefore be determined from rather simple tests like, e.g., tensile tests of smooth and notched round bars in combination with numerical analyses of these tests, or from micromechanical models. Similarly the Weibull stress model originally proposed by Beremin (1983) provides a framework to quantify their complex interaction among specimen size, deformation level and material flow properties when dealing with brittle cleavage (or intergranular) fracture.

The identification and determination of the damage parameters in the Gurson or in the Beremin model require a hybrid methodology of combined testing and numerical simulation. The full description of this methodology is out of the scope of the present chapter. More details are given elsewhere (Pineau and Pardoen 2007). Here it is enough to say that, different from classical fracture mechanics, the local approach procedure is not subject to any size requirements for the specimens as long as the same fracture phenomena occur.

This chapter is organised according to failure modes: cleavage, intergranular fracture. In the part devoted to cleavage in metals the early theories for this mode of failure are presented first. Then these theories and more recent theoretical developments are applied to ferritic steels and other metals with either a BCC or a HCP microstructure. Intergranular fracture and irradiation-induced embrittlement in steels are also reviewed. In this chapter only brittle intergranular fracture at low temperature is considered. Ductile intergranular fracture at low temperature is presented in Chap. 4. Intergranular fracture resulting from high temperature creep damage is examined in Chap. 8.

3.2 Occurrence of Cleavage

3.2.1 Crystallographic Aspects

Cleavage fracture occurs preferentially over dense atomic planes. Table 3.1 lists some cleavage planes that have been observed experimentally. Two examples of cleavage fracture observed in a low alloyed ferritic steel (a, b) and in pure zinc (c) are shown in Fig. 3.1. These photographs reveal that the orientation of cleavage plane changes when it crosses sub-boundaries, twin boundaries or grain boundaries,

Table 3.1 Cleavage planes

Structure	Cleavage plane	Some materials
BCC	{100}	Ferritic steels, Mo, Nb, W
FCC	{111}	Very rarely observed
HCP	{0002}	Be, Mg, Zn
Diamond	{111}	Diamond, Si, Ge
NaCl	{100}	NaCl, LiF, MgO, AgCl
ZnS	{110}	ZnS, BeO
CaF ₂	{111}	CaF ₂ , UO ₂ , ThO ₂

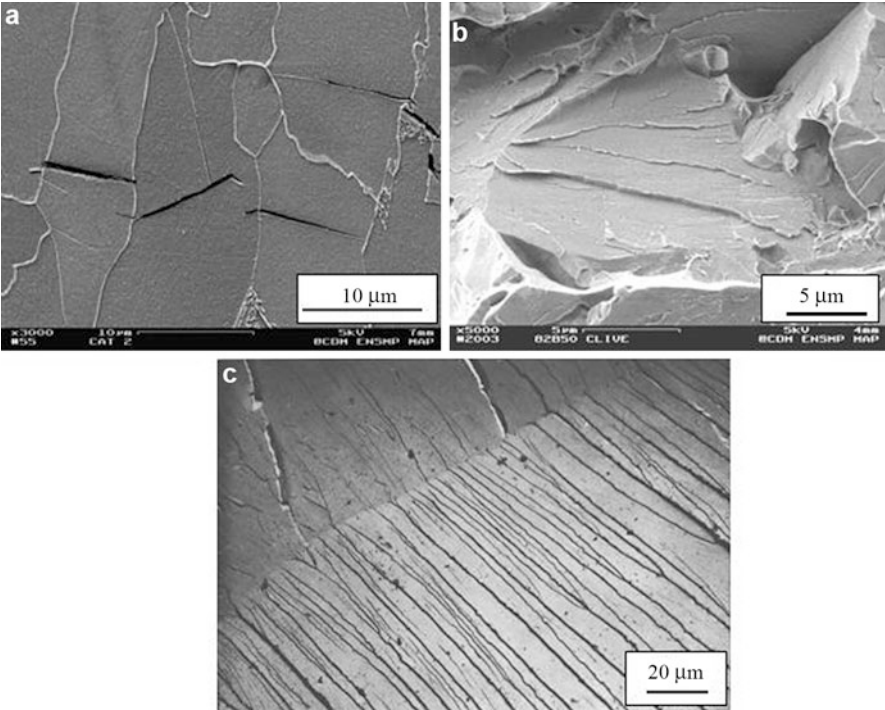
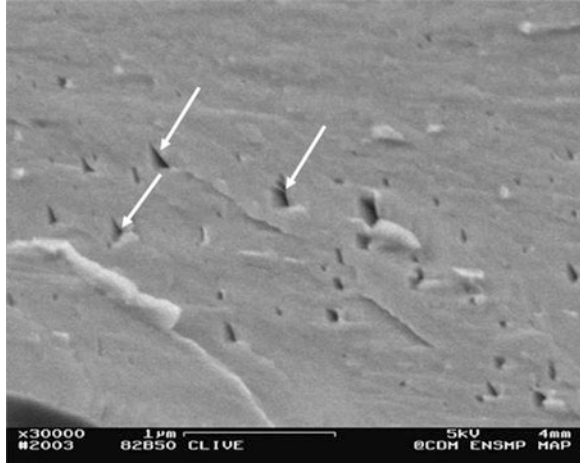


Fig. 3.1 (a) Cleavage microcracks observed on longitudinal sections in a low alloy steel (Lambert-Perlade 2001); (b) fracture surface in a low alloy steel (Lambert-Perlade 2001) showing the presence of rivers; (c) fracture in polycrystalline zinc. Rivers originating from a grain boundary

and steps or ridges appear on the fracture surface to compensate for the local misorientation, in particular at grain boundaries. In the case of mechanical twins, these steps look like indentation marks called “tongues” (Fig. 3.2). In order to maintain the equilibrium of the crack front, the nearest steps gather to form a single step of larger height leading to the formation of “rivers”, as observed in Fig. 3.1b. These rivers align with the direction of the local propagation of the cleavage cracks. On a macroscopic scale the surfaces of the cleavage facets are normal to the maximum principal stress. In fracture mechanics terminology, this is called mode I fracture (see Chap. 2).

Fig. 3.2 SEM micrograph of a fracture surface of a low alloy steel. The *arrows* indicate the presence of “tongues” corresponding to the intersection of the main (001) fracture plane with mechanical twins



3.2.2 Cleavage Versus Intergranular Fracture

Another brittle mode of fracture observed in polycrystalline metals corresponds to intergranular fracture. If a crack forms along a grain boundary having a surface energy, γ_b , the breaking energy, $2\gamma_s^{\text{int}}$, of the atomic bonds for an intergranular fracture must be reduced by γ_b . Hence, one could think that intergranular fracture will be easier than transgranular cleavage. However the anisotropy of the surface energy, γ_s , according to the crystallographic orientation, must also be taken into account. The surface energy γ_s for a crystallographic cleavage plane is always less than the surface energy of an intergranular surface by typically a factor of about 1.20. In order to characterise the transition between intergranular and cleavage fracture, the parameter R_{CI} is defined as:

$$R_{\text{CI}} = \frac{2\gamma_s^{\text{int}} - \gamma_b}{2\gamma_s} = 1.20 - \frac{\gamma_b}{2\gamma_s} \quad (3.1)$$

Intergranular fracture is favored when $R_{\text{CI}} < 1$.

Cottrell (1989, 1990a, b) showed that γ_b depends mainly on the shear modulus, μ , whilst γ_s depends on the bulk modulus, k . This means that the parameter R_{CI} is a function of the ratio μ/k and can thus be written as:

$$R_{\text{CI}} = 1.20 - \alpha\mu/k \quad (3.2)$$

where α is a numerical constant close to one. In nickel, for instance, α was found to be equal to 0.95. Table 3.2 gives the values of R_{CI} for a number of metals. This table shows that, in a large number of pure metals including Fe, intergranular fracture should be the preferential mode of fracture. However, in many cases, the segregation of impurities like carbon in iron tends to suppress intergranular brittleness.

Table 3.2 Transition parameters for fracture

Metal	Au	Ag	Cu	Pt	Ni	Rh	Ir	Nb	Ta	V	Fe	Mo	W	Cr
μ/k	0.11	0.19	0.22	0.24	0.34	0.52	0.52	0.25	0.31	0.32	0.33	0.48	0.52	0.82
R_{CI}	1.09	1.02	0.99	0.97	0.87	0.71	0.70	0.97	0.91	0.89	0.88	0.75	0.71	0.42
$(\mu/k)_{CD}$	0.36	0.43	0.57	0.38	0.49	0.39	0.32	0.59	0.55	0.65	0.56	0.35	0.45	0.68

μ/k is the ratio of the shear modulus to bulk modulus, R_{CI} quantifies the risk of intergranular fracture (if smaller than 1) versus cleavage, $(\mu/k)_{CD}$ is the ratio required to for the transition between cleavage and ductile fracture (refer to Sect. 3.2.3)

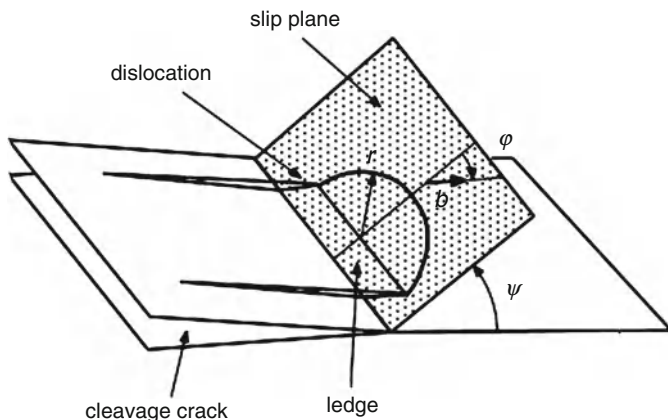


Fig. 3.3 Nucleation of a dislocation half loop of Burgers vector $\underline{b}$ in a slip plane at the tip of a crack

3.2.3 Cleavage Versus Blunting

In crystalline solids, cleavage cracks can blunt by the emission of dislocations (see Chap. 5 in this volume). Rice and Thompson (1974) investigated the conditions under which this mechanism operates. They compared the energy needed for the propagation of a cleavage crack over a unit area, that is $2\gamma_s$, twice the surface energy, to the energy U_L^* needed to nucleate a dislocation loop of radius r in a slip plane intersecting the cleavage plane (Fig. 3.3). The energy of such a dislocation loop is given by the following expression:

$$U_L = \pi r [E_{core} + \alpha b^2 \log(8mr/e^2b)] + 2rE_{ledge} - 3.5br^{3/2}K_M S_M \quad (3.3)$$

where E_{core} is the energy in the dislocation core, b the Burgers vector, E_{ledge} the energy of the ledge of width b created by the dislocation at the crack tip, K_M the stress intensity factor, S_M an orientation factor, the index M designating the loading mode. The first term represents the self energy of the half dislocation loop, so that

$$\alpha = (2 - \nu) \mu / 8\pi (1 - \nu) \approx \mu / 10 \quad (3.4)$$

μ being the shear modulus.

The core energy is related to the core radius r_0 :

$$E_{\text{core}} = \alpha b^2 \log (b / r_0) \quad (3.5)$$

The last term in Eq. 3.3 is the work done by the crack tip stress field.

The factor m is introduced to describe the elastic interaction between the dislocation and the crack. It is estimated to range between 1.2 and 2.3.

$K_{\text{M}} S_{\text{M}} / \sqrt{\rho}$ is the shear traction at a distance ρ from the crack tip.

The orientation factor S_{M} in mode I is equal to:

$$S_{\text{I}} = \sqrt{1/8\pi} \sin \varphi \cos (\varphi / 2) \cos \psi \quad (3.6)$$

the angles φ and ψ being defined in Fig. 3.3.

The condition for the nucleation of the dislocation half loop is given by $\partial U_{\text{L}} / \partial r = 0$ and $\partial^2 U_{\text{L}} / \partial r^2 = 0$, yielding:

$$(K_{\text{M}} S_{\text{M}})_{\text{disl}} = \frac{1.2 \alpha b^{1/2}}{\sqrt{r^* / b}} \quad (3.7)$$

and the critical radius r^* is given by:

$$\frac{r^*}{b} = \frac{e^3}{8m} \exp \left[-\frac{1}{\alpha b^2} \left(E_{\text{core}} + \frac{2}{\pi} E_{\text{ledge}} \right) \right] \quad (3.8)$$

The critical stress intensity factor for blunting as given by the preceding Eqs. 3.7 and 3.8 must now be compared with the critical stress intensity factor for cleavage derived from the fracture toughness $G_{\text{c}} = 2\gamma_{\text{s}}$. When this is more favorable than the blunting condition, we deal with *intrinsic brittleness*.

From such consideration, Rice and Thompson (1974) derived a criterion for intrinsic brittleness. Intrinsic brittleness would occur when the ratio between the shear modulus, μ , and the bulk modulus, k , would be less than a critical value given by:

$$(\mu / k)_{\text{CD}} = 10\gamma_{\text{s}} / bk \quad (3.9)$$

The propensity for blunted cleavage increases with increasing ratio $(\mu/k)_{\text{CD}}$. A number of values for this ratio are given in Table 3.2, which shows that, in almost all metallic materials, blunted cleavage should be the rule.

It is well to remember that in elastic isotropic materials we have $k = 2\mu (1 + \nu) / 3 (1 - 2\nu)$ (see Volume I, Table 2.2), hence the ratio μ/k . This suggests that blunted cleavage in crystalline materials will be favored by large values of the Poisson ratio, ν . In metallic metals there is no theory to predict the value of ν . In bulk metallic glasses, the value of ν can be adjusted by changing the composition (see e.g. Lewandovsky et al. 2003). In a number of these metallic glasses it was shown that the fracture toughness increases significantly with the value of ν .

Values of the Poisson coefficient as large as 0.42 were obtained in Pd based bulk metallic glasses, which exhibit a fracture toughness quite large, of the order of $100 \text{ MPa m}^{1/2}$. In these materials, cleavage along crystallographic planes does not exist and we invoke only “brittle” fracture. Similarly the mechanisms of crack blunting are different from those observed in crystalline materials. Crack blunting is associated with the formation of shear bands.

Equation 3.8 shows that the critical radius r^* of the dislocation loop is but a few b , that is of the same order of magnitude as the dislocation core. This casts doubt on the validity of the preceding calculation, so that, more recently, this theoretical transition between blunted cleavage and pure cleavage was reanalysed by Rice et al. (1992). These authors used the Peierls concept, which is a better representation of dislocation cores behaviour, (see Volume I, Sect. 3.3.7.7), to analyse dislocations nucleation from a crack tip. The shear stress on the slip plane is now a periodic function of period b . The work done to displace the two half bodies on both sides of the slip plane by $b/2$ is denoted γ_{us} . In mode I, this model yields a value of the strain energy release rate G_D for the nucleation of a dislocation loop at the crack tip given approximately by:

$$G_D = 8 \frac{1 + (1 - \nu) \tan^2 \varphi}{(1 + \cos \psi) \sin^2 \psi} \gamma_{us} \quad (3.10)$$

It is found that adding modes II and III has a strong influence on this critical condition.

This quantity must be compared with the critical G value for cleavage crack propagation, that is $2\gamma_s$. Applied to a $\{100\}$ cleavage with the tip along a $\langle 100 \rangle$ direction meeting a $\{110\}$ slip plane with a Burgers vector along $\langle 111 \rangle$, the ratio γ_s / γ_{us} must be larger than 6.3 for blunting to be easier than cleavage in pure mode I. It falls to 3.5 when modes II and III loadings are simply 10% of mode I.

With more exact calculations and considering that blunting requires only the nucleation of partial dislocations, it is finally found that in mode I and 10% shear modes the critical condition for blunting is γ_s / γ_{us} larger than 4.4 for FCC and 2.4 for BCC. Values of the ratio γ_s / γ_{us} estimated by Rice et al. (1992) are shown in Table 3.3. The values of the ratio γ_s / γ_{us} reported in Table 3.3 were obtained from Frenkel-Peierls-Nabarro potential or from embedded-atom models (E.A.M.). These values are very approximate.

This table shows that in most FCC metals, except iridium, blunting should always be observed. Lithium should not be able to cleave. Conversely, pure cleavage should be observed before the nucleation of dislocations for the BCC metals Cr, Mo and W, and also Fe, Nb, V and Ta are closer to the border line.

It must be stressed that these calculations were performed for a temperature of 0 K and that at high temperatures thermal activation will favor blunting.

Furthermore, if these theoretical calculations are useful to explain occurrence of cleavage in materials like silicon or pure chromium, which contain initially a very low density of dislocations, they do not apply to engineering materials. In these

Table 3.3 Materials properties to evaluate the ratio γ_s/γ_{us} (Rice et al. 1992)

Material	γ_s ($T=0$) (J/m ²)	μ_{slip} (GPa)	b (nm)	γ_s/γ_{us} (Frenkel)	γ_s/γ_{us} (EAM)
FCC metals					
Ag	1.34	25.6	0.166	8.8	12.5
Al	1.20	25.1	0.165	8.1	11.5
Au	1.56	23.7	0.166	11.0	15.7
Cu	1.79	40.8	0.147	8.3	11.8
Ir	2.95	198	0.156	2.7	3.8
Ni	2.27	74.6	0.144	5.9	8.4
Pb	0.61	7.27	0.201	11.6	16.6
Pt	2.59	57.5	0.160	7.8	11.2
BCC metals					
Cr	2.32	131	0.250	1.1	1.6
Fe	2.37	69.3	0.248	2.2	3.2
K	0.13	1.15	0.453	4.0	5.7
Li	0.53	3.90	0.302	7.3	10.4
Mo	2.28	131	0.273	1.0	1.5
Na	0.24	2.43	0.366	4.4	6.2
Nb	2.57	46.9	0.286	3.1	4.4
Ta	2.90	62.8	0.286	2.6	3.7
V	2.28	50.5	0.262	2.8	4.0
W	3.07	160	0.274	1.1	1.6
Diamond cubic					
C	5.79	509	0.145	2.2	3.1
Ga	1.20	49.2	0.231	2.9	4.2
Si	1.56	60.5	0.195	3.7	5.2

materials cleavage fracture is explained by the existence of cleavage initiation sites from second phase particles and the stress intensification produced by plastic deformation, as detailed below.

3.3 Cleavage Mechanisms

3.3.1 Theoretical Cleavage Stress

The normal stress, σ_c , theoretically needed to fracture a crystal by cleavage can easily be determined provided that the bonding energy, U , between the atoms located across the cleavage plane is known. The force required to separate cleavage planes is the derivative of this energy with respect to distance. As the distance between the lattice planes increases, the stress, which is zero at the distance b corresponding to inter-atomic equilibrium, goes through a maximum which corresponds to the value of the cleavage stress, σ_c . Cleavage fracture requires the energy of two new surfaces associated with the formation of a pair of new surfaces, $2\gamma_s$, per unit area

of new surfaces. Assuming that the variation of the force with displacement of the crystallographic planes is sinusoidal leads to (Volume I, Sect. 2.2.3):

$$\sigma_c = (E\gamma_s / b)^{1/2} \quad (3.11)$$

where E is Young modulus. With the typical values for iron $E = 200$ GPa, $b = 0.3$ nm, $\gamma_s \approx 0.1$ J/m², Eq. 3.11 leads to $\sigma_c = 26$ GPa $\approx E/10$. This theoretical value of $\sigma_c \approx E/10$ for the cleavage stress is much higher than the experimental values found for classical metallic samples (typically 1 GPa in steels). However, for a number of whiskers (*i.e.* small filaments free of dislocations), the measured values are of the same order as the theoretical value, providing a qualitative validation of theoretical calculation.

The reasons for the large difference between the observed and the calculated values for σ_c are linked to local stress raisers. These can be of two different kinds. In crystalline ceramic materials in which brittle fracture occurs under purely elastic conditions, the cleavage stress is related to the existence of defects, which are inherently present. In this case the fracture stress is given by the Griffith stress σ_G (see Chap. 2 in this volume):

$$\sigma_G = (E\gamma_s / \pi a)^{1/2} \quad (3.12)$$

where a is the size of the large defects.

In crystalline metallic materials the reason for the difference between the theoretical value for σ_c and the experimental cleavage stress is different. Cleavage in these materials is always accompanied by plastic deformation. In other words plastic deformation is a prerequisite to initiate cleavage fracture. The dislocations and twins produce stress concentrations, which are sufficient to reach locally the theoretical cleavage stress. Moreover the presence of heterogeneities like second phase particles or inclusions is also the source of local stress concentrations. This is the basis of the theories, which are briefly presented in the following.

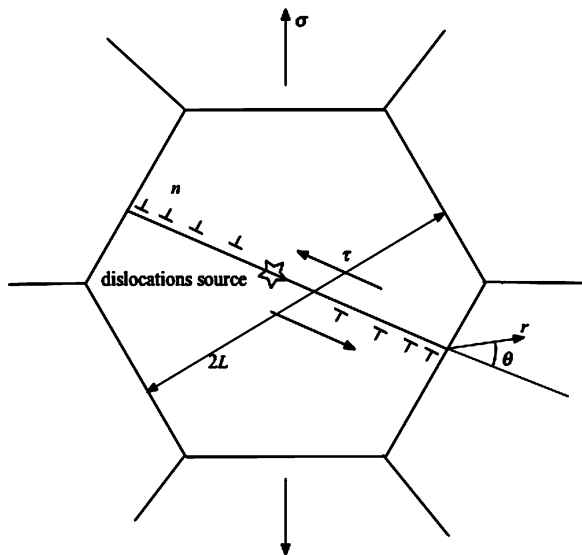
3.3.2 Local Conditions for Cleavage

3.3.2.1 Initiation Controlled Cleavage

The formation of slip bands and, under given circumstances, of mechanical twins during deformation is the source of stress concentration. This is illustrated in Fig. 3.4 where a pile-up of n dislocations is blocked by a grain boundary. Many variants have been proposed for this elementary mechanism; see *e.g.* Zener (1949) and Stroh (1954). For a pile-up of length $2L$ the normal stress at a distance r from the grain boundary in a direction, θ is given by:

$$\sigma = (\tau - \tau_i) (L / 2r)^{1/2} f(\theta) \quad (3.13)$$

Fig. 3.4 Sketch showing stress concentration at the head of a dislocation pile-up generating cleavage in a neighbouring grain



where τ and τ_i are the applied resolved shear stress and the lattice friction stress, respectively. It is assumed that cleavage is initiated when the local stress, given by Eq. 3.13, and thus the resolved shear stress τ , reaches the theoretical critical value, σ_c , over a sufficiently long distance $r = X_c$. Equation 3.13 then leads to:

$$\tau_f - \tau_i = \frac{\sigma_c (X_c/d)^{1/2}}{f(\theta)} \simeq \frac{(E/10) (X_c/d)^{1/2}}{f(\theta)} \quad (3.14)$$

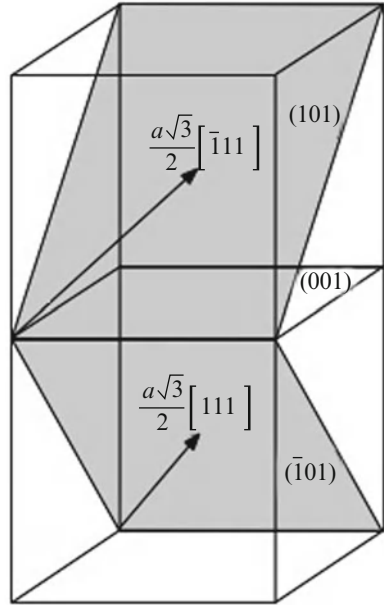
τ_f being then the overall cleavage shear stress.

Since the length, L , that characterises the size of the dislocation pile-up, is a linear function of the grain size, d , i.e. $L \sim d/2$, Eq. 3.14 predicts that the stress necessary to initiate cleavage fracture varies like $1/\sqrt{d}$. Moreover, as the friction stress, τ_i , is strongly temperature dependent, this equation also shows that the stress for cleavage initiation strongly increases with decreasing temperature. In many cases, in particular in steels, it has been shown that, within a first approximation, the cleavage stress does not depend on temperature. This strongly suggests that cleavage is not initiation controlled otherwise a temperature dependence should be observed.

3.3.2.2 Growth Controlled Cleavage

The above calculation of the stress necessary to initiate a cleavage crack has not addressed the question of whether the process is possible on energy grounds. This problem was studied by Cottrell (1958) who assumed that a $\{100\}$ cleavage was initiated in a BCC material by the so-called Lomer-Cottrell lock mechanism of two $\{110\}$ slip systems (Fig. 3.5). When a cleavage crack of length C is initiated,

Fig. 3.5 Cottrell's mechanism: (001) cleavage initiated in a BCC metal at the intersection of two {110} slip systems



the dislocations in the two pile-ups climb rapidly in the crack which can thus be considered as a sessile dislocation with a Burgers vector nb and a core whose size, according to Cottrell, is $C/2$. Simultaneously two surfaces are created, and a potential energy equal to $\pi (1 - \nu^2) \sigma^2 C^2 / 2E$ is released (ν is the Poisson ratio). Thus the change in energy is given by:

$$U_t = \frac{\mu(nb)^2}{4\pi(1-\nu)} \log\left(\frac{2R}{C}\right) + 2\gamma_s C - \frac{\pi(1-\nu^2)\sigma^2 C^2}{2E} \quad (3.15)$$

Writing the conditions of instability $\delta U_t / \delta C \leq 0$, it is found that the critical value for the growth of a cleavage crack is given by:

$$\sigma = \sigma_f = \frac{2\gamma_s}{nb} \quad (3.16)$$

As $nb = \pi(1-\nu)L(\tau - \tau_i)/\mu$ (Friedel 1964), Eq. 3.16 leads to:

$$\sigma_f(\tau - \tau_i) = \frac{2\mu\gamma_s}{\pi(1-\nu)d} \quad (3.17)$$

The effective stress $(\tau - \tau_i)$ acting on the dislocations in the pile-ups is a function of grain size according to Hall (1951) and Petch (1953):

$$\tau - \tau_i = k_y d^{-1/2} \quad (3.18)$$

Combining Eqs. (3.17) and (3.18) leads to:

$$\sigma_f = k'_y d^{-1/2} \quad \text{with} \quad k'_y = \frac{2\mu\gamma_s}{\pi(1-\nu)k_y} \quad (3.19)$$

Experimental results show that the cleavage stress, σ_f , is proportional to $d^{-\alpha}$, with α close to 1/2 but the constant k'_y that is found experimentally leads to surface energy values much greater than $2\gamma_s$. These large values are due to dissipative mechanisms, which add to the work, $2\gamma_s$, required to break the atomic bonds.

3.3.2.3 Cleavage Initiated from Brittle Inclusions

As an example, in mild steels it is assumed that cleavage fracture initiates from very brittle platelets of cementite located along grain boundaries. Dislocations pile-ups raise the stress within these carbides and nucleate a cleavage crack across them. According to Smith (1966) the corresponding necessary condition for cleavage fracture is expressed by:

$$(C_0/d)\sigma^2 + (\tau - \tau_i) \left[1 + (C_0/d)^{1/2} 4\tau_i/\pi(\tau - \tau_i) \right]^2 \geq 4E\gamma_s/\pi(1-\nu^2)d \quad (3.20)$$

where C_0 is the thickness of the platelets and d is the ferrite grain size. The terms on the left hand side of Eq. 3.20 relate to the applied stress and express first the direct effect of the applied stress (Griffith stress, Eq. 3.12), and then the indirect effect of the stress concentration resulting from the dislocation pile-up on the particles (second term). The term on the right hand side of Eq. 3.20 represents the resistance of the ferrite phase to the propagation of a cleavage crack initiated from the carbide. At the yield strength, the Petch relationship given by Eq. 3.18 can be applied in Eq. 3.20 leading to:

$$\sigma_f^2 = \frac{4E\gamma_s}{\pi(1-\nu^2)C_0} - \frac{k_y^2}{C_0} \left(1 + \frac{4\sqrt{C_0}\tau_i}{\pi k_y} \right) \quad (3.21)$$

As an order of magnitude, with $C_0 = 1 \mu\text{m}$, $\tau_i = 100 \text{ MPa}$, $k_y = 0.33 \text{ MPa m}^{1/2}$, the first term, representing the stress needed for the propagation in the ferrite grain, is equal to $(400\gamma_s \text{ MPa})^2$ (γ_s expressed in J/m^2) while the second one, representing the nucleating stress in the cementite, is equal to $(400 \text{ MPa})^2$. Since γ_s in the cementite is of the order of $1\text{--}2 \text{ J/m}^2$, the stress for the nucleation of cleavage from a cementite platelet is very low.

In the condition (3.21) for cleavage fracture, the ferrite grain size does not appear anymore, contradicting experiments, which clearly showed that the cleavage stress increases when the grain size decreases. This apparent discrepancy is often explained by the existence of metallurgical correlation between the grain size, d , and

the platelet thickness, C_0 . It should also be added that the Smith mechanism predicts that the cleavage stress σ_f is an increasing function of temperature since the lattice friction stress τ_i is strongly temperature dependent.

This mechanism envisaged for carbide platelets can operate as well for inclusions.

3.4 Statistical Aspects

3.4.1 Beremin Model and Weibull Stress

The variety of microstructural elements and of their sizes at play in cleavage nucleation and propagation results in a statistical distribution of the fracture stresses. Rather surprisingly, although the scatter in cleavage stress measurements is well known since a long time, it was only in the 1980s that models have been proposed to account for this effect (for review papers see *e.g.* Wallin et al. 1984; Wallin 1991a, b). Nowadays the most largely used models are those derived from the seminal work by Beremin (1983) (for a review of this model see Mudry 1988; Pineau 1992, 2003, 2006; see also Evans 1983). Assuming that the material contains a population of microdefects (particles or grain sized microcracks) distributed according to a simple (power or exponential) law, $p(a)$, the weakest link theory states that the probability to failure $P(\sigma)$ of a representative volume element V_0 is given by:

$$P(\sigma) = \int_{a_c(\sigma)}^{\infty} p(a) da \quad (3.22)$$

where a_c is simply given by Eq. 3.23, *i.e.*:

$$a_c = \frac{2E\gamma_s}{\alpha\sigma^2} \quad (3.23)$$

where α is a numerical constant. Knowing the distribution $p(a)$, it is therefore possible to calculate the associated distribution $P(\sigma)$.

In a volume V , which is uniformly loaded and which contains V/V_0 statistically independent elements, the probability to failure can thus be expressed as:

$$P_R = 1 - \exp \left[-\frac{V}{V_0} P(\sigma) \right] \quad (3.24)$$

As a general rule the function $p(a)$ is not known. However when the critical step for cleavage fracture is the propagation of microcracks initiated from particles of size a , the distribution $p(a)$ can be determined experimentally. Two types of laws are usually proposed:

- a power law

$$p(a) = \gamma a^{-\beta} \quad (3.25)$$

- an exponential law including, if necessary, a cut-off parameter (see *e.g.* Carassou et al. 1998; Lee et al. 2002) such that the cumulated probability is given by:

$$p(\text{size} \geq a) = \exp \left[- \left(\frac{a - a_u}{a_0} \right)^n \right] \quad (3.26)$$

where a_u and a_0 are parameters of the distribution. The simple power law leads to the well-known Weibull expression:

$$P_R = 1 - \exp \left[- \frac{V}{V_0} \left(\frac{\sigma}{\sigma_u} \right)^m \right] \quad (3.27)$$

with the Weibull shape factor:

$$m = 2\beta - 2 \quad (3.28)$$

This shape factor characterises the dispersion, which is the higher the lower m ; σ_u is roughly the mean breaking strength for an element of volume V_0 .

It should be noted that, within a first approximation, m and σ_u are temperature independent. Similarly, the exponential law (3.26) leads to (Tanguy et al. 2003):

$$P_R = 1 - \exp \left\{ - \frac{V}{V_0} \exp \left[- \left(\frac{1/\sigma^2 - 1/\sigma_u^2}{1/\sigma_{c0}^2} \right)^n \right] \right\} \quad (3.29)$$

with

$$\sigma_{c0} = (2E\gamma_s/\alpha a_0)^{1/2} \quad \text{and} \quad \sigma_u = (2E\gamma_s/\alpha a_u)^{1/2} \quad (3.30)$$

Equation 3.27 is a simplified expression since no threshold is introduced. In three dimensions and in the presence of smooth stress gradients, this equation can be written as:

$$P_R = 1 - \exp \left(- \frac{\int_{PZ} \sigma_1^m dV}{V_0 \sigma_u^m} \right) \quad (3.31)$$

where the volume integral is extended over the plastic zone (PZ), as far as in metals plastic deformation is needed for cleavage to occur. This equation can be rewritten as:

$$P_R = 1 - \exp \left[- \left(\frac{\sigma_w}{\sigma_u} \right)^m \right] \quad (3.32)$$

where σ_w is referred to as the “Weibull stress”:

$$\sigma_w^m = \frac{1}{V_u} \int_{PZ} \sigma_1^m dV \quad (3.33)$$

V_u simply replacing V_0 .

A number of investigators introduced a threshold stress, σ_{th} , directly into Eq. 3.31 (see *e.g.* Bakker and Koers 1991; Xia and Cheng 1997). Gao et al. (1999) proposed a modified form of Eqs. (3.31) and (3.32) given by:

$$P_R = 1 - \exp \left[- \left(\frac{\sigma_w - \sigma_{wmin}}{\sigma_u - \sigma_{wmin}} \right)^m \right] \quad (3.34)$$

where σ_{wmin} represents the minimum value of σ_w at which cleavage fracture becomes possible (for a full discussion, see also Gao et al. 1998a, b; Gao and Dodds 2000).

In the original Beremin model (Beremin 1983), an implicit threshold Weibull stress was also included, an aspect which is somewhat forgotten in the literature. In this model, applied to metals, it is assumed that cleavage fracture cannot occur in the absence of plastic deformation, *i.e.* below the yield strength R_p . This means that ahead of a crack tip the plastic zone (PZ) size must be larger than a critical size, X_c , or otherwise stated that there exists a threshold for the stress intensity factor, K_{Imin} , below which cleavage fracture cannot occur.¹ This threshold is simply given by:

$$K_{Imin} \approx R_p \sqrt{3\pi X_c} \quad (3.35)$$

It should also be noted that the exponential law with a cut-off parameter (3.26) used to describe the particle distribution leads also to a threshold stress given by (3.29).

The original Beremin model contains only two independent parameters, m and the product $\sigma_u^m V_u$. A number of studies showed that in structural steels $m \approx 20$ when no threshold is explicitly used, (see *e.g.* Beremin 1983; Minami et al. 2002). Similar values were reported on structural steels with yield strength between 490 and 685 MPa (Minami et al. 2002). Values of m lower than 20 are found when a threshold is introduced (see *e.g.* Gao and Dodds 2000; Petti and Dodds 2005a, b). Low values of m are also observed for brittle intergranular fracture (see Sect. 3.8.5).

¹Fracture toughness, which is introduced here simply to show the existence of a threshold, is discussed later.

3.4.2 Angular Distribution of Cracks: Batdorf's Theory

We have assumed for simplicity that all the microcracks were oriented normal to the tensile axis. This will not be so in reality and Batdorf's treatment (Batdorf and Crose 1974) takes this fact into account. This treatment introduces an effective stress σ_e which causes a crack to propagate as soon as it exceeds a critical value σ_c ; σ_e is a function of the normal stress σ_n and shear stress τ that act on the plane of the crack. Let $d\Omega$ be the solid angle containing the normals to those cracks for which σ_e lies between σ_c and $\sigma_c + d\sigma_c$. This will be a function of the stress field $\underline{\sigma}$ and of σ_c , and will take different values for uniaxial and biaxial traction, for example.

As before we denote by $P(\sigma)$ the proportion of cracks per unit volume, which propagate under an effective stress σ , and now assume these to be randomly oriented. If the stress increases from σ to $\sigma + d\sigma$ the number of cracks activated per unit volume will be $P(\sigma) d\Omega/4\pi$; so the proportion of critical cracks in a volume dV is:

$$dP_R(\sigma) = dV \int_{\sigma_c=0}^{\sigma_c=\sigma} P(\sigma_c) \frac{d\Omega(\sigma_c)}{4\pi} \quad (3.36)$$

Finally, the failure probability is given by:

$$P_R = 1 - \exp \left[- \int_V \left(\int_{\sigma_c=0}^{\sigma_c=\sigma} P(\sigma_c) \frac{d\Omega(\sigma_c)}{4\pi} \right) dV \right] \quad (3.37)$$

equivalent of Eq. 3.24.

A simplifying assumption is to say that the cracks will propagate only under the effect of the normal stress, and that the shear force has no effect; this is equivalent to putting $\sigma_e = \sigma_n = \underline{g} \cdot \underline{\sigma} \cdot \underline{g}$, where $\underline{g}$ is the unit vector perpendicular to the crack plane.

In uniaxial traction $\sigma_e = \sigma \cos^2 \varphi$, where φ is the angle between the normal to the crack and the tensile axis; then:

$$d\Omega = 2\pi \frac{d(\sigma_c/\sigma)}{(\sigma_c/\sigma)^{1/2}} \quad (3.38)$$

In biaxial traction $\sigma_e = \sigma \sin^2 \varphi$, where φ is the angle between the normal to the crack and the normal to the plane of the tractions respectively; then:

$$d\Omega = 2\pi \frac{d(\sigma_c/\sigma)}{(1 - \sigma_c/\sigma)^{1/2}} \quad (3.39)$$

With a Weibull distribution we find the probabilities of fracture for the two cases respectively

$$\begin{aligned}
 P(\sigma, 0, 0) &= 1 - \exp \left[-\frac{V}{V_0} \left(\frac{\sigma}{\sigma_u} \right)^m \int_0^1 \frac{x^m}{2} \frac{dx}{\sqrt{x}} \right] \\
 P(\sigma, \sigma, 0) &= 1 - \exp \left[-\frac{V}{V_0} \left(\frac{\sigma}{\sigma_u} \right)^m \int_0^1 \frac{x^m}{2} \frac{dx}{\sqrt{1-x}} \right]
 \end{aligned} \tag{3.40}$$

More generally, we need to know the effective stress σ_e as a function of σ_n and τ . The difficulty here is that propagation of the cracks is generally accompanied by bifurcation.

3.4.3 Some Limitations

These theories are valid only so long as the initial assumptions hold, in particular that of the independence of the volume elements. Thus if the normal stress is compressive a crack can still propagate, but in general it will bifurcate, so that the stress intensity factor decreases and the crack becomes inactive. Even so it will contribute to the progressive damaging of the material, but the “weakest link” assumption will cease to hold (Fig. 3.6).

Consider a biaxial compressive stress field (σ_1, σ_2). Cracks will propagate only in Mode II, with bifurcation, in the direction of the maximum tensile stress at their extremities. Friction between the edges is to be taken into account, which involves using the normal stress. Failure will result if a sufficient number of defects coalesce. With the assumption that the cracks are randomly oriented, the probability of failure can be calculated as a function of the distribution of the size of the cracks.

Another case in which the previous theories do not apply is that in which cracks propagate but are blocked on meeting obstacles, which arrest them. This occurs in concrete and also in certain ceramics.

A further difficulty arises when the stress gradients are very high, for example in the region of crack tip where the elastic stress varies like $r^{-1/2}$, r being the distance from the crack tip. The choice of V_0 is far from being without effect, and here we come up against the complexity that results from the fact that the stress must remain approximately constant throughout this volume, which is large compared to microcracks. The theory is not applicable if the stress gradient is large.

To avoid these difficulties more general treatments have been developed in which random boolean functions, or random dead leaf functions, are used and in which Weibull's theory appears as a very particular case. These methods use scale effects that differ from this, and effects of shape that do not appear at all in this treatment. They require more elaborate experiments than those normally used for determining the Weibull constants.

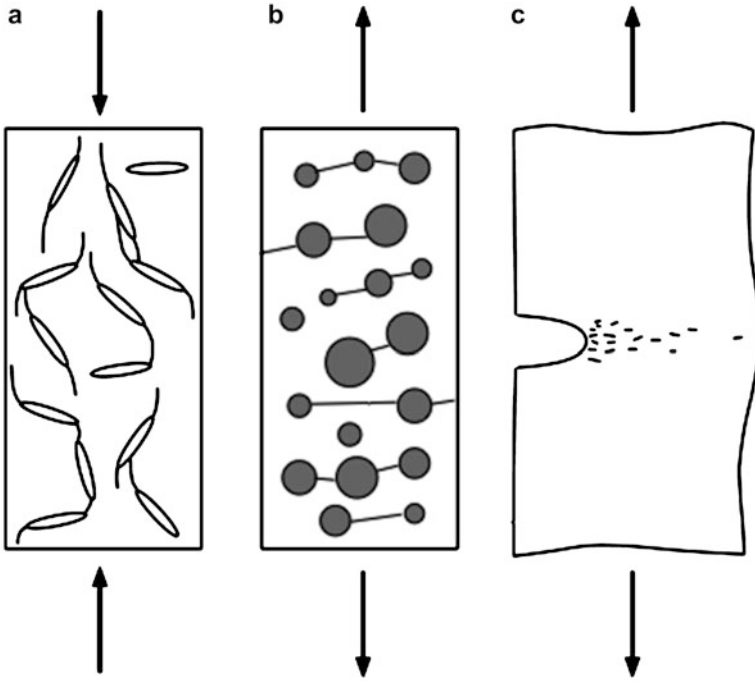


Fig. 3.6 Cases where the “weakest link” theory does not apply: (a) compressive loading; (b) cracks stopped by obstacles; (c) high gradients

3.4.4 Weibull Statistical Distribution and Fracture Toughness

3.4.4.1 Brittle Material

We have shown in Chap. 2 of this volume that the stress in a sector of small angle θ is:

$$\sigma = \frac{K_I}{\sqrt{2\pi r}} \quad (3.41)$$

where K_I is the stress intensity factor.

If we apply Weibull’s theory here in a small sector, θ , we find that, provided $m > 4$:

$$\log \frac{1}{1 - P_R} = B \int_{-\theta}^{\theta} d\theta \int_{r_0}^{\infty} \left(\frac{K_I}{\sqrt{2\pi r}} \right) \frac{r dr}{V_0 \sigma_u^m} = \frac{4B\theta K_{Ic}^m}{(m-4)(2\pi)^{m/2} V_0 \sigma_u^m r_0^{(m-4)/2}} \quad (3.42)$$

From this we can find the macroscopic toughness K_{IC} , which, for any given probability, is a function of the Weibull parameters, the length B of the crack front and a critical distance r_0 . This last distance has to be introduced to avoid the second integral diverging at the lower limit. In reality, at the head of the crack the damage shields the stress field; the cracks interfere with one another so as to make there the weakest-link theory inapplicable. The problem now is how to choose a value for r_0 . A possible course is to say that this distance is related to some characteristic dimension of the microstructure, the grain size, say, or a size related to that of the damaged zone, which will be of the order of $(K_I/\sigma_u)^2$. With this second choice we get for the fracture toughness:

$$K_{IC} = \left[\frac{(m-4)(2\pi)^{m/2}}{4} \frac{V_0}{B\theta} \log \frac{1}{1-P_R} \right]^{1/4} \sigma_u \quad (3.43)$$

This procedure is approximate, but it enables the influence of several parameters on the macroscopic toughness to be evaluated. In particular, Eq. 3.43 shows the effect of scale, with the toughness falling as the length B of the crack front increases.

3.4.4.2 Beremin Model and Its Extensions

Small Scale Yielding

The application of the Beremin model (Eq. 3.27) to the crack tip situation is straightforward if it is assumed that the stress-strain field ahead of the crack-tip under small scale yielding (SSY) is simply scaled by the ratio $r/(K/R_p)^2$ or $r/(J/R_p)$, where R_p is the yield strength and r is the distance ahead of the crack tip (Pineau 1981; Beremin 1983; Pineau 1992). The probability distribution for the fracture toughness of a specimen, containing a 2-D crack, scaled in terms of K_{IC} can simply be expressed as:

$$P_R = 1 - \exp \left(- \frac{K_{IC}^4 B \sigma_0^{m-4} C_{mn}}{V_0 \sigma_u^m} \right) \quad (3.44)$$

where B is the specimen thickness and C_{mn} is a numerical factor dependent on the work-hardening exponent of the material, n appearing in the work-hardening tensile stress-strain curve written as: $\sigma = \sigma_0 \varepsilon^n$ and on the Weibull shape factor, m . It should be noted that the size effect measured by the product $K^4 B = \text{constant}$ (for a given probability and a given temperature) derives simply from the fact that the volume of material sampled by the plastic zone of size R is given by $R^2 B$, i.e. $K^4 B$.

The above expression can be extended to a 3-D crack of length l in which the stress intensity factor, K , is a function of the curvilinear abscissa, s . It can thus be stated that, for a given probability to fracture, the following expression is satisfied:

$$\int K^4(s)ds = K_{lc}^4 l \quad (3.45)$$

Equations 3.27 and 3.44 contain only two parameters, as previously stated. However, in this theory, there is a hidden condition for the initiation of cleavage fracture, which is the initiation of plastic deformation as a prerequisite for this failure mode. This condition given by Eq. 3.35 does not explicitly appear in Eq. 3.44. However, in the original Beremin model, it was shown that this condition might be critical when the theory was applied to fracture toughness data obtained at very low temperatures (Pineau 1981).

A number of researchers (e.g. Bakker and Koers 1991; Xia and Shih 1996) introduced a threshold stress, σ_{th} , into the computation of the Weibull stress to reflect the observed macroscopic threshold toughness. A similar approach with a third parameter was introduced by Wallin (1989). Moreover this author proposed a model able to take into account ductile crack growth, which eventually precedes brittle cleavage fracture (see Chap. 5 in this volume). In the absence of ductile crack extension, the fracture probability is expressed as:

$$P_R = 1 - \exp \left[-\frac{B}{B_0} \left(\frac{K_{lc} - K_{min}}{K_0 - K_{min}} \right)^4 \right] \quad (3.46)$$

where B_0 is an arbitrary (normalised) thickness, K_0 is a parameter depending on temperature and K_{min} is a limiting value below which fracture is impossible ($K_{min} \approx 20 \text{ MPa m}^{1/2}$). In the *master curve concept*, a universal temperature dependence for cleavage fracture toughness is assumed (Wallin 1991a and b, 1993):

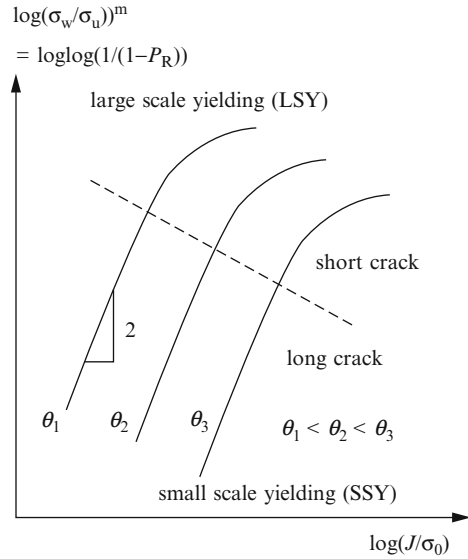
$$K_0 = 30 + 70 \exp [0.019 (\theta - \theta_0)] \quad (K \text{ in MPa}\sqrt{\text{m}}, \theta \text{ in } ^\circ\text{C}) \quad (3.47)$$

where θ_0 corresponds to the temperature at which the mean (median) fracture toughness for a 25 mm thick specimen has the value of $100 \text{ MPa m}^{1/2}$. Details about the determination of θ_0 can be found elsewhere (ASTM E1921-02). Gao et al. (1998a and b) and Gao and Dodds (2000) provided detailed discussions about the three-parameter Weibull stress model and the toughness scaling based on the Weibull stress.

Large Scale Yielding

In many situations, the analytical expression given by Eq. 3.44 cannot be directly applied to small laboratory specimens, in particular when they are tested in the ductile to brittle (DBT) regime, since this expression was derived when SSY conditions are prevailing, *i.e.* when the stress field ahead of the crack tip can be approximated by the Hutchinson-Rice-Rosengren (HRR) field (Hutchinson 1968; Rice and Rosengren 1968). The influence of the loss of constraint effect on the Weibull stress for large scale yielding (LSY) conditions is schematically indicated

Fig. 3.7 Schematic variation of the probability to failure as a function of loading parameter (J/σ_0) at various temperatures, $\theta_1, \theta_2, \theta_3$. The transition from small scale yielding (SSY) to large scale yielding (LSY) condition is shown by a dotted line. The difference between long cracks (LC) and short cracks (SC) is indicated



in Fig. 3.7. In this figure, the apparent increase in fracture toughness of specimens tested at various temperatures ($\theta_1 < \theta_2 < \theta_3$) is indicated when LSY conditions are reached. A quantitative analysis of these effects requires the use of FEM calculations.

Calibration of Weibull Parameters

These are only very few situations where analytical expressions for the Weibull stress can be derived. In most cases numerical calculations are necessary to determine this quantity.

Analytical solutions

Semi-analytical Weibull stress solutions for 2-D cracks in elastic and elastic plastic materials were given by Lei et al. (1998) and O'Dowd et al. (2000). These authors presented closed form expressions for the Weibull stress for linear elastic and power law materials. The solution for an elastic-plastic material with a Ramberg-Osgood material law is given by Eq. 3.49. For such a material, the uniaxial constitutive behaviour is expressed as:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \alpha \left(\frac{\sigma}{\sigma_0} \right)^N \quad (3.48)$$

where α is a material constant, N is the strain-hardening exponent, and σ_0 and ε_0 are reference stress and strain, respectively. The crack tip stress field for this

$m \log(\sigma_w/k)$

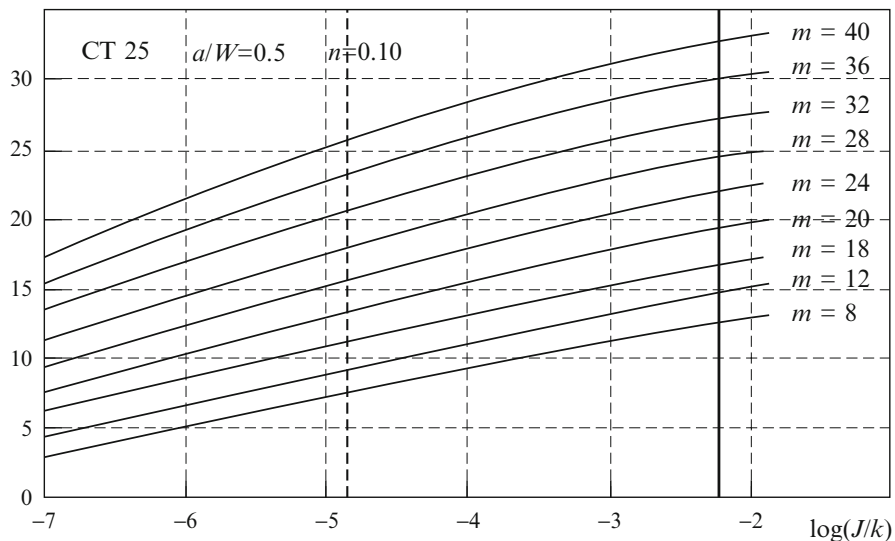


Fig. 3.8 Finite element (FE) numerical calculations on CT25 specimen. Calculations for different values of the Weibull shape factor, m . Dotted vertical line corresponds to the initial crack type opening displacement imposed in FE calculations. Solid vertical line corresponds to the plastic limit load of the specimen

material is assumed to be described by the HRR field. The following expression for the normalised Weibull stress σ_w is obtained:

$$\frac{\sigma_w}{\lambda \sigma_0} = \left[\frac{\hat{\phi}(N)}{1 - m/2(N+1)} \left(\frac{J}{\alpha \varepsilon_0 \sigma_0 L \lambda^{N+1}} \right)^2 \right]^{1/m} \quad (3.49)$$

where $\lambda > 1$ is a parameter which scales the yield stress, σ_0 , $L = (V_u/B)^{1/2}$, m is the Weibull shape parameter and $\hat{\phi}(N)$ is a function which only depends on the strain-hardening exponent N . This function can only be evaluated numerically and depends strongly on N . The introduction of this expression for the Weibull stress into the equation giving the probability to failure leads to an expression similar to (3.44).

Numerical solutions

The results of 2-D plane strain calculations on CT25 specimens are reported in Fig. 3.8. These calculations were made for different values of the Weibull shape factor, m , assuming that the elastic-plastic constitutive behaviour of the material was given by a power law $\sigma = \sigma_0 \varepsilon^n$ (Di Fant et al. 1996b). Tables corresponding to different values of the normalised crack length, a/W and the work-hardening exponent, $n = 1/N$ were established (Di Fant et al. 1996a).

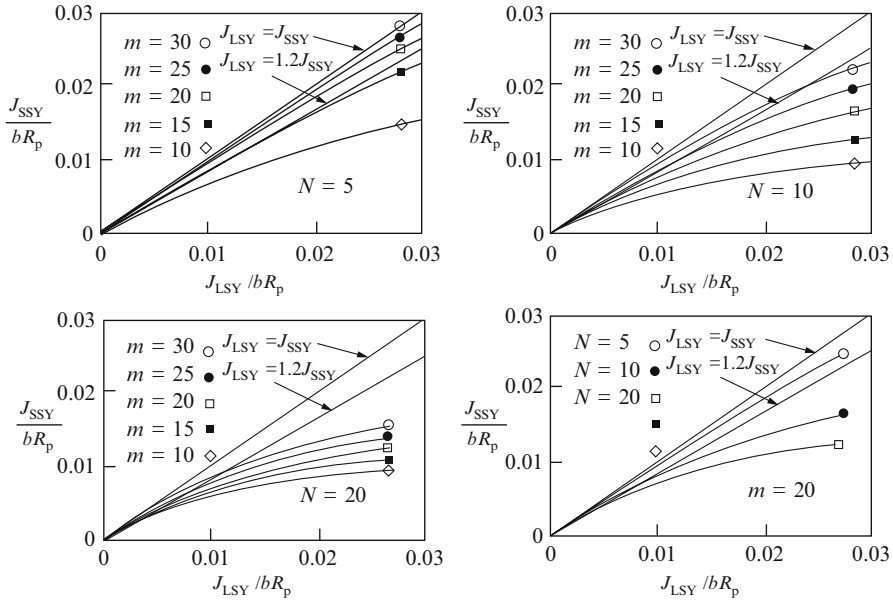


Fig. 3.9 Correction for LSY – SSY conditions using a scaling methodology based upon Weibull stress (Ruggieri et al. 2000)

A detailed study on the Weibull stress approach to assess the effects of constraint loss on cleavage fracture toughness was published by Ruggieri et al. (2000). These authors addressed the significance of the Weibull modulus m on the correlation of macroscopic fracture toughness for varying crack configurations in SENB specimens. They questioned the ability of current calibration procedures to provide reliable values for the Weibull parameters (m , σ_u) and presented arguments that non-uniqueness arises in the calibrated curves, *i.e.* many pairs of (m , σ_u) provide equally good correlation of critical Weibull stress values with the distribution of measured fracture toughness values. Results of a parametric study gave typical values of m for a wide range of material flow properties and toughness values. The results obtained for 1(T) SENB specimens with $a/W = 0.50$ calculated using 3-D FEM simulations are reported in Fig. 3.9. These computations consider specimens with different material properties ($N = 5$ with $E/R_p = 800$; $N = 10$ with $E/R_p = 500$; $N = 20$ with $E/R_p = 300$) and for varying Weibull moduli m (10, 15, 20, 25, 30). In these calculations, b is the length of the ligament, *i.e.* $b = W - a$; J_{LSY} and J_{SSY} are the values of the J integral for large scale yielding and small scale yielding conditions, respectively. In Fig. 3.9, each curve provides pairs of J values, J_{LSY} in the SENB specimen and J_{SSY} in SSY, that produce the same σ_w , and thus the same failure probability. Reference lines are drawn which define a constant ratio of constraint loss, *e.g.*, $J_{LSY} = 1.20 J_{SSY}$, which implies that the SENB average J value must be 20% greater than the SSY value to generate the same Weibull stress. One observes that when near-front stresses deviate from the plane-strain SSY levels, the σ_w curves for the SENB specimens fail to increase at the same rate

with further loading. These results clearly illustrate the gradual effect of constraint loss, especially for moderate to low-hardening materials. Further details about the strategy to determine the Weibull parameters are given elsewhere (Pineau 2003; Andrieu et al. 2012a).

The results of numerical calculations on compact tension (CT) specimens, which are widely used for determining the fracture toughness of materials have been published (Andrieu et al. 2012a). In this figure the values of C_{mn} coefficient (Eq. 3.44) are given for different values of m and n .

Directional Aspects

In a number of circumstances, crack propagation does not remain coplanar. This occurs for instance in functionally graded materials and in welds where the mechanical properties and the fracture toughness of the various regions of a weld, base metal (BM), weld metal (WM) and heat-affected zones (HAZ), (see Appendix B in Volume I for the definition of these zones) are different. An initial crack located in one of these zones, say BM, and parallel to the fusion line can deviate to propagate in the material with a lower toughness, *i.e.* the HAZ or the WM. The Beremin-Weibull model can still be used to predict the direction of crack propagation and the overall fracture toughness provided that the local properties (R_p, m, σ_u) are known. The integration in (3.33) is made over small circular sectors surrounding the crack tip to calculate the density of probability of fracture. The crack will propagate in the direction where this density is maximum and the fracture toughness can be calculated (see *e.g.* Becker et al. 2002; Bezensek and Hancock 2007; Bezensek and Banerjee 2010; Andrieu and Pineau 2012).

3.5 Application to Steels

3.5.1 Introduction

Cleavage fracture concerns ferritic steels only, as in FCC austenitic steels the number of slip planes limits stress concentrations. Many studies showed that cleavage fracture in ferritic steels is often initiated from brittle second phase particles, *e.g.* carbides (Mc Mahon and Cohen 1965; Gurland 1972; Lee et al. 2002; Hahn 1984; Yu et al. 2006). Carbide particles can be spherical as well as oblong. As a result of a fibre loading mechanism, oblong carbides experience very high stresses as the surrounding ferrite matrix is plastically deformed (Lindley et al. 1970; Echeverria and Rodriguez-Ibabe 1999). Oblong particles are thus more prone to the initiation of cleavage fracture. Non-metallic inclusions, such as manganese sulfides, MnS (Tweed and Knott 1987; Alexander and Bernstein 1989; Neville and Knott 1986; Carassou et al. 1998) or titanium nitrides, TiN (see *e.g.*, Fairchild et al. 2000a, b) also act as initiation sites for cleavage fracture in ferritic steels.

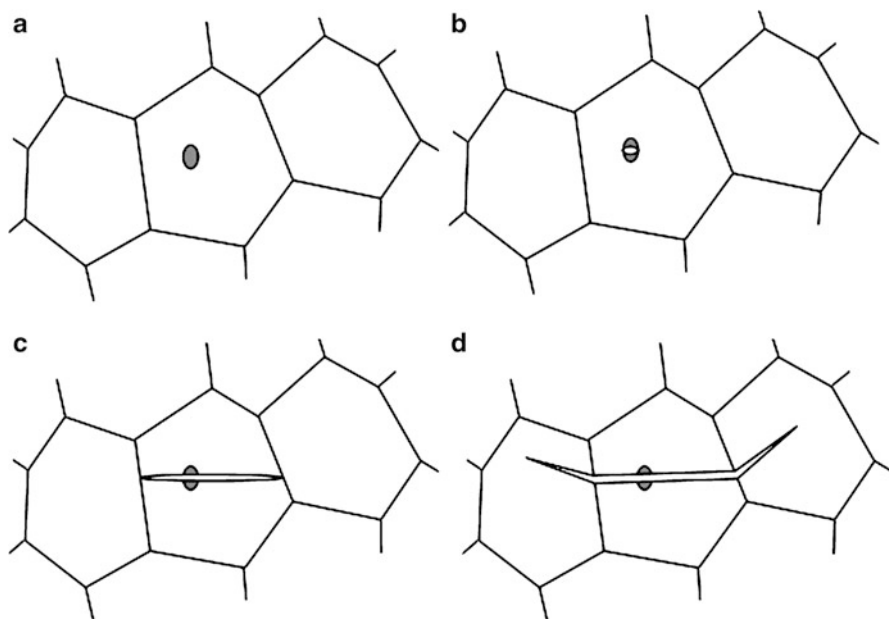


Fig. 3.10 Schematic representation of the role of microstructural barriers on fracture micromechanisms. The crack is assumed to nucleate from an intragranular particle: (a) undamaged material; (b) microcrack initiation and propagation in the particle; (c) microcrack propagation across the particle/matrix interface, and (d) microcrack propagation across a grain boundary or a bainite high angle packet boundary leading to final fracture (Lambert-Perlade et al. 2004)

A cleavage crack initiated from the fracture of a brittle particle can propagate within the adjacent ferrite forming rapidly advancing microcrack, and if the arrest fracture toughness of the ferrite is too low, the crack will penetrate into the neighbouring ferrite grains changing its propagation direction (Fig. 3.10). The grain boundaries are important obstacles for continued crack growth as discussed by a number of authors (*e.g.* Qiao and Argon 2003a and b; Crocker et al. 2005). Proper modelling of cleavage in ferritic steels requires thus to account for multiple barriers (M.B) to the propagation of cleavage cracks. The simplest models are deterministic. However more sophisticated models including the statistical aspects of the problem were also proposed, as discussed later.

Over the past few decades, there has been a steady decrease in many structural steels of the carbon content and of the impurity (P, S) level, and, as a result, typical cleavage initiators like cementite particles and non metallic inclusions have been largely reduced in number and size. This has contributed to the improvement of the brittle cleavage fracture resistance. However, despite these advances, three factors virtually guarantee that cleavage fracture in steel will unfortunately always remain a concern. First, because of continuing improvements in structural steels users are selecting these materials for more severe service conditions. Second, cleavage will always remain the intrinsic brittle mode of failure in BCC materials.

Third, structural steels are usually fusion welded and this leads to the presence of microstructures in the weld metal and in the heat-affected zone (HAZ), which are typically inferior to the highly processed base metal. In the following a special section (Sect. 3.5.4.6) is devoted to the fracture micromechanisms in welds.

3.5.2 Multiple Barriers Models

3.5.2.1 Basic Model

In many ferritic steels, it was found that the cleavage stress, σ_f , was independent of temperature. This strongly suggests that in these materials, the mechanism of cleavage fracture is *growth controlled*, as indicated previously (see *e.g.* Curry and Knott 1979; Pineau 1981, 1992). Cleavage microcracks are progressively nucleated under the influence of plastic strain. These microcracks are arrested at microstructural barriers and fracture occurs when the longest crack reaches the Griffith stress, given by Eq. 3.12. In this equation all terms are almost independent of temperature, except the term γ_s , which is much higher than the true surface energy because of the dissipated energy due to plastic deformation associated in particular with the nucleation of microcracks from second phase particles (inclusions, carbides, etc.). However, this theory is too simple since it does not recognise the different steps encountered during microcrack initiation and microcrack propagation (see *e.g.* Martin-Meizoso et al. 1994).

Schematically, fracture of ferritic steels most frequently results from the successive occurrence of three elementary events illustrated in Figs. 3.10 and 3.11:

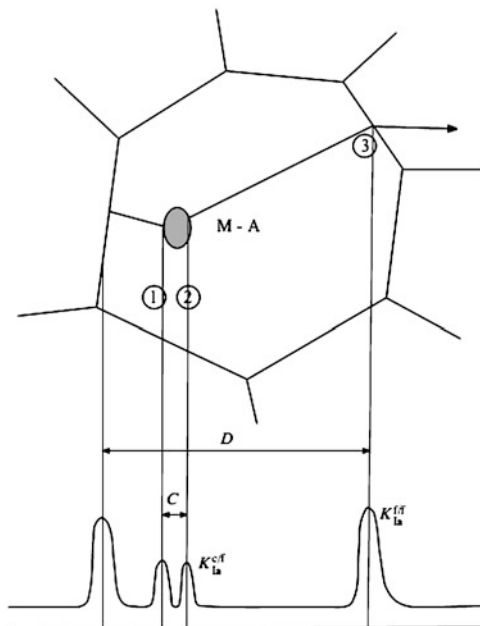
- slip induced cracking of a brittle particle
- propagation of the microcrack under the local stress state across the particle/matrix interface and then along a cleavage plane of the neighbouring matrix grain
- propagation of the grain sized (or packet sized in bainitic steels) crack to neighbouring grains across the grain boundary.

The first event which is similar to that occurring in ductile rupture is governed by a critical stress, σ_d , when the particle size is larger than $\sim 0.1\text{--}1\text{ }\mu\text{m}$ (see *e.g.* Pineau 1992).² Below this size a dislocation-based theory must be used, as indicated in Chap. 4 in this volume. It will be shown that the critical stress, σ_d , is related to the maximum principal stress, Σ_1 , the equivalent von Mises stress, Σ_{eq} , and the yield stress, R_p , by:

$$\Sigma_1 + k (\Sigma_{eq} - R_p) = \sigma_d \quad (3.50)$$

²The far-field stresses are written in uppercase letters in order to distinguish them from local stresses.

Fig. 3.11 Initiation of a cleavage microcrack from a particle (*e.g.* Martensite/Austenite (M-A) constituent in welds). The crack may eventually arrest at the interface c/f; then propagates through the matrix and is arrested at the grain boundaries



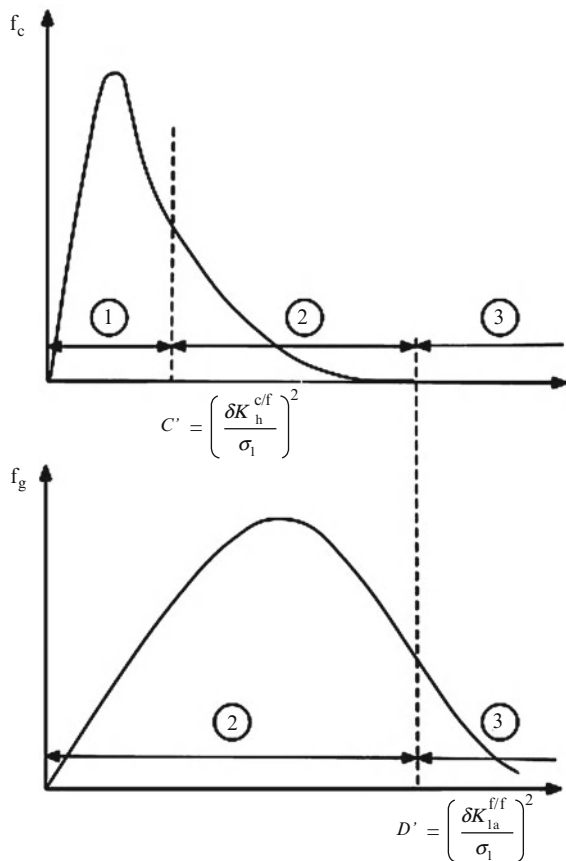
where k is a function of particle shape (Beremin 1981; François and Pineau 2001). Within a first approximation, σ_d is independent of temperature, but the values of σ_d are statistically distributed.

The simple expression given by Eq. 3.50 (see also Margolin et al. 1999 and Deyber et al. 2005) shows that for a given stress state the strain necessary to nucleate particle cracking strongly increases with temperature because of the variation of the yield strength with temperature. Figure 3.12 indicates the local values of the critical stress intensity factors, $K_{Ia}^{c/f}$ (carbide/ferrite) and $K_{Ia}^{f/f}$ (ferrite/ferrite), that must be overcome in order for the crack not to arrest. These values are also statistically distributed. Studies showed that in bainitic steels the crack arresting boundaries were those for which the misorientation between the bainite packets was large (Bouyne et al. 1998; Gourgues et al. 2000; Lambert-Perlade et al. 2004). In particular it was shown that grain-sized microcracks were preferentially arrested at grain boundaries which were misorientated with a large twist angle (Gell and Smith 1967; Qiao and Argon 2003a, b; Qiao and Kong 2004). The particle and “grain” size distribution functions (f_c, f_g) have thus to be considered, as schematically shown in Fig. 3.12. In this figure the critical values of the particle and grain size, C^* and D^* , corresponding to the different steps of cleavage fracture are simply related to the local value of the maximum principal stress, Σ_1 by a Griffith-like expression:

$$C^* = \left(\frac{\delta K_{Ia}^{c/f}}{\Sigma_1} \right)^2 \quad \text{and} \quad D^* = \left(\frac{\delta K_{Ia}^{f/f}}{\Sigma_1} \right)^2 \quad (3.51)$$

where δ is a numerical factor related to the shape of the microcrack and close to 1.

Fig. 3.12 Multiple barrier model. Three events are schematically shown with their probability of occurrence (Martin-Meizoso et al. 1994). The uderscripts h and a designate the fracture toughness of carbides and the matrix, respectively



There are few results in the literature to test the validity of the above model, in spite of the large number of studies devoted to steels. However a number of results are reported in Table 3.4. In this table the details concerning a study on a bainitic steel (Lambert-Perlade et al. 2004) are included. It is worth-noting that the local values of the calculated fracture toughness $K_{la}^{c/f}$ are much lower than the macroscopic fracture toughness, K_{lc} . Several reasons can be invoked to explain this difference. The first one lies in the calculations. Applying Eq. 3.51 necessitates, ideally, the use of the local values of the maximum principal stress which can be much larger than the macroscopic stress (used in the present calculations). The second reason could be related to the fact that these calculations apply to static conditions, which is not necessarily appropriate as discussed in the next paragraph. In Table 3.4, it is also worth-noting that $K_{la}^{f/f}$ is larger than $K_{la}^{c/f}$. This conclusion combined with other observations obtained from acoustic emission measurements (Lambert-Perlade et al. 2004) strongly suggests that the micromechanisms operating during fracture toughness measurements at increasing temperature are not necessarily the same. At very low temperature, cleavage can be controlled by the initiation of microcracks from carbides, while at increasing temperature cleavage is controlled

Table 3.4 Parameters of multiple barriers models

Parameter	Lambert-Perlade et al. (2004)	Literature data		
		Value	Microstructural unit	References
σ_d (MPa)	2,112	—	—	—
$K_{Ia}^{c/f}$ (MPa m ^{1/2})	7.8	2.5–5.0	Carbides	Martin-Meizoso et al. (1994)
		2.5	Globular carbides	Hahn (1984)
		1.8	TiN particles	Rodriguez-Ibabe (1998)
$K_{Ia}^{f/f}$ (MPa m ^{1/2})	CGHAZ-25:	5.0–7.0	Bainite packets	Martin-Meizoso et al. (1994)
	28	7.0	Bainite packets	Martin-Meizoso et al. (1994)
		7.5	Ferrite grains	Hahn (1984)
	ICCGHAZ-25:	4.8	Bainite packets	Rodriguez-Ibabe (1998)
	18	15.2	Bainite packets	Rodriguez-Ibabe (1998)

by the growth of microcracks arrested at grain boundaries. In such conditions the existence of a constant cleavage stress over a wide temperature range could appear as a simplifying assumption.

3.5.2.2 Dynamic Behaviour

The dynamic behaviour of microcracks nucleated from carbide and propagating within the ferrite matrix was studied in details by Kroon and Faleskog (2005). These authors used a unit cell-type dynamic FEM calculation (Fig. 3.13). The initiation of cleavage fracture was modelled explicitly by introducing a pre-existing small crack within the carbide. This microcrack propagated through the carbide and eventually into the surrounding ferrite. The carbide, which had a size of a few microns, was modelled as an elastic cylinder (or sphere), and the ferrite as an elastic viscoplastic material with a yield strength at vanishing zero strain rate equal to R_e . Macroscopic constitutive equations allowing for different strain rate sensitivity were adopted. The crack growth was modelled using a cohesive surface, where the tractions were governed by an exponential cohesive law. Crack growth rates as large as the Rayleigh wave velocity and thus resulting in strain rates as large as 10^4 – 10^6 s⁻¹ were simulated.

These calculations show that the critical stress required for propagating a microcrack initiated from broken carbide increases with decreasing plastic strain rate sensitivity of the matrix. Results showing the variation of the macroscopic stress, Σ_z , as a function of carbide size, C , are given in Fig. 3.14. The results were obtained for various values of the stress triaxiality measured by the ratio, T . In this figure the Griffith criterion is also included as a reference. The curve corresponding

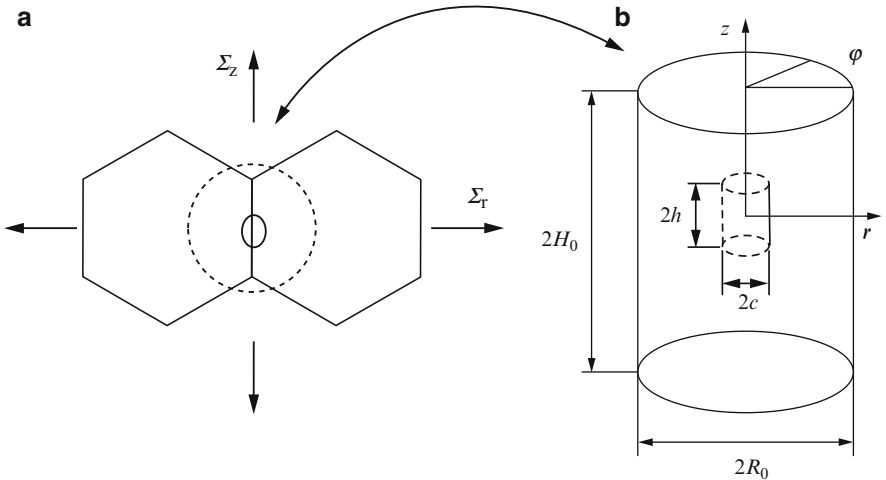
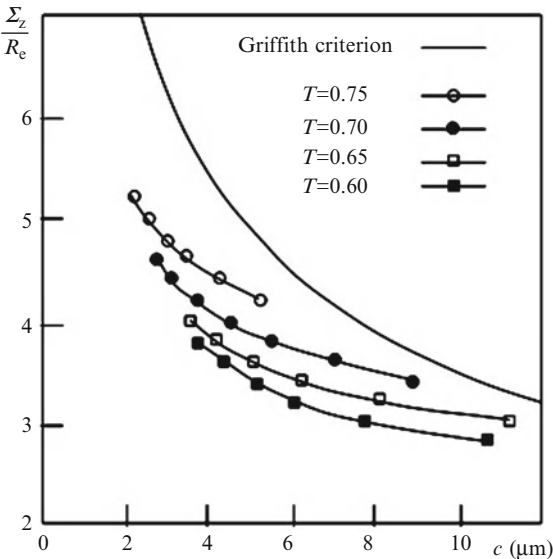


Fig. 3.13 (a) A cracked grain boundary carbide in ferrite; (b) axisymmetric model with a carbide embedded in ferrite (Kroon and Faleskog 2005)

Fig. 3.14 Overall stress Σ_z/R_e as a function of critical largest carbide size c for four levels of stress triaxiality T . Comparison with Griffith criterion (Kroon and Faleskog 2005)



to the Griffith criterion is located above all the curves corresponding to the elastic viscoplastic calculations. This situation may appear as being rather counter-intuitive since plastic flow is expected to increase the resistance to crack growth. However, as stated by the authors, the Griffith curve is valid for a stationary crack whereas in these numerical simulations the crack has a significant speed when it reaches the carbide-ferrite interface. Figure 3.14 shows also that the stress levels required to arrest a microcrack vary at a much lower rate with decreasing carbide size compared

to the Griffith stress. This implies that small carbides may play a more prominent role in cleavage fracture of ferritic steels than what might be expected from the straightforward application of the static Griffith criterion. The strain rate sensitivity and the dynamic aspect of crack growth thus come into play in the initiation and in the continued growth of a cleavage crack.

3.5.2.3 Effect of Stress Triaxiality

Figure 3.14 suggests also that if the stress triaxiality level is decreased from 0.75 to 0.60, the critical stress level required to initiate a critical microcrack decreases by an amount approximately equal to the initial yield stress of the ferrite matrix R_p . This effect of stress triaxiality on cleavage fracture initiation is opposite to what is normally seen in cleavage fracture experiments, where a decrease in crack tip constraint leads to an increase of the fracture toughness. However, this last conclusion is not valid since, in the study by Kroon and Faleskog (2005), the mechanism responsible for the initiation of carbide cracking was not considered and a pre-crack was introduced in the carbides. The deleterious effect of the increase in stress triaxiality level on the nucleation of microcracks from particles is described by Eq. 3.50. Further results were obtained in the study by Kroon and Faleskog (2005), such as a relationship between the applied axial stress Σ_z and the critical largest crack arrest length a_{cs} and this relation is independent of the carbide size and of the level of stress triaxiality.

3.5.3 Applications of the Beremin Model

3.5.3.1 Effect of Microstructure in Bainitic and Martensitic Low Alloy Steels

In the Beremin model, when the volume of V_u is fixed, the value of σ_u reflects the resistance of the material to brittle cleavage fracture. This value is therefore dependent on the microstructure of the materials. Two low alloyed steels used in the fabrication of pressurised water reactors were studied, *i.e.* A 508 (0.16 C, 1.5 Mn, 0.75 Ni, 0.50 Mo) steel and a modern $2^{1/4}$ Cr–1 Mo steel (Bouyne et al. 2001). These materials have similar yield strength at room temperature but different microstructures (Fig. 3.15).

The A 508 steel has an upper bainite microstructure, while the $2^{1/4}$ Cr–1 Mo steel shows a lower bainite microstructure. Tests on notched specimens allowed to determine the values of σ_u adopting the same value for V_u ($50 \mu\text{m}^3$). In A 508 steel, it was found that $m = 22$ and $\sigma_u = 2,600 \text{ MPa}$ (Beremin 1983), while in the $2^{1/4}$ Cr–1 Mo steel, it was found that $m = 20$ and $\sigma_u = 3,500 \text{ MPa}$ (Bouyne 1999). These values for σ_u and m explain the different DBT temperatures observed in

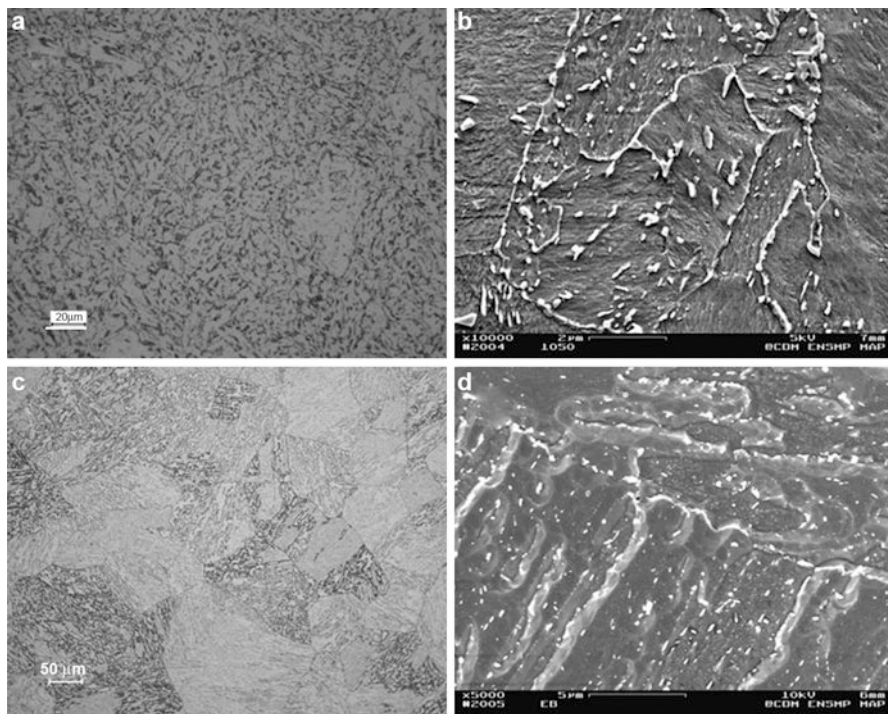


Fig. 3.15 Microstructures of pressure vessel steels. (a) and (b) A 508 (0.16C, 1.5Mn, 0.75Ni, – .0.5Mo) steel; (c) and (d) $2\frac{1}{4}$ Cr–1 Mo steel. A 508 steel exhibits an upper bainite microstructure containing relatively coarse carbide particles, while $2\frac{1}{4}$ Cr–1 Mo steel has a lower bainite microstructure with smaller carbides

those steels and the better fracture properties of lower bainite as compared to upper bainite. The ductile to brittle transition (DBT) temperature is lower by about 50°C in the $2\frac{1}{4}$ Cr–1 Mo steel than in the A 508 steel.

3.5.3.2 Extension to the Multiple Barrier Model

The Beremin model was also extended to multiple barrier models. As stated previously this model is essentially based on the description of the propagation of an existing critical defect belonging to a single population. This is a simplification, which might explain why in the application of the model over a wide range of temperatures, a number of investigators showed that the normalising stress, σ_u , tended to be an increasing function of temperature (see *e.g.* Tanguy et al. 2005a, b). This might simply reflect the existence of different critical steps depending on temperature, as indicated earlier (initiation versus growth controlled cleavage fracture).

Multiple-barrier (MB) models would therefore appear more satisfactory to account for the variation of cleavage fracture toughness over a wide temperature range. In particular MB models addressing the fracture process schematically shown in Fig. 3.12 were proposed (Martin-Meizoso et al. 1994; Lambert-Perlade et al. 2004). These models are also based on the weakest link theory. The nature of these barriers depends on temperature. Application of these models requires the knowledge of a certain number of metallurgical factors including the nucleating particle size distribution and the grain (packet) size distribution. These factors were measured in one specific structural steel in which the brittle particles were formed by M-A constituents (Lambert-Perlade et al. 2004; Davis and King 1994 and 1996), *i.e.* mixed martensite-austenite (MA) particles. The application of these models also necessitates the knowledge of the local fracture toughness, $K_{Ia}^{c/f}$ and $K_{Ia}^{t/f}$, and that of the cleavage fracture stress of particles, σ_d . There are very few results in the literature. However a number of results are reported in Table 3.4 (Sect. 3.5.2.1). In the study devoted to a bainitic steel containing M-A particles it was assumed that the local values of fracture toughness and $K_{Ia}^{c/f}$ and $K_{Ia}^{t/f}$ were not temperature dependent, which is a crude approximation. In spite of this approximation a good agreement was found between the experimental values of the fracture toughness and those inferred from this MB model, as indicated later.

3.5.3.3 Effect of Plastic Strain

We have assumed that microcracks were nucleated at the onset of plastic deformation and that they remain active during the entire loading history. This is a simplification requiring further discussion.

Detailed investigations on a number of steels showed that the number of microcracks nucleated from carbides was an increasing function of plastic strain and was increasing with decreasing temperature (see *e.g.* Kaechele and Tetelman 1969; Mc Mahon and Cohen 1965), which is in qualitative agreement with Eq. 3.50. More recently in a study devoted to the initiation of microcracks from niobium carbides and titanium nitrides in a Ni based superalloy (Inconel 718), it was shown that the probability to failure of these particles could be written as:

$$P_{\text{nucI}} = 1 - \exp \left\{ - \left[\frac{\sigma_1 + \lambda (\sigma_{\text{eq}} - R_e)}{\sigma_{\text{uN}}} \right]^{N_a} \right\} \quad (3.52)$$

where σ_{uN} is a normalising stress and N_a is a material constant (Deyber et al. 2005). Similar studies are lacking for a proper description of the nucleation of microcracks from carbides and non-metallic inclusions in ferritic steels. This is why the modifications to the Beremin model, which are presented below, are essentially based on phenomenological considerations.

Bordet et al. (2005a, b) modified the original Beremin model in order to include the effect of plastic strain on the nucleation of microcracks and the deactivation of

the latter if they are not immediately propagated. At a material point located within the plastic zone the probability of cleavage fracture is expressed as:

$$P_{\text{cleav}}(t) = \int_0^t P_{\text{prop}} dP_{\text{nuc}} \quad (3.53)$$

where t represents the “loading time”, whilst P_{prop} and dP_{nuc} are the probability of crack propagation and the increment of probability to nucleate a microcrack. In Bordet’s model, dP_{nuc} is written as:

$$dP_{\text{nuc}} = NR_p d\varepsilon_{\text{eq}}^p = N_0 (1 - P_{\text{nuc}}) R_p d\varepsilon_{\text{eq}}^p \quad (3.54)$$

where $\varepsilon_{\text{eq}}^p$ is the equivalent plastic strain, N and N_0 are respectively the remaining and the initial number of cleavage initiation sites. Equation 3.54 can be easily integrated. Normalising the yield strength by a given stress R_{p0} and the plastic strain by $\varepsilon_{\text{eq}0}^p$ which are dependent of the material, Eq. 3.54 leads to:

$$P_{\text{nuc}} = 1 - \exp \left(- \frac{R_p}{R_{p0}} \frac{\varepsilon_{\text{eq}}^p}{\varepsilon_{\text{eq}0}^p} \right) \quad (3.55)$$

which bears some similarity with Eq. 3.52. However there exists a significant difference between those two expressions since Eq. 3.55 does not include the effect of stress state which is well known to affect the nucleation of microcracks from particles (see Chap. 4 of this volume).

As for the Beremin model, assuming that the nucleated microcracks can be treated as Griffith flaws distributed according to a power law, and invoking the weakest-link principle over the plastic zone (PZ) volume, the overall fracture probability can then be expressed as:

$$P_R = 1 - \exp \left(- \int_{\text{PZ}} \int_0^t P_{\text{prop}} dN_{\text{nuc}} dV \right) = 1 - \exp \left[- \left(\frac{\sigma_w^*}{\sigma_u^*} \right)^m \right] \quad (3.56)$$

where σ_u^* is a scaling parameter and σ_w^* is a modified Weibull stress defined as:

$$(\sigma_w^*)^m = \int_{\text{PZ}(\sigma_1 \geq \sigma_{\text{th}})} \left\{ \int_0^{\varepsilon_{\text{ep}}} \frac{R_p}{R_{p0}} (\sigma_1^m - \sigma_{\text{th}}^m) [1 - P_{\text{nuc}}(t)] \frac{d\varepsilon_{\text{ep}}}{\varepsilon_{\text{ep}0}} \right\} \frac{dV}{V_0} \quad (3.57)$$

where V_0 is a reference volume as in Eq. 3.27.

Another modification of the original Beremin model was also proposed by Bernauer et al. (1999) to account for cleavage fracture in the transition region. These authors emphasised that the number of available microcracks was reduced by the number of particles around which a void had been formed. They assumed that the number of “omitted” carbides was proportional to the total number of voids formed at second-phase particles. The nucleation rate of cavities was assumed to follow the law proposed by Chu and Needleman (1980) (see Chap. 4 of this volume).

In these modifications of the original Beremin model, plastic strain appears as detrimental since the microcracks are nucleated from carbide particles or non-metallic inclusions due to plastic deformation. On the contrary, a number of other observations have also shown that the effect of a predeformation can be beneficial, (see *e.g.* Groom and Knott 1975; Knott 1966, 1967; Beremin 1981). This is why in the Beremin model a “strain correction” factor was introduced to account for the effect of plastic strain. The probability to failure was written as:

$$P_R = 1 - \exp \left[- \frac{\int_{PZ} \sigma_1^m \exp(-m \varepsilon_{p1} / \alpha) dV}{\sigma_u^m V_0} \right] \quad (3.58)$$

where ε_{p1} is the plastic strain in the direction of the highest principal stress and α is a constant close to 2.

3.5.4 Fracture Toughness

3.5.4.1 Main Characteristics of Brittle Cleavage Fracture Toughness in Steels

The *first characteristic* of cleavage fracture is the scatter in test results. This is illustrated by using the results of the largest comprehensive fracture toughness data set obtained for a single material (22 Ni Mo Cr 37 PWR steel) (see Heerens and Hellmann 2002). In this study more than 750 fracture toughness tests were performed on CT specimens of four thicknesses (12.5, 25, 50, 100 mm) tested over a wide temperature range (−154°C, 20°C). This data base represents an excellent source to check the validity of statistical failure models. The results are reported in Fig. 3.16 where the filled symbols correspond to tests in which cleavage fracture was observed without ductile crack growth and the open symbols are associated with cleavage fracture preceded by ductile tearing. These results show that the fracture toughness measured in terms of K_{JC} increases with test temperature, is largely scattered, and tends to decrease when the specimen thickness is increased. This observation on size effect illustrated in Fig. 3.17 constitutes the *second characteristic* feature of cleavage fracture toughness.

The *third important feature* in brittle fracture of ferritic steels is the so-called “short crack” effect or, more generally the geometrical dependence of the fracture toughness, which has been investigated by a number of authors (see *e.g.*, Dodds et al. 1991; Tigges et al. 1994). This effect is illustrated in Fig. 3.18 where it is shown that the DBT temperature is higher for deep cracks than for short cracks. As schematically shown in this figure, this effect can be qualitatively explained by the loss of constraint effect associated with the short cracks, which can take place much easier than on deep notches. This geometrical dependence of cleavage fracture toughness was extensively investigated by Sumpter (1993) on a low alloy steel (Fig. 3.19). This author tested a widely used structural steel (E36) at −50°C

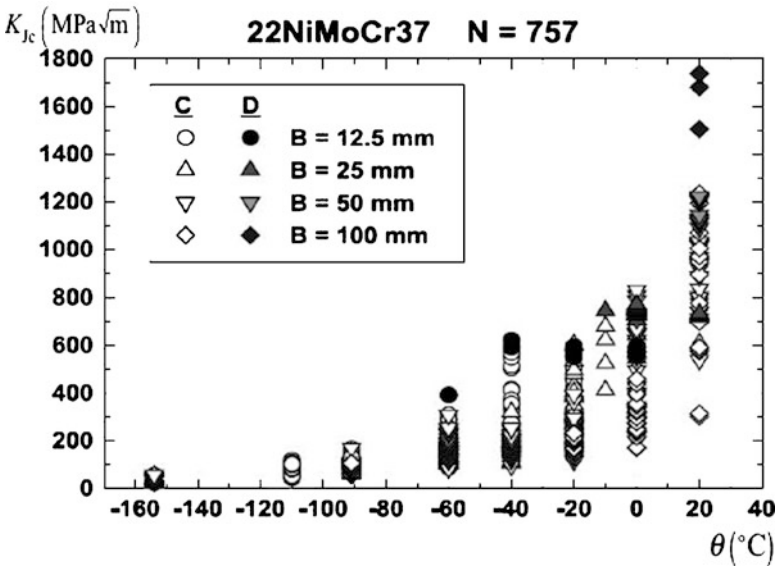


Fig. 3.16 Steel 22NiMoCr37- Tests on 757 CT specimens with 4 thicknesses (12.5, 25, 50, 100 mm). Variation of fracture toughness with test temperature and specimen thickness. *Filled symbols*: cleavage fracture without ductile tearing; *open symbols*: cleavage fracture preceded by ductile crack growth (Heerens and Hellmann 2002)

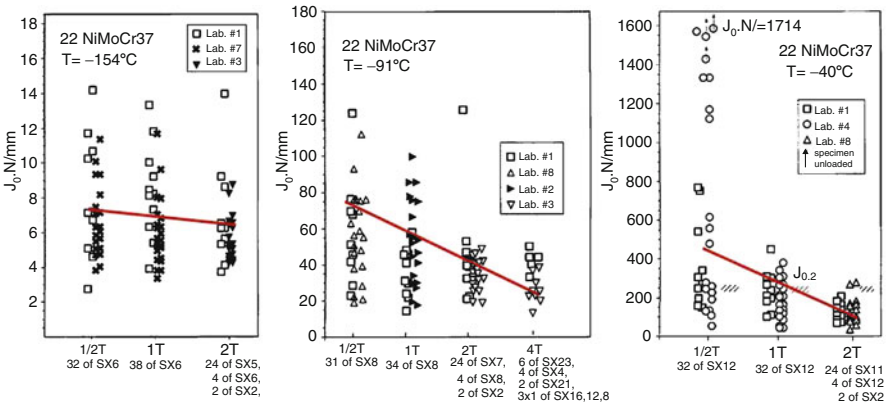


Fig. 3.17 Steel 22NiMoCr37 – Scatter and size effects on fracture toughness, J_0 , observed at three test temperatures: -154 , -91 , and -40 °C. Specimen thickness indicated in inches. The denomination SX gives the position of the specimens taken from a large pressure vessel shell (Heerens and Hellmann 2002)

with two specimen geometries leading to widely different constraint effect, single edge notch bend (SENB) specimens with varying a/W ratios between 0.03 and 0.77, and centre cracked panel (CCP) specimens. The results reported in Fig. 3.19 show that, for the same value of a/W ratio, the CCP specimens with a low constraint

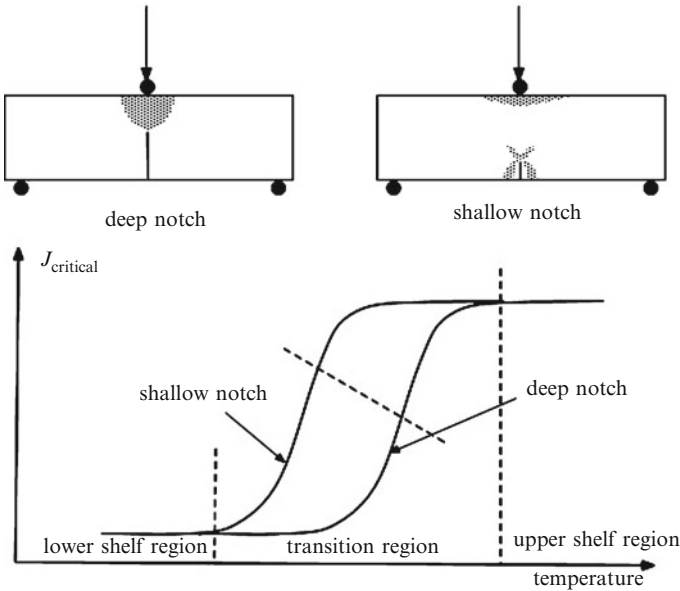


Fig. 3.18 Sketch indicating the effect of crack length in three-point bend specimens on ductile to brittle transition curves. The loss of constraint effect is related to the position and the size of the plastic zones

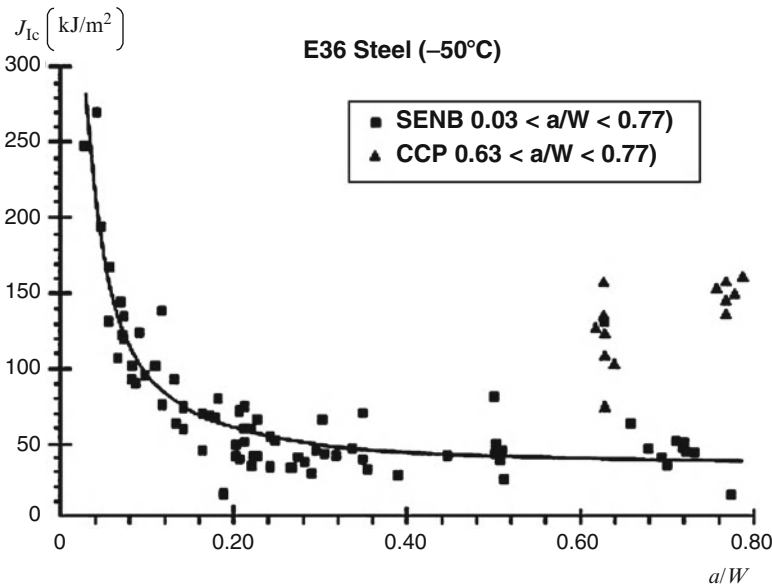


Fig. 3.19 E36 steel tested at -50°C . Variation of fracture toughness with specimen geometry (Single edge notched bend (SEN) specimen and centre cracked panel (CCP) specimen) and crack length in SENB specimens (Sumpter 1993)

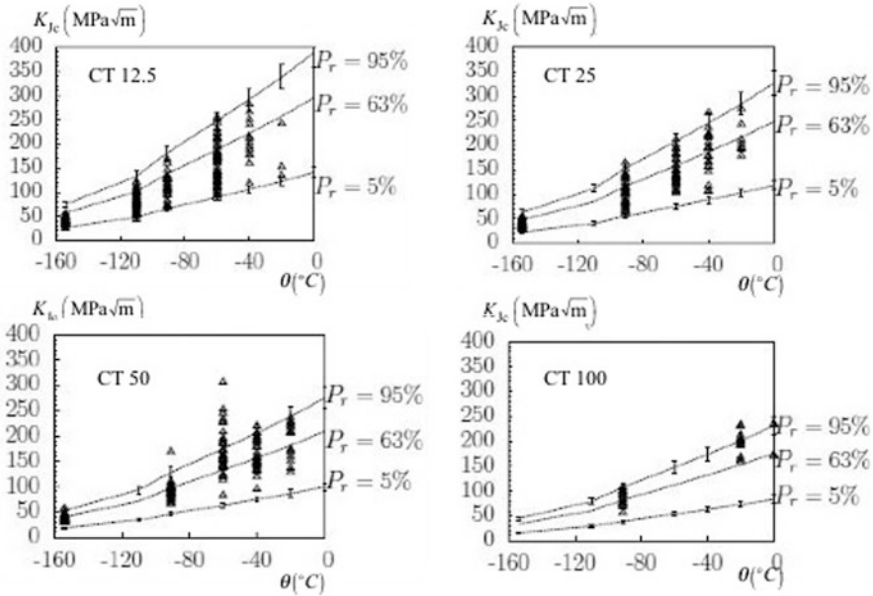


Fig. 3.20 Application of Beremin model to Euro fracture toughness data set (Heerens and Hellmann 2002). CT specimens (a) $B = 12.5$ mm; (b) $B = 25$ mm; (c) $B = 50$ mm; (d) $B = 100$ mm (Andrieu et al. 2012a)

effect lead to larger values of the apparent fracture toughness compared to SENB specimens. Moreover the fracture toughness of SENB specimens decreases with the notch depth, as shown in Fig. 3.18.

A *fourth factor* associated with brittle cleavage fracture is the effect of metallurgical variables. The grain size, the amount of inclusions, etc. are known to play a key role on brittle fracture toughness. Similarly the fracture toughness of multipass steel welds is largely dependent on the amount of local brittle zones (LBZ) which are formed in the coarse grain heat affected zones of weldments (see *e.g.* Machida et al. 1991). This metallurgical effect will be illustrated in the section devoted to welds (Sect. 3.5.4.6)

3.5.4.2 Scatter and Size Effects

Equation 3.44 predicts that the variations of the fracture toughness with a number of factors, including temperature and strain rate, are simply related to the variation of the yield strength. The application of this equation to the Euro fracture toughness data set (Fig. 3.16) is shown in Fig. 3.20. In the Euro data set (757 specimens) only the specimens which gave rise to brittle fracture without ductile tearing (491 specimens out of 757) were retained for the analysis (Andrieu et al. 2012a). In duplex stainless steels, see also Pineau and Joly (1991).

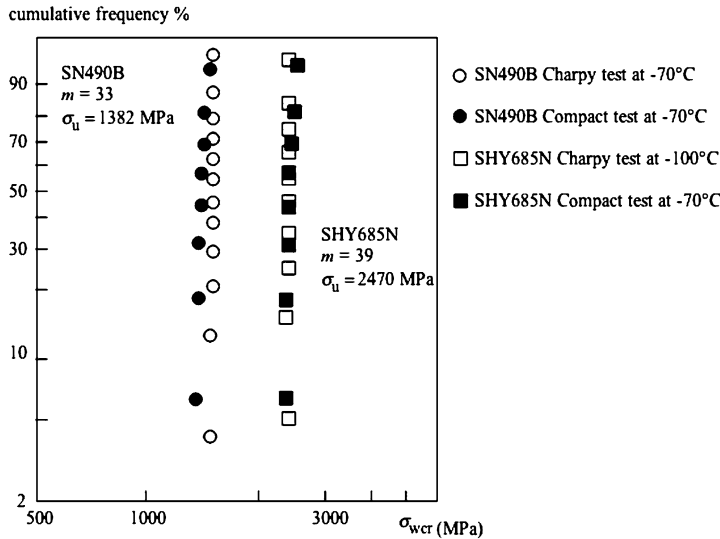


Fig. 3.21 Cumulative distribution of Weibull stress (σ_{wer}) for Charpy and CT specimens in two steels. The values of m and σ_u are given (Adapted from Minami et al. 2002)

Figure 3.20 shows that the theory accounts rather well for the scatter in test results. In particular it can be noted that Eq. 3.44 predicts that the theoretical ratio $K_{Ic}(P_R = 0.95)/K_{Ic}(P_R = 0.05)$ which is equal to 2.75 is close to the observed scatter, except at -60°C on CT 50 specimens where the scatter is anomalously large. This anomaly was attributed to the presence of macroscopic inhomogeneities in the material. The effect of these macroscopic inhomogeneities is addressed in Sect. 3.5.4.5.

The scatter in fracture toughness test results was also largely investigated by other researchers (see *e.g.* Minami and Arimochi 2001). The results obtained by these authors on two structural steels are reported in Fig. 3.21. They include dynamic impact Charpy V notch tests and quasi-static fracture toughness tests on CT specimens. Both types of specimens were calculated using FE method. The Charpy specimens were calculated using a material constitutive equation accounting for temperature and strain rate effect. The deformation of these specimens was assumed to occur under adiabatic conditions. The results of Fig. 3.21 show that, for a given steel, the same values of the Weibull parameters, m and σ_u , apply to both types of specimens tested under widely different conditions. This confirms that these parameters are characteristic of the material.

As previously stated, the Beremin theory predicts a size effect in the results of fracture toughness measurements. A number of researchers confirmed this size effect observed in the results reported in Fig. 3.17. It is now well established that measurements of fracture toughness in the DBT regime exhibit a size effect. For a given level of toughness (K_{Ic}), this may be considered as a positive shift in the transition temperature, $\Delta\theta$, with increasing specimen thickness for a particular

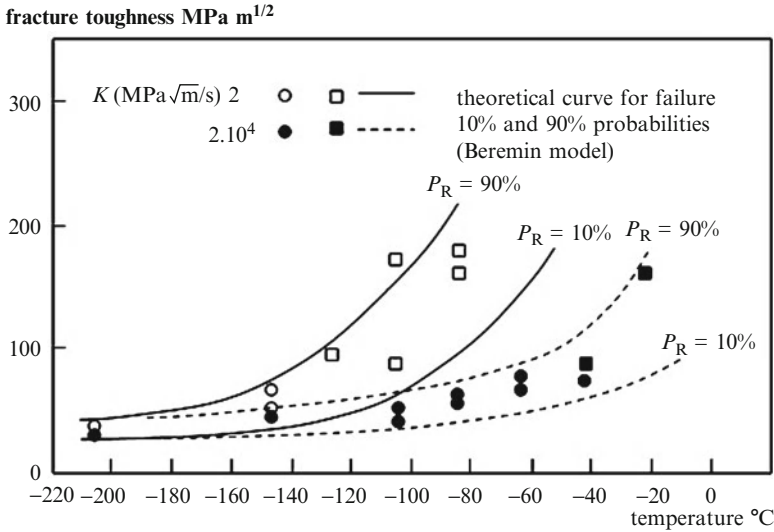


Fig. 3.22 Fracture toughness as a function of temperature and loading rate. Comparison between experiments and theory. Theoretical curves corresponding to $P_R = 10$ and 90% (Henry et al. 1985)

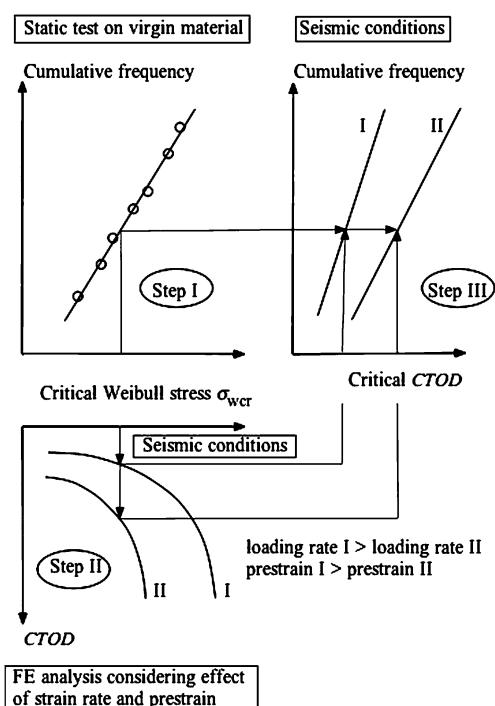
geometry. Sherry et al. (2001) numerically investigated the influences of sampling volume and constraint on $\Delta\theta$ on 10 mm thick CT specimens and 20% side-grooved pre-cracked Charpy specimens relative to 25 mm thick standard plane-sided compact tension specimens. The material investigated was a submerged arc weld metal. These authors showed that the Beremin model predicted correctly the shift in transition temperature, $\Delta\theta$, although it tended to over-predict the $\Delta\theta$ values, which were experimentally determined.

More recently, in the Euro fracture toughness data set (Heerens and Hellmann 2002), no significant size effect was observed in the lower shelf (Figs. 3.17 and 3.20). At higher temperatures, a significant size effect was shown. This observation supports a weakest-link type cleavage fracture mechanism in the ductile-to-brittle transition regime. Further statistical analysis of this large data base was also presented using a statistical procedure based on competing risks (Moskovic 2002).

3.5.4.3 Loading Rate and Prestraining Effect

It is well known that an increase in loading rate leads to a shift of the DBT towards higher temperatures. In a study on A 508 pressure vessel steel, it was shown that this shift in temperature could simply be interpreted in terms of the increase in yield strength produced by increasing strain rate, as predicted from Beremin model (Henry et al. 1985). The results are given in Fig. 3.22 where a comparison between the theoretical results inferred from Eq. 3.44 and the experimental data is made. In this application of the Beremin model, it was assumed that the deformation of the

Fig. 3.23 Methodology used for the prediction of fracture toughness at high loading rate (seismic conditions) based on Weibull stress (Adapted from Minami and Arimochi 2001)



material ahead of the crack-tip occurred under isothermal conditions, even at high loading rate. This assumption is only valid when fracture occurs after relatively low plastic strains, *i.e.* in the lower shelf regime.

Major factors promoting brittle fracture during earthquakes are prestraining and dynamic loading. The effect of these factors on the fracture toughness of a structural steel of 490 MPa strength class has been investigated by Minami and Arimochi (2001). These authors showed that prestraining and dynamic loading produced a very significant decrease of the fracture toughness measured in terms of critical crack tip opening displacement, CTOD (Fig. 3.23). The Beremin model was used to explain these effects (Fig. 3.24). The results reported in Fig. 3.24 show that the calculated Weibull stress follows the same statistical distribution, whatever the loading rate applied to the test specimens (mm s^{-1}) and the amount of prestraining ε_{pre} . In all cases the Weibull shape factor is found to be close to 21, which is a typical value for ferritic steels.

3.5.4.4 Warm Prestressing Effect

The effect of warm prestressing (WPS) on fracture toughness has already been described in Chap. 2, Sect. 2.7.1. Results obtained by Roos et al. (1998) were reported in Fig. 2.48 where two types of WPS cycles were described. A specimen or a component prestressed in the upper shelf regime exhibits an apparent increase

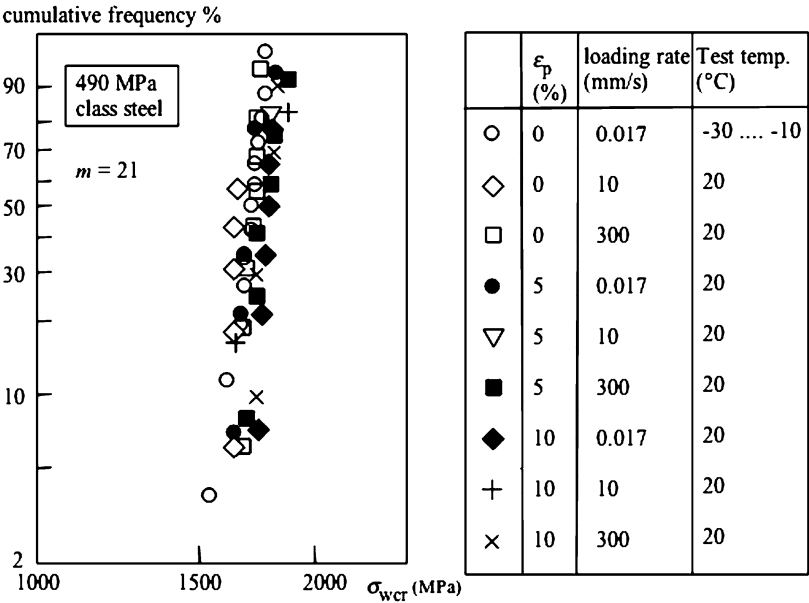


Fig. 3.24 Distribution of the Weibull stress under various conditions of prestraining and loading rate (Adapted from Minami and Arimochi 2001)

in fracture toughness when subsequently tested at lower temperature. This effect is observed for both types of loading, *i.e.* load-cool-fracture (LCF) and load-unload-cool-fracture (LUCF). These tests were performed on a pressure vessel steel which exhibited a Nil Ductility Temperature (NDT) of -45°C (see Chaps. 2 and 5 in this volume for the definition of NDT). Conventional fracture mechanics specimens were used to measure the fracture toughness and the scatter under isothermal conditions. The results of LCF and LUCF tests are given in Fig. 3.25a, b, respectively. In both cases, significant increases in apparent fracture toughness are observed. Two main factors contribute to this increase in fracture toughness, as previously discussed (Beremin 1981): (i) crack tip blunting effect, and (ii) residual stresses.

The results of the application of Beremin model are also shown in Fig. 3.25a, b. These results were obtained using in the numerical calculations $m = 16$ and $\sigma_u = 3,000\text{ MPa}$. It is seen that the model accounts reasonably well for the variations of fracture toughness with temperature in the lower shelf regime. Moreover the model accounts also for the LCF test results, but it tends to underestimate the fracture toughness results obtained with LUCF loading cycle. This might be related to the effect of large plastic strains which tend to increase the apparent brittle fracture stress, as indicated earlier (Eq. 3.58) and as shown in another study on WPS effect in welds (Stöckl et al. 2000).

The effect of plastic strain on WPS effect was also investigated using Bordet's model (Bordet et al. 2006). The load-temperature cycle was different in order to

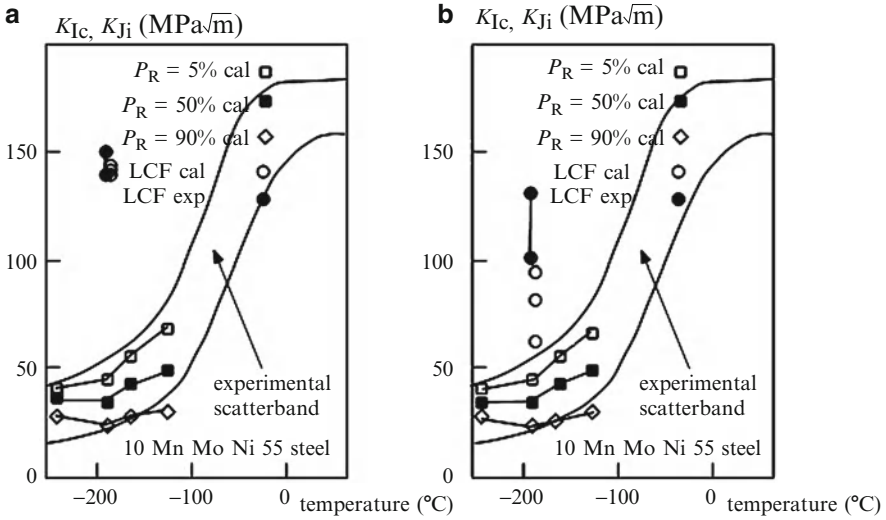


Fig. 3.25 Results of warm prestressing experiments on a Mn Mo Ni steel: (a) LCF cycle; (b) LUCF cycle. Comparison with theoretical results ($m = 16$, $\sigma_u = 3,000$ MPa) (Roos et al. 1998)

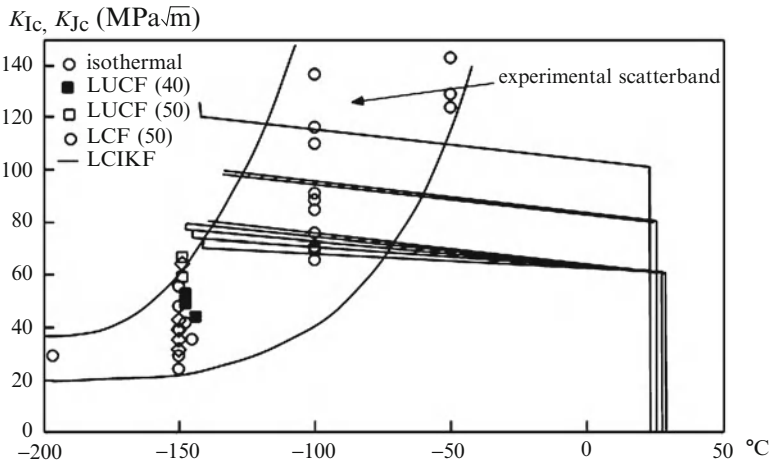


Fig. 3.26 Warm prestress effect in a pressure vessel steel (A508). LUCF, LCF and LICKF cycles on CT 25 specimens together with isothermal toughness test results. Numbers in the legend designate the value of K_{WPS} (Bordet et al. 2006)

represent situations closer to the reality (Fig. 3.26). This figure shows that after preloading at room temperature the material was slowly cooled down to -150°C with increasing K factor (LCIKF cycle). It is observed that the isothermal DBT curve can be crossed without observing failure. The results in Fig. 3.27 show a good agreement between experiments and theoretical calculations for a large number of types of WPS cycles when using Bordet's model (Eq. 3.57).

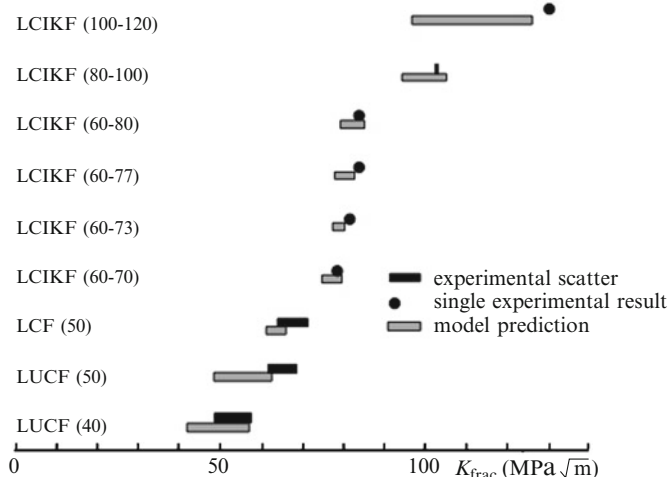


Fig. 3.27 Warm prestress effect. Comparison between experimental scatter and 5–95% predicted scatter band for all cycles (Bordet et al. 2006). The numbers in front of a given cycle (e.g. LCIF 80–100) indicate the initial value of the stress intensity factor ($80 \text{ MPa m}^{1/2}$) and the final value ($100 \text{ MPa m}^{1/2}$)

3.5.4.5 Cracks in Inhomogeneous Materials

The Beremin model and its extension were also applied to welded joints involving a mismatch effect between the weld, the heat-affected zone (HAZ) and the base metal (BM). Locating a crack in the HAZ along the fusion line of mismatched welds leads to asymmetrical crack opening. For instance the overmatching effect (yield strength of the weld larger than that of the BM) produces an elevation of the local stress in the HAZ which promotes fracture initiation at a lower crack tip opening displacement (CTOD) level than that in homogeneous materials (see e.g. Kim and Schwalbe 2001a, b). These effects were studied in detail by many authors (see e.g. Ohata et al. 1996) (for a review, see Pineau 2003).

It should be noted that the Beremin model can be used to predict not only the fracture toughness but also the crack path when dealing with inhomogeneous materials such as welds as indicated previously (Sect. 3.4.4.2). The yield strength of the material at any location is taken as deterministic and must be known, but the local cleavage parameters (m , σ_u) are spatially distributed. The probability density function is determined around the crack tip by calculating the Weibull stress in small angular sectors. Becker et al. (2002) were the first authors to apply this method in their study on functionally graded materials. More recently a similar approach was followed by Bezensek and Banerjee (2010) to predict the crack path in specimens, which were tested in the brittle regime with a temperature gradient normal to the crack surface.

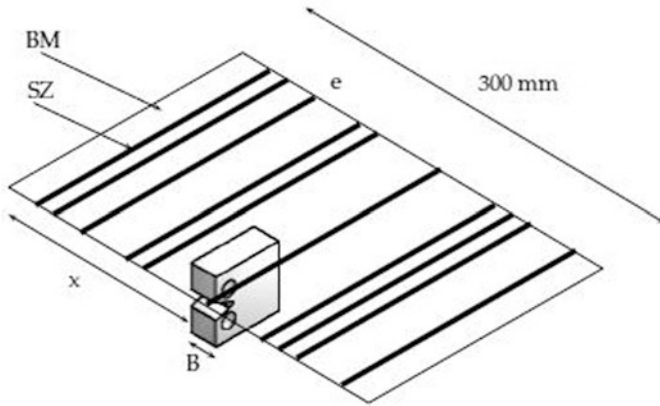


Fig. 3.28 Schematic representation of the location of segregated zones (SZ) in a thick component and position of a fracture toughness specimen (Andrieu et al. 2012b)

Other inhomogeneities are those due to the segregation taking place during solidification. Heavy steel plates are prone to this effect. They often contain bands of alternate layers with different mechanical and fracture properties as schematically shown in Fig. 3.28. These materials can be considered as macroscopically (band scale) and microscopically (carbide distribution scale) inhomogeneous. The microscopic inhomogeneities are taken into account in the Beremin-Weibull theory. The macroscopic inhomogeneities can be described by various laws, for example the Poisson's law:

$$P(X = i) = \frac{\lambda^i \exp(-\lambda)}{i!} \quad (3.59)$$

where λ is the mean distance between the bands intersecting the crack front and P is the probability that i bands of thickness, e , intersect the crack front of the specimen. Andrieu et al. (2012b) showed that, using the weakest link theory, the probability to failure of a specimen of thickness B could be written as:

$$\log \left(\frac{1}{1 - P_R} \right) = -B \left\{ \frac{\lambda_0}{B_0} \exp[-(v - u) e K^4] - \frac{\lambda_0}{B_0} - K^4 u \right\} \quad (3.60)$$

where λ_0 and B_0 are constants, K is the stress intensity factor and u and v are given by:

$$u = \frac{C_{m_1 n_1} (\sigma_0^1)^{m_1 - 4}}{V_0 (\sigma_u^1)^{m_1}} \quad v = \frac{E_2^2 C_{m_2 n_2} (\sigma_0^2)^{m_2 - 2}}{E_1^2 V_0 (\sigma_0^1)^2 (\sigma_u^2)^{m_2}} \quad (3.61)$$

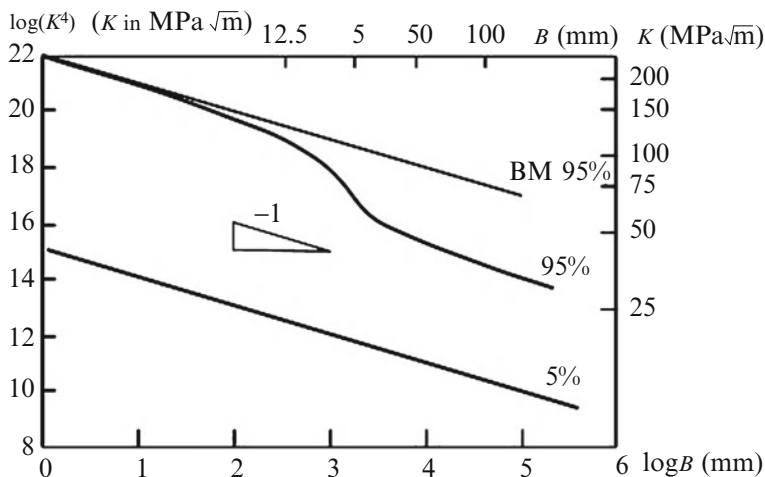


Fig. 3.29 Theoretical plot of $\log K^4$ versus specimen thickness, B in bimodal Beremin type model (Andrieu et al. 2012b). *BM* designates the base (tough) material and *SZ* the segregated material which is more brittle

where 1 and 2 refer to the matrix and the segregated material, respectively, C_{mn} is the numerical constant of the Beremin model (Eq. 3.44), V_0 the representative volume element (supposed to be the same in both materials) and σ_u the “cleavage stress”.

It is worth-noting that Eq. 3.60 indicates that the so-called “ $K^4 B$ ” size effect is no longer verified as schematically shown in Fig. 3.29. For low values of P_R (e.g. 5%), fracture is controlled by the low toughness material 2 and the size effect remains close to the usual “ $K^4 B$ ” law. The probability of finding bands of material 2 intersecting the crack front increases with specimen thickness. This is the reason why for large probabilities to failure (e.g. 95%) the probability law follows a two-slope regime as indicated in Fig. 3.29. This deviation from a unique “ $K^4 B$ ” law may have strong practical implications when one realises that experiments can be made on specimens with thicknesses lying in the transitional regime, which is not representative of the behaviour of thick components.

3.5.4.6 Cleavage Fracture in Welds: Influence of Martensite/Austenite (MA) Constituents

High strength low alloy (HSLA) steels are now widely used for structural applications. These materials combine excellent tensile strength and DBT properties. However, this combination of high strength and high fracture toughness usually deteriorates after welding thermal cycles. The degradation of the fracture toughness of HSLA steels after welding is attributed to the formation of “local brittle zones” in the welded joint (see e.g. Davis and King 1994). Significant embrittlement can be

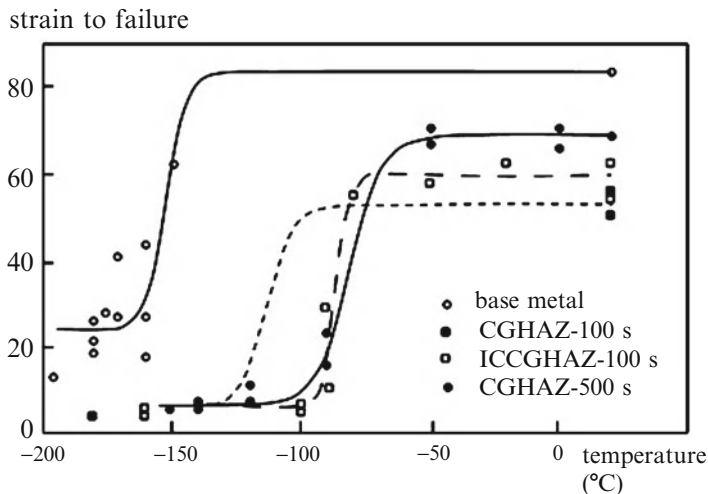


Fig. 3.30 Variation of the strain to failure measured on notched specimens as a function of test temperature in a low alloy steel with four microstructures (Lambert-Perlade 2001). See text for the definition of the simulated heat treatments (CGHAZ – 100 and 500 s, ICCGHAS – 100 s)

encountered in the coarse-grained heat-affected zone (CGHAZ) and, in particular, in the intercritically reheated CGHAZ (ICCGHAZ) of multipass welded joints (see *e.g.* Toyoda 1988; Kenney et al. 1997; Zhou and Liu 1998). (For an introduction to the terminology used in welding, see Volume I, Appendix B.)

An example of such embrittlement effects is shown in Fig. 3.30 which refers to a study performed on the micromechanisms and modelling of cleavage fracture in simulated HAZ microstructures obtained in a HSLA steel ($\sigma_0 = 430$ MPa at room temperature) (Lambert-Perlade et al. 2004). In this study as in many others devoted to welds a Gleeble simulator was used to apply thermal cycles representative of those encountered during multipass welding. The maximum temperature of the first cycle was $\theta_{p1} = 1,250^\circ\text{C}$. Cooling times from 800 to 500°C ($\Delta t^{8/5}$) were chosen to be 100 s (CGHAZ – 100) and 500 s (CGHAZ – 500) corresponding to a medium and high-input welding energy, respectively. Intercritical heating of the CGHAZ – 100 microstructure (ICCGHAZ) at maximum temperature ($\theta_{p2} = 775^\circ\text{C}$) with the same cooling conditions as CGHAZ – 500 induced partial austenitisation ($AC_1 < 775^\circ\text{C} < AC_3$) of the bainitic microstructure. Upon further cooling austenite was partially transformed into martensite leading to the formation of martensite/austenite MA constituents (Fig. 3.31). Figure 3.30 shows that the ICCGHAZ microstructure produces a shift of the DBT temperature by about 80°C . Tensile tests on notched specimens showed that the cleavage stress was reduced as compared to the ferrite-perlite microstructure (base metal), in particular in the ICCGHAZ – 100 and CGHAZ – 500 microstructures. It was observed that cleavage facets were initiated from MA constituents. This clearly illustrates the deleterious effect of the presence of MA constituents on cleavage fracture in welds.

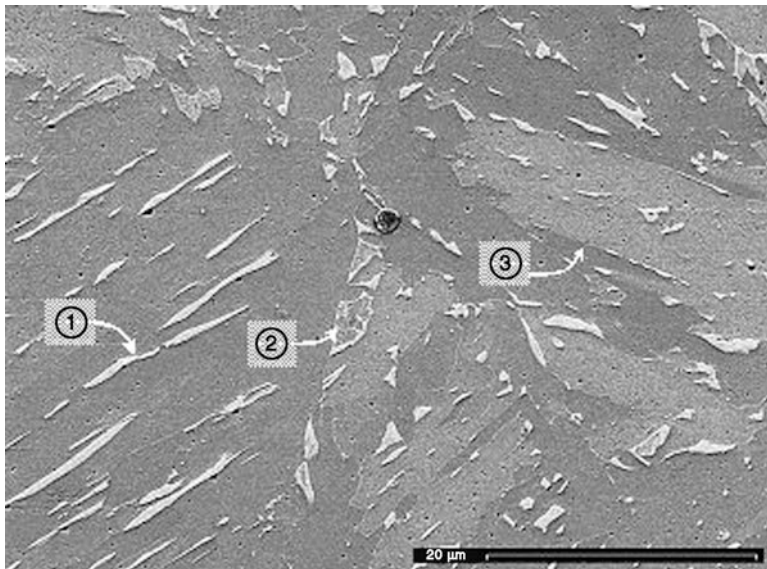


Fig. 3.31 SEM micrograph of the microstructure of a heat affected zone in a weld. (1) Residual austenite located at lath grain boundaries. (2) Martensite-Austenite mixed constituent at a former austenite grain boundary. (3) Bainitic packet boundary (Lambert-Perlade 2001)

Fracture toughness tests were carried out on specimens, which were simulated with other $\Delta T^{8/5}$ cooling conditions. Figure 3.32 shows that the transition temperature measured at $K_I = 100 \text{ MPa m}^{1/2}$ is much higher in the simulated microstructures than in the base metal since shifts as large as 150°C are measured. Cleavage crack initiation observed on a fracture toughness specimen tested at 20°C is seen in Fig. 3.33 where it is observed that cleavage was initiated from a M-A constituent (arrow). An attempt was made to explain the observed variations of the fracture toughness with temperature using the Beremin model. The results are reported on the same graphs while the values of the Beremin parameter, m and σ_u , are given in Table 3.5. Figure 3.32 shows a good agreement with the experimental results for the simulated HAZ microstructures. A similar conclusion was reached by Tagawa et al. (1993). In Fig. 3.32, we have also included the predictions obtained from the “master curve” approach, which is introduced in Sect. 3.4.4.2 (Wallin 1991a, b, 1993; ASTM E 1921-02, 2002) (for further details, see Lambert-Perlade et al. 2004).

3.5.5 Conclusion

To conclude this part devoted to ferritic steels, it appears that, in spite of its simplifications, the local approach to fracture has largely contributed to a better

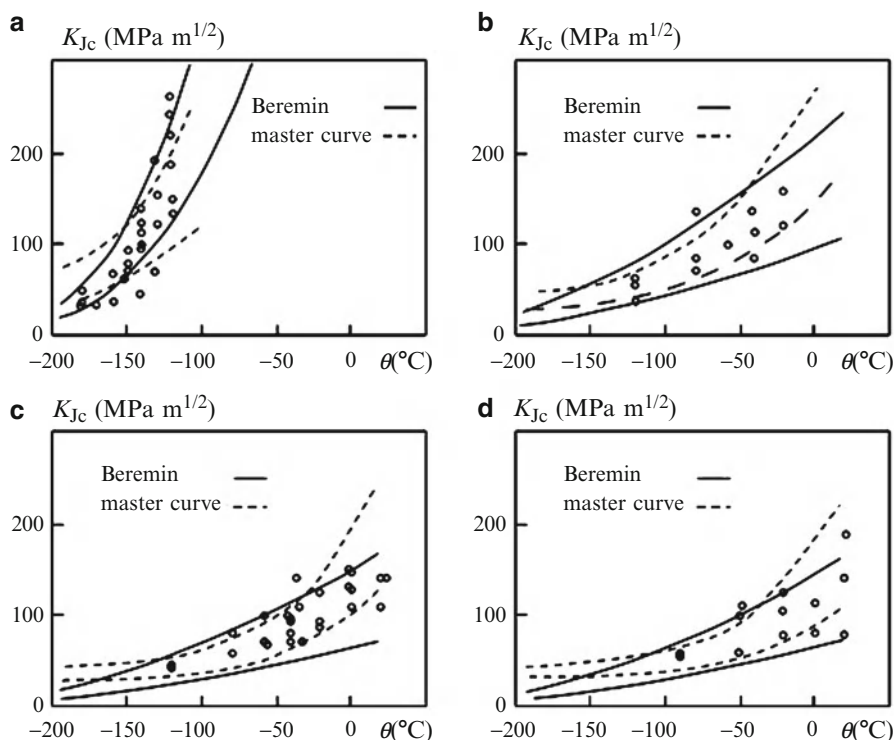


Fig. 3.32 Brittle-to-ductile toughness transition curves in a low alloy steel and in simulated heat-affected zone microstructures. (a) Base metal. (b) CGHAZ-25. (c) ICCGHAZ-25. (d) CGHAZ-120 microstructures. *Solid lines* (respectively, *dotted lines*) show fracture probabilities of 10 and 90% given by the Beremin model and the “master curve” approach. Numerical values used in the Beremin model are given in Table 3.4 (Lambert-Perlade 2001; Lambert-Perlade et al. 2004)

representation of the brittle cleavage fracture in ferritic steels. A full account of the application of this model to predict the fracture toughness of a number of ferritic steels is given elsewhere (Pineau 2003). The main criticism to the Beremin model, which is the absence of a threshold, is not really acceptable since this threshold is present although it does not explicitly appear in the expression giving the probability to fracture. The original Beremin model applied to the prediction of fracture toughness tends to underestimate the variation of fracture toughness with temperature, in particular in the transition regime. This has led a number of authors to assume that the normalising stress, σ_u , was an increasing function of temperature.

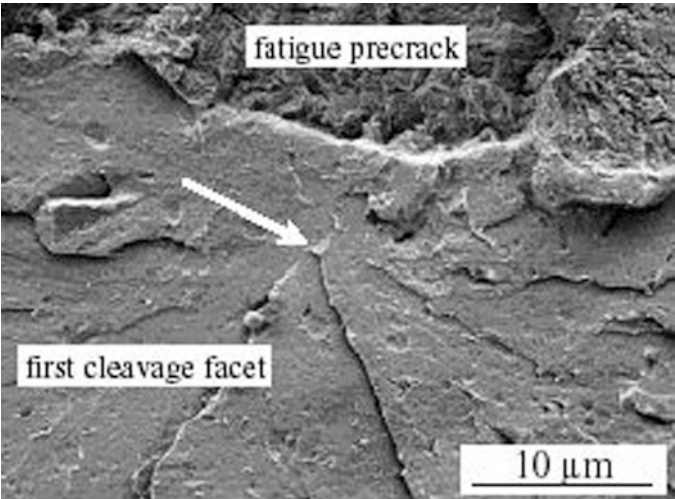


Fig. 3.33 Cleavage crack initiation after interrupted test of a fracture mechanics specimen. ICCGHAZ–25 microstructure. The cleavage crack is initiated from a M-A constituent (indicated by an *arrow*) (Lambert-Perlade et al. 2004)

Table 3.5 High strength low alloy steel

Microstructure	Beremin model		Experimental values of θ_{100}^* (°C)
	σ_u (MPa)	m	
Base metal	2,158	27	−140
CGHAZ – 25	2,670	20	−55
ICCGHAZ – 25	2,351	20	−20
CGHAZ – 120	2,085	20	−10

Parameters of the Beremin model (unit volume $V_u = (100 \text{ } \mu\text{m})^3$). The parameter θ_{100}^* provides the value of temperature for which $K_{Jc} = 100 \text{ MPa m}^{1/2}$

3.6 Cleavage in Other BCC Metals

While plastic deformation and brittle cleavage fracture behaviour of iron and ferritic steels has, for technological reasons, received considerable attention, other body-centred cubic (BCC) metals have been relatively neglected. This is partly due to the expected similarity in behaviour with α iron, and partly due to the small use in key engineering applications of the high melting refractory metals (V, Nb, Ta, Cr, Mo, W) in industry. However the increased importance of these metals, in particular in gas turbines and nuclear energy sectors, pushes for more detailed investigation of their mechanical properties. Commercially available group V-A refractory metals

(V, Nb, Ta) are considerably more ductile and have considerably lower transition temperature than commercially available group VI-A refractory metals (Cr, Mo, W). This mainly results from the much higher solubility of impurities in the V-A metals than in the VI-A metals.

Molybdenum, which is representative of the metals of group VI-A, was investigated (see *e.g.* Briggs and Campbell 1972; Koval et al. 1997). Niobium, which is representative of the metals in group V-A, was investigated in much more detail (Briggs and Campbell 1972; Samant and Lewandowski 1997a, b; Pahdi and Lewandowski 2004). Lewandowski and his co-workers have studied pure Nb and Nb-Zr solid solutions. These authors investigated the effect of grain size (~ 60 and $165 \mu\text{m}$). The cleavage stress, σ_f , was measured using either blunt notched specimens or fatigue pre-cracked fracture toughness specimens. In blunt notched specimens the Griffith and Owen solution was used to calculate the cleavage stress (Griffiths and Owen 1971).

In fracture mechanics the well-known RKR model (Ritchie et al. 1973) was used in conjunction with either the HRR field (Hutchinson 1968; Rice and Rosengren 1968) (see Chap. 2, Sect. 2.2.2.6) or the Tracey solution (Tracey 1976). This model was used by Pahdi and Lewandowski (2004). The cleavage stress was found to be a decreasing function of grain size. The identified values for σ_f appeared to be larger for fracture toughness specimens than those identified based on blunt notched specimens (typically 1,700 MPa compared to 1,400/1,500 MPa in $60 \mu\text{m}$ grain size Nb). This suggests that the sampling volumes, which were quite different in these two geometries, could also affect the values of σ_f . Additional tests and analyses are required to show if this size effect can be interpreted by using the Beremin theory.

3.7 Cleavage Fracture in HCP Metals

3.7.1 Cleavage Plane

When compared to crystal systems like body-centred cubic, hexagonal close packed (HCP) metals exhibit a wider variety of deformation modes, including slip and twinning systems. The dominant slip mode in all HCP metals involves the Burgers vector $a = 1/3 \langle 11\bar{2}0 \rangle$ whether the primary slip plane is basal (*e.g.* cadmium, zinc or magnesium) or prismatic (*e.g.* titanium or zirconium). For metals with $c/a < \sqrt{3}$ (*e.g.* beryllium, titanium, zirconium, magnesium), the $\{10\bar{1}2\}$ twinning mode is activated by compression along the c -axis and $\langle c + a \rangle$ slip is active. All these metals have a preferential cleavage plane which is the basal plane $\{0002\}$.

A major problem facing HCP metals, such as magnesium and zinc, is their limited ductility at low temperature. Even if both slip planes operate there is still no way to accommodate strains along the c -axis. Deformation twinning can help alleviating this problem but it is often insufficient to provide a large ductility.

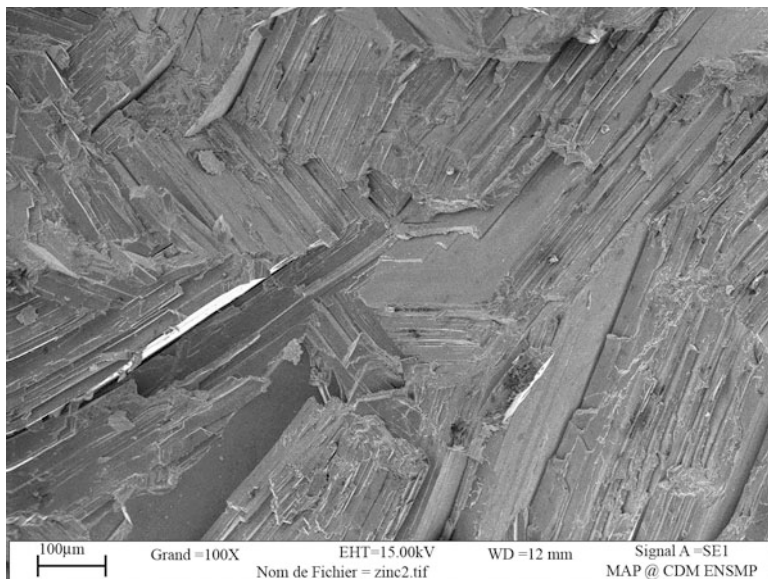


Fig. 3.34 SEM micrograph of a polycrystalline zinc specimen fractured at -196°C

In metals like titanium and zirconium, pure cleavage fracture along the $\{0002\}$ plane was not observed, except under stress corrosion conditions which are outside the scope of the present chapter (see *e.g.* for Zr alloys: Kubo et al. 1985; Schuster and Lemaignan 1989a, b; Cox 1990).

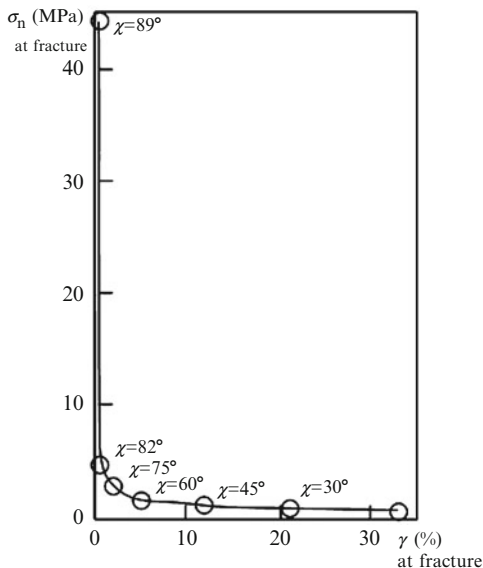
This is why in this section only cleavage fracture of zinc and magnesium are considered.

3.7.2 Cleavage Fracture of Zinc

Zinc shows brittle cleavage fracture along the basal plane and along the prismatic planes. An example of the fracture surface of a pure polycrystalline specimen broken at -196°C is shown in Fig. 3.34. The importance of prismatic cleavage in zinc and that of the accommodation required at a grain boundary as a crack propagates from grain to grain was underlined (Hughes et al. 2005). Cleavage on basal plane was studied more thoroughly, in particular on zinc single crystals (Gilman 1958; Deruyttere and Greenough 1956) (Fig. 3.35). Cleavage initiated by a glide band crossing a tilt boundary was modelled by Gilman (1954) (Fig. 3.36).

More recently, cleavage fracture on the basal plane was also studied in detail on hot dip galvanised steel sheets (Parisot et al. 2004a, b). In studies devoted to the analysis of single crystals, it was shown that the critical cleavage stress measured by the stress normal to the $\{0002\}$ cleavage plane was largely reduced when the

Fig. 3.35 Fracture in zinc. Normal stress to the basal plane at fracture as a function of basal glide for different misorientations χ between the c -axis and the tensile direction (Deruyttere and Greenough 1956; Gilman 1958)



amount of plastic strain in the basal plane, γ^{bas} , was increased (see Fig. 3.35). Similar observations were made on crack propagation behaviour in strongly textured zinc sheets (Lemant and Pineau 1981). The cleavage criterion proposed by Gilman (1958) involved the product $\sigma_n \gamma_i^{\text{bas}}$, where γ_i^{bas} is the amount of plastic slip on the basal slip system, i . The reason for the decrease of the cleavage stress when basal plastic strain is increased is likely related to the existence of local stress concentrations associated with basal (a) dislocation pile-ups, and to tilt boundaries (Fig. 3.36), that can locally reach the theoretical cleavage stress (see Sect. 3.3.2). A similar conclusion was reached in the study devoted to zinc coatings on hot-dip galvanised steel sheets, except that thresholds for the cleavage stress and the basal plastic strain were introduced, *i.e.*:

$$\sigma_f = \sigma_{\text{th}} + \frac{k}{\gamma_0 + \gamma_{\text{eq}}^{\text{pbas}}} \quad (3.62)$$

where σ_{th} , k and γ_0 are material parameters. In (3.62) the theoretical cleavage stress is, in the absence of basal slip, equal to $\sigma_{\text{th}} + k/\gamma_0$. A measure of the total basal slip activity of the three basal slip systems was defined as:

$$\gamma_{\text{eq}}^{\text{pbas}} = |\gamma_1^{\text{bas}}| + |\gamma_2^{\text{bas}}| + |\gamma_3^{\text{bas}}| \quad (3.63)$$

where 1, 2, 3 are the three slip directions in the basal plane.

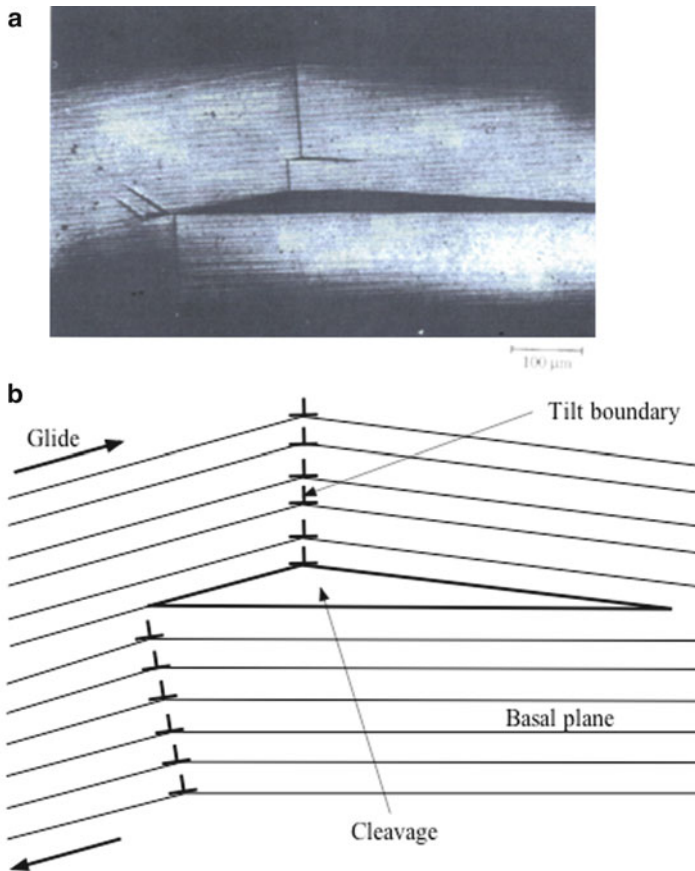


Fig. 3.36 (a) Cleavages along the basal plane in zinc (Gilman 1954); (b) Gilman's mechanism of slip crossing a tilt boundary plane was also studied

3.7.3 Cleavage Fracture of Magnesium

Magnesium cleaves also easily along the basal plane. The cleavage fracture stress of high purity magnesium is independent of temperature, but highly dependent upon grain size. The addition of lithium to magnesium alloy has an interesting effect on low temperature fracture behaviour in decreasing the DBT temperature. Several reasons were invoked to explain this effect, which is known for quite a long time (Hauser et al. 1956). Many reports were published regarding the beneficial effect of lithium on the ductility of magnesium (see *e.g.* Raynor 1960; Saito et al. 1997). In many of these studies the lithium additions resulted in a substantial volume fraction of the soft Li-rich BCC β phase, which was probably one part of the explanation for the better ductility. The increase in ductility was also observed even

in Mg-Li alpha-solid solution alloys. Another reason for the ductility improvement might be related to the reduction of the stress for prismatic slip relative to that for basal slip. More recently, Agnew et al. (2001, 2002) showed the existence of $1/3 \langle 11\bar{2}3 \rangle \{11\bar{2}2\}$ pyramidal slip in Mg-Li solid solution alloys, while pure magnesium exhibits only basal and pyramidal slip. Nevertheless, the structure of $\langle c + a \rangle$ dislocations, in particular their dissociation and decomposition are still an open issue. Very much remains thus to be done before the theories of cleavage fracture in HCP metals reach a development similar to that of BCC metals.

3.8 Intergranular Fracture

There are many causes of intergranular embrittlement. We consider only here those due to tempering and to overheating in steels.

3.8.1 *Temper-Embrittlement in Steels*

Temper embrittlement occurs in steels, which have been quenched and tempered and have been maintained for some time at a temperature in the range 350–550°C. The appearance of this mode of embrittlement is shown by an increase of the DBT temperature which can be as much as 100°C. It is fastest at 550°C and develops more quickly during cooling than at constant temperature. This embrittlement is reversible and can be eliminated by annealing above 550°C, followed by quenching. It is characterised by intergranular fracture, which usually, but not always, follows the prior austenite grain.

Analysis of the fracture surfaces by Auger spectrometry (see Appendix B in Volume I) shows the presence of metalloid impurities, the most harmful of which are, in descending number, antimony, tin, phosphorus and arsenic, but it is found that their embrittlement effect is always associated with the presence of alloying elements such as nickel, molybdenum and manganese.

Table 3.6 gives the embrittling elements, distinguishing those which favor segregation at grain boundaries and those which oppose this.

3.8.2 *Segregation of Impurities at Grain Boundaries – Thermodynamics*

A comprehensive review of grain boundary segregation in metals and that of the thermodynamics can be found in several publications (see e.g. Faulkner (1996)) and in particular in two recent books, one by Priester (2006) and the other one by Lejček (2010). In this section only a short account of the thermodynamics of intergranular segregation is given. See also Rice (1976).

Table 3.6 Intergranular embrittlement – effect of composition

 H																			He
Li	Be													B	 C	 N	O	F	Ne
Na	Mg													Al	 Si	 P	 S	Cl	Ar
K	Ca	Sc	 Ti	V	 Cr	 Mn	Fe	Co	 Ni	Cu	Zn	Ga	 Ge	 As	 Se	Br	Kr		
Rb	Sr	Y	Zr	Nb	 Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	 Sn	 Sb	 Te	I	Xe		
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	 Bi	Po	At	Rn		
Fr	Ra	Ac	Th	Pa	U														

 embrittling element

 segregation enhancement

 seems to increase grain boundary cohesion

 cosegregates with embrittling elements

 segregation inhibitor

The solubility of elements is not the same in the interior of the grains as in the boundaries. The distortion energies generated by the solute atoms are also different. A solute that reduces the surface energy of the boundary will diffuse towards the grains boundaries so as to reach an equilibrium composition θ_b defined by:

$$\frac{\theta_b}{1 - \theta_b} = \frac{c}{1 - c} \exp\left(-\frac{\Delta g_b}{RT}\right) \quad (3.64)$$

where $\theta_b = \Gamma_b / \Gamma_b^{\max}$ is the fraction of atomic sites of the boundary (or of the surface), that are occupied by solute atoms, Γ being the concentration in mole/m². When $c \ll 1$, this expression can be simplified as:

$$\theta_b = \frac{c \exp(-\Delta g_b / RT)}{1 + c \exp(-\Delta g_b / RT)} \quad (3.65)$$

In these equations, c is the atomic concentration of the solute in the alloy and Δg_b is the Gibbs free energy of segregation per mole in the boundary (Mc Lean 1957). (Δg_b is negative).

The value of Δg_b depends on the segregating element (see Table 3.7) (Lejček 2010), but is also function of the misorientation of grain boundaries (Priester 2006; Fraczkiewicz et al. 2011). This is shown in Fig. 3.37 for the segregation of C, P

Table 3.7 Parameters of grain boundary segregation in various matrixes (Lejček 2010) $\Delta g_b = \Delta H_o - T\Delta S_{oo}$

Matrix: α Fe			
Element	ΔH_o (KJ/mole)	ΔS_o (J/mole K)	Comments
P	−[21.2–50]	[0–37.3]	Polycrystal
C	−[80–37.7]	[0–(−13)]	Polycrystal
S	−51	0	Polycrystal
Sn	−[13–22.5]	[26.1–45]	Polycrystal
Sb	−[13–23]	[28–37]	Polycrystal
Matrix: γ Fe			
P	−[14.1–32]	[15–22.8]	Polycrystal
Matrix: Ni			
S	−[70–120]	–	Polycrystal
B	−10	9	Polycrystal Ni ₃ Al + 0.48%B

The extreme values reported in the literature for ΔH_o and ΔS_o are given

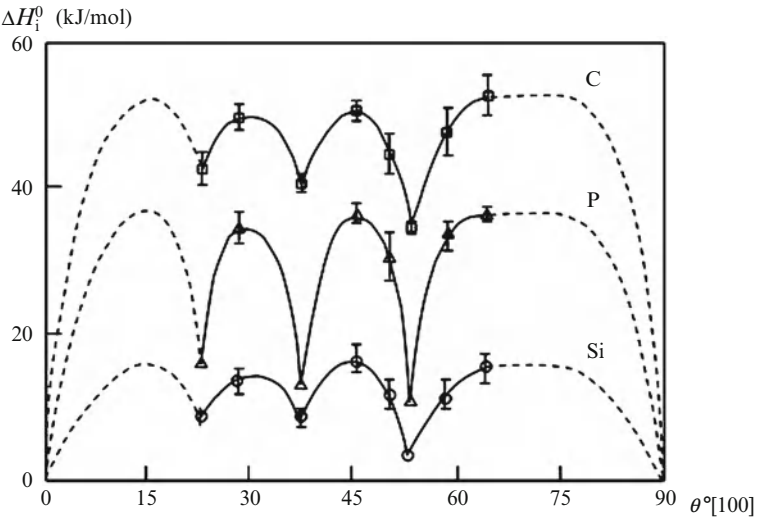


Fig. 3.37 Influence of grain boundary misorientation on the intergranular segregation enthalpy (ΔH_i , $i = \text{P, Si, C}$) in bicrystals of Fe–3.5%Si. Symmetrical tilt grain boundaries of $\langle 100 \rangle$ axis. Measurement of intergranular segregation by Auger spectrometry (Hofmann and Lejček 1996)

and Si in bicrystals of Fe-3% Si alloy (Hofmann and Lejček 1996). In this figure the enthalpy of segregation ΔH_g of these impurities is given for tilt boundaries of $\langle 100 \rangle$ axis. It is observed that large coincidence grain boundaries ($\Sigma = 5$; $\theta = 37^\circ$; $\Sigma = 9$; $\theta = 53^\circ$)³ are characterised by a much lower enthalpy than the

³Here Σ designates a type of coincidence site lattice and not a macroscopic stress. For the definition of Σ in bicrystals see *e.g.*; Priester (2006). Σ is an odd integer number in cubic crystallographic structures. Similarly here θ is the tilt angle between two neighbouring grains.

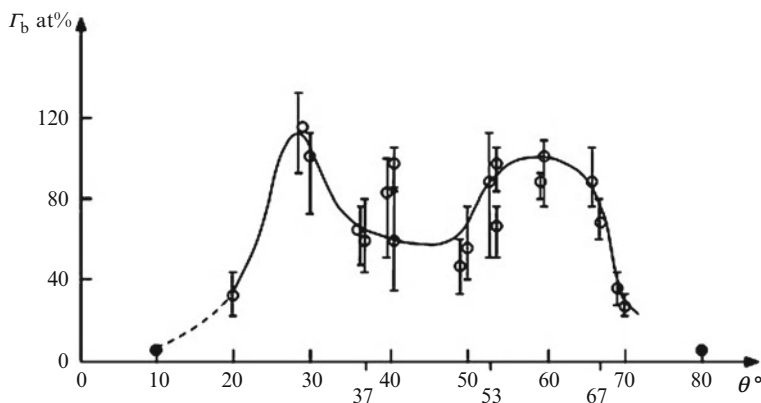


Fig. 3.38 Intergranular segregation of Bismuth in Cu bicrystals with symmetrical tilt grain boundaries of $\langle 100 \rangle$ axis. Measurements of intergranular concentration by Auger spectrometry (Fraczkiewicz and Biscondi 1985)

randomly oriented grain boundaries. Figure 3.38 shows the results of segregation measurements of Bi in symmetrical bicrystals of Cu oriented along the $\langle 100 \rangle$ axis (Fraczkiewicz and Biscondi 1985). See also Westbrook and Aust (1963). A large effect of the misorientation is observed for low values of the angle θ ($< 20^\circ$). Moreover, as in Fe-3.5Si (Fig. 3.37) the large coincidence grain boundaries ($\Sigma = 5$; $\theta = 37^\circ$) give rise to much lower values of the segregation.

Equations 3.64 and 3.65 can be used to predict the variation of Γ_b as a function of temperature for a given value of Δg_b . In iron, for phosphorus (a very detrimental impurity), the value of Δg_b is close to $-10,000$ cal/mole at $T \sim 400/500^\circ\text{C}$ (see *e.g.* Naudin 1999). These values imply an enrichment factor $\beta = \theta_b/c$ close to 500 at a temperature of 500°C . The grain boundary enrichment in phosphorus as predicted from Eq. 3.64 is schematically shown in Fig. 3.39 (Mc Lean 1957) where it is observed that Γ_b increases with the matrix concentration c at a given temperature, as expected and increases also when the temperature is decreased for a given matrix concentration.

Many studies showed that the enrichment factor at a given temperature is inversely proportional to the limit solubility c_v of the impurities in the matrix, *i.e.* $\beta = k/c_v$ where k is a proportionality factor. This simple relationship has been verified in many systems, as shown in Fig. 3.40 (Seah and Hondros 1973).

Equations 3.64 and 3.65 must be modified if there are several different elements in solution in order to take into account the interaction between these solute elements and the impurities. If this interaction is strong, as in the case of titanium and phosphorus, precipitates will be formed in the grains, eliminating the impurities and thus inhibiting the segregation. If the interaction is moderate, the two elements will diffuse together in the boundary reducing the energy.

In multicomponent systems, which were studied in detail by Guttman (1975, 1996), Guttman and Mc Lean (1979), Guttman et al. (1982), Gas et al. (1982),

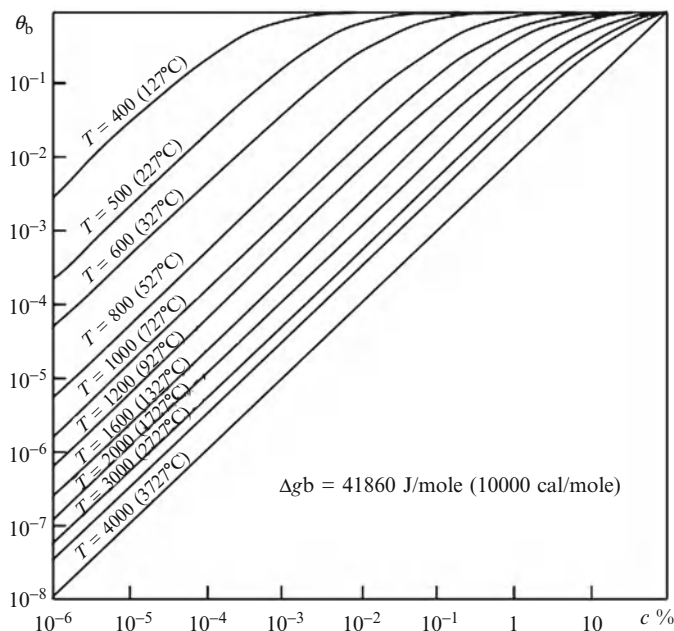


Fig. 3.39 Relation between grain boundary concentration and grain interior concentration, c , at different temperatures. The saturation grain boundary concentration of unity corresponds to about 30 atomic per cent solute. The curves are plotted assuming that $\Delta g_b = 10$ Kcal/mole (Mc Lean 1957)

and Militzer and Wieting (1986, 1987, 1989), several phenomena contribute to make the situation more complex. Let us consider a ternary system containing the matrix solute atoms, the impurity atoms (I) and the alloying solute atoms (A).

Site competition

The two segregating species can compete for sites at the grain boundary. The more surface active element, that with the higher Δg_{bi} ($i = I$ or A), preferentially occupies the available grain boundary sites, thus rejecting the other element. This mechanism has often been invoked to explain various cases of antagonistic segregation behaviours, *e.g.* (P + C) in a Fe-0.17% P alloy (Fig. 3.41). Similar effects have been observed in iron-based alloys with P + B, S + C, S + N. However this site competition mechanism cannot explain all the observed effects and alternative or additional mechanisms must be considered, in particular those associated with chemical interactions.

Chemical interactions

In general the solutions in which segregation takes place are far from ideal, *i.e.* the atoms chemically interact with each other. This leads to the introduction of a

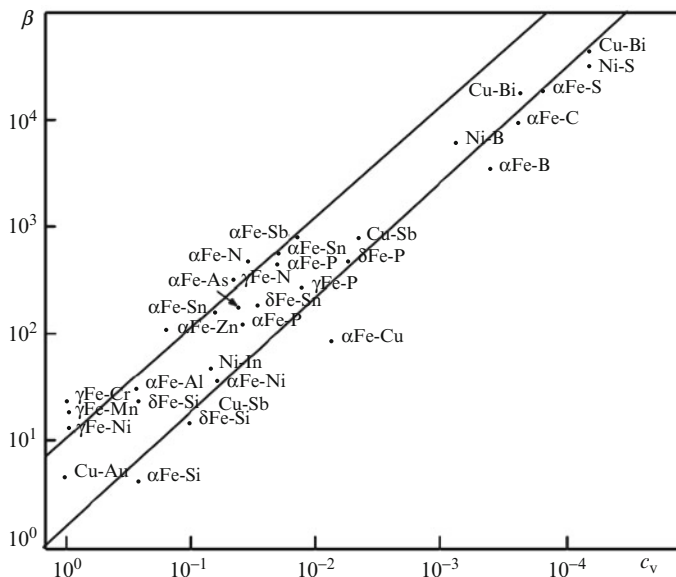


Fig. 3.40 Enrichment factor, β , as a function of the limit solubility (c_v) in various binary systems. For a system denoted M-S, M corresponds to the matrix and S to the solute (Seah and Hondros 1973) (Taken from Fraczkiewicz et al. 2011)

“ternary” chemical term α_{ij} in the segregation free energies. In a simplified form the segregation free energy can be written as:

$$\Delta g_i = \Delta g_i^0 + \alpha_{ij} c_j \quad (3.66)$$

where Δg_i^0 is the segregation free energy for element i in the absence of solute j .

Preferential attraction energy between i and j ($\alpha_{ij} > 0$) enhances their respective segregations with respect to those in the two binary systems. When the attractive i - j interaction is sufficiently high, the more concentrated solute may trap the minor solute in the matrix in the form of atomic pairs, clusters or precipitates, as indicated previously. The competition between those processes, *i.e.* enhanced co-segregation and scavenging, may thus give rise to a complex situation, which depends on all the parameters of the system. Conversely a repulsive interaction between i and j ($\alpha_{ij} < 0$) causes the less surface-active species (*i.e.* that with the lower intrinsic segregation energy), Δg_i^0 to segregate less or even to de-segregate in the presence of the other element.

Attractive segregation has been demonstrated in a number of high purity Fe based alloys, including Fe-C-P (Fig. 3.41) and Fe-Ni-Sb (Fig. 3.42). The dual effect (co-segregation and gatterting effect) of the strong impurity-metal affinity is illustrated in Fig. 3.43 (Dumoulin et al. 1980) in the case of Fe-Mo-P alloys. In this figure it is observed that the scavenging effect of P by Mo reduces the segregation of the P impurity, especially below 550°C. Similarly Ti, which strongly segregates with P at low concentration, efficiently traps this element at higher concentrations.

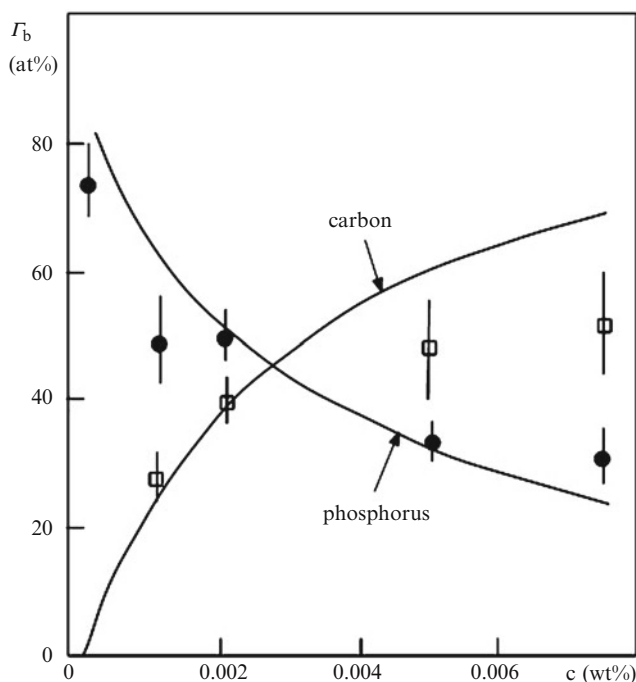


Fig. 3.41 Influence of bulk carbon content on segregation of phosphorus and carbon to the grain boundaries of Fe-0.17% P at 600°C (Erhart and Grabke 1981)

Reinforcing intergranular segregation can also take place in refractory metals (Mo, W, Ta). It has been shown that these metals are intrinsically brittle (see Sect. 3.2.2). They have a DBT temperature of the order of 300°C for Mo, 500°C for W. This DBT temperature depends on the purity of the materials, and a number of authors attempted to explain this as the result of oxygen segregation. These materials, which can easily dissolve oxygen, are extremely difficult to be prepared with a low content of this element. It is therefore very hard to test this effect of oxygen on DBT temperature. However *ab initio* simulations (see *e.g.* Krasko 1997) have shown that, if the presence of oxygen along the grain boundaries decreases the strength of these boundaries, the presence of boron or carbon can reinforce them. This means that the intergranular brittleness of these materials could be avoided with a “well-chosen” segregation phenomenon.

Intermetallic alloys rich in aluminium and in transition metals, such as Ni₃Al, NiAl, FeAl, TiAl exhibit interesting mechanical properties for high temperature applications. However the major problem with the use of these materials is their intergranular brittleness. Beneficial effects of small additions of boron on the ductility of these intermetallics were demonstrated (see *e.g.* Aoki and Izumi 1979; Liu and George 1990). *Ab initio* simulations tend to show that the interfacial cohesion is reinforced by the presence of boron (Chen et al. 1990; Wang and Wang 1998).

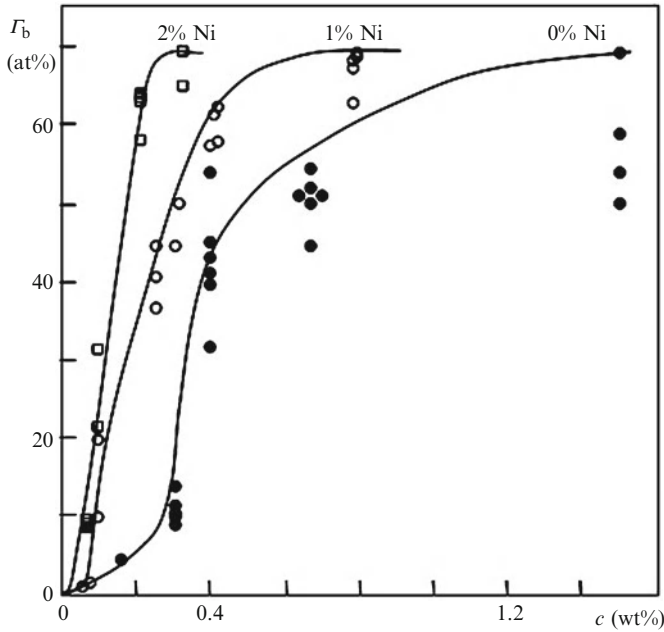


Fig. 3.42 Influence of nickel on intergranular segregation isotherm of antimony in high purity iron at 550°C (Gas et al. 1982)

3.8.3 Segregation of Impurities at Grain-Boundaries – Kinetics

In some situations the segregant atoms do not have sufficient time to reach their equilibrium level defined by Eqs. 3.64 and 3.65. The kinetics of segregation becomes a limiting factor.

Most existing models of segregation kinetics follow the Mc Lean approach (Mc Lean 1957). In the model for equilibrium monolayer segregation, the solute atoms are assumed to segregate to a grain boundary from two semi-infinite crystals or to a surface from one semi-infinite crystal. The diffusion in the crystals is described by Fick's law (see Volume I, Appendix B). The ratio of the solute concentration in the grain boundary to that in the adjacent atomic layer of the bulk is given by the enrichment ratio, β . Most models assume β to be constant, but in practice this is only true for dilute systems with low segregation levels. In this dilute approximation the enrichment factor, β , is given as:

$$\beta = \frac{\theta_b}{c} \quad (3.67)$$

The rate at which the grain boundary concentration θ_b builds up after a specimen has been suddenly cooled from a high temperature, T_1 , to a low temperature, T_2 , and is maintained at this temperature is given by:

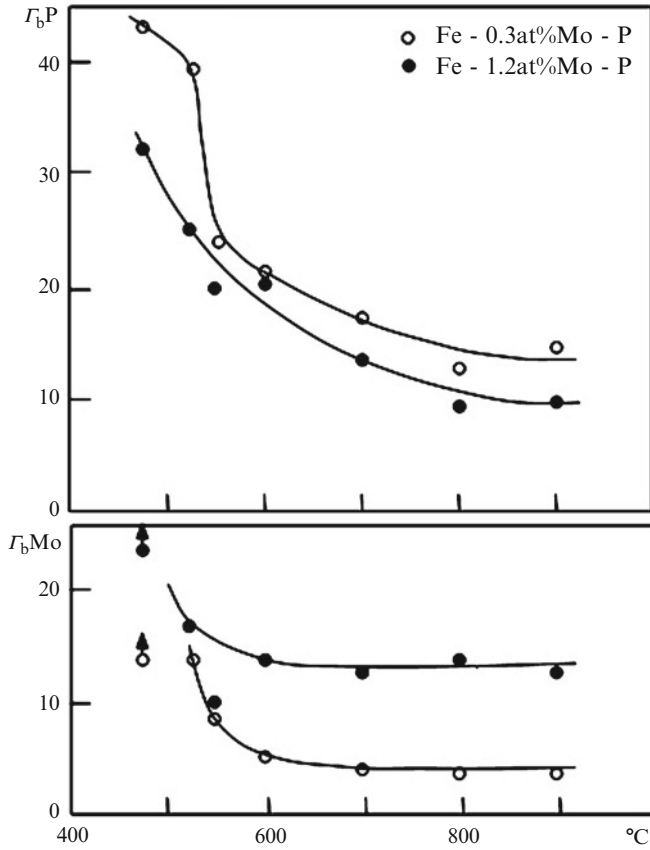


Fig. 3.43 Influence of molybdenum and phosphorus on each other's segregation at the grain boundaries of high purity iron (Dumoulin et al. 1980)

$$\theta_b(t) = \theta_b(\infty) - c(\beta_2 - \beta_1) \exp\left(\frac{4Dt}{\beta_2^2 \delta_{GB}^2}\right) \operatorname{erfc}\left(\frac{2\sqrt{Dt}}{\beta_2 \delta_{GB}}\right) \quad (3.68)$$

where $\theta_b(t)$ is the grain boundary concentration after time t at low temperature, $\theta_b(\infty)$ is the equilibrium grain boundary concentration given by Eqs. 3.64 and 3.65 and which is obtained after infinite time at T_2 , c is the grain interior concentration, assumed constant, $\beta_1 = \theta_b/c$ is the enrichment factor just before quenching, *i.e.* the equilibrium ratio $\theta_b(\infty, T_1)/c$, characteristic of the quenching temperature, T_1 , if the specimen has been maintained at this for sufficient time, β_2 is the enrichment factor at T_2 , *i.e.* $\beta_2 = \theta_b(\infty, T_2)/c$, D is the diffusion coefficient at the lower temperature, T_2 , t is the time and δ_{GB} is the thickness of the grain boundary (a few atomic planes).

The main assumptions used to derive Eq. 3.68 from the solution of the Ficks law (see Appendix B in Volume I) are that the grain diameter is much larger than the distance δ_{GB} and the lattice concentration c is not reduced as the grain boundary segregation builds up. Equation 3.68 can be written in the form:

$$\frac{\theta_b(t) - \theta_b(0)}{\theta_b(\infty) - \theta_b(0)} = 1 - \exp\left(\frac{4Dt}{\beta_2^2 \delta_{GB}^2}\right) \operatorname{erfc}\left(\frac{2\sqrt{Dt}}{\beta_2 \delta_{GB}}\right) \quad (3.69)$$

where $\theta_b(0)$ is the initial grain boundary concentration. In practice, in many situations, it can be assumed that $\theta_b(0) = \theta_b(\infty, T_1)$ when the quenching rate is sufficiently rapid. Very often it can also be assumed that $\theta_b(0) \simeq 0$. In these conditions, Eq. 3.69 can be expressed in the form:

$$\frac{\theta_b(\infty) - \theta_b(t)}{\theta_b(\infty)} = \exp\left(\frac{4Dt}{\beta_2^2 \delta_{GB}^2}\right) \operatorname{erfc}\left(\frac{2\sqrt{Dt}}{\beta_2 \delta_{GB}}\right) \quad (3.70)$$

The solution of Eq. 3.69 shows that the time to reach one half of the segregation, *i.e.* the time at which $[\theta_b(t) - \theta_b(0)] / [\theta_b(\infty) - \theta_b(0)] = 1/2$ is given by the following condition:

$$\frac{2\sqrt{Dt_{1/2}}}{\beta_2 \delta_{GB}} = \frac{3}{4} \quad (3.71)$$

or

$$t_{1/2} = \frac{9}{64} \left[\frac{\theta_b(\infty)}{c} \right]^2 \frac{\delta_{GB}^2}{D} \quad (3.72)$$

The values of the diffusion coefficient in α -iron are such that, for instance for impurity like phosphorus, $t_{1/2} \sim 20$ h at a temperature close to 500°C. Typical values of D (m²/s) in α -iron at 550°C are: $D_P = 5.1 \times 10^{-22}$, $D_{As} = 6.2 \times 10^{-19}$, $D_{Sb} = 4.75 \times 10^{-21}$, $D_{Sn} = 1.1 \times 10^{-19}$ and $D_S = 8.05 \times 10^{-19}$ (Gale and Totemeier 2004). In the case of α -iron the effect of Curie temperature on the diffusion coefficient must be taken into account as underlined by Militzer and Wieting (1989). Below the Curie temperature (770°C) bulk diffusion of phosphorus in α -iron is about two orders of magnitude lower than predicted from extrapolations of the paramagnetic values. In many studies, unfortunately, the latter values are commonly used to describe segregation kinetics of phosphorus in low-alloy steels. It should also be noted that the S impurity strongly segregates at grain boundaries (Table 3.7) and the diffusion coefficient of this element is relatively large explains why this impurity is so detrimental.

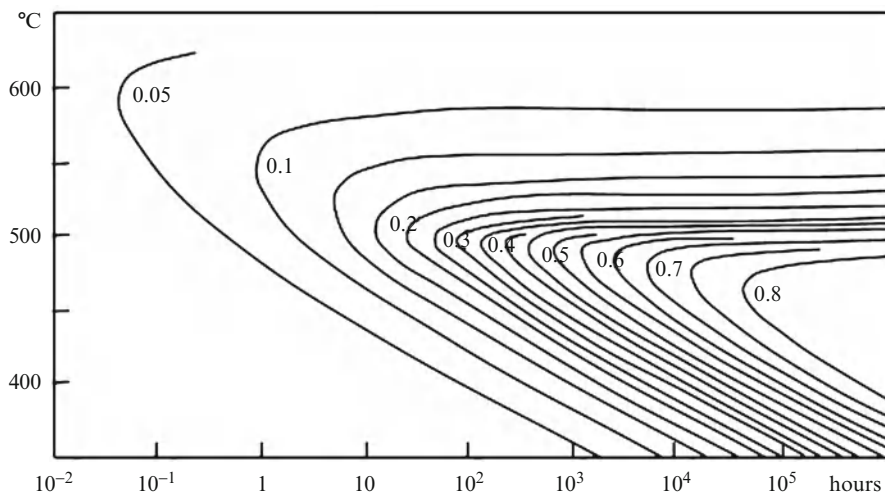


Fig. 3.44 Time-temperature diagram for segregation of phosphorus in low-alloyed steel (Carr et al. 1953). The numbers indicate the P segregation level (Seah 1977)

Equations 3.64 and 3.69 can now be combined to generate temperature-time plot of segregation of a given element, for instance P, in a given steel. This is shown in Fig. 3.44 (Seah 1977) for a Ni-Cr steel (SAE 3140). This figure shows that there is a nose in the T - t diagram close to 475°C. The correlations of Charpy transition temperature measurements in this steel (Carr et al. 1953) with those reported in Fig. 3.44 (Seah 1977) are shown in Fig. 3.45. The excellent correlation of curve shape and the relative magnitudes of the effects on DBT temperature at both 475 and 500°C indicate that Mc Leans' diffusion analysis of the kinetics of segregation is valid.

3.8.4 Micromechanisms of Grain Boundary Embrittlement

3.8.4.1 Modification of the Surface Energies

Early theories to explain the effect of solute segregation on low temperature grain boundary brittle fracture assume that the ideal work of fracture per unit area of the grain boundary, γ_c can be calculated from the relation:

$$\gamma_c = 2\gamma_s^{\text{int}} - \gamma_b \quad (3.73)$$

where γ_s^{int} is the surface energy immediately after fracture and γ_b that of the boundary. The presence of impurities in the boundary can modify the two terms and reduce γ_c sufficiently to favor fracture along the boundary. The concentration of

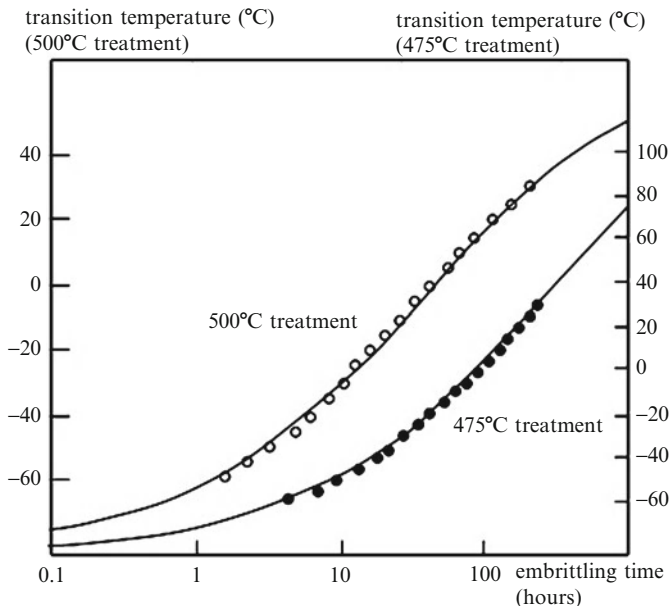


Fig. 3.45 Predicted phosphorus segregation during embrittlement at 500 and 475°C as a function of time with the experimental embrittlement data of Carr et al. (1953)

foreign atoms Γ can be different on the boundary and on the surface (Γ represents the excess concentration with respect to the bulk). Both are functions of the chemical potential μ , as Fig. 3.46 shows. Very slow separations take place at constant values of this, with Γ changing from Γ_0 to Γ_s , but during the fast separations in which we are interested, there is not time for the concentration to change; the foreign atoms on a unit area of the grain boundary remain on the two new unit areas of fracture; the concentration is divided by 2 and the potential decreases from μ_0 to $\mu_s(\Gamma_0/2)$.

The fracture energy $2\gamma_{\text{int}}$ is equal to the work of the stress σ during separation:

$$2\gamma_{\text{int}} = \int_{\delta_b}^{\infty} \sigma d\delta \quad (3.74)$$

δ being the distance of separation.

The free energy per unit area and the surface energy are given by: (Hirth and Rice 1980; Wang 2002):

$$\begin{aligned} d\bar{f} &= \sigma d\delta - \bar{s}dT + \mu d\Gamma \\ d\bar{\gamma} &= d(\bar{f} - \mu\Gamma) = \sigma d\delta - \bar{s}dT + \Gamma d\mu \end{aligned} \quad (3.75)$$

where μ is the chemical potential of the segregant.

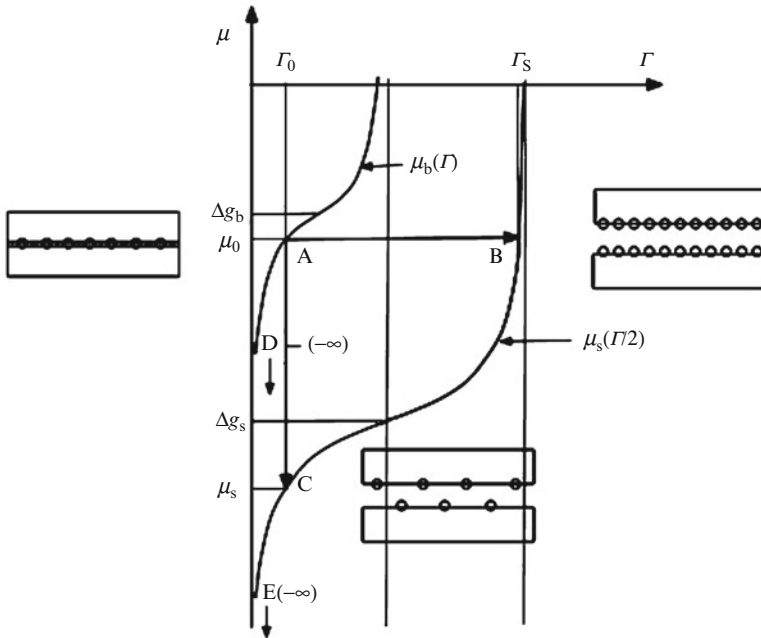


Fig. 3.46 Chemical potential μ : (1) for the grain boundaries (isotherm AD); (2) for the surface (isotherm BCE)

Hence, at constant temperature, the energy of separation $(2\gamma_{\text{int}})_{\mu}$ in slow separation, at constant value of $\mu = \mu_0$ is given by

$$(2\gamma_{\text{int}})_{\mu} = (2\gamma_{\text{int}})_0 - \int_{-\infty}^{\mu_0} [2\Gamma_s(\mu) - \Gamma_b(\mu)] d\mu \quad (3.76)$$

$(2\gamma_{\text{int}})_0$ is the energy of separation in the absence of segregant.

The integral is the area DABE in Fig. 3.46. Then:

$$(2\gamma_{\text{int}})_{\mu} = (2\gamma_{\text{int}})_0 - [2\gamma_s(\mu_0) - \gamma_b(\mu_0)] \quad (3.77)$$

For fast separation, at constant value of the concentration $\Gamma = \Gamma_0$, the change of the energy of separation is given by:

$$(2\gamma_{\text{int}})_{\Gamma} = (2\gamma_{\text{int}})_0 - \int_0^{\Gamma_0} [\mu_b(\Gamma) - \mu_s(\Gamma/2)] d\Gamma \quad (3.78)$$

The integral corresponds to the area DACE in Fig. 3.46. And then:

$$(2\gamma_{\text{int}})_{\Gamma} = (2\gamma_{\text{int}})_0 - [2\gamma_s(\Gamma_0/2) - \gamma_b(\Gamma_0)] - \Gamma_0 [\mu_b(\Gamma_0) - \mu_s(\Gamma_0/2)] \quad (3.79)$$

In general segregants decrease the surface energy and the energy of separation is decreased.

Now, we must try to quantify these expressions. As discussed by Rice and Wang (1989), if the coverage by foreign atoms of the grain boundaries and of the surface is smaller than full coverage ($\Gamma^{\max}$), the Langmuir⁴- Mc Lean expression can be used:

$$\begin{aligned}\mu_b(\Gamma_b) &= \Delta g_b^0 + RT \log \left(\frac{\Gamma_b}{\Gamma_b^{\max} - \Gamma_b} \right) \\ \mu_s(\Gamma_s/2) &= \Delta g_s^0 + RT \log \left(\frac{\Gamma_s}{2\Gamma_s^{\max} - \Gamma_s} \right)\end{aligned}\quad (3.80)$$

where the Δg^0 terms are referenced to the bulk at the same temperature μ_0 ($\mu_0 = RT \log c$).

Using these expressions of the chemical potentials in (3.79) we get:

$$(2\gamma_{\text{int}})_{\mu} - (2\gamma_{\text{int}})_{\Gamma} = \Gamma_0 (\Delta g_b^0 - \Delta g_s^0) + \Gamma_0 RT \log \frac{2\Gamma_s^{\max} - \Gamma_0}{\Gamma_b^{\max} - \Gamma_0} \quad (3.81)$$

Fast decohesion at Γ constant is easier than slow decohesion at μ constant (see Chap. 7, Sect. 7.2.5 for the case of hydrogen which is a fast moving element and which can facilitate intergranular fracture at μ constant).

It can be shown, by considering average values of the concentrations, that the last term on the right hand side of (3.81) is negligible at room temperature.

Very few values of Δg_b^0 and of Δg_s^0 have been measured (see, however, a number of results in Table 3.7). These energies can be approximated by the enthalpies of segregation, as far as the entropic terms, corresponding to vibrations, can be neglected. Rice and Wang (1989) listed the few known results at 300 K insisting on their uncertainties. They are given in Table 3.8 where it is observed that sulfur and antimony are predicted to strongly embrittle the grain boundaries in iron, as observed experimentally.

The embrittlement sensitivity ξ is the shift of the DBT temperature per atomic per cent of impurities in the grain boundaries

3.8.4.2 Intrinsic Intergranular Brittleness

In Sect. 3.2.2 we explained the Rice and Thomson model, which provides a distinction between cleavage blunting and intrinsic brittleness. This can be applied to intergranular cracking by comparing the blunting condition as given by Eqs. 3.7 and 3.8 with the critical condition for intergranular cracking $G_c = 2\gamma_{\text{int}}$. The energy

⁴Irving Langmuir (1881–1957) was an American chemist and physicist. He was awarded the 1932 Nobel Prize in chemistry for his work in surface chemistry. He was the first industrial (General Electric) chemist to become a Nobel laureate.

Table 3.8 Estimated values of the segregation energies in alpha-iron and embrittlement sensitivities ξ (Rice and Wang 1989; Anderson et al. 1990)

Impurity	$-\Delta g_s^0$ (kJ/mole)	$-\Delta g_b^0$ (kJ/mole)	$\Delta g_b^0 - \Delta g_s^0$ (kJ/mole)	ξ K/at % in g.b.
C	73–85	50–75	–2–35	–20 to –10
Sn	61–87	30–35	26–57	15–38
P	76–80	32–41	35–48	5–20
Sb	83–130	8–25	58–122	28–67
S	165–190	50–58	107–140	35–45

to create the ledge E_{ledge} is modified in the same way as the surface energy in the preceding Sect. 3.8.4.1. The core energy E_{core} is also modified, especially in the case of the dissociation of the dislocation in partials; the stacking fault energy is then modified by the segregation of impurities. The model was applied by Anderson et al. (1990) to copper embrittled by bismuth. They considered Cu bicrystals and tilt boundaries. The segregation of Bi to the grain boundaries decreases the intergranular fracture energy. However, this is enough for suppressing blunting for certain orientations of the slip systems only. This was checked experimentally.

3.8.5 Intergranular Fracture Toughness in Ferritic Steels

As stated previously theoretical calculations (see Sect. 3.2.1) indicate that intergranular fracture should be observed preferentially in many polycrystalline metals instead of transgranular cleavage fracture (see Table 3.2). However in ferritic steels, brittle fracture occurs at low temperature by transgranular cleavage. This behaviour is usually attributed to the intergranular reinforcement effect due to a number of elements, in particular C, segregated along the grain boundaries (see Table 3.7). Conversely, the segregation of other impurities, for example phosphorus, along the grain boundaries can change the brittle fracture mode from cleavage to intergranular. In ferritic steels, temper embrittlement has been – and is still – a subject of debate for the last 50 years. Many authors assumed grain boundary impurity segregation (in particular, carbon and phosphorus) to be responsible for temper embrittlement (see *e.g.* Seah 1980 and Sect. 3.8.1). Whereas often co-segregation of carbon and phosphorus is assumed (see *e.g.* Guttman et al. 1982; Gas et al. 1982), other authors (see *e.g.* Erhart and Grabke 1981) rejected this assumption and attempted to explain the enhanced impurity segregation by a decreased solubility limit of impurities or a diminished free carbon concentration. Next neighbour interactions between segregating species leading to an interfacial miscibility gap or another 2D phase transition was also discussed by several authors, in particular Militzer and Wieting (1987).

This complex situation is observed in many ferritic steels containing alloying elements like Ni, Mn and Cr. Large components like rotors for steam turbines (Cr-Mo steels) or pressure vessels (Mn-Ni-Mo steels) are sensitive to this effect. This is due to the fact that, because of the heavy sections, long times are necessary

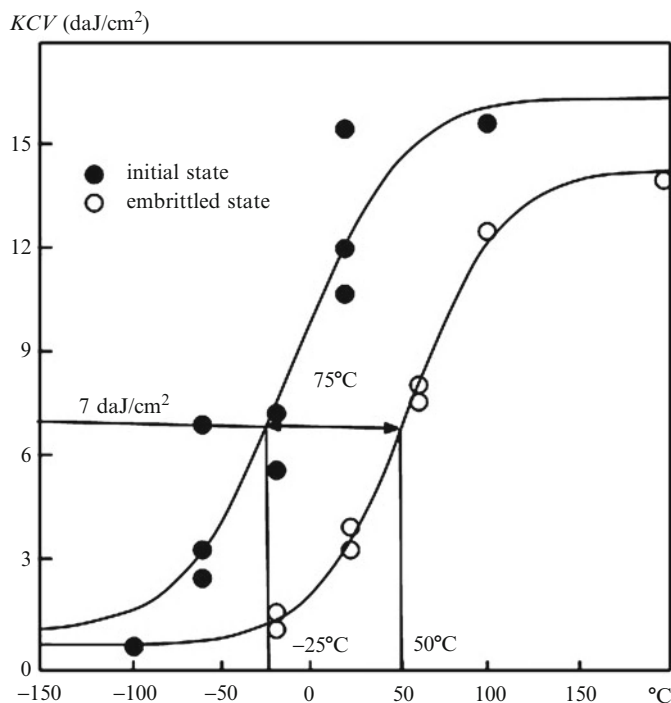


Fig. 3.47 Shift of the DBT temperature as a result of temper-embrittlement in a pressure vessel steel (Raoul et al. 1999)

to cool down these components from the normalising or tempering temperatures. If these materials remain for a too long period of time in the critical temperature range between approximately 400 and 550°C, the segregation of impurities along the grain boundaries has enough time to take place and to develop the conditions for initiating intergranular fracture when the components are tested at low temperature. This change in fracture mode is accompanied by a shift of the DBT temperature as observed in Fig. 3.47 (Raoul 1999; Raoul et al. 1999). This figure refers to results obtained on a pressure vessel steel A533 (0.18C-1.5Mn-0.75Ni-0.50Mo) containing about 80 ppm of P. It is seen that after a step-cooling heat-treatment simulating low cooling rate, the DBT temperature is shifted by 75°C. It is also noted that the level of the upper shelf energy is also decreased by the application of temper embrittlement treatment.

In these thick components the presence of small areas of low toughness corresponding to solidification segregated zones (see Sect. 3.5.4.5) may be an important source of scatter in fracture toughness values (see *e.g.* Tavassoli et al. 1983, 1989; Kantidis et al. 1994). These segregated zones can initiate intergranular brittle fracture due to temper-embrittlement effect. Intergranular brittle fracture is therefore of great practical importance.

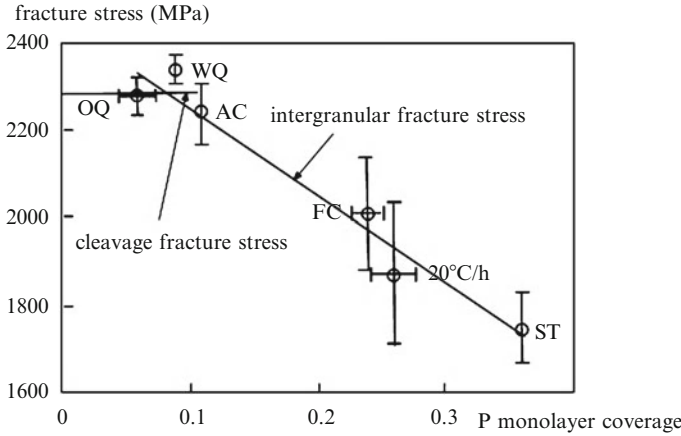


Fig. 3.48 Relationship between critical fracture stress and phosphorus monolayer coverage. Results obtained with various heat treatments (OQ oil quench, WQ water quench, AC air cooled, FC furnace cooled, 20°C/h cooling rate after austenitization, ST step cooling heat-treatment) (Naudin et al. 1999)

Unfortunately very few detailed studies were made to determine quantitatively the variation of the critical intergranular fracture stress σ_{CI} with test parameters using procedures similar to those used for the measurement of the cleavage stress. However the interesting work by Kameda and Mc Mahon (1980) should be mentioned. These authors showed that the critical intergranular fracture stress was directly related to the amount of impurity (Sb) segregated on grain boundaries. Similarly in a pressure vessel steel it was shown that σ_{CI} was decreasing linearly with the amount of phosphorus segregated on the grain boundaries. The coverage in P atoms has to be larger than a certain value (5–10%) before the intergranular stress is lower than the cleavage stress (Fig. 3.48, Naudin et al. 1999). In another recent study devoted to the statistical aspect of intergranular fracture, Wu and Knott (2004) showed that σ_{CI} was independent of test temperature and was statistically distributed according to either a normal or a Weibull law.

More detailed studies were devoted to the effect of intergranular fracture on the fracture toughness of pressure vessel steel A508 (0.16C, 1.5Mn, 0.75Ni, 0.50Mo) (Yahya et al. 1998; Kantidis et al. 1994; Raoul et al. 1999). It was shown that, in the presence of intergranular fracture, the Beremin theory should be slightly modified to account for the effect of temperature on fracture toughness, since σ_{CI} was found to be an increasing function of temperature. Kantidis et al. (1994) showed that the variation of fracture toughness with temperature could be represented using the original Beremin model (Eq. 3.44), provided that the Weibull stress included a temperature dependence, *i.e.*:

$$\sigma_{ww}^m = \int_{PZ} \sigma_1^m [1 + \lambda (T - T'_0)]^m \frac{dV}{V_0} \quad (3.82)$$

where σ_{ww} is the modified Weibull stress, $\lambda > 0$ is a material parameter and $T_0'^5$ is a reference temperature. This expression indicates that when the temperature is lower than T_0' , intergranular fracture occurs at lower stresses as compared to the original Weibull stress. In these studies it was also shown that the shape factor, m for intergranular fracture was much lower (~ 10) than for cleavage where typically $m \sim 20$. This means that the scatter in test results is larger for intergranular fracture than for cleavage.

Another aspect of segregation induced embrittlement was underlined in Fig. 3.47 where it was noted that the upper shelf energy was reduced by temper embrittlement. Detailed studies on this effect were made by Hipsley and Druce (1983). These authors investigated the effect of isothermal ageing at 500°C of a quenched-and-tempered low-alloy steel on fracture resistance. They showed that this ageing produced reductions in the resistance to ductile fracture, as measured by impact fracture upper-shelf energy, ductile fracture toughness (J_{IC}) and tensile ductility. The segregation of phosphorus to carbide/matrix interfaces during ageing was directly observed by Auger spectroscopy and was shown to be mainly responsible for this reduction of ductile fracture resistance.

3.8.6 Overheating of Steels

This type of embrittlement occurs when a steel has been heated to a high temperature, so as to dissolve all the impurities, and then cooled slowly to a temperature region in which the impurity atoms begin to be concentrated in the grain boundaries; at a lower temperature they precipitate again. This particular morphology has no effect on the cleavage phenomena and therefore on the brittle-ductile transition temperature, but it does affect the ductile fracture. Dimples form along the former austenitic grain boundaries and the energy of the upper shelf is lowered (Figs. 3.49 and 3.50). This is what happens, for example, in a steel containing nitrogen cooled slowly from 1,300°C, when precipitates of aluminium nitride form in the austenitic grain boundaries. The same effect is seen in maraging steels cooled slowly after heating to 1,200°C, due to precipitates of titanium carbo-nitrides.

The only way to remove this embrittlement is to heat the steel to a temperature high enough to dissolve the precipitates and follow this by a rapid cooling; the disadvantage is that this increases the grain size and may induce residual stresses.

⁵This temperature T_0' should not be confused with the temperature θ_0 of the *Master Curve* approach introduced in Sect. 3.4.4.1 and in Eq. 3.47.

Fig. 3.49 Ductile intergranular fracture of a steel of type 15 MND 6 subjected to overheating. Some of the dimples on the surfaces of the grains contain fine particles of manganese sulphides

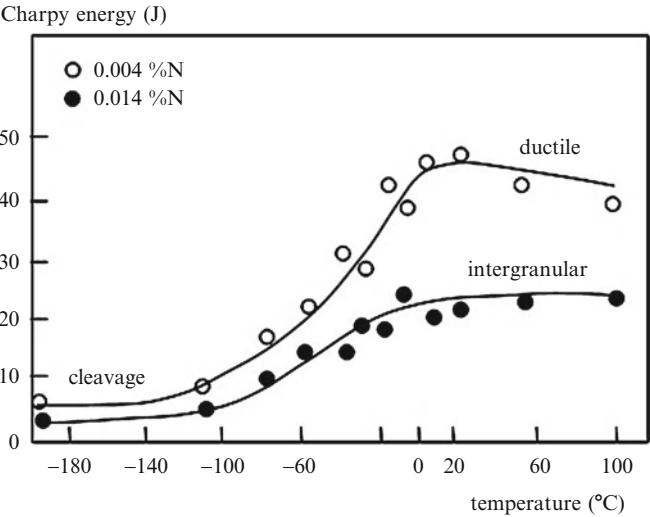
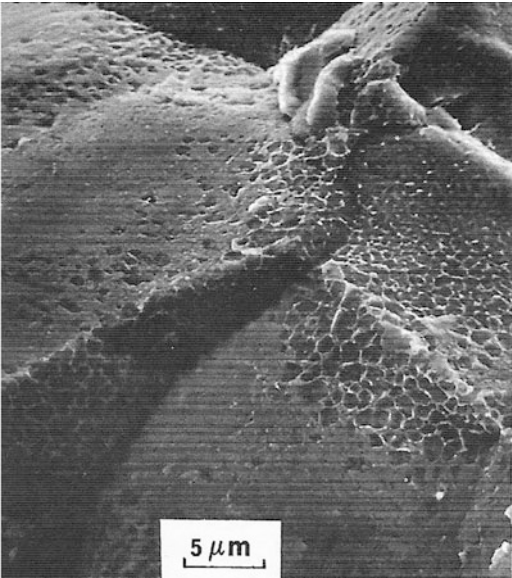


Fig. 3.50 Comparison of transition curves of Charpy energy for steels of high and low nitrogen content respectively, embrittled by intergranular aluminium nitrides

3.9 Irradiation-Induced Embrittlement in Ferritic Steels

3.9.1 *Hardening, DBTT and Reduction in the Upper Shelf Energy (USE)*

Irradiation embrittlement is one of the domains specific to the materials used in nuclear power reactors. The micromechanisms of radiation damage are described in a number of books (see *e.g.* Was 2007). The detailed description of the physical events associated with this form of damage is out of the scope of the present book. In this section we only concentrate on the effect of embrittlement associated to irradiation.

For a given fluence (ϕ n/cm²) of neutrons ($E > 1$ MeV) and irradiation temperature, irradiation-induced embrittlement is strongly dependent on the material chemical composition, and especially on Cu, Ni and P contents (Haggag 1993; Wagenhofer et al. 2001). The Cu content plays an important role in the hardening-induced embrittlement due to the irradiation-induced precipitation of Cu-rich nanoparticles (see *e.g.* Buswell et al. 1995; Auger et al. 1995; Akamatsu et al. 1995). This has led to consider the effect of Cu and other chemical elements in empirical embrittlement correlation models for the prediction of the shift of the DBT temperature (DBTT) curve (see *e.g.* Kirk et al. 2003; Hiranuma et al. 2004). On the physical point of view, irradiation produces fine scale microstructures (see *e.g.* Nikolaev et al. 2002): point defects, dislocation loops, precipitates. Their effects on the macroscopic behaviour can be separated into two groups (Nikolaev et al. 2002): (i) change of the plastic hardening capacity, and (ii) embrittlement. Another well documented effect besides Cu precipitation is phosphorus segregation at grain boundaries (Nikolaev et al. 2002; Brillaud and Hedin 1992).

Two main types of irradiation-induced hardening were reported in the literature and are schematically represented in Fig. 3.51 (Tanguy et al. 2006). In the first case (dashed curve labelled irr.1) the whole stress-strain curve of the un-irradiated material is shifted to higher stress values by $\Delta\sigma_\phi$, *i.e.* the strain-hardening rate remains unchanged whereas the yield strength is increased (see *e.g.* Al Mundheri et al. 1989). In the second case, the yield strength is increased by $\Delta\sigma_\phi$ whereas the ultimate tensile stress (UTS) remains almost unaffected by irradiation (dotted curve labelled irr.2). In that case the work-hardening rate is decreased after irradiation. This situation has been reported for steels irradiated at low temperature ($\leq 200^\circ\text{C}$) and its importance increases with decreasing irradiation temperature (Byun and Farrell 2004; Odette et al. 2003). Experimental results showing the variation of the yield strength and of the difference between the UTS and the yield strength

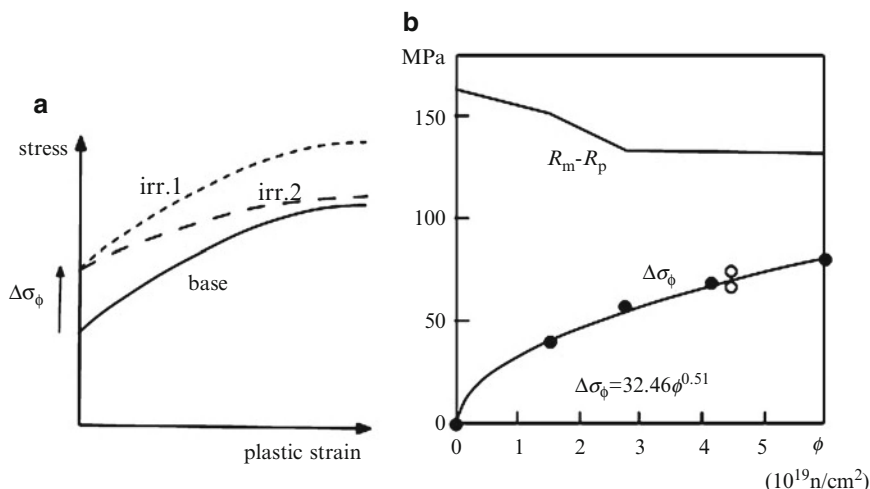


Fig. 3.51 Influence of irradiation on stress-strain curves of ferritic steels; (a) two main assumptions (irr.1 and irr.2), see text; (b) variation of yield strength with fluence in a A508 (0.16C, 1.5Mn, 0.75Ni, 0.50Mo) pressure vessel steel (Tanguy et al. 2006)

as a function of the fluence are reported in Fig. 3.51b (Tanguy et al. 2006). This variation can be expressed as:

$$\Delta\sigma_\phi = 32.46\phi^{0.51} \quad (3.83)$$

($\Delta\sigma_\phi$ in MPa and ϕ in 10^{19} n/cm^2)⁶.

Reactor surveillance programs have historically used Charpy V-notch specimens rather than fracture toughness specimens. These impact specimens are exposed in surveillance capsules, after which they are tested and the shift of the impact energy due to irradiation is calculated, as indicated schematically in Fig. 3.52. The shift is calculated for a given Charpy energy (41 J or 56 J). It is usually observed that the upper shelf energy (USE) is also decreased after irradiation.

Figure 3.53 shows that an attempt can be made to relate the change in DBTT measured in a Charpy test with the shift of the yield strength after irradiation. The methodology of the *Master Curve* (see Sect. 3.4.4.2, Eqs. 3.46 and 3.47) can be used for that purpose. These modifications in the mechanical properties induced by irradiation can be illustrated with the study by Sokolov et al. (2004). These authors performed a series of experiments on a radiation-sensitive reactor pressure steel weld with intentionally enhanced copper, nickel and manganese content, designated KS – 01. The material was irradiated to a fluence of $0.8 \cdot 10^{19} \text{ n/cm}^2$ ($E > 1 \text{ MeV}$) at

⁶Several relationships were proposed to relate the irradiation damage expressed in terms of fluence (n/cm^2) to the number of displacements per atom (dpa) (see e.g. Guionnet et al. 1982).

Fig. 3.52 Schematic effect of irradiation on the DBT curves in ferritic steels

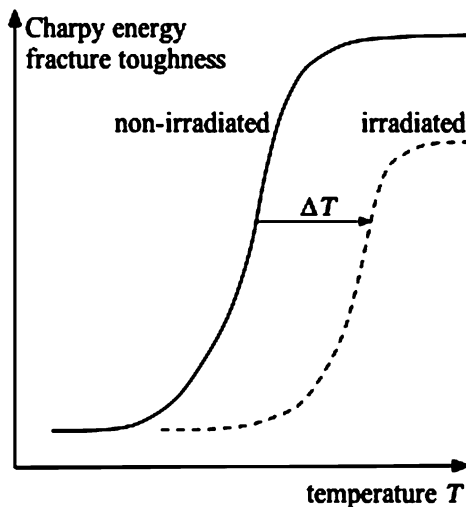
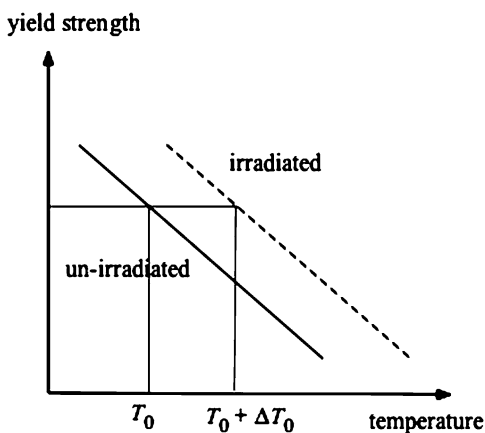


Fig. 3.53 Irradiation induced hardening. Variation of the yield strength of a ferritic steel in the un-irradiated condition and after irradiation. The shift of the DBT temperature can be predicted from the shift of the yield strength



a temperature of 288°C. This weld exhibited a large Charpy T_{41J} shift of 169°C, a large shift of the fracture toughness transition ($K = 100 \text{ MPa m}^{1/2}$) of almost 200°C, a decrease in upper shelf energy (USE) from 118 to 78 J and an increase in the yield strength from 600 to 826 MPa. Figures 3.54 and 3.55 clearly show that large effects of irradiation are observed when the content in impurity and some alloying elements is large (1.23Ni, 0.37Cu, 0.017P). These effects of impurities and alloying elements on irradiation-induced embrittlement have strong practical implications.

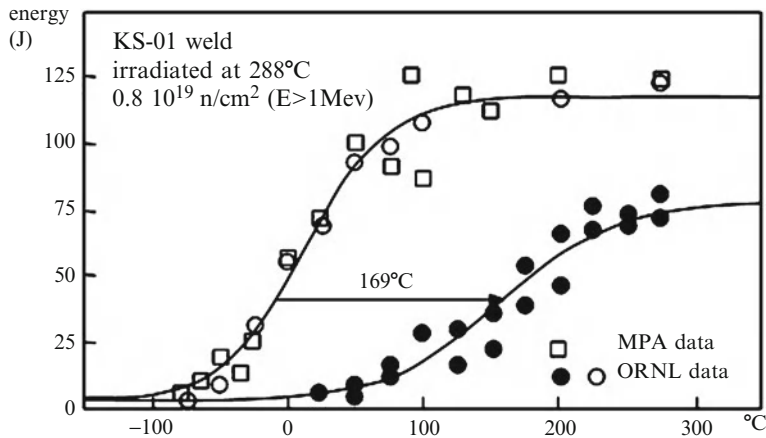


Fig. 3.54 Charpy impact energy as a function of test temperature for weld metal in the un-irradiated condition and following irradiation to $0.74 \times 10^{19} \text{ n/cm}^2$ ($E > 1 \text{ MeV}$) at 288°C (Sokolov et al. 2004)

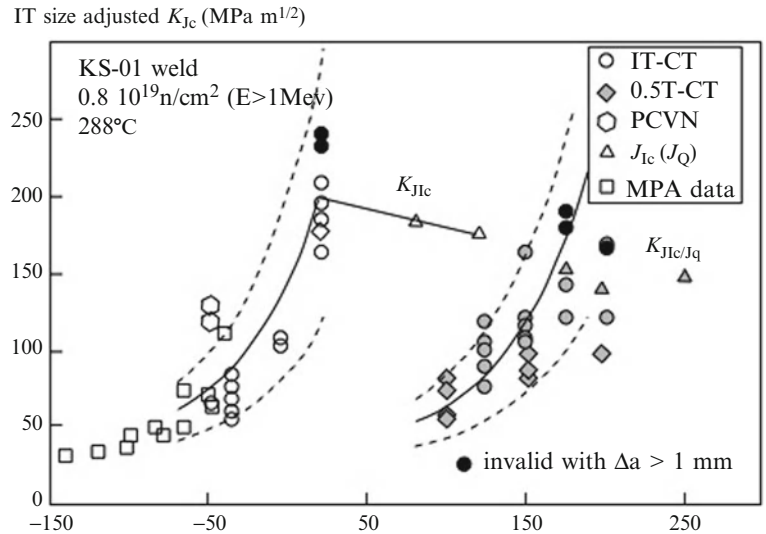


Fig. 3.55 Fracture toughness of weld metal in the un-irradiated condition and following irradiation under the same conditions as for the Charpy tests in Fig. 3.52 (Sokolov et al. 2004)

3.9.2 A Tentative Model for Predicting the Shift in DBTT

The shift of the transition temperature, ΔT , can be tentatively explained using the theories for cleavage fracture, which have been presented previously (Sect. 3.2).

This approach applies only when the “cleavage stress” is not modified by irradiation. This means that this theory can be adopted when the phosphorus content is sufficiently low to avoid the initiation of intergranular fracture in irradiated materials. In applying Eq. 3.44, the two materials (non-irradiated NI and the irradiated IR) are assumed to have the same fracture properties (m , σ_u , V_0 , C_{mn}). For a given probability to failure, the temperature shift ΔT can simply be assessed by writing that the irradiation-induced hardening is balanced by an increase of temperature, ΔT , such that the two materials have the same yield strength at T_0 (for the non-irradiated material) and $T_0 + \Delta T$ for the irradiated material (Fig. 3.53).

Several expressions were proposed for the temperature dependence of the yield strength of pressure vessel steels. The polynomial function given by Rathbun et al. (2006) can be used:

$$R_p(T) = 0.0085T^2 - 0.4402T + 481.51 \quad (3.84)$$

where R_p is expressed in MPa and T in °C. It is assumed that this function valid for non-irradiated materials can also be applied to the irradiated materials. It is also assumed that the slight variation in the work-hardening rate of the material with irradiation does not influence the value of the coefficient C_{mn} in Eq. 3.44.

Differentiating Eq. 3.84 and equating the corresponding $\Delta\sigma$ to that given by Eq. 3.83, it can easily be shown that:

$$\Delta T = \frac{32.46\phi^{0.51}}{|0.017T_0 - 0.4402|} \quad (3.85)$$

The value of T_0 can be taken as the temperature at which $K_{Jc} = 100 \text{ MPa m}^{1/2}$. In many pressure vessel steels, this temperature is close to -60°C . The calculated values of ΔT inferred from Eq. 3.85, using $T_0 = -60^\circ\text{C}$, are given in Table 3.8 for three values of the fluence, ϕ , for which the irradiation-induced hardening, $\Delta\sigma_\phi$, was measured. Full details are given elsewhere (Pineau and Pardoen 2006; Tanguy et al. 2006).

A more sophisticated model integrating a description of viscoplasticity, ductile damage and cleavage fracture based also on Beremin model was used to simulate both Charpy tests and fracture toughness tests on non-irradiated (NI) and on irradiated (IR) materials (Tanguy et al. 2006). These numerical simulations were compared to experimental results (Al Mundheri et al. 1989; Trouvain 1989). The comparison of the IR and NI predicted fracture toughness curve bounds ($P_R = 10, 50$, and 90%) with experiments is shown in Fig. 3.56. The calculated temperature shifts at $K_{Jc} = 100\text{MPa}\sqrt{\text{m}}$ referring to the un-irradiated material are reported on each graph and in Table 3.9. These fracture toughness predictions are in good agreement with experiments. These results suggest that a large part of the DBTT shift is due to irradiation-enhanced hardening effect. However this conclusion applies only to materials with a relatively low content in impurities, in particular those with a low copper content ($<0.5\%$).

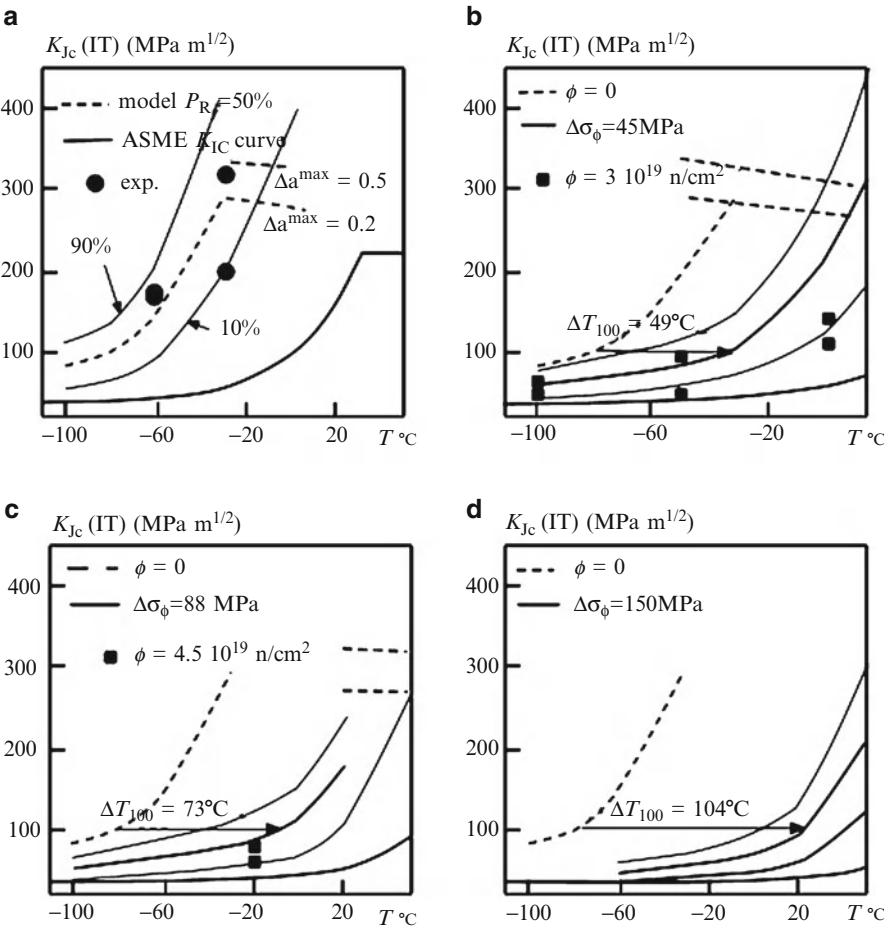


Fig. 3.56 Variation of fracture toughness with temperature on un-irradiated material and on the same material (A508 steel) with increasing fluences; **(a)** initial condition; **(b)** $\phi = 3 \times 10^{19} \text{ n/cm}^2$; **(c)** $\phi = 4.5 \times 10^{19} \text{ n/cm}^2$; **(d)** $\phi = 20.10 \times 10^{19} \text{ n/cm}^2$

Table 3.9 Influence of irradiation conditions on hardening ($\Delta\sigma_{\phi}$), shift of the Charpy impact curve at 56 J (ΔT_{56J}), shift of the temperature at which $K_{Jc} = 100 \text{ MPa m}^{1/2}$ ($\Delta T_{K_{Jc}=100}$) (Tanguy et al. 2006)

ϕ	ϕ	$\Delta\sigma_{\phi}$	ΔT_{56J}	$\Delta T_{K_{Jc},100}$	$\Delta T_{K_{Jc},100}$
10^{19} n/cm^2	dpa	(MPa)	($^{\circ}\text{C}$)	($^{\circ}\text{C}$)	calculated
1.90	0.026	45	30	49	31
7.07	0.095	88	56	73	60.3
20.10	0.27	150	95	104	103

The number of displacements per atom (dpa) has been calculated with the following expression: $1 \text{ dpa} = 1.35 \times 10^{-21} \phi$ ($E > 1 \text{ MeV}$) (Guionnet et al. 1982)

An additional consideration in assessing the structural integrity of a nuclear pressure vessel is the establishment of gradients due to its thickness (~ 200 mm). Variations in temperature and neutron fluence will cause the microstructure to differ as a function of position in the vessel. The hardening and the embrittlement will be greater at the inner wall since fluence is at a maximum at this position. This means that the fracture toughness will be lower at the vessel inner surface. Flaws initiating at the inner surface will grow into a material with increasing toughness which provides an additional margin of safety against through-wall cracking.

It can be added that one way of reversing the irradiation-induced embrittlement is to thermally anneal out the irradiation defects at a temperature of about 450°C to restore the toughness properties. However the success of annealing depends on the rate of re-embrittlement when the steel is re-irradiated (see *e.g.* Odette and Lucas 2001; Miller et al. 2006).

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Chapter 4

Ductile Fracture

Abstract Ductile fracture is the result of the nucleation, growth and coalescence of voids. These three stages can be simulated by FEM cell calculations. The nucleation of cavities takes place on second-phase particles. Dislocations interaction or Eshelby model of local stresses in inclusions allow calculation of nucleation conditions. The heterogeneous nature of the distribution of second-phase particles involves establishing nucleation rate functions. Various models (McClintock, Rice and Tracey) give the growth rate of an isolated cavity. Extensions of these models allow taking into account the interaction between cavities and others yield the behaviour of a porous material (Gurson, Rousselier). The coalescence of voids can be predicted thanks to models based on metallurgical observations or on local shear instability (Thomason). The prediction of the fracture strain should consider the heterogeneous distribution of second-phase particles.

The models of cavity nucleation and growth applied in the crack tip region together with analyses of the stress and strain fields allow calculation of the fracture toughness. This leads to simplified models for the prediction of fracture toughness.

4.1 Introduction

Ductile fracture is the most common mechanism of failure in metals at room temperature. This damage mode involves the nucleation of cavities followed by a phase of damage growth and coalescence driven by plastic deformation. This failure mode usually occurs after large plastic strains. However in some high strength alloys, “dirty” metals or metal matrix composites, the process of nucleation, growth and coalescence of cavities can take place very rapidly and lead to very low ductility, sometimes as small as 1%.

Ductile damage occurs more or less evenly in homogeneously deformed regions up to the final fracture point (Fig. 4.1a, b). Only at the very end of the process, when void coalescence takes place and a crack starts propagating in the solids, the

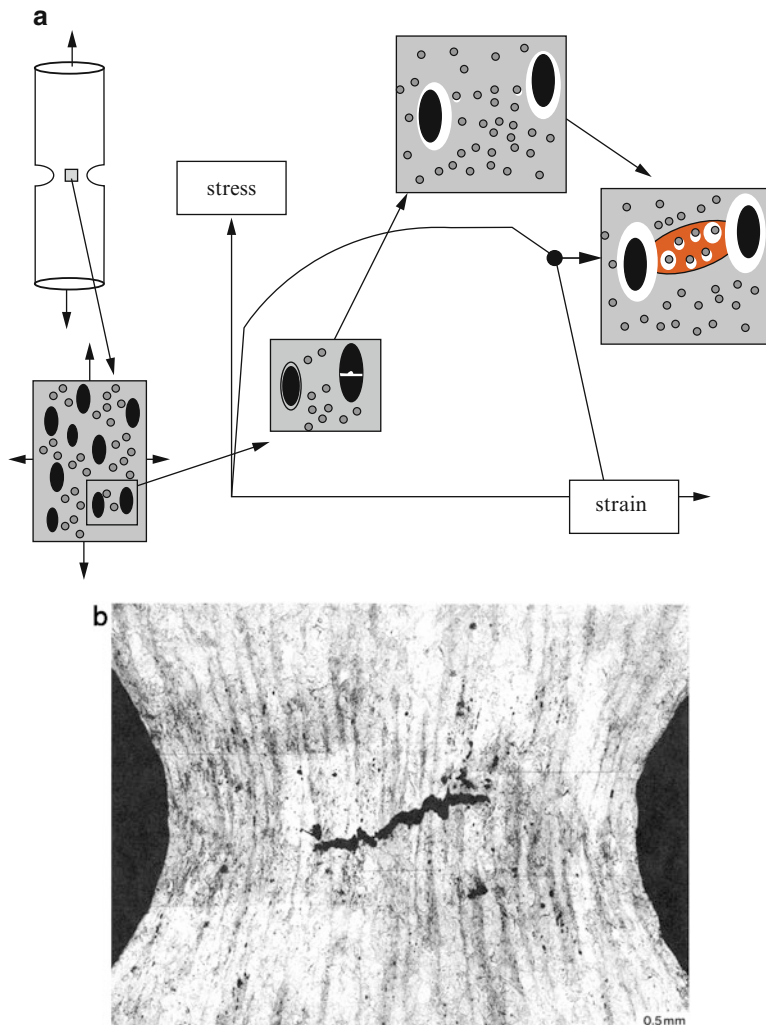


Fig. 4.1 (a) Schematic representation of the processes of nucleation, growth and coalescence of voids nucleated on second-phase particles, and the relationship with the macroscopic mechanical response; (b) Interrupted tensile test just before failure in a specimen of a pressure vessel steel (0.16C, 1.5Mn, 0.75Ni, 0.50Mo) showing the formation of a crack in the centre of a notched specimen (Lautridou 1980)

deformation can become highly heterogeneous (Fig. 4.1b). The coalescence process is accompanied by an abrupt change in the slope of the local stress strain curve.

This chapter has the same objectives as those presented in the introduction to Chap. 3, Sect. 3.1. The importance of cell simulations will be emphasised as many researches have been made in this field over the past two decades. For recent reviews, see *e.g.* Leblond (2003), Benzerga and Leblond (2011), Pineau and Pardoen (2007), and Besson (2004, 2010).

The chapter is organised as follows. The three elementary damage mechanisms of ductile fracture (cavity nucleation, growth and coalescence) are firstly described. Then it is shown how these mechanisms can be used to predict the ductility of volume elements. The last part of this chapter is devoted to the study of fracture toughness.

4.2 Cavity Nucleation

4.2.1 Observations

Void nucleation is usually not detected from the overall mechanical response of materials. In most metals, the initial second-phase content is too small to bring about significant amount of initial porosity: the initial porosity in metals ranges typically between 10^{-5} and 10^{-2} .

Void nucleation is usually associated to the presence of second-phase particles and inclusions, located either within the grains or along the grain boundaries (Puttick 1959; Argon and Im 1975; Argon et al. 1975; Argon 1976; Goods and Brown 1979; Fisher and Gurland 1981; Beremin 1981; Van Stone et al. 1985). These particles break into several fragments when aligned with respect to the main loading axis while interface separation occurs when their long axis is orthogonal to the main loading direction (Lassance et al. 2007; Beremin 1981). Moreover the mechanical response of the matrix has an effect on cavity nucleation. As shown in Fig. 4.2 (Babout et al. 2004a), a soft matrix (pure Al) favors particle decohesion while a

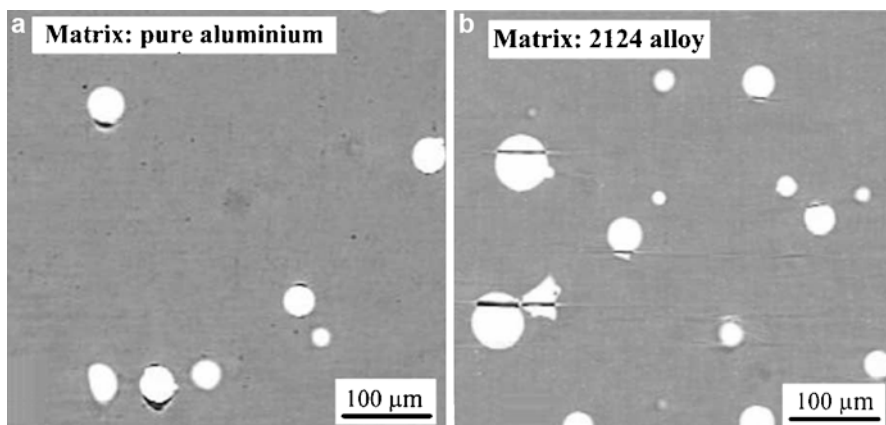


Fig. 4.2 Reconstructed images obtained by X-ray tomography during in-situ tensile test for different metal matrix composites reinforced by 4% volume fraction of $\text{ZrO}_2/\text{SiO}_2$ spherical particles. (a) Pure aluminium matrix deformed by 27.5%; (b) 2124 T6 aluminium matrix deformed by 9% (Babout et al. 2004a)

reinforced matrix (2124 Al alloy) leads to particle cracking. Void nucleation in a perfect lattice by local cleavage or specific dislocation mechanisms is not common. However these micromechanisms of void nucleation were observed in very pure single-phase metals, like Ti alloys (*e.g.* Thompson and Williams 1977).

The nucleation of cavities is usually a discontinuous process made of a succession of discrete nucleation events. Void nucleate first on the largest inclusions, which involves probably the largest internal or interfacial defects. Some materials contain different families of second-phases or inclusions (*e.g.*, manganese sulfides and carbides in steels, iron-base intermetallic compounds and Al_2Cu particles in Al alloys, etc.). One family has limited resistance to void nucleation and the other one, usually of a much smaller size, involves a much better resistance to void nucleation (see *e.g.* Cox and Low 1974; Hahn and Rosenfield 1975; Marini et al. 1985; Haynes and Gangloff 1997; Bron et al. 2004; Asserin-Lebert et al. 2005). The effect of this second population of cavities can, in some circumstances, be a dominant feature of the fracture process.

The inhomogeneity in the particle distributions can cause local stress concentrations and is certainly an important reason for heterogeneous nucleation. For instance, the inhomogeneity in local nucleation rate was thoroughly investigated in duplex stainless steels (Pineau Joly 1991; Devillers-Guerville et al. 1997; Bugat et al. 1999) using interrupted tests, as shown in Fig. 4.3 where clusters of cavities represented by Voronoï cells are clearly observed. The distribution of the cell sizes shows that a very small fraction of the surface in Fig. 4.3 is leading to large local nucleation rates compared to the mean nucleation rate.

The interfacial strength between second-phase particles and the matrix is dependent on the local chemical composition. The segregation of impurity elements similar to those which induce intergranular embrittlement (see Sect. 3.3 in this volume) can reduce the interfacial resistance. Hydrogen-induced ductility losses in steels could also be at least partly explained in this way (Chap. 7, Sect. 7.2.5.1 and Cialone and Asaro 1979; Liang and Sofronis 2003). Impurity segregation at particles interface due to irradiation effects has also been mentioned earlier (Sect. 3.4 in this volume).

Quantifying experimentally the local mechanical condition for void nucleation is a difficult task. Interrupted tests followed by the observation of longitudinal sections of notched specimens have been used by Beremin (1981). By combining the experimental determination of the locus of void nucleation with finite element (FE) calculations of the specimen, it is possible to determine the macroscopic local mechanical conditions for nucleation. The 3D *in situ* tomography study reported in Fig. 4.2 allows a more rigorous determination of the cavity nucleation conditions (Babout et al. 2004a, b; Maire et al. 2005).

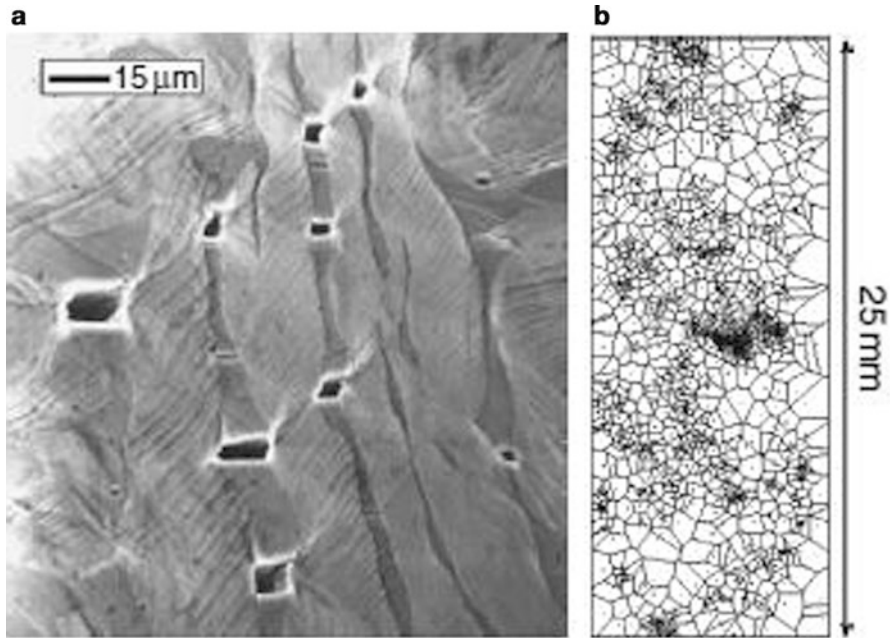


Fig. 4.3 Cast duplex (austenite + ferrite) stainless steel. **(a)** Initiation of cavities produced by the formation of cleavage microcracks in the ferrite phase; **(b)** Voronoï cells illustrating the heterogeneity in the distribution of cavities. This form of damage is only observed when the ferrite phase has been embrittled by thermal ageing (Devillers-Guerville et al. 1997)

4.2.2 Computational Cell Calculations

Lee and Mear (1992, 1999) performed a comprehensive set of calculations on ellipsoidal ($a > b = c$) inclusions embedded in viscous or elastoplastic solids. Stress concentration factors were determined for both interface decohesion (I) and particle cracking (P). Selected results from their studies are reproduced in Figs. 4.4a, b, 4.5, and 4.6. (Lee and Mear 1999) showing the evolution of the stress concentration factors (K^I for the interface stress concentration factor and K^P for the particle stress concentration factor), as a function of the applied remote strain $E_{33} = E_z$ for two different strain-hardening exponents ($N = 5$ and 10) and different Young modulus contrast ($E^P/E = 1, 4$) under uniaxial tension (Figs. 4.4b and 4.5) and under biaxial loading (Fig. 4.6), $\Sigma_{11} = \Sigma_{22} = \Sigma_r$ and $\Sigma_{33} = \Sigma_z$ with $\Sigma_z > \Sigma_r \geq 0$ ($\Sigma_r/\Sigma_z = 0.4$ and 0.7 in Fig. 4.6). These figures show the variations of K^I and that of the ratio K^P/K^I as a function of E_z for different particle aspect ratio, $\lambda = a/b$. These results show that particle fracture is favored when the particle is elongated in the direction of loading. Results obtained for remote triaxial tension indicate that the stress

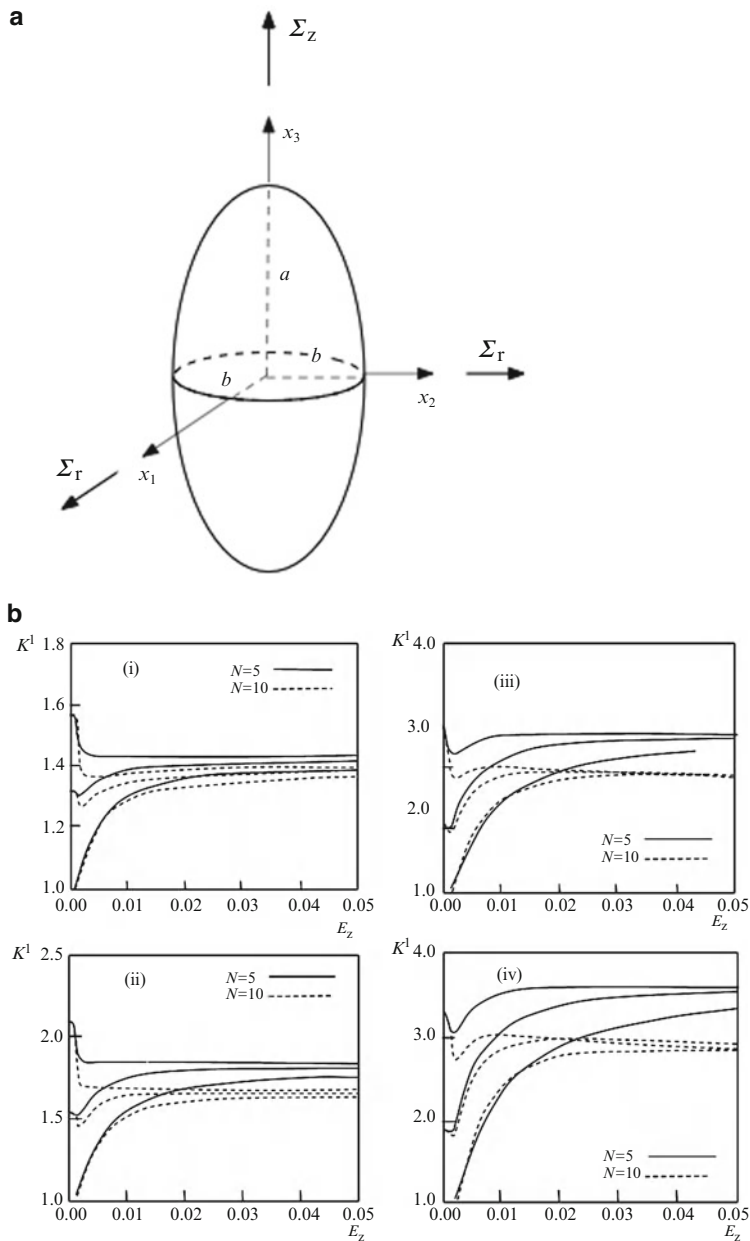


Fig. 4.4 Stress concentration factors. (a) Schematic diagram showing orientation of prolate spheroidal particle. Length of major axis is a and length of minor semi-axes is b . Remote axisymmetric stress components $\Sigma_{11} = \Sigma_{22} = \Sigma_r$ and $\Sigma_{33} = \Sigma_z$ ($\Sigma_z > \Sigma_r > 0$); (b) Stress concentration factors at particle-matrix interface K^I as a function of the remote strain, E_z . Results shown are for two values of n (0.10 and 0.20) ($n = 1/N$) and particle aspect ratio (i) $\lambda = 1$; (ii) $\lambda = 2$; (iii) $\lambda = 5$; (iv) $\lambda = 7$. The modulus contrast E^p/E is equal to 1 for the lower pair of curves, equal to 2 for the middle pair and equal to 4 for the upper pair (Lee and Mear 1999)

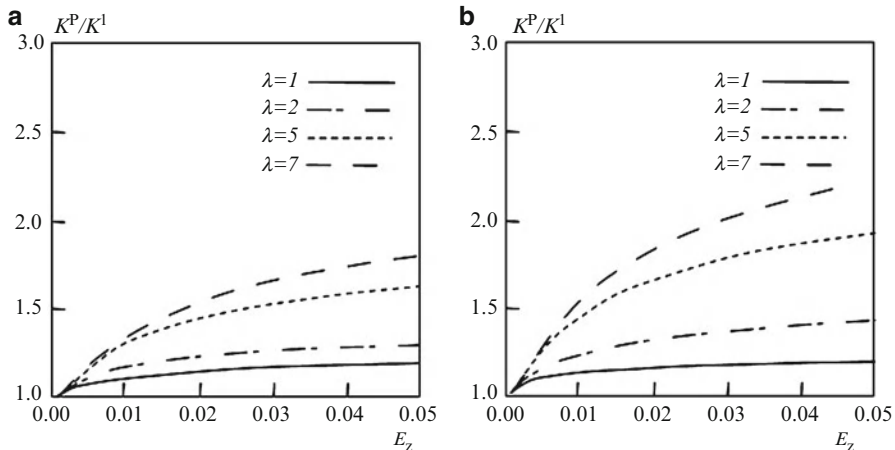


Fig. 4.5 Ratio of the stress concentration factors K^P/K^I for remote uniaxial tension as a function of the remote plastic strain, E_z for different particle aspect ratio $\lambda = a/b$; (a) $n = 0.20$; (b) $n = 0.10$. ($n = 1/N$). In both cases $E^P/E = 2$ (Lee and Mear 1999)

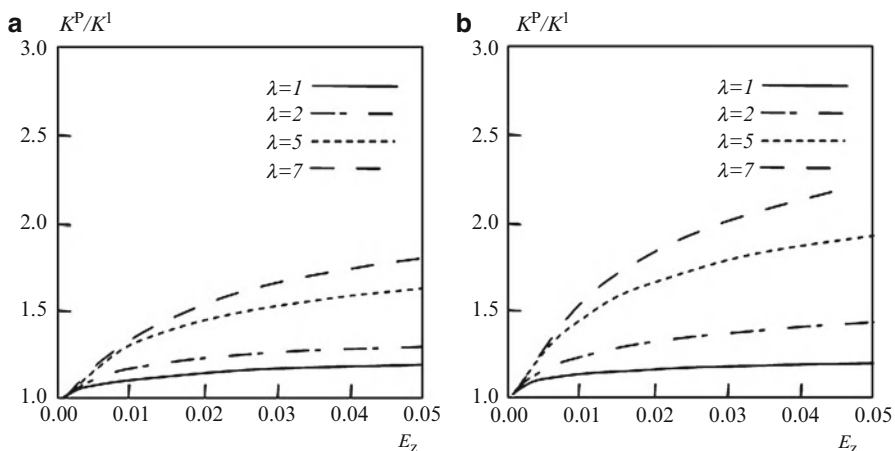


Fig. 4.6 Ratio K^P/K^I for remote triaxiality $\Sigma_r/\Sigma_z = 0.40$ (a) and $\Sigma_r/\Sigma_z = 0.70$ (b) and several particle aspect ratios $\lambda = a/b$ (1, 2, 5, 7) as a function of the remote plastic strain E_z . Modulus contrast is $E^P/E = 2$ and hardening exponent is $n = 0.10$ ($n = 1/N$) (Lee and Mear 1999)

triaxiality plays an important role in the transition from interfacial failure to particle cracking. As the stress triaxiality is increased, the ratio K^P/K^I decreases. This effect favors the initiation of interfacial failure. As the triaxiality is increased there is a larger range of particle aspect ratios for which interfacial debonding occurs in preference to particle fracture.

4.2.3 Void Nucleation Models

4.2.3.1 Nucleation from a Single Particle

Various void nucleation criteria were proposed. They are based either on dislocation theory for crystalline materials (*e.g.* Goods and Brown 1979) or on pure continuum mechanics, *e.g.* based on Eshelby (1957) theory to evaluate the load transfer between the matrix and the particle. A review of these different criteria has been made by Berdin (2004). We limit the presentation to general aspects and to one specific void nucleation criterion.

If the particle is brittle and deforms elastically, a simple one-parameter condition can be derived from linear elastic fracture mechanics arguments (see *e.g.* Kroon and Faleskog 2005). In this approach it is assumed that second-phase particles always contain tiny submicron defects. In very small and clean defect-free brittle particles, a critical stress condition based on the theoretical strength of the material is also valid.

In the case of interface fracture, a one-parameter linear elastic fracture mechanics approach is, in principle, not relevant if the matrix surrounding the particle is plastically deforming. Both the separation energy and interface strength play a role in the problem. In many instances, the energy condition can be met while enough plastic deformation must still be accumulated at the interface to raise the stress above the critical strength level. This is why both critical-strain-based models (*e.g.* Walsh et al. 1989; Pineau and Joly 1991; Bugat et al. 1999) were used as well as critical-stress-based models (*e.g.*, see Kwon and Asaro 1990). The formulation of an adequate condition for void nucleation by interface fracture is a difficult problem. This explains why many researchers attempted to solve this problem through “phenomenological” models (see Chu and Needleman 1980; Tvergaard 1990).

One relatively advanced void nucleation model has already been introduced in Chap. 3, Sect. 3.5.2.1 (Beremin 1981). Based on the Eshelby theory (Eshelby 1957) and the “secant modulus” extension to plastically deforming matrix proposed by Berveiller and Zaoui (1979), the maximum principal stress in an elastic inclusion and at the interface σ_1^{part} can be related to the overall stress state:

$$\sigma_1^{\text{part}} = \Sigma_1^{\text{max}} + k_s (\Sigma_{\text{eq}} - R_p) \quad (4.1)$$

where Σ_1^{max} is the maximum overall principal stress, Σ_{eq} the equivalent stress, R_p the yield strength and k_s is a parameter of the order unity which is function of the inclusion aspect ratio λ and of the loading direction (see Table 4.1).

Typical mean values for particle fracture stress or interfacial fracture stress are reported in Table 4.2. As explained above, the critical stress for interfacial fracture

Table 4.1 Shape factor k_s for calculating stresses in inclusions (Beremin 1981)

Loading direction	$a = b = c$ Sphere	$a \gg b = c$ Ellipsoid	Particular case	$a = b \gg c$ Disk	Particular case
		$\lambda = c/a$ $L = (1/\lambda)\log(2 - \lambda)$ $\alpha = (3/2)\lambda^2(2L - 1)$	$\lambda = 0.17$	$\lambda = c/a$	$\lambda = 0.17$
Longitudinal	1	$(1/3)(\lambda^2/\alpha + 2/\alpha - 2)$	3.2	$20/9\pi\lambda - 2/3$	3.3
Transverse	1	$(1/12)(\lambda^2/\alpha + 2/\alpha + 6)$	1.5	$8/9\pi\lambda - 2/3$	0.9

Table 4.2 Void nucleation stress reported in the literature for particle fracture or particle decohesion mechanisms

Particles	Matrix	Critical stress (MPa)	Reference
<i>Particle fracture</i>			
Elongated MnS	A508 steel	1,100	Beremin (1981)
Cuboidal TiN	4330 steel	2,300	Shabrov et al. (2004)
TiN	Inconel 718	1,280–1,540	Alexandre et al. (2005)
4% Spherical ZrO ₂ -SiO ₂ (ZS) balls	Al2124 (T6)	700	Babout et al. (2004b)
20% Spherical ZrO ₂ -SiO ₂ (ZS) balls	Al2124 (T4)	700	Babout et al. (2004b)
<i>Interface fracture</i>			
MnS	A508 steel	800	Beremin (1981)
Si	Al (cast)	550	Hubert et al. (2005)
4% Spherical ZrO ₂ -SiO ₂ (ZS) balls	Al2124 (T6)	1,060	Babout et al. (2004b)
4% Spherical ZrO ₂ -SiO ₂ (ZS) balls	Pure Al	250	Babout et al. (2004b)
20% Spherical ZrO ₂ -SiO ₂ (ZS) balls	Pure Al	320	Babout et al. (2004b)
Rounded Fe ₃ C	Spheroidised 1045 steel	1,650	Argon and Im (1975)
Cu-Cr particles	Cu alloy	1,000	Argon and Im (1975)
TiC	Maraging steel	1,820	Argon and Im (1975)
C nodules	Cast iron	80	Dong et al. (1997)

is not intrinsic to the particle but depends on the matrix (Babout et al. 2004b). Note that there are many important informations missing in this table, such as the mean particle size and particle size distribution, the flow properties of each phase. These informations are necessary for the comparison between the various systems.

The model used to derive Eq. 4.1 remains qualitative with respect to more accurate finite element calculations. These calculations could be used to provide closer estimate of the load transfer between the matrix and the particles.

4.2.3.2 Void Nucleation Rate Function

For all the reasons discussed previously, the heterogeneity of the nucleation process must be introduced through a distribution function. Following the model proposed by Needleman, Tvergaard and co-workers (see *e.g.* the review by Tvergaard 1990) a general expression for the void nucleation rate, based on Eq. 4.1, can be written as:

$$\dot{f}_{\text{nucI}} = A_N (\dot{\Sigma}_1^{\text{max}} + k_s \dot{\Sigma}_{\text{eq}}) \quad (4.2)$$

where

$$A_N = \frac{f_0}{s_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\Sigma_1^{\text{max}} + k_s (\Sigma_{\text{eq}} - R_p) - \langle \sigma_d \rangle}{s_N} \right)^2 \right] \quad (4.3a)$$

In Eq. 4.3a, f_0 is the initial volume fraction, $\langle \sigma_d \rangle$ is the average void nucleation stress, and s_N is the standard deviation.

Strain controlled nucleation rate laws are frequently preferred because they are simpler to implement numerically. The nucleation rate is then expressed by:

$$\dot{f}_{\text{nucI}} = A_N \dot{P} \quad (4.3b)$$

where A_N can be any positive function of the state variables such as the cumulated plastic strain, P , but also f . Following Chu and Needleman (1980), many authors have used a Gaussian expression for A_N :

$$A_N = \frac{f_0}{s_N \sqrt{2\pi}} \exp \left[-\frac{1}{2} \frac{(P - E_N)^2}{s_N^2} \right] \quad (4.4)$$

This nucleation law introduces three parameters: (i) the initial volume fraction of inclusions f_0 , (ii) E_N the strain for which 50% of the inclusions are fractured, s_N the standard deviation on the nucleation strain. In their numerical study, Chu and Needleman (1980) took $E_N = 0.30$ and $s_N = 0.10$. Many authors adopted these values without any microstructural justification although we have already indicated that the nucleation rate depends on the type of material, its chemical composition and heat treatments, etc.

Another possible form for A_N is to use a constant rate of nucleation between a minimum strain (P_i , strain at which nucleation starts) and a maximum strain (P_f , strain at which nucleation stops). The rate is then given by (Zhang et al. 2000; Besson 2010):

$$A_N = \frac{f_0}{P_f - P_i} \quad (4.5)$$

The two proposed forms for the nucleation rate law are illustrated in Fig. 4.7 (Besson 2004).

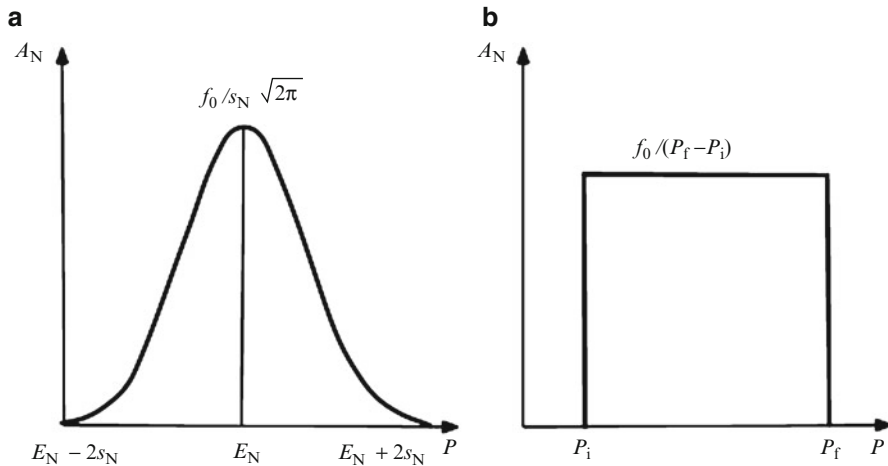


Fig. 4.7 Examples of nucleation laws: (a) Chu and Needleman (1980); (b) constant rate (From Besson 2004)

In these expressions the initial void volume fraction is equal to the volume fraction of particles generating cavities if the nucleation mechanism occurs by complete interface decohesion. For the case of partial decohesion or particle cracking, the initial void shape is very flat. It is assumed that the mechanical effect of this flat void is equivalent to that of a void having the same projected area. This assumption is supported by the analysis of penny-shape cracks in a plastic material by Gologanu et al. (1994), in particular at high stress triaxiality ratios.

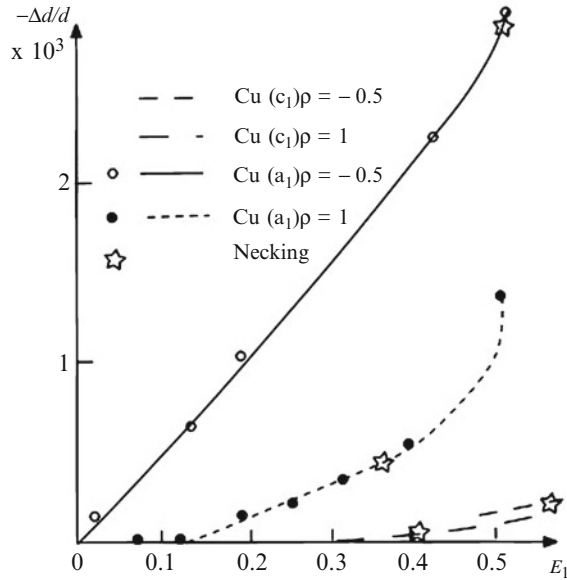
Considering the complexity involved in the chemistry, physics and mechanics of real second-phase particles embedded in a given matrix, the microstructural observations of the nucleation rate is probably the unique way of estimating model parameters. The advent of 3-D X-ray tomography should largely contribute to a better knowledge of this first stage of ductile fracture, which still remains very poorly known.

4.3 Cavity Growth

4.3.1 Observations

The macroscopic effect induced by the growth of voids in a plastically deforming matrix is relatively weak. The presence of voids can be measured macroscopically from density measurements or from other indirect methods (see *e.g.*, Montheillet and Moussy 1986; Schmitt 1981). Figure 4.8 shows a typical evolution of the density observed in samples of copper pulled in uniaxial tension or in biaxial tension. These

Fig. 4.8 Variation of density $\Delta d/d$ as a function of the principal strain E_1 in pure copper (Cu/c_1) and in an industrial grade of copper (Cu/a_1) deformed either in uniaxial tension ($\rho = E_2/E_1 = -0.5$) or under equibiaxial expansion ($\rho = E_2/E_1 = 1$) (Schmitt 1981; Montheillet and Moussy 1986)



results clearly show that, at least for moderate overall strains, the density $\Delta d/d$ changes linearly with strain, increases when the volume fraction of oxides increased (deoxidised Cu, c_1 versus industrial Cu, a_1), and increases with stress triaxiality (Schmitt 1981).

The most important macroscopic observation about ductile fracture is that the fracture strain decreases exponentially with increasing stress triaxiality, as first reported by Hancock and MacKenzie (1976), followed by many authors (for a review, see Pineau and Pardoen 2007).

This effect is directly related to a significant increase of the void growth rate with increasing stress triaxiality:

$$T = \frac{\Sigma_{kk}}{3\Sigma_{eq}} = \frac{\Sigma_m}{\Sigma_{eq}} \quad (4.6)$$

where Σ_m is the hydrostatic stress. Figure 4.9 shows the schematic evolution of the fracture strain as a function of the stress triaxiality averaged along the deformation path. The stress triaxiality takes the following values:

- 0 under pure shear
- $1/3$ under tension. Note that $T = 1/3$ in a single tension test up to the onset of necking and then steadily increases with increasing plastic deformation
- $\sqrt{3}/3$ under plane strain tension
- 0.6–1.8 in the centre of the minimum cross-section of a notched round bar
- 2.75–5 inside the fracture process zone in front of a crack tip
- ∞ under purely hydrostatic stress ($\Sigma_{eq} = 0$)

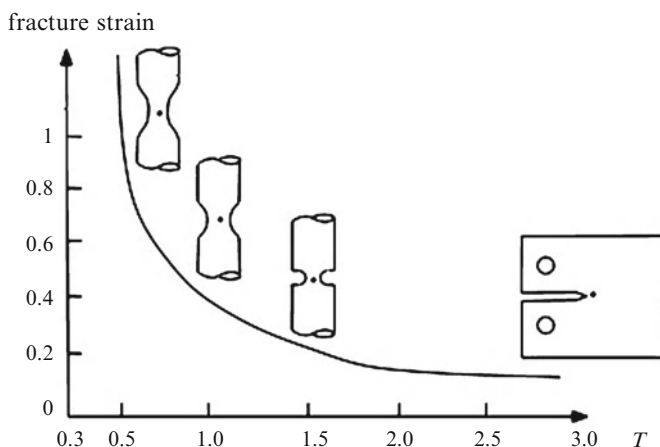


Fig. 4.9 Typical variation of the fracture strain as a function of the stress triaxiality ratio in metallic materials (Pineau and Pardoen 2007)

Usually, ductile fracture in industrial metallic alloys takes place only for T larger than 0.3–0.5. This is why most forming operations are conducted under compressive loadings, at least in the heavily plastically deformed regions of materials.

In some materials the strain to failure does not decrease monotonically with the stress triaxiality as shown in Fig. 4.9. In these materials the strain to failure under pure shear can be lower than that observed in uniaxial tension, as illustrated in Fig. 4.10 (Bao and Wierzbicki 2005). The failure mode changes from cavitation controlled ($T > 0$) to shear controlled ($-1/3 < T < 0.4$), as indicated in this figure.

Microscopic observations on void growth have been made by a number of authors using either *in situ* testing inside scanning electron microscopes or interrupted examinations on longitudinal sections of specimens. Voids nucleated by particle cracking open and become more rounded with plastic deformation. Voids nucleated by particle decohesion are initially rounded and tend to elongate in the principal loading direction, as schematically depicted in Fig. 4.1. In-situ 3D tomography experiments provided more complete information about the 3D morphology of the voids (Babout et al. 2001, 2004a, b; Maire et al. 2005). An example of a 3D reconstructed image obtained on an aluminium alloy containing ceramic particles is shown in Fig. 4.11.

A very limited number of experimental studies were devoted to the coupling between void growth and crystal orientation effects (see *e.g.* Crépin et al. 1996; Gan et al. 2006).

Experimental results on notched specimens with various triaxialities were performed to study the effect of stress triaxiality on void growth, dR/R , where R is the radius of spheroidised voids. These results (Marini et al. 1985; Beremin 1981; Bourcier et al. 1986; Barnby et al. 1984) reported in Fig. 4.12 are compared with those obtained by FE simulations or with the Rice and Tracey law (see below)

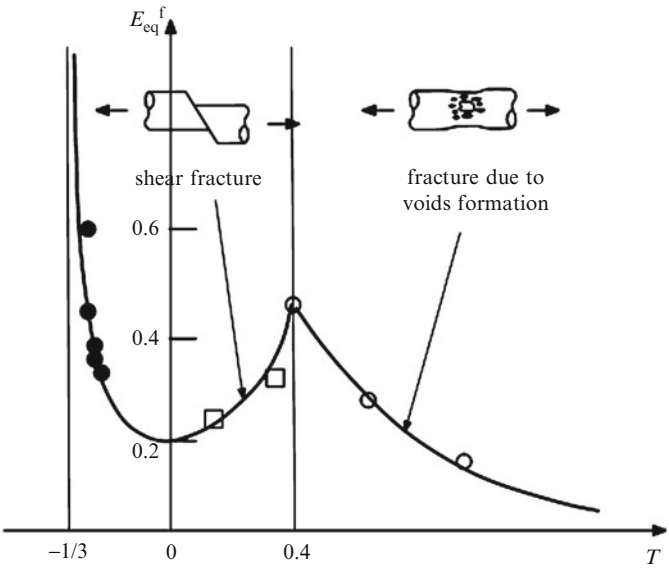
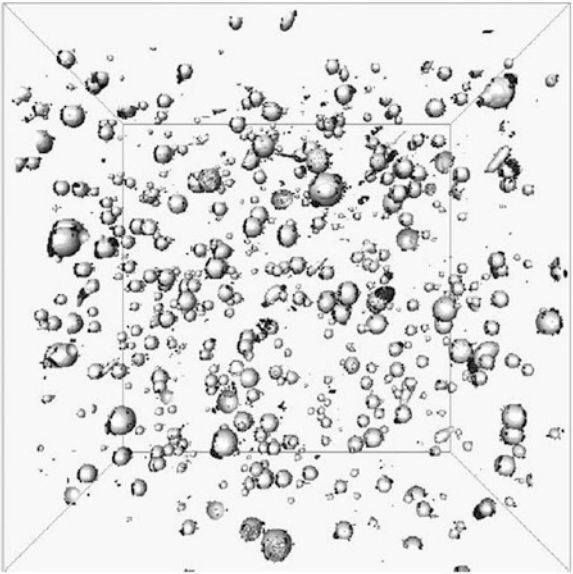


Fig. 4.10 Variation of the equivalent strain to crack formation with stress triaxiality in an aluminium alloy 2024-T351 (Bao and Wierzbicki 2005)

Fig. 4.11 3-D X-ray tomography of an aluminium matrix containing spherical ceramic particles (of a size between 20 and 60 μm) after tensile loading. The voxels in the aluminium matrix have been removed. The voids are highlighted in *black* and the particles in *white* (Maire et al. 2005)



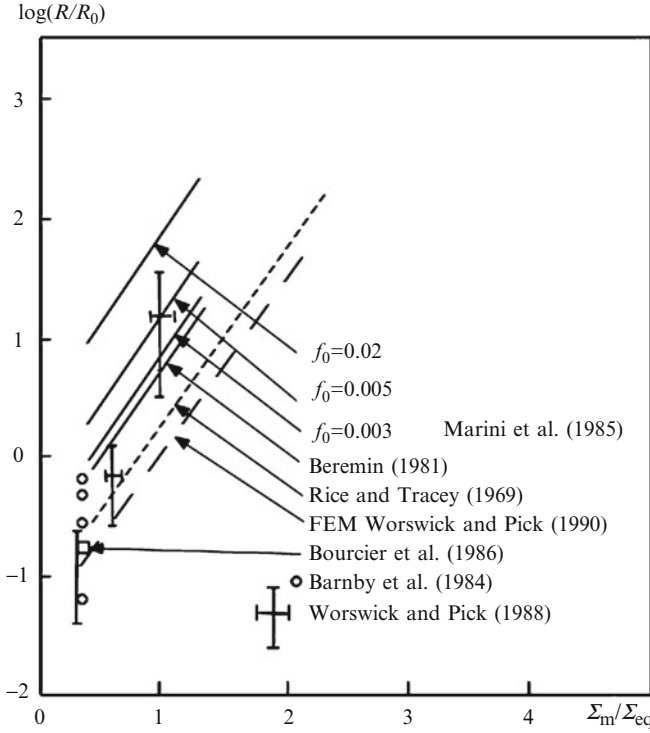


Fig. 4.12 Cavity growth $\log(R/R_0)$ as a function of stress triaxiality T (Worswick and Pick 1990)

(Worswick and Pick 1990). These results show an exponential dependence of void growth with stress triaxiality, with a slope close to $3/2$, and an acceleration in void growth when the initial volume fraction of cavities increases. Similar results are given in Fig. 4.13 where it is observed that the cavity growth rate $(1/E_{eq}) dR/R$ increases significantly with increasing initial volume fraction of second-phase particles. All these results indicate that interactions between cavities must also be taken into account.

4.3.2 Void Cell Simulations

The principle of a void cell simulation is to idealise the microstructure by considering simple arrangements of voids and to use the FE method with proper boundary conditions. The simplest arrangement is that of a periodic distribution of voids. A large number of void cell simulations were reported in the literature since the early studies by Needleman (1972a, b), Tvergaard (1981, 1982, 1990), and Koplik and Needleman (1988). For a review, see *e.g.* Pineau and Pardoen (2007).

Fig. 4.13 Rate of growth of cavities ($1/E_{eq} \times dR/R$) as a function of the volume fraction of initiating inclusions

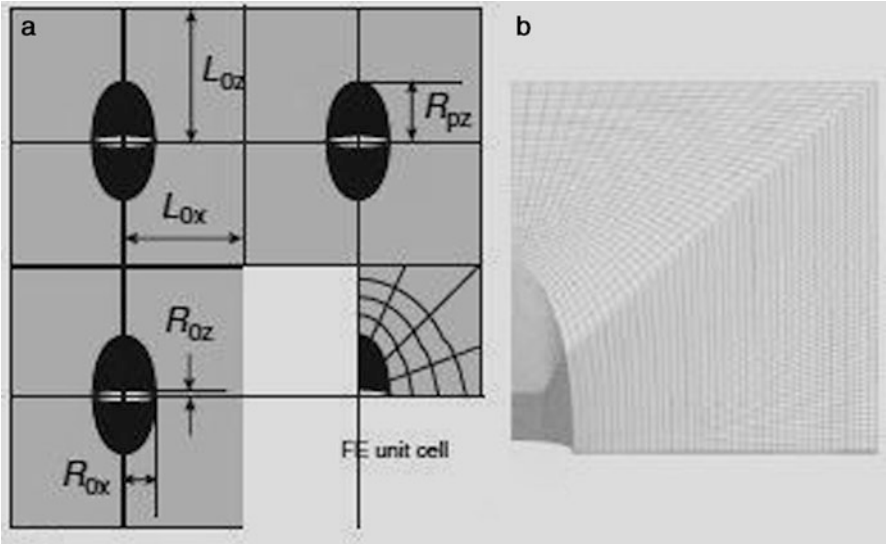
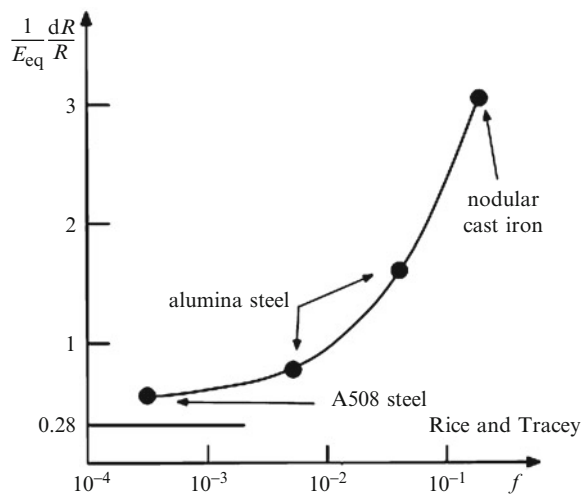


Fig. 4.14 (a) Principle of void cell simulation (and notations) based on cylindrical geometry subjected to axisymmetric loading conditions and involving a spheroidal void and a particle; (b) finite element mesh (see also Pineau and Pardoen 2007)

The key information that can be obtained from a simple cell made of a cylinder containing a spheroidal void and loaded axisymmetrically (Fig. 4.14) is presented in the following. The calculations are ideally performed at constant imposed stress triaxiality. Figure 4.15 shows the overall effective stress – effective strain curves predicted by void cell calculations performed at constant stress triaxiality $T = 1$ for

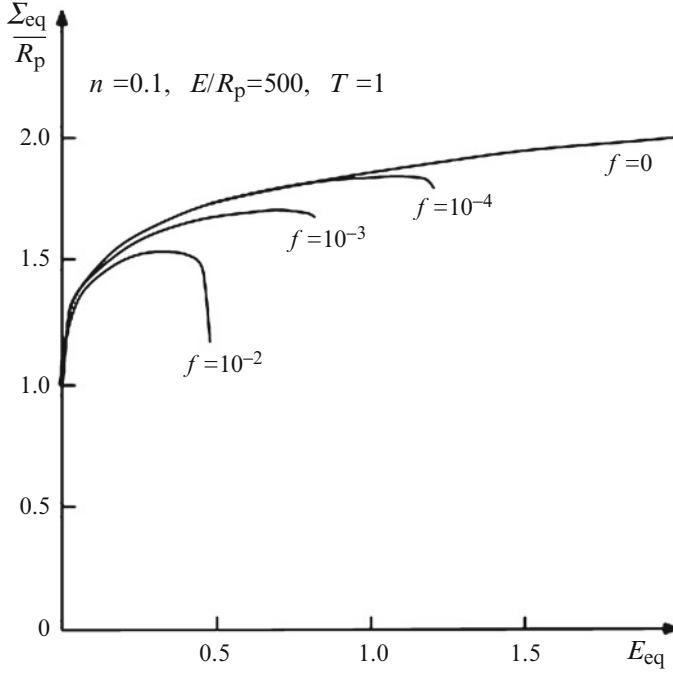


Fig. 4.15 Variation of the overall effective stress as a function of the overall effective strain predicted by a 2-D axisymmetric unit cell simulation with an initially spherical void for three different initial void volume fractions f . Comparison with the mechanical response of the nonporous material (Pardoen and Hutchinson 2000)

three initial porosities: 10^{-2} , 10^{-3} and 10^{-4} . The voids are initially spherical and the void distribution parameter, noted Λ_0 and defined as $\Lambda_0 = L_{0x}/L_{0z}$ is equal to 1. A J_2 isotropic elastic-plastic response is assumed for the matrix, characterised by the following expressions in uniaxial tension:

$$\frac{\sigma}{R_p} = \frac{E}{R_p} \varepsilon \quad \text{for } \sigma < R_p \quad (4.7a)$$

$$\frac{\sigma}{R_p} = \left(1 + \frac{E}{R_p} \varepsilon_p \right)^n \quad \text{for } \sigma > R_p \quad (4.7b)$$

where E is the Young's modulus, R_p is the initial yield stress, and n is the work-hardening exponent. Figure 4.15 shows that the softening effect due to the presence of voids appears only close to the onset of void coalescence. Until that point, the effect of porosity on the stress-strain curve is small for porosity smaller than 10^{-2} .

The various effects of the stress triaxiality, T , void aspect ratio measured by $\lambda_0 = R_z/R_x$ (Fig. 4.14), initial relative void spacing $\chi_0 = R_{ox}/L_{ox}$ and work-hardening exponent, n , are summarised in Fig. 4.16. Figure 4.16a shows the axial

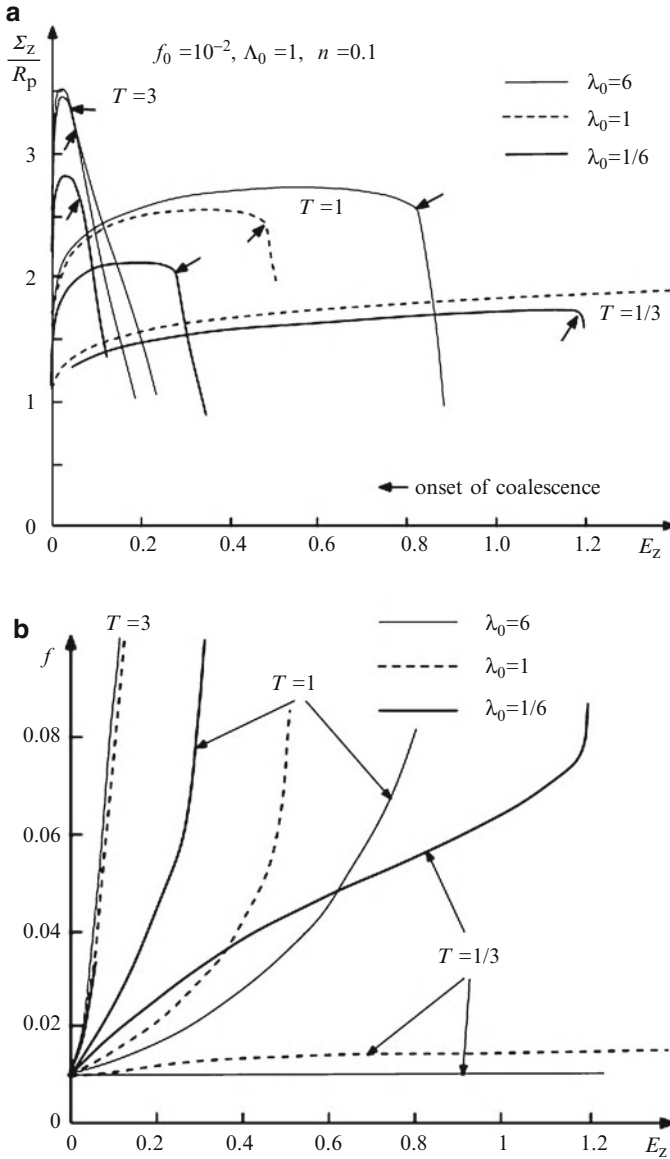


Fig. 4.16 Void cell results for $f_0 = 10^{-2}$, $R_p/E = 0.002$, $n = 0.10$ and $\lambda_0 = 1/6$ at $T = 1/3, 1, 3$. (a) axial stress versus axial strain; (b) porosity versus axial strain; (c) variation of the void volume fraction as a function of the overall strain-hardening exponents $n = 0.10$ and 0.30 under a stress triaxiality $T = 1$; (d) void cell results showing the variation of the radial strain, E_r , as a function of the axial strain, E_z (Pardoen and Hutchinson 2000)

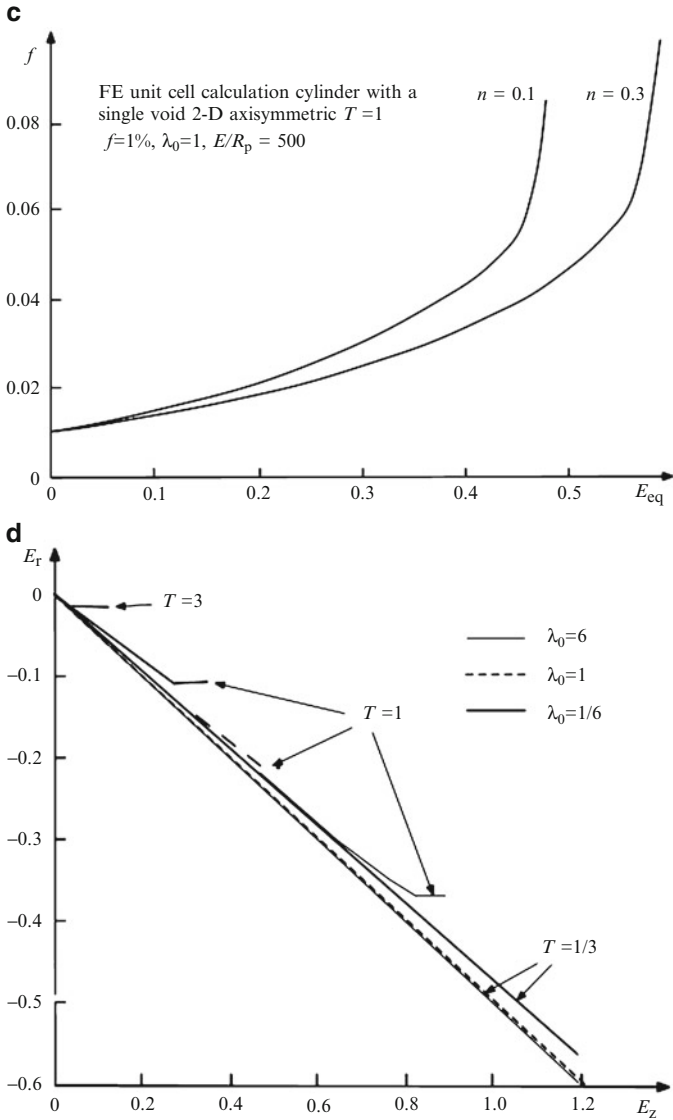
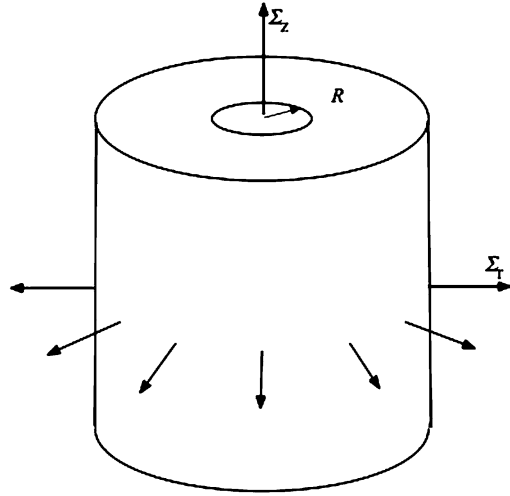


Fig. 4.16 (continued)

stress versus axial strain curves corresponding to different stress triaxialities equal to $1/3$, 1 and 3 . The fracture strain significantly drops with increasing stress triaxiality. The drop in fracture strain is directly related to an increasing rate of porosity growth with increasing stress triaxiality, as shown in Fig. 4.16b.

Figure 4.16c shows that the effect of the strain-hardening exponent on the void growth rate is not very significant. It should be noted, however, that the

Fig. 4.17 Schematic diagram of a cylindrical void of radius R



strain-hardening exponent will have an indirect effect on the damage accumulation as the prime parameter controlling plastic localisation, and on the strain distribution and stress triaxiality inside the necking zone.

Figure 4.16d shows the variation of the radial strain, E_r , as a function of the axial strain, E_z , predicted from void cell simulations performed with three different initial void aspect ratios and under three different applied stress triaxialities. This figure indicates that the transition to an uniaxial straining mode ($E_r = \text{constant}$) is almost instantaneous (see also Koplik and Needleman 1988). This transition constitutes a direct indicator of localisation and is used for quantifying the strain at the onset of void coalescence. The onset of coalescence can also be detected by an abrupt change in the overall stress-strain response (see Fig. 4.16a).

4.3.3 Void Growth Models

4.3.3.1 Isolated Cavity

A model for isolated cylindrical cavity was proposed by McClintock (1968; McClintock and Argon 1966) (Fig. 4.17).

Cavity growth results from plastic deformation of the matrix subjected to a tensile stress Σ_z and a lateral stress Σ_r . The increment of the volume v of a cylindrical cavity of radius R in a perfectly plastic material is shown to be expressed as:

$$\frac{1}{dE_z} \frac{dv}{v} = \sqrt{3} \sinh \left(\sqrt{3} \frac{\Sigma_r}{\Sigma_z - \Sigma_r} \right) \quad (4.8a)$$

where E_z is the overall strain in the Oz direction; it is also the equivalent strain. Note also that $\Sigma_z - \Sigma_r$ is the equivalent stress. As dv/v is equal to $2dR/R + dE_z$, Eq. 4.8a allows to calculate the growth of the radius of the cavity:

$$\frac{dR}{R} = dE_z \left[\frac{\sqrt{3}}{2} \sinh \left(\frac{\sqrt{3} \Sigma_r}{\Sigma_z - \Sigma_r} \right) - \frac{1}{2} \right] = dE_z \left[\frac{\sqrt{3}}{2} \sinh \left(\frac{\sqrt{3} \Sigma_m}{\Sigma_{eq}} - \frac{1}{3} \right) - \frac{1}{2} \right] \quad (4.8b)$$

The same calculation for a viscous material, with a Norton law $\sigma = \sigma_0 \dot{\epsilon}^m$, gives a result, which represents also the behaviour of a void in a plastic material with work-hardening exponent $n = m$:

$$\frac{\Sigma_r}{\Sigma_z - \Sigma_r} = \frac{1}{\sqrt{3}} \int_0^\omega (1 + x^2)^{(m-1)/2} dx \quad (4.9)$$

with $\omega = \frac{1}{\sqrt{3} \dot{E}_z} \frac{\dot{v}}{v}$.

Examination of this equation shows that the growth rate is an increasing function of the stress triaxiality ratio. It is the slower, the higher the Norton exponent m .

McClintock (1968) also developed the solution for the growth of cavities with an elliptical section on the assumption of cylindrical holes parallel to the z axis subject to generalised plane strain. The predictions of these models were partly verified using plasticine as a model material.

The Rice and Tracey model (RT model) (Rice and Tracey 1969) evaluates the growth of an initially spherical void in an infinite, rigid, perfectly plastic material subjected to a uniform remote strain field. Using well-chosen velocity fields, the RT variational analysis leads, for the assumption of spherical void growth, to the following expression for the average rate of growth:

$$\frac{dR}{R} = \frac{1}{3} \frac{df}{f} = \alpha \exp \left(\frac{3}{2} T \right) dE_{eq} \quad (4.10)$$

where f is the volume fraction of cavities and α is a constant. A value $\alpha = 0.283$ was initially computed by Rice and Tracey (1969) and re-evaluated by Huang (1991) using additional velocity fields. Huang obtained similar results but with:

$$\begin{aligned} a &= 0.427 & \text{for} & & T > 1 \\ a &= 0.427 T^{1/4} & \text{for} & & 1/3 \leq T \leq 1 \end{aligned}$$

In practice the porosity is finite and this explains why some authors (Marini et al. 1985) found higher values for α when calibrating the model using experimental measurements. It should be added that for high degrees of triaxiality McClintock and RT solutions are very close to each other.

The general analysis of Rice and Tracey took also into account the change of void shape, assuming ellipsoidal voids and plastic flow conditions in which the directions of the principal axes of the strain rates remain fixed along the strain path. The rates of change of the radii of the void, in the principal directions k , are (Rice and Tracey 1969; Thomason 1990):

$$\frac{dR_k}{R} = \frac{dR}{R} + (1 + E_v) dE_k^p \quad (4.11)$$

where E_v is a void shape parameter. Pardoen and Delannay (1998a) proposed that E_v can be written as (see also Worswick and Pick 1990):

$$E_v = \frac{(1.95 - 0.725T) \exp[(3/2) E_{eq}]}{2 \exp[(3/2) E_{eq}] - 1} \quad (4.12)$$

Budiansky et al. (1982) generalised the RT model to deal with a Norton-Hoff viscosity law $\sigma = \sigma_0 \dot{\epsilon}^m$, which applies equally to a work-hardenable material. Their result is:

$$\begin{aligned} \frac{dR}{R} = \frac{1}{2} < \text{sign} \Sigma_m > dE_{eq} \left\{ \frac{3}{2} m \frac{|\Sigma_m|}{\Sigma_{eq}} \right. \\ \left. + (1 - m) [1 + m (0.4175 + < \text{sign} \Sigma_m > 0.0144\chi)] \right\}^{1/m} \end{aligned} \quad (4.13)$$

where $\chi = +1$ if the axial stress is higher than the radial stress, and $\chi = -1$, when it is the opposite.

For a Newtonian material ($m = 1$) the exact result is:

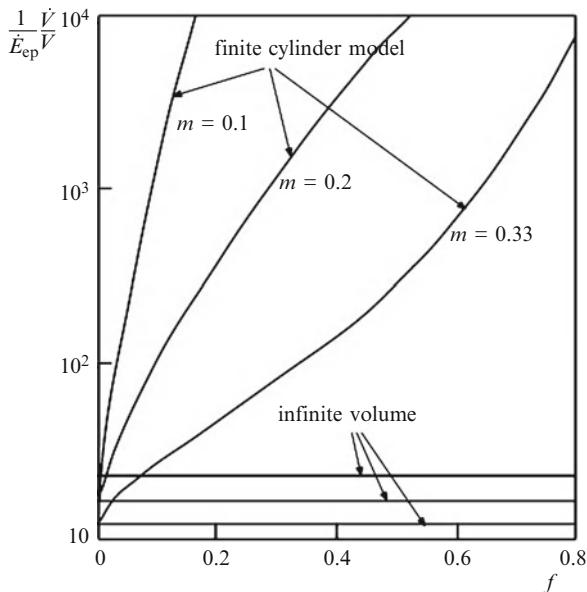
$$\frac{dR}{R} = \frac{3}{4} \frac{\Sigma_m}{\Sigma_{eq}} dE_{eq} \quad (4.14)$$

This model shows that the rate of cavity growth decreases with increasing work-hardening exponent n , the analogue of m .

4.3.3.2 Interaction Between Cavities

The models we have been discussing all concern isolated voids, not interacting with their neighbours. Michel and Suquet (1992) have investigated the growth rate of a spherical cavity of volume v enclosed in a sphere. This cavity, which occupies a

Fig. 4.18 Effect of porosity f on the growth rate of circular cylindrical cavities, axisymmetrically loaded, in a viscoplastic material for various values of m exponent ($\sigma = \sigma_0 \dot{\epsilon}^m$)



volume fraction f , is subjected to a positive hydrostatic stress. The matrix obeys a Norton-Hoff law $\sigma = \sigma_0 \dot{\epsilon}^m$. The exact solution for the growth rate is:

$$\frac{\dot{v}}{v} = \frac{3}{2} \left[\frac{3}{2} m \frac{\Sigma_m}{\sigma_0} (1 + f^m) \right]^{1/m} \quad (4.15)$$

This solution bears some similitude with the Budiansky solution (Eq. 4.14), in particular when m is close to 1. The accelerating factor in Eq. 4.15, in the region of $f \simeq 0$, takes the value $1 + \frac{1}{m} f^m$. Equation 4.15 shows that for small values of m the growth rate increases very rapidly with f : if, for example, $m = 1/3$, a volume fraction of 10^{-3} increases the growth rate by 30%. Thus it is very important to take porosity into account, as is confirmed by experimental observations (Marini et al. 1985). Figure 4.18 gives values of the growth rate of cavities calculated by Licht and Suquet (1988) for cylindrical cavities in a viscoplastic material, a problem analogous to that of McClintock. These rates, normalised by dividing by the equivalent strain rate, show clearly the rapid increase with porosity.

4.3.3.3 Behaviour of Porous Materials: Gurson Model and Extensions – Other Models

Gurson Model

The Gurson model (Gurson 1977) is the first micromechanical model for ductile fracture which introduces a strong coupling effect between deformation and

damage. The model is representative of the void growth stage only. It is derived from an analysis similar to the model of void growth by Rice and Tracey (1969). The initial model is based on a simplified representation of the voided material, which consists in a hollow sphere. The only dimensional microstructural feature in the model is the void volume fraction or porosity f . The material of the hollow sphere (*i.e.* the matrix) is assumed to be rigid, isotropic, and perfectly plastic with a yield strength R_p . The matrix obeys a standard von Mises yield criterion and associated flow rule. The main result of this model is an estimate of the yield function for the porous material which, applying the normality rule, can be used to derive the plastic flow direction. The yield surface is given by the following equation:

$$\phi = \left(\frac{\Sigma_{eq}}{R_p} \right)^2 + 2f \cosh \left(\frac{3}{2} \frac{\Sigma_m}{R_p} \right) - 1 - f^2 = 0 \quad (4.16)$$

This yield surface is identical to the von Mises (or J_2) yield surface when $f = 0$. It is very similar to the von Mises yield surface for low hydrostatic stresses ($\Sigma_m/R_p \ll 1$). The main difference is that, owing to the presence of the voids, plastic yielding takes place under a large enough hydrostatic stress, whatever the value of the deviatoric stress, while a von Mises material is incompressible and does not yield plastically under pure hydrostatic stress (Fig. 4.19). For pure hydrostatic stress Eq. 4.16 reduces to the exact solution:

$$\frac{\Sigma_m}{R_p} = \frac{2}{3} \log \left(\frac{1}{f} \right) \quad (4.17)$$

The softening effect of the growth of cavities is clearly observed in Eq. 4.16 and is illustrated in Fig. 4.19.

Owing to the normality rule for the strain increment, we can deduce from the yield surface the growth rate of the cavities. V being the entire volume, we get:

$$\frac{1}{dE_{eq}^p} \frac{dV}{V} = \frac{\partial \phi / \partial \Sigma_m}{\partial \phi / \partial \Sigma_{eq}} = \frac{3}{2} f \frac{R_p}{\Sigma_{eq}} \sinh \left(\frac{3}{2} \frac{\Sigma_m}{R_p} \right) \quad (4.18)$$

The matrix material being incompressible, the increment of the volume fraction $f = v/V$ of voids is given by:

$$\frac{df}{1-f} = \frac{dV}{V} \quad (4.19)$$

The growth rate of the cavities, which occupy a volume fV is given by:

$$\frac{1}{dE_{eq}^p} \frac{dR}{R} = \frac{1}{\kappa f} \frac{1}{dE_{eq}^p} \frac{dV}{V} = \frac{3}{2\kappa} \frac{R_p}{\Sigma_{eq}} \sinh \left(\frac{3}{2} \frac{\Sigma_m}{R_p} \right) \quad (4.20)$$

where κ is equal to 3 if the cavities are spherical and to 2 if they are cylindrical.

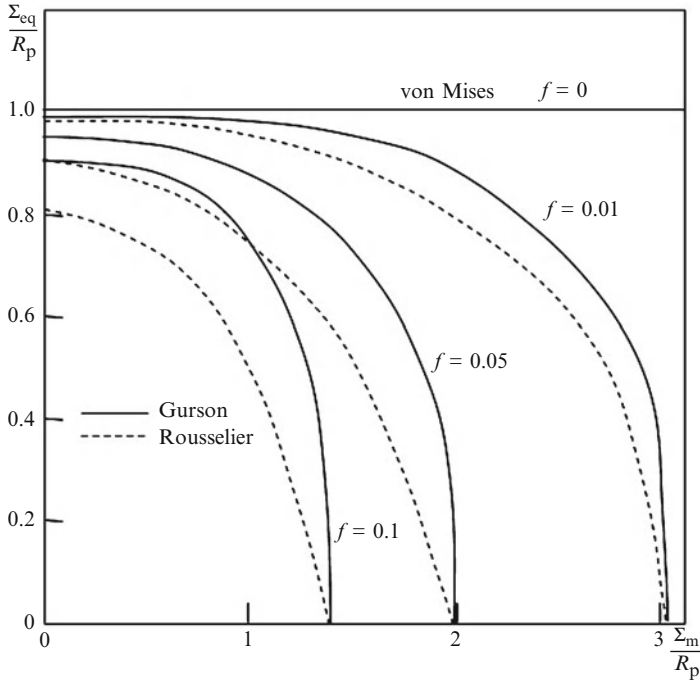


Fig. 4.19 Yield surfaces for von Mises and porous materials. The figure shows the Gurson and the Rousselier yield surfaces. The parameters of the Rousselier model are adjusted to cope with the purely hydrostatic exact solutions

It must be noted that the growth rate is then found to be independent of the porosity. This is not in agreement with experimental observations.

The result of this calculation is very similar to the ones given by the McClintock (Eqs. 4.8a) and by the Rice and Tracey (Eq. 4.10) models. For this last one the only difference is the value of the α coefficient (0.5 instead of 0.283).

Extensions of the Gurson Model

Following the work of Gurson (1977) it was soon recognised that the proposed yield surface was unable to represent fracture and coalescence. In addition, unit cell simulations showed that void growth rates were not accurately predicted. This explains why Tvergaard and Needleman (1984) modified the expression of the original Gurson yield surface as:

$$\phi = \left(\frac{\Sigma_{eq}}{R_p} \right)^2 + 2q_1 f^* \cosh \left(\frac{3q_2}{2} \frac{\Sigma_m}{R_p} \right) - 1 - (q_1 f^*)^2 = 0 \quad (4.21)$$

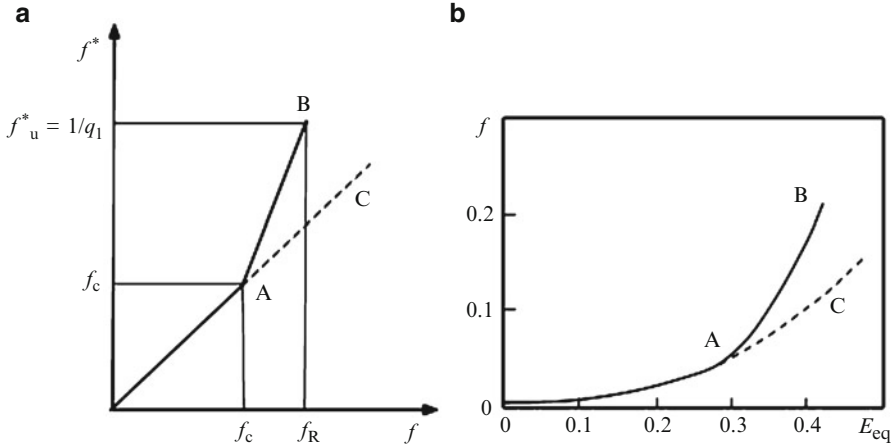


Fig. 4.20 (a) Modified Gurson model showing the acceleration in void growth rate when the volume fraction is larger than f_c ; (b) response of a FE cell with (curve B) and without (curve C) the introduction of the accelerating function

This model is often referred to as the Gurson-Tvergaard-Needleman (GTN) model in the literature. In this expression R_p is the flow stress of the matrix material and q_1 , q_2 and f^* are new parameters. The parameters q_1 and q_2 allow describing more accurately void growth rate observed in unit cell calculations. Based on early calculations (Koplik and Needleman 1988) the values of $q_1 = 1.50$ and $q_2 = 1.0$ are often used. Faleskog et al. (1998) showed that these parameters depended on the strain-hardening exponent and on the ratio of the yield stress over Young modulus. The parameter f^* , which can be considered as an effective porosity, depends on the actual porosity f , as indicated in Fig. 4.20a:

$$f^* = \begin{cases} f & \text{if } f \leq f_c \\ f_c + \left(\frac{1}{q_1} - f_c\right) \frac{f - f_c}{f_R - f_c} & \text{if } f > f_c \end{cases} \quad (4.22)$$

The above equation simply implies that before void coalescence for $f = f_c$, the void volume fraction and the decrease of load carrying capacity follow the “normal” behaviour predicted by the Gurson model. After the void coalescence process has started, the void volume fraction is accelerated to represent the sudden loss of load carrying capacity (Fig. 4.20b).

Many other extensions of the Gurson model were proposed in the literature. The format of the model remains basically the same. These modifications take into account several factors including: matrix kinematic hardening, rate dependency and viscoplasticity, plastic anisotropy and void shape effects. More details can be found elsewhere for a deeper understanding (see *e.g.* Pineau and Pardoën 2007; Benzerga and Leblond 2011; Besson 2010).

Rousselier Model

Although the Rousselier model (Rousselier 1987) was developed from quite different bases, which are those of general thermodynamic principles (Lemaitre and Chaboche 1990), its formulation remains close to the Gurson model. The yield surface of the Rousselier model is given by:

$$\phi = \frac{1}{1-f} \frac{\Sigma_{eq}}{R_p} + D_r \frac{\Sigma_\rho}{R_p} f \exp\left(\frac{3}{1-f} \frac{\Sigma_m}{\Sigma_\rho}\right) - 1 = 0 \quad (4.23)$$

where D_r and Σ_ρ are adjustable parameters. The Rousselier yield surfaces are shown in Fig. 4.19, the parameters being adjusted to cope with the exact purely hydrostatic solutions (Eq. 4.17). The normality rule (Eq. 4.18) is also assumed. It is interesting to outline some differences between Gurson and Rousselier models. Under pure hydrostatic stress, that is $\Sigma_{eq} = 0$, the Rousselier yield surface has a vertex which implies that at high stress triaxiality the plastic deformation tension retains a non-zero shear component.

A “modified” Rousselier model was introduced by Tanguy and Besson (2002). The yield surface was now expressed as:

$$\phi = \frac{1}{1-f} \frac{\Sigma_{eq}}{R_p} + \frac{2}{3} D_R f \exp\left(\frac{3}{1-f} \frac{q_R}{2} \frac{\Sigma_m}{R_p}\right) - 1 = 0 \quad (4.24)$$

where D_R and q_R are damage parameters used instead of D_r and Σ_ρ . When properly calibrated, the Rousselier model provides predictions as good as the Gurson model with the advantage of better capturing localisation.

Elliptic Models

In the so-called elliptic models, the effective stress is explicitly given by:

$$\sigma^* = \left(\frac{2}{3} \underline{\underline{\sigma}} : \mathbf{M} : \underline{\underline{\sigma}} \right)^{1/2} \quad (4.25)$$

where $\mathbf{M}$ is a fourth-order tensor. In the case of an isotropic material this reduces to $\sigma^* = \sqrt{a\sigma_{eq}^2 + b\sigma_{kk}^2}$ where a and b are scalar parameters functions of f . The tensor $\mathbf{M}$ can also depend on void shape parameters.

The elliptic form for σ^* was originally developed on a purely phenomenological basis in order to introduce a pressure dependence in the expression of the yield surface. Nonlinear estimates using homogenisation techniques (Michel and Suquet

1992) led to the same kind of expression. The main drawback of elliptic models is that they predict a cavity growth rate which varies linearly with stress triaxiality. This is true for linear viscous solids (Eq. 4.14) but wrong for plastic materials in which the dependence is exponential.

Other Models – Lode Parameter

According to RT and GTN models, the growth of a void depends mainly on the stress triaxiality T , and the effective plastic strain. When the introduction of stress triaxiality parameter is not sufficient the use of the Lode (1925) parameter can give a more complete description of the stress state. This is especially true when the second principal stress is of major importance which is a situation met in a number of forming operations.

The Lode parameter (which varies from -1 to $+1$) was introduced by Walter Lode in 1925 during his study in the University of Göttingen, Germany. Lode studied the mechanical behaviour and the failure of tubes of iron, copper and nickel, under combined tension and internal pressure. The influence of the intermediate principal stress, Σ_2 , on yielding, and the corresponding failure of Tresca's criterion, was clearly shown. The Lode parameter is defined by:

$$\mu_\sigma = \frac{2\Sigma_2 - \Sigma_1 - \Sigma_3}{\Sigma_1 - \Sigma_3} \quad (4.26)$$

The Lode parameter $\mu_\sigma = -1$ for the axisymmetric stress state; $\mu_\sigma = 1$ when the intermediate principal stress is equal to the largest principal stress ($\Sigma_1 = \Sigma_2 > \Sigma_3$); $\mu_\sigma = 0$ when the intermediate principal stress is equal to the average stress $\Sigma_2 = \Sigma_m$ which is the case of plane strain. One defines also the Lode angle which is mainly used in civil engineering.

The influence of the Lode parameter on the directional expression of a void was studied by Zhang et al. (2001). These authors showed that this parameter has a strong influence on void shape. More recently a number of authors, in particular Wierzbicki and Bai, attempted to introduce this parameter to investigate ductile fracture. Further information can be found elsewhere: Barsoum and Faleskog (2007), Danas and Ponte Castañeda (2011), and Bai and Wierzbicki (2008, 2010). Further details on the importance of this parameter for shear fracture are given in Sect. 4.6.4.5.

4.4 Void Coalescence

4.4.1 Observations

Two main mechanisms for void coalescence were evidenced. The first mode shown in Fig. 4.21a (Benzerga 2000) is produced by the internal necking mode

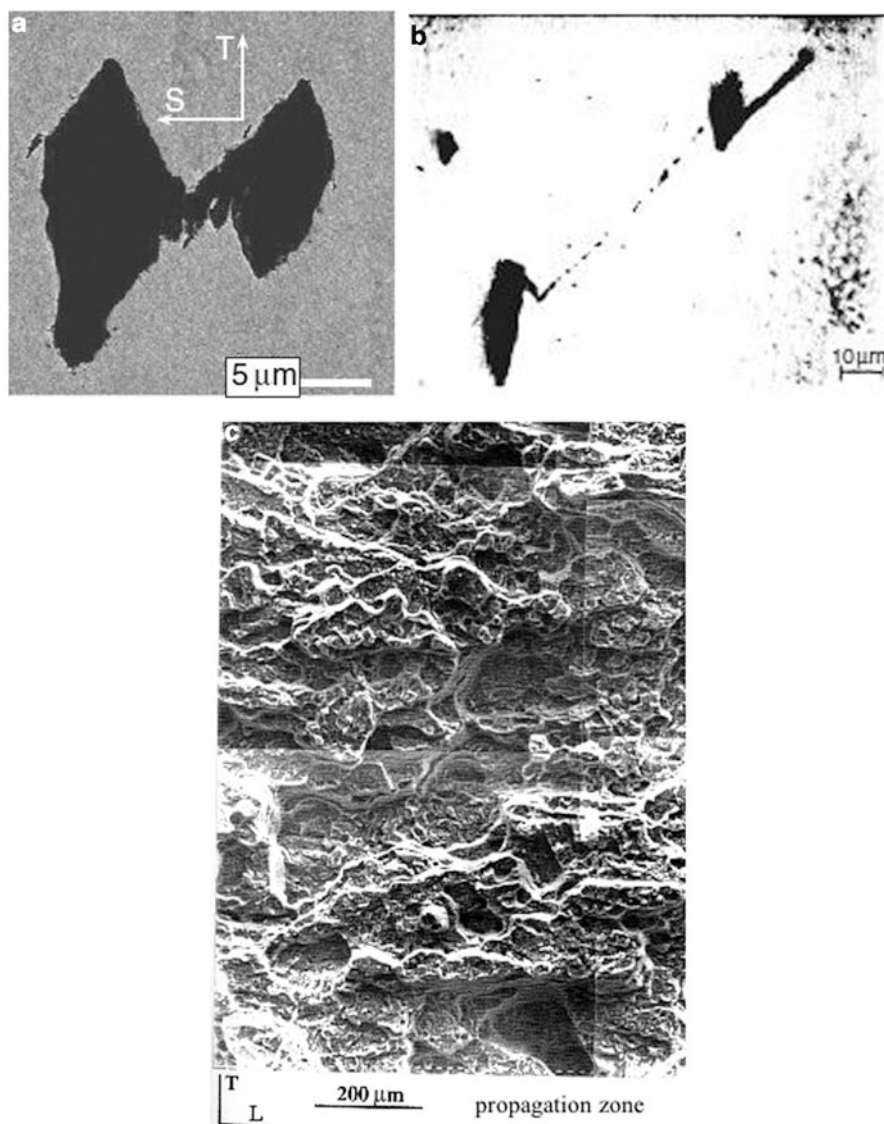


Fig. 4.21 Fracture surfaces of various alloys; (a) plain carbon steel. Longitudinal section showing the coalescence between two cavities initiated from MnS particles (Benzerga 2000); (b) void sheet mechanism by Cox and Low (1974) observed on a longitudinal section; (c) two populations of dimples observed on the fracture surface of a high strength aluminium alloy (Achon 1994)

of coalescence where the ligament between two voids shrinks with a shape typical of a necking process. During the process of coalescence the voids evolve toward a diamond shape. The fracture profile will be flat oriented at 90 degrees with respect to the main loading direction, and only one population of dimples will be observed on the fracture surfaces.

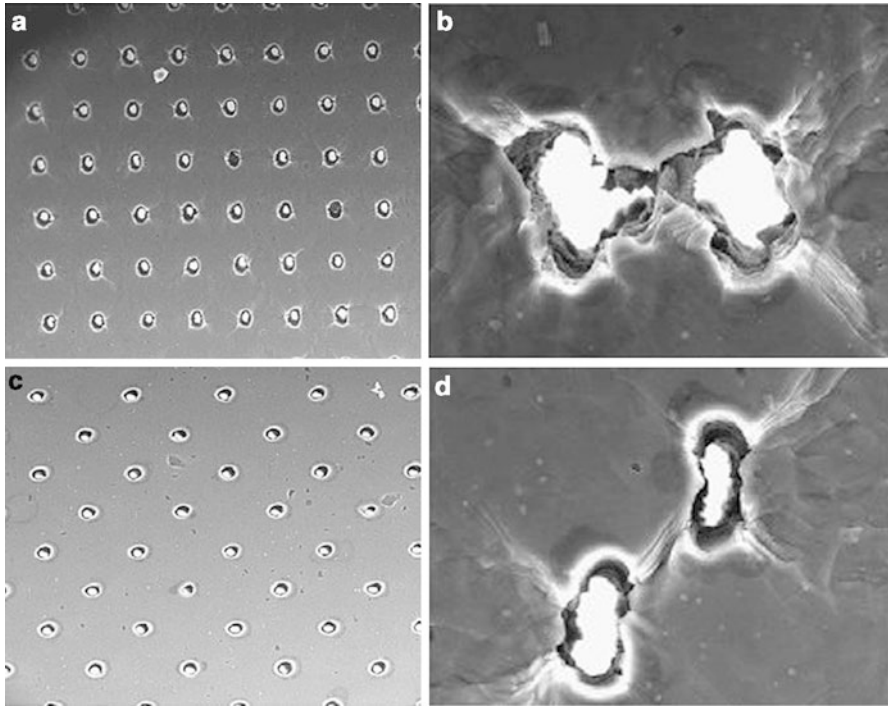


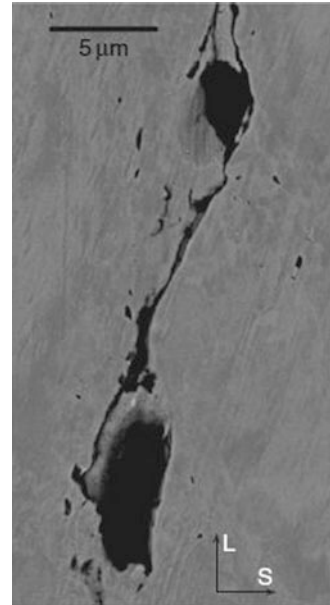
Fig. 4.22 Micrographs illustrating the micromechanisms of cavity growth and coalescence observed on laser drilled thin sheets of aluminium; (a) square array of drilled holes and (b) associated coalescence by void growth impingement; (c) holes oriented at 45 degrees with respect to the tensile axis (*vertical*) and (d) associated coalescence by void sheeting (Weck and Wilkinson 2008)

The second mode of void coalescence shown in Fig. 4.21b (Cox and Low 1974) consists in shear localisation between large primary voids. It is frequently observed in high-strength materials with low or moderate strain-hardening capacity. The fracture surface contains two populations of dimples, the larger ones initiated from the big inclusions forming primary voids, and the small ones initiated in the shear bands from second-phase particles (Fig. 4.21c, Achon 1994).

These two modes of ductile fracture were investigated using “model” systems in which holes were laser drilled into plates (Weck and Wilkinson 2008). An example is given in Fig. 4.22a, b, c, d, for different arrangements of cavities. The cavities are cylindrical. These two types of experiments allow studying the two main modes of coalescence in an ideal situation. They clearly show that the failure modes (internal necking or shear) depend strongly on the arrangement of the holes (*cf* Fig. 4.22b, c).

The transition between shear localisation and internal necking can be related to the transition between slant to flat fracture mode which will be discussed in more detail in Sect. 4.6.4.5.

Fig. 4.23 “Necklace” coalescence or coalescence in columns observed on a plain carbon steel on a longitudinal section. The tensile axis is vertical (Benzerga 2000)



There is a third mode of void coalescence, called “necklace coalescence,” which is more anecdotic. It was observed in between row of voids gathered in elongated clusters. It consists in a localisation process in a direction parallel to the main loading axis. An example is provided in Fig. 4.23 (Benzerga 2000).

4.4.2 Void Cell Simulations

The different modes of coalescence have been simulated numerically using FE void cell calculations.

Coalescence by internal necking is observed in axisymmetric void cell simulations as well as in 3D cell calculations, when the strain-hardening exponent is not too low, or when the stress triaxiality is moderate or high, and when the periodic void packing does not involve closest neighbours near to 45 degrees from the main loading direction (*e.g.* Richelsen and Tvergaard 1994). The main characteristic of this mode of coalescence is a sharp transition into a uniaxial deformation mode of the volume element imposed by the material outside of the localisation band, which unloads elastically and behaves as a rigid block (see *e.g.* Koplik and Needleman 1988). It has already been noted that the transition to a uniaxial straining mode is almost instantaneous (see Fig. 4.16d).

Void cell simulations provide important information about the state of the damage at the onset of void coalescence. Figure 4.24 shows that the value of the

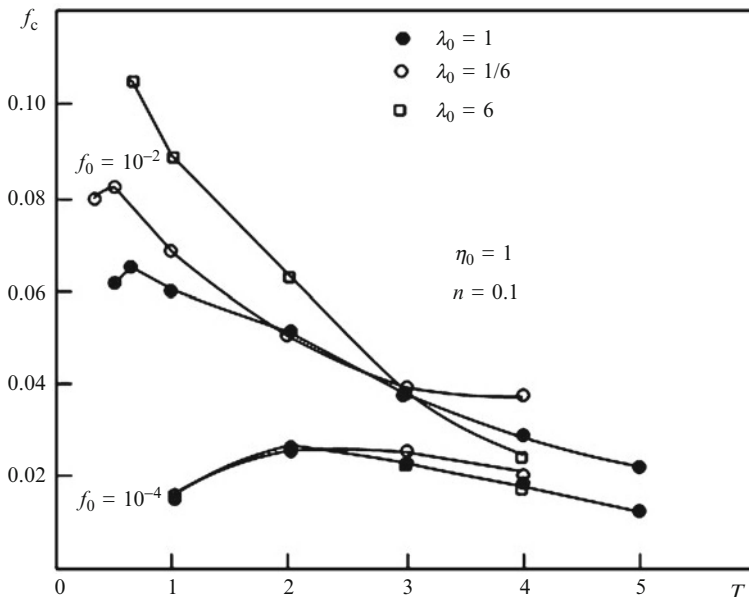


Fig. 4.24 Variation of the critical porosity at the onset of coalescence, f_c , as a function of stress triaxiality. Cell simulations $\lambda_0 = R_{oz}/R_{ox}$, $\eta_0 = L_{oz}/L_{ox}$ (see Fig. 4.14) (Pardoën and Hutchinson 2000)

porosity at the onset of void coalescence, f_c , varies with the stress triaxiality, in particular when the initial volume fraction of cavities, f_0 , is large. On the other hand, when f_0 is small ($\sim 10^{-4}$, which is a typical value of inclusion content found for instance in “clean” steels), the value of f_c does not change too much with stress triaxiality. Similar results were also been reported by Zhang and Niemi (1995). The value of the strain-hardening exponent also affects the critical porosity.

The effect of the existence of a second population of cavities was simulated using also cell calculations by different authors (see *e.g.* Fabrègue and Pardoën 2008). These authors modelled the second population using a Gurson-type model for the matrix surrounding the primary voids. Figure 4.25 shows the evolution of the volume fraction of primary voids and that of the overall stress-strain curve obtained with a fixed total initial volume fraction $f_{10} + f_{20} = 1.5 \times 10^{-3}$, once with secondary voids ($f_{10} = 1 \times 10^{-3}$, $f_{20} = 0.5 \times 10^{-3}$). The influence of the presence of a second population of particles, which are assumed to nucleate immediately, on the onset of coalescence is very clear as well as the negligible effect on the overall stress response before coalescence. It should also be noted that the void growth rate of the primary void is not affected by the presence of the second population until coalescence occurs.

Coalescence in shear was first investigated numerically by Yamamoto (1978) using FE cell calculations involving a cylindrical void under plane strain conditions. The kinematics associated to this loading mode favors the development of shear

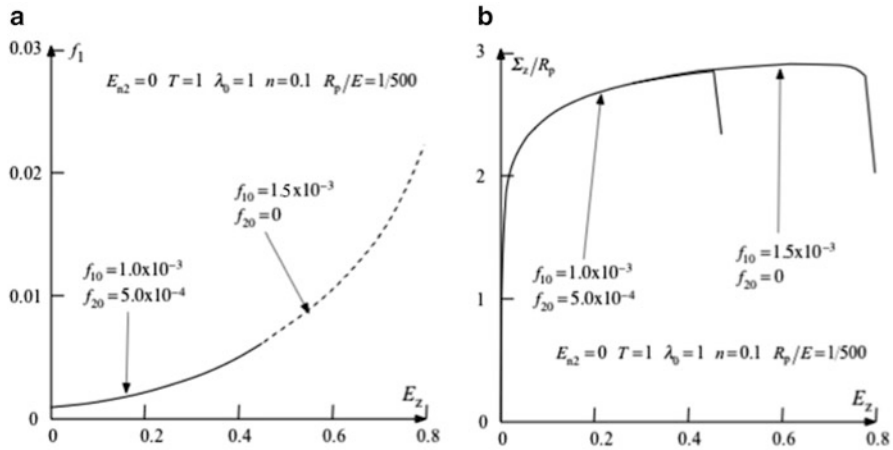


Fig. 4.25 Influence of the presence of a second population of voids on (a) the evolution of the rate of increase of the primary void volume fraction, f_1 as a function of the applied axial strain, Σ_z ; (b) the overall axial stress/axial strain response at a constant void volume fraction $f_{10} + f_{20} = 1.5 \times 10^{-3}$. The stress triaxiality T is equal to 1 and the initial void aspect ratio λ_0 is equal to 1 (Fabrègue and Pardoen 2008)

bands. Specific packing of voids where the closed neighbours are located at 45 degrees from the main loading direction also favors a shear coalescence process (Fig. 4.26, Faleskog and Shih 1998), as the experiments on laser drilled holes have shown (see Fig. 4.22c, d).

4.4.3 Models for Void Coalescence

4.4.3.1 Metallurgical and Phenomenological Models

The most widely employed criterion for the onset of void coalescence states that void coalescence starts at a critical porosity (McClintock 1968, 1971; d'Escatha and Devaux 1979; Lautridou and Pineau 1981; Pardoen Delanay 1998b). For a given material, this criterion is acceptable from a practical standpoint, especially in low-inclusion alloys, such as most modern steels. However it is clear that any general void coalescence model requires the introduction of other microstructural information related to the void/ligament dimensions and geometry.

An attempt was made by Pineau (1981) and Mudry (1982) to develop a simplified model, which took into account the presence of a second population. Figure 4.27 shows the basis of this model. The size of the “plastic zone” associated to the growth of large cavities can be evaluated as:

$$c = R \exp \left(\frac{1}{2} \frac{\Sigma_m}{R_p} - \frac{1}{3} \right) \quad (4.27)$$

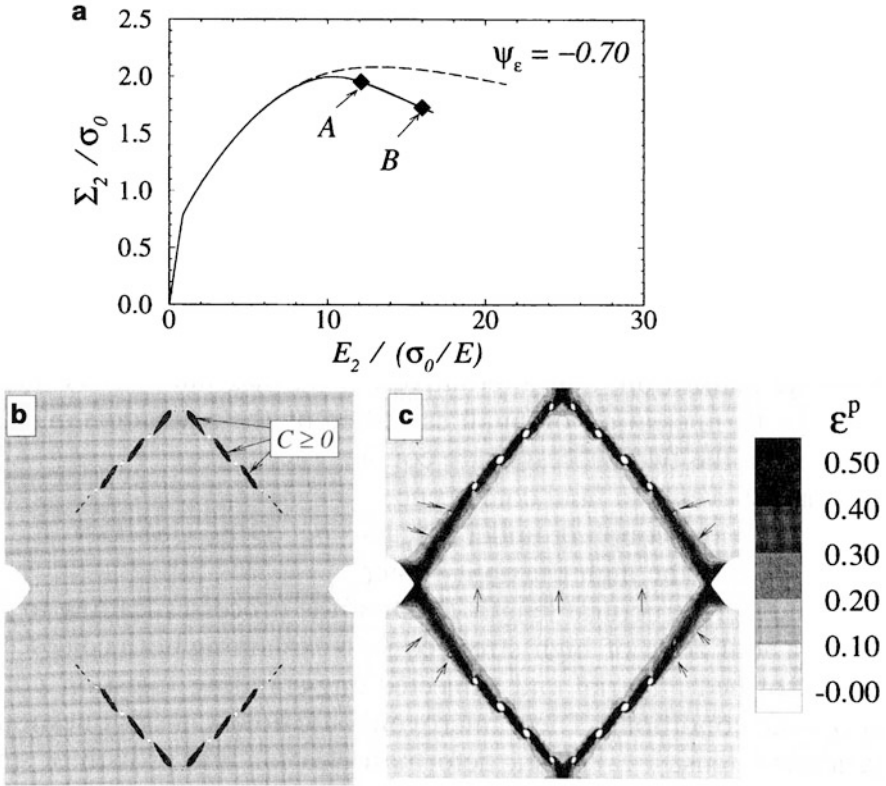


Fig. 4.26 Void sheet mode of coalescence under low stress biaxiality in a high strength material ($\sigma_0/E = 0.004$, $n = 0.025$); (a) macroscopic stress-strain response; (b) microvoids and current cavitation zones at load point A; (c) advanced coalescence at load point B, displaying contours of effective plastic strain (Faleskog and Shih 1997)

If it is assumed that localisation occurs for a critical proximity, *i.e.* $\alpha = 2c/L$, this relation can be used to calculate the critical growth rate at failure, $(R/R_0)_c$:

$$\left(\frac{R}{R_0}\right)_c = 0.80\alpha f^{-1/3} \exp\left(\frac{1}{3} - \frac{1}{2} \frac{\Sigma_m}{R_p}\right) \quad (4.28)$$

where the volume fraction of large cavities has been expressed as $f = (4\pi/3)(R_0/L)^3$. The critical hole growth rates derived from Eq. 4.28 are plotted in Fig. 4.27 for typical values of volume fraction of manganese sulfides inclusions which can be found in “clean” steels, *i.e.* $f = 5 \times 10^{-4}$ and for $\alpha = 0.50$. It is observed that this simple model predicts values of $(R/R_0)_c$ which are much higher than those measured in modern pressure vessels steels (Lautridou and Pineau 1981). This difference might be related to the fact that inclusions are not evenly distributed and that,

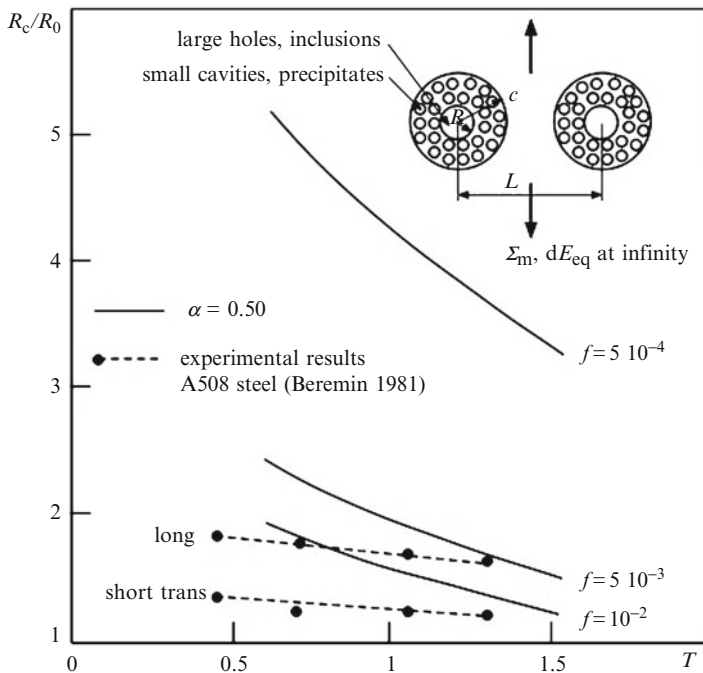


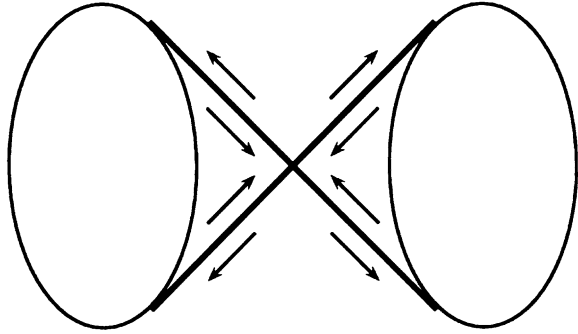
Fig. 4.27 Schematic diagram showing “plastic zones” associated to cavity growth. Variation of calculated critical cavity growth $(R/R_0)_c$ as a function of stress triaxiality and for various values of inclusion content, f . L designates the distance between cavities of radius R . Comparison with experimental results. *Long* longitudinal direction, *short trans* short transverse direction (Pineau 1981)

for a mean volume fraction of $f_0 = 5 \times 10^{-4}$, there is a large probability of finding locally much larger values of the volume fraction of inclusions ($f_0 = 10^{-2}, 10^{-3}$). Figure 4.27 shows that for these larger values of f_0 the calculated critical void growth is closer to the experimental results. This means that ductile fracture models should also consider the statistical distribution of inclusions.

Coalescence by internal necking is the most documented mode of coalescence and has, up to now, received the largest attention. Brown and Embury (1973) proposed a simple model based on an explicit micromechanical point of view. In a perfectly plastic material, internal necking is considered to initiate when it is possible for 45 degrees microshear bands to connect two neighbouring voids (Fig. 4.28). Fabrègue and Pardoen (2008) showed that this criterion could be written in terms of non-dimensional parameters as:

$$2\chi\sqrt{\lambda^2 + 1} = 1 \quad (4.29)$$

Fig. 4.28 Schematic diagram showing the mechanism of cavity coalescence of Brown and Embury (1973) (Besson 2004)



with the evolution law of the relative void spacing in the case of spheroidal voids, that is, $\chi = R_x/L_x$ expressed as:

$$\frac{\dot{\chi}}{\chi} = \frac{1}{3} \left(\dot{E}_z - \dot{E}_x + \frac{\dot{f}}{f} - \frac{\dot{\lambda}}{\lambda} \right) \quad (4.30)$$

Combining cavity growth models to evaluate the term $\dot{f}/f$ in Eq. 4.30, void cell simulations to determine the evolution of cavity shape $\dot{\lambda}/\lambda$, this model theoretically predicts the strain to failure. However there are still very few quantitative applications of this type of model in the literature.

It should be added that proper modelling of the second mode of coalescence, which is shear coalescence, is still a matter of open debate. Ideally, a micromechanics-based analysis of ductile fracture should rest on multiple coalescence criteria: one for internal necking and one for void sheeting.

4.4.3.2 Thomason Model

More accurate representations of the coalescence of voids were developed by Thomason (1985a, b, 1990) who extensively studied the transition to localisation for elastic-perfectly plastic solids using slip-line solutions leading to the following condition for the onset of void coalescence:

$$\frac{1}{1-f_2} \frac{1}{1-\chi^2} \frac{\Sigma_n}{R_p} = \frac{2}{3} \left[\alpha \left(\frac{1-\chi}{\chi\lambda} \right)^2 + 1.24 \frac{1}{\chi^{1/2}} \right] \quad (4.31)$$

where Σ_n is the critical value of the axial stress Σ_z at which coalescence starts; the volume fraction of secondary voids f_2 is equal to 0 in the original criterion and the parameter α was fitted as a function of the average value of the strain-hardening exponent, n (see Pardoen and Hutchinson 2000):

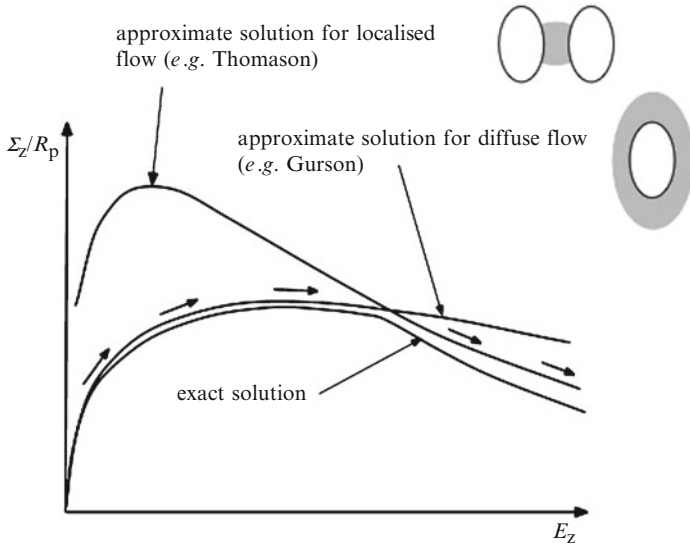


Fig. 4.29 Schematic description of the competition between the two modes of plasticity corresponding to “diffuse” plasticity and localised flow

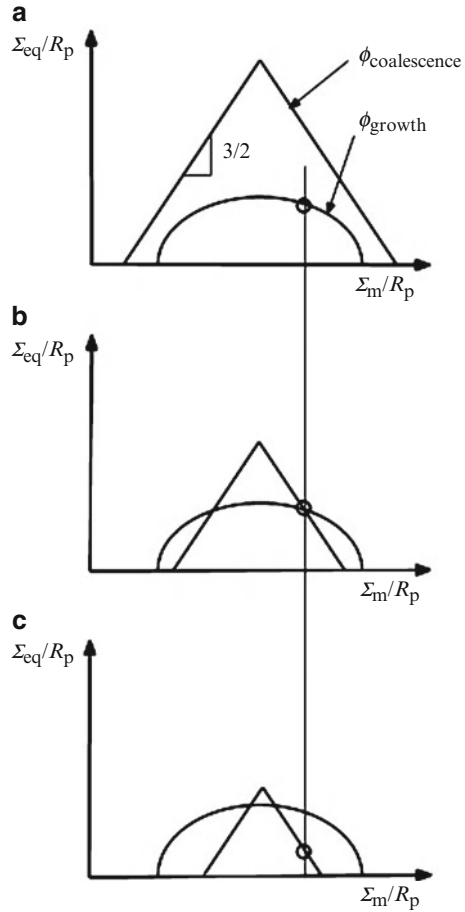
$$\alpha(n) = 0.10 + 0.22n + 4.8n^2 \quad (0 < n < 0.3) \quad (4.32)$$

The relationship between Σ_z/R_p and the overall strain E_z is shown qualitatively in Fig. 4.29 for a cell of material containing a finite porosity made of voids that grow and coalesce through an internal necking process. At low overall strain, Σ_z/R_p from Eq. 4.30 is much greater than the actual value, which is better predicted by a Gurson-type model. There is a transition point from which plasticity is localised within the ligament between voids. The velocity fields used to build the Gurson model are not realistic anymore.

The effect of secondary voids on the onset of coalescence by internal necking was introduced heuristically into Eq. 4.31 by multiplying the current mean yield stress R_p of the matrix materials by $(1 - f_2)$. An estimate of the evolution of f_2 in the region next to the surface of primary voids was presented by Fabrègue and Pardoen (2008). These authors showed that by applying a constitutive model involving a void nucleation law (e.g. Beremin criterion) and a void growth law (e.g. Gurson) it was possible to estimate the evolution of f_2 and use the extended coalescence condition (Eq. 4.31) to predict the onset of fracture in the presence of two populations of voids.

The Thomason model (Eq. 4.31) is not only a condition for the onset of coalescence but also provides the evolution of the stress during the full coalescence process as a function of the evolution of the shape and spacing of cavities.

Fig. 4.30 Schematic diagram showing the transition from plasticity/void growth (a) to plasticity/void coalescence (b, c) in terms of the yield surfaces



It constitutes the basis for elaborating a constitutive model for the full coalescence process. This idea was followed by Pardoën and Hutchinson (2003), Benzerga et al. (2004), Besson (2010), and Benzerga and Leblond (2011). In order to use the solution of the Thomason model as a full constitutive model, Eq. 4.31 was transformed into a yield surface.

Figure 4.30 illustrates the competition between the two modes of plastic deformation (diffuse plasticity during void growth versus localised plasticity during void coalescence) generalising Fig. 4.29 to a general stress state. Two potentials are introduced ϕ_{growth} and $\phi_{\text{coalescence}}$. The potential $\phi_{\text{coalescence}}$ is written as:

$$\phi_{\text{coalescence}} = \frac{\Sigma_{eq}}{R_p} + \frac{3}{2} \frac{|\Sigma_m|}{R_p} - F(\lambda, \chi) = 0 \quad (4.33a)$$

with

$$F(\lambda, \chi) = \frac{3}{2} (1 - \chi^2) \left[\alpha \left(\frac{1 - \chi}{\chi \lambda} \right)^2 + 14 \left(\frac{1}{\chi} \right)^{1/2} \right] \quad (4.33b)$$

Equation 4.33a yields a straight line of slope $-3/2$ in the plane $\Sigma_{eq} - \Sigma_m$. The transition to coalescence occurs when the two yield surfaces intersect at the current loading point (Fig. 4.30b). With increasing deformation ϕ_{growth} tends to expand due to hardening and then to contract due to void growth softening, and the coalescence yield surface $\phi_{coalescence}$ tends to contract very rapidly towards the zero stress state (Fig. 4.30b, c).

4.5 Prediction of the Fracture Strain

4.5.1 Homogeneous Distribution of Voids: Introduction of the Effect of Heterogeneity

The equations governing the three steps of ductile fracture have been assembled. It is now theoretically possible to calculate the strain to failure. Assuming immediate void nucleation ($E_N = 0$) and axisymmetric loading conditions, Pardoën has shown that the strain to failure E_f can be written as:

$$E_f = G(f_0, \zeta) \exp \left(-\frac{3}{2} \frac{\Sigma_m}{\Sigma_{eq}} \right) \quad (4.34)$$

where ζ is a parameter measuring the void packing geometry ($\zeta = 0.5$ for simple cubic and $\zeta = 0.60$ for hexagonal). This expression was obtained using the Brown and Embury criterion for void coalescence (Eq. 4.29).

The model can be qualitatively assessed by comparing the predictions from Eq. 4.34 with a wide set of results collected by Edelson and Baldwin (1962), as shown in Fig. 4.31. The model captures relatively well the experimental results. However it underestimates the fracture strain for large volume fraction of inclusions. This simple model can be improved in different ways, for instance by introducing the Beremin condition (Eq. 4.1) for void nucleation and by using the more accurate coalescence model by Thomason (Eq. 4.31).

In many cases the prediction of the fracture strain remains an open question because of the absence of a good knowledge of the physical mechanisms initiating ductile damage. Two extreme situations can be envisaged.

The first situation corresponds to a number of alloys in which the critical stage is void initiation. This is the case of many high strength modern steels and that of a number of high purity aluminium alloys. In these materials the void nucleation stage

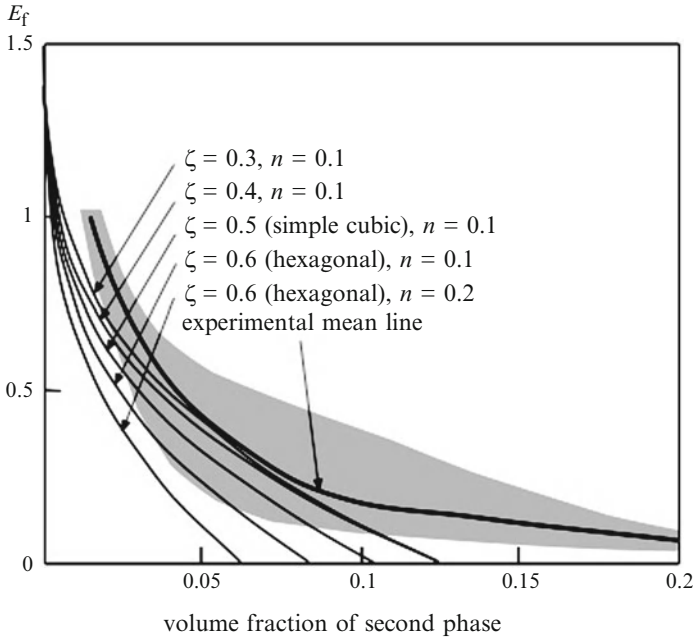


Fig. 4.31 Variation of the fracture strain as a function of the volume fraction of second-phase particles. Comparison between experimental results collected by Edelson and Baldwin (1962) and the predictions from a simple analytical model (Pineau and Pardoen 2007)

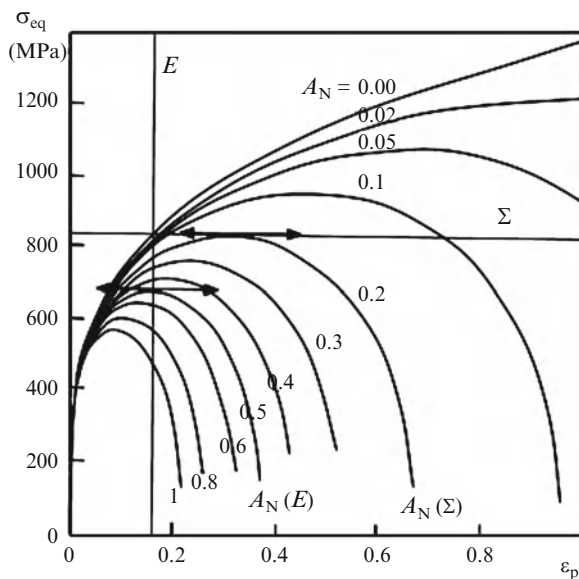
is essentially strain-controlled and occurs after large plastic strains. This means that the strain to failure is relatively independent of stress triaxiality. This is the situation which was clearly observed in duplex stainless steels (Fig. 4.3) (Pineau and Joly 1991).

In these materials it was shown that the nucleation rate A_N (Eq. 4.3b) was statistically distributed. It is therefore possible to predict the statistical distribution of the fracture strain.

Let us consider a local zone containing second-phase particles with a given nucleation rate; it is possible to derive the local stress strain curve in a given cell, which takes into account the softening effect due to cavity growth using the GTN model (Fig. 4.32). The macroscopic failure strain can be computed assuming that either the local strain is equal to the macroscopic applied strain or that the local stress is equal to the macroscopic applied stress (see below). Macroscopic failure is assumed to occur as soon as the local stress strain curve reaches a maximum as indicated in Fig. 4.32. These predictions of the failure strain have a statistical meaning if the distribution of A_N is known as it is the case in duplex stainless steels (Pineau and Joly 1991).

These calculations showed that the local volume fraction of cavities in the cell (assuming isostress condition), which gives rise to the failure of the specimen (smooth tensile or notched bar), can be as large as 5–10 times the mean volume

Fig. 4.32 Schematic diagram illustrating the variation of the strain to failure as a function of the nucleation rate, A_N . Two extreme cases are considered to predict the overall strain to failure



fraction. Further results on the same materials are reported in Fig. 4.34 where it will be shown that this approach gives prediction in good agreement with experiments, as shown below. In particular these results show a very small influence of stress triaxiality on strain to failure, a size effect (the tensile specimens, $T = 1/3$, are much bigger than the notched bars in which $T \gg 1/3$) and a scatter comparable to the observed dispersion.

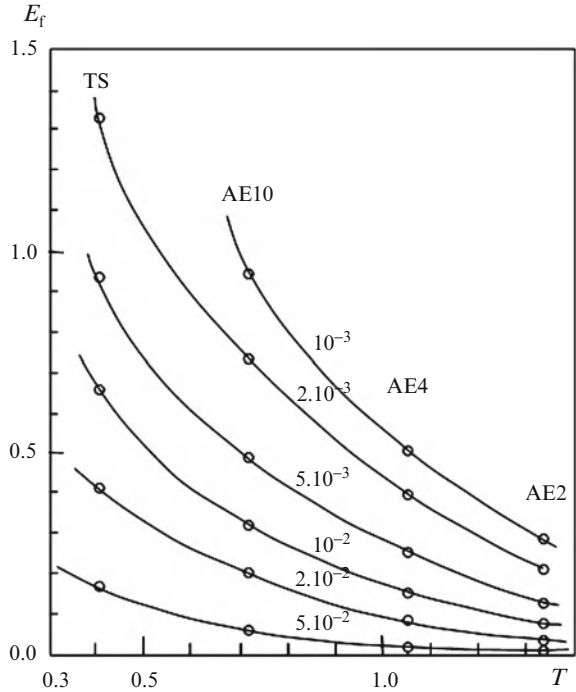
The second situation corresponds to materials in which cavity nucleation occurs easily and the fracture strain is essentially strain controlled. All the models which have been introduced previously indicate that in these conditions the fracture strain is strongly dependent on stress triaxiality ratio and is inversely proportional to $\exp[-(3/2)(\Sigma_m/\Sigma_{eq})]$. These materials include conventional steels with a “high” sulfur content, pre-deformed steels, cast iron and aluminium alloys, etc.

It should be added that when void nucleation is partly stress-controlled the variation of the fracture strain with stress triaxiality can follow the strain-controlled regime at low values of T and the growth-controlled regime at large values of T . Moreover in many materials, several populations of voids and the statistical distribution of cavities have to be taken into account, as in duplex stainless steels.

4.5.2 Further Considerations on the Effect on Heterogeneity

The Gurson potential without the accelerating effect in the volume fraction of cavity, f , introduced by Tvergaard and Needleman (1984) largely overestimates

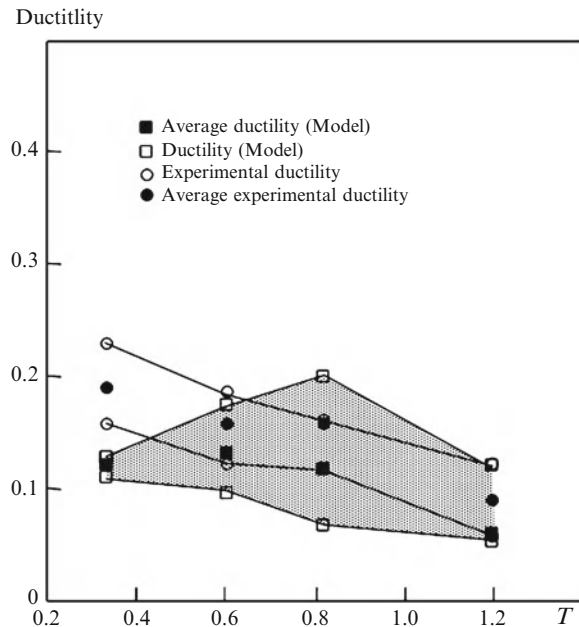
Fig. 4.33 Theoretical variation of ductility with stress triaxiality ratio, T . The numbers indicated on the curves refer to various values of the ratio f_0/n where f_0 is the initial volume fraction of cavities and n is the work-hardening exponent. The symbol AEX designates axisymmetrically notched specimens with a notch radius of X ; TS stands for smooth tensile specimens (Mudry 1982)



the ductility of materials when the cavities are assumed to be homogeneously distributed. However it was shown by Mudry (1982) that the Gurson model can account for the salient features of ductile fracture, *i.e.* the strong dependence of the strain to failure with stress triaxiality, initial void volume fraction, f_0 , and work-hardening exponent n , when the inhomogeneity in the distribution of inclusions is taken into account.

Mudry (1982) assumed that the inclusions were distributed according to a Poisson law. He assumed that macroscopic failure occurred when the Considere instability condition was reached in a given cell containing a large local volume fraction of voids (Fig. 4.32). The results of these calculations are shown in Fig. 4.33 where it is observed that, within a first approximation, the variations of the ductility E_f with stress triaxiality ratio, $T = \Sigma_m/\Sigma_{eq}$, is only dependent on the ratio between the initial volume fraction of inclusions, f_0 , and the work-hardening exponent, n , of the stress-strain curve, $\sigma = k \varepsilon^n$. In Fig. 4.34 only the curves corresponding to a failure probability, P_R , of 50% are drawn because it was found that the difference between the curves corresponding to $P_R = 10\%$ and $P_R = 90\%$ was small. These calculations indicated that the local volume fraction in the cell which gave rise to the failure of the specimen could be as large as five or ten times the mean volume fraction, as already underlined in the study devoted to duplex stainless steels and by other authors (Pineau 1981; Pineau and Joly 1991).

Fig. 4.34 Duplex stainless steels. Results of a model with a nucleation rate A_N statistically distributed and using the Taylor assumption. Variation of the strain to failure with stress triaxiality; *scatter bands* corresponding to failure probabilities of 10 and 90%. Comparison with experimental results (Joly 1992)



A similar approach was developed by Joly (1991, 1992) in his study on duplex stainless steels, as indicated earlier. The ductility of these materials is mainly controlled by the nucleation of cavities in the ferrite phase, which are distributed very inhomogeneously as shown in Fig. 4.3. The nucleation rate was described by a simple linear law, $f = A_N E_p$, where the statistical distribution of the nucleation rate, A_N , was determined experimentally. The strain to failure of axisymmetrically notched specimens was calculated using either the Taylor or the Sachs (actually Reuss) assumption, as indicated in Fig. 4.32. The comparison with experiments is made in Fig. 4.34 where the scatter band corresponding to $P_R = 10\%$ and $P_R = 90\%$ is shown. It is observed that this simple model, which does not include a coalescence criterion, is able to reproduce a large number of the observed variations. It should be added that very few authors have attempted to include these statistical aspects in ductile fracture (see however Young et al. 2002) in spite of their importance.

4.6 Ductile Fracture and Fracture Toughness

4.6.1 Introductory Comments

This section is devoted to the fracture resistance of cracked structures. The emphasis is laid on “thick components” or “thick test specimens,” which is the classical

topic of elastoplastic fracture mechanics (see Chap. 2, this volume). By thick, it would mean that the thickness is larger than the plastic zone (PZ) size and that plane strain or near plane strain conditions prevail along the crack front. However it was shown that the determination of elastoplastic fracture toughness in terms of J_{Ic} or K_{Ic} corresponding to predominantly small scale yielding (SSY) conditions, is such that in many situations these quantities are measured under large scale yielding (LSY) conditions and the values obtained for J_{Ic} or K_{Ic} are not intrinsic since they depend on the specimen geometry. Moreover only mode I fracture, which is the most important for a majority of practical applications, is considered.

In this section we do not deal with the fracture of “thin sheets,” a subject which has recently received a recrudescence of interest. In “thin” sheets, as discussed in Chap. 2 of this volume, the thickness plays a key role in controlling the dissipation of energy at the crack tip through its direct effect on the energy spent in localised necking and an indirect effect on the rate of damage evolution in the fracture plastic zone. While in “thick” plane strain specimens, loss of constraint results from in-plane effects, in “thin” sheets one must also consider out-of-plane effects. In “thin” sheets, mixed mode I and III fracture is also an important issue. Some details on specific topics of thin sheets can be found elsewhere (Pineau and Pardoen 2007).

We start with a number of observations and results on two families of structural alloys: steels and aluminium alloys. Then a second section is devoted to the results of fracture mechanics applied to the notch tip of ductile materials. In a third section the strategies which are used to model ductile crack initiation and crack growth are presented. The last section is devoted to the presentation of models for fracture toughness. These models are based on the mechanical analysis of the notch tip and on metallurgical considerations.

4.6.2 Observations

Examinations of sections perpendicular to the crack front show that ductile crack initiation is preceded by crack tip blunting as observed in Fig. 4.35a (Lautridou 1980) and as indicated in Chap. 2 of this volume. Cavity formation occurs preferentially ahead of the crack tip. These cavities grow and coalesce with the blunted crack as schematically shown in Fig. 4.35b.

Crack propagation occurs by a different mechanism, which involves an unzipping process from one inclusion to another one. This process gives rise to a crack tip which is much sharper than at crack initiation, as observed in Fig. 4.36a and schematically shown in Fig. 4.36b. This modification in crack profile leads to an increase of the normal stress ahead of the crack tip when the crack propagates. This phenomenon has an important effect on the slope of the J - Δa curve (Volume II, Chap. 2, Sect. 2.4). It is also a key factor in the analysis of the brittle-to-ductile transition (Volume II, Chap. 5).

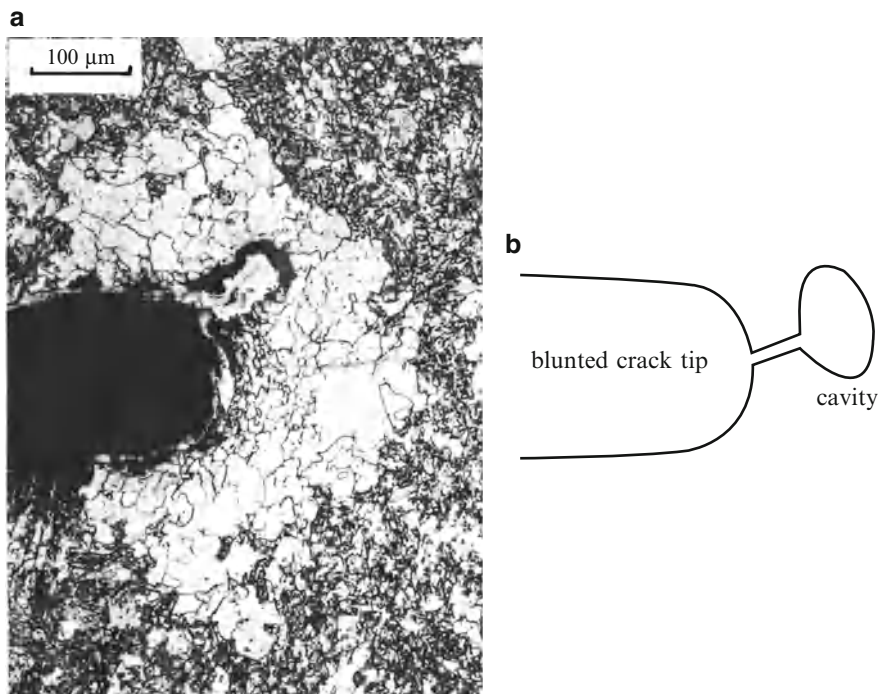


Fig. 4.35 Observations on sections of fracture toughness specimens in a low strength steel (0.16C, 1.5Mn, 0.70Ni, 0.50Mo) ($R_p \sim 500$ MPa), (a) crack tip blunting (the material was subjected to a recrystallisation heat treatment after being tested in order to estimate the local plastic strains); (b) schematic diagram showing *cavity growth* ahead of the crack tip

In structural alloys the main factor controlling the value of the fracture toughness measured in terms of J_{Ic} , K_{Ic} , K_{Jc} (for the definition, see Chap. 2 in this volume) is the volume fraction of inclusions. The examples of aluminium alloys and steels can be used to illustrate this effect.

The classical results collected by Hahn and Rosenfield (1975) are reproduced in Fig. 4.37. These results obtained essentially on aluminium alloys (see also some results on high strength steels) show that the fracture toughness K_{Ic} (divided by the square root of the product $R_p E$) decreases with increasing volume fraction of inclusions, f . In these alloys the ductile process of fracture involves large, ~ 1 – 10 μm, Fe, Si and Cu – bearing inclusions which break easily, and the growth of voids at the cracked particles. The linking-up of the voids is accomplished by the rupture of the intervening ligaments, and this is affected by the fine, ~ 0.01 μm precipitate particles that strengthen the matrix. The 0.1 μm Cr, Mn and Zr – rich intermediate particles are more resistant to cracking and enter the process in the linking-up stage.

The results obtained on various heats of pressure vessel steels (A508 type) are reported in Fig. 4.38. They illustrate the influence of the volume fraction of

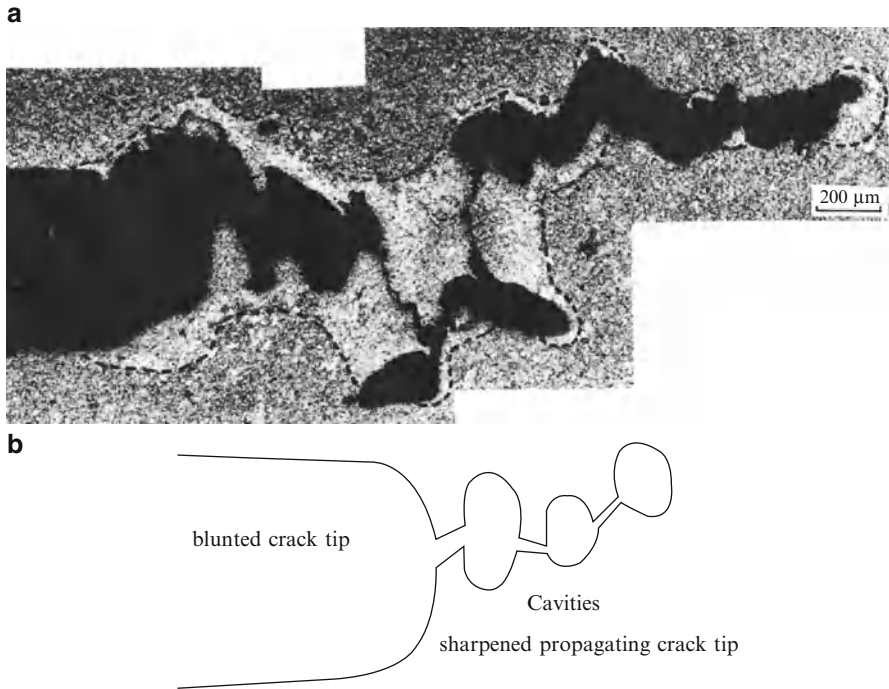


Fig. 4.36 Sections of fracture toughness specimens; (a) illustration of the unzipping process during crack propagation; (b) schematic diagram showing a crack jumping from one inclusion to another one

inclusions on the fracture toughness at crack initiation, J_{Ic} , and on the slope of the $J-\Delta a$ curve (Lautridou and Pineau 1981). These results show also the effect of the distribution of inclusions on the anisotropy of the fracture toughness. In these low alloy (0.15C, 1.50Mn, 0.70Ni, and 0.50Mo) quenched and tempered steels ($R_p \sim 450$ MPa) the inclusions are formed by manganese sulfides, MnS. The volume fraction of these inclusions is directly related to the content in sulfur, S (see Table 4.3) which can be determined using a simple relation established by Franklin (1969): $f = 5.45 (\text{Wt}\%) - 0.001/\text{Mn} (\text{Wt}\%)$.

These MnS inclusions are elongated along the longitudinal direction of forging (L) and have smaller dimensions along the transverse (T) and short transverse directions, as shown in Table 4.3. The number of inclusions per unit are (N_a) in a plane perpendicular to the crack front (L, T, S) and per unit volume (N_v) were also measured using quantitative metallography.

These materials were tested at room temperature using thick ($B = 30$ and 50 mm) CT type specimens (see Chap. 2 of this volume) which were cut along the three main directions, as indicated in Fig. 4.39.

The $J-\Delta a$ curves determined in these steels and reported in Fig. 4.38 are normalised by plotting the ratio J/J_{Ic} as a function of $\Delta a/\delta_c$, where δ_c is the

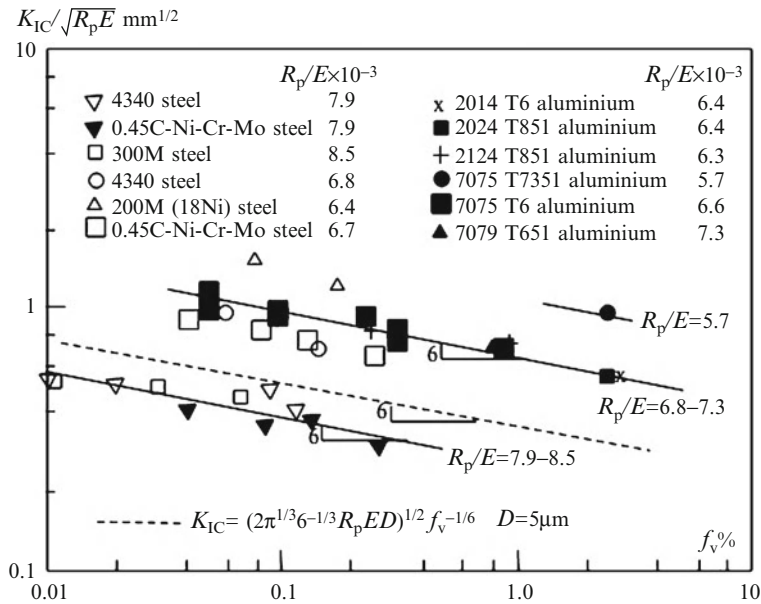


Fig. 4.37 Influence of the volume fraction of large ($\geq 1 \mu\text{m}$) second-phase particles on the plane strain fracture toughness of various high strength alloys (Hahn and Rosenfield 1975)

value of the critical crack tip opening displacement measured at crack initiation using metallographical observations similar to the picture shown in Fig. 4.35. The results of the δ_c measurements are given in Table 4.3. These normalised curves are shown in Fig. 4.40. It is observed that the δ_c and the J_{IC} values change by a factor of approximately 2 between steels B ($S = 0.020\%$) and C ($S = 0.013\%$) tested in two transverse directions and between the longitudinal and the short-transverse directions in steel A. This strong anisotropic effect is also observed on the slope of the $J-\Delta a$ curves (Fig. 4.38). These results highlight the importance of the two main factors for interpreting the fracture toughness, *i.e.* the volume fraction of inclusions and their distribution. An attempt will be made in Sect. 4.6.5 to show how these results can be interpreted using the theories of ductile fracture.

4.6.3 Notch Tip Damage Analysis in Terms of Cavity Growth

The detailed analysis of the crack tip blunting process in the framework of finite strain J_2 plasticity was achieved first by McMeeking (1977) using the finite element method and extending the slip line analysis of Rice and Johnson (1970) limited to perfectly plastic materials (see Chap. 2, Sect. 2.3.4.2 in this volume).

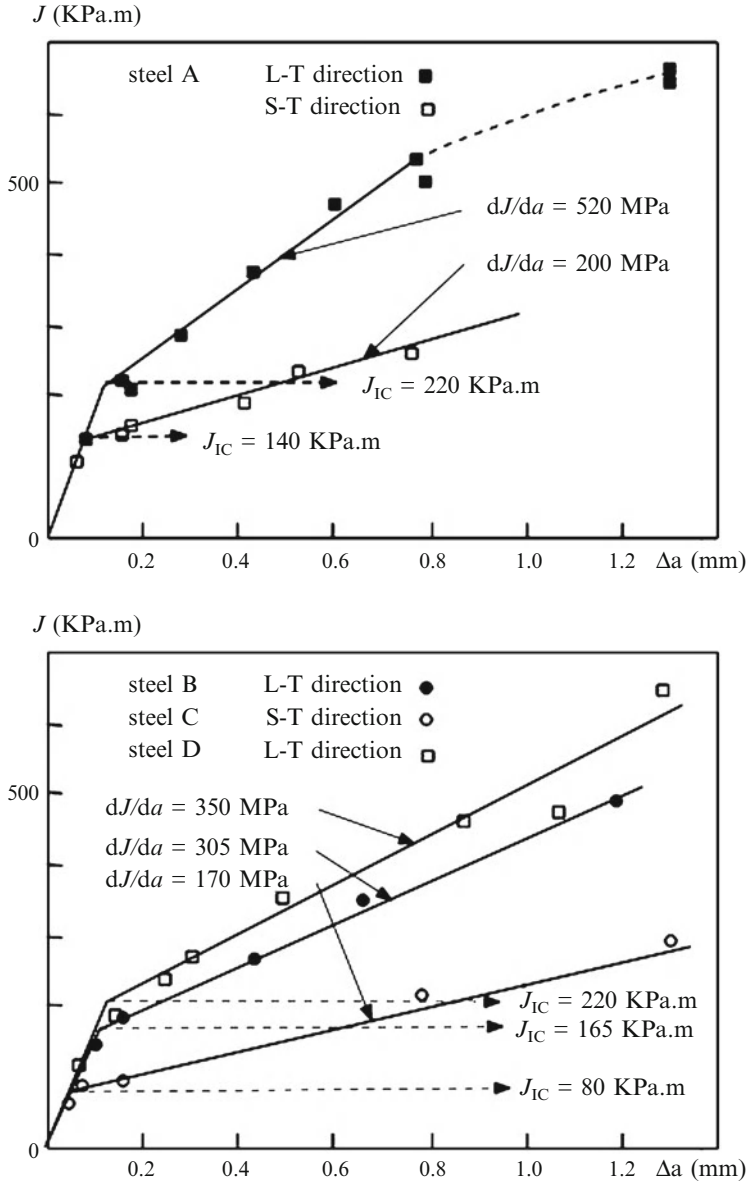


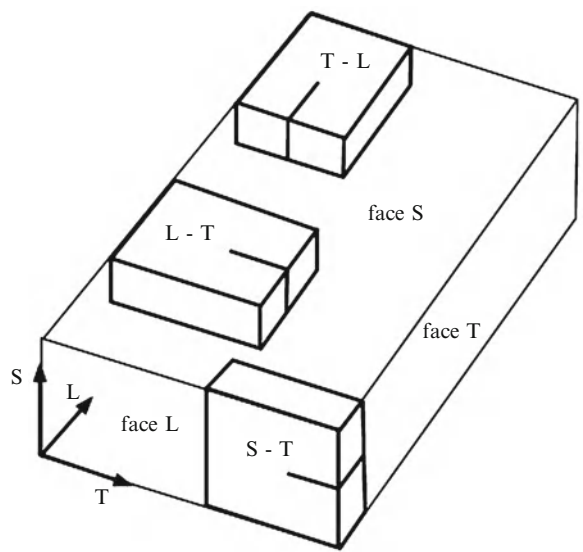
Fig. 4.38 *J*-Δ*a* resistance curves on various heats of A508 steel (0.15C, 1.5Mn, 0.70Ni, 0.50Mo) (*R_p* ~ 450 MPa). See Table 4.3 for the presentation of the four steels. The letters indicate the orientation of the specimens with the notations of Fig. 4.39 (Lautridou and Pineau 1981)

Table 4.3 Composition, inclusion characteristics and critical CTOD, δ_c (Lautridou and Pineau 1981)

Steel	S wt %	f %	$\bar{L}$ μm	$\bar{T}$ μm	$\bar{S}$ μm	N_a (S) mm^{-2}	N_a (L) mm^{-2}	N_a (T) mm^{-2}	N_v mm^{-3}	δ_c μm
A	0.010	0.05	15	10	5	4.8	11.8	10.8	943	
L-T										210
S-T										125
B	0.020	0.104	17.5	16	6	3.8	12.9	13.4	736	
T-L										190
C	0.013	0.067	7.6	7.9	2.6	11.3	22	18.5	3,200	
S-T										115
D	0.005	0.025	9	6.3	4	10.2	15.7	14.6	2,100	
L-T										200

N_a (S) = on face S

Fig. 4.39 Schematic diagram showing the orientation of the fracture toughness specimens used to determine the fracture resistance curves



The evolution of the key parameter controlling the ductile damage process, that is, the stress triaxiality, is schematically shown in Fig. 4.41a as a function of the distance to the crack tip normalised by δ_p . As shown in Fig. 4.41b, the maximum stress triaxiality is equal to 2.75 for perfectly plastic materials ($n = 0$) and increases with increasing strain-hardening exponent. These values of the stress triaxiality are correctly predicted by Hutchinson (1968) and Rice and Rosengren (1968) (the so-called HRR theory, see Chap. 2, Sect. 2.2.2.6 in this volume). The stress triaxiality drops down near the blunted crack tip due to the presence of the free surface and reaches the plane strain tension value $T = \sqrt{3}/3 \simeq 0.58$. The plastic strains become larger than a few percent at a distance equal to about 1–3 the CTOD, δ . The fracture plastic zone will thus extend in a zone equal roughly to 1–3 δ . The plastic strains up to a value equal to 0.10–0.20 are correctly predicted by the HRR field.

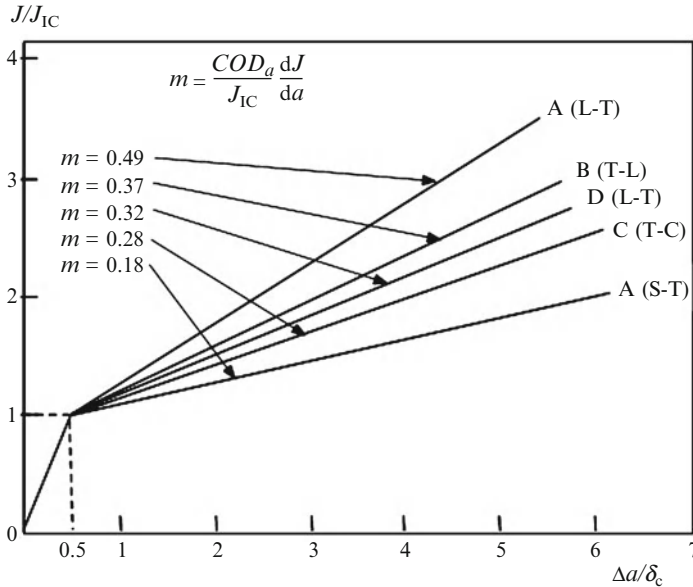


Fig. 4.40 Normalised fracture resistance curves obtained on four heats of A508 steel (0.16C, 1.5Mn, 0.70Ni, 0.50Mo) (see Table 4.3). δ_c designates the value of the critical CTOD at crack initiation (Lautridou 1980)

The fact that the stress triaxiality reaches large values ahead of the crack tip means that cavity nucleation will take place easily when this nucleation stage is stress-controlled (see *e.g.* Eq. 4.1). This suggests that, in many materials, the fracture toughness will be related to cavity growth and cavity coalescence.

The observation that J keeps increasing with the loading is extrinsic to the fracture process taking place in the near-tip region (see Fig. 4.42). J increases owing to the plastic dissipation taking place in the crack wake due to the progressive elastic unloading related to crack growth, to the non-radial loadings in the active PZ of a propagating crack and to changes in the crack-tip geometry as indicated earlier. In very large samples, a steady-state regime should in principle be reached, as indicated in this figure, but it is almost never observed because of specimen size limitations.

The results obtained by McMeeking (1977) for a non-hardening material with $R_p/E = 1/300$ are reported in Fig. 4.43a. These results show that the voids at 45 degrees experience faster earlier growth due to the larger amount of plastic strain occurring in this direction. The voids at 0 degree grow faster when they are close to the crack tip owing to the larger local stress triaxiality. The growth rates in a hardening material with $n = 0.10$ and $R_p/E = 1/300$, shown in Fig. 4.43b, are not much different from those in the non-hardening materials when measured in terms of the CTOD as in Fig. 4.43. However, when measured in terms of J , the growth rates in the hardening materials are lower than the rates in the non-hardening material (see Eq. 2.99b in Chap. 2 of this volume).

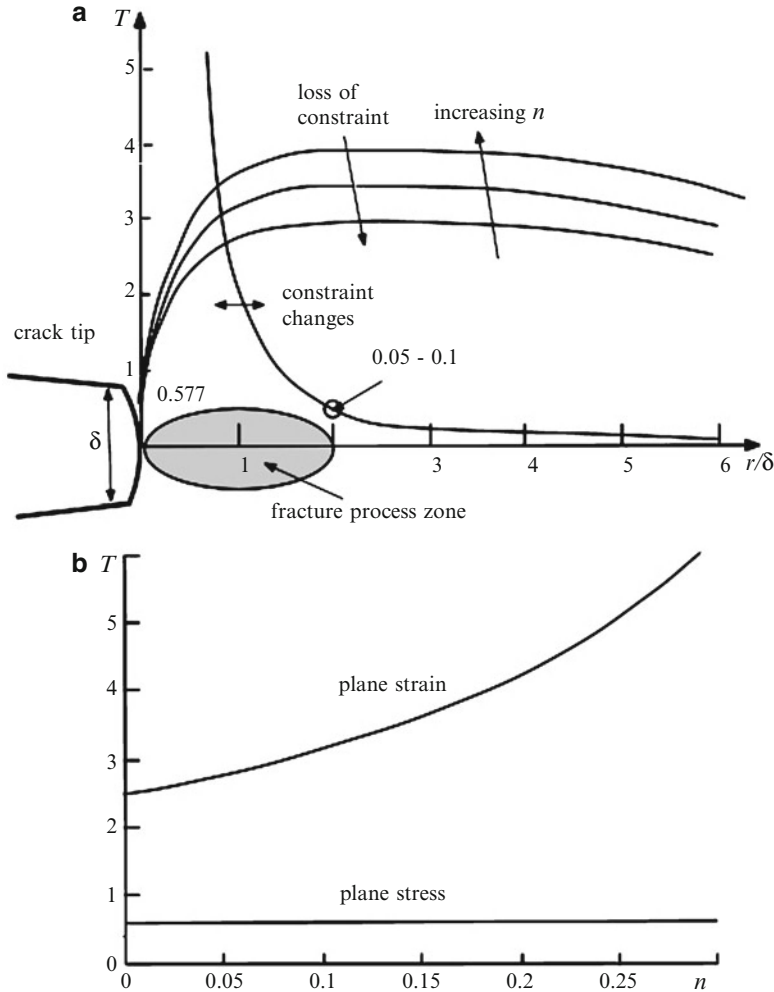


Fig. 4.41 Variation of the stress triaxiality as a function of (a) the normalised distance to the blunted crack tip; (b) the strain-hardening exponent for plane strain and plane stress conditions

These results do not include the effect of the distribution of inclusions. The probability of finding one inclusion in a given direction and at a given distance from the blunted crack tip must be taken into account. This statistical effect was considered in the analysis of crack tip ductile damage by Lautridou and Pineau (1981). These authors calculated the mean cavity growth rate using the McMeeking results and the Rice and Tracey expression (Eq. 4.10). They found the following mean value for the cavity growth:

$$\left\langle \frac{R}{R_0} \right\rangle = \int_0^x \frac{R}{R_0}(x/\delta, \theta) P(x, \theta, h) dV(x) \quad (4.35)$$

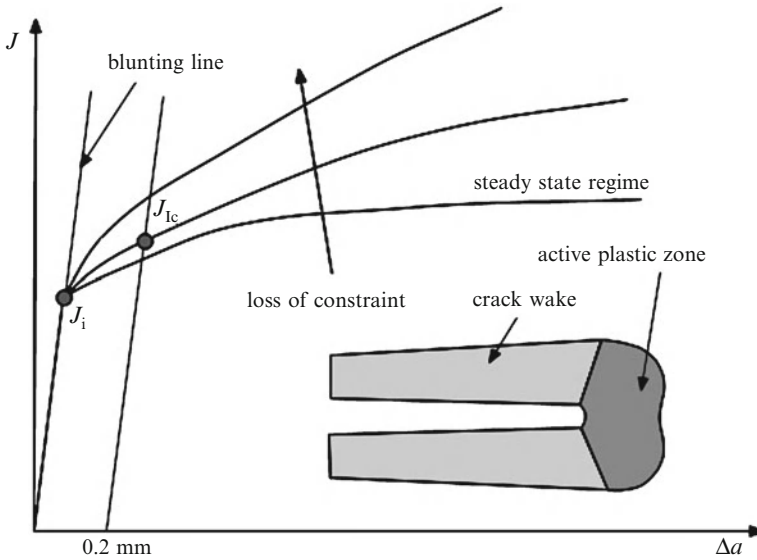


Fig. 4.42 Schematic J_R curve and plastic zone size extension during crack propagation. The loss of constraint produces an apparent increase of the slope of the J - Δa resistance curves

where $P(x, \theta, h)$ is the probability of finding one inclusion in the volume dV of the thickness h defined in Fig. 4.44a. In this figure, the angle 2θ was taken equal to 90 degrees since McMeeking reported the distribution of plastic strains and stresses in this sector.

The cavity growth R/R_0 was calculated as a function of the loading parameter, CTOD at each position in front of the crack only for $\theta \sim 0$, since the numerical results by McMeeking tend to indicate that the cavity growth does not change too much with θ in this sector. The elementary thickness parallel to the crack tip was defined as:

$$h = \frac{(N_{aL}N_{aT}N_{aS})^{1/3}}{N_V} \quad (4.36)$$

where N_{ax} is the number of inclusions per unit area in a plane perpendicular to the direction x and N_V is the number of inclusions per unit volume.

These calculations were applied to the steels presented in Table 4.3. The Rice and Johnson (1970) analysis was also used. The results are given in Fig. 4.44b, c. In these figures we have also reported the values of $(R/R_0)_c$ which were measured on notched specimens. These measurements were made by integrating the Rice and Tracey expression (Eq. 4.10) along the stress-strain path in the centre of notched specimens. Assuming that crack initiation takes place for the same value of $(R/R_0)_c$, it was then possible to compare the values of the CTOD, which were experimentally

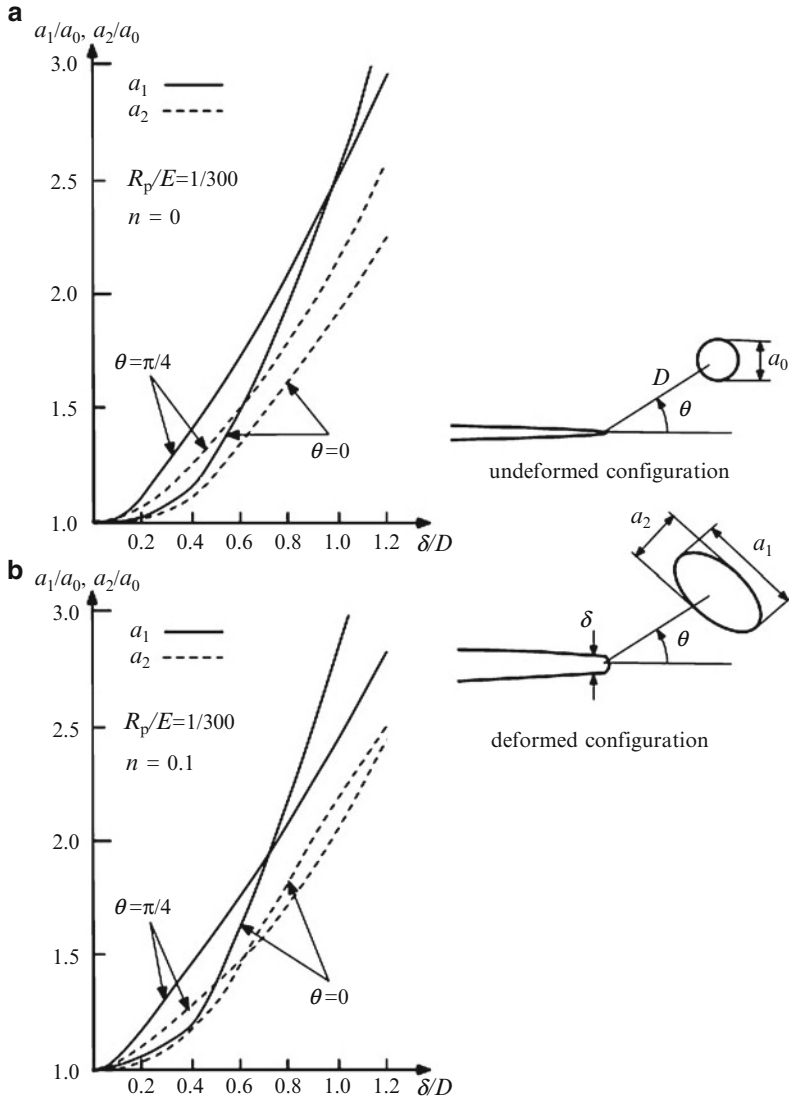


Fig. 4.43 Dimensions of a void growing in the near-tip field in a material $\sigma_0/E = 1/300$ as a function of crack tip opening CTOD and the distance from the crack front; (a) $n = 0$; (b) $n = 0.10$ (McMeeking 1977)

measured (from sections on CT specimens), and those which were calculated from Eqs. 4.35 to 4.36. The results are given in Fig. 4.45 where a good agreement between the calculated CTOD and the experimental CTOD is observed, except for material C tested in the short transverse direction. This might be related to the fact that in

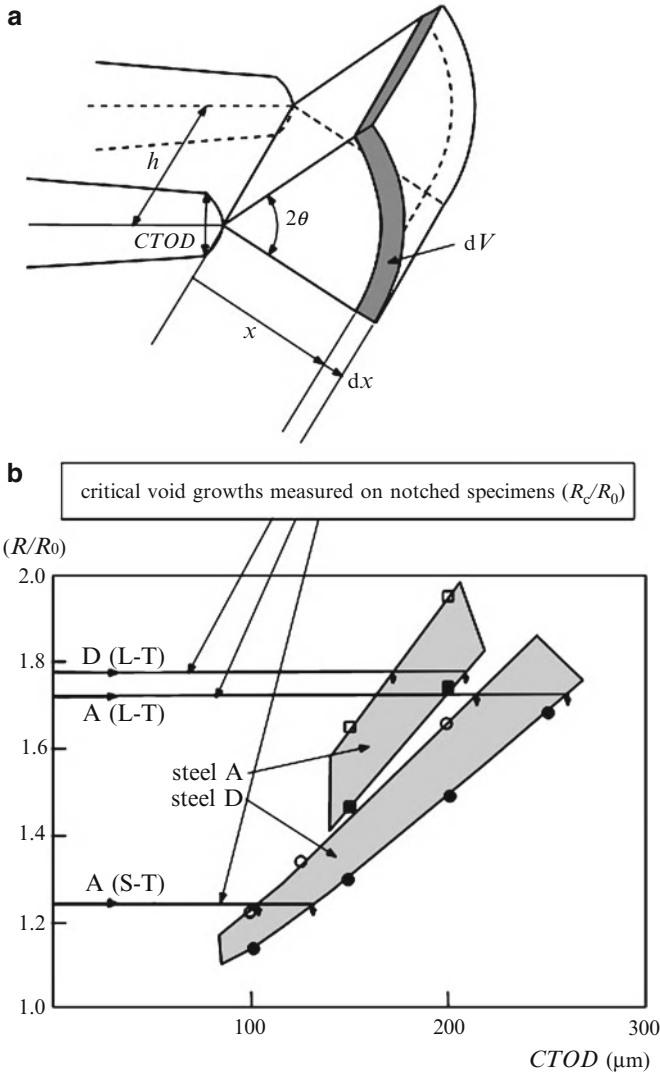


Fig. 4.44 (a) Definition of the elementary volume in front of a crack tip to determine the probability of finding one inclusion; (b) and (c) calculated variations of mean cavity growth in front of a blunted crack as a function of the crack tip opening displacement. *Full symbols* correspond to Rice and Johnson (1970) analysis. *Open symbols* refer to McMeeking (1977) calculations. The critical cavity growth measured on notched specimens is indicated horizontally

this material the inclusions were not randomly distributed. However these results remain encouraging because they indicate that, to some extent, it is possible to predict the fracture toughness of a given material knowing its strain to failure in notched specimens.

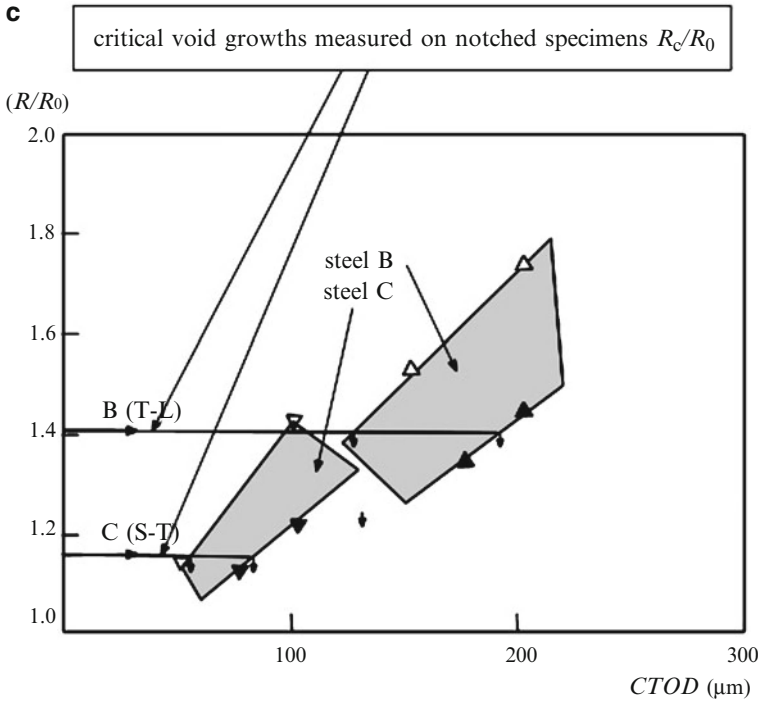


Fig. 4.44 (continued)

4.6.4 Computational Strategies to Simulate Ductile Crack Propagation

We have only dealt with ductile crack initiation up to now. Modelling ductile crack growth is far more difficult, essentially because of the lack of well accepted theories describing the stress-strain fields in front of a propagating crack. This explains why the strategies to simulate ductile crack propagation are based on numerical calculations. In this section four strategies are briefly presented. More details can be found elsewhere (Bilby et al. 1993; Pineau and Pardoen 2007; Pineau 2008).

4.6.4.1 The First Strategy

The “first strategy” consists in using the void nucleation, growth and coalescence laws introduced earlier in Sects. 4.2, 4.3, and 4.4. In this method the coupling effect between the mechanical fields and the damage evolution is neglected. This approach allows relating the fracture resistance to the microstructure (see *e.g.* the

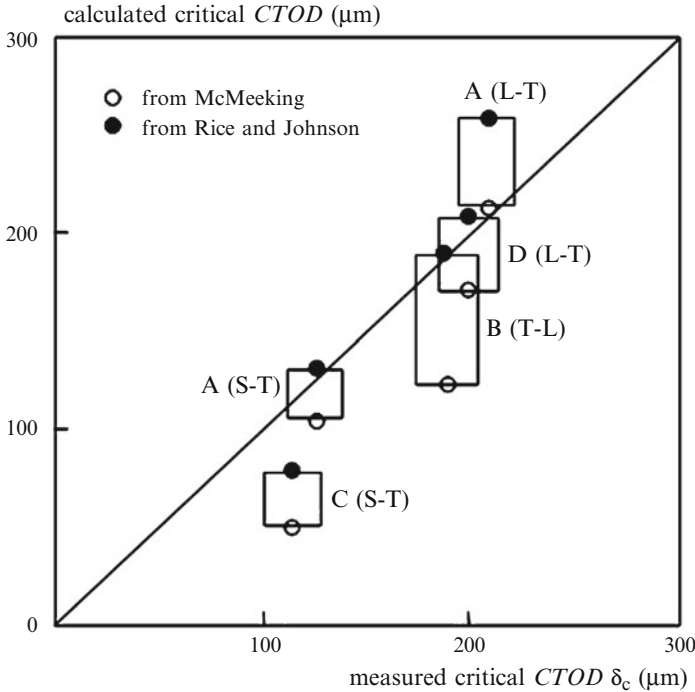


Fig. 4.45 Comparison between calculated and observed values of the critical crack tip opening displacement δ_{ci} in four steels (Lautridou and Pineau 1981)

model in terms of critical cavity growth presented in Sect. 4.6.3) and also allows simulating crack growth using FE methods with a node release technique. See *e.g.* the pioneering work by d’Escatha and Devaux (1979), which used a criterion of critical void growth to simulate ductile crack propagation. This approach has been largely used to simulate the effect of the loss of constraint effect on the shape of the J - R curves (Fig. 4.42). See also Rousselier et al. (1989).

4.6.4.2 The Second Strategy

The “second strategy” is illustrated in Fig. 4.46. FE calculations are performed with the voids explicitly modelled using refined meshes (see *e.g.* Aravas and McMeeking 1985a, b; Petti and Dodds 2005; Xia et al. 1995a, 1995b). This approach still requires a criterion for the final fracture of the intervoid ligament. This method can only be used to account for a few voids and is thus not attractive for simulating large structures, especially in three dimensions. Nevertheless such simulations provide useful results for assessing the validity of other approaches.

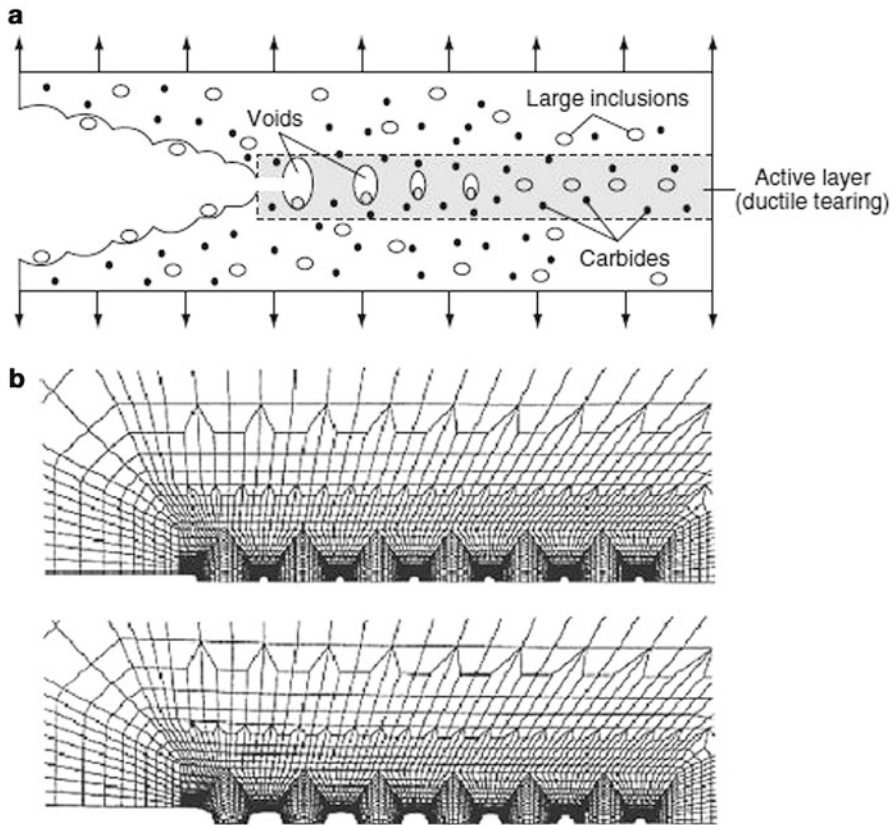


Fig. 4.46 A coupled model using discrete voids to simulate a more realistic situation: (a) realistic situation; (b) discrete voids in front of the crack tip with increasing CTOD (Gullerud et al. 2000; Tvergaard Hutchinson 2002)

4.6.4.3 The Third Strategy

The “third strategy” has been mainly developed in France, Germany, the United Kingdom and the United States. This approach uses a constitutive model such as GTN model, which accounts for the damage-induced softening effect (Fig. 4.47) (see *e.g.* Mudry et al. 1989; Ruggieri et al. 1996; Gao et al. 1998a, b; Koppenhoefer and Dodds 1998; Zhang et al. 2000; Rivalin et al. 2001a, b; Gullerud et al. 2000; Pardoen and Hutchinson 2003).

This strategy relies on a constitutive model implemented in a FE code to simulate the initiation and growth of the crack. A micromechanical damage model, such as the GTN model (and its extensions), is used in such a way that it should adequately reproduce the behaviour of a material cell. Near a crack tip strong stress and strain gradients develop at the scale of the void cell size. These gradients are roughly averaged by only using a single FE mesh.

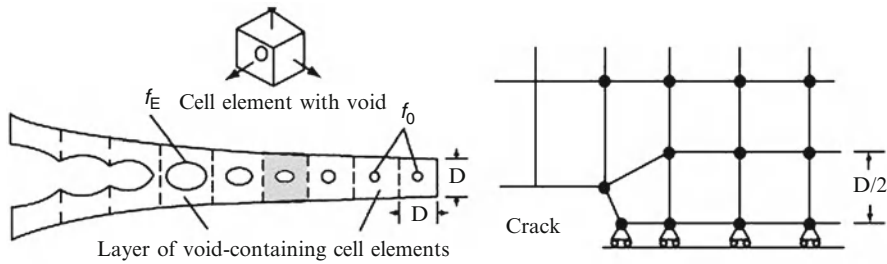


Fig. 4.47 Computational cell model to simulate ductile crack growth (Gullerud et al. 2000)

This approach requires thus the introduction of a length scale in the model, which is related to the mean spacing between cavities. This simple approach has the disadvantage to artificially tie a physical length to a numerical parameter. This is the reason why a number of research teams are working on formulating non-local constitutive models.

This technique based on computational cell models was used by Gao et al. (1999) to simulate the R curve of a low-strength ($R_p = 260$ MPa) high-hardening ($n = 0.20$) pressure vessel steel (2 ¼ Cr – 1 Mo) taken from a decommissioned chemical reactor. Details on the finite element meshes used by these authors are shown in Fig. 4.48.

Wallin (1993) reported an extensive set of test data on this steel including 105 tests on CT25 specimens. The Gurson model was used. The calibration of the model suggested the following values for the model parameters $q_1 = 2$, $q_2 = 0.77$, $f_0 = 0.0045$, $f_c = 0.20$. A linear force reduction versus cell elongation relationship for cell extinction was used to simulate cavity coalescence.

Figure 4.48 displays the FE mesh. A zone of highly refined mesh stretches out on the ligament ahead of the crack tip where crack propagation occurs (Fig. 4.48b). A row of uniformly sized void-containing cell elements with the dimensions $D/2 \times D/2$ ($D = 300 \mu\text{m}$) is embedded in the highly refined mesh directly ahead of the initial crack. Figure 4.49 compares the resistance curve predicted from the model with the experimental data. The agreement between model prediction and experimental data is quite satisfactory even for crack extensions as large as 5 mm. These results clearly show the efficiency of this third strategy to model ductile crack growth.

4.6.4.4 The Fourth Strategy

The “fourth strategy,” schematically illustrated in Fig. 4.50, was initiated by Tvergaard and Hutchinson (1996), Siegmund and Brocks (1999, 2000), Roy and Dodds (2001), Brocks et al. (2003), Cornec et al. (2003), Chen and Kolednik (2005), Pardoen et al. (2004), and Scheider et al. (2006). The response of the fracture process zone is modelled by a traction-separation curve as shown in Fig. 4.50. The main characteristics of this curve are the work of separation, G_0 and the maximum stress, σ_c , also named the cohesive stress.

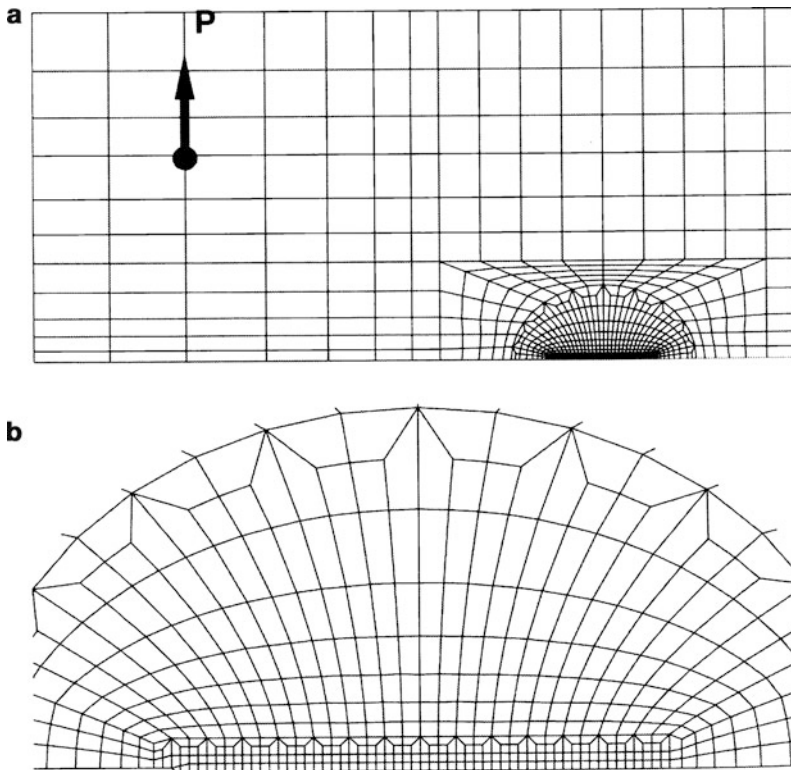


Fig. 4.48 Finite element model of a compact tension CT specimen for plane strain analysis and for simulating ductile crack propagation with a computational cell model; (a) half of the specimen; (b) region near the initial crack tip with cell elements with the dimensions $D/2 \times D/2$ (Gao et al. 1999)

4.6.4.5 Some Limitations

The four modelling approaches have all attained a relatively high level of maturity. However comparisons with experimental data similar to those shown in Fig. 4.49 are still extremely scarce. This explains why simple models as those introduced in the next session will continue to be largely used even if their predictive capability is poorer.

The approaches 1 and 3, which have just been introduced, are very comparable. Both of them have some difficulties. Their results are dependent on the mesh size, which has to be considered as a model parameter. The philosophy behind these models is that the cell size must be of the order of the mean distance between inclusions. These two approaches cannot predict the direction of crack growth, which can be a problem in the presence of mixed mode loadings or in the presence of strength inhomogeneities such as those found in welds. The direction

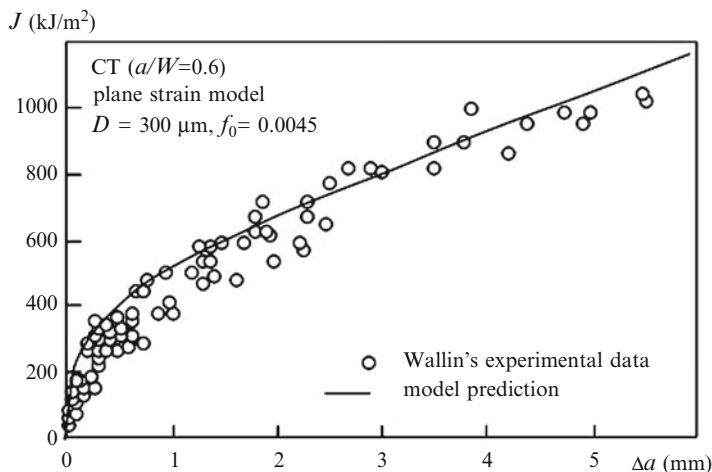


Fig. 4.49 Comparison of the predicted crack growth resistance curve from the plane strain model with the experimental data (Gao et al. 1999)

of crack growth is imposed by the geometry of the FE mesh. Moreover these approaches cannot fully account for one of the most challenging problem in ductile fracture linked to the competition between flat and slant fractures (Fig. 4.51). This competition is shortly described below.

The most important structural metallic materials (Al alloys, high strength steels, etc.) exhibit a slant fracture mode when they are tested in the shape of thin sheets. As schematically shown in Fig. 4.51a, and as observed in Fig. 4.51b, the crack starts propagating in a flat mode I with significant tunneling effect adopting triangular shape. After moderate amount of crack growth, shear localisation occurs at the free surfaces and the crack plane tilts at an angle, which depends on the plastic anisotropy of material. As a result, the fracture surface shows a first “triangle” of flat fracture before the profile becomes slanted. The conditions, which favor this slant fracture mode, are discussed elsewhere in some detail (Pineau and Pardoen 2007).

The determination of a general predictive criterion for slant fracture and shear localisation in thin sheets appears to be a difficult task. The complexity is related to the 3-D character of the boundary value problem and to the complexity of realistic (in particular anisotropic) elastoviscoplastic constitutive laws of metallic materials. Hence only full 3-D numerical simulations can provide quantitative insights into the competition between slant and flat fracture in thin sheets. There were only a few number of studies devoted to the simulation of the slant fracture able to capture the initial complex crack tilting. However one can refer to the work by Mathur et al. (1996) for 3-D dynamic simulation involving two populations of voids as well as adiabatic heating and associated thermal softening effect. See also Rivalin et al. (2001a, b) for the simulation of crack growth over relatively large distances and at high crack propagation velocity, Besson et al. (2001) for the comparison of

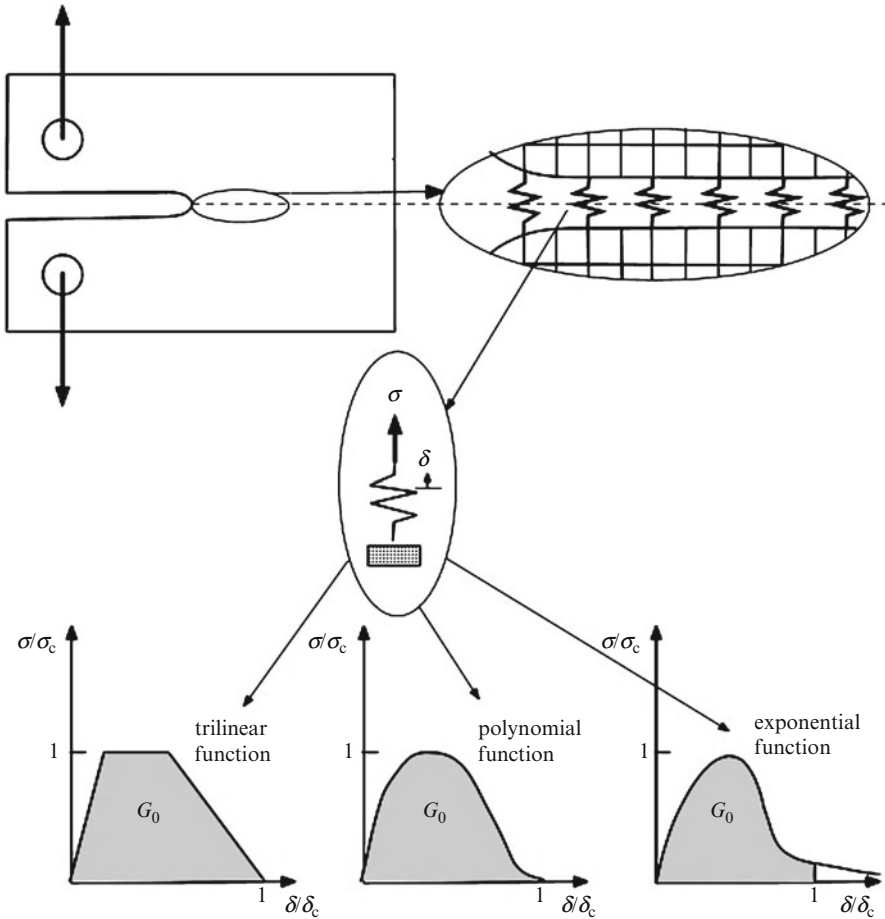


Fig. 4.50 Simulation of ductile crack growth based on cohesive zone models

the simulation of slant fracture in plane strain specimens using Rousselier (1987), Gurson (1977) models, and Morgeneyer and Besson (2011) for the simulation of flat to slant ductile fracture transition.

4.6.5 Simplified Models for Predicting the Fracture Toughness

4.6.5.1 Metallurgical Models

The simplest models are those based on the knowledge of the volume fraction of inclusions. One of the first models of this type was introduced by Hahn and

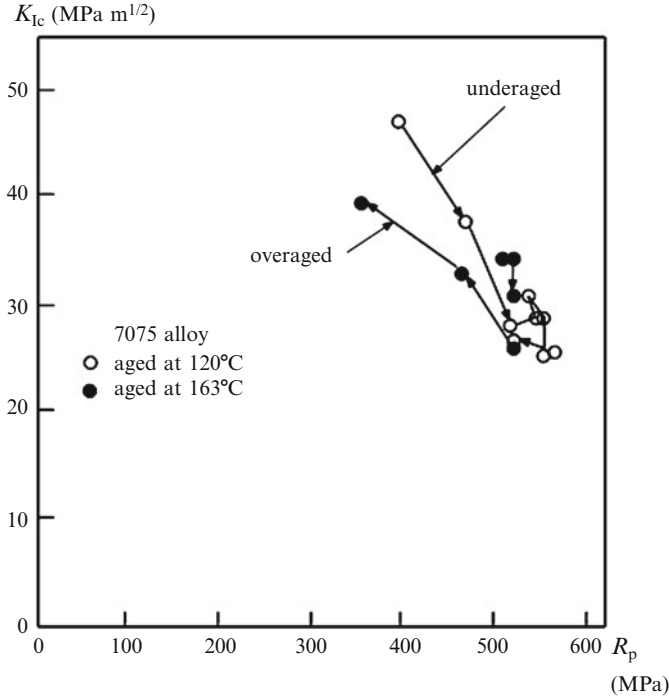


Fig. 4.52 Effect of ageing on the fracture toughness and the yield strength of 7075 aluminium alloys. Arrows indicate trends with increasing ageing time (Hahn and Rosenfield 1975)

where D is the particle diameter and f the volume fraction of inclusions. A relatively good agreement was found when the results predicted from this equation were compared to experimental results (Fig. 4.37).

This simple analysis predicts a relatively small effect of the volume fraction of inclusions. It predicts also that the fracture toughness increases with increasing yield strength, which may appear to be contradictory. However the situation is much more complex as observed in a 7075 aluminium alloy and as shown in Fig. 4.52 (Hahn and Rosenfield 1975). This figure indicates that for the same yield strength the under-aged material leads to larger values of K_{Ic} as compared to the over-aged material. This might be related to the fact that in over-aged alloys ductile intergranular fracture also takes place.

Ductile crack initiation can be idealised as a 2-D problem. It seems therefore more appropriate to compare the critical CTOD, δ_c , to the mean distance, λ_a , between inclusions in a plane perpendicular to the crack front. Stereology teaches that this distance can be expressed as (Underwood 1972; de Hoff 1963):

$$\lambda_a = \frac{1}{2\sqrt{N_a}} \simeq \delta_c \simeq \frac{1}{2} \frac{K_{Ic}^2}{E R_p} \quad (4.38)$$

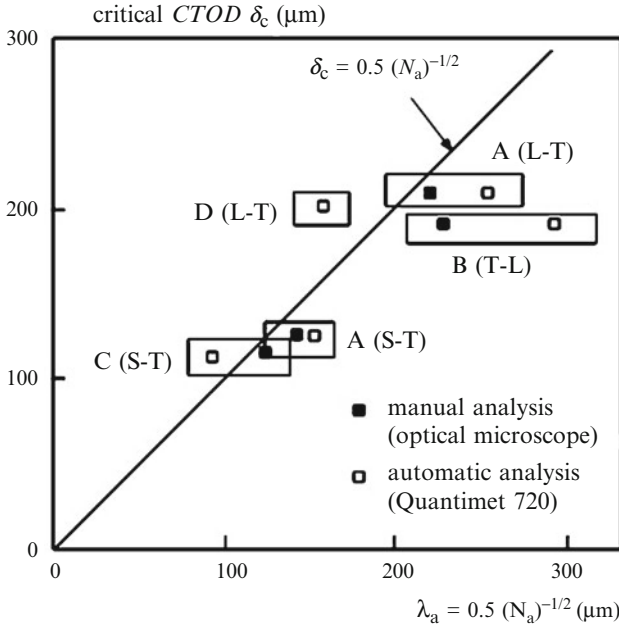


Fig. 4.53 Correlation of the critical CTOD δ_c with the mean distance λ_a between inclusions in a plane perpendicular to the crack front. The letters *A, B, C, D* indicate the steels (Table 4.3). The pairs of letters such as *(L-T)* designate the orientation of the specimens (see Fig. 4.39) (Lautridou and Pineau 1981)

where N_a is the number of particles per unit area in that plane. The values of N_a and δ_c were measured in the four steels which were studied in detail (see Table 4.3, Lautridou and Pineau 1981). The comparison between the two quantities is made in Fig. 4.53 where a relatively good one-to-one correspondence is observed. However this correlation has strong limitations since it predicts the same δ_c for specimens cut in such a way that the thickness direction of the specimens is the same, but with different directions of crack propagation, that is for instance LT and TL or TS and SL (see Fig. 4.39). It is clear that differences between these orientations can be found in strongly textured materials.

In spite of this difficulty it is worth adding that a similar approach was developed to predict the slope of the crack growth resistance curve, $J-\Delta a$. It was shown that the non-dimensional slope $(\delta_c/J_{Ic}) dJ/da$ correlates very well with the product $R_s (N_v)^{1/3}$ with $R_s = (s_p/\pi)^{1/2}$ where s_p is the mean area of an inclusion projected onto the fracture surface. This means that:

$$\frac{\delta_c}{J_{Ic}} \frac{dJ}{da} \simeq R_s (N_v)^{1/3} \quad (4.39)$$

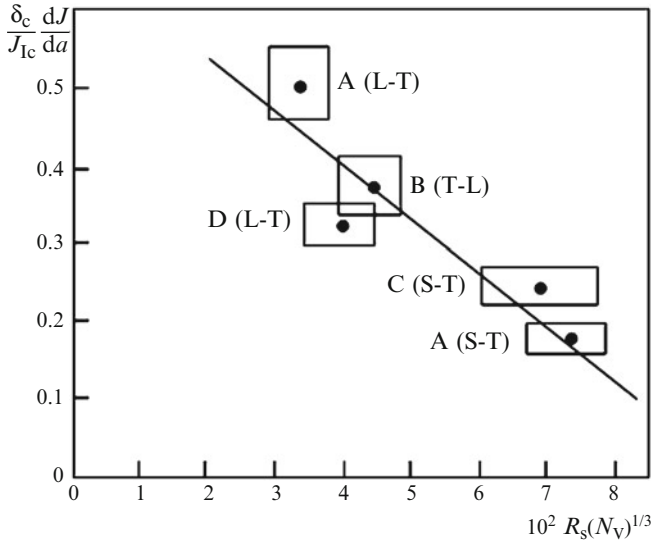


Fig. 4.54 Relation between a non-dimensional crack resistance parameter and the product $R_s(N_v)^{1/3}$ (Lautridou and Pineau 1981)

as shown in Fig. 4.54. These results indicate that it is possible to predict at least the tendency of the variations for the critical CTOD and for the slope of the resistance curves, $J-\Delta a$, provided that a detailed analysis of the inclusions is made. It should be kept in mind that this conclusion applies to a given class of materials (*e.g.* low strength steels with MnS inclusions).

4.6.5.2 A Simplified Criterion Based on Critical Void Growth Rate

As stated previously the most widely used criterion for the onset of void coalescence states that cavity coalescence starts at a critical porosity. Several experimental and numerical studies (*e.g.* Koplik and Needleman 1988; Tvergaard 1990; Brocks et al. 1995; Marini et al. 1985; Becker 1987; Pardoen and Delanay 1998a, 1998b) assessed the validity of this attractive fracture criterion (see *e.g.* Figs. 4.24 and 4.45). Figure 4.24 showed that the volume fraction of voids at fracture was almost independent of the initial volume fraction, at least when f_0 is sufficiently small ($\sim 10^{-4}$). This criterion is attractive because of its simplicity and it can easily be calibrated from tests on notched specimens with stress triaxiality lower than those corresponding to the crack tip situation. In the absence of tests on notched bars, simple tensile tests can be used provided that the variation of the stress triaxiality ratio $T = \Sigma_m/\Sigma_{eq}$ in the centre of the neck is taken into account.

As already discussed a full analysis requires the statistical distribution of inclusions. The results on steels presented in Fig. 4.44 and showing the calculated

variation of the mean void rate $\langle R/R_0 \rangle$ with the applied CTOD can then be used to determine the critical CTOD at which the calculated $\langle R/R_0 \rangle$ is equal to the value determined from notched specimens. The comparison between the calculated values of δ_c with the experimentally determined values of this parameter was made in Fig. 4.53 (Lautridou and Pineau 1981) where a good agreement was observed.

These results indicate that a simple criterion in terms of critical void growth rate calibrated from tests on notched specimens can be used to estimate the values of the fracture toughness at crack initiation.

4.6.6 Recapitulation

Rough estimates of the ductile fracture toughness can be obtained from simple equation relating the critical crack tip opening displacement, δ_c , to the mean distance between inclusions or second-phase particles.

The ductile fracture toughness can also be evaluated from tests on notched specimens assuming that the calculated critical void growth rate $(R/R_0)_c$ obtained from finite element calculations is the same, that is that $(R/R_0)_c$ is independent of stress triaxiality. This assumption is verified for low volume fraction of inclusions thanks to finite element calculations.

These predictions apply to materials in which cavity nucleation from inclusions occurs easily at the onset of plastic deformation. It is much more challenging to predict the ductile fracture toughness in materials exhibiting a strong resistance to cavity nucleation.

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Chapter 5

Ductile-Brittle Transition

Abstract For many materials impact tests display a ductile-brittle transition when temperature is lowered. Analyses of the limit moment of notched test pieces and of the stress distribution are given, especially for the Charpy specimen. Testing procedure including instrumented impact testing are described. Various ductile-brittle transition temperatures (DBT) can be defined. They are related to metallurgical variables. Drop weight tests are more representative of in-service situations. The failure analysis diagram puts together the results. Correlations exist between them. Modelling of test pieces, especially of the Charpy specimen, provides ways to predict the DBT.

5.1 Introduction

Ductile-brittle transition is mostly studied by impact tests. The name of Charpy¹ remains associated with impact testing on notched specimens. He published his first results in 1901 at a congress in Budapest, as well as in the Société des Ingénieurs civils de France (Charpy 1901). Originally this test was used mainly as a quality control test (see Volume I, Sect. 1.3.5.5).

During World War II a great deal of attention was directed to the brittle failure of welded Liberty ships and T-2 tankers (Williams 1954). Out of 4,694 ships made during the war, 1,289 experienced serious or potentially serious fractures. The ship plates where the cracks had initiated showed generally a lower impact energy (C_V) at the failure temperature (see *e.g.* Wallin et al. 2002a, b). This led to the introduction of the *transition temperature* concept, which is the subject of this chapter. Failures due to brittle fracture are still occurring and remain a subject of great concern as illustrated in Fig. 5.1. For a review of case studies, see *e.g.* Pelloux (1991).

¹Augustin Georges Albert Charpy (1865–1945) was a French engineer, scientist and entrepreneur. He proposed the pendulum which bears his name in 1901.

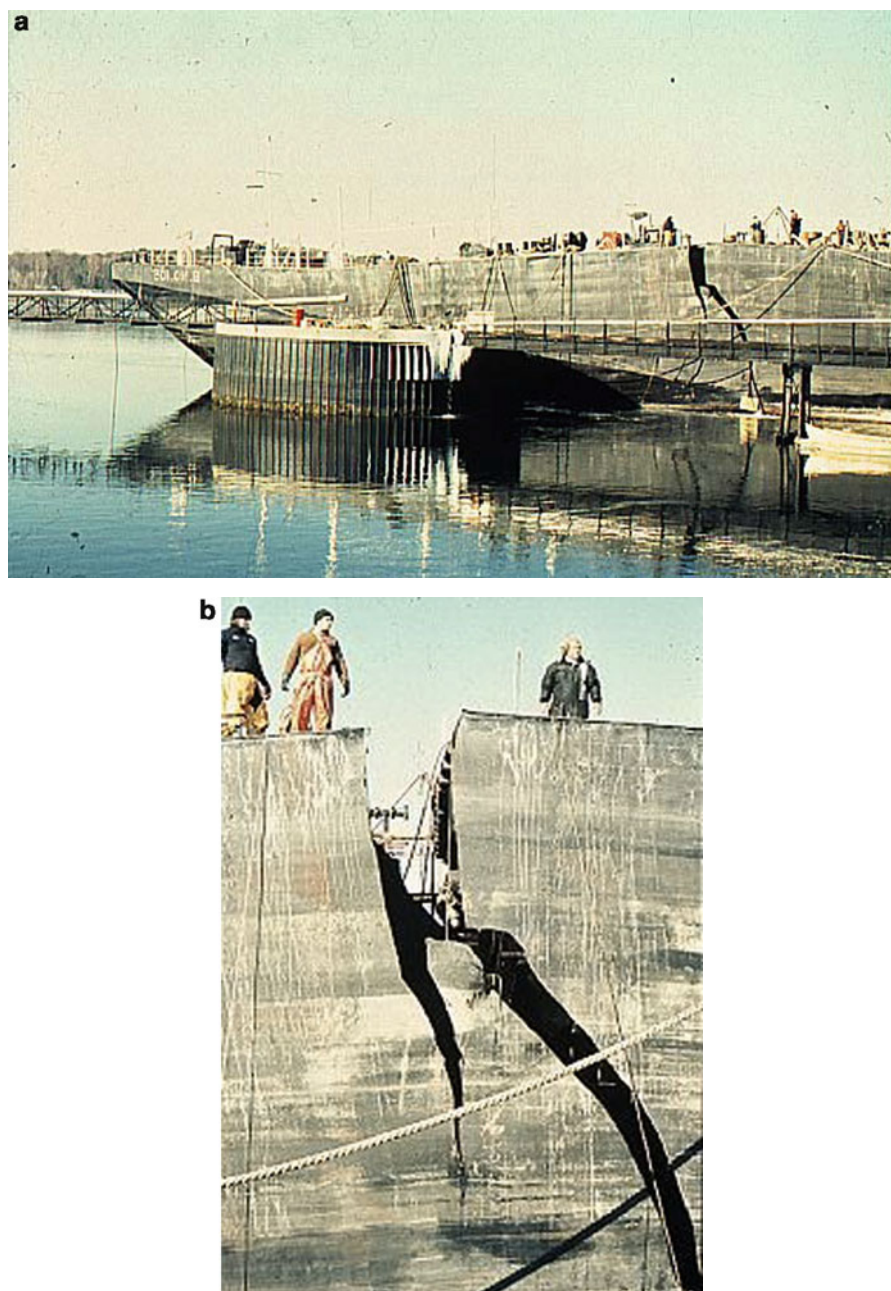


Fig. 5.1 (a) Fracture of a barge; (b) Enlargement (courtesy R.M. Pelloux). For similar case studies, see *e.g.* Pelloux (1991)

As indicated in Chaps. 3 and 4 of this volume, three basic factors contribute to a brittle-cleavage type of fracture. They are: (i) a triaxial state of stress, (ii) a low temperature, and (iii) a high strain (or loading) rate. The stress triaxiality is largely dependent on the geometry and is favoured by the existence of notches, as underlined in Chap. 1 of Volume I (Sect. 1.3.3.3) as well as in Chap. 2 of this volume (Sect. 2.7.3). This explains why in the present chapter we start with the analysis of notched specimens, in particular bend specimens, since the Charpy tests are performed under bending stress. In the transition regime a ductile crack is initiated from the notch (or from the crack tip when fracture mechanics specimens are tested), propagates in a stable manner through the remaining section of the specimen over a given distance (which is largely dependent on test temperature) and then abruptly leads to a cleavage crack. Both ductile and cleavage fractures have been studied in Chaps. 3 and 4 of this volume. In these chapters we essentially considered stationary cracks or notches. The present chapter is devoted to a more complex situation, *i.e.* that of a ductile propagating crack leading to ultimate brittle-cleavage fracture. Recent advances in the modelling of this phenomenon largely observed in ferritic steels have been made over the past decade. These results are presented in the third part of the present chapter.

In a number of materials the ductile-to-brittle transition (DBT) is related to the nucleation and the mobility of dislocations from a stressed crack tip. This is the situation in materials like *silicon* or *tungsten*. Armstrong (1966) and Kelly et al. (1967) advanced the viewpoint of brittle versus ductile response as the competition between Griffith cleavage and plastic shear straining at the crack tip. Kelly et al. (1967) proposed that the response of a crystal should be treated by comparing the ratio of the largest tensile stress to the largest shear stress close to a crack tip with the ratio of the theoretical cleavage stress (Sect. 3.2.2.1) to the ideal shear stress (Volume I, Sect. 3.3.2.2). Armstrong (1966) compared the applied stress necessary to satisfy the Griffith condition with the stress to shear apart a dislocation dipole near a crack tip, and thereby noted the importance of the dimensionless number $\gamma_s/\mu b$ where γ_s is the surface energy, μ the shear modulus and b the magnitude of the Burgers vector. Subsequently Rice and Thompson (1974) specifically investigated the shear process as the nucleation of dislocations from a stressed crack tip. This model of intrinsic brittleness specific to a number of materials has been presented in Chap. 3 (Sect. 3.2.3). We will deal again with this aspect of the DBT in the present chapter.

The need for statistical modelling of cleavage fracture initiation has been acknowledged during the past decades (see Chap. 3 in this volume). In this chapter a number of models describing the behaviour of fracture toughness in the cleavage fracture temperature region have been presented. Most of them are based on the assumption of weakest link statistics, *i.e.* one single critical event is sufficient to cause macroscopic failure. In the DBT region, the cleavage fracture initiation is more complex. In this region the fracture initiation is affected by large-scale yielding effects as well as by the effect of ductile tearing prior to cleavage fracture initiation.

Some attempts have been made to develop statistical models to also describe this temperature region. The increased complexity is reflected upon the existing models which are no longer analytical but essentially numerical. This explains why there remain so many empirical relations describing this transition.

5.2 Notched-Bar Impact Testing

5.2.1 Mechanics of Notched Bend Bars

The Charpy test piece is the subject of an international standard (ISO 148-1). It is a square section, 10×10 mm specimen; the notch is V-shaped with angle 45° , depth 2 mm and radius of curvature at the root ρ of 0.25 mm (Fig. 5.2).

We now give an approximate treatment of this specimen geometry under static loading, although Charpy specimens are more usually subjected to the impact of a heavy swinging pendulum (the impact velocity is approximately 5 m/s; the specimen is forced to bend and fracture at a high strain rate on the order of 10^3 s^{-1}).

The limit load for a test piece with a deep notch can be determined by conceiving a kinematically admissible stress field consisting of two plastic hinges around two circular arcs of angle 2α (Fig. 5.3a). If k is the yield strength in shear, B the thickness, W the height and a the notch depth, an upper bound for the limit moment is found to be:

$$M_L = 0.69kB(W - a)^2 \quad (5.1)$$

For comparison, the limit moment for a beam of height $(W - a)$ is:

$$M_{L0} = 0.50kB(W - a)^2 \quad (5.2)$$

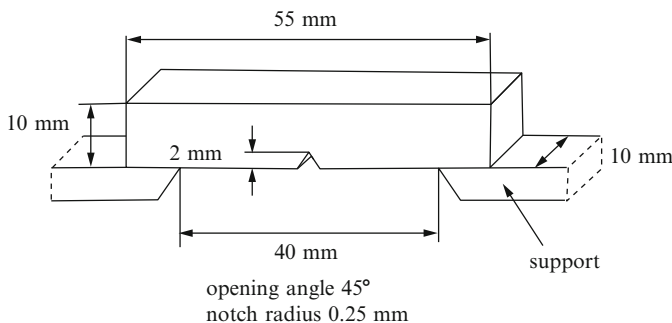


Fig. 5.2 Charpy V notch specimen

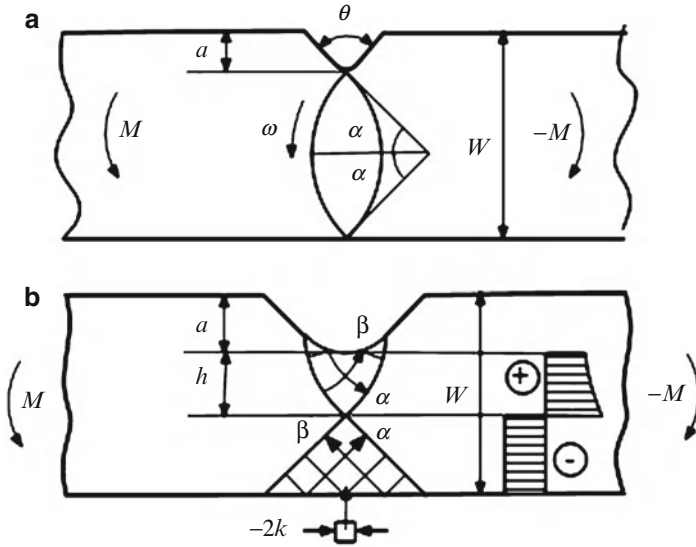


Fig. 5.3 Slip lines in a bend test specimen with a deep notch; two suggested solutions: (a) and (b)

With this geometry the value of the plastic constraint factor M_L/M_{L0} is 1.38. A more complex field of slip lines can be constructed as in Fig. 5.3b. The Prandtl slip line field has already been introduced in Chap. 2 of this volume (Sect. 2.3.4.2). The complex slip line field leads, by more complex calculations, to a value of $M_L/M_{L0} = 1.28$.

The slip line field in the region of the root of the notch enables the stress distribution to be calculated, assuming plane strain conditions and perfect plasticity. This field consists of logarithmic spirals, forming an angle of 45 degrees with the isostatic lines normal to the surface of the notch. The result on the axis of symmetry is:

$$\sigma_{\theta\theta} = 2k \left[1 + \log \left(1 + \frac{x}{\rho} \right) \right] \quad (5.3)$$

where x is the distance from the notch root of radius ρ .

This stress reaches a maximum at a distance x_t , which depends on θ , the angle of the notch:

$$\sigma_{\theta\theta}^{\max} = 2k \left(1 + \frac{\pi}{2} - \frac{\theta}{2} \right) \quad (5.4)$$

with

$$\frac{x_t}{\rho} = \exp \left(\frac{\pi}{2} - \frac{\theta}{2} \right) - 1 \quad (5.5)$$

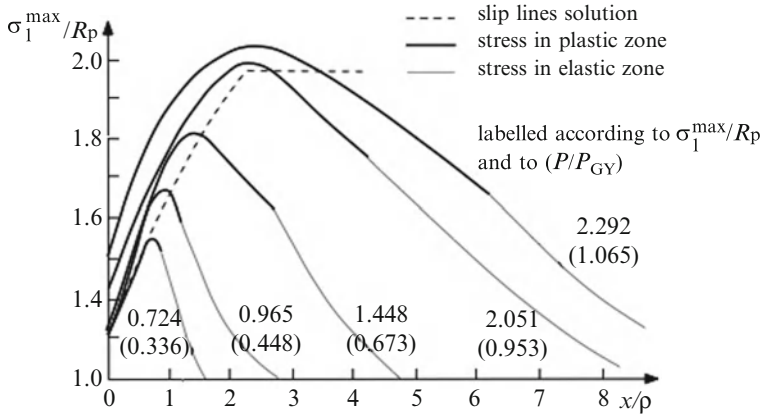


Fig. 5.4 Results of finite element and analytical calculation for a bend notch bar; variation of the principal stress with distance from the notch tip; full curves are for finite element calculations, others are based on slip lines calculations (Griffiths and Owen 1971)

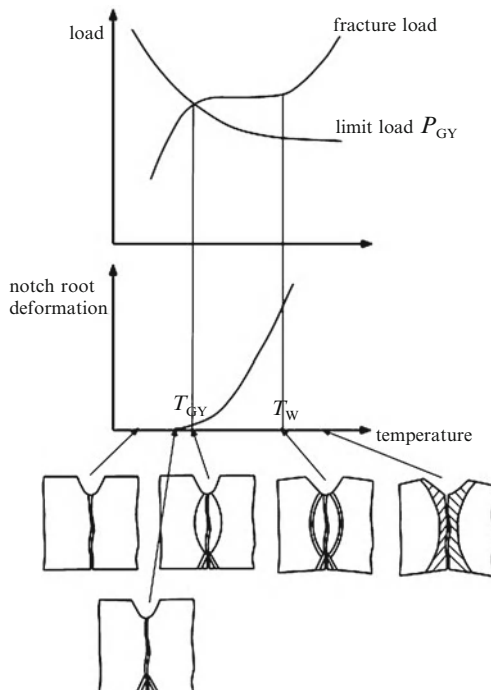
Equations (5.4) and (5.5) lead, for the Charpy V test piece, to $\sigma_{\theta\theta}^{\max} = 2.18 R_p$ (Tresca criterion) or $\sigma_{\theta\theta}^{\max} = 2.51 R_p$ (von Mises criterion) and $x_t = 1.53$ mm.

Figure 5.4 compares results calculated in this way with those of finite element (FE) calculations (Griffiths and Owen 1971) obtained on geometry similar to that of Charpy V specimens. Plastic constraint at the notch produces a triaxial state of stress. Thus, the standard Charpy V-notch specimen provides a severe test for brittle fracture.

This static analysis of notched bend specimens was largely used to measure the cleavage stress in mild steels. The principle is indicated in Fig. 5.5 (Knott 1973). The general yield curve gives the load P_{GY} (or the limit moment, M_{LY}) needed to spread yielding completely across the net section of the specimen. M_{LY} can simply be related to the uniaxial yield stress (Eq. 5.1 or results of FE calculations). The temperature T_{GY} , is the temperature below which the fracture is macroscopically (or *mechanically*) brittle. The sequence of graphs in Fig. 5.5 shows how the amount of yielding preceding fracture increases with temperature. To a first approximation, this temperature, T_{GY} may be considered as analogous to the nil-ductility temperature (NDT) in impact testing (see Sect. 5.2.4.1). Assuming that fracture occurs at T_{GY} , it is possible to calculate the cleavage stress, using Eq. 5.4. Changing the radius of the notch, ρ , or the flank angle, θ , of the notch, it is possible to measure the cleavage stress as a function of temperature.

In mild steels, the cleavage stress was found to be independent of test temperature (Knott 1973). However this method has some limitations. In several steels it is observed that fracture occurs after large macroscopic deformation, at a temperature T_W higher than T_{GY} . Moreover the analysis is based on plane strain assumption, although the mechanics of notch-bend specimen is 3-D. This is one reason why axisymmetric specimens are preferred by a number of researchers to measure the cleavage stress (see Chap. 2, Sect. 2.7.3 in this volume).

Fig. 5.5 Variation with temperature of limit load (P_{GY}), fracture load and deformation at the notch tip for a Charpy V notch specimen. Critical temperatures, T_{GY} and T_W , are defined. Schematic variation of the plastic zones with temperature



5.2.2 Charpy V-Notch Impact Tests

The principal measurement from the impact test is the energy absorbed in fracturing the specimen, C_V . After breaking the test bar, the pendulum swings through to a height, which decreases as the amount of energy absorbed in fracture increases. This energy is recorded directly on a dial gauge. Sometimes this energy is divided by the surface area of the specimen (0.8 cm^2). This gives the resilience, KC_V (J/cm^2), which by definition is equal to $1.25 C_V$ (Joules). It is important to remember that the Charpy test is not a specific test because the values that are measured cannot be put into a fracture mechanics equation that can be directly used for design purposes. However it will be shown that a number of correlations between the Charpy energy and the fracture toughness were proposed (see Sect. 5.2.7).

Typical transition-temperature curves are illustrated in Fig. 5.6. This figure shows how reliance on impact energy at only one temperature can be misleading. Steel A shows higher notch toughness at room temperature. However its transition temperature is higher than that of steel B. The material with the lowest transition temperature is to be preferred.

Notched bar impact tests are subject to considerable scatter, particularly in the region of the transition temperature. Most of this scatter is due to local variations in the properties of the materials, while some is due to difficulties in preparing perfectly reproducible notches and to the proper placement of the specimen in the impact machine.

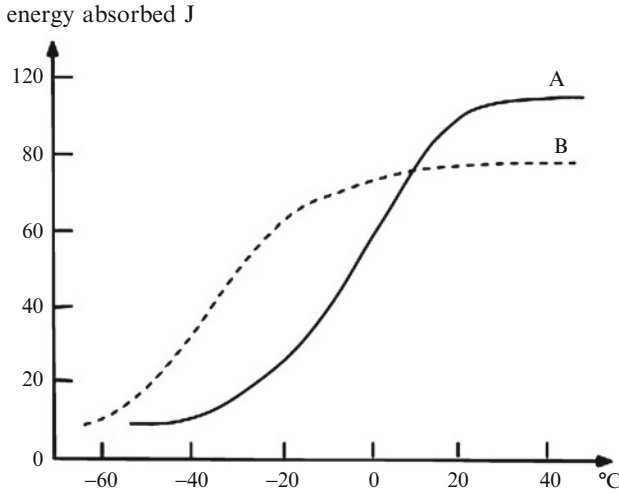


Fig. 5.6 Transition-temperature curves for two steels

Analytical expressions are used to represent the shape of the Charpy curves. The most widely used is:

$$C_V = A + B \tanh\left(\frac{T - T_0}{C}\right) \quad (5.6)$$

where $A + B$ is the Charpy energy at the upper shelf, $A - B$ is the Charpy energy at the lower shelf, T_0 is the temperature in the middle of the transition ($C_V = A$) and C represents the slope of the transition.

Fracture surfaces of Charpy specimens tested at different temperatures are shown in Fig. 5.7. One can observe a gradual decrease in the granular region and increase in lateral contraction at the notch with increasing temperature. At low temperature, in the lower shelf, the fracture surface is covered by cleavage facets. Intergranular fracture can also be observed in certain circumstances (see Chap. 3, Sect. 3.3). At high temperature, in the upper shelf regime, the fracture surface is covered by ductile dimples. In between these two extreme conditions, one observes ductile crack initiation and growth from the notch followed by cleavage fracture. There are few results in the literature giving the crack growth Δa , as a function of the Charpy impact energy. Results obtained on pressure vessel steel (0.16C, 1.6Mn, 0.7Ni, 0.50Mo) are reported in Fig. 5.8. These $C_V - \Delta a$ curves are, to some extent, analogous to the J_R curves for ductile fracture. Figure 5.8 shows that the slope of these curves is steeper with increasing impact rate, v_0 . This observation is in qualitative agreement with the theories on ductile fracture developed in Chap. 4 of this volume.

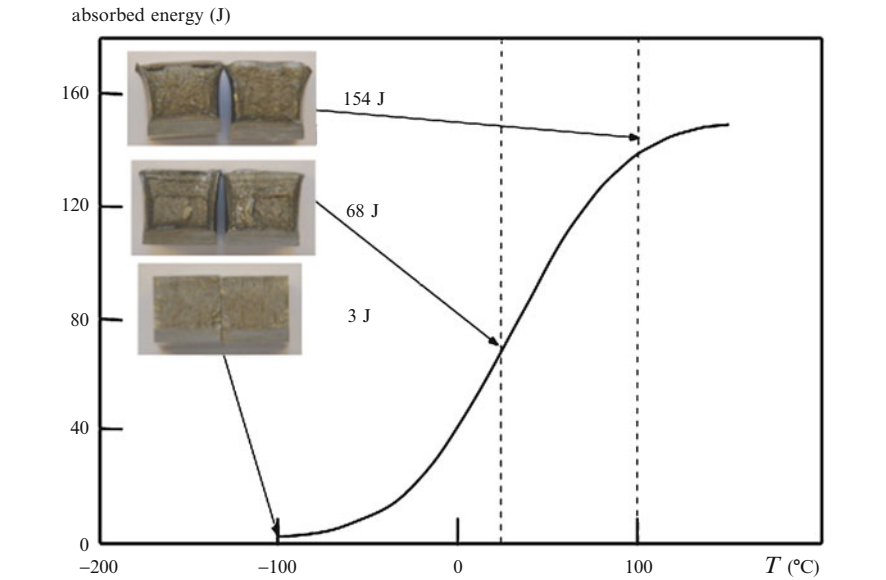


Fig. 5.7 Fracture surfaces of Charpy specimens tested at different temperatures (courtesy of A. Andrieu)

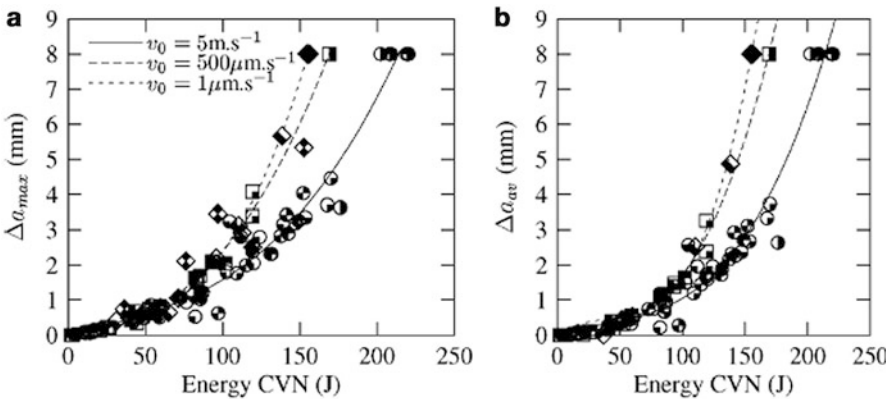
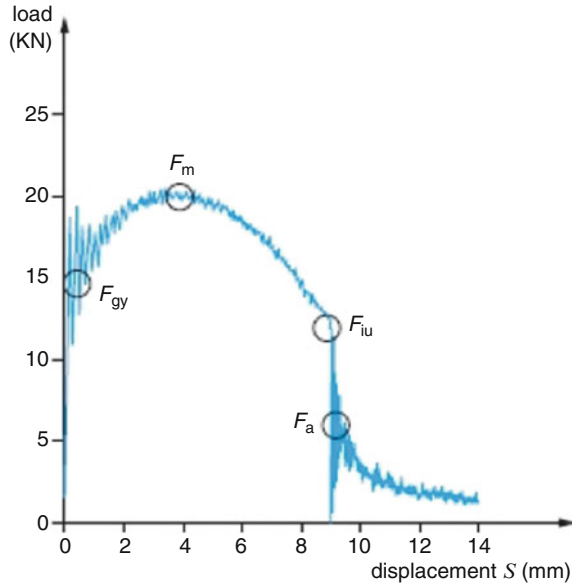


Fig. 5.8 Ductile crack growth, Δa , in Charpy V specimens tested at various impact rate, v_0 , as a function of energy applied to the specimens, C_V (J); (a) maximum ductile crack length in the centre of the specimens; (b) average ductile crack length (Tanguy et al. 2005a, b)

5.2.3 Instrumented Impact Tests

The ordinary Charpy test measures the total energy absorbed in fracturing the specimen. Additional information can be obtained if the impact tester is instrumented

Fig. 5.9 Example of the results of an instrumented Charpy test. Definition of the different parameters (in Besson 2004)



to provide a load-time history of the specimen during the test. Figure 5.9 shows an idealised load-time for an instrumented Charpy test (François 2007). One can determine with this type of record the energy required for initiating fracture and the energy required for propagating fracture. It is also possible to determine the load for general yielding, F_{GY} , which reflects the effect of high strain rate on the yield strength of the material. The load F_{GY} can be used to determine the *dynamic* elastic limit of the material R_p :

$$R_p = \alpha F_{GY} \frac{L}{B(W-a)^2} \quad (5.7)$$

In this expression $L = 20$ mm, $B = 10$ mm, $(W-a) = 8$ mm and α is a factor equal to 2.9 (Tresca criterion) or 2.5 (von Mises criterion). The value of α is determined from Eq. 5.1.

This curve yields also information on the maximum load, F_m , which coincides with crack initiation from the notch, the fracture load F_{iu} which corresponds to the load at crack arrest, F_a . The load F_a gives a useful correlation, which allows the determination of the temperature $T_{K_{IA}}$ at which the fracture toughness at crack arrest is equal to $100 \text{ MPa m}^{1/2}$. It was shown empirically (Wallin et al. 2002a, b) that:

$$T_{K_{IA}} = T_{Fa4KN} + 13.5 \text{ } ^\circ\text{C} \quad (5.8)$$

where T_{Fa4KN} is the temperature at which the crack arrest load is equal to 4KN.

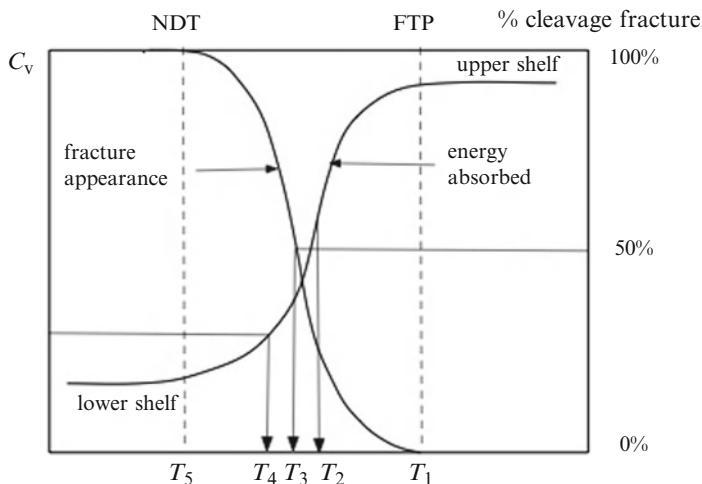


Fig. 5.10 Various criteria of transition temperature obtained from Charpy test

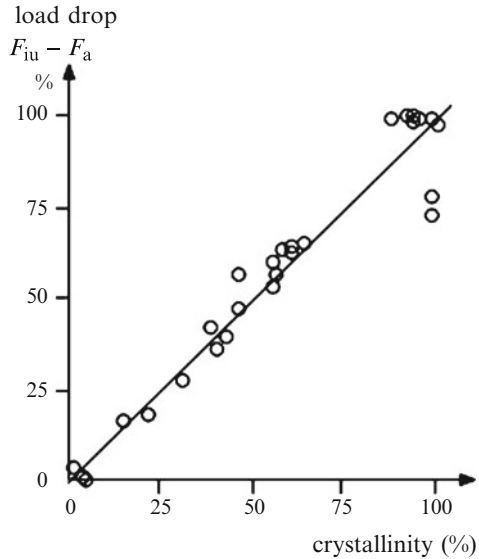
Many studies are still made on instrumented impact testing. More details can be found in the international standard, ISO 148-1. Because the root of the notch in a Charpy specimen is not as sharp as a crack in fracture mechanics tests, there has been a trend toward using standard Charpy specimens which are pre-cracked by the introduction of a fatigue crack at the tip of the V notch. These pre-cracked specimens are used in the instrumented Charpy test to measure dynamic fracture toughness (K_{Id}). It has been shown in Sect. 3.5.4.3 that K_{Id} is lower than the “static” fracture toughness, K_{Ic} .

5.2.4 Ductile-to-Brittle Transition (DBT) Temperature

5.2.4.1 Definition of the DBT Temperatures

The design philosophy using transition-temperature curves centres about the determination of a temperature above which brittle fracture will not occur at elastic stress levels. Obviously, the lower is this transition temperature, the greater is the fracture toughness of the material. The shape of the typical C_v versus temperature curve (Fig. 5.6) shows that there is no single criterion to define the transition temperature. The various definitions of transition temperature obtained from Charpy V energy versus temperature curve or from a fracture appearance versus temperature curve are shown in Fig. 5.10. Correlations were established from the load drop ($F_{iu} - F_a$) in Fig. 5.9 and the fracture appearance (FA) which gives the relative amount of the fracture surface covered by cleavage (Fig. 5.11) (François 2007). However in most cases the FA curve is measured from visual observation.

Fig. 5.11 Relation between the fracture appearance (crystallinity, %) and the *load drop* observed in instrumented Charpy test (François 2007)



The most conservative criterion for transition temperature is to select T_W corresponding to the upper shelf in fracture energy and the temperature above which the fracture is 100% ductile (or fibrous) (0% cleavage) (Fig. 5.10). This transition is called the fracture transition plastic (FTP). The FTP is the temperature below which the fracture changes from totally ductile to substantially brittle. The probability of brittle fracture is negligible above the FTP.

The use of the FTP is very conservative and is not used in practical applications. An arbitrary, but less conservative, criterion is to determine the transition temperature as that corresponding to 50% on the FA curve, T_2 (Fig. 5.10). This temperature is called a *fracture-appearance transition temperature* (FATT). Roughly similar results are obtained by defining the transition temperature as the average of the upper and lower shelf values, T_3 (Fig. 5.10).

A practical and widely used criterion is to define the transition temperature T_4 on the basis of selected low value of energy absorbed C_V as indicated in Fig. 5.10. This is often called the *ductility transition temperature*. A 20 J, C_V transition temperature has become an accepted criterion of low-strength ship steels. However it is important to realise that for other materials $C_V 20$ bears no special significance. For higher strength steels, in particular for those used in nuclear industry for the fabrication of pressure vessels, $C_V 41$ or $C_V 56$ are used.

Another well defined criterion is to base the transition as the temperature, T_5 , (Fig. 5.10) at which the fracture becomes 100% cleavage. This temperature is defined as the *nil ductility temperature* (NDT). The NDT is the temperature at which fracture initiates with essentially no prior plastic deformation. Below the NDT the probability of ductile fracture is negligible.

Examples of Charpy curves obtained on low strength ship steel with a ferrite-pearlite microstructure are reproduced in Fig. 5.12 (Morrison and Wu 2002). These

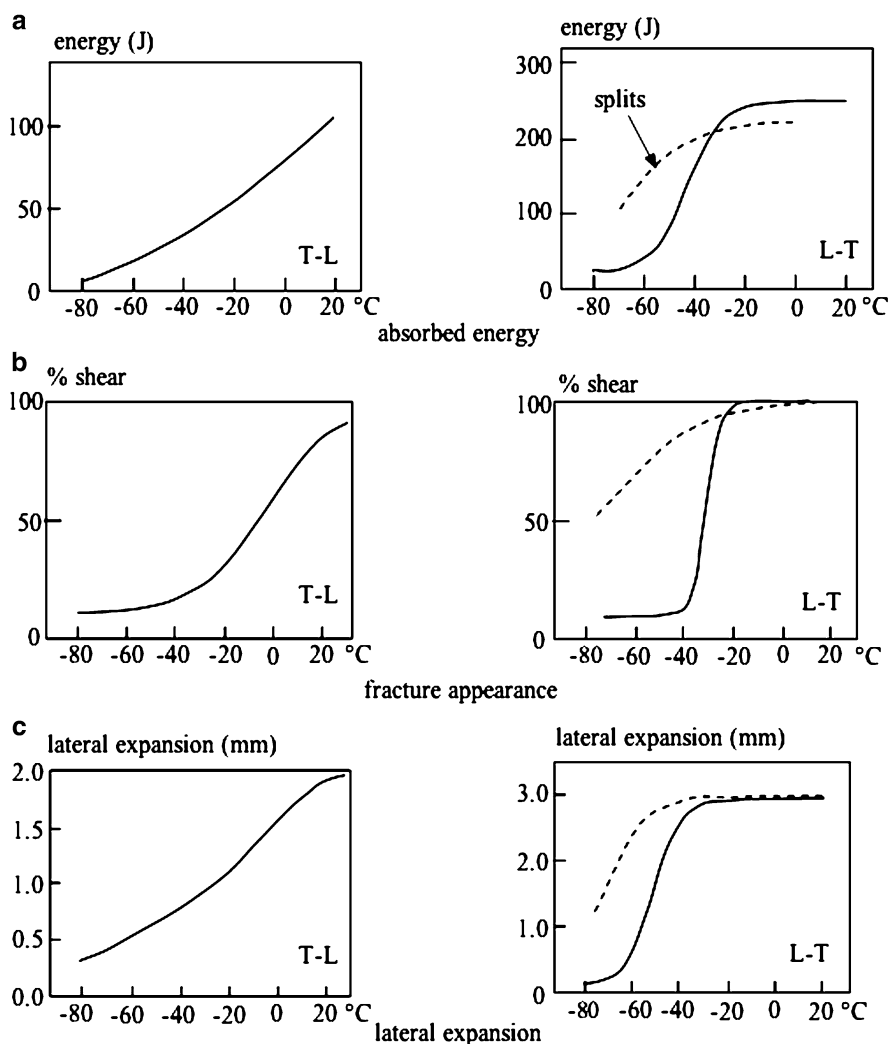
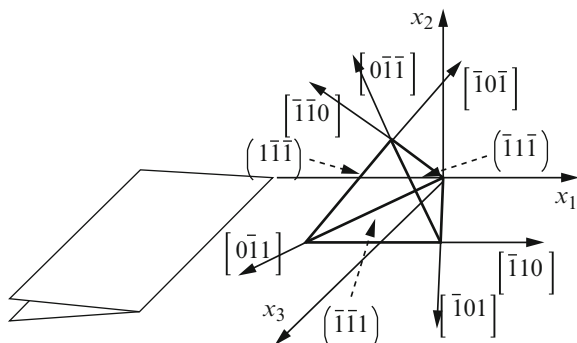


Fig. 5.12 Charpy V notch transition curves in a ship steel tested in two orientations LT and TL. In LT orientation the formation of delaminations produces various values of the energy (Adapted from Morrison and Wu 2002)

curves were obtained using specimens tested in the T-L orientation (T: longitudinal direction of the specimen parallel to the transverse direction of the plate; L: direction of crack propagation parallel to the longitudinal direction of the plate and in the L-T orientation.) (For the notation of the orientation of fracture mechanics specimens see Fig. 4.39 in Chap. 4 in this volume). A strong anisotropy effect is observed whatever the definition for the transition-temperature curves. This behaviour is expected in this type of steel, which exhibits a strong banded (ferrite + pearlite) microstructure.

Fig. 5.13 Orientation of the crack with respect to the dislocation glide planes in a Si single crystal (St John 1975)



5.2.4.2 Brittle-Ductile Transition and the Nucleation of Dislocations at the Crack Tip

Some materials like diamond and mica apparently undergo *pure brittle cleavage* with no observable plasticity associated with the process. Some of these materials, like silicon, are purely brittle at low temperature ($\leq 700^\circ\text{C}$) and exhibit a very sharp transition over a few degrees (St John 1975). Above this transition temperature cleavage cracks are surrounded by dislocations. The behaviour of monocrystalline silicon tested according to the orientations shown in Fig. 5.13 is illustrated in Fig. 5.14. This figure shows that the transition is largely dependent on the loading rate. The activation energy for the transition was found to be 1.9 eV (43 Kcal/mole). This value compares well with a value of 1.8 eV for glide of screw dislocations in silicon at similar temperatures (Kannan 1969; Kannan and Washburn 1970). The transition from purely brittle to ductile fracture in these crystals can be associated to the emission of dislocations from an atomically sharp cleavage crack, as suggested by a number of authors (Amstrong 1966; Kelly et al. 1967; Rice and Thomson 1974; Rice 1992).

As studied in Chap. 3 (Sect. 3.2.3) Rice and Thomson (1974) calculated the stability of a sharp cleavage crack against emission of a blunting dislocation for a number of crystals and crystal types and the energy to form a stable loop of dislocation from the crack tip. Rice (1992) also investigated dislocation nucleation from a crack tip using an approach based on the Peierls concept (see Volume I, Sect. 3.4.2.1).

5.2.4.3 Effect of Metallurgical Variables

The transition behaviour of many materials falls into the three categories shown in Fig. 5.15. Medium and low strength FCC metals have high notch toughness and brittle fracture by cleavage is not a problem (see Chap. 3, Sect. 3.2.1 in this volume). High strength BCC materials have low notch toughness and the transition temperature curves are very flat. The notch toughness of low and medium strength

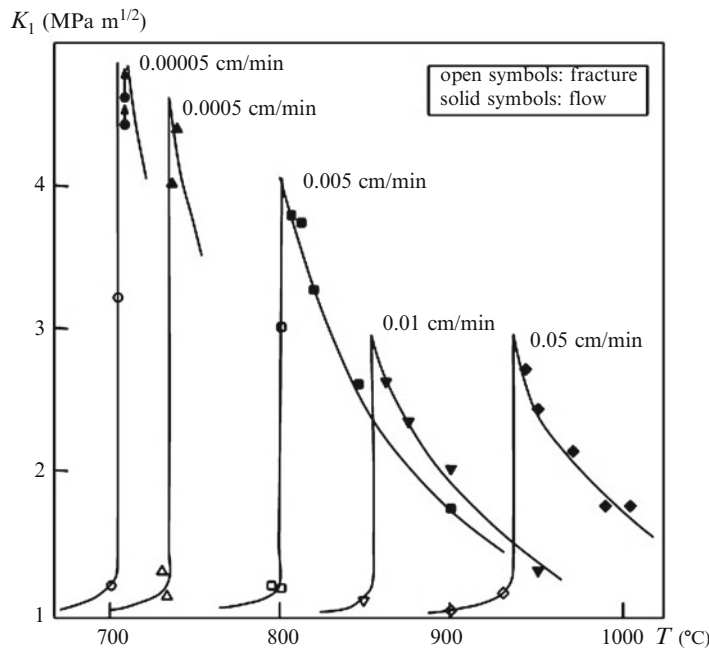
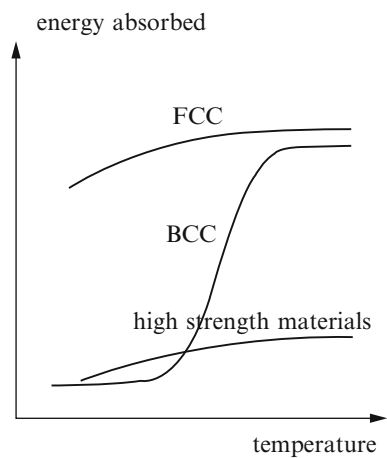


Fig. 5.14 Critical stress-intensity factors at fracture or flow versus temperature for five cross-head speeds showing the ductile-to-brittle transition in Si single crystals (St John 1975)

Fig. 5.15 Effect of temperature on notch toughness in three types of steels – schematic



BCC metals, as well as some HCP metals, like Be and Zn, exhibit a strong temperature dependence. At low temperature the fracture occurs by cleavage while at high temperature the fracture occurs by ductile rupture. Thus there is in these materials a transition from notch brittle to notch tough behaviour with increasing temperature.

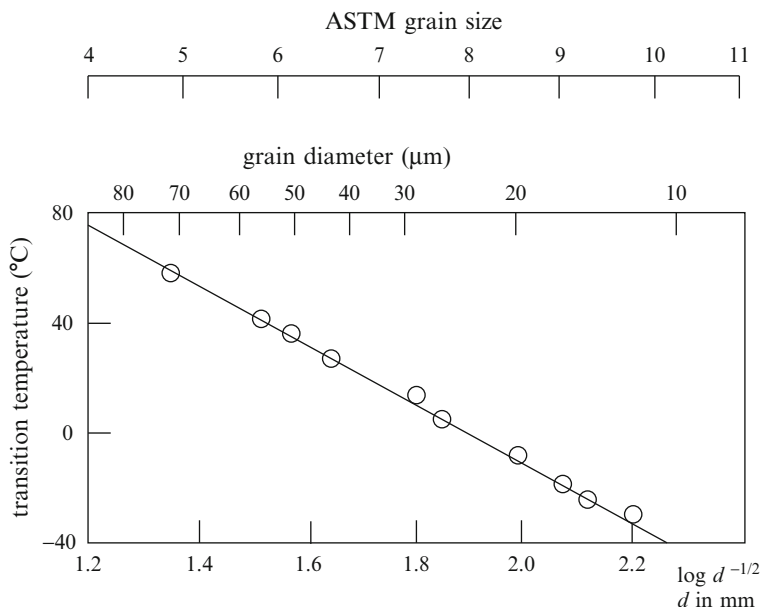


Fig. 5.16 Effect of grain size on the ductile-to-brittle transition temperature for a 0.11% C steel (Rivalin 1998)

Grain size is the most important metallurgical parameter. Reducing grain size, d , raises the yield strength (the Petch relation, see Volume I, Sect. 3.4.2.3) and the cleavage stress even more resulting in a lowering of the transition temperature (Fig. 5.16) (Constant et al. 1992).

Gamma-forming elements, by lowering the temperature of the austenite-ferrite transformation temperature, favour the formation of small ferritic grains and consequently lower the DBT temperature. An example is *nickel* (Fig. 5.17), which is added to steels intended for use at low temperatures; *manganese* has a similar effect, and is less costly. In contrast, *alpha-forming* elements such as chromium and molybdenum raise this transition temperature.

Interstitials such as carbon and nitrogen, by increasing the value of the friction stress (see Volume I, Sect. 3.4.2.3) in the Petch relation, increase the yield strength and so raise the transition temperature. Figure 5.18 shows that carbon has a very strong effect on the transition temperature. In addition it reduces the elongation at fracture and reduces the level of the upper shelf. If the steel has been heat-treated at around 300°C the carbon and nitrogen atoms can form *atmospheres* which anchor the dislocations, as we saw in the first volume (Volume I, Sect. 3.4.3). This effect, known as *blue brittleness* because of the colour of the iron oxide which forms at this temperature, makes the steel very brittle.

Grain size is dependent on the temperature of hot rolling: the lower temperature at the end of this operation, the smaller will be the grain size and the lower the transition temperature (Fig. 5.19).

Fig. 5.17 Effect of nickel on the ductile-to-brittle transition temperature in steels

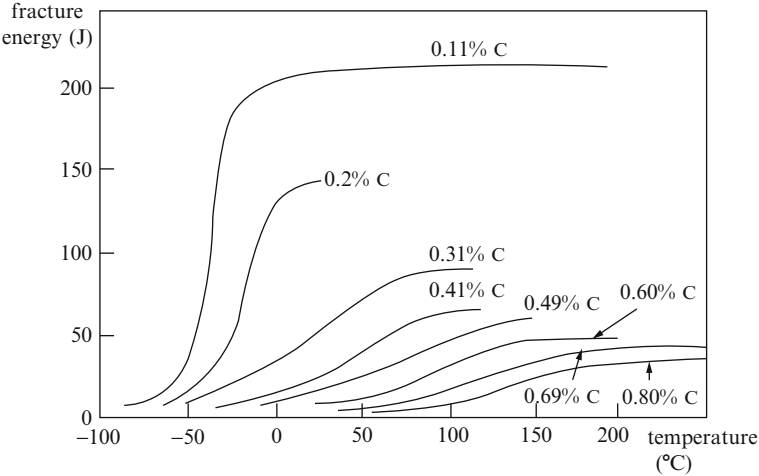
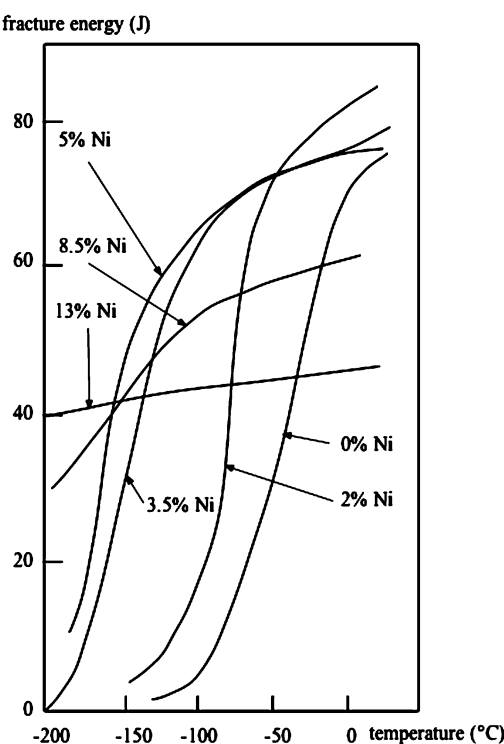
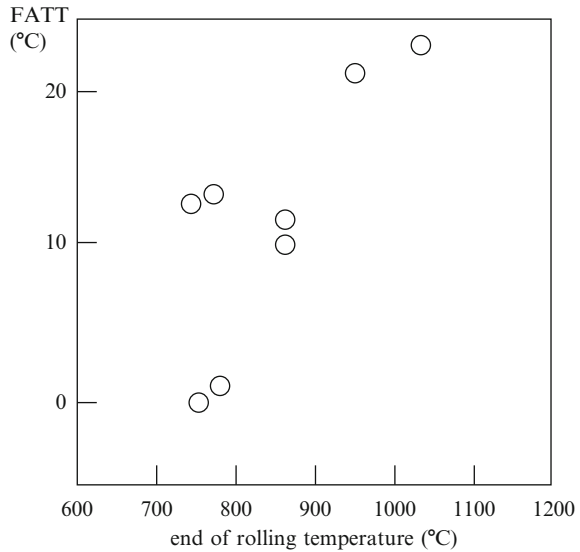


Fig. 5.18 Effect of carbon on the ductile-to-brittle transition curves for steels

Generally speaking, every factor that tends to distribute slips more uniformly by reducing local deformation heterogeneities will be beneficial. Thus a high stacking fault energy, which will favour cross-slip, will be beneficial. In contrast, increasing

Fig. 5.19 Variation of the fracture appearance transition temperature (*FATT*) with final temperature of hot rolling, for steel



the degree of long-range order will decrease the waviness of slip and make cleavage easier. Cold forming increases the transition temperature (see *e.g.* Porter et al. 2004).

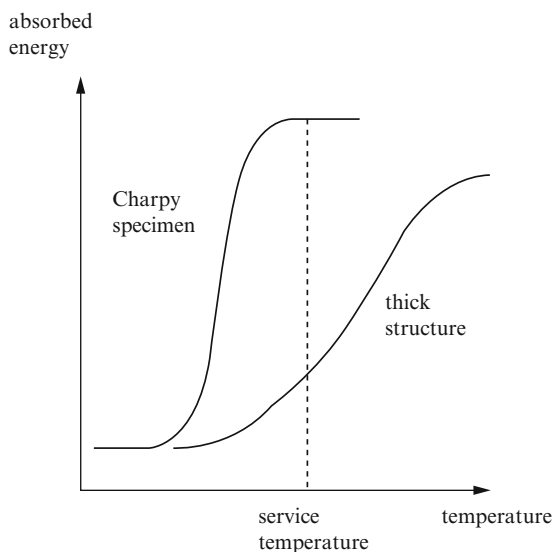
5.2.5 Drop Weight Tests and Other Large-Scale Tests

The main deficiency of the Charpy impact test is that the small ($10 \times 10 \text{ mm}^2$) specimen is not always a realistic representation of the situation, which is met with thick components. The situation that can result is illustrated in Fig. 5.20. At a particular temperature the Charpy specimen gives values of the energy, which are too conservative.

There are two logical approaches to this problem. The first one, which has been the most widely used for a long time, consists in testing thicker specimens. The development of such tests was chiefly the work of Pellini and his co-workers at the U.S. Naval Research Laboratory (Pellini² 1971). The second approach is based on modelling. In particular it was shown that the Weibull theory accounts for the size effect observed in brittle fracture toughness (Chap. 3, Sect. 3.2.3 in this volume). This approach will be subsequently presented in more detail (Sect. 5.3).

²William S. Pellini (1917–1987) was an engineer working for the Naval Research Laboratory in USA. He has largely contributed to solve the mystery of brittle fractures of World War II *Liberty* ships.

Fig. 5.20 Effect of section thickness on transition-temperature curves



The first experimental development was the *explosion-crack-starter test*, which featured a short, brittle weld bead deposited on the surface of a $350 \times 350 \times 25 \text{ mm}^3$ steel plate. Tests are carried out over a range of temperature and the appearance of the fracture determines the various transition temperatures.

The *drop weight test* (DWT) was developed specifically for the determination of the NDT on full thickness plates (see *e.g.* Tetelman³ and McEvily 1967). It is essentially a Charpy-type impact test carried out on large plates ($450 \times 125 \times 25 \text{ mm}^3$) (Fig. 5.21). A small notch is introduced in the weld bead and the specimen is supported as a simple beam in a constant temperature bath (ASTM E208). The brittle weld bead is fractured at near yield-stress levels as a result of dynamic loading from a falling weight (40–100 kg, height < 4 m). The anvil stop restricts the deflection of the test specimen. This limits the stress on the tension face of the plate to a value that does not exceed the yield stress. If the starter-crack propagates across the width of the plate from the tension face, the test temperature is below the NDT. Complete separation on the compression side of the specimen is not required. The NDT is the highest temperature at which nil ductility break is produced. Reproducibility is restricted by the dispersion in the weld bead fabrication.

The *Battelle drop weight tear test* (BDWTT) is a giant Charpy type test, which was developed at Battelle (Columbus, Ohio, USA) to study the brittle fracture of pipelines (Fig. 5.22). The specimen is a rectangular plate (305 mm $\times$ 76 mm) with a height equal to the thickness of the steel product. A sharp (45 degrees, depth = 5 mm, radius < 0.02 mm) notch is machined or pressed on one face of

³Alan Stephen Tetelman (1936–1978) was a world renowned American expert in the field of fracture mechanics and its application to the failure in engineering applications.

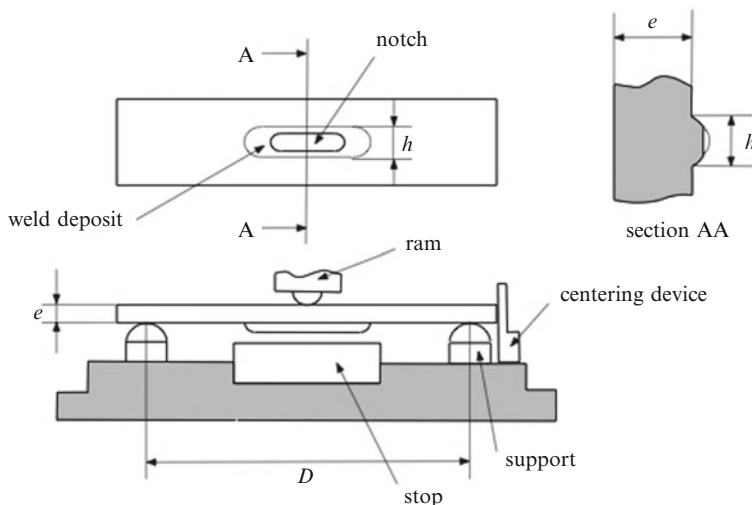


Fig. 5.21 Schematic diagram of Pellini test

the specimen. The Pellini pendulum can be used to test these BDWTT specimens. Several criteria are used to determine the toughness: (i) the ratio of the crystalline surface (C_r %), the height of the brittle fracture below the notch (H) or the ratio of the thickness at mid height occupied by ductile sharp lips (L %). Two examples of results obtained on plain carbon steel are shown in Fig. 5.23 where the benefit of a normalising heat treatment on the transition temperatures is clearly observed.

In addition to these large-scale tests one should mention the *Robertson test*, which is used to evaluate the temperature of crack arrest (Robertson 1953). The principle of this test in which a thermal gradient is imposed is indicated in Fig. 5.24. A uniform stress is applied across a plate of given thickness (12.5–25 mm) containing a notch at one end. The notch is located in the cold end. This end is impacted by a blow from a bolt gun, which causes the formation of a cleavage crack that spreads into the hotter (tougher) end of the plate. The crack spreads until it becomes arrested at some distance (temperature) that varies with applied stress and plate thickness. The Robertson test (and its subsequent modifications) is the only test that directly measures the *crack arrest curve*. The difficulty with this test is that it is complicated, expensive and impractical for use, except as a research tool. Moreover this test does not provide a measurement of the *fracture toughness at crack arrest* (K_{Ia}). This is usually performed using other specimen geometries, like the compact-crack arrest (CCA) specimen. More recently, tests on disks containing a notch at their periphery and heated internally were also used to measure K_{Ia} (see e.g. Bouyne et al. 2001). These authors tested two pressure vessel steels, 2¼Cr – 1Mo steel and A 508 steel (0.16C-1.5Mn-0.70Ni-0.50Mo). These thermal shock tests revealed a greater temperature shift (as large as +100°C) between K_{Ic} and K_{Ia} in 2¼Cr-1Mo steel than in A 508 steel.

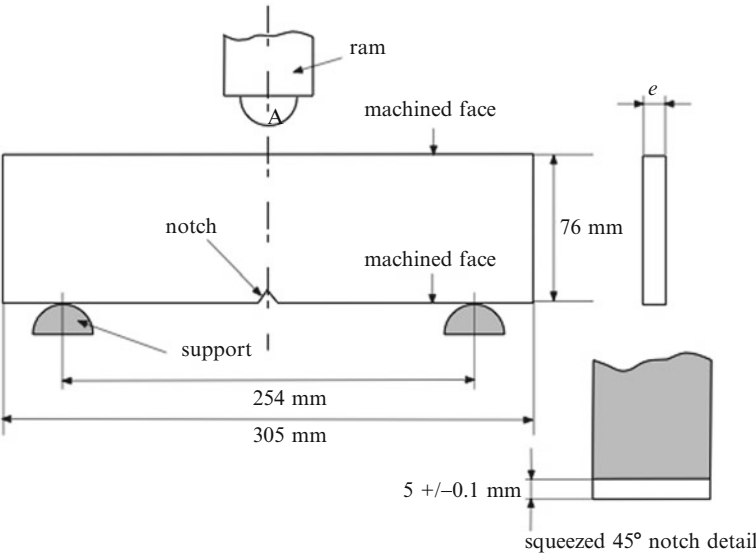


Fig. 5.22 Schematic diagram showing Battelle drop weight tearing test (BDWTT)

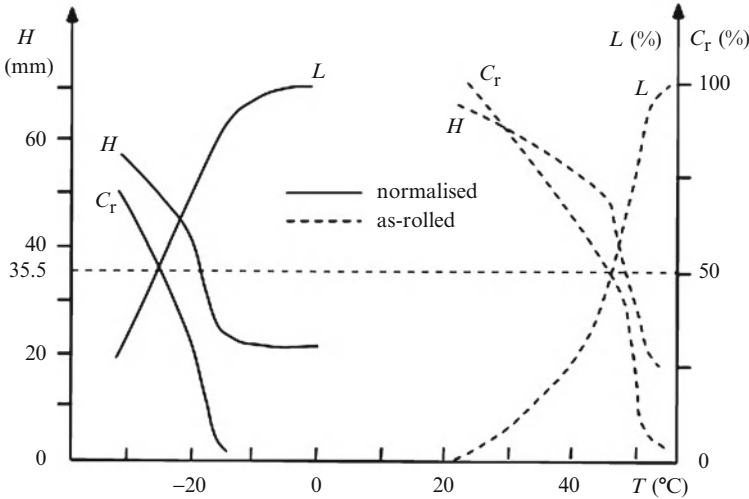


Fig. 5.23 Results of BDWTT: crystalline height (H), thickness of ductile shear lips (L), ratio of cleavage fracture (C_r) as a function of test temperature. Two conditions: normalised (I) and as-rolled (II) (François 2007)

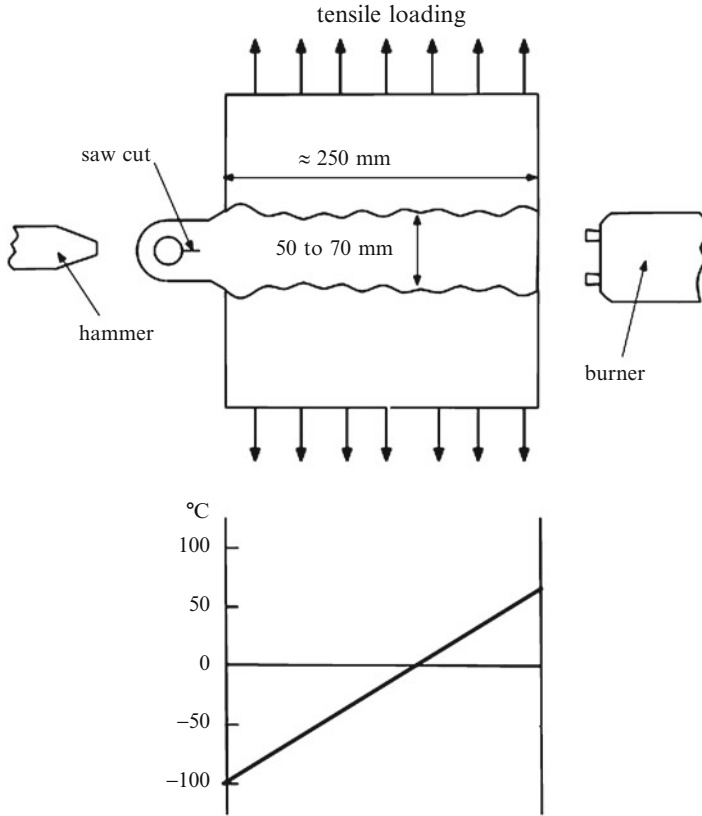


Fig. 5.24 Principle of Robertson test to measure crack arrest temperature (CAT) (François 2007)

5.2.6 Failure Analysis Diagram

The previous sections have introduced a number of terms dealing with brittle fracture, such as NDT, FTP, etc. The tests for determining the transition temperatures have been described. Before seeing how these temperatures are used in engineering applications through the failure analysis diagram, we redefine these transition points through basic properties determined from a tensile test.

The variations of the yield strength, R_p , and ultimate tensile strength, R_m , in a BCC metal with temperature are shown in Fig. 5.25. For an un-notched specimen the material is ductile down to a very low temperature, point A, where $R_p = R_m$. The temperature at point A represents the NDT temperature for a flaw-free material. The curve BCD represents the fracture strength of a specimen with a small flaw (for instance 2 mm). The temperature corresponding to point C is the highest temperature at which the fracture stress $R_p = R_m$. Point C represents the NDT for a specimen with a small crack or flaw. Increasing the flaw size decreases the fracture stress curve (curve EFG) until with increasing flaw size a limiting curve of fracture

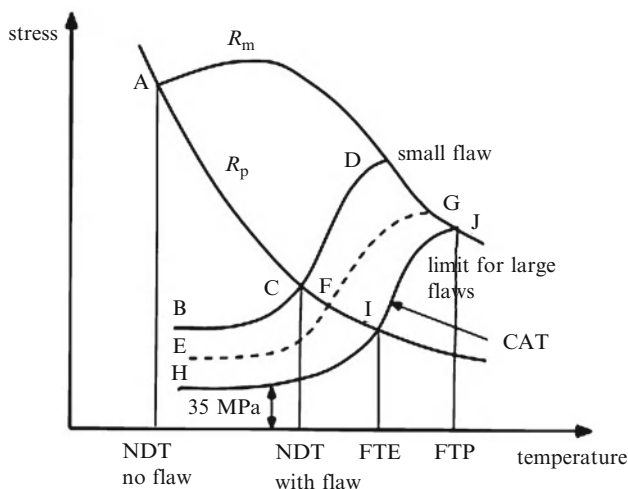


Fig. 5.25 Temperature dependence of yield strength R_p , tensile strength R_m , and fracture stress of steel containing flaws of different sizes

stress HIJ is reached. This is the crack arrest temperature curve (CAT). The CAT defines the highest temperature at which unstable crack propagation can occur at any stress level. Fracture will not occur for any point to the right of the CAT curve.

Results obtained from the BDWTT and other large scale fracture tests were assembled by Pellini and co-workers (Pellini and Puzak 1964) into the failure analysis diagram (FAD) shown in Fig. 5.26. The basis for this diagram is the CAT curve. At the temperature NDT, fracture mechanics can be used to determine the critical size of the defect (horizontal lines). All these curves meet together to the ultimate strength. This diagram can be used to determine the shift in temperature with reference to the NDT.

In many pressure vessel steels without stress concentration the applied stress is not larger than $R_p/2$. For a plate thickness of 25 mm, the NDT of the steel to fabricate the pressure vessels must be lower by 15°C than the maximum in-service temperature for defects having a dimension lower than 30 cm.

5.2.7 Correlations

The parameters introduced previously are reported in Table 5.1 with their significance. A number of correlations were determined between these parameters (see Techniques de l'ingénieur M4520). They were established for a wide variety of steels. These correlations are reported in Table 5.2.

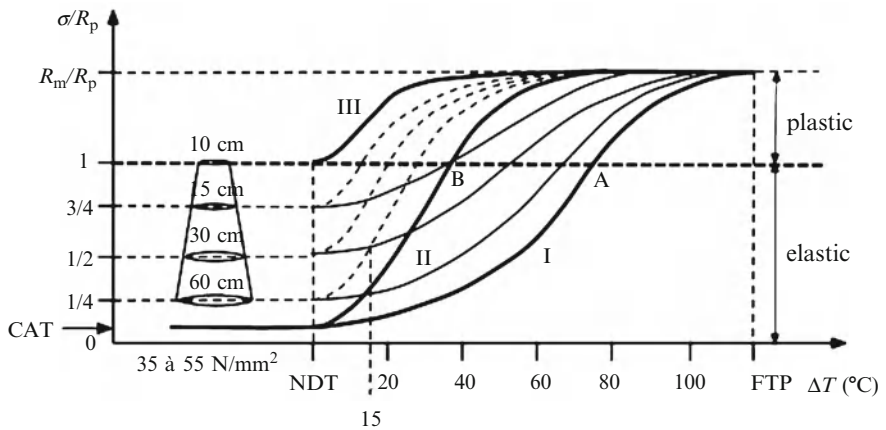


Fig. 5.26 Fracture analysis diagram (FAD) showing influence of various initial flaw sizes (François 2007)

Table 5.1 Criteria and denominations for various tests

Type of test	Name	Criterion	Denomination
Impact test	Charpy V	Transition temperature at $C_V = 28 \text{ J}$	TK_{28}
Impact test	Pellini	Nil Ductility temperature	NDT
Impact test	Battelle	Temperature for 50% crystallinity	TB 50
Tension under thermal gradient	Robertson	Crack arrest temperature	θ_s

Table 5.2 Correlations between test results

Correlation	Equation	Correlation coefficient r	Number of results N
Pellini-Charpy V	$(NDT) = -16 + 0.51(TK_{28})$	0.80	67
Battelle-Charpy V	$(TB 50) = 30 + 0.94(TK_{28})$	0.86	36
Robertson-Charpy V	$(\theta_s) = 17 + 0.64(TK_{28})$	0.90	56
Pellini-Robertson	$(NDT) = -29 + 0.84(\theta_s)$	0.90	58
Battelle-Robertson	$(TB 50) = 10 + 1.29(\theta_s)$	0.94	28

5.3 Modelling the DBT Behaviour in Fracture Mechanics Tests and in Charpy V Tests

5.3.1 Introduction

The competition between ductile tearing and cleavage fracture controls the fracture behaviour of ferritic steels in the upper part of the DBT regime. As indicated in Fig. 5.27 ductile crack propagation occurs under increasing load, and the structure

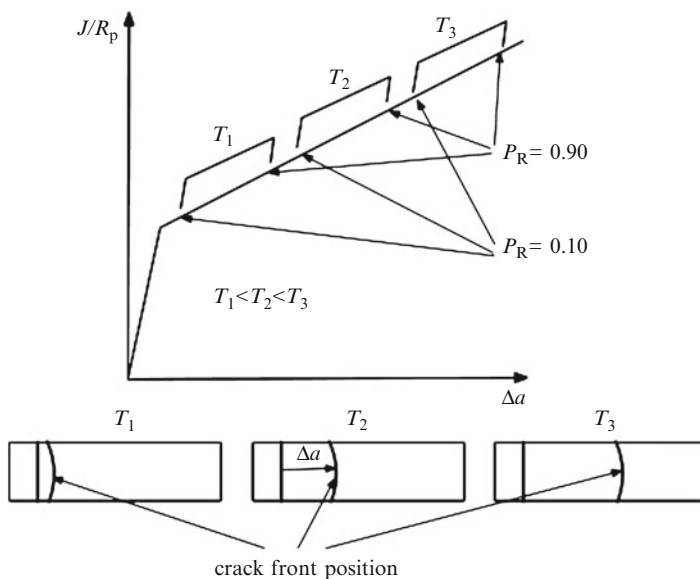


Fig. 5.27 Schematic J-R curve and probabilities to initiate front cleavage fracture at increasing temperatures $T_1 < T_2 < T_3$. The position of the fracture surface is schematically shown

can withstand a significant amount of stable ductile crack propagation, Δa , without substantial loss of load-bearing capacity. On the other hand, cleavage fracture leads to catastrophic failure, and the initiation of cleavage is limiting the load-bearing capacity of the structure. Figure 5.27 schematically shows that the transition is characterised by a large scatter (the values of the probability to cleavage fracture of 10 and 90% are reported), and the amount of ductile crack growth increases with increasing temperature. Typical results obtained on the 22NiMoCr37 steel tested in the Euro fracture toughness data set (Heerens and Hellmann 2002) are reported in Fig. 5.28.

The transition at relatively low loading rate is usually investigated using conventional fracture toughness tests. The transition in these specimens corresponds to relatively simple loading conditions, which are quasi-static and strictly isothermal. The DBT temperature can be defined for a given value of the stress intensity factor (for instance, $100 \text{ MPa m}^{1/2}$).

The DBT behaviour at high loading rate is usually investigated by using impact Charpy V specimens and, as indicated in the first section of this chapter, several transition temperatures are defined. Charpy impact testing remained for a long time essentially a technological tool and the correlations between C_V and K_{Ic} largely empirical. It is only recently that the *local approach to fracture* (see Chaps. 3 and 4 in this volume) has been used for a better understanding of the Charpy test and for relating K_{Ic} and C_V (see François and Pineau 2002).

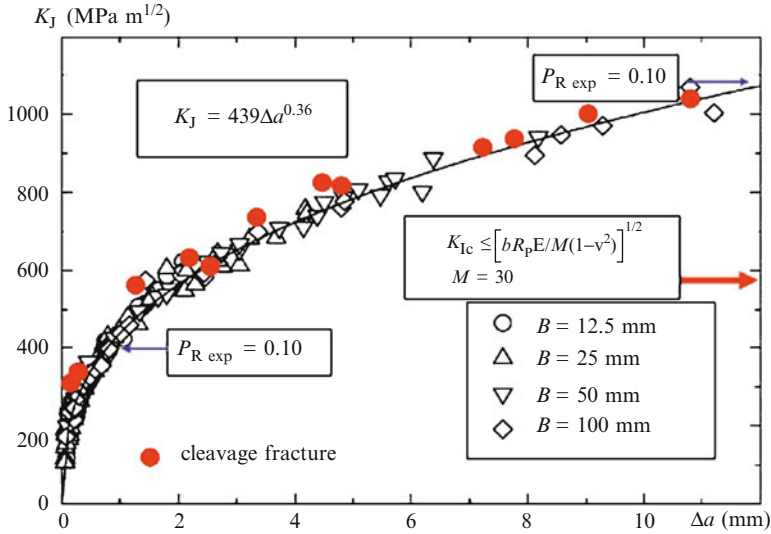


Fig. 5.28 Steel 22NiCrMo35 – J-R curve obtained with CT specimens of various thickness and cleavage fracture in CT 100 specimens tested at 20°C. The observed probabilities to failure of 10 and 90% are indicated (Heerens and Hellmann 2002)

In this section the DBT behaviour observed in testing fracture mechanics specimens is described first. Advanced models based on the study and the numerical simulation of cleavage (Chap. 3 in this volume) and ductile fracture (Chap. 4 in this volume) are presented. Then recent results on modelling the Charpy impact test are shown.

5.3.2 DBT in Fracture Toughness Tests

A simple explanation for the existence of a transition lies in the temperature dependence of the stress-strain curves. Increasing test temperature results in a decrease of the normal stress ahead of the crack-tip until this stress becomes lower than the cleavage stress. However this situation is more complex and no single explanation for this transition can be given. A combination of additional causes exists:

1. the stress level ahead of a growing ductile crack is higher than the stress ahead of a stable blunted crack
2. the volume of material sampled during ductile crack growth increases with crack extension. This results, according to the weakest link theory for cleavage fracture (see Chap. 3, Sect. 3.2.3 in this volume) in an increasing probability of cleavage.

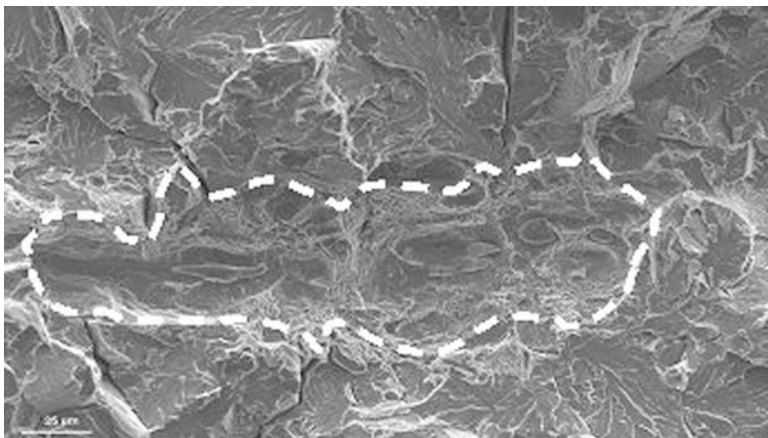


Fig. 5.29 Cleavage fracture observed in a notched specimen of pressure vessel steel (0.16C, 1.5Mn, 0.70Ni, 0.50Mo) tested at -150°C . Cleavage is initiated from ductile dimples containing manganese sulphide inclusions (Tanguy, 2007, private communication)

- the number of cleavage initiation sites can decrease owing to the elimination of eligible particles for brittle fracture (Chen et al. 1996; Wang and Chen 2001; Koers et al. 1995). This elimination is due to the formation of cavities from inclusions or second-phase particles. However these cavities can also produce stress concentrations and initiate cleavage fracture, as illustrated in Fig. 5.29 (Tanguy, 2007, private communication; Pineau 2008).

5.3.2.1 A Simplified Approach

One simple approach to predict cleavage fracture after some ductile crack extension is to consider the existence of a unique J_R curve (or better $(J/R_p)_R$ curve to account for slight temperature dependence). The Beremin model (Beremin 1983) allows calculating the probability to failure for various stationary crack lengths, as schematically indicated in Fig. 5.27. This simple approach is easy to apply using a post-processing FE routine. Figure 5.30 shows that this simplified approach applied to a pressure vessel steel (0.16C, 1.5Mn, 0.70Ni, 0.50Mo) (Amar and Pineau 1987) gives theoretical predictions in good agreement with experiments. Similar conclusions were obtained by Koers et al. (1995) on a C-Mn steel (Fe 510Nb) tested at -170°C .

This simplified approach has strong limitations since it does not account for the modifications in the stress field ahead of a propagating crack and does not account also for a possible change in the nature of cleavage initiating sites during crack propagation. Further details on the applicability of this simplified method can be found elsewhere (Pineau and Pardoën 2007; Pineau 2008).

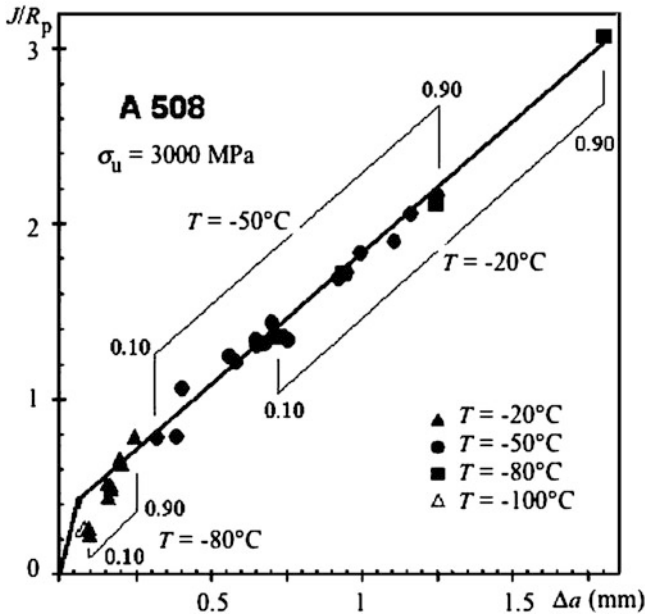


Fig. 5.30 J_R curve and ductile-to-brittle transition. Experimental and theoretical results on a pressure vessel steel tested at -100 , -80 , -50 and -20°C

5.3.2.2 Stress Profiles Ahead of a Propagating Crack

Detailed simulations by a number of authors (see *e.g.* Xia and Shih 1995a, b, 1996) using the methodologies introduced in Chap. 4, Sect. 4.6.4, showed that during ductile crack propagation, the maximum tensile stress, σ_{22} , ahead of a simulated growing crack increases with crack extension, as shown in Fig. 5.31. These results were obtained from numerical simulations of three-point bend specimens ($W = 50$ mm) with three increasing crack lengths, $a/W = 0.10$, 0.25 and 0.60 . The mesh size, D , which is one of the parameters in the simulation of ductile damage, was introduced using the GTN model (see Chap. 4, Sect. 4.3.3.1 in this volume). In this specimen geometry, the constraint effect is largely dependent on the crack length. Figure 5.31 shows that, at the early stage of crack growth, $\Delta a = D$, σ_{22} for $a/W = 0.25$ is lower than the value for $a/W = 0.60$. When the stress level for $a/W = 0.25$ is examined for increasing crack lengths, it is observed that σ_{22} increases with crack extension, and at $\Delta a = 20D = 6$ mm, the peak stress has reached the level for $a/W = 0.60$. A similar effect is observed for $a/W = 0.10$, although the effect is less pronounced. This elevation of the tensile stress ahead of the propagating crack-tip can thus increase the probability of cleavage fracture.

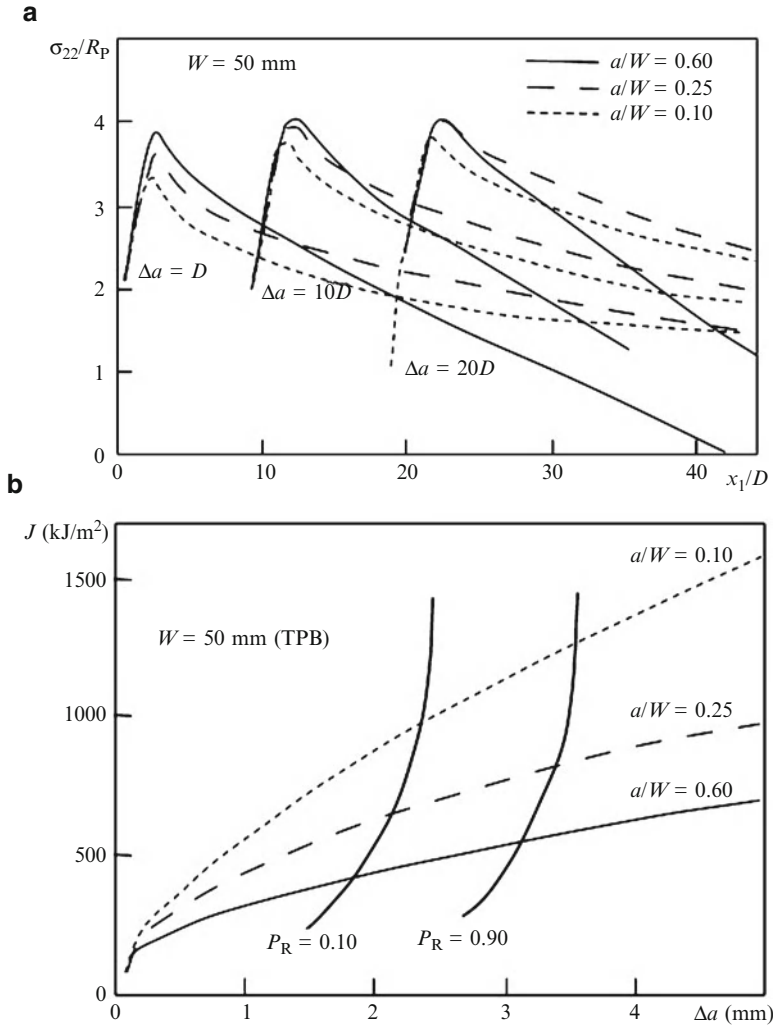
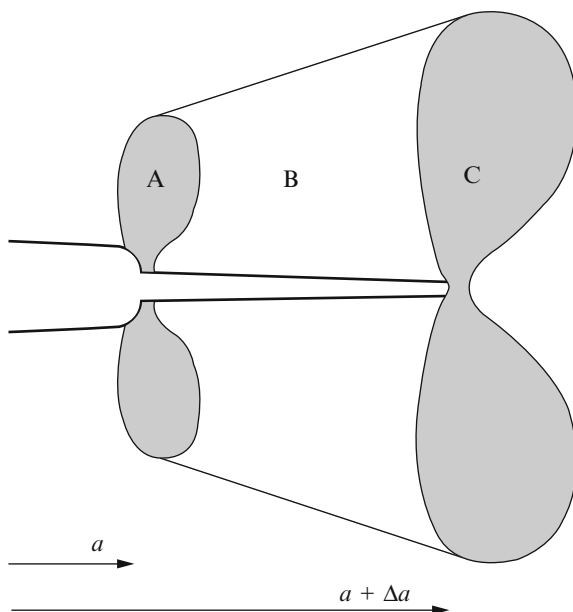


Fig. 5.31 (a) distribution of tensile stress σ_{22} ahead of a propagating crack for three crack extensions ($\Delta a = D, 10D, 20D$) and three initial crack lengths ($a/W = 0.10, 0.25, 0.60$); (b) ductile crack growth resistance curves for three point bend specimens with $W = 50$ mm, $E/R_p = 500$, $n = 0.10$, $\nu = 1/3$, $D = 200$ μm (Xia and Shih 1995a, 1996). The positions corresponding to $P_R = 10$ and 90% are schematically shown (Pineau 2008)

This effect of the stress elevation during crack growth is less pronounced in specimen geometry in which the constraint effect is less dependent on crack length, such as tensile bars with one single edge crack, as shown by Xia and Cheng (1997). For further discussion on the effect of a growing crack on stress profiles, see also Dodds et al. (1997), Tanguy (2001), and Tanguy et al. (2002a, b).

Fig. 5.32 Sketch showing the plastic zones ahead of a propagating crack. The plastic wake *left* behind the growing crack is indicated



This indicates that the DBT behaviour is strongly dependent on the specimen geometry. Cleavage fracture will be favoured in bend specimens in which the initial crack size is relatively small. This is the situation corresponding to Charpy V notch specimens.

The introduction of a damage softening law such as the GTN model produces a reduction of the “local” stresses. This softening effect and its consequence on the calculation of the Weibull stress were quantified by Busso et al. (1998). A detailed discussion of this effect can be found elsewhere (Pineau and Pardoen 2007).

The fracture process zone for a propagating crack includes three distinct zones denoted A, B and C in Fig. 5.32. The material unloaded in the zone A located behind the current position of the crack-tip has experienced severe stress and strain fields without triggering cleavage fracture. The material in zone C experiences increased stresses leading to the formation of new cleavage initiation sites. Catastrophic cleavage fracture is eventually initiated in this zone not only because the stresses are increased as indicated in the previous section, but also because of an increasing probability to contain larger microdefects which were not sampled during the earlier crack extension. In the calculation of the probability of failure the total volume of material, which experienced plastic deformation, $(A + B + C)$ in Fig. 5.32 must be taken into account (Brückner and Munz 1984; Wallin 1989, 1991a, b, 1993). This leads to the introduction of a ductile crack growth (DCG) “correction.”

The basic assumptions for this correction were presented by Wallin (1989). The volume increment due to the increase of $K = f(\Delta a)$ and to crack growth, Δa , leads to

an increase of the Weibull stress (Eq. 3.33, Chap. 3 in this volume). The probability to failure, P_R , can thus be written as:

$$\log \left(\frac{1}{1 - P_R} \right) = \left[\frac{f(\Delta a)}{K_0} \right]^4 + \frac{2\sigma_c^2}{K_0^4 B} \int_0^{\Delta a} [f(\Delta a)]^2 d\Delta a \quad (5.9)$$

where K_0 is a normalising value for the stress intensity factor and σ_c is the *cleavage stress*, while B is the specimen thickness.

Another expression for this correction was proposed by Brückner and Munz (1984):

$$\log \left(\frac{1}{1 - P_R} \right) = \left(\frac{K_i}{K_0} \right)^4 + \frac{1}{K_0^4 W_i} \int_0^{\Delta a} [f(\Delta a)]^4 d\Delta a \quad (5.10)$$

where K_i is the initial value of the stress intensity factor at ductile crack initiation and W_i is a constant related the size of the active volume.

Both equations are very similar. The difference lies in that Wallin assumes that the effective volume grows as a function of K_i^4 , whereas Brückner and Munz assume that the size of this volume is constant during crack growth. In both cases, Eqs. 5.9 and 5.10 require that the $J - \Delta a$ curve be known. This is the reason why Wallin (1991a, b) proposed a simplified expression for the DCG correction, which can be used without knowledge of the $J - \Delta a$ curve:

$$\log \left(\frac{1}{1 - P_R} \right) = \left(\frac{K_i}{K_0} \right)^4 \left(1 + \frac{2\Delta a \sigma_c^2}{K_i^2 \beta} \right)^4 \quad (5.11)$$

where $\beta \simeq x / (K_i / \sigma_c)^2$ defines the cleavage fracture process zone size. These expressions can be useful to predict in an analytical way where cleavage fracture occurs along the $J - \Delta a$ curve. However, as shown later, this prediction is usually achieved in a numerical way.

5.3.2.3 Elimination of Eligible Particles

The original Beremin model was modified to account for continuous nucleation of cleavage microcracks from particles, such as carbides in ferritic steels (see *e.g.* Chen et al. 1996, 2003) (see Chap. 3, Sect. 3.5.3.3 in this volume). The idea is that a void can be formed from a particle with a given probability, P_{void} . The particle cannot then act as an initiation site for a cleavage microcrack. The Weibull stress must therefore be modified and written as:

$$\sigma_w = \left[\sum_{i=1}^n (1 - P_{\text{void}}) (\sigma_i)^m \frac{V_i}{V_0} \right]^{1/m} \quad (5.12)$$

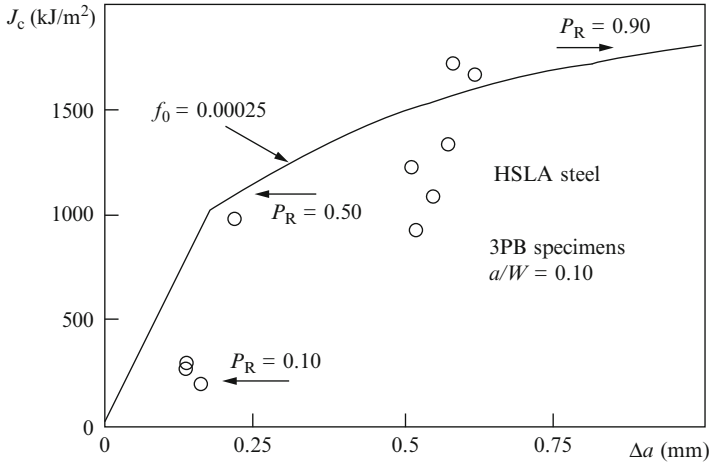


Fig. 5.33 J_c curve and cleavage fracture in a high strength low alloy (HSLA) steel. Experimental results (Toyoda et al. 1991); numerical simulation (Ruggieri and Dodds 1996). The calculated probabilities to failure of 10, 50 and 90% are indicated by arrows (Pineau 2008)

where V_i is the volume of the i -th element and σ_1^i is the maximum principal stress in V_i . This modification was implemented by Koers et al. (1995). However in the absence of quantitative measurements of void nucleation rate in the literature, the elimination of eligible particles by void formation is usually neglected.

5.3.2.4 Applications

Two applications of these “advanced” models to predict the DBT behaviour are presented. The large development in numerical tools should lead to many more applications in the near future.

1. High strength low alloy steel (HSLA steel)

This first application based on computational cell methodology (see Chap. 4, Sect. 4.6.4 in this volume) was presented by Ruggieri et al (1996). These authors used the results obtained on a HSLA steel ($R_p = 663$ MPa) tested at -120°C using three-point bend specimens with shallow and deep cracks ($a/W = 0.10$). These results are reported in Fig. 5.33. The predicted J_R curve is also shown in this figure. These predictions were obtained using the following set of parameters: $f_0 = 0.00025$, $f_c = 0.15$, $D = 0.2$ mm, $m = 15.6$, $\sigma_u = 1,557$ MPa, $V_u = 1$ mm³. Figure 5.33 shows that the agreement between the calculated and the observed $J - \Delta a$ curve is only qualitative. It is worth noting that in these experiments ductile crack extension preceding cleavage fracture was rather small ($\Delta a < 0.6$ mm). This would suggest that these results could be analysed with the simplified approach developed in Sect. 5.3.2.1.

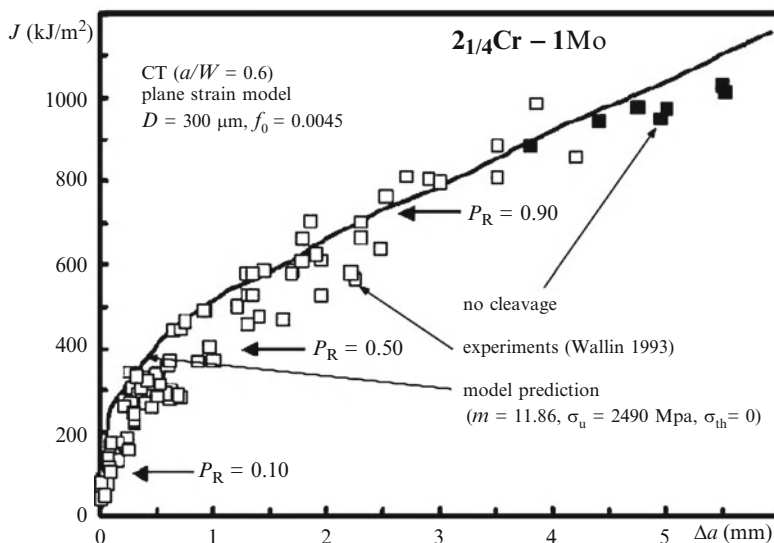


Fig. 5.34 J_R curve and cleavage fracture in $2\frac{1}{4}$ Cr-1Mo steel. Experiments (Wallin 1993) and numerical simulations (Gao et al. 1999; Pineau 2008)

2. Fracture toughness of $2\frac{1}{4}$ Cr-1Mo steel

This material was taken from a decommissioned 20-year old chemical reactor. All the experiments were performed at room temperature, which fell in the middle of the DBT interval determined from Charpy tests (Nilsson et al. 1992). The tensile properties of the material were: $R_p = 300$ MPa, $R_m = 530$ MPa. The fracture toughness tests were performed on 25 mm thick compact tension (CT) specimens with 20% side grooves and deep cracks ($a/W = 0.60$). Details on the test procedures and the results were reported by Wallin (1993) who conducted the statistical analysis. These experiments represent a wide data basis obtained from 105 tests with identical specimens. The results shown in Fig. 5.34 have already been reported in Chap. 4, Sect. 4.6.4.3. Seven of the specimens failed to initiate cleavage before the end of the test. These results are included in Fig. 5.34 with full symbols but they were omitted from the statistical analysis.

The numerical simulation of these tests was achieved by Gao et al. (1999). These authors also used the computational cell methodology (see Chap. 4, Sect. 4.6.4.3) with the following parameters: $f_0 = 0.0045$, $f_c = 0.20$, $D = 0.30$ mm, $m = 11.86$ and $\sigma_u = 2,490$ MPa.

Figure 5.34 shows that the model provides a good representation of the J curve, except for low values of ductile crack growth where the model tends to overestimate the ductile tearing resistance of the material. The calculated values for the probability to cleavage fracture are also in reasonable agreement with the experiments. It can thus be considered that these results validate the computational cell methodology to predict with a reasonable accuracy the DBT behaviour even after large crack extension.

5.3.3 Modelling Charpy V Notch Impact Tests

Charpy V impact tests remain largely used to determine the DBT temperature. Several definitions are used for this temperature as indicated earlier. In industry the Charpy test remains as a qualitative tool for quality control. However when properly instrumented and numerically modelled this test can also be used as a tool to measure the intrinsic fracture properties of a given steel. This was shown by a number of authors (Minami et al. 2002; Tanguy et al. 2002; Tanguy et al. 2005a, b; Schmitt et al. 2002; Rossol et al. 2002a, b). This is now briefly illustrated using the results of a study performed on pressure vessel steel (A 508 steel) (Tanguy 2001; Tanguy et al. 2005a, b).

The main difficulty encountered with modelling the Charpy test is the three-dimensional aspect. Contact between the striker and the support must also be taken into account. Inertial effects can also lead to another difficulty. However these effects affect failure only at very low temperature in the lower shelf regime and not in the transition regime where plastic strains are sufficiently large to damp the oscillations on the load-displacement curves (Tvergaard and Needleman 1988; Tahar 1998; Rossol 1998; Rossol et al. 1999, 2002a, b). The Charpy test in the transition regime can thus be simulated under quasi-static conditions.

Local strain rates as large as 10^3 s^{-1} occur during the deformation of the notch and during crack propagation. Modelling the mechanical response of the specimen requires properly capturing the strain rate effect in the constitutive equations. This was achieved using a double isotropic hardening law with a viscoplastic flow function including two strain rate components representing the Peierls friction forces at moderate strain rates and the phonon drag effect at higher strain rates ($\sim 10^{-3} \text{ s}^{-1}$) (see Tanguy et al. 2002a, b). The effect of temperature on the constitutive equations was also taken into account since temperature increases as large as 100°C were measured and obtained from numerical simulations.

Ductile damage was simulated using the GTN model (see Chap. 4, Sect. 4.3.3.3.2). The Rousselier (1987) model introduced in Chap. 4 was also used. In both cases the parameters of these models were calibrated from tests on axisymmetrically notched specimens (Tanguy et al. 2002a, b). Cleavage fracture was simulated using the Beremin model (Eq. 3.27, Chap. 3 in this volume).

Experimental results are shown in Fig. 5.35 where the calculated Charpy energy C_V corresponding to a failure probability of 10, 50 and 90% is also plotted as a function of test temperature. The normalising stress σ_u (Eq. 3.27) was initially assumed to be constant. The results reported in Fig. 5.35a show that the predictions are in good agreement with the experiments for temperatures lower than -80°C . Above this temperature the model largely underestimates the Charpy energy. Similar results were reported in the literature (Bernauer et al. 1999; Rossol et al. 2002a, b). The transition curve was adjusted with a value of σ_u , increasing with temperature (Tanguy et al. 2005a, b). As observed in Fig. 5.35b a better description of the Charpy transition curve is thus obtained.

This numerical simulation can be used to determine the amount of energy spent in crack propagation before cleavage initiation. Figure 5.36 shows that this energy

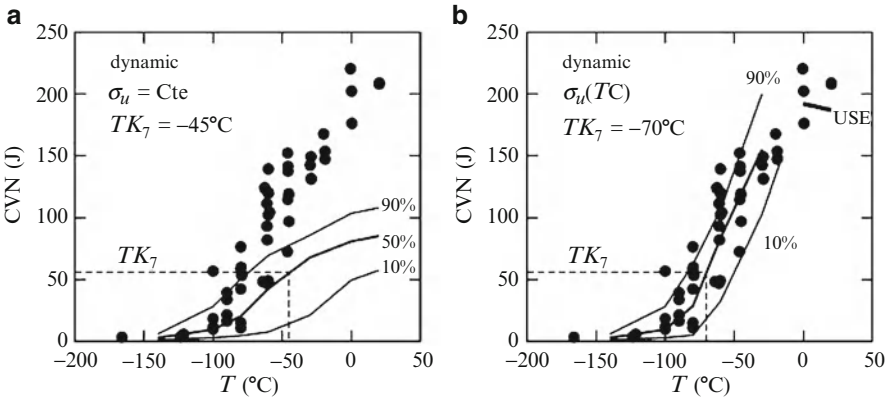


Fig. 5.35 Charpy V notch transition curve. Experiments and simulations assuming (a) a constant value of the normalising stress, σ_u of the Beremin model; (b) an increasing value of σ_u with temperature (Tanguy et al. 2005a, b; Pineau 2008)

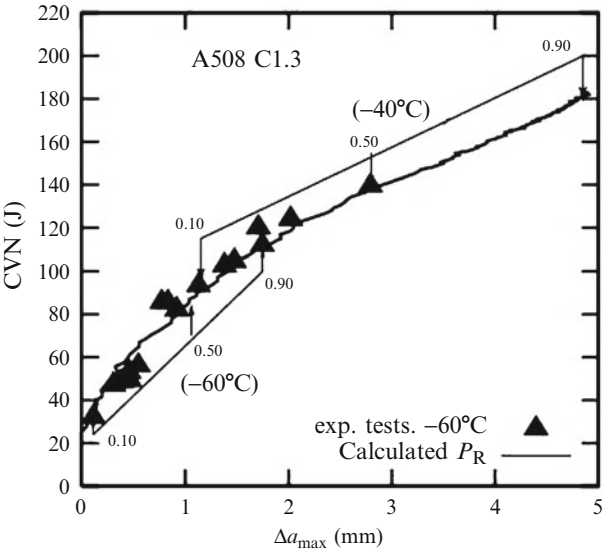


Fig. 5.36 Variation of the Charpy energy with crack extension from the notch tip. Pressure vessel steel (0.16C, 1.5Mn, 0.70Ni, 0.50Mo). Experimental results at -60°C . Calculated probabilities to initiate cleavage fracture ($P_R = 0.10, 0.50, 0.90$) for two temperatures (-60 and -40°C) (Tanguy et al. 2005a, b)

is largely dependent of temperature in the transition regime, as expected. In this figure, we have also included the experimental results obtained from interrupted Charpy tests performed at -40 and -60°C (Tanguy 2001). In both cases a very good agreement with the calculated curve is observed. See also Mathur et al. (1993, 1994), Tvergaard and Needleman (1998, 2004), and Schmitt et al. (2002).

This example clearly shows that it is now possible to adapt the Charpy impact experiment from a purely technological test to a more quantitative tool for evaluating the intrinsic fracture properties of materials and structures.

5.3.4 Correlations and Recapitulation

On the upper shelf the Charpy V notch energy C_{VUS} relates to ductile fracture resistance and it is possible to correlate the C_{VUS} to the J_R curve (Wallin et al. 2002a, b). These authors analysed 112 multi-specimen J_R curves from a wide variety of materials and proposed a simple power law based description of the J_R curves. These curves were correlated with the C_{VUS} energy. This correlation corresponds essentially to a 5% lower bound and gives a conservative tearing resistance estimate in terms of the J -integral as follows:

$$J = J_{1mm}(\Delta a)^m \quad (\text{kJ/m}^2, \text{ mm}) \quad (5.13)$$

with

$$m = 0.133 C_{VUS}^{0.256} \exp\left(-\frac{T-20}{2000}\right) - \frac{R_p}{4674} + 0.03 \quad (\text{J}, ^\circ\text{C}, \text{ MPa}) \quad (5.14)$$

These expressions can be used either to estimate J_{Ic} from Eq. 5.13 or to estimate the J -integral corresponding to any amount of ductile tearing between 0 and 6 mm for temperatures lower than 300°C:

$$\frac{J_{Ic}}{2\sigma_p} + 0.2 \text{ mm} - \left(\frac{J_{Ic}}{J_{1mm}}\right)^{1/m} = 0 \quad (5.15)$$

with the flow stress approximated by:

$$\sigma_p \simeq R_p \left[1 + \left(\frac{150 \text{ MPa}}{R_p} \right)^2 \right] \quad (5.16)$$

In Eq. 5.15 the first term corresponds to the blunting line.

The value of the Charpy energy of 28 J (or 41 J), T_K , is related to the temperature T_0 at which $K_{Ic} = 100 \text{ MPa m}^{1/2}$ (ASTM E 1921-02, 2002), *i.e.*:

$$T_0 = T_K + C \quad (5.17)$$

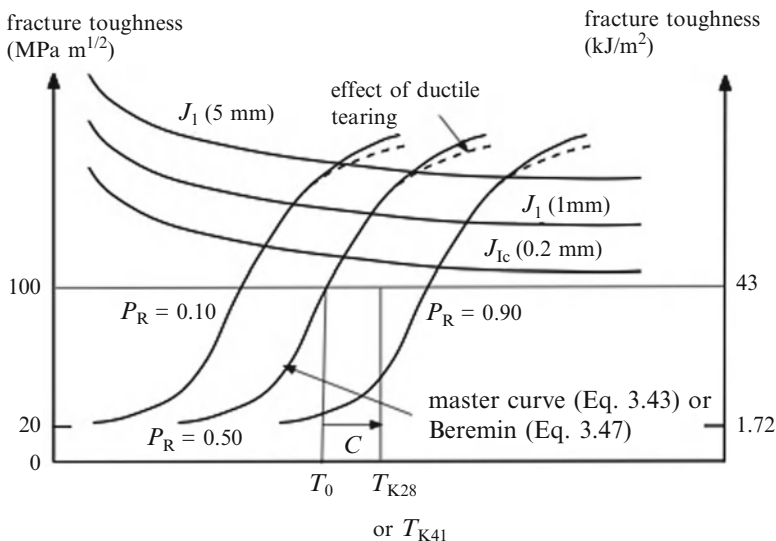


Fig. 5.37 Fracture toughness J , K versus temperature. The variations of J_R at upper shelf temperature are shown for three crack extensions ($\Delta a = 0.2$ mm, 1 mm and 5 mm). The “theoretical” fracture toughness curves inferred from Master Curve or Beremin model are drawn for probabilities $P_R = 0.10$, 0.50 and 0.90. Definition of T_0 , the temperature at which $K_{J\text{median}} = 100 \text{ MPa m}^{1/2}$. Correlation between T_0 and T_{K28} or T_{K41}

where the value of C is tabulated. For 25 mm thick CT specimen, $C = -18^\circ\text{C}$ at $C_V = 28 \text{ J}$, and $C = -24^\circ\text{C}$ at 41 J. Knowing the value of T_0 , it is possible to use the “Master Curve” methodology or the Beremin expression to draw the K_{Ic} – temperature curves (or J_{Ic} – temperature curve, with $J_{Ic} = K^2/E(1 - \nu^2)$) (see Chap. 2, Sect. 2.2.5 in this volume). These curves can be drawn for different values of the probability for brittle cleavage fracture. Assuming that the simplified method indicated in the previous Sect. 5.3.2.1 can be applied, it is then theoretically possible to estimate the probability of cleavage fracture, as schematically shown in Fig. 5.37. In the absence of FE simulations it is possible to use Eqs. (5.13) or (5.14) to account for ductile crack growth before cleavage. This methodology which, in principle, should allow drawing the J_R curve scaled in terms of the probability for cleavage fracture at increasing temperatures (Fig. 5.30) does not seem to have been yet largely used.

Even though the correlations presented here take into consideration the physical aspects of the fracture parameters being correlated, they are still empirical in nature.

The next generation of C_V correlations will take the step forward to a fully theoretical interpretation of the Charpy V test. Already, efforts are undergoing to model the Charpy V impact energy with the help of the French local approach (see Chaps. 2 and 3) and other similar micromechanisms based models. Eventually, fracture assessment may change so that conventional fracture mechanics parameters like K_{Ic} and J may become obsolete and only material parameters used in the micromechanical models are needed. Then the Charpy V test may become a true materials evaluation tool.

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Chapter 6

Fatigue

Abstract The main fatigue tests: rotating bar, axial force and strain controlled, torque controlled fatigue tests, as well as crack propagation fatigue tests are described. Gigacycle fatigue tests method is also explained. Fatigue crack nucleation results from strain irreversibility. Cyclic loading modifies the dislocations arrangements. Inclusions can be initiation sites in fatigue. The propagation of long cracks is linked with the cyclic plastic zone at the crack tip. Striations on the fracture surface result. Short cracks propagation is influenced essentially by the microstructure. Notches are preferential sites for fatigue cracks nucleation; the strain field at the notch root modifies the way they propagate. The Wöhler (or SN) curves can be described by empirical equations. Various formulations exist to account for the mean stress effect. Multiaxial loading is discussed. It is shown how cumulative damage can be represented. In low cycle fatigue, cyclic strain-hardening and the laws for the fatigue strength are described. There exist methods to account for the influence of notches. The Paris law and the way it is influenced by crack closure are described. Short cracks behave in a different manner from long ones. Over-loads slow down or even stop crack propagation. Testing under vacuum displays the intrinsic crack propagation behaviour. Surface treatments can improve the resistance to fatigue. Metallurgical factors influence fatigue behaviour. The case of titanium alloys is discussed in particular.

6.1 Introduction

Fatigue is a very detrimental kind of damage resulting from cyclic loading. It keeps affecting a great number of components and has been responsible for major disasters. An historical introduction can be found in “Introduction to Fatigue, Fundamentals and Methodology” by Bathias and Pineau (2010). It may be reminded here that the first acknowledged accident due to fatigue was that of the Paris-Versailles railway on the 8th of May 1842, which killed 55 people (Fig. 6.1). It was due to the fracture of an axle.



Fig. 6.1 Picture of the Paris-Versailles railway catastrophe by an unknown artist

As many such axle fractures plagued transportation systems, systematic investigations were initiated (Smith 1990). The pioneering work of Wöhler¹ (1870) showed that the higher the number of loading cycles the smaller the life and that for loads well below the static fracture resistance. This is one of the reasons why fatigue fractures are so frequent: people forget that a component safe under static conditions can fail when much lower loads are applied for a sufficient number of cycles. This is true even for human bones!

Since, a great deal of research has been devoted to fatigue. Indeed, it is obvious that the number of parameters coming into play is large: constant or variable cyclic amplitudes, mean stress, frequency, state of stress, multiaxiality, environment, temperature, surface condition and the great variety of metallurgical variables and of materials. We can cover here but the main aspects. Environment effects will be treated in Chap. 7 and fatigue at elevated temperature in Chap. 8 including creep-fatigue-oxidation interactions. We will give some indications concerning fatigue of non-metallic materials in Chap. 10. In the present chapter, we will first describe the most important fatigue tests. We will then deal with fatigue mechanisms. The various parameters affecting fatigue life will be discussed. Finally we will analyse fatigue crack propagation.

¹August Wöhler (1819–1914) was a German engineer who studied the mechanical properties of steels.

6.2 Fatigue Tests

6.2.1 General Considerations

These considerations can be found in a number of text-books, including Weibull (1961) and Lieurade et al. (2008).

6.2.1.1 Notations

In a cyclic fatigue test at *constant amplitude*, the load varies between the minimum load $F_{\min}$ and the maximum load $F_{\max}$. The load ratio is by definition the algebraic ratio $R = F_{\min}/F_{\max}$. The same applies to the stress and to the strain. The stress ratio is by definition the algebraic ratio $R = \sigma_{\min}/\sigma_{\max}$. (In a load controlled test it is denoted R_S according to the ISO standard). In fatigue tests involving noticeable plastic deformation, the stress ratio can be different from the strain ratio. They are then distinguished by denoting R_σ the stress ratio and by R_ϵ the strain ratio. The mean stress is given by:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = \sigma_{\max} \frac{1 + R}{2} \quad (6.1)$$

$\sigma_{\max} - \sigma_{\min}$ is the range, twice the stress amplitude denoted S (or σ_a) (Fig. 6.2).

The R ratio is usually less than 1. (It can be larger than 1 when $\sigma_{\max}$ is negative). A purely alternating cyclic test corresponds to $R = -1$; a repeated cyclic test in tension to $R = 0$, in compression to $R = \infty$.

The fatigue strength at N cycles is denoted σ_N .

Things are much more complicated in *variable amplitude loading*. In those tests, the maximum and minimum loads can vary in a random fashion, or by successive blocks of constant amplitude, or by an exact reproduction of the service loads, or in any other manner. We will not deal here with such tests, which are described by

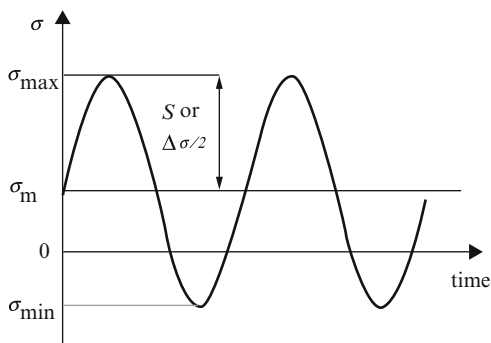


Fig. 6.2 Notations for fatigue tests

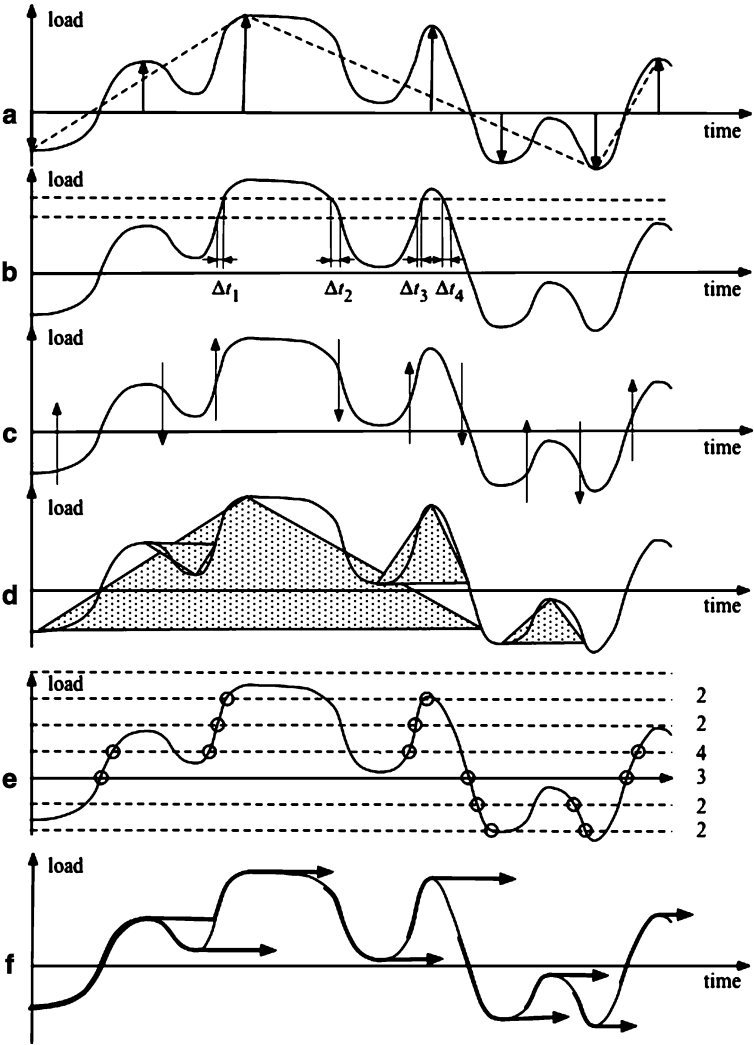


Fig. 6.3 Variable amplitude counting methods (a) peak counting (b) time counting (c) simple range counting (d) range-pair counting (e) level crossing counting (f) rain-flow counting (Adapted from Palin-Luc 2010)

Palin-Luc (2010). Let us simply briefly mention that counting the number of cycles is obviously not straightforward. There are six main methods (ASTM 2011): peak counting, time counting, simple range counting, range pair counting, level crossing counting and rain-flow counting (Fig. 6.3). This last method is the most widely used.

Complex loading tests are also performed in various manners: tension-compression-torsion tests, bending-torsion tests for instance. We will not describe them here, limiting ourself to standardised tests.

6.2.1.2 Preparation of the Test Pieces

Fatigue depends very much on the *surface preparation*. We will see why later when we study the mechanisms. Test pieces should have a roughness less than $0.2\text{ }\mu\text{m}$. The striations left by turning must be removed by resurfacing. Grinding and polishing should be achieved in the direction of the maximum principal stress, which tends to open cracks. In this way microscopic striations left do not influence the fatigue crack initiation. The corners of square section test pieces must be rounded. Would they remain sharp they would lead to premature fatigue crack initiation.

The tests must be done on materials in their final stage of in-service thermal treatment.

6.2.2 Load Controlled Fatigue Testing

6.2.2.1 Rotating Bar Bending Fatigue Testing

Rotating bar bending fatigue testing is the first method, which has been employed and it is still widely used (Manson and Halford 2006). The principle is to apply a bending moment to a cylindrical test piece, which is clamped in a rotating jaw. In this way, a point on the external diameter is alternatively loaded in tension and in compression (Fig. 6.4). The R ratio is equal to -1 .

There exist many different machines and test pieces for testing in rotating bending (ISO 1143/2010). The tests are performed in succession with different loads. The result of the test is simply the number of cycles at fracture, that is the fatigue life N_f , according to the stress amplitude on the external diameter, which is often denoted S . ISO standard includes a table giving formulae to calculate this stress for various geometries. In this way are plotted the stress amplitude as a function of the logarithm of N_f , that is the $S-N$, or *the Wöhler curve*.

6.2.2.2 Axial Force Controlled Method

The test piece is mounted on an axial compression-tension machine (ISO 1099:2006). Descriptions were given in Sect. 1.3 of Volume I. The alignment must be rigorous. The test piece can be either cylindrical or prismatic. In the first case, the diameter of the gauge length should be larger than 3 mm, while the overall diameter D should be larger than $2d$, the test section length L_c less than $8d$. The

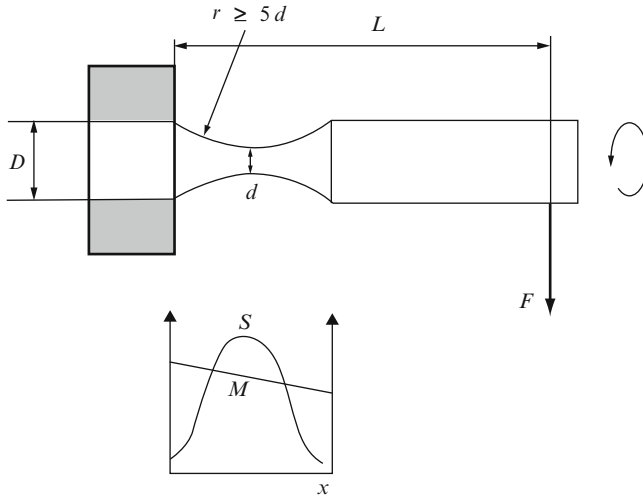


Fig. 6.4 Hour-glass test piece for rotating bar bending fatigue testing. The *bottom* diagram relates to the bending moment M and to the stress S

transition curve radius r should be larger than $2d$. These requirements concerning the dimensions are mainly there to avoid buckling in compression.

The test consists in applying a cyclic stress between $\sigma_{\min}$ and $\sigma_{\max}$ until fracture of the test piece. The mean stress σ_m can be either positive, negative or zero. The results allow to draw the Wöhler $\sigma_a - \log N_f$ curve or to use either the *Haigh*, the *Goodman-Smith* or the *Rös* diagrams, which allow to visualise the effect of the mean stress (Fig. 6.5).

Axial force control allows *variable amplitude testing*.

6.2.2.3 Torque Controlled Fatigue Testing

Torque controlled fatigue tests are cyclic torsion tests, which allow obtaining fatigue life in shear (ISO 1352:2011). They are performed on solid or tubular cylindrical test pieces. Tubular test pieces have the advantage that the shear stress is almost constant in the wall.

The notations are similar to the one given above (Sect. 6.2.1.1) replacing the normal stress σ by the shear stress τ . The torque is denoted T .

The diameter d is such that $5 \text{ mm} \leq d \leq 12 \text{ mm}$. The other dimensions of the test piece must respect the following specifications: for a full section, the test section length $L_c \leq 5d$, the overall diameter $D \geq 2d$, the transition curve radius $r \geq 3d$; for a tubular section the wall thickness $(d_0 - d_i)/2$ between 0.05 and $0.1d_0$, L_c between 1 and $3d_0$, $r \geq 3d_0$ and $D \geq 1.5d_0$.

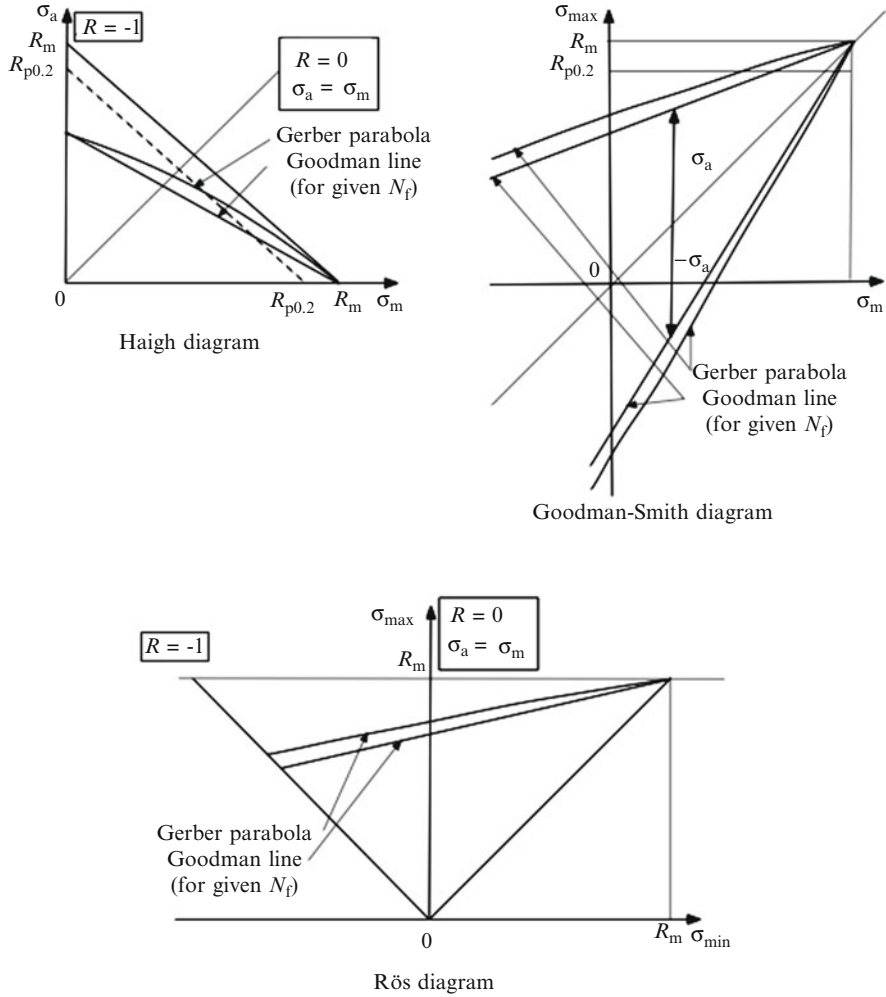


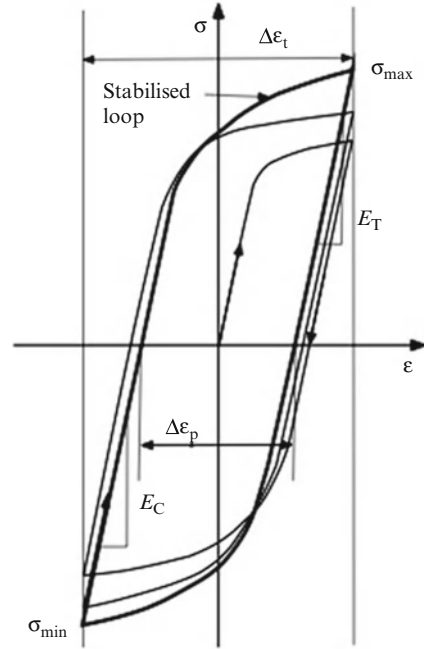
Fig. 6.5 The Haigh, Goodman-Smith and Rös diagrams to represent results of tests for various R ratios

The following formulae give the maximum shear stress τ as a function of the torque:

$$\begin{aligned} \text{full specimen } \tau &= \frac{16T}{\pi d^3} \\ \text{tubular specimen } \tau &= \frac{16T d_0}{\pi (d_0^4 - d_i^4)} \end{aligned} \quad (6.2)$$

d_i being the inside diameter.

Fig. 6.6 Evolution of the stress under a strain controlled fatigue test, until a stabilised loop is obtained after complete cyclic strain-hardening



6.2.3 Axial Strain Controlled Fatigue Testing

There exist many situations in components where cycling is strain controlled: this is the case in stress concentration areas, which can experience plastic deformation while the rest of the component remains elastic. We find such a situation at a notch root for instance. We need then to perform strain controlled fatigue tests. An extensometer must be placed on the specimen to measure the strain. It is most often a longitudinal one. In those tests it is important to record the evolution of the stress as cycling goes on. The kind of *hysteresis loops* obtained and some of the notations used are shown in Fig. 6.6. For the R ratio, the strain ratio R_ϵ is distinguished from the stress ratio R_σ .

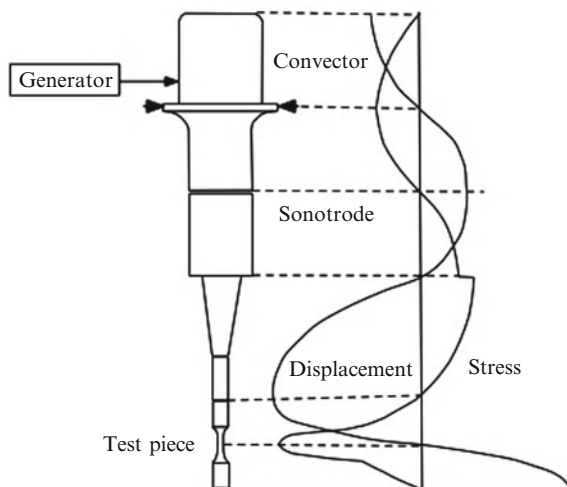
The stress-strain law can be expressed as:

$$\sigma = K' \epsilon^{n'} \quad (6.3)$$

with K' the cyclic stress coefficient and n' the cyclic strain-hardening exponent, σ_f , the fatigue strength coefficient, b , the fatigue strength exponent, ϵ_f , the fatigue ductility coefficient and c the fatigue ductility exponent are also defined by the standard (ISO12106:2003). These quantities are introduced in Sect. 6.4.2.2.

The *standard dimensions of the test piece* are as follows: diameter $d = 5$ mm, gauge length $L_0 = 2d$, $r = 2d$, $D = 2d$ and the distance L_T between the shoulders must be less than $8d$. These requirements again are there mostly to avoid buckling in compression.

Fig. 6.7 Sketch of an equipment to test in the gigacycle range



The evolution of the maximum stress usually displays an increase during the first cycles up to a stabilised plateau. When a crack appears in the test piece the maximum stress falls. The fatigue life is taken at some value of the percentage of maximum stress decrease. This is often chosen to be 25%.

The fatigue life results are plotted as the strain amplitude as a function of the fatigue life in a bi-logarithmic diagram. The strain amplitude may be partitioned in the plastic and elastic components. In such a diagram, a straight line when plotting the plastic strain amplitude corresponds to the so-called Manson-Coffin law (see Sect. 6.4.2.2).

6.2.4 Gigacycle Fatigue

Testing with usual machines at about 1 Hz takes 11.5 days to get 10^6 cycles. Now, there are many equipments and parts, which should have a longer fatigue life. To test under those conditions without spending too long times, vibrating machines must be used. Figure 6.7 is a sketch of such an equipment. A convactor excites a sonotrode, which amplifies the vibrations. The test piece is placed so that its centre is at a displacement node and at a maximum of the stress. It is possible to calculate the various dimensions to achieve these conditions and to get the values of the stress and strain in the test piece. It is assumed that the material is linear elastic.

The equipment can be adapted to load the test piece with a non-zero static component and also in torsion.

Often the temperature of the test piece increases, so that blowing air on it is needed to cool it down. This casts some doubt on the hypothesis that the behaviour is purely elastic.

6.2.5 Fatigue Crack Propagation Testing

6.2.5.1 Aim of Fatigue Crack Propagation Tests

The aim of fatigue crack propagation testing is to determine the crack velocity da/dN as a function of the load. As we saw in Chap. 2, the stress and strain conditions at the crack tip depend essentially on the stress intensity factor (SIF). This is usually well verified for fatigue cracks because the loading being low, often lower than the proof strength $R_{p0.2}$, the plastic zone at the crack tip is small and well embedded in the elastic singularity. Small-scale yielding conditions are easy to be fulfilled. In fatigue crack propagation testing, it is then required to be able to determine the stress intensity factor at any time during the test, and more precisely the SIF amplitude ΔK . Thus, besides the measurement of the applied load and of the crack length, well calibrated test pieces must be used.

6.2.5.2 Test Pieces

The most widely used test pieces are the compact tension (CT) and the single edge notched bend (SENB) ones (ISO12108:2002). Figure 6.8 shows the dimensions of the CT and of the SENB test pieces. They refer to the width W . There is no reason not to adopt other shaped specimens, provided the SIF is well known as a function of the crack length.

These test pieces are notched for crack initiation control. The notch should be as narrow as possible in order to make the fatigue crack initiation easier thus reducing the time spent in precracking. The notch length is denoted a_n . Before measuring the crack velocity, the fatigue crack must be initiated from the notch root of the test piece and precracking of length a_p achieved in order to get rid of perturbations due to the notch and the initiating load. This is the reason why the value of the maximum of the SIF, $K_{\max}$, for crack initiation should be as small as possible, and in any case less than its value during the test. $K_{\max}$ at initiation should be 30–60% of the fracture toughness K_{Ic} .

6.2.5.3 Determination of the Crack Length and Orientation

The crack length is denoted a ; it is the distance between a reference plane in the test piece and the crack tip. a_{fat} is the crack length between the notch root and the crack tip as it can be observed on the face where the crack emerges. The crack front not being rigorously straight, a *correction* a_{cor} must be added to the crack length a . This correction is the extra length obtained by taking the average of the distance to the curved front. Then $a = a_n + a_{\text{fat}} + a_{\text{cor}}$.

There are many different ways to measure the crack length. The most obvious one is to follow the crack tip by an *optical device*. Other methods are, among others,

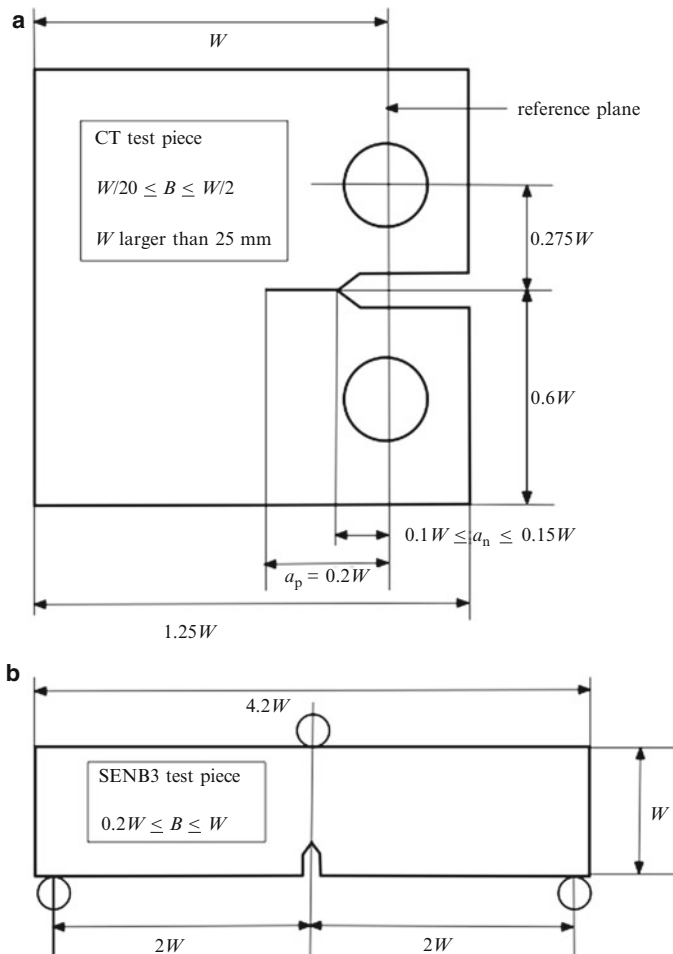


Fig. 6.8 Dimensions of the CT and of the 3PSENB test pieces

the measurement of the test piece *compliance* (Sect. 2.2.1.2), by partial unloading at some predetermined intervals; measurement of the *electric potential drop* between two points on both sides of the crack tip when a direct or alternating current is fed to the test piece; measurement of the *deformation* of the test piece face opposite to the notch.

The compliance method requires a precise measurement of the load point displacement.

When dealing with textured material the fatigue crack propagation velocity depends of the *orientation*, and it is important to note the position of the crack with respect to the texture. Figure 6.9 shows how to designate this orientation, referring to the coordinate axes X, Y and Z. In a steel sheet X would be the rolling direction

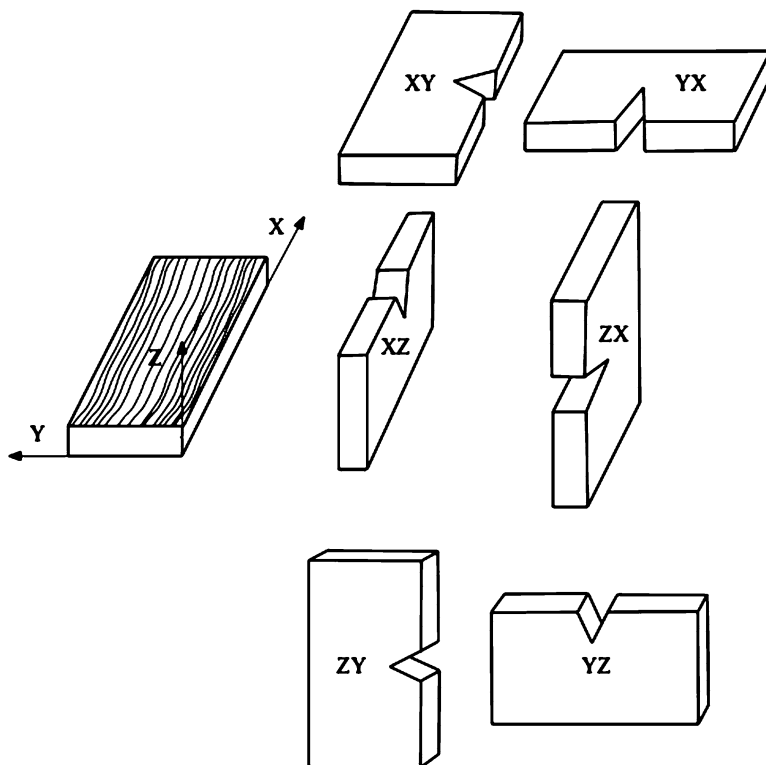


Fig. 6.9 Designation of the orientation of the test pieces with respect to the axes of a textured material

L, Y the transverse direction T and Z the short transverse direction S. The test piece orientation is designated by two letters, the first being the direction of the opening of the crack and the second the direction of propagation.

The propagating crack does not always remain in the same plane and it can deviate mostly owing to texture effects. For the test to be valid, this *deviation* must remain within a “corridor” of width two times $0.05 W$ on both sides of the plane of symmetry of the notch.

6.2.5.4 Determination of the Stress Intensity Factor Amplitude

Once the evolution of the crack length has been determined in a given test piece, it is possible to deduce the connected evolution of the SIF using the following formulae for the CT and for the 3PSNB specimens (ISO12108:2002) (ISO standard gives the following formula using mm as a unit for lengths. Written with m as a unit the factor $10^{1.5}$ must be replaced by 10^6)

$$K = \frac{F}{BW^{1/2}} g(a/W) 10^{1.5} \quad \text{lengths in mm} \quad \text{force in kN} \quad (6.4)$$

$g(a/W)$ being given by:

$$\text{CT} \quad 0.2 \leq a/W \leq 1.0$$

$$g(a/W) = \frac{(2 + \alpha)(0.886 + 4.64\alpha - 13.32\alpha^2 + 14.72\alpha^3 - 5.6\alpha^4)}{(1 - \alpha)^{3/2}}$$

$$3\text{PSENB} \quad 0 \leq a/W \leq 1.0$$

$$g(a/W) = \frac{6\alpha^{1/2}}{(1 + 2\alpha)(1 - \alpha)^{3/2}} [1.99 - \alpha(1 - \alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)] \quad (6.5)$$

In these equations $\alpha = a/W$.

For the tests to be valid, a long enough ligament ($W - a$) ahead of the crack tip must remain in order that small scale yielding is insured. The conditions are:

$$\begin{aligned} \text{CT} \quad W - a &\geq \frac{4000}{\pi} \left(\frac{K_{\max}}{R_{p0.2}} \right)^2 \\ 3\text{PSENB} \quad W - a &\geq \left(\frac{6000WF_{\max}}{BR_{p0.2}} \right)^{1/2} \end{aligned} \quad (6.6)$$

When testing at constant ΔF , ΔK is increasing. Now, below a *threshold* denoted ΔK_{th} , the crack stops propagating. For its determination, tests must be performed at decreasing ΔK . In that case, the threshold can depend on the decreasing rate $(1/K)(dK/da)$. It is recommended to keep it larger than -1 mm^{-1} . ΔK_{th} is taken for a crack velocity of about 10^{-8} mm/cycle .

Results are plotted as the crack propagation rate, da/dN , N being the number of cycles as a function of ΔK , on a bi-logarithmic diagram. A straight line on such a diagram corresponds to the propagation law:

$$\frac{da}{dN} = C \Delta K^m \quad (6.7)$$

This is the so-called Paris law since it was given by Paul Paris at Lehigh University.

6.3 Fatigue Mechanisms

6.3.1 Fatigue Initiation Mechanisms

6.3.1.1 Fatigue Is the Result of Strain Irreversibility

Let us first recall that we define damage as the creation of new surfaces. Under those conditions, would the applied load be such that the deformations would remain perfectly elastic, reversibility would be perfect and there could not appear any fatigue damage. On the contrary, if the yield limit of the material is exceeded, defects activity at the microscopic level produces an irreversible plastic deformation, provided we neglect recovery effects. In crystalline materials these defects are dislocations and they lead to slips, in particular within grains at the surface. This creates steps. Further cycling with the same load amplitude would then remain elastic. This would not produce any further plastic deformation and we would still predict that there should not be any fatigue damage. If now we reverse the load until the elastic limit is exceeded, the slip steps, which had been created, could simply disappear, reversibility would be perfect and again we would not expect any fatigue damage. However, the number of dislocations has been increased, and local strain-hardening prevents reversibility of slips. Now, damage, an increase of the free surface due to the steps, has occurred. *Fatigue is the result of slip irreversibility.* We now look into more details at this phenomenon. The ease of plastic deformation being different in the grains which are within the solid and in those which are at the free surface, we will treat them separately.

6.3.1.2 Dislocation Activity and Stress Evolution

In Volume I (Sect. 3.4.2.2 Fig. 3.60c) we showed that plastic straining increases the number of dislocations and that they gather to form clusters of higher density than the average. Cyclic straining increases this heterogeneity of the distribution of dislocations (see Mughrabi 1988). After a sufficient number of cycles, *dislocation cells* appear, the walls of which become more and more perfect as cycling goes on (Fig. 6.10). The walls contain dipoles of edge dislocations, while screw dislocations go back and forth in the channels between.

However, when cross-slip is reduced, such as in alloys of low stacking fault energy, *planar arrangements of dislocations* are created rather than cells, at least for low strain amplitudes (Fig. 6.11).

Figure 6.12 is a diagram enhancing these evolutions, showing the influences of the stacking fault energy, which is decreased by alloying copper with zinc, and of the strain amplitude (Lukas and Klesnil 1973) (see a review by Grosskreutz 1971).

In any case, the slip is concentrated in thin bands. They are called *persistent slip bands* (PSB)

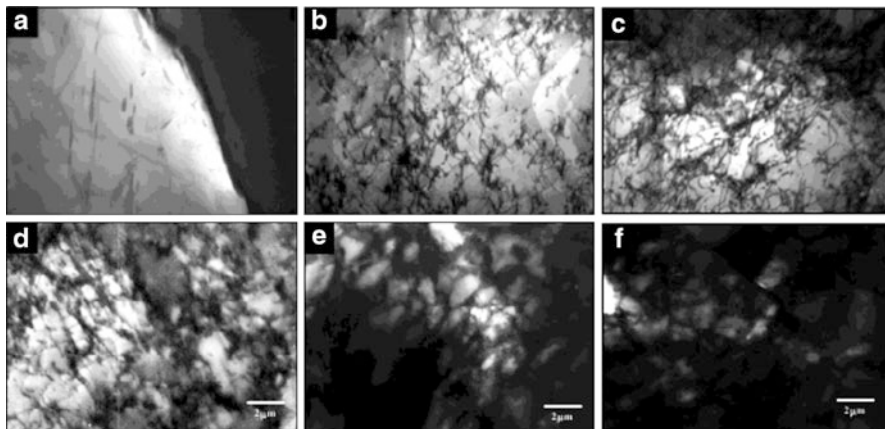


Fig. 6.10 Cell structure of dislocations in aluminium cycled at $\Delta\varepsilon = 0.010$ (a) for $N = 30$ cycles; (b) for $N = 11,300$ cycles at failure (Plumtree and Abdel-Raouf 2001)

This multiplication of dislocations and this evolution of their arrangements are accompanied by *cyclic strain-hardening* and sometimes by *strain-softening* (Pineau 2010). Figure 6.6 shows stress-strain hysteresis loops and that Eq. 6.3 can be used to represent the cyclic behaviour. Figure 6.13 shows how evolves the stress as the test piece is loaded at constant strain amplitude. It shows strain-hardening for 304 stainless steel and on the contrary strain-softening for high strength 4340 steel. Often, the number of cycles is plotted on a logarithmic scale. This can be misleading in enhancing too much the first cycles and hiding somewhat the long stabilised plateau. To draw the strain-hardening (or strain-softening) curves several specimens can be tested at different strain amplitudes, or starting with a low strain amplitude and increasing it after stabilisation of the stress, or applying successive blocks of linearly increasing and decreasing strain amplitudes.

The strain-softening of the 4340 steel depends on the tempering temperature. It is the highest when this temperature is 400°C. A possible explanation is that the martensitic structure is, before cycling, the more strain-hardened in this case, the cyclic strain destroying the initial arrangements of dislocations.

In alloys containing precipitates that can be sheared such as Al-Cu or Waspaloy, after an initial strain-hardening due to the multiplication of dislocations in a similar way as observed in solid solutions, softening is observed. This can be explained by an evolution of the precipitates by the back and forth crossing by dislocations (Calabrese and Laird 1974).

Pineau (2010) gives a thorough discussion of the behaviour of steels of various microstructures showing how these considerations apply. For further information see also Polak (1991), Suresh (1998), and Doquet (2009).

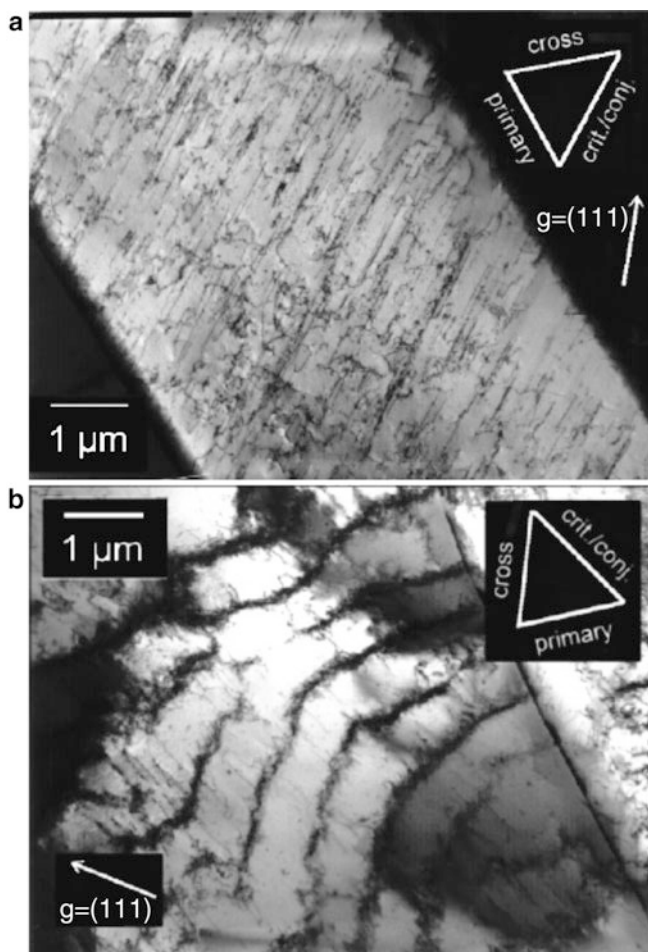


Fig. 6.11 Cell structure of dislocations in a Cu-15% Zn cycled at: (a) $\Delta\epsilon_p = 5.510^{-4}$ for $N = 50,000$ cycles; (b) $\Delta\epsilon_p = 3.310^{-3}$ for $N = 8,333$ cycles (Lee et al. 2006)

6.3.1.3 Crack Initiation

The cyclic slip activity is easier in the grains which are at the free surface, because the deformation there is less restricted by neighbouring grains than in the interior. It is then understandable that crack initiation takes place on the surface. We will see however that this is not the case in gigacycle fatigue.

Figure 6.14 shows how successive irreversible slips form steps on the surface and tongues sticking out, called *extrusions*, or slits called *intrusions*.

Multiplication of extrusions and intrusions in a slip band initiates a crack, which begins to propagate in a 45° direction to the load axis, in mode II of fracture

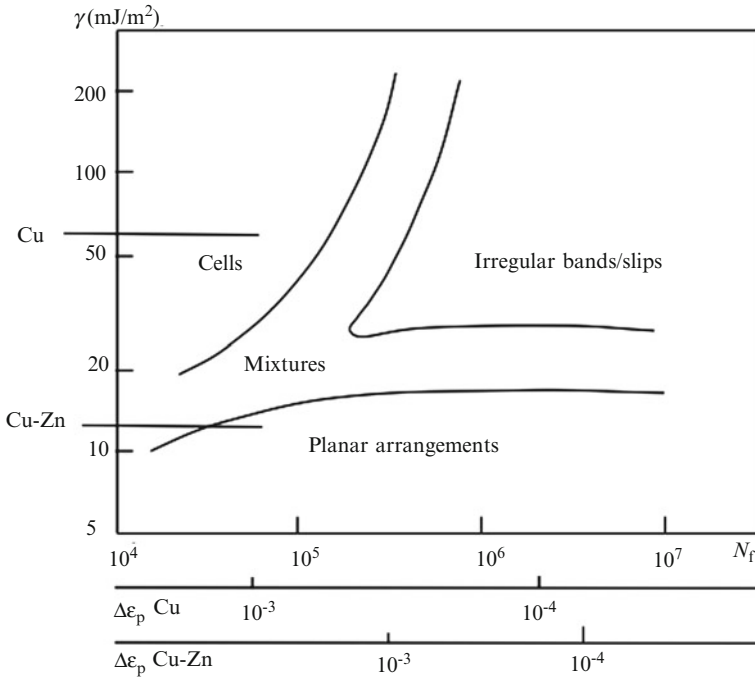


Fig. 6.12 Different arrangements of dislocations as a function of the stacking fault energy γ and of the fatigue life related to strain amplitude $\Delta\epsilon$ for copper and copper-zinc (Lukas and Klesnil 1973)

mechanics. This is called stage I of fatigue crack initiation. When the crack reaches a grain boundary it changes orientation progressively to align perpendicular to the load axis, in mode I of fracture mechanics. This is stage II of fatigue crack initiation (Fig. 6.15).

Now, crack initiation in PSBs is far from being the only mechanism. *Intergranular cracking* due to the accumulation of dislocations along grain boundaries is often observed, as underlined recently by Pineau and Antolovich (2011) (see also Chap. 8).

In *multiaxial loading* the initial propagation of the fatigue crack depends on the principal stress orientation at the free surface (Fig. 6.16). When the loading is torsion with shear parallel to the free surface, the crack propagation direction in stages I and II is parallel to the free surface. This is *case A* of Brown and Miller² (1973) (Fig. 6.16). When the loading is in traction-compression parallel to the free surface, the crack propagation direction in stages I and II is away from the free surface into

²Professor Keith John Miller (1932–2006) from Sheffield University has largely contributed to a better understanding of fatigue in metals.

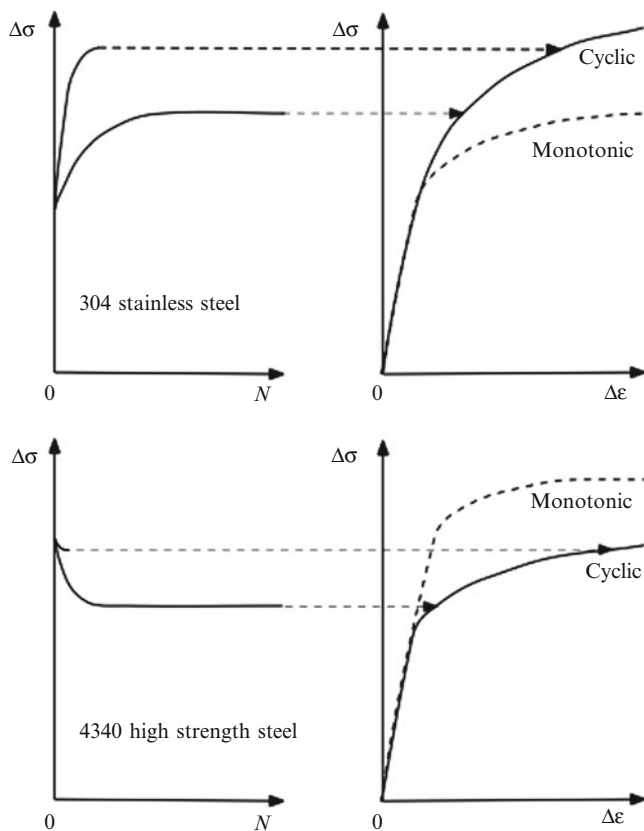


Fig. 6.13 Sketch of the evolution of the stress in cyclic straining of a strain-hardening material (304 stainless steel) and of a strain-softening one (4340 high strength steel)

the solid at first following the shear direction. This is *case B*. It is more dangerous than case A since the crack depth keeps increasing. It can be also anticipated that a tensile normal stress on the shear plane will facilitate crack initiation.

The presence of an *inclusion* close to the surface, a site of stress concentration, can easily initiate a shear band, which emerging at the surface starts a fatigue crack. These inclusions are quite detrimental to fatigue life. Murakami and his co-workers (Murakami and Endo 1983, 1986; Murakami and Usukuki 1989; Murakami et al. 1990; Murakami 2002) proposed to consider an inclusion as a small crack since initiation is almost instantaneous at its interface. Hence, in their model, the fatigue strength is propagation controlled and corresponds to the threshold for short cracks. Fracture mechanics considerations and some empirical observations led them to an equation relating the fatigue limit σ_F to the size of inclusions, characterised by the parameter $\sqrt{\text{area}}$, the square root of the area perpendicular to the maximum principal stress, whatever the shape:

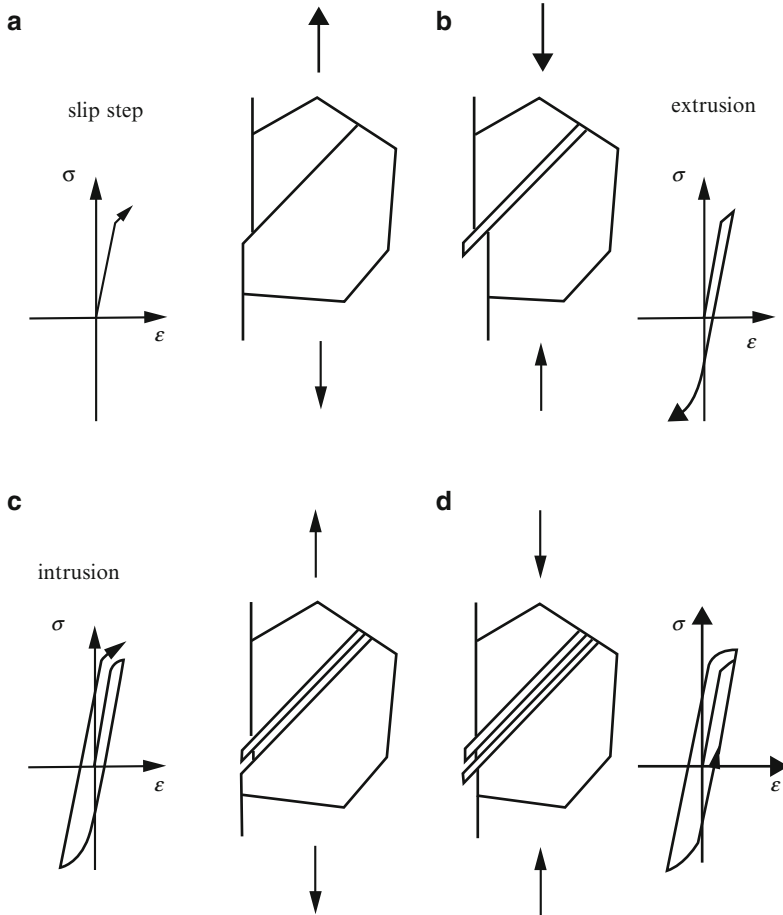


Fig. 6.14 Sketch showing successive irreversible slips initiating (a) a step on the surface (b) an extrusion (c) an intrusion (d) another extrusion upon unloading

$$\sigma_F = C (HV + 120) \sqrt{\text{area}}^{1/6} [(1 - R)/2]^{\alpha_{ME}} \quad (6.8)$$

where $C = 1.56$ for an internal inclusion, $= 1.43$ for a surface inclusion, $= 1.41$ for an inclusion touching the surface; HV is the Vickers hardness in kg/mm^2 ; $\sqrt{\text{area}}$ is in μm ; R is the R ratio; $\alpha_{ME} = 0.226 + HV \cdot 10^{-4}$. The most severe inclusion, the one which determines the fatigue life, is the largest inclusion touching the surface. Murakami et al. (1989) give statistical methods to determine this.

In *gigacycle fatigue*, it is observed that cracks no longer initiate at the free surface, but rather from an inclusion deep inside the solid (Fig. 6.17). At this location strain-hardening is higher owing to the blocking of slip by surrounding grains. When the applied stress is larger than the yield strength on the free surface the strain there is larger than inside the solid and this is the site of crack initiation.

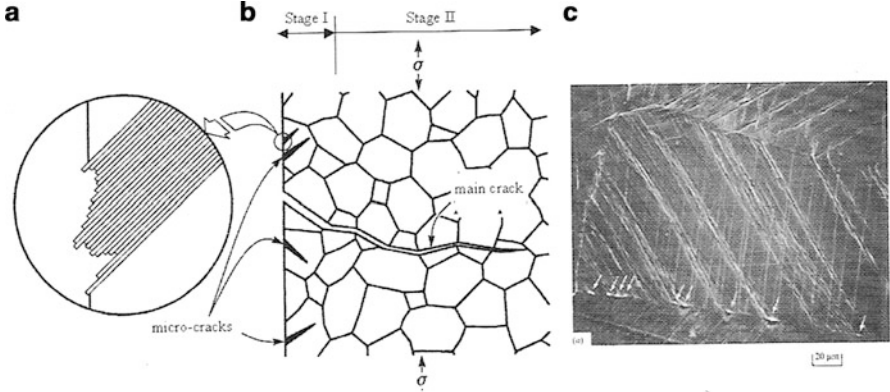


Fig. 6.15 (a) Initiation of a fatigue microcrack due to the formation of intrusions and extrusions on the free surface; (b) stages I and II of fatigue crack initiation; (c) intrusions and extrusions on the surface of a fatigued specimen of copper (Bathias and Pineau 2010)

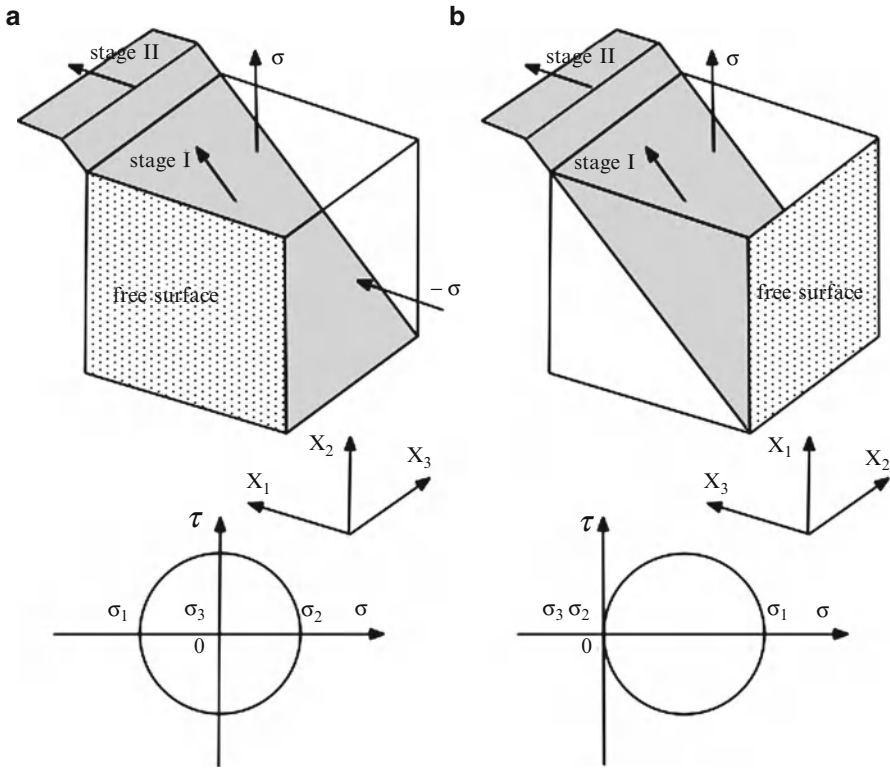


Fig. 6.16 A and B facets according to the state of stress at the free surface. Case A torsion, case B tension-compression. The corresponding Mohr circles are represented

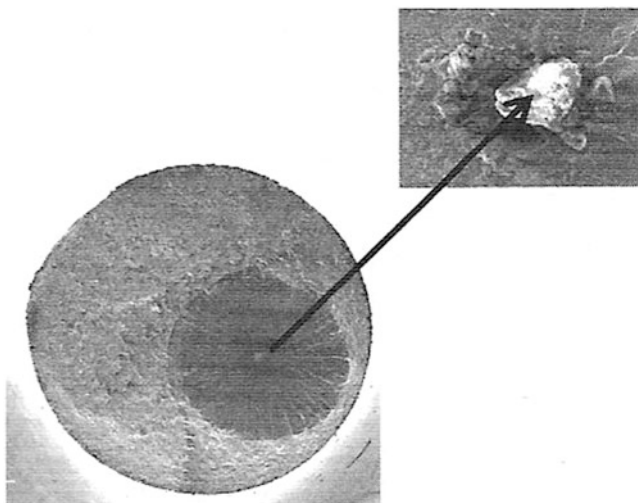


Fig. 6.17 Gigacycle fatigue crack initiated from an inclusion inside the test piece in a martensitic 42CD4 steel ($R_m = 1,530$ MPa, $\Delta\sigma = 760$ MPa, $R = -1$, 20 kHz, $N_f = 5.7510^8$ cycles) (Bathias 2010a)

However, when the stress at the free surface drops down so that there is no more slip activity on the free surface there is no more crack initiation there, whereas inside the solid, at the stress concentration of an inclusion, there is still enough plastic deformation to initiate a crack. It must also be noted that, in spite of the cooling of test pieces by blowing air on the surface, the interior might remain quite hot when cycling at high frequencies. This could well modify the crack initiation mechanism, shifting from transgranular at low frequencies to intergranular at high frequencies.

As a consequence of the crack initiation mechanisms at the free surface, it is easy to understand why the *surface condition* plays a very important role in fatigue. *Machining grooves*, producing strong stress concentrations, are very detrimental. This effect is the more pronounced when plastic deformation is low. Resurfacing brings a great deal of improvement.

An efficient way to prevent fatigue crack initiation is to introduce compressive *residual stresses* on the surface. This can be achieved by *quenching*, producing a volume increase due to the superficial martensitic transformation. *Shot peening* is also an efficient way to introduce compressive stresses on the surface. It is a common method employed in particular after welding.

The superficial nature of fatigue crack initiation explains also why it is so affected by *environment*. In particular, it is slowed down in vacuum. This will be expanded in Chap. 7.

6.3.2 Mechanisms of Fatigue Crack Propagation

6.3.2.1 Long Cracks

(a) *Cyclic plastic zone and crack closure*

It is observed that long cracks take an orientation such that they open in mode I. If the piece is large enough, a small scale yielding zone is created at the crack tip but its size is modified with respect to monotonic loading owing to the reversal of the stress. The plastic deformation elongates the material in a direction perpendicular to the crack plane. When the load is released, the piece, which as a whole is elastic, acts like a pair of pliers on this plastically deformed material and puts it in compression. Figure 6.18 shows the monotonic Irwin plastic zone in plane stress, and, upon unloading, the creation of a plastic zone in compression by the superposition of a negative stress singularity in $r^{-1/2}$. The size R_{cyc} of the cyclic plastic zone is found by equating the result of this superposition at a distance $x_1 = R_{\text{cyc}}$ with the yield strength $-R_p$, assuming it is the same in compression as in traction (no Bauschinger effect), and by expressing that, upon unloading, the integral of the stress σ_2 must be equal to zero. Then

$$R_{\text{cyc}} = \frac{1}{4\pi} \left(\frac{\Delta K_I}{R_p} \right)^2 = \frac{R}{4} \quad (6.9)$$

This is a quarter of the monotonic zone size R (Eq. 2.93 in Sect. 2.3.3.1). The result is approximate as far as the constitutive behaviour is supposed to be perfectly plastic and stress equilibrium is insured only along the Ox_1x_2 plane.

The situation is more complicated in plane strain because the stress is not constant in the plastic zone, but there will be again a cyclic plastic zone embedded within the monotonic one. The condition of small scale yielding is thus more easily met in cyclic loading.

In the cyclic plastic zone the flow stress R'_p of the material is different from the one in monotonic loading owing to strain-hardening and sometimes strain-softening phenomena as explained in Sect. 6.3.1.2. This modifies the cyclic plastic zone size calculated assuming perfect plasticity.

The existence and the size of the cyclic plastic zone were checked by various experimental techniques. In their pioneering work, Hahn et al. (1972) used a chemical etching technique for an iron-silicon alloy. Micro-hardness measurements have the advantage that they allow determining the level of strain-hardening. Various X-ray methods were also employed. Experiments are in good agreement with the calculations (Bathias 2010b).

After unloading, there remains a zone in compression close to the crack tip. Thus there is a *crack closure effect*: to reopen the crack a certain level of SIF K_{op} must be reached (Elber 1971). This has led to the introduction of the effective stress intensity factor $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}}$ (Sect. 6.4.3.2 for further discussion).

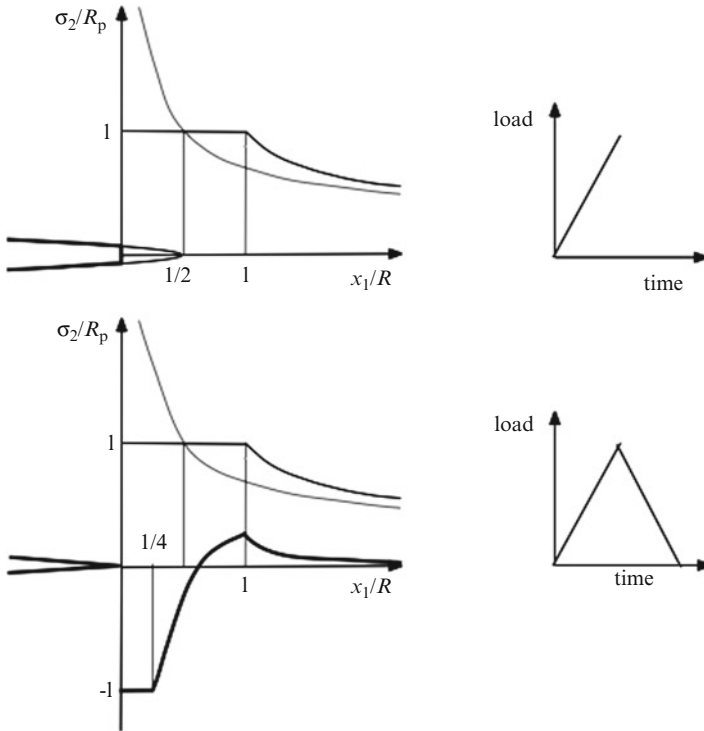


Fig. 6.18 Cyclic plastic zone. (a) monotonic plane stress Irwin plastic zone (b) cyclic plane stress plastic zone by superposition of the negative stress elastic singularity

Other effects can contribute to crack closure. The crack path is not completely straight (Fig. 6.19). Irreversibility prevents complete closure upon unloading. When loading again, at least the SIF corresponding to the coming in contact of the crack faces must be overcome. The oxide formed on the freshly opened surfaces might also prevent complete crack closure with a similar effect. As the plastic zone size is smaller in the centre of a plate than at its surface, the opening of the crack will appear sooner in the centre than at the surface.

(b) *striation mechanisms*

Irreversible slips produce the plastic deformation at the crack tip. Figure 6.20 is a sketch giving an idea about what is taking place. A positive load opens the crack and the crack tip blunting produces a crack opening displacement (CTOD) δ . At the same time there is a slight advance of the crack by an amount of the order of $\delta/2$. Not only does this mechanism produce a crack growth step, but it also leaves a striation behind. Indeed, this is what can be observed on fatigue fracture surfaces (Fig. 6.21). Each striation should correspond to one cycle, and counting them might give the number of cycles applied. By applying regular loading blocks with overloads, Pelloux (1969) was able to clearly relate each

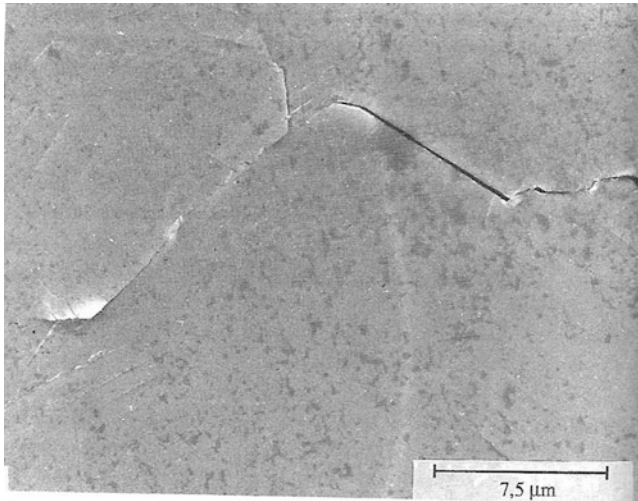


Fig. 6.19 Tortuous fatigue crack path in a nickel-based alloy displaying incomplete closure

striation to one cycle. However, care is needed, as at small load amplitudes it might take several cycles for a striation to appear. At large load amplitudes, ductile tearing and dimple mechanisms explained in Chap. 4 appear at the same time as the striations.

(c) *Non-metallic materials*

In non-metallic materials the crack growth in fatigue is due to brittle mechanisms taking place at the crack tip (Ritchie 1999). We will deal again with this in Chap. 10.

6.3.2.2 Short Cracks (Verreman 2010)

Microstructurally short cracks initiated from slip bands on the surface grow, in tensile testing, in planes inclined at 45° . When they come close to grain boundaries they slow down and many of them stop. Depending on the orientation of the next grain one of these short cracks can go through the grain boundary and continue growing in a slightly different crystallographic orientation. Grain boundaries are not the only barriers to short cracks propagation (Miller 1993). In ferritic-pearlitic steels for instance, the strongest barriers are pearlite bands. The crack propagation rate increases with the grain size in the cases of Ti and of a Ti-Al alloy (see Sect. 6.5.2.2), and of low carbon steel (Tokaji et al. 1994). However it was found to be grain-size independent in the case of Ti-6Al-4V, and decreasing when the grain size was increased in the case of the aluminium alloy 7075 T6 and of A533B steel (James and Sharpe 1989).

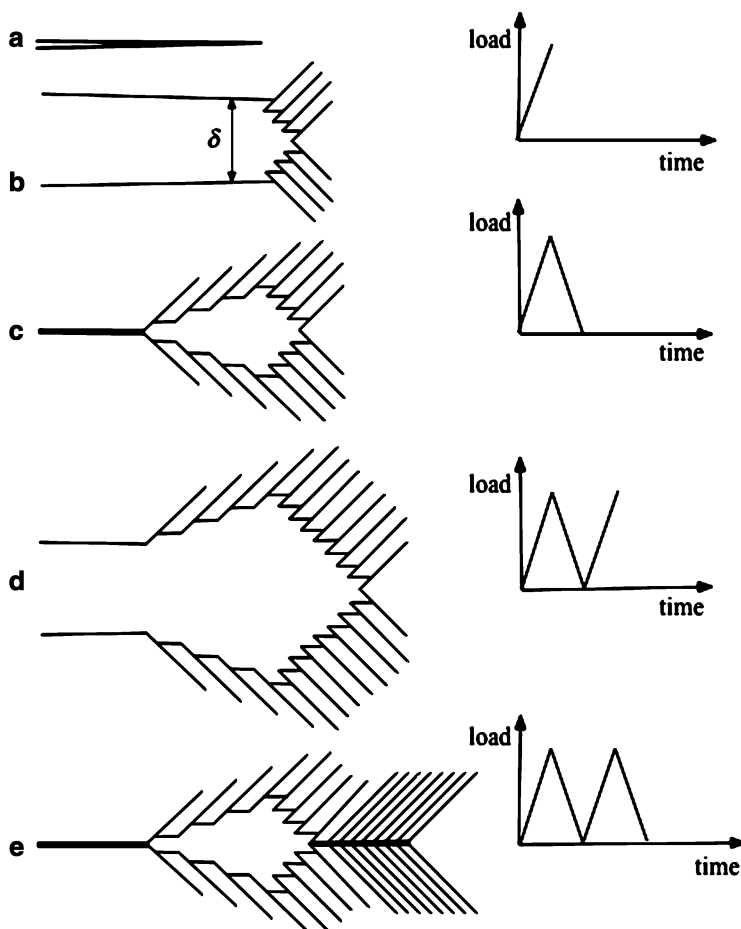


Fig. 6.20 Irreversible slips at the crack tip. (a) no load, the crack is closed (b) positive load, the crack is blunted by slips (c) unloading, the crack is closed but a striation is left behind the tip (d) positive load, the crack is blunted again (e) unloading, the crack tip has moved forward

Progressively, after crossing several grains the cracks propagate in mode I. They are considered as *mechanically short cracks* as far as their behaviour escapes normal fracture mechanics description. Three reasons are formulated to explain this:

- The condition of small scale yielding does not prevail for instance because the crack is completely embedded in a large plastic zone in a notch.
- The plastic zone at the crack tip emerges out on the free surface on the back of the crack, which is completely embedded in its plastic zone.
- The conditions for crack closure are absent.

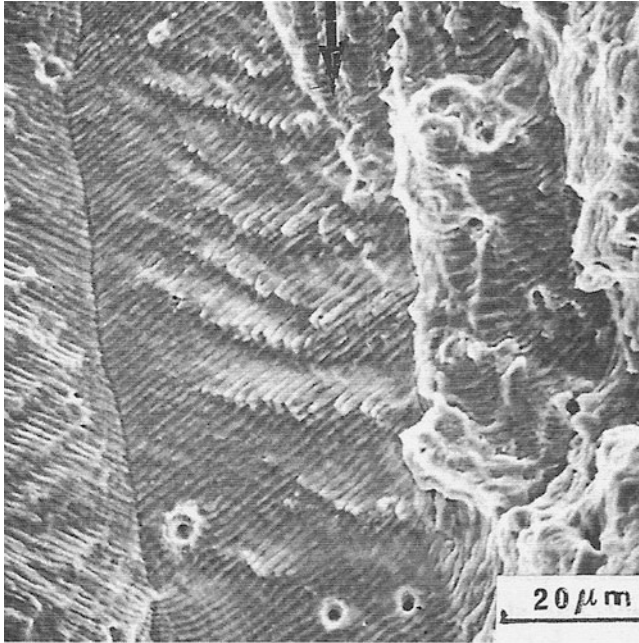


Fig. 6.21 Scanning electron micrograph displaying striations on the fatigue fracture surface of an austenitic stainless steel. Note the change of orientation of the striations when crossing a grain boundary

6.3.2.3 Kitagawa Diagram

The Kitagawa diagram (Kitagawa and Takahashi 1976) represented in Fig. 6.22 summarises the crack propagation conditions and quantifies the limit between short crack and long crack propagation.

In this diagram is first plotted the horizontal line representing the limit of short cracks propagation, that is the endurance limit for a large number of cycles N , 10^8 for instance. The second line with a slope $-1/2$ represent the threshold for long cracks propagation:

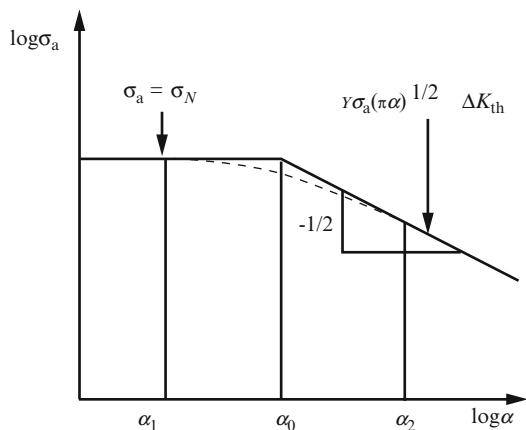
$$Y\sigma_a\sqrt{\pi a} = \Delta K_{th} \quad (6.10)$$

where Y is a factor function of the geometry (1.12 for a straight front crack, 0.728 for a semi-circular crack), ΔK_{th} the SIF threshold.

The critical length a_0 is found at the intersection of the two lines:

$$a_0 = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{Y\sigma_a} \right)^2 \quad (6.11)$$

Fig. 6.22 Kitagawa diagram. σ_N is the endurance limit, the fatigue strength for N cycles, 10^8 for instance. The line with the slope $-1/2$ represents the threshold for long cracks propagation



This critical length is strongly dependent on the material. It is for instance lower than $10 \mu\text{m}$ for high strength steels whereas it reaches 1 mm in the case of mild steels (Verreman 2010).

When approaching this critical length a_0 , the threshold line gives non-conservative results. A deviation appears below a critical length a_2 . Taylor (1989) proposed and checked that the value of a_2 corresponds to the highest of $10 d$ (d being the grain size) or of $10 R_{th}$ (R_{th} being the cyclic plastic zone size at the propagation threshold).

6.3.2.4 Notch Effect – Frost Diagram

When the cracks initiate from a notch root they slow down as they grow. If the notch root has a stress concentration factor K_T , the stress amplitude needed to initiate a crack there is σ_N/K_T . The corresponding hyperbola, below which there is no crack initiation, is represented in Fig. 6.23, called the Frost diagram (Frost 1959). An initiated crack, which has reached the size of the notch depth D (neglecting the length a of the crack with respect to D), will not propagate if the SIF amplitude is less than the SIF threshold. The corresponding stress amplitude is a constant in the Frost diagram.

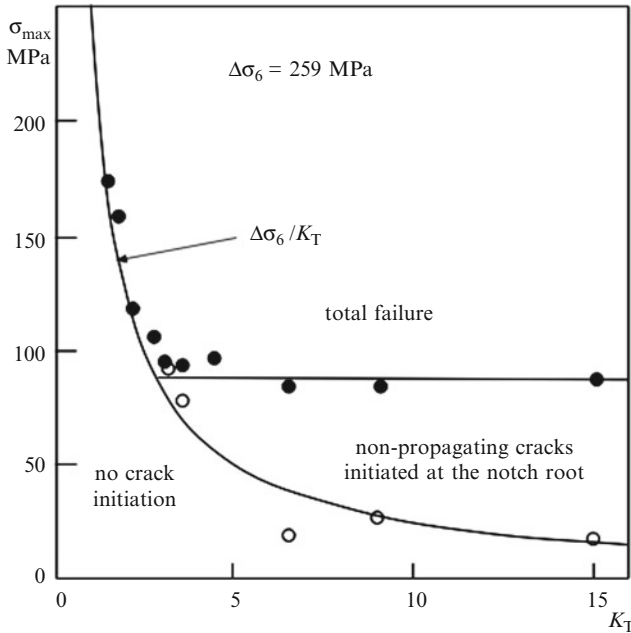


Fig. 6.23 Frost diagram for mild steel for a V notched specimen. The *open symbols* represent the stress needed to initiate a crack; the *full symbols* for propagation until complete fracture (According to Verreman [2010](#))

6.4 Fatigue Behaviour and Modelling

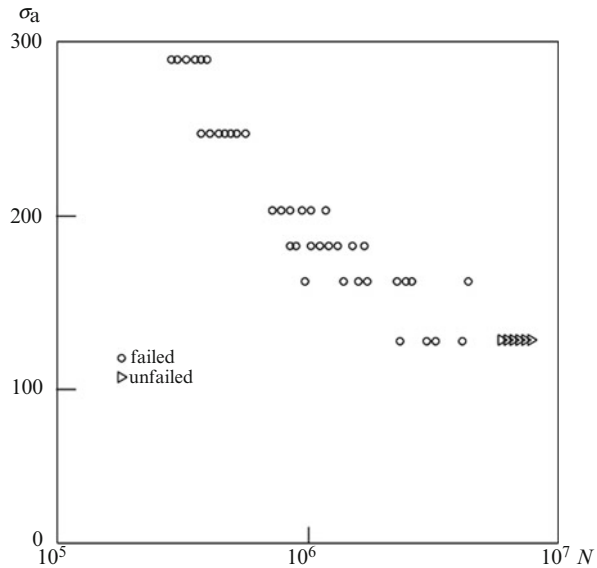
6.4.1 High-Cycle Fatigue

6.4.1.1 The Wöhler Curve and the Endurance Limit

The high-cycle fatigue behaviour is represented by the *Wöhler (or S-N) diagram*. The stress amplitude is plotted as a function of the number of cycles (Fig. [6.24](#)). When the ratio R is equal to -1 , the ordinate of the diagram is the maximum stress. Figure [6.24](#) shows results on steel (Lieurade et al. [2010](#)).

Below a certain stress amplitude level, specimens do not fail before 10^6 cycles. This is the endurance limit σ_6 at 10^6 cycles. The *endurance limit* is also denoted σ_d . We cannot be certain that testing for a larger number of cycles the specimens would not fail at this level. In fact, gigacycle testing usually displays lower endurance limits.

Fig. 6.24 Fatigue life for various levels of the stress amplitude (According to Lieurade et al. 2010)



Various mathematical representations of the Wöhler curve were proposed. The oldest one is simply:

$$\log N = a - b\sigma_N \quad (6.12)$$

In 1910 Basquin proposed the equation:

$$\log N = a - b \log \sigma_N \quad (6.13)$$

This does not incorporate a fatigue endurance limit.

So, in 1914 Stromeier introduced the equation:

$$\log N = a - b \log (\sigma_N - \sigma_d) \quad (6.14)$$

Figure 6.25 shows the adjustment of this equation with a large number of experimental results.

We omit various other representations (see Lieurade et al. 2010).

It can be seen that the dispersion of the results is not negligible. The *statistical distribution* of the fatigue life for a given stress amplitude level follows a log-normal law; that of the fatigue strength for a given number of cycles a normal law.

A number of methods were proposed for the *determination of the endurance limit*: the probit method, the staircase method, the iteration method and the method with K non-failed specimens (ISO 2003). They are described by Lieurade et al. (2010). The probit method is the most accurate, but it needs to test a number of

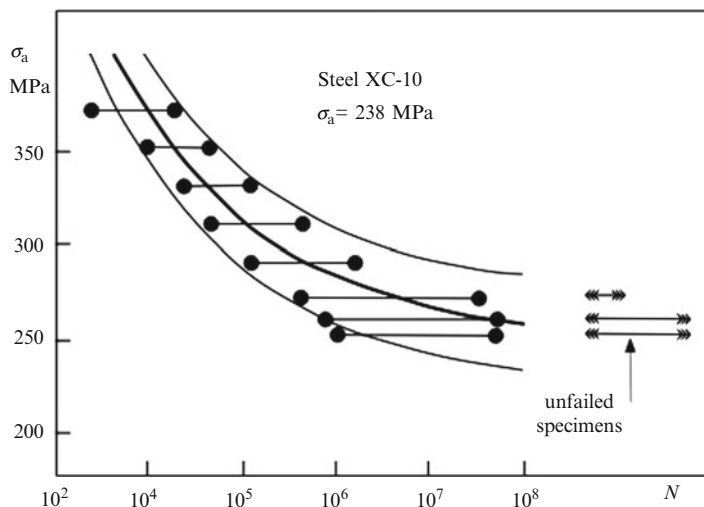


Fig. 6.25 Adjustment of the Stromeier equation (Eq. 6.14) with a large number of experimental results on XC-10 steel. The bars with points represent the experimental scatter (Adapted from Lieurade et al. 2010)

specimens larger than 40. The simplest is the staircase method. It is of approximate precision when performed with one test per level using a total of 10–15 specimens; but, performed with groups of two to four tests at each level, it can reach average precision. Otherwise, the iteration and the identical K results method (K equal to 3 for instance) need but 10–15 tests providing approximate estimations.

Many empirical relations were proposed to deduce the endurance limit from the tensile properties. As an order of magnitude the endurance limit is half the UTS (R_m). Four hundred and eighty nine results in rotating bending at 10^7 cycles provided the relation:

$$\sigma_7 = 0.32 R_m + 121 \quad (\text{in MPa}) \quad (6.15)$$

Another relation proposed for steels is (Buthod and Lieurade 1986):

$$\sigma_7 = 0.41 R_m + 2 \quad (\text{in MPa}) \quad (6.16)$$

Table 6.1 gives a few results.

The fatigue life includes the initiation stage as well as the propagation stage. We will see that for a number of cycles larger than 10^4 the initiation stage is predominant. Therefore *size effects* are related with the free surface area and to the increased probability to pick up grain oriented for easiest crossing of barriers, or an inclusion. The larger the piece, the more reduced the fatigue life.

Table 6.1 Endurance limit for various alloys

Alloy	Yield strength R_p	UTS R_m	Endurance limit σ_d	
	MPa	MPa	MPa	Ratio σ_d/R_m
Mild steel	260	395	232	0.59
Medium strength steel	360	590	270	0.46
High strength steel	580	735	410	0.56
Cast iron SG	—	600	230	0.38
10Ni6Cr steel	900	1,000	530	0.59
35Cr4Mo steel	1,430	1,640	660	0.40
Maraging steel	1,725	1,840	758	0.41
304 stainless steel	240	586	240	0.41
316 stainless steel	205	515	270	0.52
Brass		460	130	0.28
2024 Al	275	395	125	0.31

6.4.1.2 Mean Stress Effects and Multiaxial Loading

In Sect. 6.2.2.2 we described the ways to represent tests for various R ratios (Fig. 6.5).

Multiaxial testing results are most of the time referred to the von Mises stress. However, it is obvious, from the results presented in the Goodman diagram for instance, that the hydrostatic stress Σ_m has a large effect, which is not taken into account by the von Mises equivalent stress Σ_{eq} alone. Sines proposed the equivalent stress $A + B\Sigma_m$; Crossland $A + B\Sigma_{m(max)}$ where A and B are empirical parameters. Dang Van (1973) proposed to follow the loading path in complex multiaxial situations such as found in wheel. Figure 6.26 shows the limit within which the stress path should remain for not exceeding the endurance limit.

The figure shows that if the Goodman criterion is valid the Dang Van criterion is non-conservative for high values of the hydrostatic stress.

An extension of the Dang Van criterion was proposed by Papadopoulos (1999) based on the average shear stress on active slip planes.

6.4.1.3 Cumulative Damage

How can we predict the fatigue life for successive loadings at different levels? The classical method is that of *Miner* (Miner 1945). It consists in evaluating the fatigue damage at a certain level by the parameter N/N_f , N_f being the fatigue life at this particular level. Then the fatigue life is reached when this parameter is equal to 1. When testing at various successive levels, the resulting fatigue damage is now simply given by adding the corresponding damage parameters. The fatigue life is supposed to be reached when the cumulative parameter takes the value 1. As a consequence of this linear rule the order of application of the various loadings

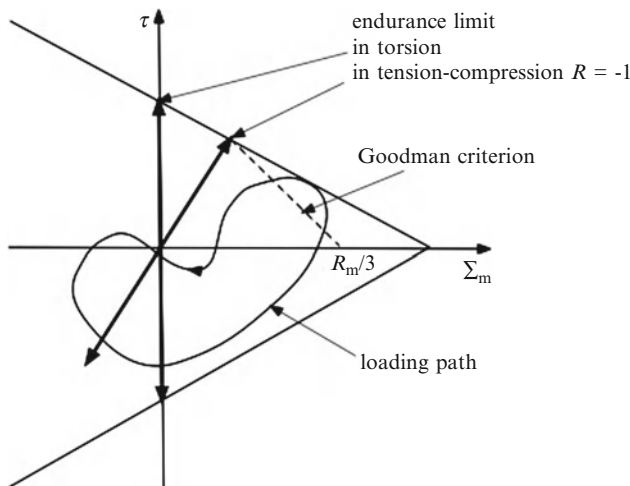


Fig. 6.26 Dang Van diagram of the shear stress τ as a function of the hydrostatic stress Σ_m for complex loading paths. They should remain within the endurance limit lines determined from results in tension-compression and in torsion

should not play any role. Experiments show that this is rarely the case. Starting at a low load level and continuing at a higher one usually leads to a longer life than the one predicted, and vice-versa.

This rule can be extended to more general variable amplitude loadings based on the rain flow counting method.

Notwithstanding the inadequacy of the Miner rule, which has no real physical basis, it is much used because of its simplicity.

Different methods were proposed for *nonlinear cumulative damage* (Chaboche 2011). We cannot discuss them all in the present book. Fatemi and Yang (1998) give a complete literature review. It is worth mentioning the Chaboche model (1974). The evolution of the damage parameter D is given by a differential equation:

$$\frac{dD}{dN} = \frac{D^\alpha}{(1 - \alpha) F(\sigma_N)} \quad (6.17)$$

where $F(\sigma_N)$ is any equation representing the Wöhler curve, the Basquin one (6.13) for instance. Integration from $D = 0$ to $D = 1$ at each level indeed yields the fatigue life at that level. For two successive levels for instance, N_1 cycles at level 1 produce a damage D_1 :

$$D_1 = \left(\frac{N_1}{N_{fl}} \right)^{\frac{1}{1-\alpha_1}} \quad (6.18)$$

and the integration of (6.17) between D_1 and 1 yields:

$$1 - D^{1-\alpha_2} = \frac{N_2}{N_{f2}} \quad (6.19)$$

Combining the two preceding equations:

$$\frac{N_2}{N_{f2}} = 1 - \left(\frac{N_1}{N_{f1}} \right)^{\frac{1-\alpha_2}{1-\alpha_1}} \quad (6.20)$$

Note that if $\alpha_1 = \alpha_2$, Eq. 6.20 reduces to the linear cumulative Miner rule.

6.4.2 Low-Cycle Fatigue

6.4.2.1 Cyclic Strain-Hardening

Low-cycle fatigue tests are usually performed at constant strain amplitude as explained in Sect. 6.2.3. Stress-strain loops are recorded. They can be represented by the cyclic strain-hardening law (6.3). Their shape provides information about the nature of hardening: isotropic and kinematic. Isotropic hardening is characterised by rather square loops, kinematic hardening by tapering ones.

In solid solution alloys, strain-hardening increases during the first cycles and it finally saturates. This stable strain-hardening is particularly high for austenitic stainless steels (Brun et al. 1976). In alloys containing shearable precipitates, such as aluminium-copper alloys or nickel base alloys, the strain-hardening goes through a maximum before decreasing. Whereas the increasing strain-hardening results from the multiplication of dislocations the subsequent decrease is attributed to a destruction of the order within the precipitates, which eases the crossing by dislocations (Calabrese and Laird 1974; Fournier and Pineau 1977). When the same alloys contain non-shearable larger precipitates, a strain-hardening plateau is observed and there is a strong Bauschinger effect with a kinematic behaviour (see Sect. 3.4.3.2 in Volume I).

In low-carbon steels, when the stress amplitude is less than the lower yield strength a cyclic strain-softening can be observed (Klesnil and Lukas 1965). This is similar to the lessening of the heterogeneous deformation due to the escape of the dislocations from the carbon and nitrogen atmospheres observed in monotonic loading. This early softening is then replaced by cyclic hardening once the deformation has become homogeneous.

In martensitic steels, which can be considered as strongly strain-hardened structures, strain-softening is observed (Thielen et al. 1976; Gallet and Lieurade 1974; Van Swam et al. 1975) (see Pineau 2010 for a summary).

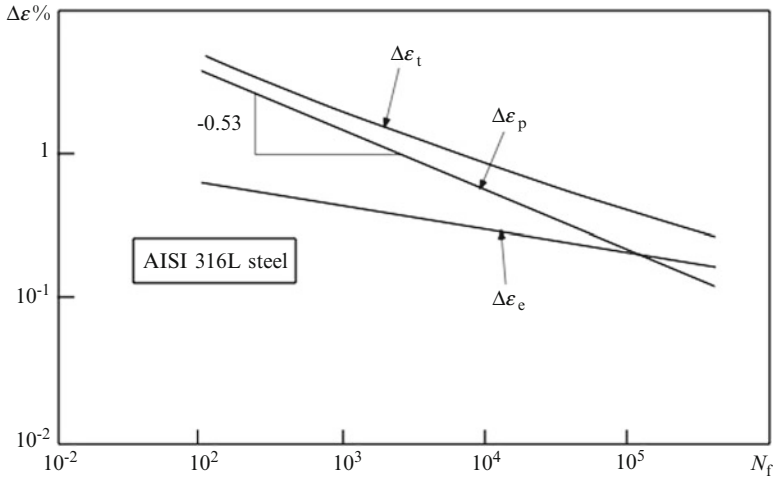


Fig. 6.27 The Manson-Coffin and the Basquin laws for AISI 316L stainless steel. Data points corresponding to results obtained in three different laboratories are not reported (see Bathias and Pineau 2010, Figure 4.7)

6.4.2.2 Low-Cycle Fatigue Strength

Fatigue life follows the Manson-Coffin law (Fig. 6.27) (Manson 1952; Coffin 1954). In a bilogarithmic diagram data points align along two straight lines, if the strain range is partitioned in the elastic component $\Delta\epsilon_e$ and the plastic component $\Delta\epsilon_p$. The two following relations result:

$$\begin{aligned}\Delta\epsilon_e &= \epsilon_{ef} N_f^{-a} \\ \Delta\epsilon_p &= \epsilon_{pf} N_f^{-b}\end{aligned}\quad (6.21)$$

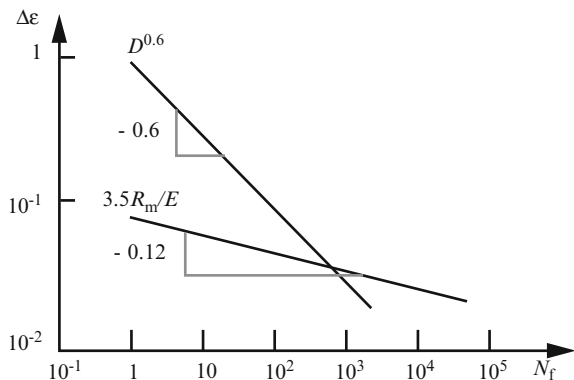
The first relation can be written in term of the stress range, showing that it is nothing else than the Basquin law (Eq. 6.13). In this equation $\epsilon_{ef} = \sigma_f/E$ where σ_f is the fatigue strength coefficient. The slope b is usually close to 0.12.

The second relation is the Manson-Coffin law. The slope a is close to 0.5.

Data obtained on a wide variety of materials were gathered by Baumel and Seeger (1990).

The first quarter of cycle representing the monotonic stress-strain curve, the coefficient ϵ_{pf} would be related to the ductility of the material. According to Manson the best fit is obtained for $a = 0.6$ and $\epsilon_{pf} = D^{0.6}$ with the ductility $D = \log[100/(100 - Z)]$, with Z the percent reduction in area. This is basis of the

Fig. 6.28 Universal slopes method to deduce the fatigue resistance curves from tensile properties



method of *universal slopes*, which allows deducing the fatigue resistance curve from simple tensile properties. This is depicted in Fig. 6.28. According to this method:

$$\Delta \varepsilon_t = \left(\frac{N_f}{D} \right)^{-0.6} + 3.5 \frac{R_m}{E} N_f^{-0.12} \quad (6.22)$$

where R_m is the UTS and E the Young modulus.

Later Muralidharan and Manson (1988) proposed a similar improved formula.

6.4.2.3 Low Cycle Fatigue Crack Propagation

A number of studies have shown that the LCF crack propagation rate can be written as:

$$da/dN = Aa(\Delta \varepsilon_p)^\alpha \quad (6.23)$$

where A and α are constants for a given material (see *e.g.* Levaillant and Pineau 1983; Skelton 1975). The value of the coefficient α is related to the exponent n' of the cyclic stress-strain curve (Eq. 6.3), as suggested by the model proposed by Tomkins (1968). In this model, the crack growth rate is supposed to be proportional to the crack tip opening displacement and is written as:

$$\frac{da}{dN} = \frac{\pi^2}{8} \frac{\Delta \sigma^2}{2R_m'^2} \Delta \varepsilon_p a \left(1 + \frac{\pi^2}{8} \frac{\Delta \sigma^2}{4R_m'^2} \right) \quad (6.24)$$

where R'_m is the “ultimate tensile strength” of the cyclically deformed material. When the term $(\pi^2/8)(\Delta \sigma^2/4R_m'^2)$ is small compared to 1, the above equation leads to:

$$\frac{da}{dN} \approx \frac{\pi^2}{8} \frac{\Delta \sigma^2}{2R_m'^2} \Delta \varepsilon_p a \quad (6.25)$$

which is of the same form as (6.23) with $\alpha = 1 + 2n'$. This model which is rather well in agreement with experiments indicates that fatigue crack propagation in LCF is controlled by continuum mechanics. Thus in LCF the crack propagation stage is less sensitive to physical aspects than the initiation stage.

6.4.2.4 Crack Initiation from Notches

The question is now raised to predict the low-cycle fatigue life in the presence of a notch or of a hole. In fact the material near the notch root can be plastically cycled even if the rest of the structure remains elastic. It is possible to compute by finite elements the maximum strain range in the concentration zone and then use the Manson-Coffin relation to calculate the number of cycles for initiation there. A simplified method is to use the Neuber relation:

$$K_\sigma K_\varepsilon = K_T^2 \quad (6.26)$$

K_σ and K_ε being respectively the stress concentration coefficient and the strain concentration coefficient and K_T the elastic stress concentration coefficient (3 for instance for a circular hole). With the help of this approximate equation, assuming the cyclic strain-hardening law (Eq. 6.3) is known, the strain range can be calculated from the range $\Delta \Sigma$ of the applied stress on the structure:

$$\frac{\Delta \varepsilon_t \Delta \sigma}{\Delta E \Delta \Sigma} = \frac{\Delta \varepsilon_t K' \Delta \varepsilon_t^{n'} E}{\Delta \Sigma^2} = K_T^2 \quad (6.27)$$

6.4.3 Fatigue Crack Propagation

6.4.3.1 The Paris Law

In Sect. 6.3.2.1 we saw that the fatigue propagation was due to cyclic plastic deformation at the crack tip. In small scale yielding conditions it is an increasing function of the SIF. It is therefore logical to correlate the crack propagation rate to this loading parameter. This is what Paris and Erdogan (1963) did. They found that in a bi-logarithmic diagram the data points aligned on a straight line and the crack propagation law could be written as Eq. 6.3. This is called the Paris law. Figure 6.29 shows results for A 533 B steel. In that case the parameters of the Paris law are $C = 3.710^{-12}$ and $m = 3$ in m/cycle and MPa m^{1/2}.

It can be observed a deviation for decreasing crack propagation rates, and the existence of a crack propagation threshold ΔK_{th} . For high values of ΔK there is an acceleration of the crack propagation rate owing to the superposition of ductile tearing mechanisms.

Fig. 6.29 Crack propagation rate for A533 B steel as a function of the stress intensity factor range follows the Paris law (test at R ratio equal to 0.1). Crack propagation stops at the threshold

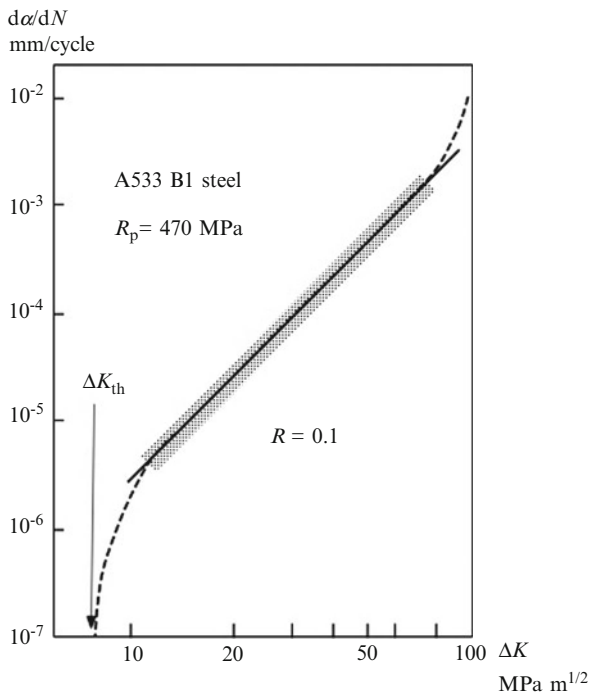


Figure 6.30 shows that most materials, including non-metallic ones, follow the Paris law.

When lacking precise data the standard BSI PD 6493: 1991, “Guidance on methods for assessing the acceptability of flaws in fusion welded structures” provides a useful estimate for the parameters of the Paris law for ferritic steels: $C = 3.10 \cdot 10^{-13}$ and $m = 3$ with units mm and MPa.

Table 6.2 gives values of the Paris law parameters for various alloys.

Empirical correlations were found between the parameters C and m . For instance, taking average values, from the Paris equation at the onset of the upward deviation:

$$\left(\frac{da}{dN} \right)_{CR} = [(1 - R) K_{Ic}]^m \quad (6.28)$$

Carpinteri and Paggi (2007) proposed:

$$\begin{aligned} \log C &= \log (7.6 \times 10^{-7}) + m \log (1.67 \times 10^{-2}) & \text{for steels} \\ \log C &= \log (2.5 \times 10^{-6}) + m \log (2.86 \times 10^{-2}) & \text{for Al alloys} \end{aligned} \quad (6.29)$$

In view of the crack advance mechanism, we would expect the crack propagation rate to be related to the *crack opening displacement* CTOD. It is proportional to

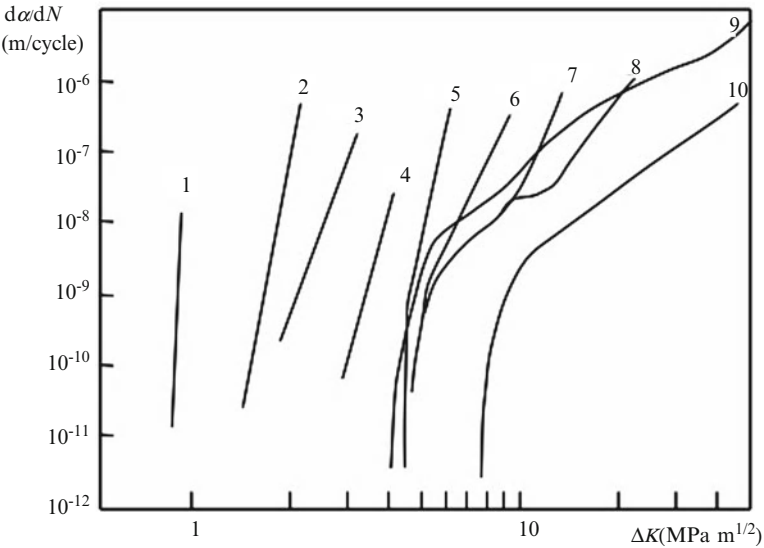


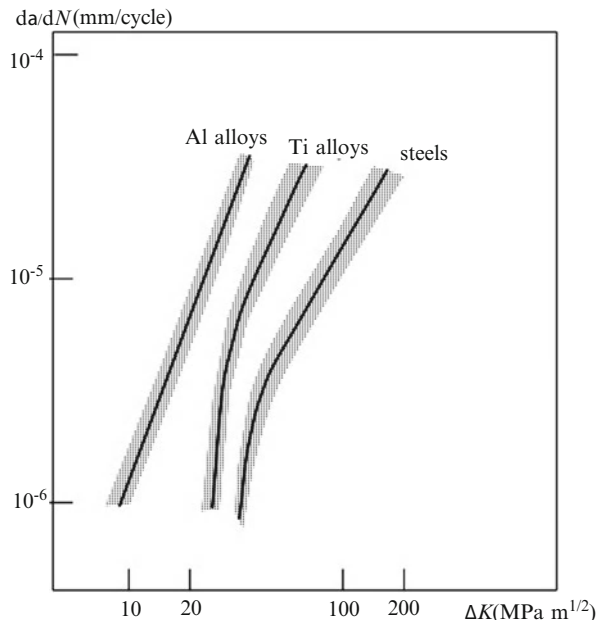
Fig. 6.30 Crack propagation rate da/dN as a function of the stress intensity factor range ΔK for various materials (1) pyrolytic carbon graphite composite (2) overaged Mg-PSZ (3) $\text{MoSi}_2 + \text{Nb}$ (4) $\text{Al}_2\text{O}_3 + \text{SiC}_w$; (5) $\text{TiAl} + 5/\text{TiNb}$ (6) $\text{TiAl} + 20/\text{TiNb}$ (7) $\alpha_2\text{-Ti}_2\text{Al}$ (8) 2090-T8 AlLi (9) α/β Ti-550 (10) high strength steel (Adapted from Ritchie 1999)

Table 6.2 Paris law parameters C and m for various alloys (units m and MPa)

Alloy	C (multiplied by 10^{11})	m
Aluminium alloy 2219 T62 (L-T)	24.0	2.87
Aluminium alloy 2219 T87 (L-T)	6.27	3.30
Aluminium alloy 6061 T62 (L-T)	16.3	3.20
Aluminium alloy 7075 T73 (L-T)	18.1	3.41
Pure Ti	1.95	3.41
Ti-6Al-4V RT	3.80	3.11
Steel PH 13-8Mo H 1000	0.5	3.40

$\Delta K^2/ER'_p$, (E the Young modulus and R'_p the cyclic flow stress), so that, on this basis, we would anticipate an exponent m equal to 2. That it is not the case might be due to the fact that the crack front does not grow by one step as a whole. The crack rather advances only at certain locations along the front and then at others, so that the entire crack propagation rate is lower than the local rate. Note also that at slow rates as already mentioned it takes several cycles for local propagation of the crack. The influence of the CTOD shows that the crack propagation rate depends on the *Young modulus* and indeed Figs. 6.30 and 6.31 display such a trend, the higher the modulus the lower the rate.

Fig. 6.31 Paris law for various alloys



Testing in *vacuum* provides Paris curves, which are lower than the ones recorded in air (Petit and Sarrazin-Baudoux 2010). This demonstrates that this behaviour must already be considered as environment assisted fatigue discussed in Chap. 7.

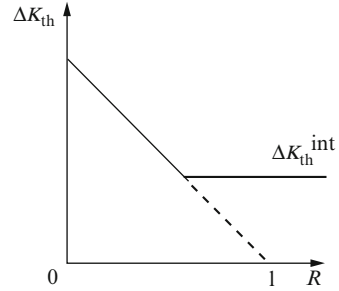
Increasing the *R* ratio translates the Paris curve towards higher values of the crack propagation rate. Many more or less empirical equations were proposed to take this effect into account (Masounave et al. 2010). To quote only one example:

$$\frac{da}{dN} = C \frac{(\Delta K - \Delta K_{th})^m}{(1 - R) K_{Ic} - \Delta K} \quad (6.30)$$

6.4.3.2 Crack Closure

We saw that there are a number of crack shielding mechanisms, which prevent the crack from opening before reaching a sufficient level of SIF: K_{op} (Elber 1971). These mechanisms were described by Ritchie (1999). K_{op} can be determined by recording the crack opening displacement, putting an extensometer on both sides of the crack, as close to the tip as possible. We can define an effective SIF range: $\Delta K_{eff} = \Delta K - K_{op}$. Now the Paris law can be expressed as a function of ΔK_{eff} . If the *R* ratio is sufficiently high the applied SIF always remains above the opening SIF and there is no more any crack closure effect. The variation of ΔK_{th} with *R* ratio

Fig. 6.32 Variation of ΔK_{th} for long cracks versus R ratio



is schematically shown in Fig. 6.32. ΔK_{th} decreases with R ratio, as expected from the existence of a crack closure effect. Above a critical value of the R ratio, R_t , there remains a positive value of ΔK_{th} , ΔK_{th}^{int} , which corresponds to the intrinsic threshold measured in the absence of crack closure effect. In low alloy steels, typically $\Delta K_{th}(R=0)$ is of the order of $8 \text{ MPa m}^{1/2}$, and ΔK_{th}^{int} is of the order of $4 \text{ MPa m}^{1/2}$.

Crack closure effect depends on crack length. It is an increasing function of crack length. K_{op} can be expressed as (McEvily et al. 1984; Minakawa et al. 1984):

$$K_{op}(a) = K_{op}(\infty) [1 - \exp(-ka)] \quad (6.31)$$

In this expression k is a simple parameter (mm^{-1}) which measures the length over which the closure of short cracks has an effect. $K_{op}(\infty)$ is the threshold for infinitively long cracks. Typical values of $K_{op}(\infty)$ and k are the following: in an aluminium alloy 2024-T351 $K_{op}(\infty) = 6.65 \text{ MPa m}^{1/2}$ and $k = 10 \text{ mm}^{-1}$, in nodular cast iron (NCI) $K_{op}(\infty) = 4 \text{ MPa m}^{1/2}$ and $k = 1.5 \text{ mm}^{-1}$ and in a 9Cr-1Mo steel $K_{op}(\infty) = 2.4 \text{ MPa m}^{1/2}$ and $k = 13 \text{ mm}^{-1/2}$. These values indicate that the short crack effect due to crack closure will occur over much longer distances in NCI compared to the other two materials (Journet et al. 1989)

An approximate calculation of K_{op} can be performed by the use of the Dugdale-Barenblatt model of the plastic zone (Sect. 2.3.3.2). At any point in the plastic zone the displacement u_3 is known., and so the compressive stress induced by unloading. (See Pommier 2010).

6.4.3.3 Overloads

When an overload is applied while testing at constant amplitude, it can be observed that the crack propagation rate drops and then slowly increases to reach the value which was achieved before the overload application. If the overload is sufficiently high the crack can stop all together. Figure 6.33 shows an example of experimental results on a carbon steel (Hamam et al. 2007). The retardation effect lasts until the crack tip remains within the overload plastic zone (about 0.5 mm in the case of the 1.8 overload). The progressive increase of the crack propagation rate lasts for about 4 mm of propagation for the 1.8 overload. The Wheeler's (1972) model quantifies

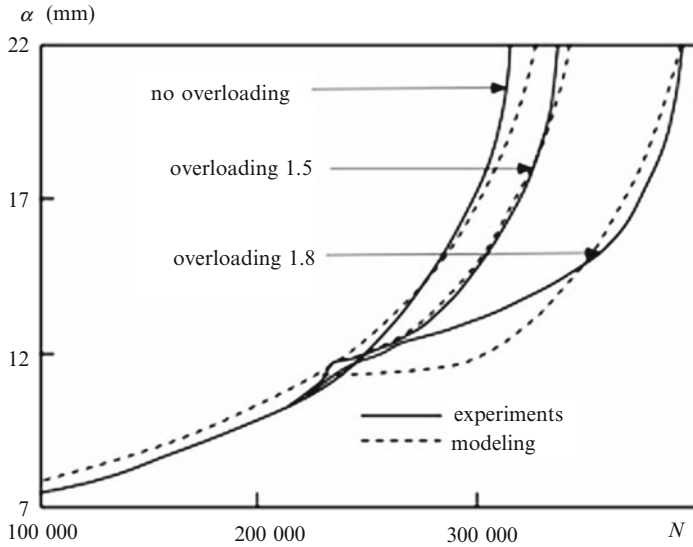


Fig. 6.33 Fatigue crack growth tests on a pearlitic-ferritic steel 0.48%C with a stress amplitude of 100 MPa at $R = 0.05$ with two different one cycle overloads (Adapted from Hamam et al. 2007)

this concept by multiplying the crack growth rate by a delay function ϕ lower than 1, such that:

$$\phi = \left(\frac{R_i}{a_{\text{overload}} + R_{\text{overload}}} \right)^{m_w} \quad (6.32)$$

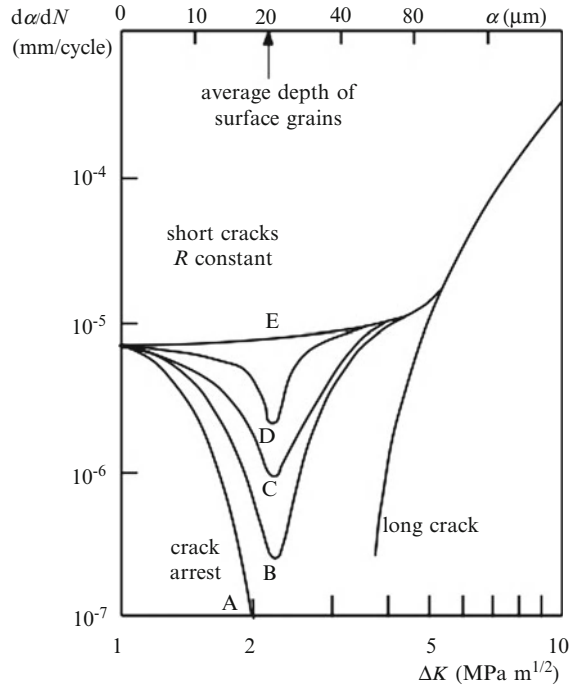
where R_i is the size of the plastic zone of the crack of length a_i , and a_{overload} and R_{overload} its length and plastic zone size respectively when the overload is applied. The Willenborg's (1971) model follows the same idea, but applying the delay correction on the SIF. This is enlightened considering the Elber crack closure, K_{op} , increasing when the overload is applied since the plastic zone size increases and so does the compressive stress.

Pommier (2010) modelled the propagation with history effects by a detailed analysis of the plastic deformation at the crack tip. This is the scale-up method. Some results of this modelling are shown in Fig. 6.33.

6.4.3.4 Short Cracks

The presence of a crack propagation threshold does not mean that no propagation can be observed at lower values of the SIF range. The crack propagation threshold of long cracks is found at decreasing values of the SIF. However short cracks propagate at very small levels of SIF ranges (Fig. 6.34).

Fig. 6.34 Propagation rates of microstructurally short cracks in aluminium 7075 T651 alloy. The curves labelled A, B, C, D, E correspond to increasing levels of $\Delta\sigma$, kept constant for each curve recording (Adapted from Lankford 1985)



It can be observed that cycling at constant stress range, the SIF range increasing since the cracks grow, the propagation rate first decreases as long as the cracks remain within the surface grains. After a minimum observed for this particular size, the rate keeps increasing until reaching the long cracks behaviour. If the stress range is insufficient, crack arrest is observed.

This kind of behaviour was modelled by Hobson et al. (1986) writing:

$$\frac{da}{dN} = B\Delta\sigma^s (D - a) \quad (6.33)$$

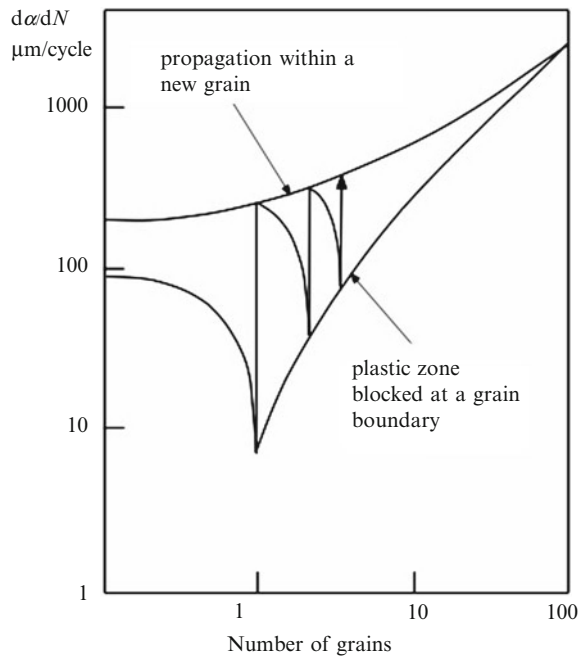
where B and s are material constants and D the distance of the crack tip to the grain boundary.

Navarro and de Los Rios (1988) related the propagation rate to the crack tip displacement δ_{II} in mode II for a crack within a PSB blocked at a grain boundary:

$$\Delta\delta_{II} = \frac{1}{\pi M} \frac{\sqrt{1 - \alpha^2}}{\alpha} a \Delta\sigma \quad (6.34)$$

with $M = \mu/2\pi(1 - \nu)$ for an edge dislocation and $M = \mu/2\pi$ for a screw, μ and ν being the shear modulus and the Poisson ratio respectively; α is the ratio between the crack length a and the crack length plus the size of the plastic zone. The crack propagation rate is considered to be proportional to this crack tip displacement.

Fig. 6.35 Crystallographic short fatigue crack propagation rate according to the model of Navarro and de Los Rios (1988)



Following the evolution corresponding to Eq. 6.33 the crack slows down as its tip approaches the grain boundary (Fig. 6.35). Once it has reached it, it propagates in the next grain, following again the evolution corresponding to Eq. 6.33 but now with D the distance to the next grain boundary. This is repeated each time a grain boundary is met. The rate of crack propagation thus oscillates with a decreasing amplitude, until the crack ceases to feel the microstructural barriers and behaves as a mechanically short crack.

The propagation rate of mechanically short cracks, while increasing with the SIF range, can be larger than the one of long cracks. This is what is shown on Fig. 6.36. This behaviour is due to the development of the plastic zone, which in proportion to the plastic zone at the tip of a long crack, is larger because it is not in small scale yielding condition. In particular it joins the free surface at the back of the crack.

Espinosa (1995) and Verreman and Espinosa (1997) measured the crack closure for these mechanically short cracks. They discovered that there was no closure when the R ratio was equal to 0, and that the ratio $U = \Delta K_{\text{eff}}/\Delta K$ was always larger than 0.5 when R was equal to -1 . This is more than for long cracks for which this ratio U is 0.1. This closure effect explains why mechanically short cracks grow faster than long ones.

There is no really satisfactory method to model the propagation of mechanically short cracks. Using a stress intensity or strain intensity factor would assume the validity of LEFM whereas the small scale yielding conditions are not met. Nevertheless, El Haddad et al. (1979) could match experimental data on steel using

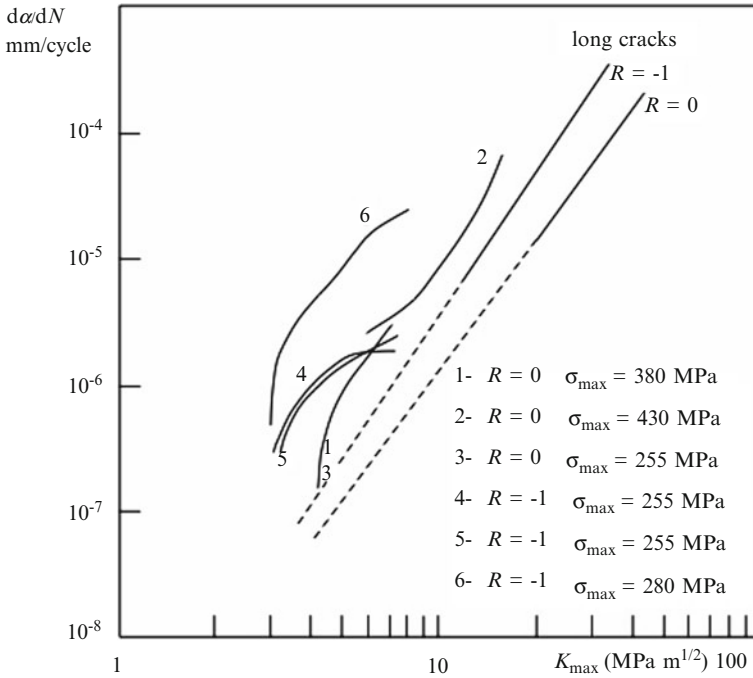


Fig. 6.36 Propagation rates as a function of $K_{\max}$ for semi-elliptical cracks at the surface of smooth specimens of structural steel ($R_p = 265$ MPa). The depth of the cracks ranging between 100 and 500 μm exceeds the grain size, which is about 30 μm (Adapted from Espinosa 1995)

a strain intensity factor obtained by adding to the crack length a , a length a_0 such that $a_0 = (1/\pi)(\Delta K_{\text{th}}/\Delta\sigma_N)^2$, $\Delta\sigma_N$ being the stress range for the endurance limit at N cycles.

In principle, a better solution would be to use EPFM with the right estimation of the J integral (Dowling 1977; Leis 1982; Laue and Baumas 2006).

6.4.3.5 Intrinsic Crack Propagation Under Vacuum

Petit et al. (2003) tested fatigue crack propagation under vacuum and confirmed the observation that the crack growth rate was lower than in air. They then called intrinsic the behaviour under vacuum and they distinguished three stages.

1. Intrinsic stage I corresponds to slip localisation giving rise to rapid crack propagation along the maximum shear direction (short crack behaviour).
2. Intrinsic stage II corresponds to crack propagation in mode I following a Paris law. Plotting da/dN as a function of $\Delta K_{\text{eff}}/E$ (E is Young modulus) puts together with a small dispersion all the data for various alloys. The Paris exponent is equal

- to 4. The value of the constant in the Paris equation written with $\Delta K_{\text{eff}}/E$ in $\text{m}^{1/2}$ and da/dN in mm/cycle is about 3×10^8 .
3. Intrinsic stage I-like is observed at slow cracking rates. It corresponds to crack propagation limited to a grain in a polycrystal, the plastic zone size being smaller than the grain size. It is delayed by the number of cycles needed to cross the barrier constituted by the grain boundary.

6.5 Improving the Fatigue Life

6.5.1 Surface Treatments

As fatigue cracks initiate from the free surface (except in gigacycle fatigue), a good way to improve the fatigue life is by surface treatments.

The surface should be as smooth as possible so as to avoid strain concentration at the root of tool marks.

Further, creating compressive stresses in layers close to the surface prevents cracks from opening. This can be achieved by *shot-peening* (SAE standard J2441 200008 Shot peening). 1,000% fatigue life improvement can be obtained. It should be done with rounded particles and not with sharp edges as the ones of particles used in grit blasting. One of the best media is cut wires. The intensity of the blasting operation can be controlled with Almen strips: thin strips. Shot-peened under the same conditions as the worked piece, they bend more or less according to the compressive stress introduced on one side. Care should be taken not to over- blast, which could strain-harden too much and introduce cracks. The amount of coverage is an important parameter of the operation.

Another way to introduce compressive stresses close to the surface and to improve the surface finish at the same time is by *burnishing*. This is accomplished by pressing a rolling ball on the surface.

6.5.2 Metallurgical Factors

6.5.2.1 Crack Initiation and Endurance Limit

We saw that one source of strain concentrations, which can initiate fatigue cracks, is the presence of *inclusions*. It is therefore interesting to make the choice of alloys as clean of inclusions as possible.

We are then faced with two contradictory requirements. On the one hand, to hamper crack initiation at the surface we should introduce there as many barriers to crack propagation as possible. This means we should reduce the grain size. In fact

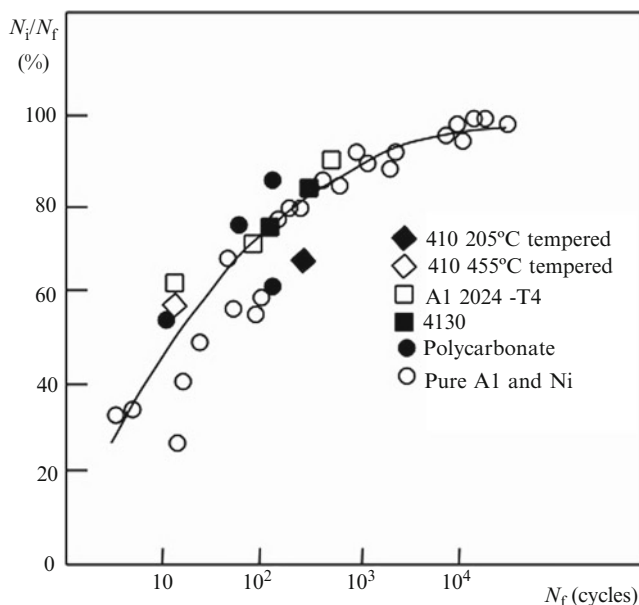


Fig. 6.37 Ratio of the number of cycles needed to create a crack of length 50 μm to the number of cycles to fracture N_f , as a function of N_f for various materials (Adapted from Manson 1972)

the endurance when it is controlled by fatigue crack initiation more or less follows a Hall-Petch relation with the grain size. On the other hand, when the endurance is controlled by fatigue crack propagation alloys should have large grains because this makes the crack path more tortuous and promotes crack closure.

Figure 6.37 shows the relative parts of initiation and of propagation controlled fatigue. For long fatigue lives the number of cycles spent in crack initiation is large and those for crack propagation negligible. It is the contrary for low-cycle fatigue. As we are mostly interested for industrial applications in long fatigue lives, we should privilege the improvement of surface properties.

The *grain size* should then be reduced.

We showed also that crack initiation took place in persistent slip bands. It is then desirable to promote wavy slip. Increasing the *stacking fault energy* is beneficial. We want also to avoid shearable precipitates and thus age enough the alloys for *precipitation coarsening*. Increasing the *yield strength* reduces the crack tip plasticity and thus the short crack propagation rate. It must be nevertheless remembered that what counts is the cyclic yield strength. It might be reduced by cyclic softening especially for high strength alloys. The yield strength increase can be limited to surface layers by *case hardening* and by *quenching*. Furthermore these treatments create superficial compressive stresses.

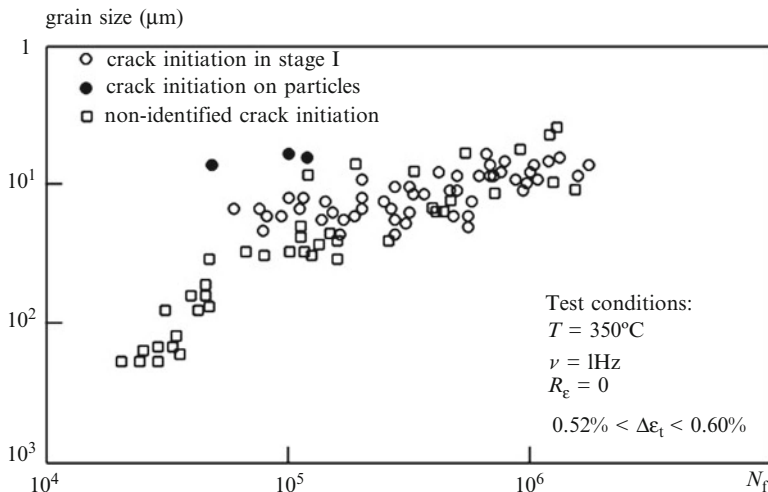


Fig. 6.38 Influence of grain size on fatigue life under low-cycle fatigue in In718 alloy. The test conditions are indicated. The nature of the initiation sites (stage I, crack initiation from particles) is also indicated (Alexandre et al. 2004)

Many models have been proposed in the literature in order to estimate the number of cycles spent in crack initiation, N_i , more precisely in stage I cracking, which corresponds to a microcrack whose size is of the order of the grain size. Most of these models have been proposed after referring to the model proposed by Tanaka and Mura (1981). This model leads to a simple equation relating N_i and the plastic strain amplitude, $\Delta\varepsilon_p$. This model has been slightly modified to account for grain size effect (Alexandre et al. 2004)

$$N_i \Delta\varepsilon_p^2 = A / d \quad (6.35)$$

where A is a constant for a given temperature and d is the grain size.

This model has been used to define the optimum grain size when there is a competition between crystallographic stage I crack initiation and initiation from inclusions or coarse carbides in In718 alloy (Alexandre et al. 2004). This optimum grain size corresponds to the inclusion size. This is the grain size just below which the fatigue endurance does not increase anymore when the grain size is reduced and the scatter in test results increases, as shown in Fig. 6.38 where the results obtained on a nickel-based superalloy (In718) are reported. In this case the optimum grain size is of the order of a few microns.

6.5.2.2 Crack Propagation and Fatigue Threshold

When the concern is the increase of the long crack propagation threshold, large grains are favourable. For some alloys the cyclic crack tip plasticity would produce phase changes: this is the case for *deformation induced martensitic transformation* in some austenitic stainless steels. The volume increase of the transformation products creates compressive stresses at the crack tip and closure.

The effect of microstructural parameters on fatigue crack growth behaviour has been largely investigated in *titanium and titanium alloys*.

Pure titanium, as well as the majority of titanium alloys, crystallises at low temperature in a modified ideally hexagonal close packed (HCP) structure called α titanium. The lattice parameters of α titanium ($a = 0.295$ nm and $c = 0.468$ nm) gives a c/a ratio of 1.587 which is lower than the c/a ratio of an ideally HCP lattice (1.63). This explains why in α titanium prisms planes rather than basal planes are preferred slip planes. At high temperatures, however, the body-centered cubic (BCC) structure is stable and is referred to as β titanium. The β -transus temperature for pure titanium is $882 \pm 2^\circ\text{C}$. Alloying with either α forming elements (such as Al) or β forming elements (Fe, M, Cr, Si, etc.) it is possible to obtain dual-phase ($\alpha + \beta$) materials in which the α microstructure is very much dependent on the annealing temperature and the cooling rate from the solutionising heat treatment. The two extreme cases of phase arrangement of α and β phases in ($\alpha + \beta$) alloys are the lamellar microstructure, which is generated upon cooling from the β phase field, and the equiaxed microstructure, which results from a recrystallisation process. Both types of microstructures can have a fine as well as a coarse arrangement of their two phases.

Commercially pure titanium (which can contain some impurities, in particular oxygen) and α alloys consist only of α grains. Therefore, grain size, interstitial oxygen content, and degree of work-hardening can be varied. The influence of these parameters on the fatigue strength of commercially pure titanium is shown in Fig. 6.39 (Wagner and Bigoney 2003). In Fig. 6.39a it is observed that the reduction of grain size from 100 to 6 μm increases the fatigue strength from 160 to 210 MPa. Increasing the oxygen content increases significantly the fatigue strength (Fig. 6.39b). Similarly the fatigue strength increases with work-hardening (Fig. 6.39c).

The influence of grain size and oxygen content on the growth rate of long cracks is reported in Fig. 6.40 (Wagner and Bigoney 2003). Large grains and increased oxygen content reduce crack propagation rates. These relationships of grain size and oxygen content on $da/dN - \Delta K$ behaviour observed for long cracks are not necessarily the same for short surface cracks. This is illustrated in Ti-8.6 Al alloy in Fig. 6.41 where it is observed that at a given value of ΔK , microcracks propagate faster in coarse-grained structures than in fine-grained material. This is likely a consequence of the higher grain boundary density, since grain boundaries are known to effectively hinder microcrack growth as indicated previously. Macrocracks propagate much more slowly in coarse-grained structure

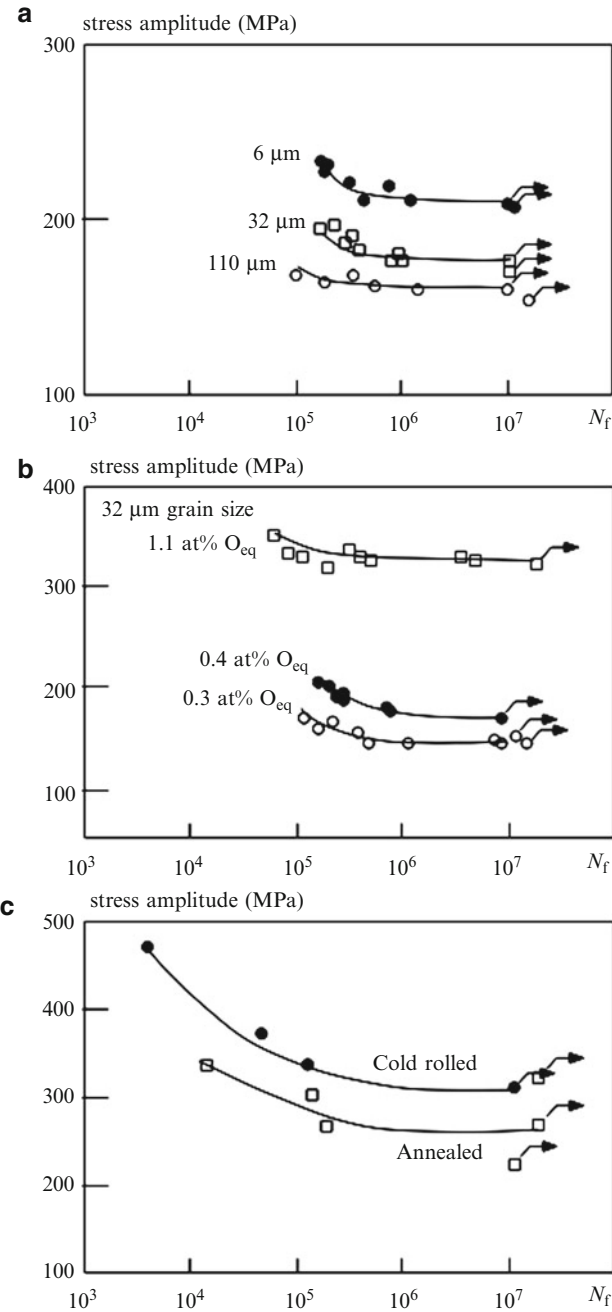


Fig. 6.39 High cycle fatigue behaviour of commercially pure titanium (a) influence of grain size ($R = -1$); (b) influence of oxygen content ($R = -1$); (c) influence of cold work ($R = 0.10$) (Adapted from Wagner and Bigoney 2003)

Fig. 6.40 Fatigue crack propagation in commercially pure titanium for two different grain sizes and two different oxygen contents at $R = 0.01$ (Adapted from Wagner and Bigoney 2003)

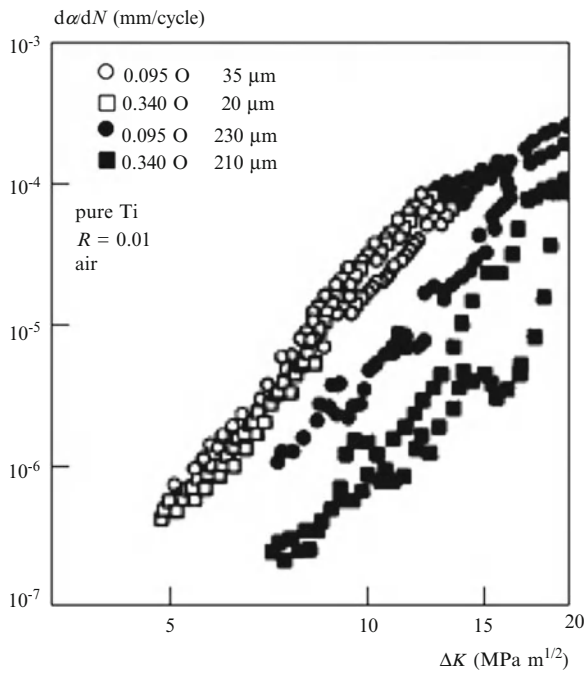


Fig. 6.41 Comparison of microcrack and macrocrack propagation in Ti-8.6Al (Adapted from Wagner and Bigoney 2003)

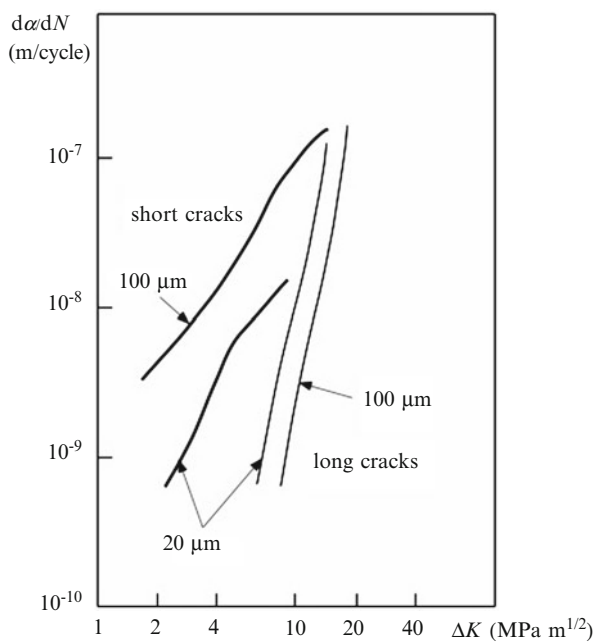
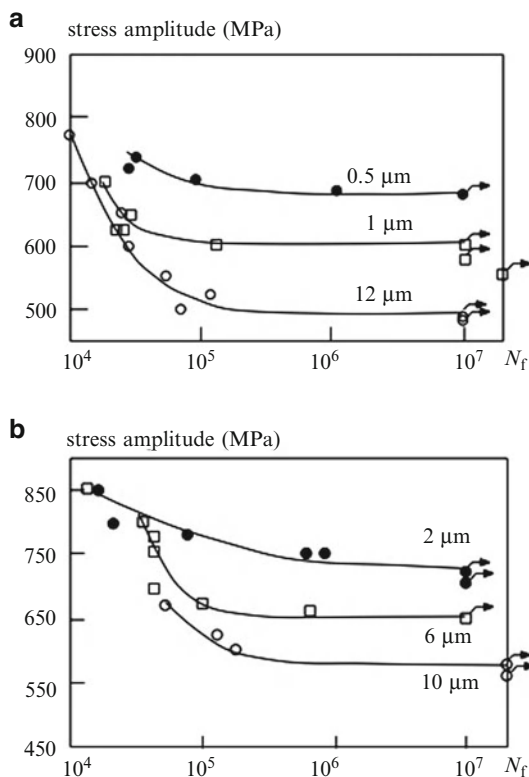


Fig. 6.42 High cycle fatigue behaviour ($R = -1$) of Ti-6Al-4 V (a) influence of lamellae width (lamellar microstructure) (b) influence of α grain size (equiaxed microstructure) (Adapted from Wagner and Bigoney 2003)

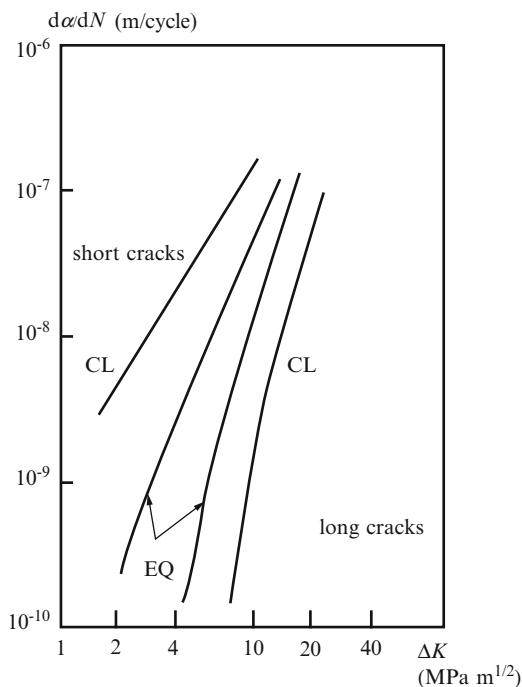


than in fine-grained material. This difference was previously related to the so-called crack closure effect which is particularly important at low R values.

The alloy Ti-6Al-4V which represents about 50% of all the titanium alloys is largely used in the ($\alpha + \beta$) structure. The high cycle fatigue behaviour of these microstructures is shown in Fig. 6.42. Reducing the α lamellae width from 10 to 0.5 μm in lamellar microstructures raises significantly the fatigue strength from 480 to 675 MPa (Fig. 6.42a). Similarly, reducing the grain size from 10 to 2 μm in equiaxed microstructures increases the fatigue strength from 560 to 720 MPa (Fig. 6.42b).

The comparison of crack growth rate measured with lamellar or equiaxed microstructure is reported in Fig. 6.43 where opposite effects of the microstructure on fatigue crack growth rate are observed for short and long cracks.

Fig. 6.43 Comparison of microcrack and macrocrack propagation in Ti-6Al-4V (CL coarse lamellar, EQ equiaxed) (Adapted from Wagner and Bigoney 2003)



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Chapter 7

Environment Assisted Cracking

Abstract Hydrogen embrittlement is due to the introduction of hydrogen either from a gaseous or from a liquid environment. The solubility of hydrogen in iron is temperature dependent. So is the diffusion. At low temperatures traps control the diffusion of hydrogen. Supersaturation of hydrogen results in the precipitation of cavities. The modification of the surface energy results in hydrogen enhanced decohesion; its interaction with dislocations in hydrogen enhanced local plasticity. Crack propagation takes place above a stress intensity factor threshold. When it is high enough stress-independent crack propagation is controlled by the diffusion of hydrogen. Embrittlement of hydride forming metals is due to the brittleness of the hydrides. Stress corrosion cracking initiation and propagation depend on anodic dissolution and cathodic hydrogen introduction. This interacts with slip mechanisms. Crack propagation stops below a critical stress intensity factor threshold. It becomes stress independent at higher levels of stress intensity factor. Similar mechanisms are responsible for corrosion fatigue. The cases of long and of short cracks are distinguished. Liquid metal induced embrittlement is due to modification of the surface energy. Liquid embrittling metals also reduce the cohesion energy and the core energy of dislocations. The same embrittling metals are active in the solid state resulting in solid metal induced embrittlement.

7.1 Introduction

Rare are the structures working in vacuum. Numerous are cases where the environment interferes with mechanical loading in the initiation and propagation of damage. The active environment can be gaseous or liquid. The loading can be static or cyclic, at rates ranging from very slow to fast. The temperature can be constant at various levels or changing.

Environment alone can damage structures in the form of corrosion. This constitutes a vast discipline, which we will not study here. We are only concerned in combinations of corrosion and mechanical loading.

We are faced with the difficulty of the specific nature of these interactions. Each couple of a particular material and of a particular active environment creates its own problem. A given atmosphere can generate great damage on one material and no harm at all on another or even on this same material treated in a different way. All kinds of materials can be affected by environment assisted cracking (EAC): metallic alloys, glasses and ceramics, polymers, composites. The environment can be aqueous solutions, non-aqueous solvents, gases, molten salts, liquid metals. This great variety of problems makes it difficult to come to a rational description of the damage mechanisms. This chapter is limited to metals. EAC in other materials will be treated in Chap. 9.

However, their study is of utmost importance in view of the many manifestations of environment assisted damage, too often catastrophic. We will give examples of such wrecks in the following sections. They can be found in aeronautics, in chemical and petroleum industries, in construction and civil engineering, in nuclear industry among others. Thirty percent of failures in chemical industries are due to environment enhanced cracking. The danger comes from the often unexpected nature of the phenomena and of the difficult detection of the cracks created.

Table 7.1, borrowed from Guillot and Béranger (2009), contains combinations of materials and environments leading to EAC.

In many cases hydrogen is held responsible for the damage. We will thus begin by studying the embrittlement due to this very small atom able to diffuse quickly in alloys and to interact in a variety of ways with different defects: dislocations, twin and grain boundaries, precipitates and inclusions, free surfaces.

We will then see that hydrogen intervenes in many cases of stress corrosion cracking, a kind of damage occurring under static loading in an aggressive environment, or in corrosion fatigue under cyclic loading.

We will then turn to the case of liquid metal embrittlement due to mechanisms of different nature than those acting in stress corrosion or in corrosion fatigue.

The damage due to hot corrosion, although not devoid of interactions with stresses, is really of purely chemical nature and it will not be studied here. However the damage due to environmental effects, in particular oxidation, in creep-fatigue will be examined in Chap. 8.

7.2 Hydrogen Embrittlement

7.2.1 Importance of Hydrogen Embrittlement

Hydrogen embrittlement is a dreadful phenomenon. Hydrogen is present in great many environments, liquids, starting with water, as well as gas. There are then many opportunities for its introduction in metals. Adsorption on surfaces modifies the surface properties. Hydrogen can penetrate in metals and there interact with point

Table 7.1 Combinations of materials and environments leading to EAC

Material	Environment	Remarks, temperatures
Armco iron	Bi, Cd, Sn	$T < 450^{\circ}\text{C}$
	Pb	$350\text{--}400^{\circ}\text{C}$
Steels	Molten soda, caustic media	$>80^{\circ}\text{C}$
	Nitrates solutions NO_3^-	$>50\text{--}60^{\circ}\text{C}$
	Carbonated solutions	$>50\text{--}60^{\circ}\text{C}$
	Liquid ammonia NH_3	At room temperature with water
	Cyanidric acid HCN	traces
	H_2O , H_2 , HCN	Medium strength steels
	Tin	High strength steels
	Sodium	Quenched and tempered
	Mercury	$250\text{--}575^{\circ}\text{C}$
	Zinc	Bainitic, martensitic $200\text{--}400^{\circ}\text{C}$
	Lead, lead-bismuth	Quenched and tempered Fe-5 to 12 at % Si $660\text{--}975^{\circ}\text{C}$ 9% Cr steel $150\text{--}500^{\circ}\text{C}$
Stainless steels		
Austenitic	Neutral aerated chlorinated media	$>80\text{--}100^{\circ}\text{C}$
	Acid chlorinated media + H_2S	Room temperature
Ferritic	Caustic media	$>80\text{--}120^{\circ}\text{C}$
Martensitic	Br^-	$>80\text{--}120^{\circ}\text{C}$
	Liquid metals (Ga, Zn etc . . .)	
	Hydrogen	
	Hydrogen, water	
Nickel alloys	Caustic media, molten soda	$>80\text{--}120^{\circ}\text{C}$
Pure nickel	Water, hydrogen	$>250\text{--}280^{\circ}\text{C}$
	HF vapor	700°C
	Bismuth	Room temperature
	Mercury, hydrogen	
	Lead	
Titanium alloys		
6 %Al-4 %V	Cl^- in alcoholic media	Room temperature
	N_2O_4	
8 %Al-1 %Mo-1 %V	Cl^- , Br^- , I^- , H_2O , CH_3OH , CCl_4	$375\text{--}700^{\circ}\text{C}$
Alpha titanium	Cadmium	50°C
	Mercury + 3 at.% Ga	
Zirconium alloys		
Zircaloy 2	Gaseous iodine	$>300^{\circ}\text{C}$
	Alcoholic solutions of iodine	Room temperature
Zr-2.5 %Nb	Cesium, cadmium	
	Mercury, gallium	
Copper alloys	Ammoniacal solutions	Room temperature
	Nitrites	Room temperature
	Gallium	Room temperature

(continued)

Table 7.1 (continued)

Material	Environment	Remarks, temperatures
Aluminium alloys	NH ₃ or H ₂ O vapours	
	Mercury salts	
	Soft water, sea water	
	0.6 mole/l NaCl, chlorides	
	Bismuth	300–600°C
	Indium	600–900°C
	Chlorinated media	Room temperature
	NaCl/H ₂ O, NaCl/HCl	
	Gallium	Room temperature
	Tin	600°C
Magnesium alloys	Indium	600°C
	Bi-Pb-Sn-In-Cd eutectic	60°C
	H ₂ O	
Glass	Distilled water, K ₂ Cr ₂ O ₇	
Polymers	Water	
Polycarbonates	Alkalis	
Polyesters	Acids	
Elastomers	Ozone	

defects, dislocations, stacking faults, grain boundaries and solute atoms. Its modes of action are therefore varied. They all lead to the formation and propagation of cracks.

Hydrogen atoms are very small. In the lattice of metals they are in interstitial positions. They can diffuse very fast even at moderate temperatures. Thus the conditions for hydrogen embrittlement to appear are easy to be met.

A great number of researches were devoted to hydrogen embrittlement (Troiano 1960; Oriani 1972; Speidel 1975; Hirth 1980; Wei 1981; Pressouyre 1983; Dietzel et al. 1989; Birnbaum 1990; Coudreuse and Bocquet 1995; Gangloff 2003; Lynch 2003). But, the hydrogen embrittlement mechanisms are complicated to analyse because several can coexist in experiments. The potential energy of dissolved hydrogen atoms depends on the stress state, particularly on the hydrostatic stress and, being present as protons in metals, electronic interactions also play a role. Other solutes interact also strongly with hydrogen. Thus, the strain-hardening level, the concentration of defects and of alloying elements or impurities need to be controlled with great care. So must be also the temperature evolutions owing to the high diffusivity. Moreover, the plastic strain and plastic strain rate interact with the diffusion of hydrogen. These complications and the scale at which the hydrogen interactions take place explain why hydrogen embrittlement mechanisms are yet not fully understood. In the recent years, nanoindentation techniques have helped (among others Barnoush and Vehoff 2008; Gao 2005). *Ab initio* calculations have provided also some better understanding (among others Domain et al. 2004; Udagawa et al. 2010; Psiachos et al. 2011; von Pezold et al. 2011).

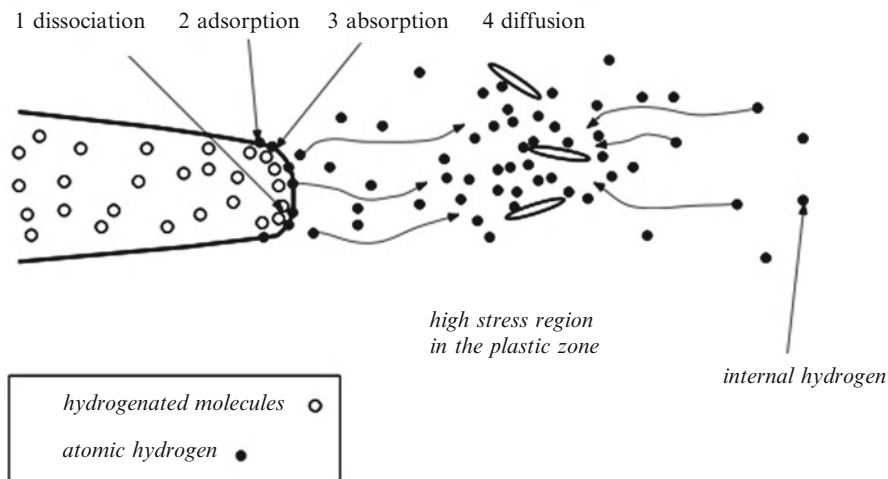


Fig. 7.1 Sketch showing the successive steps in the penetration of hydrogen atoms in a metal (at the *tip* of a crack). After sorption at the surface, the atoms diffuse to the regions of high hydrostatic stress

Hydrogen embrittlement can take place in many different industrial operations: in molding and forging; in welding; in electroplating. It can be encountered in the storage of hydrogen gas, in nuclear fuel cladding for instance. It is often present in stress corrosion cracking and in fatigue corrosion, which will be studied in later sections.

Hydrogen embrittlement plagues all sorts of materials, but is particularly harmful for high resistance alloys.

In the following study of hydrogen embrittlement we first see how hydrogen can penetrate in a metal. We then give some indications about its solubility and diffusion. The various hydrogen embrittlement mechanisms are described, distinguishing between hydrides forming metals and those which do not form hydrides. For the latter, we envisage hydrogen enhanced decohesion (HEDE) and hydrogen enhanced local plasticity (HELP), the two models which appear to meet some common agreed understanding. We describe both crack nucleation and crack propagation.

7.2.2 Introduction of Hydrogen in Metals

7.2.2.1 Introduction of Gaseous Hydrogen in Metals

Let us consider the surface of a metal within a hydrogen rich gaseous environment. This surface is usually covered with an oxide layer. Figure 7.1 provides, for the tip of a crack, a sketch of the successive steps of the penetration of hydrogen atoms in a

(Fig. 7.2b). The presence of an oxide layer increases this energy. Hydrogen atoms are then adsorbed with energy of chemical adsorption E_c . The chemical adsorption is proportional to the pressure. The adsorption of hydrogen can be modified by the presence of other gas molecules whose constituents occupy sites on the surface, and then forbidden to hydrogen atoms. Moreover these other molecules can modify the adsorption energy of hydrogen atoms.

- The second step in the introduction of hydrogen is absorption. The *absorption* of the adsorbed hydrogen atoms from a site on the surface to an interstitial site in the metal can be endothermic or exothermic according to the difference of the absorption energy ΔE with the dissociation energy. When it is exothermic, the *hydrides* are stable. This is the case with Ti, Zr and Ta.

According to the *law of Sievert* the concentration of absorbed atoms is proportional to the square root of the hydrogen pressure.

- Once introduced in the metal, the hydrogen atoms diffuse with a diffusion energy E_{dif} .

A particular case is the dissolution of hydrogen in *liquid metals*. We will see that the solubility in that case is very high. This dissolution process is easy. To avoid it care must be taken to keep atmospheres dry, the more so when molecular hydrogen is dissociated as in plasma. This is to be feared in arc melting operations and particularly in welding.

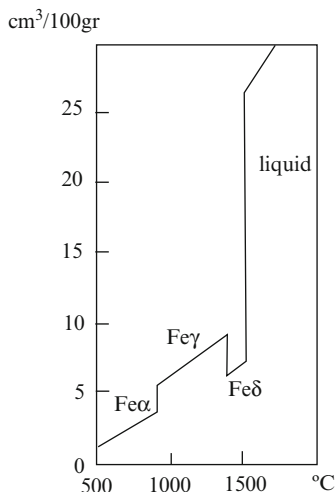
7.2.2.2 Introduction of Hydrogen in Metals from a Liquid Phase

In liquid phase electrochemical phenomena operate. The dissociation step of the hydrogen molecule is by-passed. Along the surface of the metal are zones, which are cathodic, while others are anodic. This is due to local variations of strain-hardening or of composition, however slight. Of course, the entire surface can be cathodic when a pile is created, when another kind of metal is immersed in the liquid solution. The cathodic reduction produces hydrogen atoms, which can be absorbed in the metal. At the same time, there must exist anodic oxidation reactions. Some nascent hydrogen atoms can also combine to form gaseous hydrogen molecules. The kinetics of the penetration of hydrogen results from equilibrium between these various reactions.

It is modified by the presence of alloying elements or impurities in the metal. Those which slow down the penetration are the ones which displace the corrosion potential towards more noble values. Such are, in steels, Cu, Zr, Mo, Si, Cr, Ni, or phosphates. On the contrary, metalloid impurities P, S, Se, Sb favor hydrogen penetration by hampering the reaction of formation of hydrogen molecules at the surface.

Elements present in the liquid can also modify the rate of penetration of hydrogen. Obviously oxygen, creating oxides on the surface, slow down the rate. So do organic nitriles. On the contrary, metalloids as well as ions such as Cl^- , I^- , CN^- accelerate hydrogen penetration, by decreasing the chemi-sorption energy. Very small concentrations are sufficient.

Fig. 7.3 Variation of the solubility of hydrogen in iron as a function of the temperature (the ordinate scale is approximate)



Sorption of hydrogen is facilitated by *plastic deformation*, because the fresh steps created on the surface by slips are free from oxidation. This plays a fundamental role in stress corrosion and in fatigue corrosion.

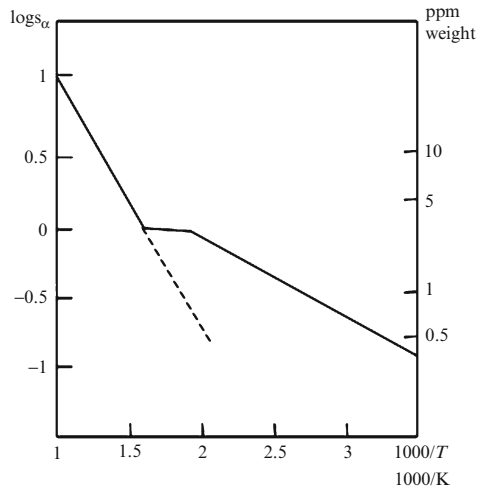
7.2.3 Hydrogen Solubility in Iron

Figure 7.3 shows the evolution of the solubility of hydrogen in iron as a function of the temperature. As a general rule it increases with the temperature. At room temperature it is $3 \cdot 10^{-6}$ at.%; it increases with temperature reaching $1.6 \cdot 10^{-2}$ at.% at 900°C. We note, as mentioned previously, that it is very high in the liquid phase (0.13 at.% at 1,540°C). In solid phases, the solubility is higher in FCC gamma iron than in BCC delta or alpha iron. At 900°C the solubility in γ iron is $2.3 \cdot 10^{-2}$ at.%. This evolution follows the same trend as C or N atoms in insertion in the iron lattice, in spite of the larger compactness of the FCC packing. This is so because of the dissymmetric shape of insertion sites in the BCC lattice. This difference in solubility in austenite and in ferrite is an important aspect of hydrogen embrittlement of steels. During cooling, when the transformation from austenite to ferrite takes place, hydrogen ejected out of the first phase over-saturates the second one. This creates conditions for embrittlement.

The solubility of hydrogen in alpha iron follows an Arrhenius¹ law (Fig. 7.4). The deviation at low temperatures comes from trapping on defects.

¹Svante August Arrhenius (1859–1927) was a Swedish chemist who won the Nobel Prize in 1903.

Fig. 7.4 Arrhenius plot of the hydrogen solubility in alpha iron



Variation of the hydrogen solubility in alpha iron at the Curie² point shows that magnetic interactions play a role in the dissolution energy of these atoms (Yu et al. 1999).

7.2.4 Diffusion of Hydrogen

7.2.4.1 High Temperature Diffusion

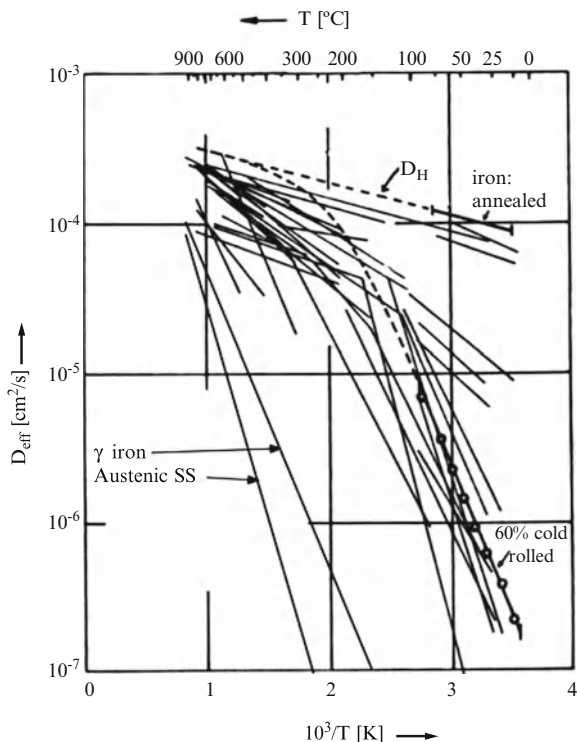
The position of hydrogen atoms as interstitial atoms, diffusion taking place from an interstitial site to the next, results in fast diffusion. As usual, it follows an Arrhenius law (Fig. 7.5). The diffusion coefficients D_α and D_γ for the iron alpha and gamma phases respectively in m²/s are given by:

$$\begin{aligned}
 D_\alpha &\approx 7.8 \cdot 10^{-8} \exp\left(-\frac{1900}{RT}\right) \\
 D_\gamma &\approx 4.35 \cdot 10^{-7} \exp\left(-\frac{12350}{RT}\right)
 \end{aligned}
 \tag{7.1}$$

the activation energies being expressed in cal/mole, R being the perfect gas constant and T the temperature in K.

²Pierre Curie (1859–1906) was a French physicist winner of the Nobel Prize with his wife Marie Curie in 1903.

Fig. 7.5 Arrhenius plot of diffusivity (D_{eff}) of hydrogen in various steels compared with the diffusivity (D_{H}) in pure iron as compiled by Grabke and Riecke (2000) on which was added the data on γ iron and an austenitic stainless steel given by Interrante and Raymond (2005)



It can be checked that diffusion is quite faster in the alpha phase.

In one second, an hydrogen atom travels $(Dt)^{1/2} = 85 \mu\text{m}$ at 400 K.

These diffusion coefficients are very dependent on the purity and increase the purer the metal.

7.2.4.2 Low Temperature Diffusion in the Presence of Traps

In the same way that traps increase the solubility of hydrogen at low temperatures, they slow down diffusion (Fig. 7.5). There exist a variety of traps for hydrogen atoms: solute atoms, dislocations, stacking faults, twin and phase boundaries, grain boundaries, interfaces. The trapped hydrogen atoms are freed when the temperature is sufficient for the binding energy to be overcome. Table 7.2 gives the interaction energies for various traps.

Dislocations as they move are able to transport hydrogen atoms. Hydrogen atoms interacting with dislocations form atmosphere around them. These are dragged by moving dislocations if the velocity is less than a critical value. This value is the higher the higher the temperature, since the diffusion of hydrogen atoms increases with the temperature (Sirois and Birnbaum 1992). The corresponding critical strain rate is about $6.8 \cdot 10^4 \text{ s}^{-1}$ at room temperature.

Table 7.2 Binding energies of hydrogen atoms with various traps in iron

Type of trap	Binding energy (eV)
Interstitial atoms C, N	Few 1/100
Substitution atoms Ni, Cr . . .	Few 1/10
Dislocations [□]	0.28
Twin and grain boundaries	0.4
Inclusions (carbides, nitrides, oxides, sulfides)	
Fe ₃ C	0.7
TiC	0.8

[□]Interactions with dislocations result from the elastic stress field and from the core

Hydrogen atoms can also diffuse within traps, especially along the core of dislocations by *pipe diffusion*. This accelerates the transport of hydrogen.

The variety of traps complicates the modelling of hydrogen diffusion (McNabb and Foster 1963). A statistical analysis of Leblond and Dubois (1983) takes account of the jump frequency from traps and from an atomic site to the next in the metal lattice. There is also an effect of the plastic deformation, which modifies the number of traps (Krom et al. 1997, 1999). In the presence of traps the diffusion differential equation, neglecting the pipe diffusion, can be written (Falkenberg et al. 2010):

$$\frac{c_L + c_T(1 - \theta_T)}{c_L} \frac{\partial c_L}{\partial t} - \nabla \cdot (D_L \nabla c_L) + \nabla \cdot \left(\frac{D_L c_L V_H}{RT} \nabla \sigma_h \right) + \theta_T \frac{dN_T}{d\varepsilon_p} \frac{\partial \varepsilon_p}{\partial t} = 0 \quad (7.2)$$

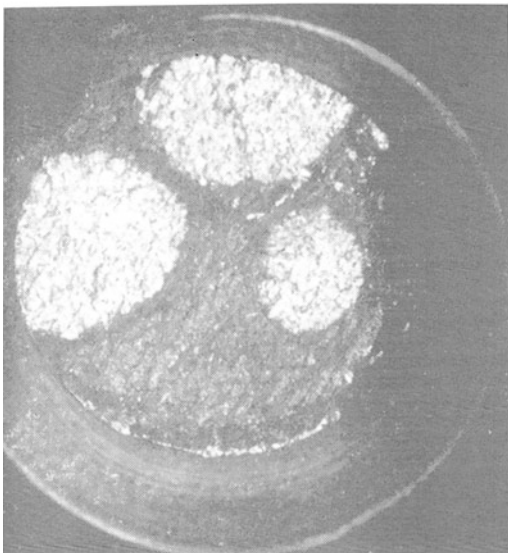
where c_L is the concentration of hydrogen in the lattice, c_T the trapped hydrogen concentration, θ_T the occupancy of the trap sites, D_L the diffusion coefficient, V_H the partial molar volume of hydrogen, σ_h the hydrostatic stress, N_T the trap density, ε_p the plastic deformation ($c_L = \alpha_L \theta_L N_L$ is the product of the number of interstitial sites per solvent atom, by the occupancy of the available lattice sites, by the number of lattice solvent atoms per unit volume; $c_T = \alpha_T \theta_T N_T$ is the product of the number of hydrogen atom sites per trap, by the occupancy of the trap sites, by the trap density).

This equation shows how the influence of the hydrostatic stress and of the plastic strain and strain rate modify the diffusion equation (see Appendix B in Volume I). It is implemented as a user supply routine in the finite elements code ABAQUS (2008).

7.2.5 Hydrogen Embrittlement Mechanisms

As for any damage mechanism we must distinguish two steps in hydrogen embrittlement: the nucleation of cavities or cracks and their growth.

Fig. 7.6 Flakes as observed on a fracture surface (Zapffe and Sims 1941)



7.2.5.1 Nucleation of Cavities or Cracks

We deal with a classical problem of precipitation, the precipitate in that case being a gas. We are then faced, as usual, with a problem of energy balance: the gain of energy from the nucleation of a cavity or of a crack, whether elastic strain energy or chemical energy, must counter balance the creation of surface energy. As the first depends on the cube of the dimension and the second on the square, there exists a critical dimension. It must be remembered that the energy of the dissolved hydrogen atoms depends on the hydrostatic stress. Hydrogen can also decrease the surface energy, which facilitates the crack nucleation. The hydrogen atoms migrate to regions of high hydrostatic stress, such as the ones which exist in the plastic zone ahead of the tips of cracks, or at the head of dislocation pile-ups, or in the heat affected zone in welds. The accumulation of hydrogen in these regions is likely to produce embrittlement.

In any case hydrogen super-saturation is the source of chemical energy. As it is particularly high after solidification of a liquid in which a large quantity of hydrogen was dissolved by accident, nucleation of cracks is easy in that case. This is the explanation of the formation of *flakes* in arc melted ingots (Fig. 7.6).

In the same way, hydrogen dissolved during a welding operation leads to the phenomenon of *cold cracking* of welds, so called because it does not need high temperatures to appear owing to the high diffusivity of hydrogen. Nevertheless, it can take days for cold cracks to appear. In that case, non-destructive testing performed immediately after welding does not detect any defect and the welds, which are declared good, undergo delayed fracture. The catastrophic failure of the Grande Dixence shaft as an example (see Sect. 1.1.3 in Volume I) was attributed

to cold cracks. Cold cracking is linked with the internal stresses in the welds, due to local heating and cooling. Furthermore, fast cooling produces brittle phases by martensitic transformations.

To prevent these detrimental phenomena, care must be taken to keep atmospheres as dry as possible. Vacuum melting is a good way to avoid water vapour. Whatever the welding process avoidance of humidity is essential. In manual arc welding the electrodes must be well protected against humidity and stoved before use.

When there is no overall super-saturation of hydrogen, nucleation of cavities or cracks can appear owing to high local concentrations of hydrogen. As stated before, this is produced by the existence of *high local hydrostatic stresses* where hydrogen atoms accumulate. The gain of elastic energy due to the precipitation of hydrogen adds to the gain in chemical energy.

The modification of surface energy or of interface energy or of grain boundary energy by the presence of hydrogen should be also considered. Hydrogen can segregate at interfaces, in particular at grain boundaries. It can then act as many other impurities and contribute to a modification of the surface energies (Rice 1976). This is called *hydrogen enhanced decohesion* (HEDE) if the effect is a decrease of these energies. A thermodynamic treatment of this is given by Kirchheim (2010). The basis of the theory of grain boundary embrittlement by the segregation of impurities is presented in Chap. 3 (Sect. 3.8.4). The main difference between hydrogen and other impurities such as S or P in α -iron is due to the high mobility of hydrogen. It is so high that hydrogen can diffuse on the fracture surfaces allowing separation at constant chemical potential μ instead of constant chemical composition Γ . In Fig. 3.46 the separation is from A to B instead of from A to C. The intergranular energy $2\gamma_{\text{int}}$ for “slow” separation at fixed μ is always less than $2\gamma_{\text{int}}$ for “fast” propagation at fixed Γ . The difference is represented by the area ABC in Fig. 3.46 and is given by:

$$(2\gamma_{\text{int}})_{\Gamma=\Gamma_0} - (2\gamma_{\text{int}})_{\mu=\mu_0} = \int_{\Gamma_0}^{2\Gamma_s(\mu_0)} [\mu_0(\Gamma) - \mu_s(\Gamma/2)] d\Gamma \quad (7.3)$$

if $d\mu_s(\Gamma_s)/d\Gamma_s > 0$ (see Eq. 3.78). It can be shown that the term $(2\gamma_{\text{int}})_{\mu=\mu_0}$ is given by (Rice and Wang 1989):

$$(2\gamma_{\text{int}})_{\mu=\mu_0} = (2\gamma_{\text{int}})_0 - RT \left[2\Gamma_s^0 \log \frac{\Gamma_b^0 + (\lambda - 1)\Gamma}{\Gamma_b^0 - \Gamma} \right]$$

with $\lambda = \exp \left(\frac{\Delta g_b - \Delta g_s}{RT} \right)$ (7.4)

See Sect. 3.8.4 for the definitions of the various terms in those expressions. For a review of the thermodynamic models of interfacial embrittlement, see Hirth

and Rice (1980) and Anderson et al. (1990). In the case of hydrogen at room temperature, these last authors showed that the right hand side of Eq. 7.3 is of the order of 1.9 J/m^2 , when the concentration of hydrogen is in equilibrium with a pressure of hydrogen gas phase of 1 atm. This reduction of the intergranular energy $2\gamma_{\text{int}}$ is quite high as compared with the value of $\approx 0.2 \text{ J/m}^2$ when fast fracture occurs at fixed concentration.

Moreover, hydrogen atmospheres can shield the interactions between dislocations and between dislocations and obstacles of various kinds. Various observations by transmission electron microscopy showed that the dislocation velocity was increased in the presence of hydrogen (Sirois and Birnbaum 1992). This corresponds to an increase of the rate of relaxation and a reduction of the yield strength (Beachem 1972 for an early observation). Hydrogen reduces the activation area as well as the activation energy for the overcoming of obstacles. It was also observed that the distance between dislocations in pile-ups in nickel and in austenitic stainless steel was reduced, demonstrating the reduction of the interaction forces (Ferreira et al. 1998; Abraham and Altstetter 1995). This shielding effect can be partly explained by the rearrangement of the hydrogen atmospheres by elastic interactions between the hydrogen atoms and the stress field of other dislocations (Sofronis and Robertson 2002), or more generally of obstacles. However this requiring a high concentration of hydrogen, von Pezold et al. (2011) showed by calculation that H–H interactions had also to be considered to explain that the effect could be observed even with low concentrations. In any case, neither the modification of the shear modulus nor of the Peierls force negligible in FCC crystals could explain the shielding effect.

Thus the distance between dislocations in pile-ups is reduced in the presence of hydrogen and the stress concentration at the head is increased. This enhanced local plasticity facilitates the nucleation of cracks: this is what is called the *hydrogen-enhanced local plasticity* mechanism (HELP) (Sirois and Birnbaum 1992). Connected with the shielding effect is the difference of behaviour in FCC crystals between the edge dislocations, around which atmospheres are formed, and screw dislocations, which have no hydrostatic stress field. The first, keeping their edge orientation, are unable to cross-slip. The result is that the slips remain planar, another factor promoting stress localisation and embrittlement.

7.2.5.2 Crack Propagation

Once nucleated, cracks propagate by further penetration of hydrogen at their tip where there is a high stress concentration. Furthermore the slip planarity favours the formation of fresh slip steps, which ease the penetration of hydrogen.

The arrival of hydrogen at the tip of cracks or its transport to regions of high stress concentration can be discontinuous. The crack then grows by successive jumps, leaving striations on the fracture surface. They can be confused with fatigue cracks. Figure 7.7 is an example of discontinuous crack propagation due to jumps when the hydrogen pressure built up by diffusion of hydrogen atoms becomes sufficient and to the crack stopping when this pressure is decreased enough by the increase of the volume of the crack.

Fig. 7.7 Discontinuous propagation of a crack by hydrogen embrittlement of a Fe-Si alloy. The slips are visualised at the successive positions of the crack tip (Tetelman and Robertson 1963)

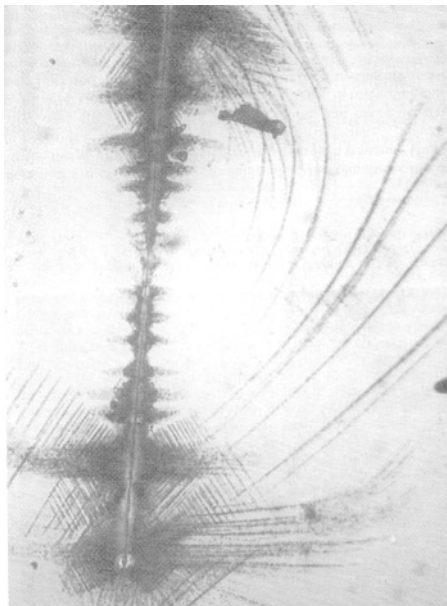
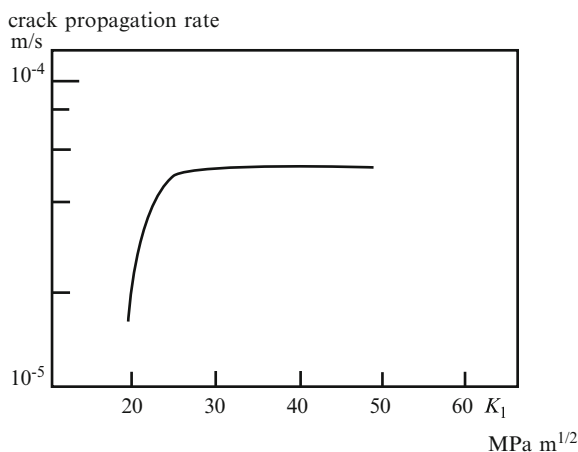
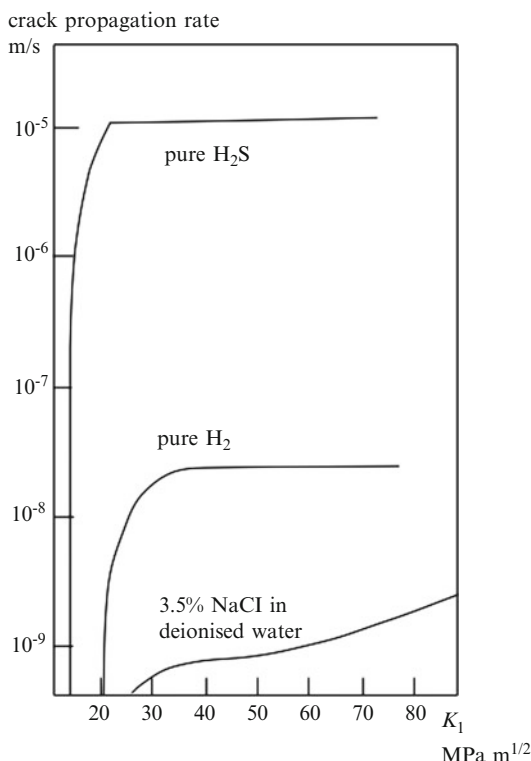


Fig. 7.8 Crack propagation rate as a function of the stress intensity factor in 18 Ni maraging steel in purified hydrogen gas at low pressure and 23°C under constant load (Adapted from Gangloff and Wei 1977)



The crack propagation rate as a function of the stress intensity factor displays two stages, as shown in Fig. 7.8. Below a *threshold* denoted K_{TH} there is no propagation. When it is exceeded, the crack propagation rate is a power function of the stress intensity factor. It then reaches a plateau, the crack propagation rate becoming independent of the stress intensity factor. This two-stage behaviour is interpreted by a crack tip stress controlled advance in the first stage, while hydrogen diffusion dominates in the plateau region. The threshold K_{TH} is measured in experiments

Fig. 7.9 Crack growth rate as a function of the stress intensity factor for tempered maraging steel ($dK/dt = 0.18 \text{ MPa m}^{1/2} \text{ s}^{-1}$) in three environments (Adapted from Kerns et al. 1977)



with either increasing or decreasing (at constant CMOD) applied SIF. This must be specified in reporting the measured values as they often differ. They increase with the rate of K_I . Indeed time must be sufficient for hydrogen to accumulate and very slow rates need to be achieved for limit values of K_{TH} to be obtained.

7.2.5.3 Influence of Various Factors

The *nature of the environment* plays a major role. Figure 7.9 shows this effect on the crack propagation rate. H₂S is seen to be very detrimental.

Hydrogen embrittlement takes place in a certain *temperature range*. When it is low, diffusion is too slow and when it is high, concentration homogenisation and even escape out of hydrogen prevent embrittlement. Note that strain rate does not have the usual effect. Its increase, in the case of hydrogen embrittlement, lowers down the temperature of the higher brittle-ductile transition.

The finer the *microstructures* the better the resistance to hydrogen embrittlement. The sensitivity to the phenomenon in steels increases going from annealed martensite, containing very fine precipitates, to bainite or pearlite. The higher the

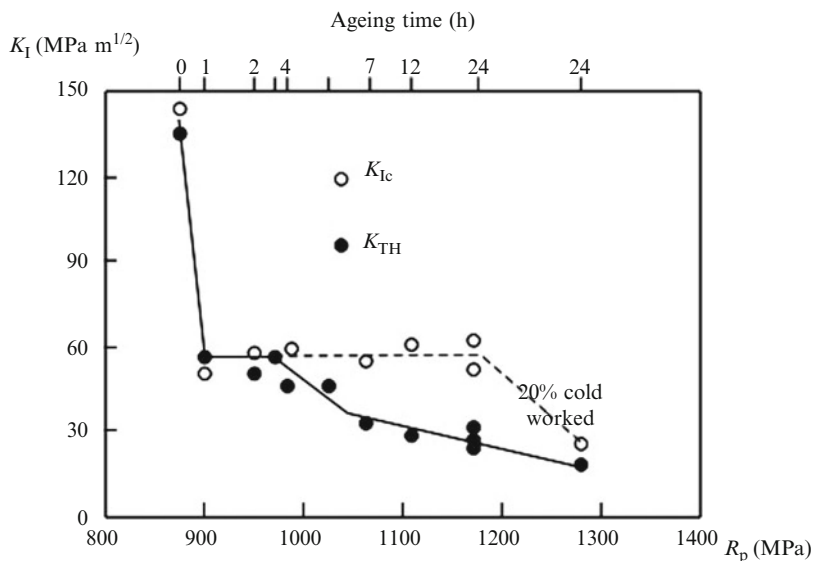


Fig. 7.10 The yield strength and iso-thermal ageing time at 500°C dependencies of the fracture toughness and threshold for the onset of hydrogen enhanced cracking in solution treated Beta-C (Ti-3Al-8V-6Cr-4Mo-4Zr) stressed under 30 $\mu\text{m/s}$ rising CMOD loading in moist air and in aqueous chloride solution at constant applied electrode potential of -150 V and 25°C (Adapted from Gangloff 2003 quoting Somerday 1998)

temperature of heat treatment of steels containing titanium carbides, the larger they are and the higher the sensitivity to hydrogen embrittlement.

The higher the yield strength, the more sensitive are alloys to hydrogen embrittlement. Figure 7.10 is an example for a titanium alloy. Heat treatments which increase the yield strength lower the threshold K_{TH} .

Phosphorus, which facilitates the introduction of hydrogen, is particularly harmful. This element, as well as tin and antimony accelerate the diffusion of hydrogen to grain boundaries, favouring intergranular cracking.

Hydrogen increases M_s temperature and thus the amount of the brittle *martensite*.

7.2.6 Embrittlement of Hydride Forming Metals

The metals of the columns IVB and VB of the periodic table: Ti, Zr, V, Nb, Ta and also Mg and Al are hydride forming. Vanadium has the particularity that the hydride is not thermodynamically stable at ambient temperature but its energy of formation is decreased by the stress field. In all cases the hydrides acquire an orientation perpendicular to the tensile direction. They are brittle and give birth to cracks, the

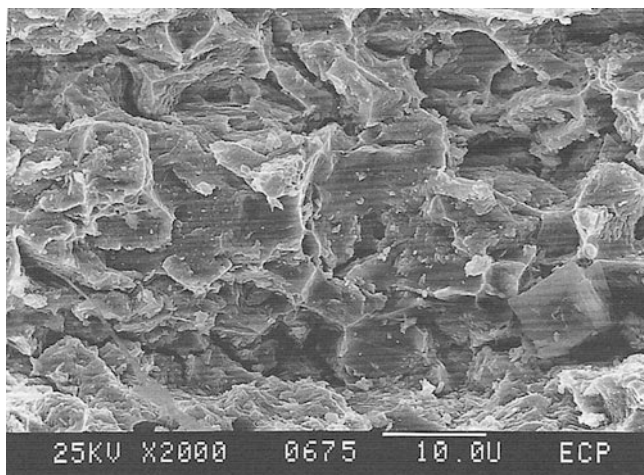


Fig. 7.11 Fracture surface of a zirconium alloy (Zircaloy 4) containing 1,055 ppm of hydrogen, broken at 475°C, showing *cracks across hydrides*

orientation of which in this direction is quite detrimental. Figure 7.11 shows such phenomenon in the case of a zirconium alloy.

7.3 Stress Corrosion Cracking

7.3.1 Introduction

Table 7.1 lists a number of systems prone to stress corrosion cracking (SCC). The first documented occurrence was the cracking of brass cartridges in India in the early nineteenth century. This was called *season cracking* because it occurred in the spring or summer during the rainy hot season. Ammonia from dung of horse manure decomposition was one of the possible causes.

The acuteness of stress corrosion cracking can be illustrated for instance by the collapse of the eye bar suspension Silver Bridge at Point Pleasant (West Virginia) in December 1967 killing 46 people. The failure was due to a stress corrosion 2.5 mm deep crack in a main chain joint. It is also worth mentioning the collapses of indoor swimming pools: that of Ulster in Switzerland in 1985 and that of Steenarijk in the Netherlands in 2001. They were due to stress corrosion cracking of fastening components made of stainless steels by chlorine added to the water for sanitary purpose. Stress corrosion cracking problems in nickel-based alloys in

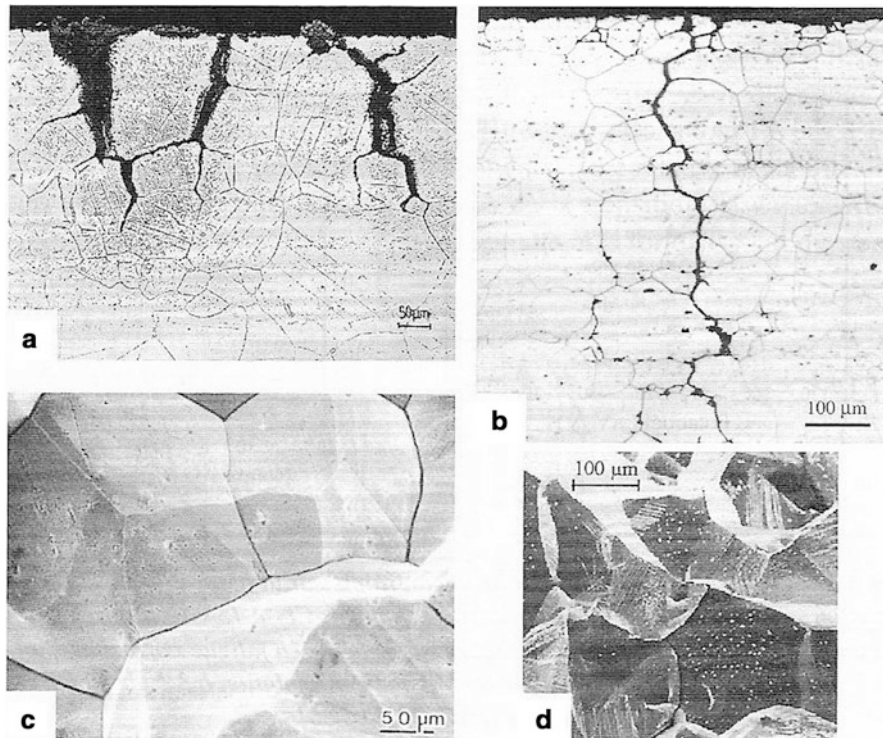


Fig. 7.12 (a) Intergranular cracking of alloy 800 at 920°C in combustion gas (Rie and Klingelh 1993) (b) Intergranular SCC of Hastelloy C (17 %Mo-16.1 %Cr-4.4 %Fe-3.8 %W-Ni remaining) at 316°C in soda 50% solution (McIlree and Michels 1977) (c) Intergranular SCC of nickel alloy 182 (14.3 %Cr-7.55 %Mn-7.25 %Fe-1.67 %Nb aged 5 h at 600°C) in boiling 50% nitric acid (Briant and Hall 1987) (d) Fractograph of a nickel specimen stressed in liquid bismuth at 700°C (Marié et al. 2000) (From Guillot and Béranger 2009)

vapour generators tubes worried the nuclear industry. The aeronautic industries fear SCC of aluminium alloys in humid air.

Stress corrosion cracking is due to local chemical reactions, which initiate cracks and accelerate their propagation. As a general rule the higher the strength of the alloy the more severe the damage by this phenomenon. It is particularly acute when the surface of the alloy is covered by a very *thin protective layer* as it is the case in stainless steels.

The active stresses without surprise are tensile. The *internal stresses* must not be forgotten. They are very often the reason for SCC. An efficient way to prevent it is the introduction of compressive stresses at the surface of the material for instance by *shot-peening*.

The cracks are either trans- or inter-granular with a brittle character (Figs. 7.12 and 7.13). They can be also of mixed character. As a general rule they display much *branching*.

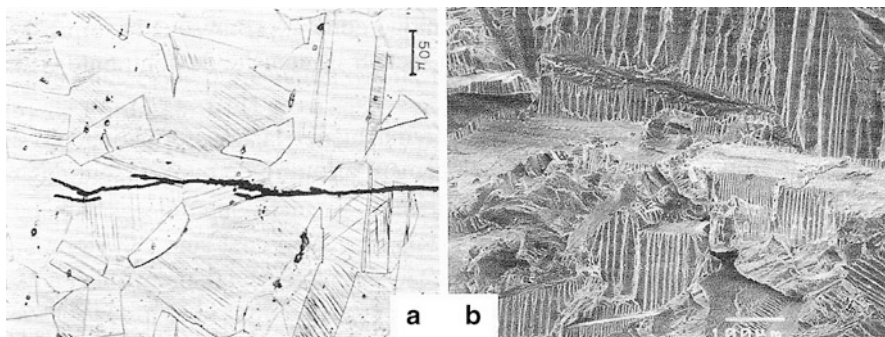


Fig. 7.13 (a) Transgranular SCC of a stainless steel AISI 304L in a 42% MgCl_2 solution at 130°C (Speidel 1977); (b) Fractograph of SCC of a zirconium alloy in an alcoholic solution of iodine at 300°C (Schuster and Lemaignan 1989) (From Guillot and Béranger 2009)

A slight variation of one of the parameters of SCC, whether stress, oxidation potential, pH, thermal treatment, concentration or temperature, can be sufficient to modify the morphology of the cracks.

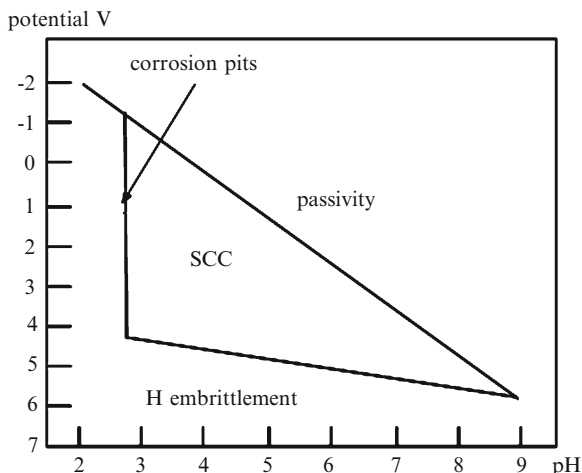
7.3.2 Stress Corrosion Cracking Initiation

The two main causes of SCC initiation are anodic dissolution and hydrogen embrittlement. They are the result of local differences of electric potential. A very frequent case is that due to the fracture of the thin protective oxide layers which form at the surface of alloys like stainless steels or aluminium alloys. Slip lines emerging at the surface break those layers and the fresh surfaces created are anodic. This is where *anodic dissolution* takes place, the positive metal ions reacting with negative ions coming from the environment, OH^- in the case of water. The protective film can also be locally weakened owing to the presence of precipitates or of inclusions, or to local corrosion pits. In the absence of strain the protective films are rebuilt and the anodic dissolution ceases. It is the plastic deformation which keeps breaking the protective films. Thus, there is a competition between the two phenomena and the *strain rate* is an important parameter: at very slow rates the protective film builds up faster than it is destroyed; at very fast rates the anodic dissolution is not fast enough. In any case, the most sensitive test to evaluate the stress corrosion susceptibility of a material is the *slow tensile test*.

When negative ions are taken out of the environment H^+ ions move to the cathodic zones on the surface and the introduction of hydrogen in the material produces *hydrogen embrittlement* as discussed in Sect. 7.2.

Figure 7.14 shows schematically how the pH of the solution and the potential affect the various phenomena. A low pH favours the formation of corrosion pits, a

Fig. 7.14 Schematic potential-pH graph showing the domain of SCC for carbon-manganese steels in phosphoric acid solution (Adapted from Parkins 1992)



high potential passivity. Low potentials correspond to hydrogen embrittlement. In between is found SCC.

More precisely, the *polarisation curves* as shown in Fig. 7.15 display three critical zones for SCC. Potentials below a critical value produce negative current densities corresponding to cathodic reactions and hydrogen embrittlement (zone A). High potentials produce a steep increase of the current density corresponding to the formation of pits and anodic dissolution by local weakening of the protective layer (zone C). Unstable passivity occurs when there is a peak of the current density as in Fig. 7.15a and the ease with which the protective layer can be destroyed favours SCC (zone B). This does not happen in the case of stable passivity (Fig. 7.15b). A method to counteract anodic dissolution is thus *cathodic protection*, consisting in applying a negative potential to the item to be preserved.

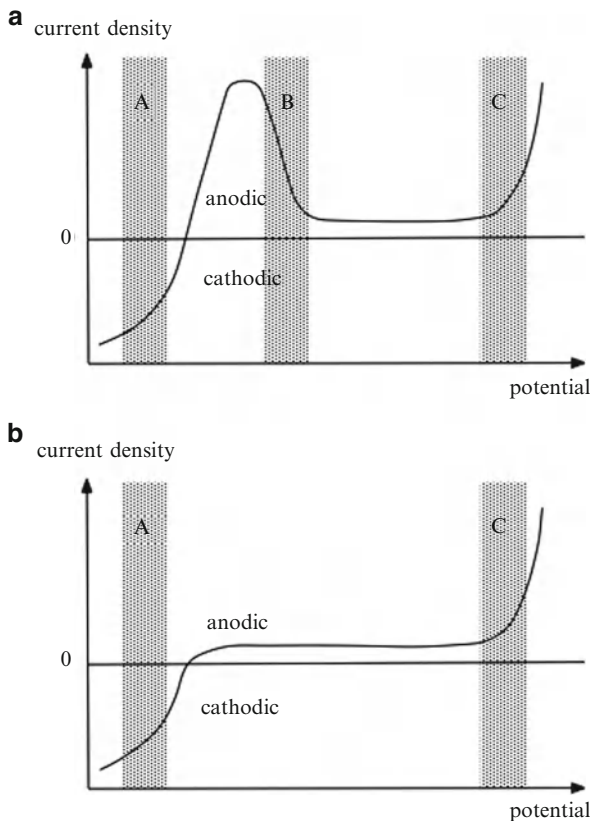
The metallurgical state of the alloy influences SCC. A first factor to be considered is the *stacking fault energy*. When it is low, the *slip planarity* is favoured and this eases the breaking of the protective layer. It also increases the strength of the dislocation pile-ups, which facilitates hydrogen embrittlement.

In the same way, the presence of *shearable precipitates* favours slip planarity and this again eases the breaking of the protective layers.

Another important factor is the *homogeneity of the composition* of alloys. In this respect, *grain boundaries* are particularly critical because impurities can concentrate there. This explains why cracking is very often intergranular. An example is the reduction of the concentration of chromium close to grain boundaries in stainless steels where chromium carbides Cr_{23}C_6 precipitated.

For SCC to take place the stress must be larger than a critical value. This *critical stress* is linked with the yield strength since it is the plastic strains which initiate SCC. It is comprised between 60 and 110% of the yield strength. It is then not

Fig. 7.15 Polarisation curves (current density – potential)
 (a) case of unstable passivity
 (b) case of stable passivity



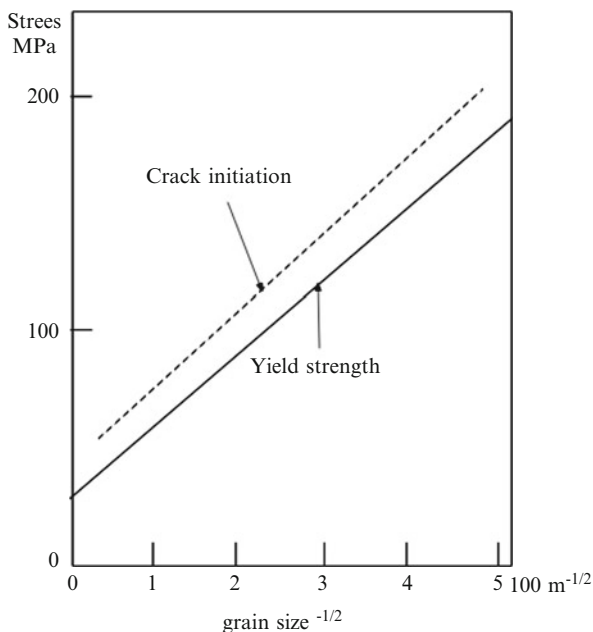
surprising that this critical stress follows a Hall-Petch law as a function of the grain size as does the yield strength. This is shown for instance by Fig. 7.16 for brass in a solution of sodium nitrate.

7.3.3 Stress Corrosion Cracking Propagation

7.3.3.1 Chemical Conditions at the Crack Tip

The mechanical and chemical phenomena taking place at the tip of propagating crack are the same as those envisaged on a smooth surface as discussed before (Sects. 7.2 and 7.3.2). But the active environment at the crack tip is of very limited volume, so that the chemical reactions there can be different from those existing on a smooth surface. In particular, acidification depolarises the cathodic reaction and increases the corrosion current density i ; this accelerates the crack propagation rate.

Fig. 7.16 Stress needed to initiate SCC and yield strength as functions of grain size for 70/30 brass in molar solution of NaNO_2 (pH 9, -0.1 V) (Adapted from Parkins 1992)



The crack propagation results from a process of successive slip induced creations of fresh surfaces followed by anodic dissolution and formation of hydrogen, and of repassivation (Ford 1984). Figure 7.17 shows such a phenomenon in cyclic loading of an austenitic stainless steel in a 3% NaCl solution at pH 6. Strain variations produce an increase of the corrosion current density, while relaxation periods allow repassivation. Observation of arrest lines on the fracture surface as well as bursts of acoustic emission suggests that the propagation can be discontinuous (Combrade and Magnin 2002).

7.3.3.2 Dissolution Models and Strain Interactions

The crack propagation rate is related in the first place to the dissolution rate at the crack tip. It was indeed found that it is proportional to the current density for a large range of materials (Parkins 1979). On this basis Ford (1992) developed a *slip dissolution model*. In this model, the crack propagation rate depends on the strain rate $\dot{\epsilon}_{CT}$ at the crack tip, which creates slips, and on the electric charges Q_f needed for repassivation between two ruptures of the protective film

$$Q_f = \int_0^{t_f} i(t) dt \quad (7.5)$$

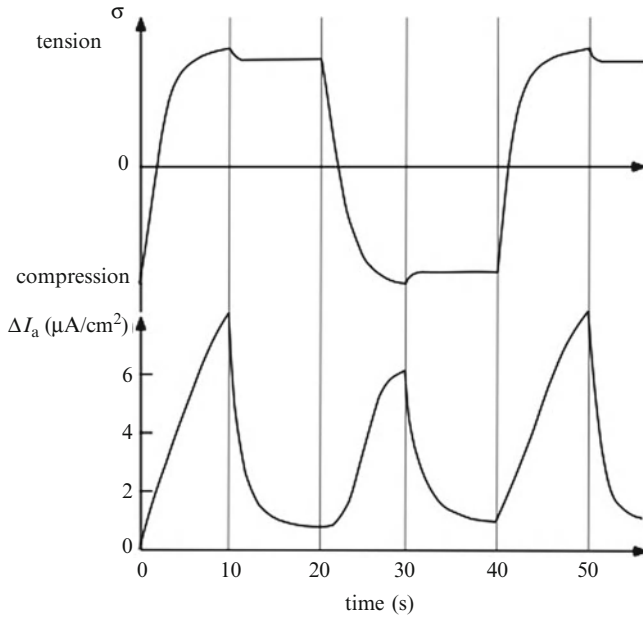


Fig. 7.17 Depassivation during strain variations and repassivation during relaxation in cyclic loading of an austenitic stainless steel in 3% NaCl solution at pH 6 and imposed potential equal to the free potential. $\Delta\epsilon_T/2 = 2 \times 10^{-2}$, $\dot{\epsilon}_T = 10^{-3} \text{ s}^{-1}$ (Adapted from Amzallag et al. 1981)

where t_f is the time between two depassivation sequences. It is estimated from the critical strain for the rupture of the film ϵ_f :

$$\epsilon_f = \dot{\epsilon}_f t_f \quad (7.6)$$

Estimating that the dissolution rate at the crack tip is equal to the mean film rupture rate, we obtain the equation:

$$\frac{da}{dt} = \frac{M}{z\rho F} \frac{Q_f}{\epsilon_f} \dot{\epsilon}_f \quad (7.7)$$

where M is the atomic weight, ρ the density, z the number of electrons in the reaction and F the Faraday constant.

This model was successfully used to predict SCC of austenitic stainless steels in boiling water reactors.

Now, SCC fracture surfaces of FCC metals often display transgranular features of cleavages and shear steps linking them, as well as crack arrests lines. This morphology cannot be explained by the slip dissolution model. This led to propose the *film induced model*. It considers that the brittle fracture of the metal is due to

the sudden rupture of the protective film (Sieradzki and Newman 1985). Another source of brittle crack initiation can be the de-alloyed layer created by corrosion in some alloy systems such as copper or gold-silver alloys. Indeed, tensile stresses were measured in such layers and TEM observations showed easier dislocation nucleation in them for brass (Lu et al. 1998) and similarly in the passive film for α Ti (Lu et al. 2000). This hypothesis is supported by molecular dynamics studies showing that the presence of these de-alloyed layers lowered the SIF needed for emission of dislocations from the crack tip as well as K_{Ic} (Li et al. 2002). As in those cases there is no sudden fracture of the crack tip surface layers, but emission of dislocations, the mechanism cannot be the same as the one proposed by Sieradzki and Newman (1985). For Flanagan et al. (1991) these dislocations open the Cottrell-Lomer locks thus nucleating cleavage cracks.

Corrosion reactions nucleate vacancies so that some theories assume that they interact with dislocations and increase their mobility. However, dislocation climb, which the vacancies would produce, was not observed by TEM observations. Another suggested mechanism involving the vacancies was their surface migration along the crack, from anodic dissolution zones, towards the tip, favouring crack advance (Galvele 1993).

We thus see that there exist a variety of mechanisms and models based on dissolution at the crack tip and on interaction with the plastic deformation there. There is also a great number of stress-corrosion situations each one possibly corresponding to a different prevailing mechanism, notwithstanding simultaneous occurrence of them. Furthermore we will now see that hydrogen introduction in the metal plays an obvious role in stress corrosion crack propagation. We then consider that Eq. 7.5 cannot support any particular mechanism.

7.3.3.3 The Role of Hydrogen

The mechanisms of hydrogen embrittlement taking place at the crack tip are the same as the ones described above (Sect. 7.2.5): H precipitation, decrease of the surface energy, H enhanced local plasticity. The formation of brittle hydrides is also a reason for stress corrosion crack propagation in the case of hydride forming metals (Sect. 7.2.6).

The presence of a corrosive environment adds effects absent in pure hydrogen embrittlement. A model including at the same time anodic dissolution and cathodic hydrogen penetration is the *Corrosion Enhanced Plasticity Model* (CEPM). It was mostly developed by Magnin et al. (1990, 1994) (see also Lépinoux and Magnin 1993). It consists in five stages schematically shown in Fig. 7.18. The first one is the rupture of the protective film by emerging slip lines and anodic dissolution on the fresh steps; the second is the enhancement of the mobility of dislocations by the introduction of hydrogen (and also of vacancies), leading to increased localisation of slips; creation of dislocation pile-ups and zones of high stress concentration; nucleation of micro-cracks there; lowering of the local fracture toughness by hydrogen adsorption. The crack follows $\{111\}$ planes.

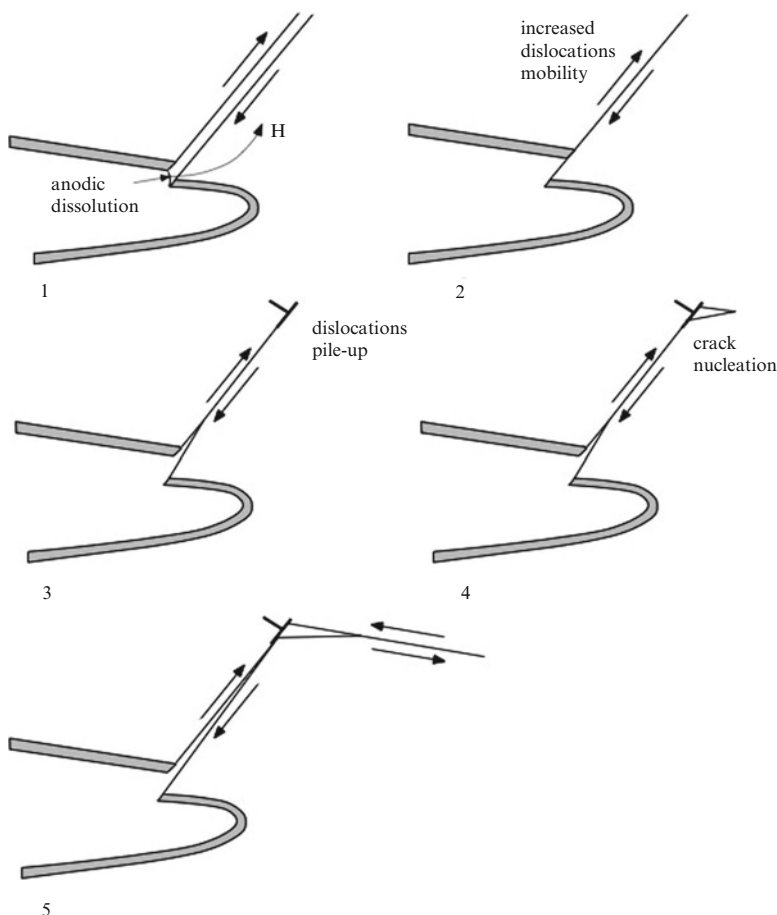


Fig. 7.18 Corrosion enhanced plasticity model. (1) rupture of the protective film; (2) increased mobility of dislocations; (3) dislocations pile-ups; (4) crack nucleation; (5) lowering of the local fracture toughness by H adsorption (Adapted from Delafosse and Magnin 2001)

This model can be extended to intergranular cracking.

7.3.3.4 Kinetics of Crack Propagation

If the crack propagation rate is plotted as a function of the stress intensity factor, in the same way as for hydrogen embrittlement (Sect. 7.2.5.2), three stages can be distinguished. (Owing to frequent crack branching in SCC it is sometimes difficult to calculate the SIF). Below a critical value of the SIF denoted K_{Isc} there is no crack propagation (Fig. 7.19). Above this value, the crack propagation rate increases very

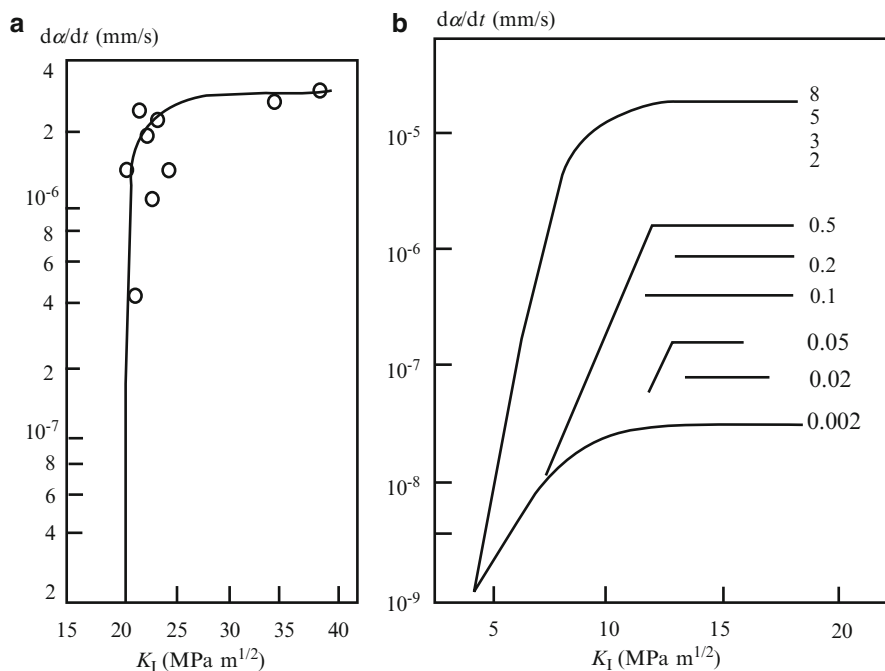


Fig. 7.19 Crack propagation rate in SCC as a function of the SIF (a) mild steel in CO_3/HCO_3 at -650 V and $75^\circ C$ (b) aluminium alloy 7079 T 651 at 0.7 V, pH 6, $23^\circ C$ showing the effect of the concentration in NaI (Adapted from Parkins)

rapidly when the SIF is increased until a plateau is reached. Its existence is attributed to the control of the crack propagation by the flow of the aggressive environment to the crack tip. This flow is limited by the small value of the CMOD. It is controlled by hydrodynamics forces.

The values of $K_{I_{SCC}}$ are usually found between 8 and 25 MPa $m^{1/2}$.

The highest crack propagation rates are those found in austenitic stainless steels in acid concentrated chlorinated media. Figure 7.20 shows the influence of the yield strength on the crack propagation rate at the plateau for a number of steels in water at $100^\circ C$. As for crack initiation the stress corrosion sensitivity increases with the yield strength. Low strength steels are practically unaffected in this environment. The highest rate is found for a chromium-molybdenum steel.

The composition of a given material can drastically change the SCC susceptibility. Such an effect is shown in Fig. 7.21 for molybdenum in an austenitic stainless steel.

Fig. 7.20 SCC crack propagation rate at the plateau vs. SIF as a function of the yield strength for various steels in water at 100°C (Adapted from Parkins 1992)

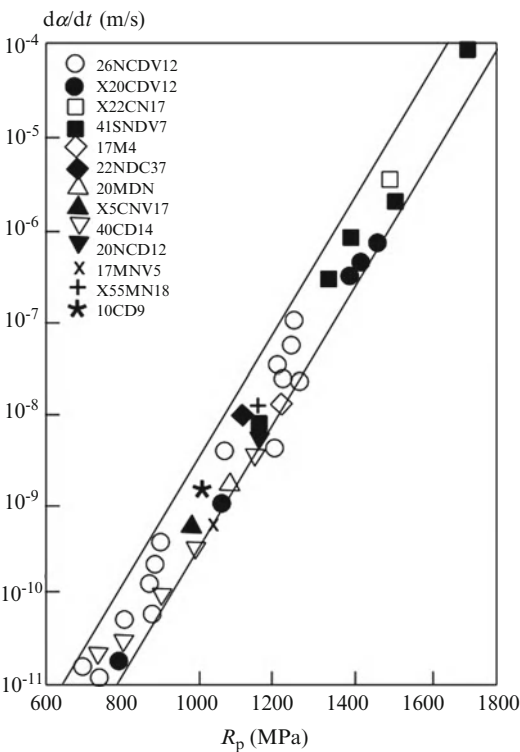
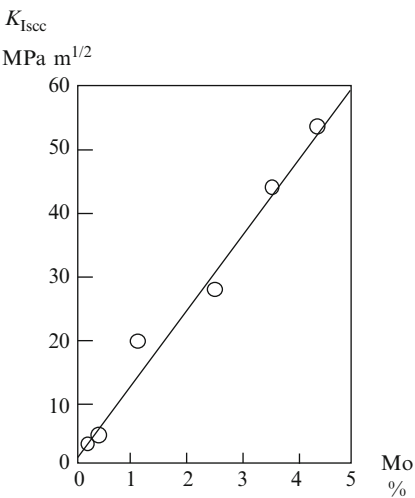


Fig. 7.21 Effect of molybdenum on K_{Isc} of an austenitic stainless steel in a 22% NaCl solution at 105°C (Speidel 1981)



7.4 Corrosion Fatigue

7.4.1 Initiation of Corrosion Fatigue

Corrosion can take different forms (generalised corrosion, pitting, crevices, intergranular) according to the nature of the metal, the treatments applied, the nature of the environment (pH, oxygen concentration...), the electric potential... On these varieties of phenomena are superimposed the great number of parameters operating in fatigue. It complicates what we have discussed about stress corrosion cracking. It is therefore difficult to highlight general characteristics of corrosion fatigue. Concerning initiation we can nevertheless distinguish three different types of behaviour.

1. *Electrochemical passivity* in which case the metal surface is protected by a passive film. We have seen that, in stress corrosion cracking (Sect. 7.3.3.2), plastic deformation slips break locally this film, producing dissolution followed by repassivation (Fig. 7.17). In cyclic loading, the concentration of shear strain in PSBs accentuates this phenomenon. Note that it is observed already in air, in which environment the fatigue life is shorter than under vacuum (Petit and Sarrazin-Baudoux 2010; Mendez and Violan 1988). These mechanisms allow to understand the deleterious effect of the planarity of slip, as it was mentioned in stress corrosion cracking, due to low stacking fault energy or to the presence of shearable precipitates.
2. *Preferential dissolution* corresponds to dissolution of anodic zones, which can be found in heterogeneities of plastic strain. These are enhanced in cyclic deformation. PSBs are anodic zones.
3. Local disappearance of passivity initiates corrosion most often by *pitting*. The pits are then strain concentrators and fatigue cracks initiate there (Fig. 7.22). In fact they can be assimilated to semi-elliptical cracks. What counts then is the kinetics of pitting compared with that of the other corrosion fatigue modes (Kondo 1989).

Hydrogen embrittlement further reduces the fatigue life. The mechanisms have been described in the preceding sections of the present chapter. The influence of *inclusions* close to the surface should not be forgotten (Fig. 7.23). They create galvanic effects and corrosion takes place preferentially at their interface. The inclusion is dissolved if it is anodic with respect to the matrix, or it is the inverse if it is cathodic.

Depending on the corrosion fatigue conditions the same material can display different initiation mechanisms. As examples: for an Al-Zn-Mg alloy in a 3.5% NaCl solution Magnin and Rieux (1987) observed that an increase of the strain rate changes initiation from intergranular to transgranular. Similarly, for an Al-Li-Cu alloy in a 3.5% NaCl solution, Rebière and Magnin (1990) showed that decreasing the strain rate shifts the initiation mechanism from preferential dissolution to pitting and then to electrochemical passivity.

Fig. 7.22 SEM image of corrosion fatigue crack intergranular initiation inside a pit in 316L austenitic stainless steel (stress amplitude 400 MPa and $N = 7.2 \times 10^4$ Hank's solution) (Xie et al. 2002)

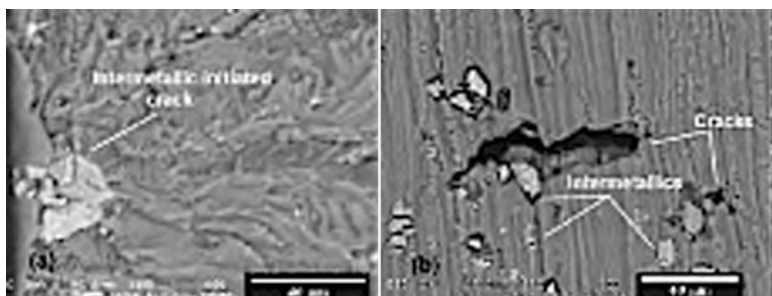
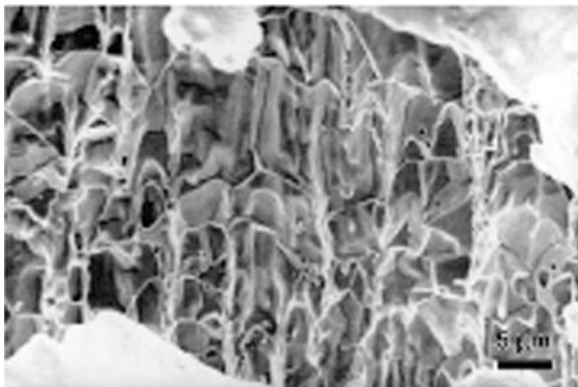


Fig. 7.23 Corrosion fatigue crack initiation at the interface of second-phase particles in 5083H111 aluminium alloy (3.5% NaCl solution) (Mutombo and du Toit 2011)

7.4.2 Corrosion Fatigue Crack Propagation

7.4.2.1 Propagation of Long Cracks

If the loading frequency is very low, it could be expected that corrosion fatigue would be the result of cumulation of stress corrosion at successive varying stresses. More generally, the crack propagation rate might be the superposition of pure fatigue and of stress corrosion according to the equation:

$$\frac{da}{dt} = \left(\frac{da}{dt} \right)_{\text{SCC}} + \frac{1}{T} \left(\frac{da}{dN} \right)_{\text{F}} \quad (7.8)$$

where $(da/dt)_{\text{scc}}$ is the crack propagation rate in stress corrosion, $(da/dN)_{\text{F}}$ the pure fatigue propagation rate and T the period of the cyclic loading. This behaviour corresponds to case (b) in Fig. 7.24.

However synergetic effects often operate and it is then found that the preceding equation underestimates the corrosion fatigue crack propagation rate (Fig. 7.24c).

Typical behaviour for steels, titanium and aluminium alloys is exemplified by Fig. 7.25.

Figure 7.26 shows the case of a well defined SCC threshold. The effect of stress corrosion is the highest at low frequencies (at propagation rates higher than the stress corrosion cracking threshold). At low frequencies repassivation has more time to operate the lower the frequency. This explains why in the aqueous environment the crack propagation rate is the lowest at 1 Hz frequency.

In gaseous environments the crack propagation rate is higher than the one measured in vacuum, for the same amplitude of the stress intensity factor (Pelloux and Genkin 2010; Petit and Sarrazin-Baudoux 2010). Figure 7.27 shows as an example the influence of the partial water pressure on the crack propagation rate for a 5070 aluminium alloy. The tests were at constant stress amplitude, so that the amplitude of the stress intensity factor is proportional to the square root of the crack length. The figure displays crack propagation rates independent of the water pressure below and above a critical value, the rate at low water pressures being smaller than at high water pressures.

When approaching the threshold by decreasing the amplitude of the stress intensity factor, closure effects become pronounced. The *closure due to the oxide layer* (see Sect. 6.3.2.1) lowers the threshold in air with respect to vacuum. There is then an interval below the fatigue threshold in air where the crack propagation rate is higher under vacuum than in air, the oxide layer closure effect predominating. Figure 7.28 gives an example of such a behaviour for E460 steel (Petit and Sarrazin-Baudoux 2010). Testing at a R ratio equal to 0.7 eliminates the closure effect. Under those conditions, crack propagation is faster in air than under vacuum and the fatigue threshold is lower. In nitrogen the crack propagation rate is about the same as in air. For a R ratio equal to 0.1, closure effects raise the fatigue threshold in air. The effect is much lower in nitrogen because the oxidation is not so high.

7.4.2.2 Propagation of Short Cracks

Two important differences distinguish short crack from long crack behaviour in corrosion fatigue:

- The environment at short crack tips is not very different from the one at the crack mouth on the surface, in contrast to long cracks.
- For short cracks the closure effect is small or even non-existent.

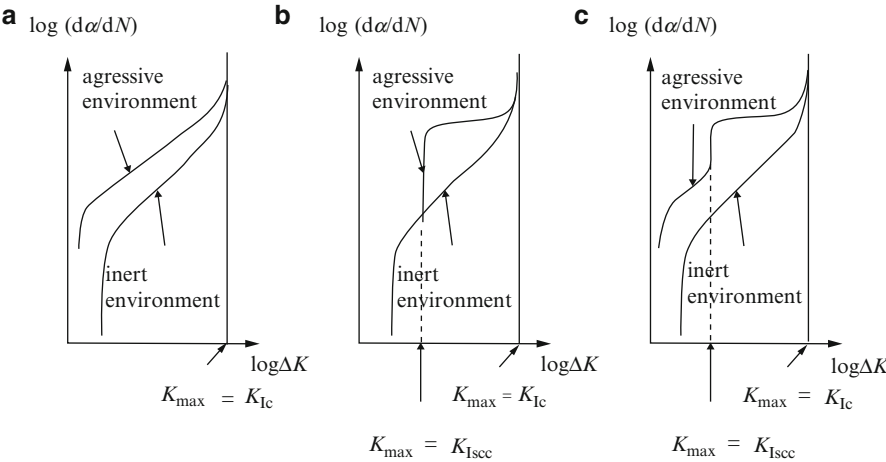


Fig. 7.24 Schematic typical corrosion fatigue crack propagation rate as a function of the amplitude of the stress intensity factor. (a) Pure corrosion fatigue; (b) Superposition of fatigue and stress corrosion cracking; (c) Synergetic effects

Fig. 7.25 Cracking rates in corrosion fatigue of a 12Ni-5Cr-3Mo stainless steel for various load frequencies (Adapted from Pelloux and Genkin 2010)

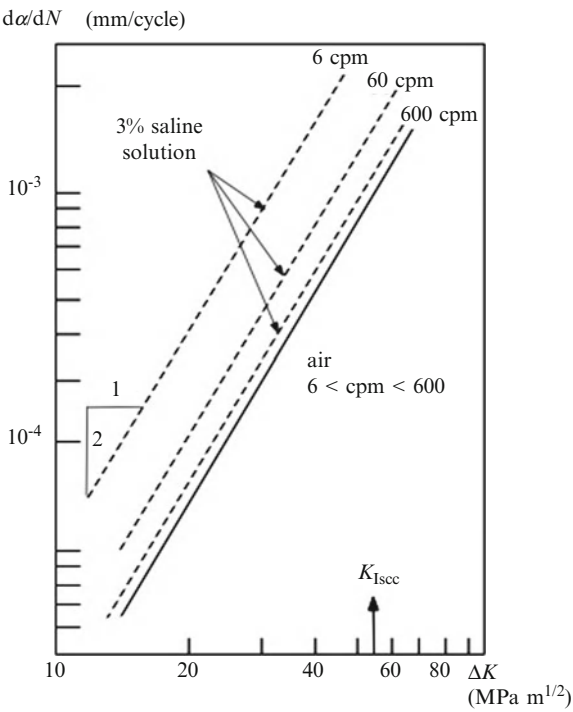
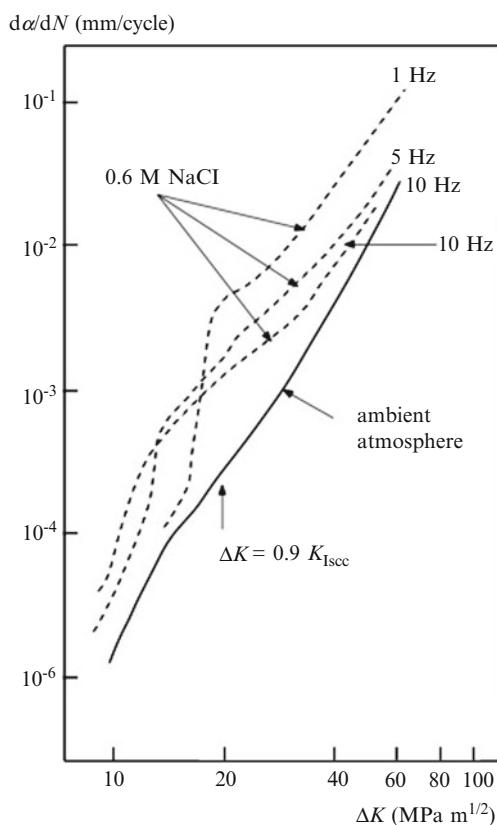


Fig. 7.26 Crack growth rate in corrosion fatigue of a Ti-6Al-4V mill annealed alloy (Adapted from Dawson and Pelloux 1974)



As it is the case in pure fatigue short cracks propagate below the threshold of long cracks and for the same SIF amplitude their growth rate is larger. The effect is more pronounced in corrosion fatigue. This was first observed by Gangloff (1985) in 4130 steel. Piasick (1994) found that for 2024 aluminium alloy in synthetic sea water the short crack effect was simply a disappearance of the threshold.

The same is observed for atmospheric corrosion fatigue. For E460 steel Petit (1999) measured about ten times faster long crack propagation rates in air than under vacuum and even faster short crack propagation rate, whereas short cracks and long cracks displayed the same behaviour under vacuum.

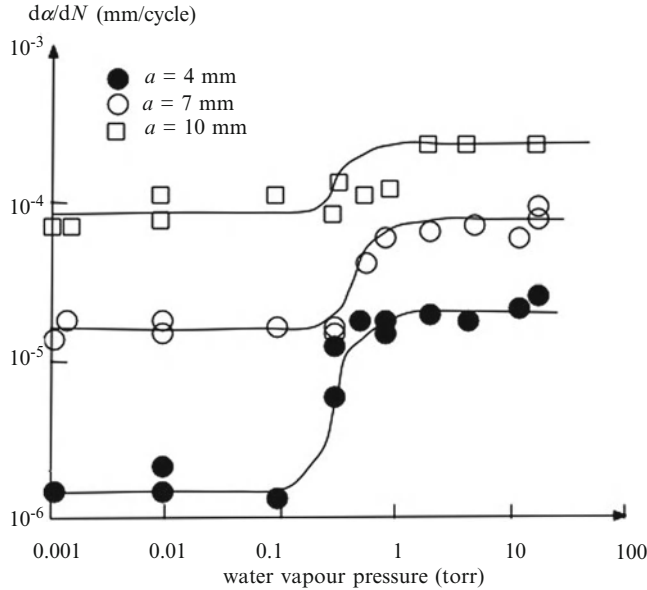


Fig. 7.27 Influence of the partial water vapour pressure on the fatigue crack propagation rate for 5070 aluminium alloy (Adapted from Bradshaw 1967)

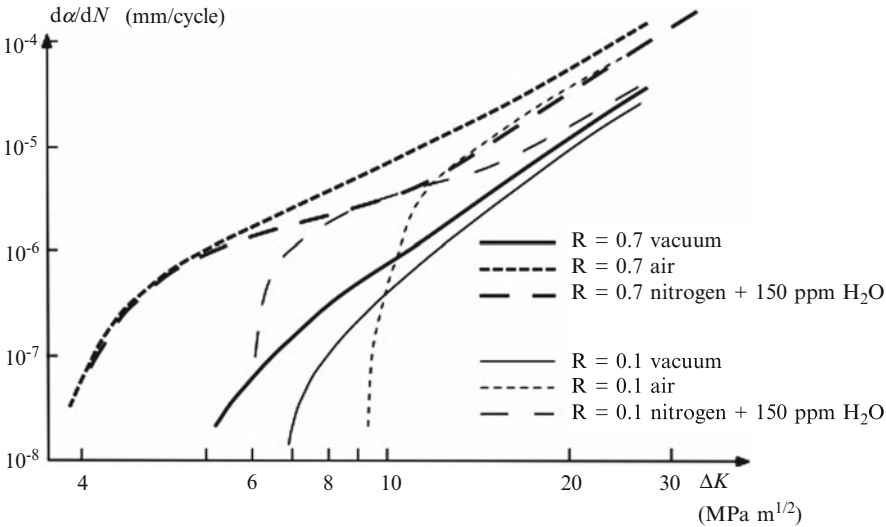


Fig. 7.28 Different da/dN versus ΔK curves for two R ratios and different environments for E460 steel (at 35 Hz) (Adapted from Petit and Sarrazin-Baudoux 2010)

7.5 Liquid and Solid Metal Induced Embrittlement

7.5.1 Occurrence of Liquid Metal Induced Embrittlement

In the presence of some liquid metals other metals can have an extremely brittle behaviour. This is the case for instance for zinc in liquid indium, aluminium in liquid gallium (liquid at room temperature as mercury), steels in liquid lead. The very fast brittle fracture is preceded by an incubation time. The phenomenon occurs close to the melting point of the liquid metal.

A spectacular case of liquid metal induced embrittlement (LMIE) was the fire of the natural gas processing plant at Moomba in Australia on the first of January 2004. It was due to the failure of a heat exchanger inlet nozzle made of an aluminium alloy, which was embrittled by mercury. The nuclear industry is concerned by liquid metal embrittlement as far as liquid metals can be used as coolant. For this purpose, research has focussed recently on martensitic 8.5%Cr steel T91 in the eutectic lead-bismuth.

The fracture is most often intergranular. However single crystals can be embrittled by liquid metals. Liquid metal embrittlement of aluminium alloy 6061 by low melting point inclusions of Bi, Cd, Pb was also observed (Roth et al. 1980). Although cracking can occur in the absence of stress, tensile loading accelerates greatly the embrittlement.

The mechanisms of liquid metal embrittlement are not yet fully understood.

7.5.2 Liquid Metal Induced Embrittlement Mechanisms

7.5.2.1 Rehbinder³ Effect

The first explanation which comes to mind is the influence of the *surface energy*. At the surface of a material wetted by a liquid grooves form at grain boundaries. The angle θ of these grooves is given by

$$\cos \frac{\theta}{2} = \frac{\gamma_b}{2\gamma_s} \quad (7.9)$$

γ_b being the grain boundary energy and γ_s the surface energy. When γ_s decreases and tends to $\gamma_b/2$ this angle tends to zero, and the groove becomes crack-like.

In Sect. 3.8.4.1 we have written the fracture energy γ_c for intergranular fracture as (Eq. 3.73):

³Piotr Alexandrovitch Rehbinder (1898–1972) was a Russian physico-chemist. He discovered the “Rehbinder effect” in 1928.

Table 7.3 Surface energy γ_s in J/m² of aluminium in various liquid metals (Old and Travena 1979)

Metal and embrittling liquid metal	Al	AlPb	AlCd	AlBi
Surface energy γ_s (J/m ²)	1.0	0.344	0.298	0.288

$$\gamma_c = 2\gamma_s^{\text{int}} - \gamma_b \quad (7.10)$$

where γ_s^{int} is the surface energy of the broken surface immediately after fracture and γ_b the energy of the grain boundary. Now, a liquid metal modifies the surface energy and, if it decreases γ_s^{int} , the intergranular fracture energy is that much reduced. In fact this is the Rehbinder effect, originally discovered in 1928 from investigations on the fracture of calcite and rock salt in water. Table 7.3 shows the reduction of the surface energy of aluminium in various liquid metals. It shows indeed reductions, the most embrittling media being liquid bismuth.

We can also refer to the model of Rice of crack tip blunting by emission of dislocations as expressed in Eqs. 3.7 and 3.8 (Sect. 3.2.3). In these equations, the liquid metal can modify the energy of the ledge due to the emission of a dislocation as well as the core energy of the dislocation.

Linked to this role of the surface energy, empirical criteria for liquid metal embrittlement are:

- no tendency to form intermetallic stable components
- little mutual solubility

7.5.2.2 Adsorption Induced Reduction in Cohesion

Adsorption induced reduction in cohesion is a more elaborate model. It considers that liquid metal atoms at the crack tip reduce the cohesion energy. This would tie up with the modification of the core energy of dislocations at the crack tip. Nam and Srolovitz (2009) studied by molecular dynamics calculations the penetration of gallium in aluminium. They showed that it was controlled by the climb within the grain boundary of edge dislocations. These dislocations were created to release the stress concentration due to the penetration of Ga in the grain boundary.

7.5.3 Initiation and Propagation of Cracks Due to Liquid Metal Embrittlement

In liquid metal induced embrittlement crack propagation is preceded by a delay time. Figure 7.29 shows that it is the shorter the higher the stress. A critical stress level is needed.

Fig. 7.29 Initiation time for liquid metal embrittlement of aluminium alloy 2024 T4 by mercury at 100°C (Adapted from Rostoker et al. 1960)

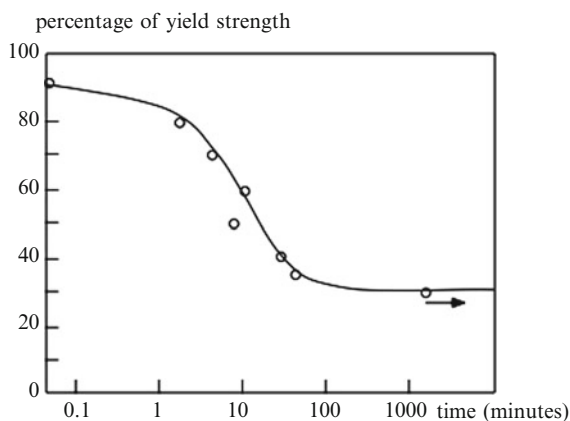
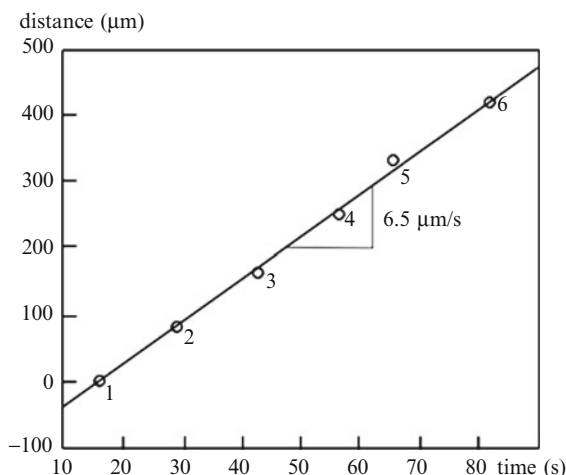


Fig. 7.30 Penetration of Ga in the grain boundary (110) of an Al bicrystal (Adapted from Pereiro-Lopez et al. 2005)



Pereiro-Lopez et al. (2003, 2005, 2006) investigated the wetting by gallium of the grain boundaries of bicrystals and polycrystals of aluminium. They used micro-radiography with the synchrotron radiation. They were able not only to follow the penetration rate of Ga but also the thickening of the Ga layer. They found that in a bicrystal the rates were constant (Figs. 7.30 and 7.31). On the contrary, in polycrystals, it was slow at first and then increased suddenly in an irregular fashion. Applying a stress increased the rate of penetration (Fig. 7.32). The molecular dynamics simulations of Nam and Srolovitz (2009) reproduce the bicrystal behaviour, demonstrating the influence of the applied stress (Fig. 7.33).

Fig. 7.31 Thickening of Ga in the grain boundary (110) of an Al bicrystal. The number on the curves correspond to the location along the grain boundary as labelled in Fig. 7.30 (Adapted from Pereiro-Lopez et al. 2005)

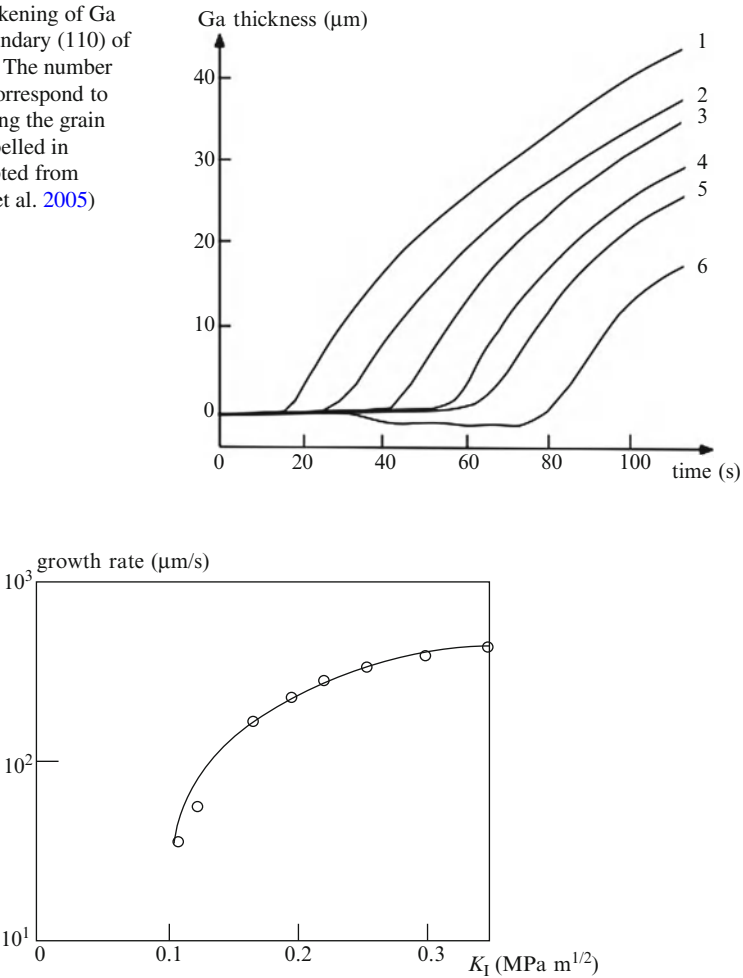


Fig. 7.32 Rate of penetration of Ga in the grain boundary of an Al bicrystal as a function of the applied stress (Adapted from Pereiro-Lopez et al. 2006)

LMIE requires a direct contact with the liquid metal. Oxide layers prevent it and difficulties can be met to determine the liquid metal embrittlement susceptibility. This was the case for T91 martensitic steel in lead-bismuth eutectic studies by Auger et al. (2004). These authors had to apply special surface preparation to remove the oxide layer before they could investigate LMIE.

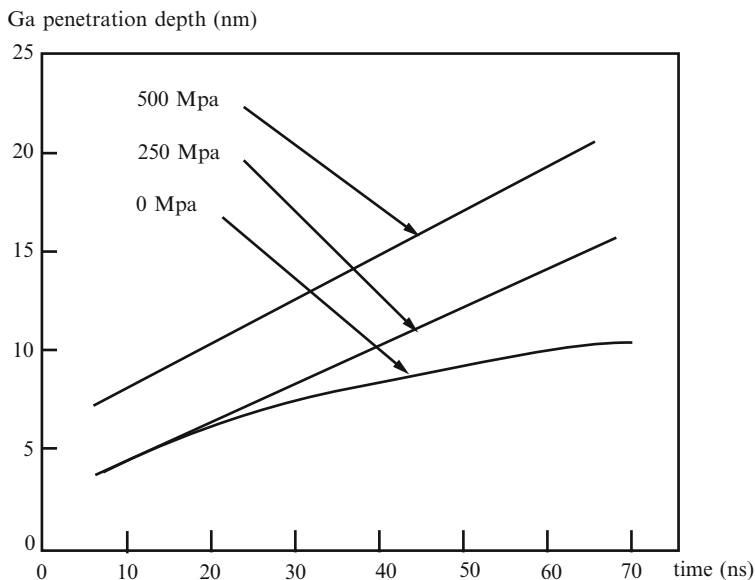


Fig. 7.33 Simulation of the penetration of Ga in a grain boundary of an Al bicrystal at 600 K (Adapted from Nam and Srolovitz 2009)

7.5.4 Solid Metal Induced Embrittlement

The embrittling metals in the liquid state can have the same action in the solid state but at a much slower rate (Lynn et al. 1975; Lynch 1989). This was observed for high strength steels embrittled by Cd, Ag, Au, Zn, Pb, In, Sn, for titanium alloys embrittled by Cd, Ag, Au, for zirconium alloys embrittled by Cd. The practical consequences are found for instance in the need to use cadmium plated steel bolts below 230°C; in embrittlement of zircaloy in nuclear fuel elements by cadmium in the coolant liquid; in embrittlement of inconel vacuum seals by indium; in embrittlement of leaded steels by lead inclusions (ASM 2002).

Solid metal induced embrittlement (SMIE) does not occur below a threshold stress intensity factor. Lynch (1989) determined this to be about 8 MPa m^{1/2} for aluminium alloy 7075 embrittled by indium. For embrittlement to occur the temperature must be high enough. Lynch reported $T/T_M > 0.38$ (T_M being the absolute melting temperature) for Ti alloys embrittled by Ag and Au, $T/T_M > 0.59$ for Ti alloys and high strength steels embrittled by Cd (Fager and Spurr 1970, 1971). SMIE is accelerated when temperature is increased. As an example Fig. 7.34 shows results of Lynch (1989) on the cracking rate for aluminium alloy 7075 embrittled by solid indium.

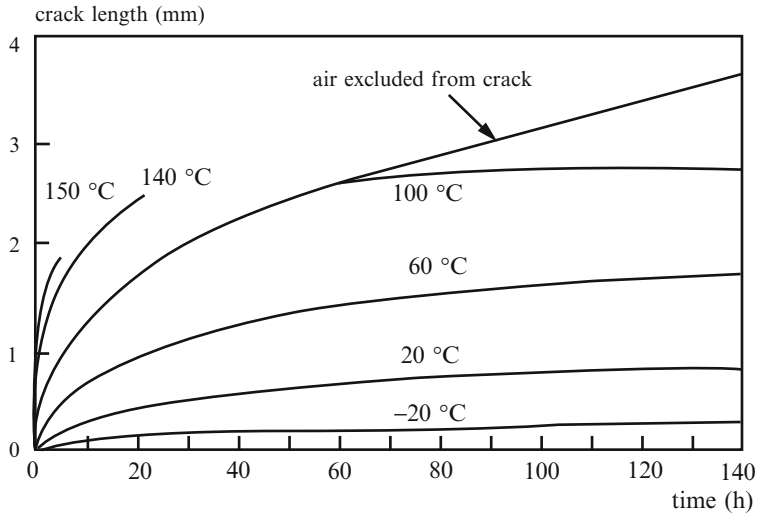
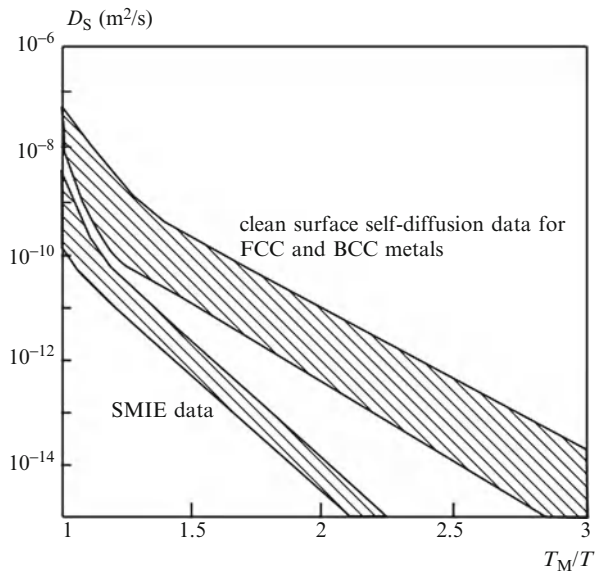


Fig. 7.34 Crack propagation in aluminium alloy 7075 embrittled by solid indium at various temperatures (Adapted from Lynch 1989)

Fig. 7.35 Comparison of the surface diffusion coefficients obtained from SMIE data with surface self-diffusion coefficients for FCC and BCC metals (Adapted from Lynch 1989)



SMIE is independent of the stress level above the threshold. This shows that the controlling mechanism is the diffusion of the embrittling metal. In fact the diffusion coefficient deduced from SMIE kinetics is close to the one measured for surface diffusion as shown in Fig. 7.35. The fact that the later values are larger is thought to be due to the roughness of the SMIE cracks, which increases the distance to travel

for embrittling atoms compared with the distance on the projected surface. Thus the apparent diffusion coefficient as measured on the projected surface is lower than the real one. The higher cracking rate measured in the absence of air (Fig. 7.34) suggests that diffusion is slowed down by the presence of oxide on the surface.

The mechanism of embrittlement is thought to be the same as for LMIE: decohesion at the crack tip.

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Chapter 8

Creep-Fatigue-Oxidation Interactions

Abstract The fracture maps of Ashby display the various modes of creep fracture. Creep damage results from the nucleation of cavities on grain boundaries. Viscoplastic deformation can be the controlling mechanism. Hull and Rimmer model is at the basis of diffusion controlled nucleation. Viscoplasticity and diffusion can be coupled in the nucleation of creep cavities. Furthermore, overall deformation can constrain this phenomenon. The Monkman-Grant law is a phenomenological expression relating the time to fracture to the strain rate. Time to fracture contours in multiaxial loadings are found to lie between the von Mises and the maximum stress criteria. Continuous damage mechanics was introduced to predict creep fracture. This is exemplified by the cases of copper, Nimonic 80A and austenitic stainless steels. The behaviour of three types of alloys, 9–12 Cr steel, austenitic stainless steel and Ni-base superalloys, is explained by creep-fatigue-oxidation interactions. Various engineering methods allow predicting the creep fatigue life: limit load analysis, crack initiation and propagation related to the stress intensity factor in the case of creep brittle materials and to the C^* parameter in the case of creep ductile materials, frequency modified fatigue life prediction, strain range partitioning, linear and nonlinear damage accumulation. Long crack propagation is linked with creep-fatigue interactions. Thermal barriers coatings and ceramic matrix composites are two examples of high temperature materials.

8.1 Introduction

High temperature service often combines extreme conditions of aggressive environments, and high stresses having fluctuating components. Fracture under these conditions is usually the performance limiting phenomenon and is of primary concern to the designer. In many cases the loading is stationary and components can be protected from the aggressive environments by the application of protective coatings, so that the problem becomes often one of accumulation of creep damage.

Ashby et al. (1979) distinguished different types of creep damage mechanisms in a number of important engineering alloys. The information which is similar to the *deformation maps* (see Volume I, Chap. 4, Sect. 4.3.2.3) is presented in *fracture mechanisms maps* which are complementary as shown in Fig. 8.1a, b for the case of Nimonic 80A. This material is representative of wrought nickel-based superalloys in use today. A brief description of the microstructure of these alloys was given in Chap. 1 of Volume I (Fig. 1.41). The heat treatment typically applied to the alloy consists of a solution treatment at 1,000 °C followed by ageing at 700 °C. This leads to precipitates of γ' ($\text{Ni}_3(\text{Ti}, \text{Al})$) in a nickel-chromium rich matrix of γ . In addition these superalloys contain both a general dispersion of $\text{M}(\text{C}, \text{N})$ carbides where M is usually titanium, and a grain boundary dispersion of carbides M_{23}C_6 or M_7C_3 where M is essentially chromium. This dispersion helps limiting boundary sliding which is a deformation mechanism operating at elevated temperature.

The results reported in Fig. 8.1a, b are purely experimental. In Fig. 8.1a the isochronal fracture data are presented on a field of the normalised tensile stress (σ/E , where E is Young's modulus) and homologous temperature (T/T_m where T_m is the melting temperature, taken as equal to 1,638 K in Nimonic 80A). This type of presentation can be taken together with the corresponding deformation mechanism map for the same material (Frost and Ashby 1982). It relates the fracture information process to the creep fracture mechanisms. The fields outlined by the time-contours characterise the regions in which one of the specific different forms of fracture depicted in Fig. 8.2 dominates. Figure 8.1b gives the information in terms of isothermal contours on a field of normalised tensile stress and time to fracture. This representation is more familiar to the design engineers.

Figure 8.1a, b represent almost entirely the various forms of fracture encountered in creeping alloys. The type of fracture at high stress and high strain rates, leading to short times to fracture, is transgranular and occurs by micro-void coalescence with features almost identical to those in ductile fracture at low temperature (see Chap. 4 in this volume). As the stress is decreased, fracture becomes intergranular and is dominated by wedge cracks leading to the accumulation of grain facet fractures, as depicted in Fig. 8.2a. At lower stresses leading to long times to fracture, typically encountered in service, the fracture is dominated by the coalescence of individual grain boundary cavities (Fig. 8.2a). These intergranular fracture modes, that are dominant in service conditions, are the primary subject of this chapter.

It should be emphasised again that the transition of fracture from a transgranular form of micro-void coalescence to intergranular cavitations is largely influenced by grain boundary sliding which has been described in Volume I (Sect. 4.3.2.2c) and by many authors (see e.g. Chen and Argon 1979, 1981).

In this chapter we shall first describe the mechanisms that give rise to intergranular cavitations. This is a field in which great progress was made in the 1980s (see e.g. Argon et al. 1980; Argon 1982; Beere 1981; Cocks and Ashby 1982a, b; Evans and Wilshire 1985; Riedel 1987). However the first two stages, nucleation and growth of cavities, only were studied in depth. Creep cavity coalescence has not been studied in such detail. After this, we shall discuss the results of damage measurements made on various alloys and we shall attempt to locate these in

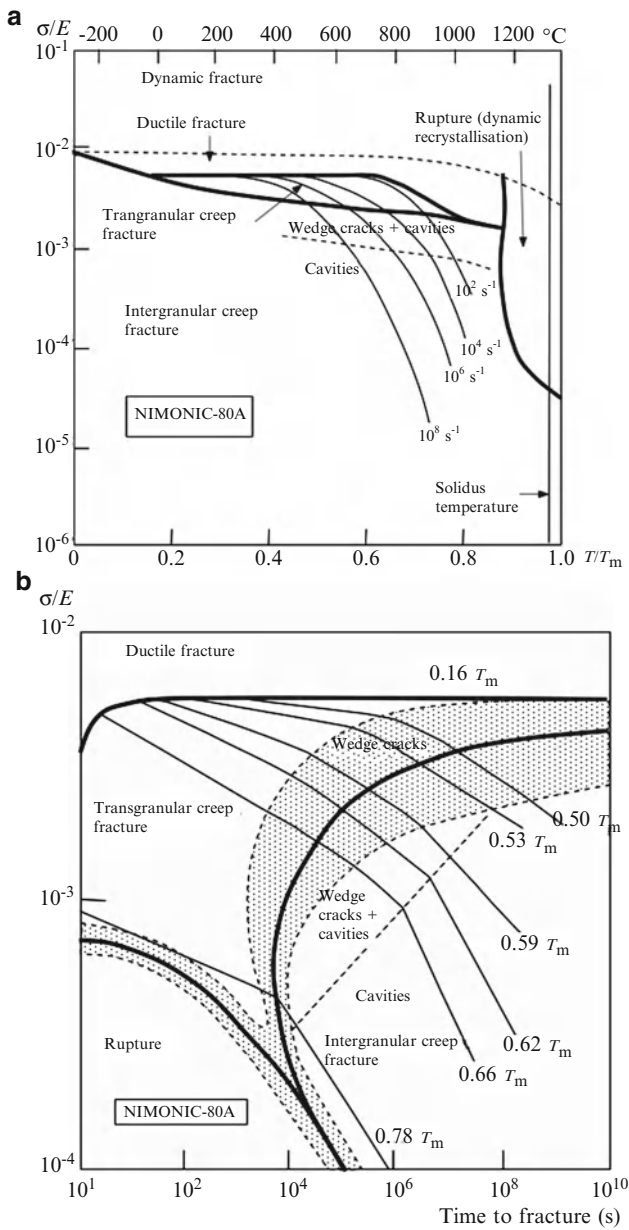


Fig. 8.1 Fracture maps of Nimonic 80A. (a) fracture map of the first type: stress-temperature; (b) fracture map of the second type: stress-time to failure. The melting temperature of this alloy is taken as equal to 1,638 K (Adapted from Ashby et al. 1979)

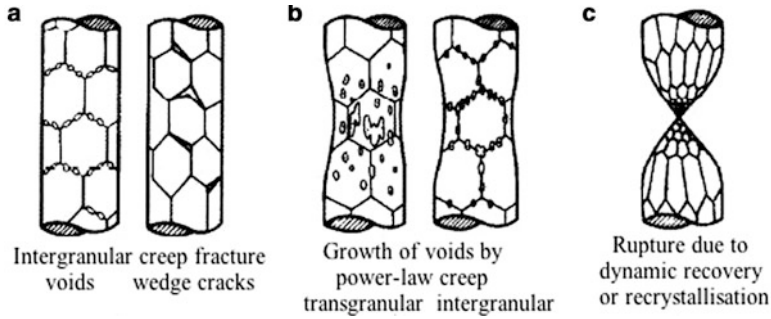


Fig. 8.2 Classification of fracture mechanisms at elevated temperature. The mechanisms associated with intergranular fracture are described in the text (Adapted from Ashby et al. 1979)

the framework of phenomenological macroscopic approaches. This will lead us to present the techniques which are in use for life assessment of components working at elevated temperature. Finally a short presentation of the materials that have been developed for high temperature applications will be given.

8.2 Nucleation of Creep Cavities

8.2.1 Theory of Cavity Nucleation

As in plasticity, cavities can be initiated from inclusions or second-phase particles under the effect of stress concentrations (refer to Sect. 4.2.1 in this volume). This mechanism, which is not thermally activated, occurs less frequently in the case of creep deformation because of the low levels of applied stress. At elevated temperature cavity nucleation occurs mainly at grain boundaries and is thermally activated because it involves diffusion. We consider first cavity nucleation in a homogeneous solid before dealing with intergranular cavity nucleation.

In a homogeneous solid under hydrostatic tension σ_m , the change in free enthalpy when a spherical cavity of radius R and surface energy γ_s appears is:

$$\Delta G = 4\pi R^2 \gamma_s + \Delta E_e - \frac{4}{3}\pi R^3 \sigma_m \quad (8.1)$$

where ΔE_e is the change in elastic energy equal to $\alpha [3(1 - 2\nu)/E] \sigma_m^2 (4\pi/3) R^3$ where α is a coefficient representing the perturbation of the stress field by the cavity (see Sect. 2.7.4.1 in Volume I). As creep stresses are low, the term ΔE_e , which is proportional to σ_m^2 , is negligible.

The energy barrier to be overcome in order to create a cavity is the value of ΔG , denoted ΔG^* , corresponding to the radius R^* for which $\partial \Delta G / \partial R = 0$. Neglecting

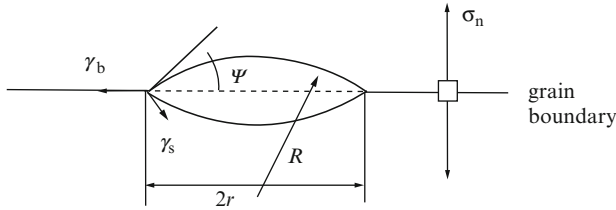


Fig. 8.3 Lenticular cavity located on a *grain boundary*. γ_s is the surface energy and γ_b the grain boundary surface energy

ΔE_e in (8.1), we find:

$$R^* = \frac{2\gamma_s}{\sigma_m}, \quad \Delta G^* = \frac{16\pi}{3} \frac{\gamma_s^3}{\sigma_m^2} \quad (8.2)$$

It can easily be checked that the energy ΔG^* is much greater than ΔE_e . In (8.2) R^* is the radius of curvature of a surface in equilibrium under a pressure σ_m and surface tension γ_s .

The rate of nucleation of cavities will be proportional to $\exp(-\Delta G^*/kT)$, which must be multiplied by the frequency with which vacancies cross the cavity surface, of area $4\pi (R^*)^2$. For a single vacancy the frequency is $\exp[-(\Delta E_f - \sigma_m \Omega)/kT]$, where ΔE_f is the energy of formation of the vacancy and Ω is its volume, that is $D_v/\Omega^{2/3} \exp(\sigma_m \Omega/kT)$ where D_v is the coefficient of self-diffusion. Since there are $4\pi(R^*)^2/\Omega^{2/3}$ atomic sites on the surface of the cavity we have finally for the rate of nucleation:

$$\dot{N} = \frac{4\pi (R^*)^2}{\Omega^{2/3}} \frac{D_v}{\Omega^{2/3}} \exp\left(\frac{\sigma_m \Omega}{kT}\right) \exp\left(-\frac{\Delta G^*}{kT}\right) \quad (8.3)$$

8.2.2 Cavity Nucleation at Grain Boundaries

Cavities nucleate most easily on the grain boundaries because these have a surface energy γ_b which vanishes when a cavity is formed; thus for a spherical cavity under hydrostatic stress σ_m the change in free energy is:

$$\Delta G = 4\pi R^2 (\gamma_s - \gamma_b/4) - \frac{4}{3}\pi R^3 \sigma_m \quad (8.4)$$

However at equilibrium the cavity is not spherical, since, at the point where it meets the grain boundary, the surface tensions are in equilibrium (Fig. 8.3). Therefore:

$$\gamma_b = 2\gamma_s \cos\psi \quad (8.5)$$

where ψ is the angle at which the two halves of the cavity meet; for usual surface energy of grain boundaries ψ is equal to about 60 degrees. Let us call $S = F_s R^2$ the area of cavity surface, $S_b = F_b R^2$ the area of boundary intersected by cavity and $V = F_v R^3$ the volume of cavity, where F_s , F_b and F_v depend on the geometry of the cavity and R is the radius of curvature of the cavity. Then:

$$\Delta G = 2\gamma_s (F_s/2 - F_b \cos\psi) R^2 - \sigma_n F_v R^3 \quad (8.6)$$

where σ_n is the normal stress acting on the grain boundary.

The critical radius R^* is the value for which $\partial\Delta G / \partial R = 0$; hence:

$$R^* = \frac{4\gamma_s}{3\sigma_n} \frac{F_s/2 - F_b \cos\psi}{F_v}, \quad \Delta G^* = \frac{32\gamma_s^3}{27\sigma_n^2} \frac{(F_s/2 - F_b \cos\psi)^3}{F_v^2} \quad (8.7)$$

For a cavity on a planar boundary, as in Fig. 8.3:

$$F_s = 4\pi (1 - \cos\psi), \quad F_b = \pi \sin^2\psi, \quad F_v = (2\pi/3) (1 - \cos\psi)^2 (2 + \cos\psi) \quad (8.8)$$

$$R^* = \frac{2\gamma_s}{\sigma_n}, \quad \Delta G^* = \frac{8\pi}{3} \frac{\gamma_s^3}{\sigma_n^2} (1 - \cos\psi)^2 (2 + \cos\psi) \quad (8.9)$$

It follows that R^* and ΔG^* decrease with decreasing ψ , that is with increasing values of the ratio γ_b / γ_s .

This explains the role played by *segregation of impurities* on the grain boundaries: by changing the surface energy these impurities can greatly reduce the activation energy for the nucleation of intergranular cavities, as is the case for example for antimony in copper and sulphur and phosphorus in steels (see Chap. 3, Sect. 3.8.4 in this volume).

Another important effect is that of a gas under pressure in the cavity: this pressure will add to the applied stress σ_n , resulting again in a large decrease in the activation energy. An example is the *helium* generated by nuclear reactions with boron atoms during experiments on *irradiation-induced creep*.

It follows also that the critical size and the activation energy are less if the cavity forms at the intersection of two boundaries which meet at an acute angle. This explains why cavities form more easily at triple junctions and on precipitates or particles lying along the boundaries.

Once the activation energy ΔG^* is known, the nucleation rate can be calculated; the method is analogous to that for the homogeneous case, with the difference that the transport of vacancies is now taken to be by diffusion along the grain boundary, with coefficient D_{GB} and over an effective thickness δ_{GB} . The result is:

$$\dot{N} = \frac{2\pi R^* \sin\psi}{\Omega^{2/3}} \frac{\delta_{GB} D_{GB}}{\Omega^{2/3}} \exp\left(\frac{\sigma_n \Omega}{kT}\right) \exp\left(-\frac{\Delta G^*}{kT}\right) \quad (8.10)$$

Fig. 8.4 Results of the theory of nucleation for creep cavities in γ iron. Nucleation rate (s^{-1}) is plotted against normalised stress (σ_n/E) on the boundary, where E is Young modulus

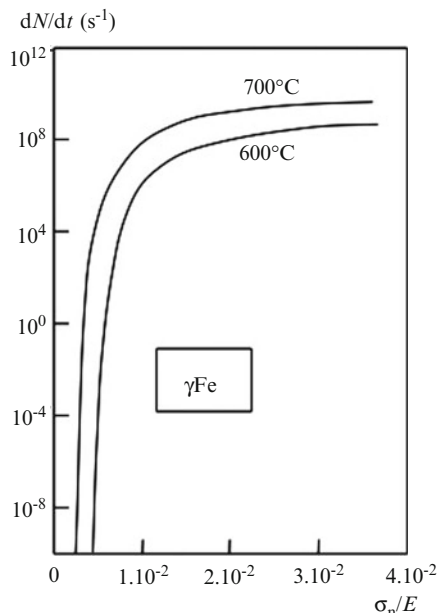


Figure 8.4 (Argon 1982) gives the results of the calculation for γ -iron taking the values: $\Omega = 1.21 \times 10^{-29} \text{ m}^3$, $\delta_{GB} D_{GB} = 7.5 \times 10^{-14} \exp(-159 \times 10^3/kT) \text{ m}^3 \text{ s}^{-1}$, $\psi = 1$ radian, $\mu = 1.40 \times 10^2 \text{ GPa}$. k is the Boltzmann constant. The activation energy for diffusion is expressed in J/mole.

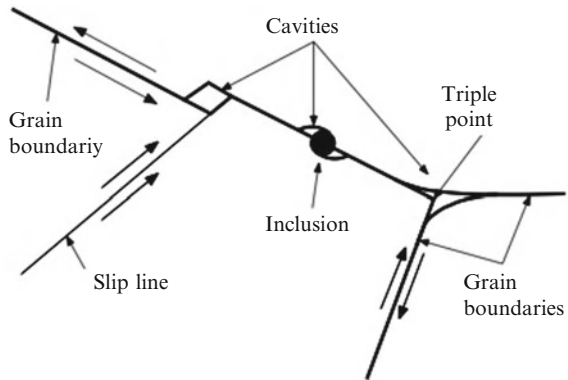
Figure 8.4 shows that the nucleation rate reaches a constant value, varying only slightly with temperature, for values of σ_n/E of the order of 5×10^{-3} that is for stresses of the order of 700 MPa. Creep stresses are about ten times smaller than this value.

This indicates that for reasonable rates of nucleation of cavities on the grain boundaries there must be stress concentration factors of the order of 10. However we must recall that, as indicated earlier, segregation of impurities on the grain boundaries or presence of gas in the cavities can also largely reduce the nucleation threshold.

8.2.3 Importance of Stress Concentrations

Several mechanisms have been postulated to account for stress concentrations high enough to nucleate cavities at certain sites along the grain boundaries (Fig. 8.5): pile-up of dislocations, intergranular sliding, in particular at triple nodes, twins or precipitates. It is important to take into account the ease with which these stresses can relax in the presence of creep.

Fig. 8.5 Sketch showing various sources of stress concentration at *grain boundaries* during creep deformation



This is a complex problem which involves transport of material along the boundaries (see Coble creep, Volume I, Chap. 4, Sect. 4.3.2.2b) and creep deformation of the grains themselves. This problem was analysed in detail by Argon et al. (1980) and Argon (1982). A rough estimate of the distance over which stress relaxation by diffusion can take place can be determined using (4.76) in Chap. 4 of Volume I. This distance Λ is thus defined by:

$$\Lambda = \left[\frac{D_{GB} \delta_{GB} \sigma \Omega}{kT \dot{\epsilon}} \right]^{1/3} \quad (8.11)$$

Argon et al. (1980) and Argon (1982) concluded that, in the presence of particles located along the grain boundaries, the local stresses can be increased by a factor of 10–30 when the size, p , of the particles located along the grain boundaries is much larger than the distance Λ . Otherwise, when $p \ll \Lambda$, the local stresses will be largely reduced by diffusion.

8.2.4 Concluding Remarks

The above models for the nucleation of intergranular cavities emphasise the role played by normal stress and intergranular sliding. If the second factor dominates it seems reasonable to expect that in a uniaxial test the maximum damage will occur at grain boundaries oriented at 45 degrees to the tensile axis; but in practice most of the cavities are found to occur on boundaries at right angles to this axis. Two explanations for this have been suggested. The first explanation is related to the fact that both the normal stress and the intergranular sliding are important; the second explanation, equally plausible, is that the methods of observation that are available do not enable cavities to be examined at the nucleation stage, when the critical radius R^* is a few nanometres, but only at the later stage of cavity growth.

8.3 Growth of Creep Cavities

Many laws describing the growth of creep cavities have been established. For a review see Raj (1978), Bensussan (1984) and Riedel (1987). This is related to the diversity of mechanisms that can be involved:

1. viscoplastic deformation
2. diffusion
3. coupling between (1) and (2)
4. constrained growth by creep in the matrix

We discuss each of them, giving their physical bases and the relevant laws.

8.3.1 Growth Controlled by Viscoplastic Deformation

If the rate of deformation is great enough there will not be time for the diffusion effects to appear. Consequently the laws established for the plastic regime (see Chap. 4 of this volume) are easily transposed to apply here, with strains replaced by strain rates. Thus the Rice and Tracey (1969) law (see Eq. 4.10) becomes:

$$\frac{1}{R} \frac{dR}{dt} = \alpha \dot{\varepsilon}_{eq} \exp \left(\frac{3}{2} \frac{\sigma_m}{\sigma_{eq}} \right) \quad (8.12)$$

where $\dot{\varepsilon}_{eq}$ is the von Mises equivalent strain rate, R is the radius of cavities which are assumed to be spherical and the coefficient α depends on stress triaxiality (Huang 1991).

The expression proposed by Budiansky et al. (1982) established for a Norton-Hoff viscosity law can also be applied (Eqs. 4.13 and 4.14).

8.3.2 Growth Controlled by Diffusion

This growth mode is specific to creep. It is more likely to occur at low creep rates than at high ones, implying that it is more likely to apply to real components tested for long periods at low stress than in laboratory creep tests which are more often performed under accelerated conditions.

This growth mode was first studied by Hull and Rimmer (1959) for a spherical cavity. Their treatment, though approximate, is presented below. We start with their model in which it is assumed that spherical or ellipsoidal cavities are in equilibrium; afterwards we will consider the case in which the cavities have the shape of a crack.

- the concentration of vacancies is in thermal equilibrium with the local normal stress acting across the grain boundaries, which are assumed to be ideal sources of vacancies.
- the normal stress across the grain boundaries varies from the equilibrium sintering stress $2\gamma_s\Omega/R$ at the cavity tips to a maximum value halfway between nearest neighbour cavities.
- diffusion of vacancies at the surface of the cavity is sufficiently rapid to keep the radius always at equilibrium value.
- the grains are rigid and their relative displacement is prevented.
- the cavities are assumed to be on regular arrays in the plane of the grain boundaries. Each cavity can be considered to be the centre of an isolated cylinder or sphere of diameter equal to the cavity spacing and of axis parallel to the far-field stress direction.

If g is the potential of the vacancies, L the distance between the centres of the cavities and ρ a radial coordinate with the origin at the centre of the void, one can write using the Einstein equation $J = (D_b / \Omega kT) \text{grad } g$:

$$J = \frac{D_{GB}}{\Omega kT} \text{grad } g \simeq 2 \frac{D_{GB}}{kTL} [\sigma_n - 2\gamma_s/R] \quad (8.13)$$

The vacancies diffuse in the cavity across a surface $2\pi R\delta_{GB}$ where δ_{GB} is the width of the boundary. The change in volume of the cavity is thus:

$$\frac{dv}{dt} = 2\pi R\delta_{GB} J \Omega = 4\pi R\delta_{GB} D_{GB} (\sigma_n - 2\gamma_s/R) \Omega / kTL \quad (8.14)$$

If the stress is sufficiently high, one can neglect the term $2\gamma_s/R$ and the final result is:

$$\frac{dR}{dt} = \frac{\delta_{GB} D_{GB}}{kT} \frac{\sigma_n \Omega}{L R} \quad (8.15)$$

8.3.2.2 Lenticular Cavities in Equilibrium

Creep cavities have not necessarily a spherical shape. The growth of lenticular cavities (Fig. 8.6) was also studied. This shape satisfies more easily the equilibrium of the surface energies.

In the stationary state each element of the boundary emits the same number of vacancies per unit time. If this rate is λ the conservation of mass gives:

$$\operatorname{div} |J| + \lambda = 0 \quad (8.16)$$

and the flux J , proportional to the gradient of the potential, g , is given by (8.13). This leads to a differential equation for the potential g with the following boundary conditions:

$$dg/d\rho = 0 \quad \text{for } \rho = L/2; \quad g = -2\gamma_s\Omega/R \quad \text{for } \rho = r \quad (8.17)$$

where $r = R \sin\psi$ is the radius of the cavity in the plane of the boundary. Integrating (8.16) it was shown (Riedel 1987) that for an axisymmetric cavity the potential g can be written as:

$$g = \frac{\lambda kT\Omega}{4D_{GB}\delta_{GB}} \left[\frac{L^2}{2} \log\left(\frac{r}{\rho}\right) - \rho^2 + r^2 \right] - \frac{2\gamma_s\Omega}{R} \quad (8.18)$$

with a corresponding normal stress on the plane of the boundary related to the potential by $g = -\sigma_n\Omega$.

These stresses, which vary along the length of the boundary, are in equilibrium with the applied stress σ_a , such as:

$$\pi L^2 \sigma_a / 4 = \int_r^{L/2} 2\pi \rho \sigma_n d\rho \quad (8.19)$$

from which we can calculate the rate λ of emission of vacancies.

The rate of change of the volume of the cavity is given by:

$$dv/dt = \pi \lambda \Omega (L^2/4 - R^2) \quad (8.20)$$

which, if $R_c = 2\gamma_s/\sigma_a$ is the critical radius for growth, becomes (Raj and Ashby 1975):

$$\frac{dv}{dt} = 2\pi\Omega \frac{D_{GB}\delta_{GB}}{kT} \left[1 - \left(\frac{2r}{L} \right)^2 \right] \left[\frac{(\sigma_a - 2\gamma_s/R) (1 - 4r^2/L^2)}{-3/4 + \log(L/2r) + (2r/L)^2 (1 - r^2/L^2)} \right] \quad (8.21)$$

Fig. 8.7 Schematic representation of a flat crack-like intergranular cavity

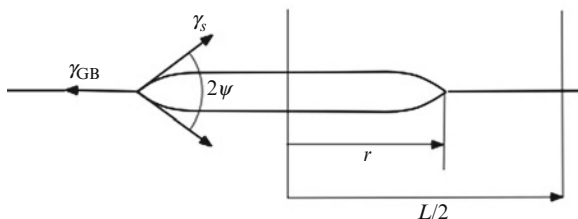


Table 8.1 Values of the ratio $\Delta = D_{GB}\delta_{GB}/D_s\delta_s$ for various metals

Metal	Ag	Cu	Fe α	Fe γ	Ni	Zn
Δ	570	0.08	3	2	28	13,000

8.3.2.3 Growth of Flat Cavities Controlled by Surface Diffusion

In deriving the above results we have assumed that the rate of diffusion of the atoms along the cavity surface is great enough to compensate for the flux of vacancies from the boundary, and therefore to keep the radius of curvature always at the equilibrium value. If this rate is too low the cavities will flatten and become as cracks (Fig. 8.7). The cavities will then adopt a non-equilibrium shape. If δ_s is the atomic diameter $\Omega^{1/3}$ and D_s the surface diffusion coefficient this will be the case when the ratio $\Delta = D_{GB}\delta_{GB}/D_s\delta_s \gg 1$.

Table 8.1 gives the values of Δ for various pure metals. It shows that in many materials the value of Δ is much larger than 1. This would suggest that in many cases creep cavities will grow with non-equilibrium shapes.

Chuang et al. (1979) calculated the rate of growth of such flat cavities controlled by surface diffusion. The equations of conservation of matter and continuity of flux determine the shape of the cavity for a given flux at its extremity. The result can be approximately written for an axi-symmetric void (Bensussan 1984) as:

$$\frac{dr}{dt} \simeq \frac{3}{8 \sin^3(\psi/2)} \frac{D_s \delta_s \Omega \gamma_s}{(L/2)^3 kT} \left(\frac{\sigma_a}{2\gamma_s/L} \right)^3 \left[1 - (2r/L)^2 \right]^{-3} \quad (8.22)$$

The angle ψ is determined by the equilibrium of the surface tension (Fig. 8.7). Integrating (8.22) shows that the time to fracture varies like σ_a^{-3} if fracture occurs for a critical volume of cavities.

8.3.3 Coupling Diffusion and Viscoplasticity

In the models which we have developed no effect is played by creep of the grains, since the atoms emanating from the cavities cover the entire grain boundaries and the grains are supposed to be rigid. We can consider a situation in which the

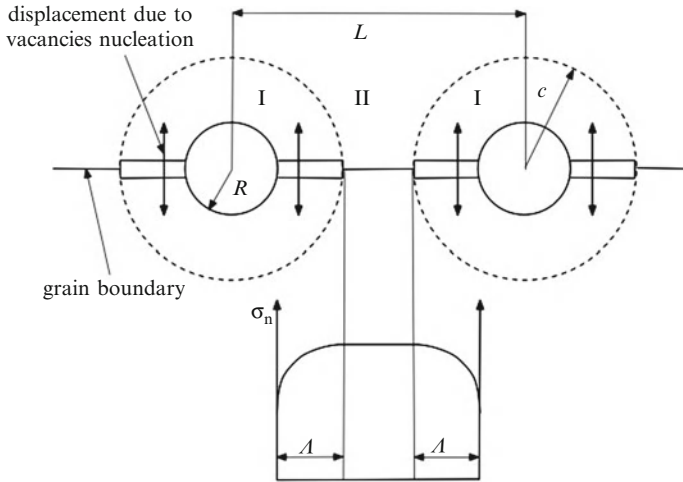


Fig. 8.8 Model for the growth of intergranular cavities with coupling between intergranular diffusion and transgranular viscoplasticity

vacancies necessary to ensure the growth of the cavities are assumed to come only from the immediate neighbourhood of these cavities. The diffusion distance will then be small and the effect will be to increase the rate of growth. This was the approach of several authors, notably Speight and Beere (1975). This requires that the deformation far from the cavities is due to creep of the grains themselves.

Figure 8.8 is a schematic presentation of the model that results from coupling between diffusion and viscoplasticity, with the cavities assumed spherical for simplicity.

In region I, close to the cavities, vacancies appear at a uniform rate and diffuse towards these, helping to relax the stresses; here the displacements are due mainly to the formation of vacancies. In contrast, in region II, distant from the cavities, the rate at which vacancies are created is zero and the displacements are due to transgranular creep. The size of the first region, $c - R$, is comparable to the characteristic distance introduced in (8.11).

A simple approximation is to equate the displacement rate $\lambda\Omega$ in region I to that due to creep $\simeq 2c\dot{\epsilon}$, in region II. Since we have:

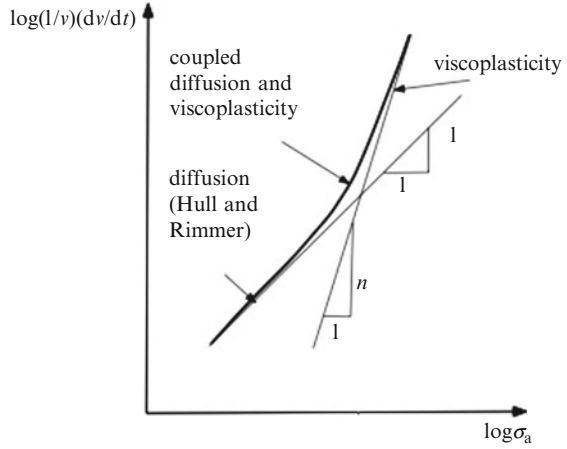
$$\frac{dv}{dt} \simeq \pi c^2 \lambda \Omega \quad (8.23)$$

we find, with $c = \Lambda$ given by (8.11):

$$\frac{dv}{dt} \simeq 2\pi \Lambda^3 \dot{\epsilon} = 2\pi D_{GB} \delta_{GB} \Omega \sigma_a / kT \quad (8.24)$$

This equation predicts a time to fracture inversely proportional to the applied stress σ_a .

Fig. 8.9 Variation of cavity growth rate with applied stress for (i) diffusional creep (Hull and Rimmer 1959); (ii) plastic regime; (iii) coupled diffusion and viscoplasticity



Chen and Argon (1981) gave a more exact treatment of the problem in which the reduction in diffusion distance was taken into account. They arrived at the following expressions:

1. for cavities in equilibrium ($\Delta < 1$)

$$\frac{1}{r} \frac{dr}{dt} = \frac{\dot{\epsilon}}{2h(\psi)} \left(\frac{\Delta}{r} \right)^3 \left\{ \log \left(\frac{r + \Delta}{r} \right) + \left(\frac{r}{r + \Delta} \right)^2 \left[1 - \frac{1}{4} \left(\frac{r}{r + \Delta} \right)^2 \right] - \frac{3}{4} \right\}^{-1} \quad (8.25)$$

where $h(\psi) = \left(\frac{1}{1 + \cos\psi} - \frac{\cos\psi}{2} \right) \frac{1}{\sin\psi}$

Typical numerical value for $h(\psi)$ is 0.61 for $\psi = 70$ degrees

2. for flat cavities ($\Delta \gg 1$)

$$\frac{1}{r} \frac{dr}{dt} = \left(\frac{1}{4 \sin\psi/2} \right)^{3/2} \left(\frac{D_{GB}\delta_{GB}}{D_s\delta_s} \right)^{1/2} \frac{D_{GB}\delta_{GB}\Omega\gamma_s}{\pi^4 kT} \left(\frac{\sigma}{\gamma_s/r} \right)^{3/2} \{ \}^{3/2} \quad (8.26)$$

where the last expression in braces is the same as in (8.25).

Figure 8.9 shows the rate of growth of cavities $(1/v)(dv/dt)$ as a function of the applied stress. Two extreme regimes are considered: (i) the pure diffusional regime, and (ii) the purely viscoplastic regime. This figure shows that, as would be expected, the first regime predominates at low stresses.

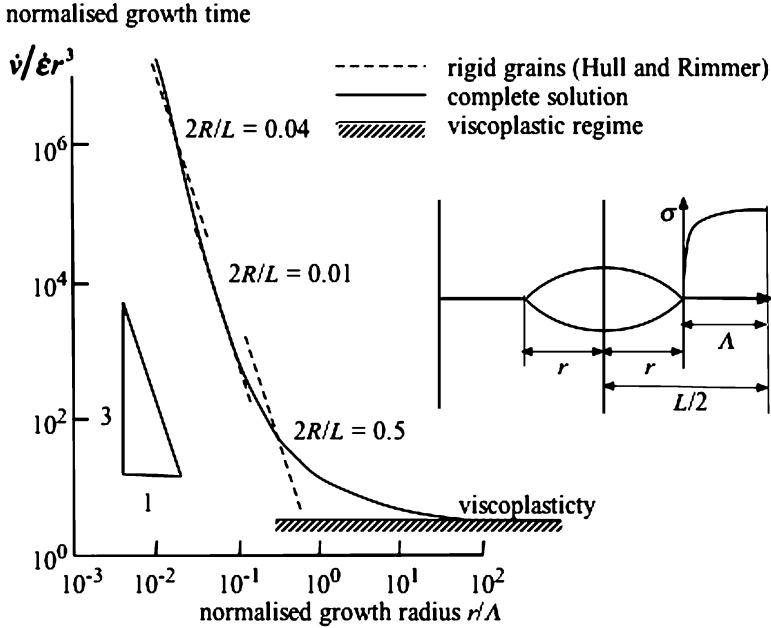


Fig. 8.10 Cavity growth rate normalised by $r^3(d\epsilon/dt)$ as a function of normalised radius r/Λ , where Λ is a diffusion length given by (8.11)

Figure 8.10 (Chen and Argon 1981) shows the normalised cavity growth rate, $(dv/dt)/\dot{\epsilon}r^3$ as a function of normalised cavity radius, r/Λ . In this figure, the dotted lines correspond to the simplified model (8.15) in which the grains are assumed to be rigid. These lines are drawn for different values of the ratio $2R/L$. It is observed that this approximation becomes closer as $2r/\Lambda$ decreases and for small values of the ratio r/L , as expected. The horizontal asymptote for large values of the ratio r/Λ corresponds to the purely viscoplastic regime.

8.3.4 Constrained Cavity Growth

We have assumed that intergranular cavitation is the same for all grain boundaries. However, practical experience has shown that this is not so, and that the situation represented by Fig. 8.11 is closer to what is often observed, especially in alloys with intergranular precipitates.

If cavity formation occurs on an isolated grain, oriented perpendicular to the applied stress, the load will be partly removed from the boundary and will be transferred to the undamaged neighbouring grains. Thus void growth will be partly

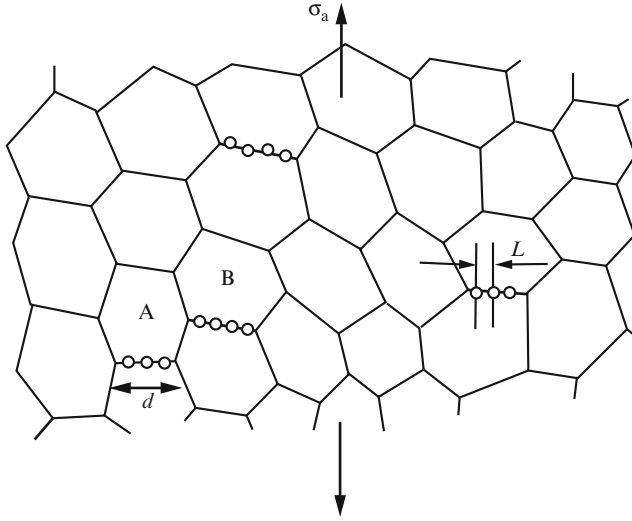


Fig. 8.11 Cavity growth in boundaries damage at A and B is retarded by creep deformation in the bulk of the matrix

controlled by the manner in which traction forces on the boundary can be maintained by the surrounding grains. This problem was addressed by several authors (see *e.g.* Dyson 1976, 1979; Argon 1982). In the limit, for high creep rates, that is high stresses, cavity growth is not retarded and will proceed at the rates previously established in the purely diffusional regime; whilst for low creep rates it can be partly controlled by viscoplastic relaxation of the stresses.

This problem was analysed by Rice (1981) by matching the thickening rate of a cavitating sheet along the grain boundary under an unknown traction, to the opening rate of a penny shaped crack of length d in the creeping surrounding, carrying the same unknown traction on the crack faces. The unknown traction determined from this compatibility match is then used in the cavity growth law to determine the actual cavity growth rate, constrained by the surrounding creeping solid, as:

$$\frac{dr}{dt} = \frac{1}{r^2 h(\psi)} \frac{(D_{GB} \delta_{GB} \Omega \sigma_a / kT)}{[(D_{GB} \delta_{GB} \Omega \sigma_a / kT) (16\alpha \dot{\epsilon} L^2 d) + Q (2r/L)]} \quad (8.27)$$

where α is a numerical constant slightly less than unity (0.9 for a stress exponent of 5) and $Q (2R/L)$ stands for:

$$Q (2R/L) = \log (L/2R) + (2R/L)^2 \left[1 - (R/L)^2 \right] - 3/4 \quad (8.28)$$

When the matrix strain rate is small, the load shedding relaxation is so strong that the first term in the denominator of (8.27) dominates over the second term, leading to an asymptotic growth rate of cavities, and a corresponding separation time of the facet as follows:

$$\frac{dr}{dt} = \frac{\alpha d}{16 h(\psi)} \left(\frac{L}{r} \right)^2 \dot{\epsilon} \quad (8.29)$$

and

$$t_f = \frac{8h(\psi)}{3\alpha} \frac{2L}{d} \left[1 - \left(\frac{r_i}{L} \right)^3 \right] \frac{1}{\dot{\epsilon}} \quad (8.30)$$

where r_i is the initial cavity radius. If it can be assumed that isolated facet fracture is synonymous with overall fracture, then (8.30) can be used to explain the experiments of a number of authors (see *e.g.* Dyson and Rodgers 1977).

8.3.5 Recapitulation of Results – Comparison with Observations

The results of the theoretical models for creep cavity growth are summarised in Table 8.2 (Cocks and Ashby 1980, 1982a). In this table the surface fraction of cavities along the grain boundaries is defined as $f_b = r^2/L^2$ where $2r$ is the hole diameter and L their spacing. The creep power law is written as $\dot{\epsilon} = \dot{\epsilon}_0(\sigma/\sigma_0)^{n_2}$.

Cocks and Ashby (1982a) calculated also the time and strain to fracture. The differential equations which were established for the various mechanisms of cavity growth were integrated. For simplicity they assumed that the voids all nucleated at time t_N (and strain ϵ_N). To evaluate the integrals a fracture criterion is required: a critical value f_b^c for f_b at the instant of failure. Clearly, the material is failed when $f_b = 1$. But in reality it will fail sooner than this because, as f_b approaches 1, the true stress on the remaining ligaments increases rapidly and they will fail by ductile tearing. Cocks and Ashby (1982a, b) adopted the fracture criterion $f_b^c = 0.25$. Further details can be found in their publication.

In three dimensions, $\dot{\epsilon}$ must be replaced by the equivalent von Mises strain rate. If the law of steady-state creep under uniaxial stress has the form:

$$\dot{\epsilon} = \dot{\epsilon}_0(\sigma/\mu)^m \quad (8.31)$$

Table 8.2 Approximation for the rate of growth of the surface fraction f_b of the cavities

Growth controlled by	Growth law
Viscoplastic deformation	$df_b/dt = 2\alpha f_b \dot{\epsilon} \exp(3\sigma_m/2\sigma_{eq})$
Grain boundary diffusion Spherical cavities in equilibrium	$df_b/dt = \frac{2}{f_b^{1/2} l_n(1/f_b)} \frac{D_{GB}\delta_{GB}\Omega\sigma}{kTL^3}$
Surface diffusion Flat cavities	$df_b/dt = \frac{f_b^{1/2}}{\sqrt{2}(1-f_b)^2} \frac{D_s\delta_s\Omega\sigma^3}{kTL\gamma_s^2}$
Coupled diffusion viscoplasticity	$df_b/dt = \frac{8}{3f_b^{1/2} \log(1/f_b)} \frac{D_{GB}\delta_{GB}\Omega\sigma_c}{kTL^3}$
with $\frac{\sigma_c}{\sigma_0} = \frac{\frac{1}{(1-d/2l)^{n_2}} \left(\frac{\sigma}{\sigma_0}\right)^{n_2}}{\left[\frac{1}{(1-d/2l)^{n_2}} - 1\right] \left(\frac{\sigma}{\sigma_0}\right)^{n_2-1} + \frac{16}{3d \log(1/f_b)} \frac{D_{GB}\delta_{GB}\Omega\sigma_0}{kTL^2\dot{\epsilon}_0}}$	

$2L$ distance between cavities, $R_c = 2\gamma_s/\sigma$, $\cos\psi = \gamma_b/2\gamma_s$, $2l$ spacing of clusters of cavities, d grain size, σ is the normal stress on a grain boundary

we have:

$$\dot{\epsilon}_{ij} = \frac{3}{2} (\dot{\epsilon}_0/\mu^m) \sigma_{eq}^{m-1} s_{ij} ; \quad \dot{\epsilon}_{eq} = \sqrt{\frac{2}{3}} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij} \quad (8.32)$$

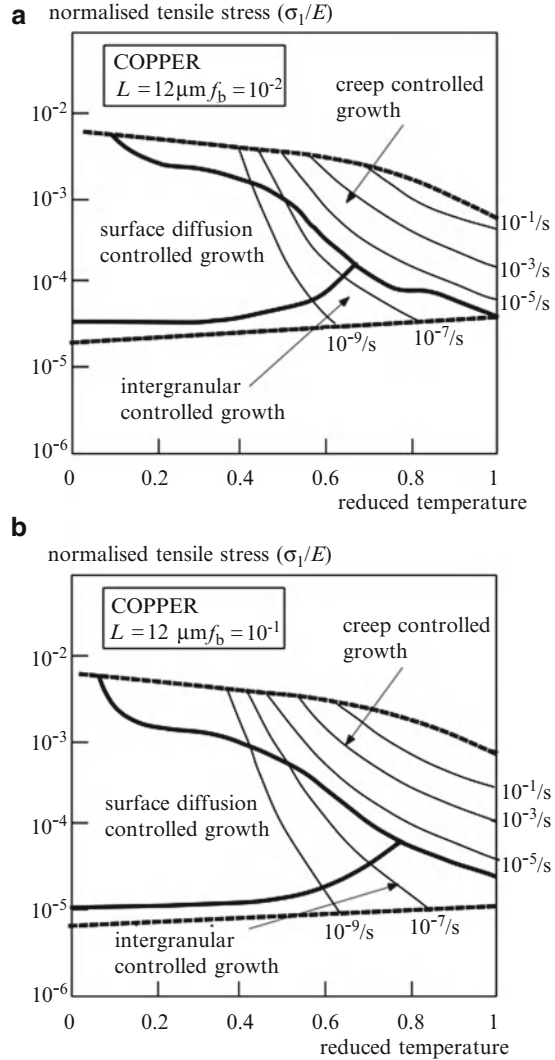
For a polycrystal under multiaxial loading it is usual to take for σ_a the maximum principal stress, σ_1 .

Wray (1969) first suggested the possibility of displaying failure modes in the form of a schematic diagram in strain rate-temperature space, and the concept was later improved and developed by Ashby and Raj (1975) to give a fracture map for copper, as shown in Fig. 8.12. These maps plot the normalised stress against the homologous temperature, from absolute zero to the melting point, and show fields in stress-temperature space within which a particular fracture mechanism is dominant. It is assumed that fracture occurs for a critical surface or volume fraction of cavities. The slight influence of the value adopted for the critical surface fraction of cavities f_b is observed when comparing Fig. 8.12a ($f_b = 10^{-2}$) and Fig. 8.12b ($f_b = 10^{-1}$).

Ashby (1977) described in detail the construction of these fracture maps, and the acceptance of this procedure is demonstrated by the inclusion of fracture maps for a number of different engineering materials (Ashby 1983). Example of a superalloy Nimonic 80A has been given in Fig. 8.1. Other examples of stainless steels are shown in Fig. 8.13a, b for 304 (18Cr-10Ni) and Fig. 8.14a, b for 316 (18Cr-12Ni-2.5Mo). For 316 stainless steel, see also Miller and Langdon (1979). These maps are still largely experimental because of the lack of precise physical input data necessary to apply the nucleation and the growth laws which have been presented previously.

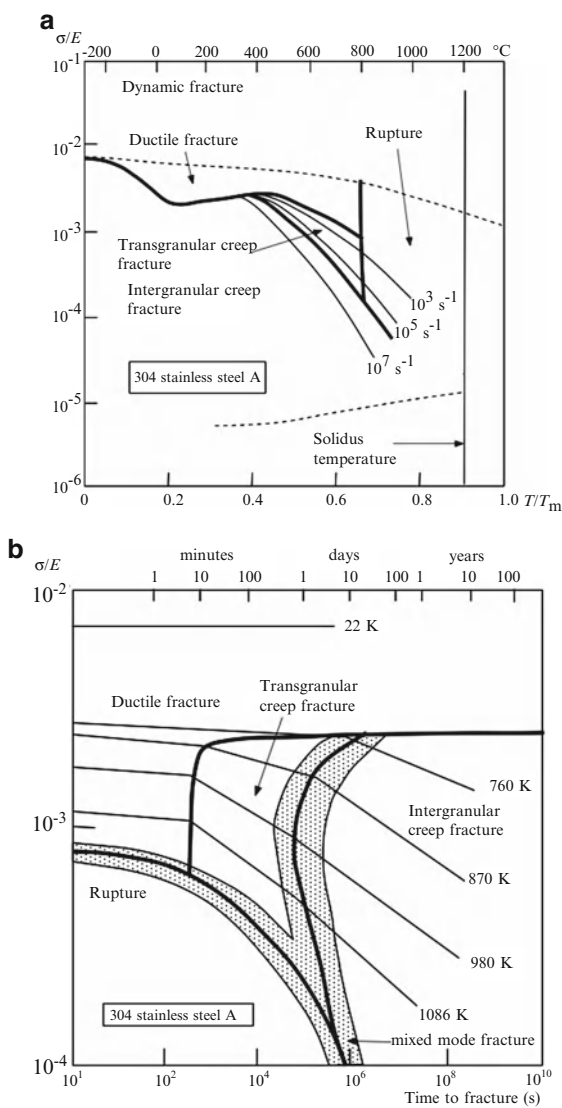
The comparison of theoretical predictions with experimental observations is more difficult in the regime of intergranular fracture than in that of low temperature

Fig. 8.12 Modes of creep failure for two initial values of the surface fraction f_b of intergranular cavities for copper with grain size $d = 12 \mu\text{m}$. Temperature is given as the reduced temperature T/T_m where T_m is the melting temperature. The charts give *lines* corresponding to equal creep rate (Adapted from Frost and Ashby 1982)



plastic failure. Direct measurements of intergranular damage are difficult. This explains why most comparisons are indirect, through the intermediary of the time to fracture. This requires strict control of the nucleation rate. This is the case of silver or copper in which the cavities can be controlled by implanting bubbles of water vapour before the creep test. Another method is to create cavities by means of a preliminary creep test at high stress, followed by creep under low stress. The diagrams of Fig. 8.12 give some results obtained with these methods.

Fig. 8.13 Fracture maps of low boron 304 type stainless steel. (a) fracture map of the first type: stress-temperature; (b) fracture map of the second type: stress-time to failure (Adapted from Fields et al. 1980)

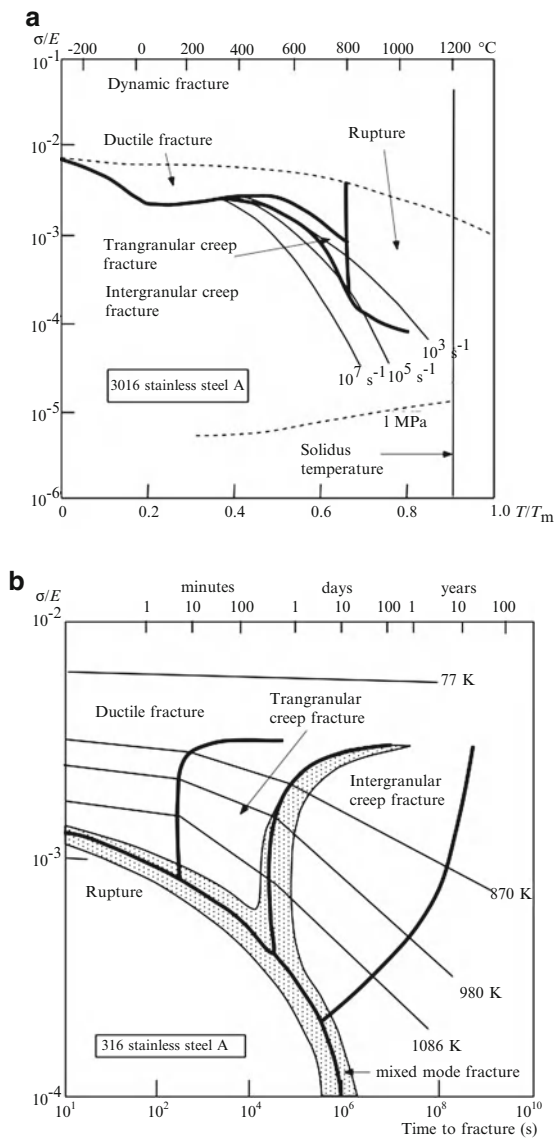


8.4 Phenomenological Approaches to Creep Damage

8.4.1 Introductory Remarks

As stated previously, damage measured in terms of, for example, the density and the average size of intergranular cavities, consists in general of two components, nucleation and growth. Formally we can write that the total volume fraction of

Fig. 8.14 Fracture maps of low boron 316 type stainless steel. (a) fracture map of the first type: stress-temperature; (b) fracture map of the second type: stress-time to failure (Adapted from Fields et al. 1980)

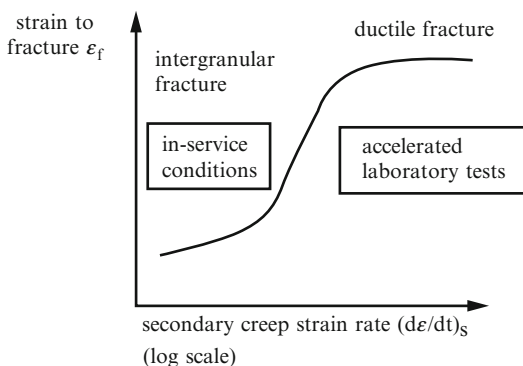


cavities is given by:

$$V(t) = \int_0^t \dot{N}(t') dt' \int_{t'}^t \frac{dv}{dt}(t', t'') dt'' \quad (8.33)$$

where dv/dt is the rate of growth of the cavities which were nucleated at the instant t' and $\dot{N}$ is the nucleation rate. In view of the variety of regimes that we have

Fig. 8.15 Creep ductility as a function of creep strain rate. A transition between transgranular ductile fracture and intergranular fracture is indicated



described, there is a great difficulty in deriving a general law governing the overall kinetics of damage established from a detailed knowledge of the micromechanisms.

This explains why most attempts to describe creep damage were made at the macroscopic and phenomenological levels, with damage measured most often in terms of time to failure, t_f .

These macroscopic approaches are described first. Then a number of measurements of intergranular creep damage are reviewed with the aim of positioning them with respect to the macroscopic approach.

8.4.2 Monkman-Grant Law

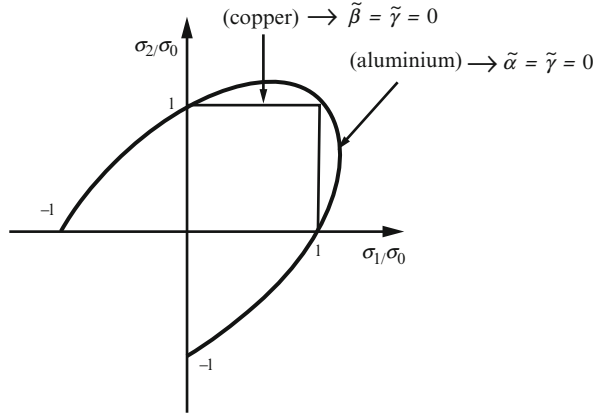
In many materials the data for creep fracture under uniaxial tension follow a simple law, established by Monkman and Grant¹ (1956), which can be written:

$$\dot{\epsilon}_s^q t_f = C_{MG} \quad (8.34)$$

where $\dot{\epsilon}_s$ is the steady-state creep rate, t_f the time to failure and C_{MG} a constant varying little with temperature and applied stress. The exponent q does not differ greatly from 1, lying between 0.7 and 2. A value of 1 indicates that to a first approximation the creep ductility of the material is constant, at least when the deformation in the tertiary creep stage can be neglected. In many cases, a knee on the $\epsilon_f(\dot{\epsilon}_s)$ curve is observed (Fig. 8.15), meaning that at low stationary creep strain rates $\dot{\epsilon}_s$ the creep ductility is reduced. This reduction in ϵ_f is associated with the appearance of intergranular damage.

¹Nicholas J. Grant (1916–2004) was professor at MIT.

Fig. 8.16 Curves of equal time to creep fracture for the extremes of equivalent stress criterion (aluminium) and maximum principal stress criterion (copper)



8.4.3 Time-to-Fracture Contours in Creep Under Multiaxial Loading

The elementary damage laws introduced in the above sections apply essentially to uniaxial loading. A number of authors attempted to extend these damage laws to multiaxial loading since the early 1980s (see *e.g.* Leckie and Hayhurst 1977). The existing data refer most often to plane stress conditions obtained with tubular specimens loaded in tension-torsion. If $\tilde{\sigma}_0$ is the uniaxial stress under which the time to fracture has a given value, t_f , a curve can be constructed in the (σ_1, σ_2) plane where σ_1 and σ_2 are the maximum principal stresses. The time to fracture t_f is the same on all points of this curve. The equation for this curve can be written as:

$$\Delta(\sigma_{ij}/\tilde{\sigma}_0) = 1, \quad \text{where} \quad \Delta = \tilde{\alpha} \frac{\sigma_1}{\tilde{\sigma}_0} + \tilde{\beta} \frac{\sigma_{eq}}{\tilde{\sigma}_0} + \tilde{\gamma} \frac{\sigma_1 + \sigma_2 + \sigma_3}{\tilde{\sigma}_0} \quad (8.35)$$

If $\tilde{\gamma}$ is small as compared to $\tilde{\alpha}$ and $\tilde{\beta}$ the curves will lie between two surfaces which in plasticity correspond respectively to the criteria of von Mises equivalent stress ($\tilde{\alpha} = 0$) and of maximum principal stress ($\tilde{\beta} = 0$). These boundaries are shown in Fig. 8.16 where the two extreme behaviours are seen in aluminium ($\tilde{\alpha} = \tilde{\gamma} = 0$) and in copper ($\tilde{\beta} = \tilde{\gamma} = 0$) respectively.

8.4.4 Introduction to Continuum Damage Mechanics (CDM)

The mechanics of viscoplastic damage is a macroscopic approach in which damage is expressed in terms of an internal variable, D , which can be a scalar or a tensor

(Kachanov² 1958; Rabotnov³ 1969; Leckie and Hayhurst 1977; Lemaitre and Chaboche 1985, 1990). In this section we give only an introduction to the scalar quantities used to provide a measure of creep damage. Further details can be found elsewhere (Lemaitre and Chaboche 1990).

The continuous damage parameter, D , has not necessarily a precise physical meaning such as the nucleation and growth of intergranular cavities. It is assumed that the damage parameter D and the creep strain rate $\dot{\epsilon}$ depend not only on the stress and temperature but also on the value of D itself. Thus in simple traction:

$$\dot{\epsilon} = f(\sigma, T, D) \quad \text{and} \quad \dot{D} = g(\sigma, T, D) \quad (8.36)$$

The lifetime under creep, the deformation at fracture and the shape of the creep curves $\epsilon(t)$ are all obtained by integrating these equations between appropriate limits.

Various forms were proposed for the functions f and g . Those due to Kachanov (1958) are usually written as follows:

– uniaxial stress:

$$\dot{\epsilon} = \dot{\epsilon}_0 (\sigma / \mu)^m (1 - D)^{-n} \quad \dot{D} = \dot{D}_0 (\sigma / \mu)^k (1 - D)^{-q} \quad (8.37)$$

– multiaxial stress:

$$\dot{\epsilon}_{ij} = \frac{3}{2} \frac{\dot{\epsilon}_0}{\mu^m} \sigma_{\text{eq}}^{m-1} s_{ij} (1 - D)^{-n} \quad \dot{D} = \dot{D}_0 \Delta^k (s_{ij} / \sigma_0) \left(\frac{\sigma_0}{\mu} \right)^k (1 - D)^{-q} \quad (8.38)$$

where Δ is defined in (8.35)

The time to fracture, t_f , is found integrating the appropriate equation for D with the boundary conditions $D = 0$ (no damage) at $t = 0$, $D = 1$ (fracture) at $t = t_f$. One obtains for uniaxial loading:

$$t_f = \frac{1}{\dot{D}_0 (q + 1) (\sigma / \mu)^k} \quad (8.39)$$

²Lazar Markovich Kachanov (1914–1993) was a Russian scientist.

³Yuri Nikolaevich Rabotnov (1914–1985) was a Russian scientist.

From t_f one can calculate the strain to failure:

$$\varepsilon_f = \frac{1}{q - n + 1} \frac{\dot{\varepsilon}_0}{\dot{D}_0} \left(\frac{\sigma}{\mu} \right)^{m-k} \quad (8.40)$$

Very often $m < k$, so that the creep ductility is a decreasing function of the applied stress. The Monkman-Grant law holds in its simplest form (8.34) when $m = k$.

8.4.5 Continuum Damage Mechanics (CDM) and Physical Measurements of Intergranular Damage

There is still very little detailed information on the physical intergranular damage associated with creep deformation, as already stated. However copper, a nickel-based alloy, Nimonic 80, and austenitic stainless steels were investigated in some detail. The results on these three types of materials are briefly presented, and we show in each case how the results of the phenomenological analysis of damage can be compared with those obtained from metallographical observations.

8.4.5.1 Results for Copper

A detailed study of this metal was made by Hanna and Greenwood (1982) using creep tests under uniaxial tension at 500°C. These authors measured the time to fracture and also the number, size and total volume of the cavities. If for a test at stress σ_0 the time to fracture is t_0 , the deformation is ε_0 , the number of cavities is N_0 and their average volume is V_0 , Hanna and Greenwood's results can be expressed as follows:

$$\begin{aligned} d(\varepsilon / \varepsilon_0) / dt &= (\sigma / \sigma_0)^5 / t_0 \\ d(N / N_0) / dt &= (\sigma / \sigma_0)^2 d(\varepsilon / \varepsilon_0) / dt \\ d(V / V_0) / dt &= (\sigma / \sigma_0) / t_0 \end{aligned} \quad (8.41)$$

Comparing these results with those derived from the theoretical models (8.14) we see that the kinetics of cavity growth follows essentially the law corresponding to intergranular diffusion. A comparison with the laws postulated from CDM (8.37) shows that, in this particular case, damage has a negligible effect on creep strain rate.

The damage parameter⁴ can be defined as the surface fraction of cavities:

$$D = NV^{2/3} / N_0 V_0^{2/3} \quad (8.42)$$

If the tests are performed under constant uniaxial tension it follows from (8.41) that:

$$D = (N / N_0) (V / V_0)^{2/3} = (t / t_0)^{5/3} (\sigma / \sigma_0)^{23/3} \quad (8.43)$$

and with the condition for fracture $D = 1$

$$(t_f / t_0) (\sigma / \sigma_0)^{23/5} = 1, \quad (\varepsilon_f / \varepsilon_0) = (\sigma / \sigma_0)^{2/5} \quad (8.44)$$

This last equation shows that creep ductility is a decreasing function of the applied stress and therefore does not follow the simple Monkman-Grant law (8.34).

Expression (8.43) can be written as a differential equation:

$$dD / dt = (5 / 3 t_0) (t / t_0)^{2/3} (\sigma / \sigma_0)^{23/3} = (5 / 3 t_0) (\sigma / \sigma_0)^{-23/5} D^{2/5} \quad (8.45)$$

which is a particular form of the Kachanov-Rabotnov law (8.37).

For the case of multiaxial loads, Hayhurst et al. (1983) gave a generalisation of (8.41), based on simple concepts suggested by the theories and models of the nucleation and growth of grain-boundary cavities. These equations are:

$$\begin{aligned} d(\varepsilon_{eq} / \varepsilon_0) / dt &= (\sigma_{eq} / \sigma_0)^5 / t_0 \\ d(N / N_0) / dt &= (\sigma_1 / \sigma_0)^2 d(\varepsilon_{eq} / \varepsilon_0) / dt \\ d(V / V_0) / dt &= (\sigma_1 / \sigma_0) / t_0 \quad \text{with} \quad D = (N / N_0) (V / V_0)^{2/3} \end{aligned} \quad (8.46)$$

Proceeding as before we find that:

- the time-to-fracture contour has the form

$$\Delta = (\sigma_1 / \sigma_0)^{8/23} (\sigma_{eq} / \sigma_0)^{15/23} = 1 \quad (8.47)$$

- the kinetics of damage accumulation is given by:

$$dD / dt = (2 / 3 t_0) \Delta^{23/5} (s_{ij} / \sigma_0) (1 - D)^{-1/2} \quad (8.48)$$

Figure 8.17 shows the corresponding time-to-fracture contour which lies between the equivalent stress and the maximum principal stress contours.

⁴This surface fraction was denoted previously f_b . We prefer to use here another notation, because in the CDM approach, damage defined by D has not necessarily a clear physical meaning.

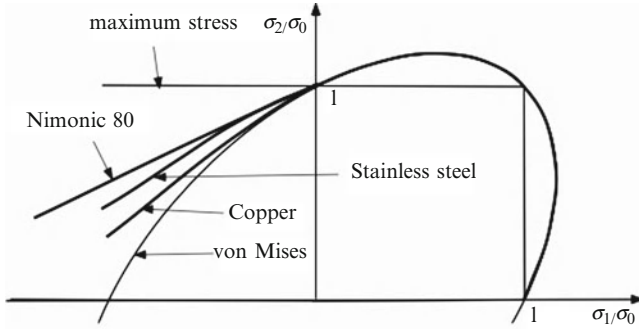


Fig. 8.17 Time-to-fracture contours for copper, austenitic stainless steel and nickel-based alloy Nimonic 80A compared with maximum stress and von Mises criteria

8.4.5.2 Results for Nimonic 80A

Dyson and McLean (1977) made a detailed study of this alloy which was briefly presented in Sect. 8.1 (see also Table 8.3). These authors performed creep tests at 700°C under uniaxial stress and also under tension-torsion. They measured the time to failure, the number of cavities per unit area of grain boundary, N , and the total volume of cavities, V . The following equations for the multiaxial case were obtained:

$$\begin{aligned} d(\varepsilon_{eq}/\varepsilon_0)/dt &= 3(\sigma_{eq}/\sigma_0)^4 (V/V_0)^{4/9} / t_0 \\ d(N/N_0)/dt &= 1/2(\sigma_1/\sigma_{eq})^2 (\varepsilon_{eq}/\varepsilon_0)^{-1/2} d(\varepsilon_{eq}/\varepsilon_0)/dt \\ d(V/V_0)/dt &= (\sigma_1/\sigma_{eq})^{0.7} d(\varepsilon_{eq}/\varepsilon_0)/dt \end{aligned} \quad (8.49)$$

The first equation expresses the coupling between damage and creep rate and is therefore of the same type as the first equation proposed by the CDM approach (8.37). The third equation does not seem to match any of those established for cavity growth. However in uniaxial loading this equation reduces to the form associated with constrained cavity growth since then dV/dt is directly proportional to $d\varepsilon/dt$.

The criterion for fracture adopted by Dyson and McLean is:

$$NV/N_0V_0 = \sigma_0/\sigma_1 \quad (8.50)$$

The same procedure as before gives the form of the time-to-fracture contour as:

$$(\sigma_1/\sigma_0)^{1/9} \Gamma(s_{ij}/\sigma_0) = 1 \quad \text{where} \quad \Gamma = \left(\sigma_1^{1.8} \sigma_{eq}^{2.2} / \sigma_0 \right)^{1/4} \quad (8.51)$$

This contour is plotted in Fig. 8.17. For proportional loading the ratio σ_1/σ_2 is constant. Equation (8.49) can then be integrated. This gives:

$$\begin{aligned} d(\varepsilon_{eq}/\varepsilon_0)/dt &= (3/t_0) (\sigma_{eq}/\sigma_0)^4 D^{4/9} \\ dD/dt &= (9/2t_0) \Gamma^4 D^{7/9} \end{aligned} \quad (8.52)$$

which are again particular forms of the Kachanov-Rabotnov relations.

A criticism that could be made against this treatment is that according to the first of these equations (8.49) the creep rate is zero at $t=0$ (when $D=0$), which is not in agreement with observation. In fact the expression of the creep rate in the Dyson-McLean treatment takes into account only the tertiary creep. The strain corresponding to the steady-state creep should be added when calculating the total strain.

8.4.5.3 Results for Austenitic Stainless Steels

Austenitic stainless steels are extensively used because of their good corrosion resistance and also because of their high creep resistance at elevated temperature (500–600°C). The most widely used stainless steels are those of the 316 series (18%Cr-12%Ni-2.50%Mo). These materials are prone to intergranular damage as shown in the fracture map of Fig. 8.14 and as illustrated in Fig. 8.18 (Chabaud-Reytier et al. 2001; Poquillon et al. 2001).

Since the 1980s the development of a correlation between the degree of intergranular damage and creep or creep-fatigue life has been sought (see *e.g.* Levaillant and Pineau 1982; Levaillant et al. 1988; Yoshida et al. 1986, 1990; Argence and Pineau 1996). Intergranular damage D^i was defined as:

$$D^i = L_f/L_t \quad (8.53)$$

where L_f is the cumulative length of cavitated grain boundaries per unit area and L_t is the total grain boundary length per unit area. An incremental multiaxial expression was developed having the form:

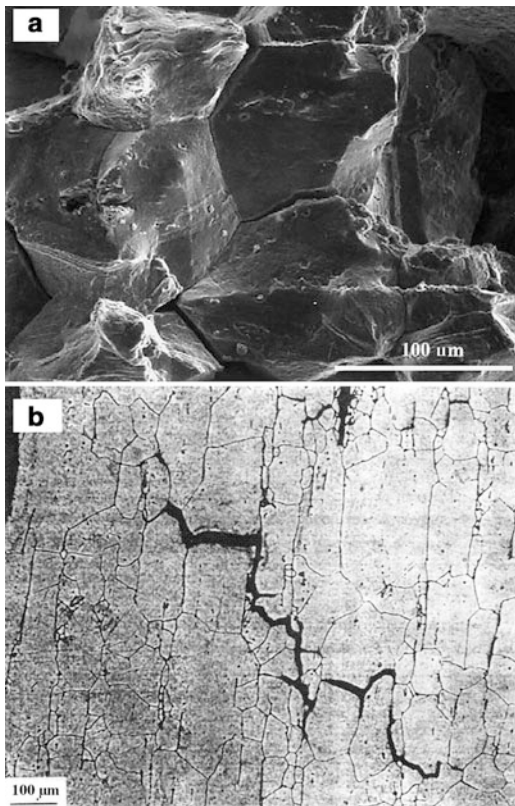
$$dD^i = B\sigma_1^\gamma (\varepsilon_{eq})^\delta d\varepsilon_{eq} \quad (8.54)$$

where B varies only with temperature for a given composition, γ is a constant ($2 < \gamma < 4$) and $\delta = 0.50$.

If we assume that in a uniaxial test fracture occurs when the metallographical damage D^i reaches a critical value, D_c^i , integration of (8.54) leads to a simple expression between the strain to failure, ε_f , and the applied stress, σ , of the form:

$$\varepsilon_f \sigma^8 = \text{const} \quad (\text{with } \gamma = 4) \quad (8.55)$$

Fig. 8.18 (a) Fracture surface of a CT type specimen tested at elevated temperature and low strain rate (AISI 321 steel); (b) longitudinal section of a notched specimen tested at 600°C for 475 h (AISI 321 steel) (Chabaud-Reytier et al. 2001)



In reality, in steels of this type, the development of creep ductility follows a more complex path due to ageing effect and to the precipitation of carbides at grain boundaries.

If we assume that the creep law is not affected by the damage, as it is the case for copper (8.41), and limit our analysis to steady-state creep, we have also the relation:

$$\dot{\varepsilon} = \dot{\varepsilon}_0 (\sigma / \mu)^m \quad \text{with} \quad m \simeq 8 \quad (8.56)$$

In the multiaxial case it can be shown that the kinetics of damage, in Kachanov and Rabotnov form, can be expressed as:

$$\frac{dD}{dt} = \frac{1}{2} \left(\frac{\sigma_{eq}}{\sigma_0} \right)^8 \left(\frac{\sigma_1}{\sigma_0} \right)^8 \frac{1}{D} \frac{1}{t_0} \quad (8.57)$$

The fracture condition $D = 1$ then leads to a time-to-fracture contour given by:

$$(\sigma_{eq} / \sigma_0) (\sigma_1 / \sigma_0) = 1 \quad (8.58)$$

This contour is also plotted in Fig. 8.17. It can be noted that this last expression differs little from that given by Dyson and McLean treatment (8.51).

It should be added that the time fraction rule for the estimation of creep damage is widely used in design codes such as ASME III (2001) and RCC-MR (2002). The time fraction rule for creep damage is:

$$D = \int_0^t dt / t_f(\sigma, T) \quad (8.59)$$

For the time fraction approach the creep damage under compressive (and multiaxial) states of stresses is calculated using a multiaxial rupture stress σ_{rup} . For Type 304 (18%Cr-10%Ni) and Type 316 (18Cr-12Ni-2.5Mo) stainless steels, ASME III (2001) uses (Spindler 2004, 2007):

$$\sigma_{rup} = \sigma_{eq} \left[0.24 \left(\frac{3\sigma_m}{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)^{1/2}} - 1 \right) \right] \quad (8.60)$$

RCC-MR (2002) uses:

$$\sigma_{rup} = 0.133 \times 3\sigma_m + 0.867 \sigma_{eq} \quad (8.61)$$

In conclusion we have shown, with three specific examples, that there are strong similarities between two apparently very different approaches to the problem of damage: one, microscopic and based on physical measurements of intergranular damage, the other, macroscopic, postulating the existence of laws having a stated form. In practice, it often proves difficult to discriminate between the various laws of damage proposed, especially when the tests performed under biaxial loading, such as tension – torsion, are limited to values in the first quadrant of the stress plane. One of the main advantages of the physical approach lies in the fact that it can be used in non-destructive inspection for the assessment of the remaining life of a given component.⁶ Further details on microstructural parameters for creep damage quantification can be found elsewhere (Eggeler 1991; Cane 1982; Needham and Cane 1983; Riedel 1987).

⁶ Industry has at its disposal replica techniques to detect superficial microcracks as well as devices to sample but a few tenth of a millimetre of the surface of a component, sufficient to evaluate the microstructural damage.

8.5 Creep-Fatigue-Oxidation Damage

8.5.1 Introduction and Overview

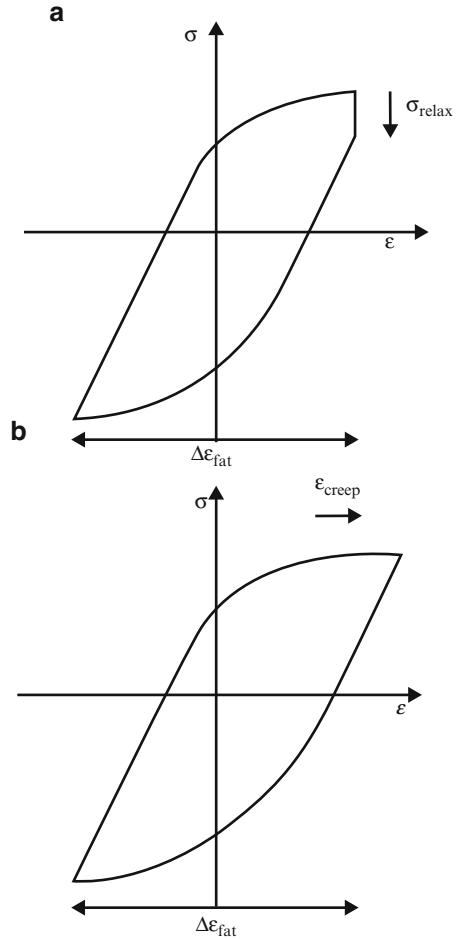
In the power generation industry (nuclear industry, coal-fired and/or oil-fired plants), any increase in working temperature leads to a decrease in fuel consumption, pollution and operating costs. In the jet engine industry, increased operating temperatures lead to improved performances. However, as operating temperatures are increased, additional problems to those encountered at lower temperatures arise. Regardless of its type, all engines have moving parts that experience variable loading during each operating cycle. In general, loading above a certain level causes important damage, which involves an interaction between creep, fatigue and oxidation. With continued operation (*i.e.* cyclic loading) damage accumulates and eventually leads to the fracture of the component.

In addition to the damaging effects of reversed plasticity (see Chap. 6 in this volume) creep and environmental effects (usually in the form of oxidation) must be considered at high temperatures. Furthermore, interactions among these damage mechanisms must be measured as well as the stability of the underlying microstructure. It should be emphasised that creep damage is essentially a bulk damage mechanism, while fatigue and oxidation are surface damage mechanisms.

In this section the emphasis is laid on physically-based models of interaction. Most creep-fatigue tests are performed under strain control conditions as indicated in Fig. 8.19 where a hold time (either in tension or in compression) is superimposed to the fatigue cycle. The corresponding regime is that of the low-cycle fatigue (see Chap. 6 in this volume). Two types of tests are shown in Fig. 8.19. In stress relaxation fatigue tests (Fig. 8.19a), a hold period is applied at the maximum strain (tension) or the minimum strain (compression). During this hold period, stress relaxation occurs by a mechanism of viscous deformation processes in which the elastic strain is partly converted into creep strain. In an actual creep-fatigue test a hold time is imposed at the maximum (or minimum stress) which is held constant, until the desired level of maximum (minimum) strain is attained, see Fig. 8.19b. During this hold time, a creep strain, ε_c , occurs either in tension or in compression. These creep-fatigue tests are more revealing of the behaviour, because an increasing amount of creep strain is applied on a cycle basis. These tests lead to the development of a mean stress in tension for a compressive hold and a compressive mean stress for a tensile hold.

A very schematic yet attractive method for representing the various regimes of fatigue, creep and creep/fatigue interactions was put forward by Hales (1980) and is shown in Fig. 8.20. More details on these interactions have been given by Antolovich and Pineau (2010).

Fig. 8.19 Schematic stress-strain hysteresis loops. (a) fatigue-stress-relaxation and (b) creep-fatigue (Fournier et al. 2006)



In representing the cycles or time to failure as a function of the applied strain (or stress) four regions may be identified, as indicated in Fig. 8.20.

- In the first region $N_{\text{if}} < N_f < N_{\text{ic}}$, the cycles to initiate a fatigue crack N_{if} , the cycles to failure, N_f , and the cycles to initiate a creep crack, N_{ic} , respectively. The time to creep failure, t_f , can simply be converted into a number of cycles N_{ic} when the frequency, ν , is known, by writing:

$$t_f = N_{\text{ic}} / \nu \quad (8.62)$$

In this region, fatigue failure occurs far earlier than creep damage develops and failure occurs essentially by transgranular fatigue, as described in Chap. 6 in this volume. This does not mean that in this regime creep has no effect since,

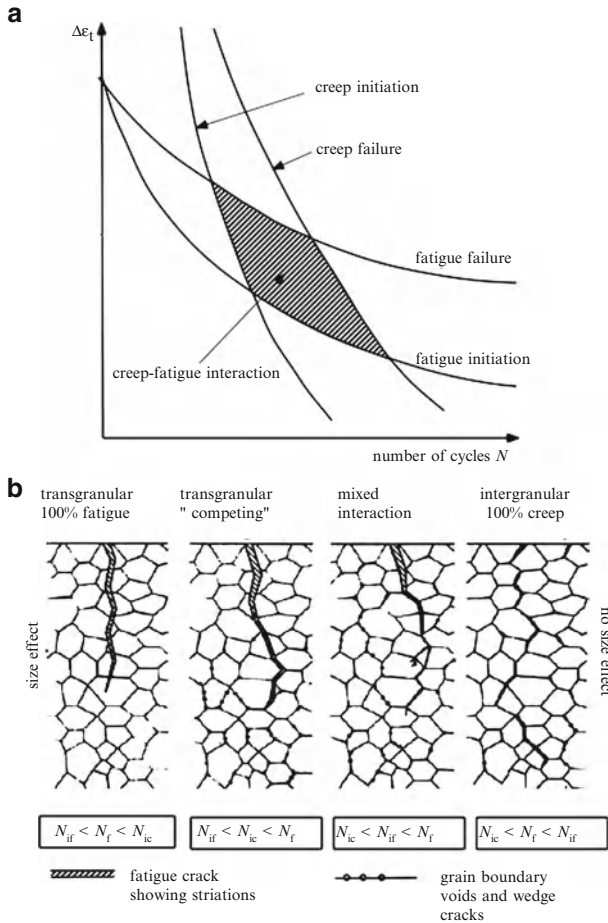


Fig. 8.20 (a) Schematic representation of creep-fatigue interactions; (b) physical interactions between crack growth and intergranular damage in creep-fatigue loading (Adapted from Hales 1980)

as indicated in Fig. 8.19, the application of a hold time (stress relaxation or constant stress) produces an increase of the plastic + viscoplastic strain range. This situation is met in 9–12%Cr steels which are presented below.

- In the second region, $N_{if} < N_{ic} < N_f$, there is a competition between intergranular cavitation and propagation of a fatigue crack. Basically a fatigue crack initiates first, but during its propagation creep cavities begin to appear. Damage generally remains transgranular.
- In the third region $N_{ic} < N_{if} < N_f$, there exists a strong interaction between the creep and fatigue damage modes. The crack growth rate is accelerated by intergranular damage. Likewise, intergranular cavitation is itself accelerated by

the stress and strain concentrations around the crack tip. Damage is essentially intergranular. This situation will be illustrated below with the case of austenitic stainless steels.

- In the fourth region, $N_{ic} < N_f < N_{if}$, damage is primarily by creep (and oxidation) and is completely intergranular. This case corresponds to that of a number of nickel-based alloys presented below.

A large number of authors (Skelton 1993; Levaillant and Pineau 1982; Pineau 1982, 1983a, b) showed that the propagation of a short fatigue crack can be described by an equation of the form:

$$da/dN = Ba^Q \quad (8.63)$$

where Q is a constant that depends on the material and B is a function that depends on the strain amplitude (see *e.g.* Tomkins model (1968) in Chap. 6 in this volume, (6.24)). In Tomkins model $Q = 1$.

$$D = \frac{1}{N^*} = \frac{1}{N} + D_c \quad (8.64)$$

where N^* and N are life in creep fatigue and in pure fatigue, respectively and D_c is the creep damage per cycle.

Equation (8.62) corresponds to a simple linear damage accumulation law. Skelton (1993) showed that this law, when applied to the crack propagation rate in creep fatigue, leads to an acceleration of the crack growth rate, which can be written as:

$$(da/dN)^* = (da/dN) (1 + ND_c)^2 \quad (8.65a)$$

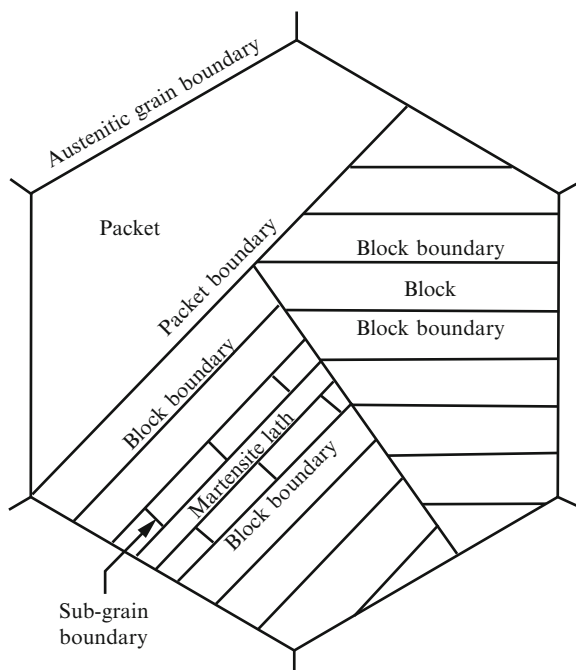
This expression is quite similar to the equation suggested by Levaillant and Pineau (1982); Pineau (1996a, b) which has the following form:

$$(da/dN)^* = (da/dN) / (1 - N^* D_c^i)^2 \quad (8.65b)$$

where D_c^i designates the amount of physical intergranular damage per cycle defined by (8.53).

This model can be applied in the second and the third regions of Fig. 8.20. It addresses the actual damage mode in austenitic stainless steels and gives satisfactory life prediction results. Other models were developed for superalloys, as shown below. There is no universally applicable model that describes damage processes in all alloys. Instead, in developing a lifespan prediction model, each alloy must be considered in terms of the precise damage mechanisms, which will depend on the details of the material, environment and applied loading profile.

Fig. 8.21 Sketch showing the microstructure of quenched and tempered steels with lath martensite morphology observed in 9–12% Cr alloys (Fournier 2007)



8.5.2 Creep-Fatigue-Oxidation Interactions in Three Types of Alloys

8.5.2.1 9-12Cr Steels: Importance of Work-Softening

Microstructure of 9–12%Cr Steels

The microstructure of these low carbon martensitic steels must be considered at several scales (Fig. 8.21) (Fournier et al. 2008a, b, c, 2009a, b): the prior austenite grains ($d \sim 10\text{--}60\ \mu\text{m}$), the packets ($\sim 5\text{--}10\ \mu\text{m}$) and the laths ($\sim$ a few microns). During quenching, subgrains ($\sim 500\ \text{nm}$) are formed within these laths (see Fig. 8.22a). All these steels contain carbides after ageing whose dimensions vary significantly with composition and heat treatment (chromium carbides of the M_{23}C_6 type along the grain boundaries and very fine carbo-nitrides (MX) distributed uniformly).

Mechanical Behaviour

In pure (without hold time) fatigue these steels exhibit a strong work-softening effect, as illustrated in Fig. 8.23. This softening becomes more pronounced as

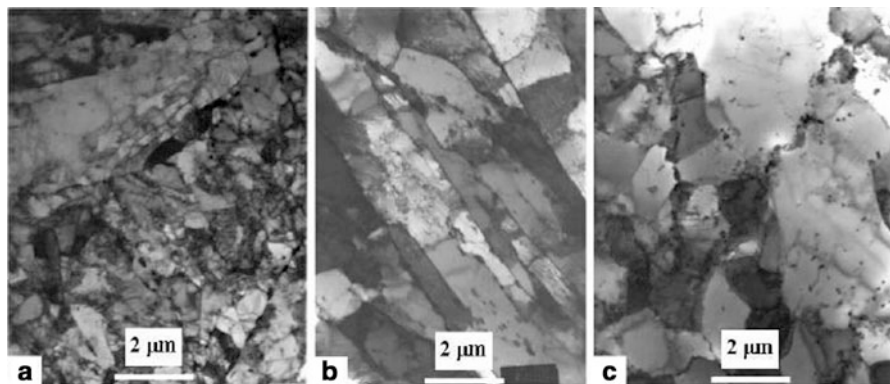


Fig. 8.22 9Cr-1Mo steel. Transmission electron microscopy observations. (a) initial (quenched and tempered) condition; (b) cyclically deformed in pure fatigue: ($\Delta\epsilon_{\text{fat}} = 0.70\%$), $T = 550^\circ\text{C}$; (c) tested with a hold time at 550°C . Coarsening of the microstructure from (a) to (c) is noted (Fournier et al. 2008a, b, c)

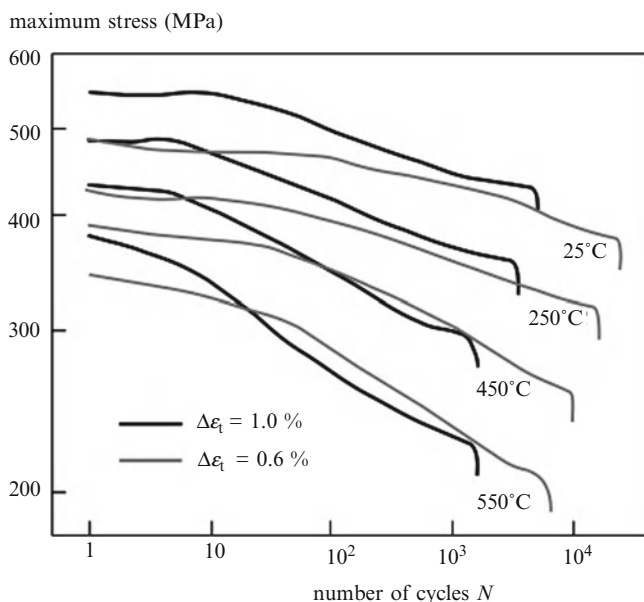


Fig. 8.23 Reduced activation Eurofer alloy. Low cycle fatigue tests at various temperatures (25, 250, 450, and 550°C) and at two strain amplitudes: 0.60 and 1%. Variation of the maximum tensile stress with the number of cycles (Adapted from Armas et al. 2004)

the temperature increases. Transmission electron microscopy (TEM) examinations reveal a significant modification of the dislocation microstructure, especially when a creep component is imposed on a pure fatigue test, as illustrated in Fig. 8.21b, c where it is observed that dislocations rearrange themselves into larger cells. It thus

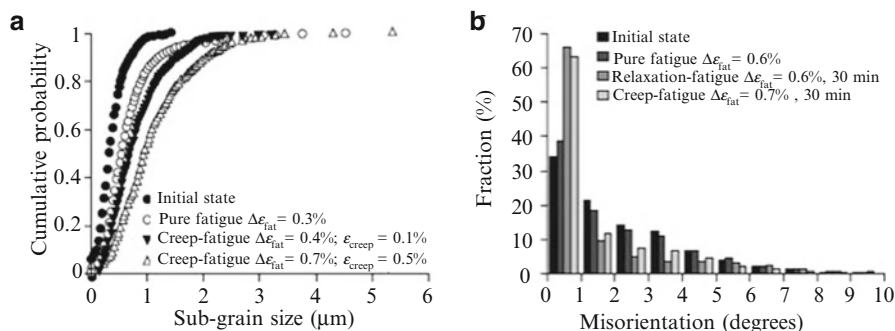


Fig. 8.24 9Cr-1Mo steel tested under various conditions at 550°C (a) distribution of subgrain sizes as measured by transmission electron microscopy; (b) distribution of average misorientations between blocks of laths. The test conditions are indicated (Fournier et al. 2006)

appears that the microstructure of 9–12%Cr steels is highly unstable under cyclic loading. Quantitative measurements of subgrain sizes and distribution of average misorientations between blocks of laths are shown in Fig. 8.24a, b. It is observed that the coarsening of the microstructure is more pronounced in creep-fatigue than in pure fatigue.

Physical Damage

The effect of tensile hold time during creep fatigue testing of P91 (9Cr-1Mo- V- Nb) steel is shown in Fig. 8.25 where it is observed that the fatigue life decreases with increasing creep strain, ϵ_c , at a given $\Delta\epsilon_{\text{fat}}$ (for the definition of these quantities, see Fig. 8.19).

A detailed study of damage in P91 steel tested at 550°C was carried out by Fournier et al. (2008a, b, c). No significant intergranular damage was observed in any case. However it was found that hold times (in tension or compression) profoundly affected crack initiation. Specimens tested in air always had an oxide on the surface whose thickness increased with increasing hold time. For thin oxide films, that is, for short hold times and low strain amplitudes, crack initiation was the same as in non-oxidising environments and occurred in a classical Stage I mode. For thicker oxide layers (*i.e.* longer hold times and higher strain levels) the consequence was brittle cracking in the oxide. These propagated as Stage II cracks. For the definition of Stages I and II, see Chap. 6 in this volume (Sect. 6.3.1.3).

This difference in the fatigue crack initiation mechanisms leads quite naturally to the notion that two regions of behaviour exist that are separated by a transition (Fig. 8.26). In region I, oxidation-related phenomena play a minor role. Nevertheless it was shown that the fatigue life in vacuum was about three times longer than in air. In region II, initiation takes place by brittle fracture of the oxide layer and is

Fig. 8.25 9Cr-1Mo steel. Influence of a tensile hold time on the creep-fatigue life at 550°C compared to results for continuous fatigue cycling. The creep strains are indicated (Fournier et al. 2008a)

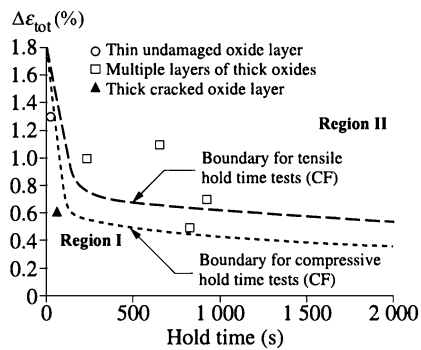
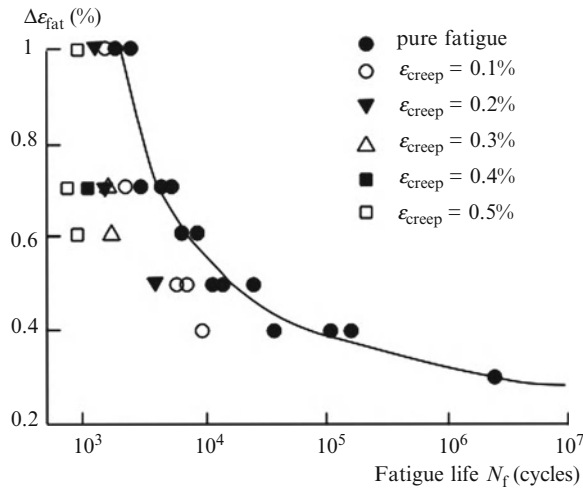


Fig. 8.26 9Cr-1Mo steel. Definition of two regions corresponding to different mechanisms of crack initiation. In region I slight effect of oxidation. In region II initiation is controlled by brittle fracture of the oxide layer. The total strain range is defined by $\Delta\epsilon_{tot} = \Delta\epsilon_{fat} + \Delta\epsilon_{creep}$. The boundary between the two regions varies with hold time (tension or compression) (Fournier et al. 2008a, b, c)

accelerated by internal oxidation at the crack tip. The transition between these two regions is defined in Fig. 8.26 for both tensile and compression hold times.

Damage Model and Life Prediction

The classical notion of dividing fatigue life into an initiation stage (N_i) and a propagation stage (N_p) was used:

$$N_f = N_i + N_p \tag{8.66}$$

Depending on whether the strain range and hold time place the part of interest in region I or II, the initiation phase can be schematically represented as follows:

- in region I, the model proposed by Mura (1994) and Tanaka and Mura (1981) is expressed as:

$$N_i = \frac{\alpha}{d} \frac{1}{\Delta \varepsilon_p^2} \quad (8.67)$$

where d is the grain size assumed in this case to be 20 μm , and $\Delta \varepsilon_p$ is the plastic strain range. The constant of proportionality α was calculated from an extensive data set taken at $\Delta \varepsilon_{\text{tot}} = 0.40\%$ to be 0.0385.

- in region II, N_i can be taken to be zero. Crack propagation in both cases was described by an equation proposed by Tomkins (1968) (see (6.24) in Chap. 6 in this volume):

$$\frac{da}{dN} = \frac{\pi^2}{8} \frac{\Delta \varepsilon_p \Delta \sigma^2}{(2R'_m)^2} a \left[1 + \frac{\pi^2}{8} \left(\frac{\Delta \sigma}{2R'_m} \right)^2 \right] \quad (8.68)$$

where a is the length of the microcracks and R'_m is the tensile strength of the cyclically deformed materials, which was taken to be 680 MPa. The number of cycles spent in crack propagation is obtained by integration of (8.68) between two limits a_0 (taken to be 50 μm) and a_f (taken as the radius of the specimen):

$$N_p = \int_{a_0}^{a_f} \frac{da}{(da/dN)} \quad (8.69)$$

This simple model, which incorporates the softening effect in (8.68) and the environmental effect (Fig. 8.26 and assumptions on crack initiation), is able to account for the experimental results. This conclusion shows that the improvement in creep-fatigue behaviour of 9–12%Cr steels requires improved microstructural stability.

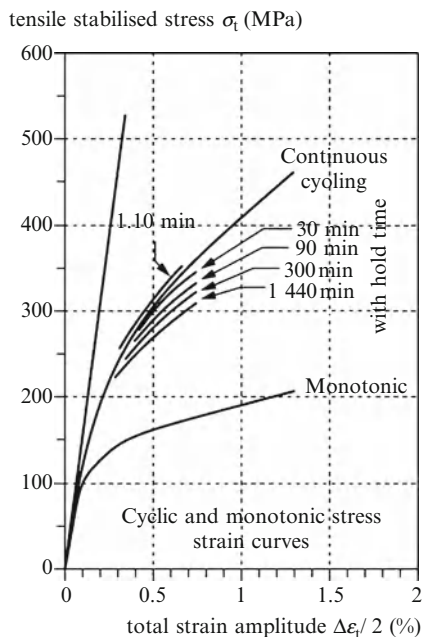
8.5.2.2 Austenitic Stainless Steels – Effect of Bulk Intergranular Damage

Mechanical Behaviour and Microstructure

Austenitic stainless steels exhibit a remarkable cyclic hardening behaviour in fatigue as seen in Fig. 8.27. This strong cyclic strain-hardening capacity persists when they are subjected to relaxation-fatigue, even for long times, as shown in this figure.

TEM observations showed that at high temperature dislocation cells are favoured by thermal activated slip. Examples of such structures in 316 (17Cr – 12Ni – 2.5Mo) steel are shown in Fig. 8.28 (Rezgui 1982). With a hold time at high temperature,

Fig. 8.27 Influence of tensile hold times on the cyclic behaviour of 316L (18Cr-12Ni-2,75Mo) stainless steel tested at 600°C (Adapted from Motto et al. 1982)



intergranular precipitation of chromium carbides of the $M_{23}C_6$ type is seen that is dependent on the level of carbon present (Fig. 8.29) (Rezgui 1982). Similarly cellular precipitation occurs at the triple points of the grains. This particular form of precipitation gives rise to very pronounced local heterogeneous deformation and grain boundary sliding which is one of the origins of intergranular damage in these steels (Antolovich and Pineau 2010).

Life and Microstructural Damage

The influence of frequency or strain rate on the fatigue life of a 304 (17Cr-10Ni) stainless steel is reported in Fig. 8.30 (Yamaguchi and Kanazawa 1980). As expected the fatigue life decreases with decreasing cyclic frequency and with increasing test temperature. A transition between transgranular and intergranular fracture is observed when the strain rate is sufficiently low. The appearance of intergranular fracture is favoured by large grain sizes.

The effects of a hold time in tension for tension-relaxation fatigue on the high temperature fatigue life of austenitic stainless steels have been the object of numerous studies. A detailed work was carried out at Ecole des Mines de Paris on five different compositions including alloys containing a controlled amount of carbon and nitrogen (Levaillant and Pineau 1982; Levaillant et al. 1988; Weiss and Pineau 1993; Argence and Pineau 1996). The results obtained for a 316L (low carbon) steel tested with long hold times are shown in Fig. 8.31. Isochronal

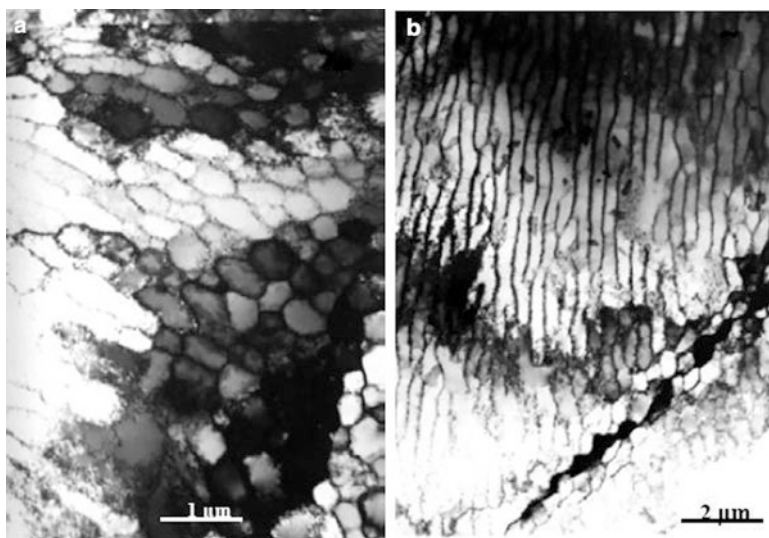


Fig. 8.28 TEM observations in 316 (18Cr-12Ni-2.75Mo) stainless steel tested in continuous fatigue at 600°C and $\Delta\varepsilon_p/2 = 0.86\%$. (a) grain containing dislocation cells; (b) grain containing dislocation veins (Rezgui 1982)

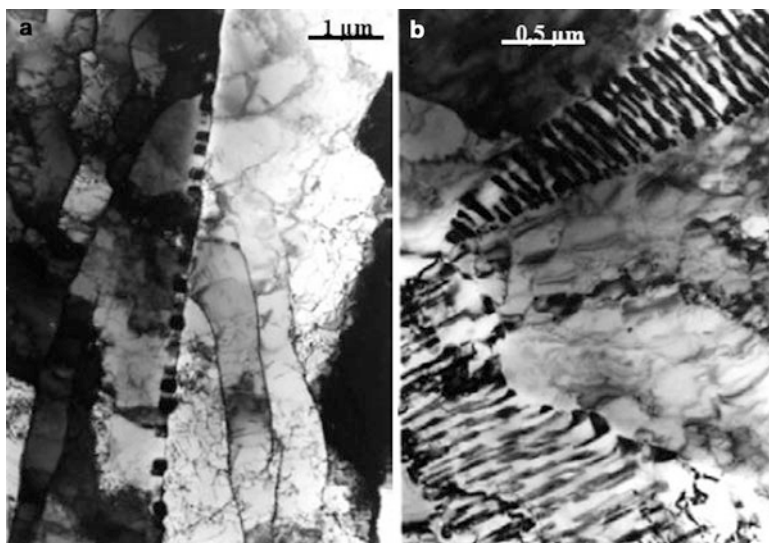


Fig. 8.29 TEM observations in 316 (18Cr-12Ni-2.75Mo) stainless steel tested in fatigue-stress relaxation with a tensile hold time, t_h , at 600°C. (a) $\Delta\varepsilon_t/2 = 2.3\%$, $t_h = 30$ min, time to failure $t_f = 35$ h; (b) intergranular carbide precipitates observed in a specimen tested with $\Delta\varepsilon_t/2 = 0.70\%$, $t_h = 100$ min, $t_f = 380$ h (Rezgui 1982)

Fig. 8.30 Low-cycle fatigue life as a function of strain rate for 304 (18Cr-10Ni) stainless steel tested at 600 and 700°C with $\Delta\epsilon_t = 1\%$ (Adapted from Yamaguchi and Kanazawa 1980)

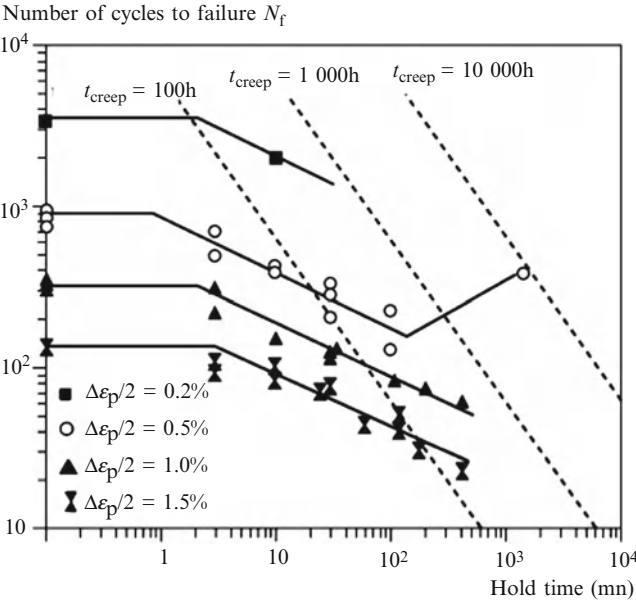
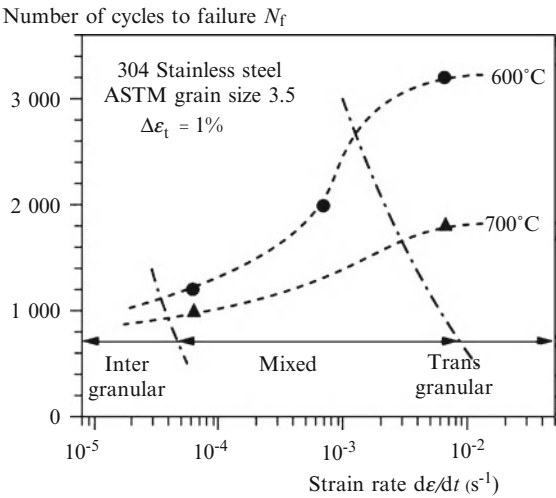


Fig. 8.31 316 (18Cr-12Ni-2.1Mo) stainless steel. Influence of a tensile hold time on fatigue life, N_f . Tests at 600°C with various levels of plastic strain amplitudes. The isochronal curves giving the time to failure, t_{creep} , as a function of the hold time and number of fatigue cycles are also shown (Argence and Pineau 1996)

curves for rupture are also drawn in this figure. It can be seen that for short hold times (several minutes) at 600°C the number of cycles to failure for a given strain amplitude varies only slightly, whereas at longer hold times the number of cycles

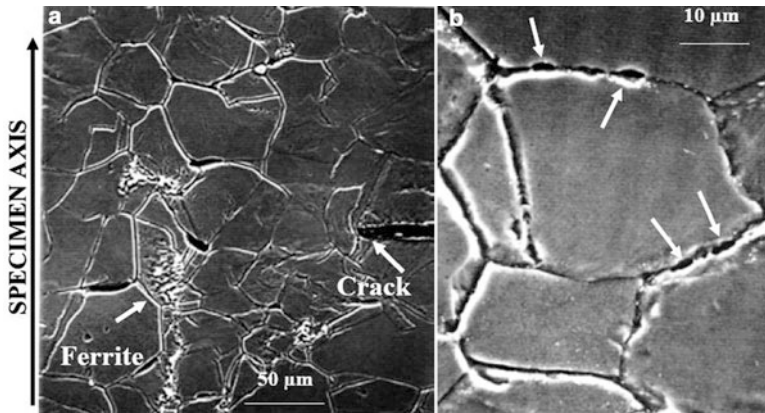


Fig. 8.32 Examples of intergranular damage observed in austenitic stainless steels tested under creep-fatigue conditions. (a) optical microscopy of a longitudinal section showing the presence of intergranular cracks (see arrows); (b) scanning electron micrograph showing intergranular cracks (see arrows) (Argence and Pineau 1996; Weiss and Pineau 1993)

to failure decreases regardless of the level of applied strain. It must be emphasised that the time to fracture increases, which indicates that failure does not only depend on time. A test with very long duration (about 1 year) with one cycle every 24 h shows that the number of cycles tends to increase. Similar behaviour was reported for other heats of 316L stainless steel (Levaillant et al. 1988; Wareing 1981).

The reduction in fatigue life is closely related to the appearance of intergranular fracture as illustrated in Fig. 8.32. This form of damage is not only present around the major crack but appears to be uniformly distributed throughout the sample. This observation indicates that, at least during the crack propagation phase, intergranular damage in creep-fatigue of austenitic stainless steels is not related to environmental effects, contrary to what is seen in superalloys. In the case of stainless steels, intergranular damage is due to creep, which occurs during hold times, and not only just in front of the crack tip but also in the bulk of the specimen.

Intergranular damage per cycle D_{ci} as defined by (8.53) was measured in various heats. The results are reported in Fig. 8.33 where it is observed that D_{ci} increases with hold time for given steel, as expected. The values of D_{ci} were also calculated using the creep damage law indicated earlier (8.54). The integration of this equation with $\gamma = 4$ along the entire stress relaxation curve yields

$$D_{ci} = \frac{2B}{\sqrt{E}} (\sigma_{t \max})^4 (\sigma_{\text{relax}})^{1/2} \quad (8.70)$$

where E is Young's modulus, $\sigma_{t \max}$ is the maximum value of the stress at the onset of stress relaxation and σ_{relax} is the relaxed stress (see Fig. 8.19) (Levaillant et al. 1988). This expression was applied to three variants of 316L steel and good results were obtained, as shown in Fig. 8.34.

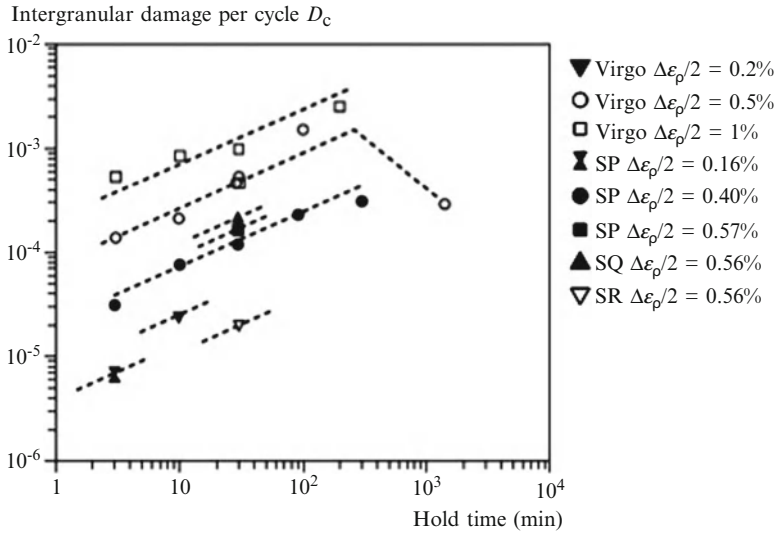


Fig. 8.33 Austenitic stainless steels. Influence of a tensile hold time on intergranular damage per cycle, D_c . Tests results from various compositions of 316(18Cr-12Ni-2.75Mo) stainless steels tested under various conditions are indicated in the figure

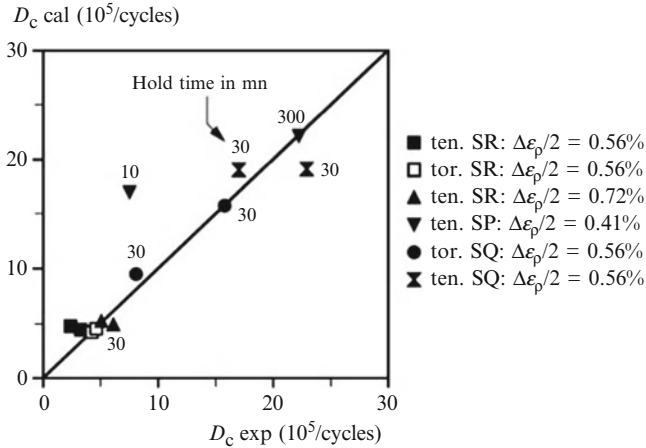
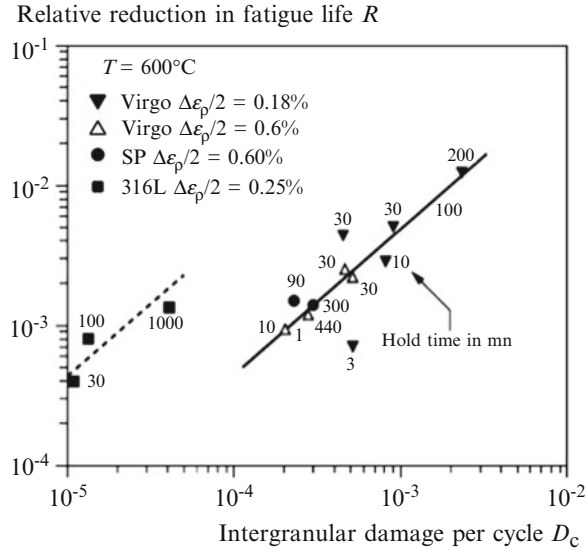


Fig. 8.34 Comparison between calculated and measured intergranular damage per cycle, D_c , for three variants of 316 (18Cr-12Ni-2.75Mo) stainless steels (Argence and Pineau 1996)

Modelling Creep-Fatigue Life

Measurements of intergranular damage D_c^i served as a basis for the development of a crack propagation model for creep fatigue (Levaillant et al. 1988; Pineau 1982, 1983a, b). It was assumed that the application of a hold time introduces oxidation

Fig. 8.35 Correlation between intergranular damage per cycle, D_c , and the relative reduction in life, R , for two different compositions of austenitic stainless steels tested under low-cycle fatigue with a hold time at 600°C (Argence and Pineau 1996)



and intergranular damage such that, to a first approximation, crack initiation occurs very rapidly and that the fatigue life is essentially spent in crack propagation. Moreover it was postulated that intergranular damage produced acceleration in crack propagation rate which could be described using (8.66). Integration of this equation leads to a simple expression giving the relative reduction of propagation life in creep-fatigue N_p^{CF} , compared to that in continuous fatigue (*i.e.* without hold time), N_p^{PF}

$$R = \frac{N_p^{PF} - N_f^{CF}}{N_p^{PF} N_f^{CF}} = k(D_c^i)^p \quad (8.71)$$

where N_f^{CF} is the lifespan in creep-fatigue cycling; k and p are parameters which have to be determined ($p \simeq 1$).

The correlation established between D_c^i and the reduction in lifespan, R , is shown in Fig. 8.35 for two steels of the 316L family. It is observed that the results are grouped around the same straight line, which allows determining the exponent p ($p \sim 0.923$), whatever the applied strain and the hold time. This justifies the use of (8.66) to describe the acceleration in crack growth rate linked to the presence of intergranular damage. Hales results (Hales 1980) are also shown in Fig. 8.35. This author used a definition for intergranular damage different from (8.64), which explains the shift in the two sets of data. The results corresponding to the longest hold times (1,440 min) are well-situated on the curve, in spite of the fact that an increase in lifespan is seen for the longest hold times (see Fig. 8.31).

The reduction in fatigue life of austenitic stainless steels tested under creep-fatigue conditions results from the acceleration of crack initiation and the acceleration of the propagation rate of fatigue microcracks, connected with the free surface, due to bulk creep intergranular damage.

8.5.2.3 Ni-Base Superalloys – Effect of Environment and Intergranular Damage

Microstructure

As mentioned in Appendix B of Volume I, superalloys (both Ni-base and Ni-Fe base) are used in the hot sections of jet engines and, although less frequently, also in the power generation industry. Microstructurally, the Ni-base alloys consist primarily of small coherent precipitates embedded in an FCC or γ matrix. The precipitates, denoted γ' , have an $L1_2$ crystal structure based on the Ni_3Al ordered compound as seen in table A1.3 of Appendix A in Volume I. Some alloys, such as In718, are strengthened by γ'' precipitates which have a DO_{22} crystal structure and are based on the Ni_3Nb ordered compound.

Representative compositions of largely used superalloys are shown in Table 8.3. Cast materials, in particular single crystals, as well as wrought materials, are used. Two review studies have been devoted to the high temperature fatigue behaviour of these alloys (Antolovich and Pineau 2010; Pineau and Antolovich 2009).

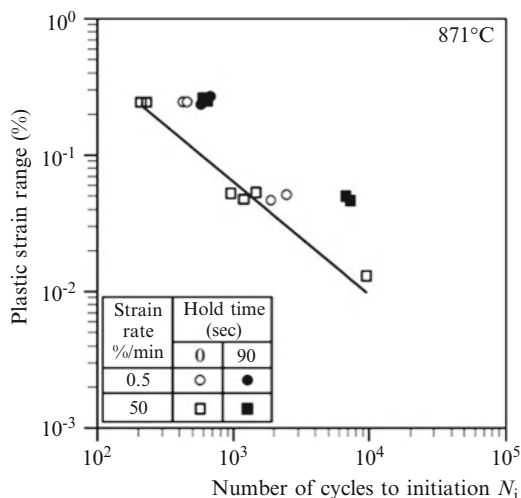
As the temperature is increased thermal activation enhances cross-slip in superalloys as in other FCC alloys (see Chap. 4 in Volume I). The formation of planar slip bands indicates that γ' (or γ'') shearing still occurs even at temperatures as large as 800°C. Increases in temperature may also lead to microstructural changes and hence to deformation mechanisms which are not normally observed at ambient temperatures. A full account of these mechanisms is given by Pineau and Antolovich (2009).

High Temperature Low-Cycle Fatigue

Low-cycle fatigue (LCF) limits life in turbine blades where temperatures may range from about 750°C at the platform to more than 1,200°C near the blade tips. It is also an important process for disks as well, especially near the rim in the region of attachment points. While most modern turbine blades are made of [001]-oriented single crystals, many engines are still in service which use polycrystalline cast alloys such as René 80. These alloys contain a γ matrix and a high volume fraction of γ' precipitates with a mismatch parameter of about 0.3%.

René 80 provides a prototype of high temperature LCF behaviour. Extensive studies were carried out at different temperatures, strain rates and hold times (Antolovich et al. 1981) on this alloy. In these studies continuous softening was

Fig. 8.36 Low cycle fatigue behaviour of René 80 (see Table 8.3 for the composition) tested at 871°C for the conditions indicated on the graph (Antolovich et al. 1981)



observed. Important results were: (a) when presented on the basis of plastic strain, the LCF life at 871°C was not reduced by tensile hold time and in fact was even increased (Fig. 8.36), and (b) there was continuous softening throughout the duration of the test. TEM studies provided a basis for understanding this anomalous behaviour. It was observed that: (a) the initial precipitate structure significantly coarsened and small precipitates were consumed by large ones (Fig. 8.37); (b) oxidation along boundaries was frequently seen (Fig. 8.38a); (c) internal cracking of grain normal to the stress axis was also observed (Fig. 8.38b).

Dynamic changes in the material structure have thus major effects on the fatigue life and cannot be ignored. In addition to the changes in precipitates, other changes also occur. In a study to show such effects using Waspaloy as a model material it was observed that carbides precipitate on slip bands, which also impinge upon oxidised boundaries to nucleate cavities as illustrated in Fig. 8.39 (Lerch et al. 1984). Basically carbides lock the dislocations in the slip bands thereby blocking relaxation of the high stresses at the tip of the slip band. These stresses in turn crack the oxidised grain boundaries causing crack initiation.

Co-base superalloy was investigated (Reuchet and Remy 1983a, b; Remy et al. 1987). The composition (wt%) of the alloy was 0.59C, 11Ni, 23.5Cr, 6.95W, 3.32Ta, 0.37Zr, 0.22Ti, 0.17Fe, 0.008B and the balance was cobalt. These alloys are widely used in the cast form as vanes in jet aircraft and, although not used in rotating components, are subjected to thermally-induced loading. They have a microstructure that includes a high fraction of carbides in the boundaries and the matrix, which undergo oxidation in use, especially along the grain boundaries. These studies demonstrated that in high-temperature fatigue, oxidation is a primary damage mode. Extensive measurements were made that demonstrated that the oxidation rate depended on the inelastic strain amplitude ($\Delta\varepsilon_{in}$) and that the depth of grain boundary carbides oxidation was proportional to $t^{1/4}$. A model was proposed in

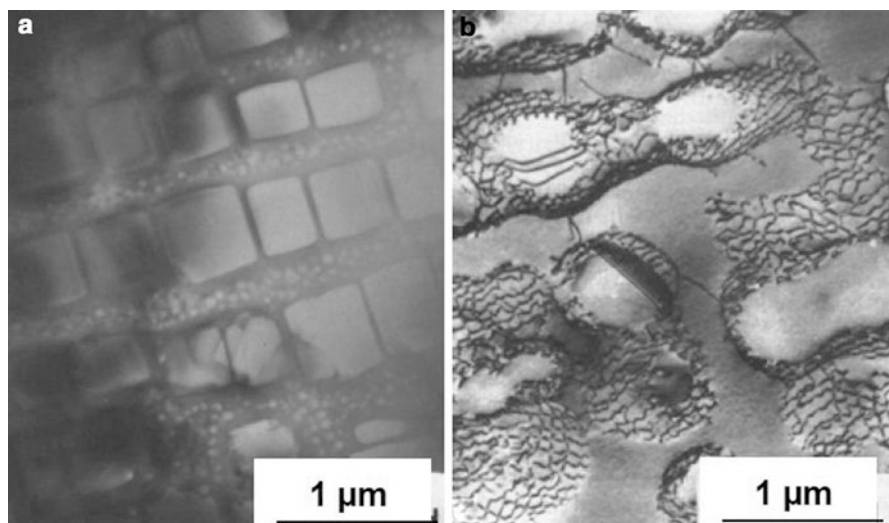


Fig. 8.37 Dark field transmission electron micrographs showing: (a) the initial microstructure of René 80 (see Table 8.3 for the composition) and (b) the microstructure after low-cycle fatigue testing at 981°C. Note the bimodal distribution of γ' precipitates in (a) and the interfacial dislocations in (b). The small precipitates in the initial condition have been dissolved during low-cycle fatigue testing at $\Delta\varepsilon_p = 0.196\%$ and $d\varepsilon/dt = 0.46\%/min$ (Antolovich et al. 1981)

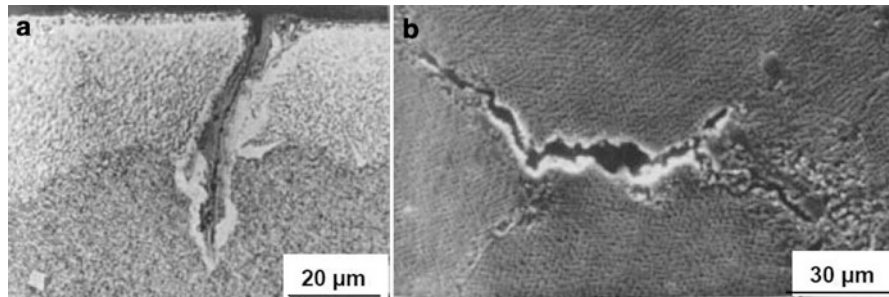


Fig. 8.38 René 80 (see Table 8.3). Oxidation during low cycle fatigue testing at 871 °C. (a) oxide spike observed at $\Delta\varepsilon_p = 0.25\%$, $d\varepsilon/dt = 0.50\%/min$; (b) boundary cracking normal to load axis (vertical), at $\Delta\varepsilon_p = 0.50\%$ and $d\varepsilon/dt = 0.50\%/min$ with a hold time of 90s at the maximum strain (Antolovich et al. 1981)

which damage was assumed to be a linear combination of the fatigue crack growth rate without oxidation and of the oxidation rate. While this formulation may seem to exclude interactions between both forms of damage, such is not the case because, as was clearly demonstrated, the carbide oxidation rate increased with inelastic strain. Tomkins (1968) model for plastic crack propagation was used. The equation of this model was:

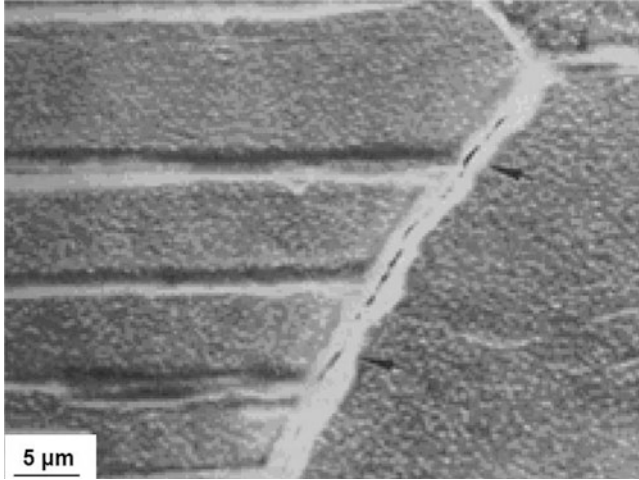


Fig. 8.39 Waspaloy (see composition in Table 8.3). Slip band-induced cracking of oxidised grain boundaries. Low-cycle fatigue at 527°C. Microcracks are indicated by *dark arrows* (Lerch et al. 1984)

$$\begin{aligned} \frac{da}{dN} = & 0.51 \Delta \varepsilon_p \left[\frac{1}{\cos(\pi \sigma / 2 R_m^c)} - 1 \right] a + (1 - f_c^*) \alpha_M^0 \left(1 + K_M \frac{\Delta \varepsilon_p}{2} \right) \Delta t^{1/2} \\ & + f_c^* \alpha_c^0 \exp \left(b \frac{\Delta \sigma}{2} \right) \Delta t^{1/4} \end{aligned} \quad (8.72)$$

where the first term is related to pure fatigue, the second term to matrix cracking due to oxidation and the third term to carbide oxidation along the boundaries. In this equation, f_c^* refers to the fraction of carbides along the crack path. The α coefficients are oxygen diffusion constants for the matrix α_M^0 and carbide α_c^0 and depend on temperature through an Arrhenius law. The cycle time is denoted Δt .

This model can be integrated by noting that the components of cracking for both matrix and boundary oxidation are independent of crack size. Thus (8.72) has the form:

$$\frac{da}{dN} = Ba + C \quad (8.73)$$

where C is the sum of the oxidation terms, $(da/dN)_{ox}$, and B is the crack length multiplier in (8.72). This equation may be integrated quite easily to give the cyclic life, N_f :

$$N_f = \frac{1}{B} \log \left[1 + \frac{Ba_f}{(da/dN)_{ox}} \right] \quad (8.74)$$

This equation has been used with some success in predicting the lifespan of Co-base alloys.

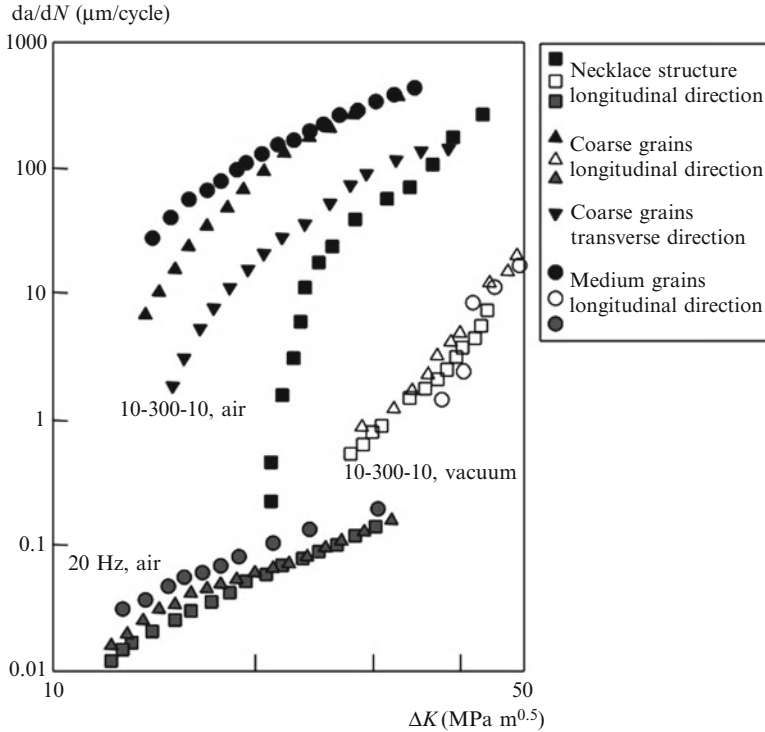


Fig. 8.40 Inco 718 (see composition in Table 8.3) tested at 650°C in air and under vacuum. Fatigue cycle 10 s – 300 s – 10 s. Three microstructures are investigated (*small and large grains and necklace microstructure*) (See also Fig. 8.41) (Pédrón and Pineau 1982)

Fatigue Crack Propagation Behaviour of Superalloys

The crack growth rate of long cracks has been largely investigated in many superalloys. These results were obtained using fracture mechanics CT type specimens. Results obtained on In718 alloy are shown in Fig. 8.40. This alloy is largely used for the fabrication of turbine disks. Various microstructures were investigated at 650°C under air and vacuum environments. These microstructures included coarse grained material (ASTM 3–4), fine-grained material (ASTM 6–8) and a material with a necklace microstructure (ASTM grain sizes 3–4 and 8–10), as shown in Fig. 8.41 (Pédrón and Pineau 1982). Continuous fatigue (frequency between 5×10^{-2} and 20 Hz) and creep-fatigue conditions (hold times of 10 and 300 s superimposed on a 5×10^{-2} Hz triangular wave shape signal) were investigated at 650°C. The results of 300 s hold time under air and vacuum are shown in Fig. 8.40 and are compared to those obtained at room temperature. A strong deleterious effect of oxidation is observed in particular in the coarse grained microstructure. In this microstructure the crack growth rate in air is increased by more than two orders of magnitude as compared to vacuum. Oxidation damage produces intergranular brittle fracture. The

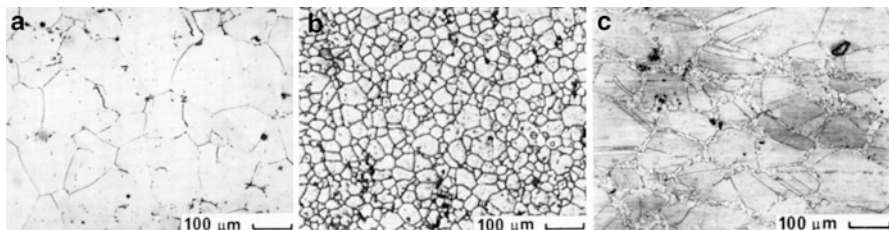


Fig. 8.41 Inco 718 (see composition in Table 8.3). Optical micrographs showing three microstructures. (a) large grains; (b) small grains; (c) necklace microstructure (Pédrón and Pineau 1982)

superiority of the necklace microstructure is mainly due to its very nature which facilitates crack deviation.

Air and vacuum testing for different model Ni-base alloys showed that during continuous cycling the environmental and microstructural effects often act in opposite directions such that the net fatigue crack growth rate is the result of a decrease due to a favourable wavy slip mode and an increase due to the environment. In some cases these effects are such that there is no apparent environmental effect when in fact it can be very large (see Pineau and Antolovich 2009).

In conclusion the high temperature fatigue properties of Ni-base superalloys are largely dependent on the environment and it is clear that in those circumstances modelling of creep-fatigue-oxidation interactions must take into account the damage by oxidation.

8.6 Lifting Techniques for High Temperature Components

8.6.1 Introductory Remarks

This section deals with the techniques used for life prediction of high temperature components subjected to pure creep or to creep-fatigue interactions. The emphasis is laid on the methods which are largely used by the engineers more than the materials scientists who prefer to develop physically-based models, such as those presented for austenitic stainless steels (Sects. 8.5.1. and 8.5.2.2) or cobalt-based superalloys (Sect. 8.5.2.3). These approaches are more complementary than contradictory, as already shown for instance in the case of stainless steels.

In many engineering applications crack initiation is defined as the number of cycles or as the time to nucleate a crack with a length of the order or 1 or 2 mm which is comparable to the section of test specimens. This explains why in many cases engineers take as a definition of crack initiation the number of cycles or the time to fail a specimen with a section of a few millimetres. This definition will be

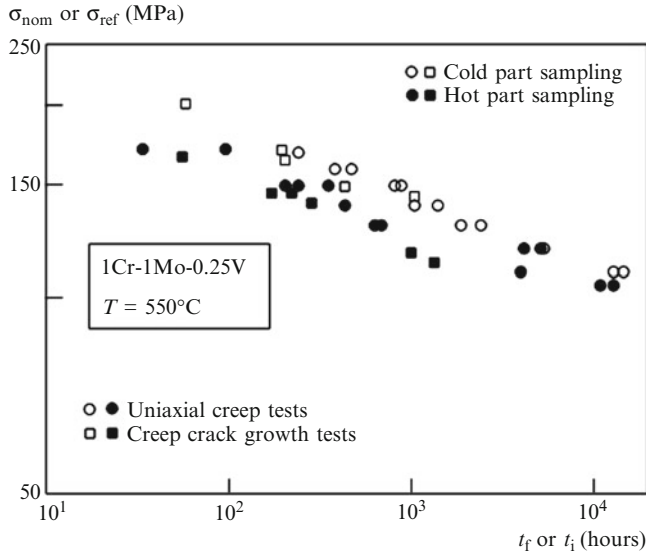


Fig. 8.42 1Cr-1Mo-0.25V ferritic steel tested at 500°C. Comparison of creep resistance as measured either on smooth bars (σ_{nom} , t_f), or on fatigue precracked specimens (σ_{ref} , t_i). The specimens were taken from a decommissioned turbine. Materials from cold part and hot part aged for 150,000 h were used (Pineau 1996a)

adopted in this section. The materials scientists subdivide this number and this time in those necessary to initiate a microcrack (~ 1 grain size) and to propagate this microcrack through the section of the specimen.

8.6.2 Time to Failure Predicted from Limit Load Analysis

This time t_{LL} involves widespread creep damage in the ligament ahead of the crack tip. This failure mode, which is not ruled out by the concepts of viscoplastic fracture mechanics (Chap. 2, Sect. 2.6 in this volume), occurs only in creep ductile materials. This time t_{LL} is simply estimated as:

$$t_{\text{LL}} = t_f(\sigma_{\text{ref}}) \quad (8.75)$$

where t_f is the rupture time from uniaxial creep rupture data obtained at a stress equal to the reference stress, σ_{ref} , which was introduced in Chap. 2, Sect. 2.6.4 in this volume (Pineau 1996a).

The results obtained on smooth bars, expressed in terms of σ_{nom} , and those giving the time to initiation determined on cracked CT specimens, expressed in terms of σ_{ref} , are reported in Fig. 8.42 (ferritic 1Cr-1Mo-0.25steel) and Fig. 8.43 (austenitic

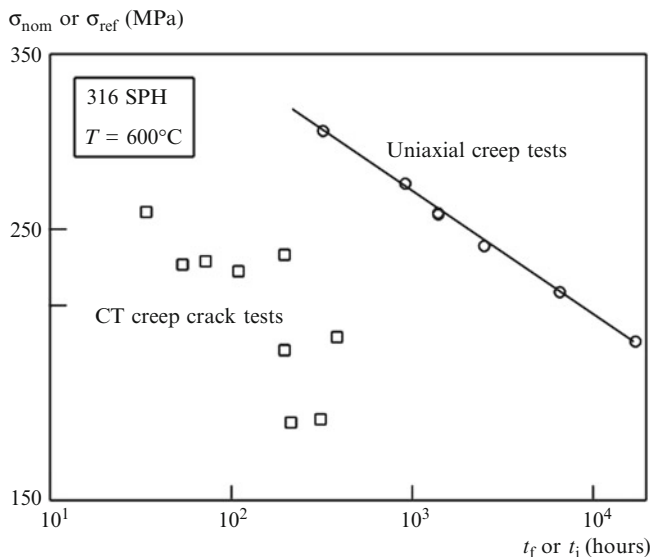


Fig. 8.43 316 (18Cr-12Ni-2.75Mo) austenitic stainless steel tested at 600°C. Creep resistance determined either on smooth specimens (σ_{nom} , t_f) or on fatigue precracked specimens (σ_{ref} , t_i) (Pineau 1996a)

316L stainless steel) (Pineau 1996a). It is seen that the austenitic stainless steel is more sensitive to notch effect than the ferritic alloy. In the procedures developed in the United Kingdom by the Central Electricity Generating Board (CEGB), now EDF Energy Nuclear Generation Limited, (R5 code) it is recommended to calculate both the time to initiation, t_i , and the time to growth t_g ($t_f = t_i + t_g$) when the ratio t_i/t_{LL} is lower than a critical value (~ 0.80), which is the situation observed in both materials, in particular in 316L stainless steel for which it is seen that $t_i/t_{LL} < 0.10$.

8.6.3 Engineering Definition of Time to Creep Crack Initiation

When the creep of a component is considered, two types of behaviours must be distinguished: *creep brittle* (for which K is the relevant loading parameter) and *creep ductile* (for which C^* is the relevant loading parameter). For the definition of these parameters see Chap. 2, Sect. 2.6 in this volume. Typical creep ductile materials are stainless steels, whereas Ni-base superalloys are creep brittle. High strength aluminium alloys, such as 2219-T851 (Al-6.3Cu-0.3Mn-0.18Zr, Wt%) are also creep brittle materials when tested at elevated temperature ($\sim 175^\circ\text{C}$) (Pineau 1992).

As already indicated in Chap. 2 (Sect. 2.6.5 in this volume) in creep brittle materials the time to initiate a creep crack from a pre-existing defect can be

represented by a power law:

$$t_i K^\alpha = \text{constant} \quad (8.76a)$$

where α is a constant which may be a function of temperature. In creep ductile materials a similar correlation was proposed between C^* and t_i :

$$t_i (C^*)^\beta = \text{constant} \quad (8.76b)$$

For the definition of C^* , primary creep must be taken into account when this creep regime is predominant, which is the case of austenitic stainless steels (see Chap. 2, Sect. 2.6.4 in this volume for the definition of C_h^* in this regime; see also ASTM E1457 for “test method for measurement of creep crack growth times in metals”).

8.6.4 Engineering Definition of Time to Creep Crack Growth

In creep brittle materials, the creep crack growth (CCG) rates are theoretically correlated with the stress intensity factor K . Examples of such correlations are shown in Fig. 8.44, where the results obtained on a Ni-base superalloy, N18 alloy, prepared by powder metallurgy (for the composition, see Table 8.3) are reported. Three sizes of tertiary γ' particles were tested. More details are given elsewhere (Pineau 1996b). The CCG rate can be represented by a power law within a first approximation:

$$da/dt = AK^\gamma \quad (8.77)$$

where A is a proportionality factor.

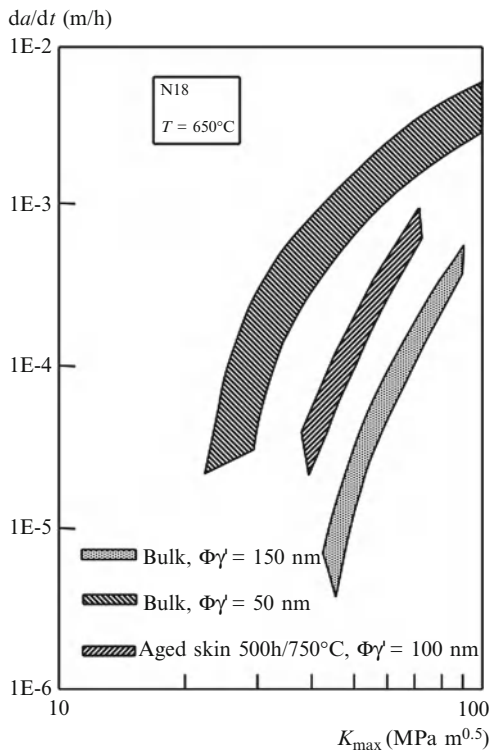
The growth time t_g is then simply calculated as:

$$t_g = \int_{a_0}^{a_f} \frac{1}{AK^\gamma} da \quad (8.78)$$

where a_0 and a_f correspond to the initial crack length and the crack length at failure, respectively.

In creep ductile materials many authors have similarly suggested to correlate da/dt with the C^* parameter, in particular with the C^* parameter calculated from the load-line displacement rate (see Eq. 2.142 in Chap. 2 of this volume). See ASTM E1457-92 (1992) standards. Examples of CCG rates – C^* curves obtained on a 1Cr-1Mo-0.25V steel are given in Fig. 8.45 (Pineau 1996a). Similar results have been published on 316L stainless steel (Bensussan et al. 1988). In calculating the load-line displacement, special care must be taken to take into account the component due to the crack growth itself and not to the creeping behaviour of the crack specimen.

Fig. 8.44 Alloy N18 (see composition in Table 8.3). Creep crack growth rate as a function of the stress intensity factor K . Influence of microstructure. The size of γ' precipitates is indicated on the graph (Pineau 1996a)



A correlation similar to that of (8.77) has been proposed:

$$da/dt = B(C^*)^\delta \quad (8.79)$$

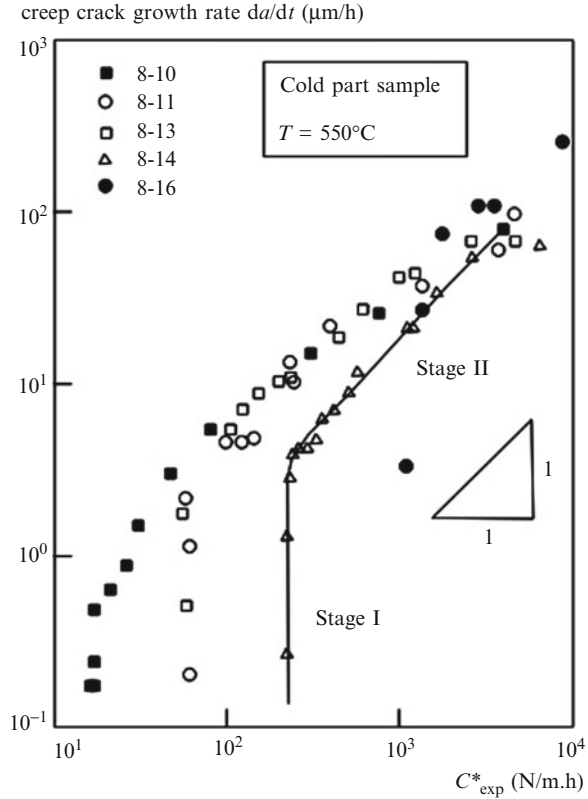
where B is a proportionality factor. Integrating this equation between appropriate limits allows to calculate t_g as:

$$t_g = \int_{a_0}^{a_f} \frac{1}{B(C^*)^\delta} da \quad (8.80)$$

8.6.5 Life Prediction for Creep-Fatigue Cracking

Before dealing with the engineering approach to creep-fatigue crack growth it is necessary to define crack initiation which, this time, involves both a fatigue (N_f) and a creep (t_f) component. In the previous section it was shown how N_f or t_f could

Fig. 8.45 1Cr-1Mo-0.25V ferritic steel tested at 550°C. Material taken from the cold part of a decommissioned turbine. Creep crack growth rate as a function of the C^* parameter. Two stages on the $da/dt - C^*$ curves are distinguished (Pineau 1996a)



be determined from physically-based models. In this section we consider only the phenomenological approaches.

8.6.5.1 Creep-Fatigue Crack Initiation – Phenomenological Models

Three main types of phenomenological models have been proposed. They are presented successively:

Frequency-Modified Fatigue Life

The first phenomenological model is the frequency modified fatigue life (FMFL) which was introduced by Coffin (1972). In the FMFL method, cycle frequency is introduced into the basic Coffin-Manson equation (Chap. 6, Sect. 6.4.2.2) by recognising in an intuitive way that decreasing the cycle frequency will both promote creep and oxidation and thereby reduce the fatigue life. The original form

of the equation was put in terms of the inelastic strain range in a hysteresis loop multiplied by a frequency factor, ν , that was called the frequency-modified strain range. It was later formulated in terms of the total strain, similar to work by Manson (1965), and took the following form:

$$\Delta \varepsilon_t = \Delta \varepsilon_e + \Delta \varepsilon_p = C_1 (N_f \nu^{k_1-1})^{-\beta_1} + C_2 (N_f \nu^{k_1-1})^{-\beta_2} \quad (8.81)$$

In the total strain version there are six constants that may be viewed as simple fitting parameters. In an attempt to consider “unbalanced” cycles with different loading and unloading rates, a method called “frequency separation” was developed by Coffin. The tension-going and compression-going frequencies, ν_1 and ν_2 , were separated and adjustments were made for individual cases in which, for example, the more damaging nature of compressive hold time effects was recognised.

It should be recognised that with six adjustable parameters this methodology is really more of a curve fit to existing data and tends to lose any physical significance, even with its phenomenological roots. However, it is still useful to represent experimental data in well-defined regimes. It can only be applied to lifespan prediction of components with caution, and then only in situations where there is a considerable experimental basis. The interested reader is referred elsewhere for a more complete discussion of this approach and its subsequent modifications (Coffin 1972).

Strain-Range Partitioning (SRP) Model

Another approach is called SRP and was introduced by the NASA group (Manson et al. 1971). In this approach, damage is presumed to result from creep and “plasticity”. Idealised cycles are shown in Fig. 8.46. The four basic types of cycles are:

- creep reversed by creep
- plasticity reversed by plasticity
- creep reversed by plasticity
- plasticity reversed by creep

In the applications, the hysteresis loop is partitioned into these various modes and the lifespan is computed from the following equation:

$$\frac{1}{N_f} = \frac{F_{pp}}{N_{pp}} + \frac{F_{cc}}{N_{cc}} + \frac{F_{pc}}{N_{pc}} + \frac{F_{cp}}{N_{cp}} \quad (8.82)$$

In this equation the F coefficients represent cycle fractions (*e.g.* plasticity reversed by creep) and the N coefficients represent the corresponding lives taken from strain amplitude-life curves, the type of damage being considered. For example, in considering the “pc” component, the corresponding N_{pc} versus $\Delta \varepsilon_{pc}$ curve is

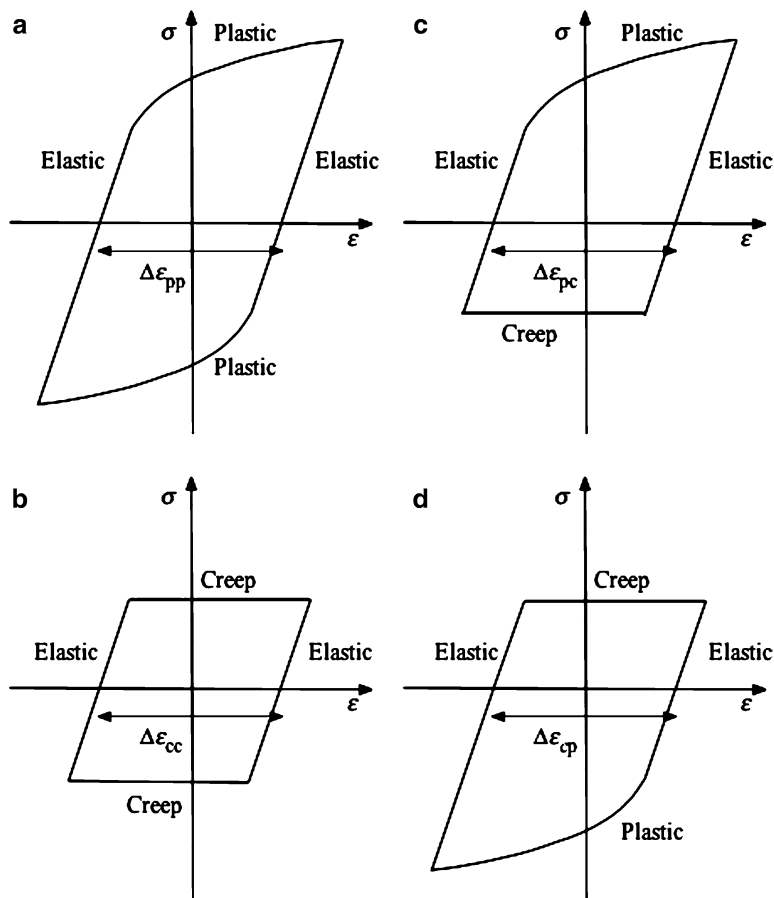


Fig. 8.46 Idealised strain range partitioning loops. (a) plasticity is offset by plasticity ($\Delta\epsilon_{pp}$); (b) creep is offset by creep ($\Delta\epsilon_{cc}$); (c) plasticity is offset by creep ($\Delta\epsilon_{pc}$); and (d) creep is offset by plasticity ($\Delta\epsilon_{cp}$) (Adapted from Manson et al. 1971)

used to determine N_{pc} as if all of the inelastic strain were of the “pc” type. This approach is repeated for all the other damage types present. This is an important detail that is sometimes not understood when applying this methodology and that is essential in using this method. The fractions are determined from the hysteresis loop by first accounting for the “pure” damage types “pp” and “cc”. Once these have been partitioned, the remaining component(s) are subsequently partitioned. This approach, like the FMFL, underwent some modifications (Saltsman and Halford 1988). In general, the “pc” cycles are the most damaging in a superalloy because the compressive hold is associated with oxide cracking on the loading cycle. The pure fatigue cycle is generally the least damaging because of the reduced time for oxidation. The baseline curves are obtained experimentally with known degrees of

the component of interest present. However, when the curves are obtained with uncontrolled amounts of the “other” damage form, this represents an uncontrolled variable that can lead to some dispersion in the predictions.

The same factors that were considered in developing the FMFL and SRP approaches operate at the tips of cracks; so these concepts may be carried over to the case in which the main interest is the propagation of a fatigue crack. Whereas strains are directly measured during traditional creep-fatigue tests, such is not the case for crack propagation. Thus, some kind of indirect approach is required to numerically compute the inelastic strains, both in terms of magnitude and type. This requirement obviously makes it more difficult to apply the SRP approach to crack propagation than to smooth bar fatigue.

Damage Accumulation Concepts

The basic idea of these phenomenological models is to accumulate two elementary damage modes, the first one related to fatigue, D_f , which is independent of time, and the second one related to creep and/or oxidation, D_c , which is time dependent. In the literature, several damage accumulation rules have been proposed that will now be discussed.

Linear damage rule

The simplest law describing damage accumulation is known as the time and cycle fraction rule and is written as:

$$D = D_f + D_c = \frac{n}{N_f} + \frac{t}{t_f} \quad (8.83)$$

where n is the number of cycles under a set of conditions, N_f is the fatigue life for the imposed conditions, t is the time for which the component of interest is under load and t_f is the time for creep rupture under these conditions. This equation is the basis of the Robinson (1952) and Taira (1962) rule.

As discussed previously, it is not always straightforward to select either the appropriate time in a cycle or the total time to creep rupture, since the conditions vary during the course of a cycle. There are instances in which $D = 1$ at failure in (8.83). However, that might, in some sense, be regarded as fortuitous. As seen in Fig. 8.47, there are many cases in which $D < 1$ at failure. Very significant negative deviations from the linear damage accumulation law are frequently seen for reasons that will be explained. Thus, in practice, bi-linear damage accumulation equations are frequently used, such as those indicated in Fig. 8.47.

These deviations from the simple linear damage rule of (8.83) have several causes. The principal reason for such deviations is related to the fact that in order to

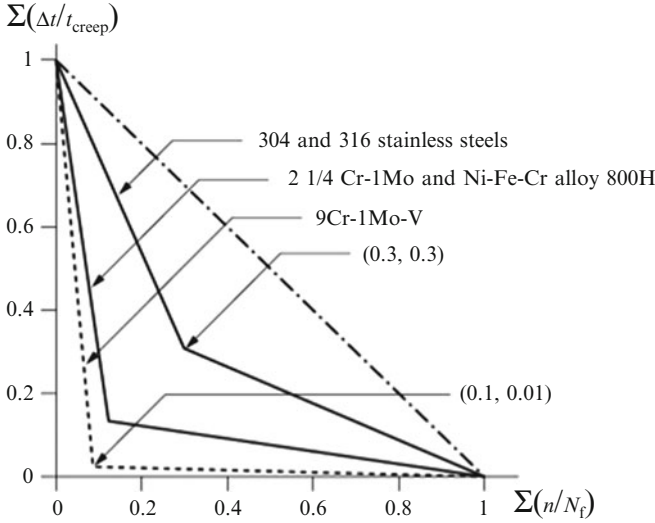


Fig. 8.47 Cumulative damage diagram of fatigue damage D_f versus macroscopic creep damage, D_c . In addition to the linear damage rule (*straight line 1-1*) three other bi-linear damage laws are represented. These are the bi-linear rules recommended by the ASME code for austenitic stainless steels (304 and 316) and for 2 1/4 Cr-1Mo steels and alloy 800H. Finally the bi-linear rule for 9Cr-1Mo steel is also shown (Antolovich and Pineau 2010)

define a reference state, N_f and t_f , virgin material is tested. Thus, all microstructural changes caused by creep-fatigue are ignored. Moreover, if this rule is applied to fatigue relaxation tests, the relaxed stress is different for each cycle, depending on whether the material hardens (*e.g.* austenitic stainless steels) or softens (*e.g.* 9–12%Cr steels). For superalloys the situation can be even more complicated (hardening followed by softening).

Finally, for hold times at maximum strain the creep damage, as previously mentioned, must be defined in a state of continually decreasing stress. The simplest damage parameter for these conditions was defined by Spera (1973) and uses a simple integration during the hold period:

$$D_c = \int_0^{t_f} \frac{dt}{t_f(\sigma_T(t))} \quad (8.84)$$

where $\sigma_T(t)$ is the time-dependent stress during the hold period at some temperature T and t_f is the time to failure.

Some researchers prefer to compute the creep damage D_c in terms of strain rather than in terms of time. The damage increment is then represented by:

$$dD_c = \frac{d\varepsilon}{\varepsilon_{f,T}(\dot{\varepsilon})} \quad (8.85)$$

where ε_f is the creep ductility at temperature T and a viscoplastic strain rate $\dot{\varepsilon}$. We can also introduce the fact that in many materials creep ductility decreases with the creep strain rate (Fig. 8.15). This is referred to as “creep ductility exhaustion concept”. Another reason for this formulation is related to the fact that multiaxial creep damage is more easily formulated in terms of strain. For example, Spindler (2004) suggested the following:

$$\frac{\varepsilon_f^{\text{multi}}}{\varepsilon_f^{\text{uni}}} = \exp \left[p \left(1 - \frac{\sigma_1}{\sigma_{eq}} \right) + q \left(\frac{1}{2} - \frac{3}{2} \frac{\sigma_m}{\sigma_{eq}} \right) \right] \quad (8.86)$$

where p and q are parameters.

This equation is completely analogous to that suggested by Auzoux et al. (2005), who modified (8.85) to obtain:

$$\frac{\varepsilon_f^{\text{multi}}}{\varepsilon_f^{\text{uni}}} = \exp \left[p \left(\frac{1}{3} - \frac{\sigma_m}{\sigma_{eq}} \right) \right] \quad (8.87)$$

These deviations from the linear damage accumulation rule prompted some researchers to suggest phenomenological, nonlinear, damage accumulation expressions.

Nonlinear damage accumulation

We will simply outline the formalism developed by Kachanov (1958) and by Lemaitre and Chaboche (1985).

In a formal way, a process involving two damage mechanisms may be described by two total differentials:

$$dD_{\text{fat}} = f_{\text{fat}}(\sigma_M, \bar{\sigma}, T, D_f + D_c) dN \quad (8.88)$$

$$dD_c = f_{\text{cr}}(\sigma, T, D_f + D_c) dt \quad (8.89)$$

where for the uniaxial case σ_M is the maximum stress in a cycle, $\bar{\sigma}$ is the mean stress, T is the temperature and the other terms have been previously defined. Creep-fatigue coupling is introduced by writing that the increment for each damage mode depends on the current total damage ($D_f + D_c$) at cycle N or time t . The expressions introduced for the damage functions f_{fat} and f_{cr} were given explicitly in a comprehensive review by Chaboche (2011).

A model in the same vein was developed by Maiya and Majumdar (1977) for austenitic stainless steels. Their model, which is essentially a model for fatigue crack growth accelerated by creep damage, has the following form:

$$\frac{1}{a} \frac{da}{dN} = \left\{ \frac{T}{C} \right\} |\varepsilon_p|^m |\dot{\varepsilon}_p|^k \left(1 + \log \frac{c}{c_0} \right) \quad (8.90a)$$

$$\frac{1}{c} \frac{dc}{dt} = \left\{ \frac{G}{-G} \right\} |\varepsilon_p|^{m_c} |\dot{\varepsilon}_p|^{k_c} \quad (8.90b)$$

The notations between parentheses in (8.90a) indicate that in tension the RHS (Right Hand Side) is to be multiplied by T , and in compression by C . Similarly, for (8.90b). Thus, the first term is not a true fraction but a symbolic notation.

Fatigue damage is represented by crack length, whereas creep damage is represented by c , the fraction or size of cavities calculated from (8.90b). In these equations, the absolute values of strain and strain rate are used. However, other quantities such as T , C , G , k , k_c , m , m_c , a_0 (initial crack length) and c_0 (initial cavity size) are determined experimentally. If it is assumed that $m = m_c$ in symmetric cycles in strain control and in continuous cycling without hold time, these equations take on the following form, which can be used for integration:

$$\left(\frac{da}{dN} \right)_{CF} = a \left(\frac{T + C}{m + 1} \right) (\Delta \varepsilon_p)^{m+1} (\dot{\varepsilon}_p)^{k-1} \quad (8.91)$$

This equation is analogous to that suggested by Tomkins (1968) from a totally different starting point, with the difference that in (8.90a and 8.90b) the strain rate appears explicitly, which is not the case for Tomkins expression. For creep-fatigue cycles with hold times at the maximum load (strain) this equation takes on the following form:

$$\left(\frac{da}{dN} \right)_{CF} = \left(\frac{da}{dN} \right)_{PF} (1 + \delta GIN) \quad (8.92)$$

where: $I = \int_{\text{cycle}} |\varepsilon_p|^m |\dot{\varepsilon}_p|^{k_c} dt$.

Here, N is the number of cycles and δ is a coefficient that is related to parameter G . This equation, exactly like the one we introduced during the development of physically-based models for interactions between fatigue and intergranular creep damage (8.65a) and (8.65b) for stainless steels, shows a multiplicative effect of creep damage on the crack growth rate.

8.6.5.2 Creep Fatigue Growth Rate for Long Cracks

We assume that a cracked component is submitted to a trapezoidal wave load including a hold time, t_h . It is also assumed that the loading and unloading rates

are sufficiently large to avoid any creep damage during these parts of the cycle, that is these parts correspond to pure fatigue in a material which may be creep damaged. Only isothermal loading is considered. The material is assumed to follow a stationary Norton law written as:

$$\dot{\epsilon} = B\sigma^{n_2} \quad (8.93)$$

This creep law should account for the softening effect observed in steels such as 9–12%Cr (see Sect. 8.5.2.1) or the hardening effect in materials like austenitic stainless steels (see Sect. 8.5.2.2). Unfortunately in many cases the effect of cyclic loading on the creep rate of materials is not known so that the Norton law will be that of the non-cycled material. The discussion of the previous section and the viscoplastic fracture mechanics introduced in Sect. 2.6 in Chap. 2 of this volume lead to the following conclusions:

1. when t_h is larger than the transition time (t_{tr}) calculated from (2.127) (Chap. 2 of this volume) and defined by:

$$t_{tr} = \frac{J}{(n_2 + 1) C^*} \quad (8.94)$$

it can reasonably be assumed that the life will only depend on the creep cracking behaviour. This life can be assessed using (8.76b) and (8.80).

2. when $t_h \ll t_{tr}$ and, in particular, when the analysis is made on creep brittle materials, two situations may be encountered. In both cases the relevant loading parameter is the stress intensity factor, K . These two situations are dependent on the value of t_h compared to that of t_i calculated from (8.76a).
 - (a) When $t_h > t_i$ it can be assumed that most of the crack growth increment occurs during the creep part of the cycle. It can therefore be assumed that the CCG rate, $(da/dN)_{CF}$, results from a simple cumulative law:

$$(da/dN)_{CF} = t_h(da/dt)_{PC} + (da/dN)_{PF} \quad (8.95)$$

where $(da/dN)_{PC}$ represents the CCG rate measured under pure creep conditions (constant load) whereas $(da/dN)_{PF}$ is the crack growth rate per cycle measured under pure (or continuous) fatigue conditions. A number of results obtained on Ni-base superalloys and showing a good agreement with this approach were given elsewhere (Pineau 1996b).

- (b) When $t_h < t_i$ it can no longer be assumed that a creep crack growth increment is associated to each cycle. In many situations it seems necessary to determine experimentally the crack growth rates corresponding to this situation. However in a number of circumstances it is possible to model the interactions between creep and fatigue damage using a simplified approach which is presented below.

In the linear damage accumulation rule, the incremental fatigue damage per cycle of continuous fatigue is simply the reciprocal of the cycles involved, *i.e.* $1/N$. This creep damage is simply added to creep damage D_c such as the total damage per cycle can be expressed as:

$$1/N_{CF} = 1/N_{PF} + D_c \quad (8.96a)$$

or

$$N_{PF} = N_{CF} / (1 - N_{CF} D_c) \quad (8.96b)$$

Differentiating with respect to crack length, a , leads to:

$$(da/dN)_{CF} = (da/dN)_{PF} \left[1 / (1 - N_{CF} D_c)^2 \right] \quad (8.97)$$

This equation was originally derived by Levillant and Pineau (1982) in a different way. This equation, which is used in the R5 code (Ainsworth et al. 1987), expresses the acceleration of the fatigue crack growth rate due to the presence of creep damage. Skelton (1993, 2008) presented a simple approach to derive an explicit expression for $(da/dN)_{CF}$. The continuous pure fatigue growth rate is given by the Paris relation:

$$(da/dN)_{PF} = C_0 \Delta K^m \quad (8.98)$$

To determine the total creep damage in the process zone the relevant number of cycles is calculated as the number to propagate the crack by fatigue over a critical distance which can be taken as equal to the plastic zone size (R_{cyc}) at the crack tip. Therefore it can be written that, in plane stress:

$$N_{CF} = R_{cyc} / (da/dN)_{CF} = \frac{\Delta K^2}{4\pi R_p^2} \frac{1}{(da/dN)_{CF}} \quad (8.99)$$

where R_p is the yield strength of the material.

Substituting (8.98) and (8.99) into (8.97) and solving the quadratic equation for $(da/dN)_{CF}$, Skelton (1993, 2008) obtained:

$$\left(\frac{da}{dN} \right)_{CF} = \frac{1 + 2M + \sqrt{1 + 4M}}{2} \left(\frac{da}{dN} \right)_{PF} \quad (8.100)$$

with $M = (1 / 4\pi R_p^2) (D_c / C_0 \Delta K^{m-2})$.

This equation shows that the predicted crack growth rate approaches that obtained under pure fatigue when the creep damage D_c tends to zero. Skelton (1993) applied this equation to a number of published results and thus obtained likely values for the D_c parameter. Various materials including 304 and 316 stainless steels, cast

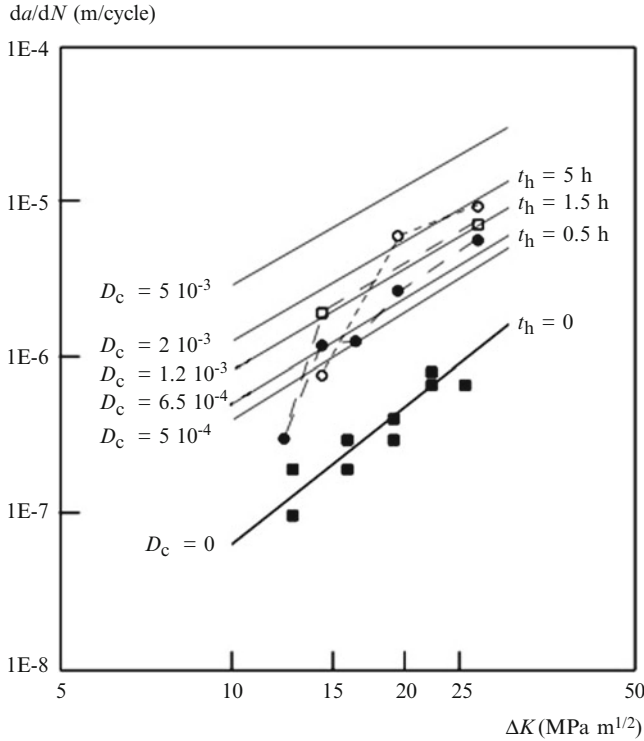


Fig. 8.48 Austenitic stainless steel. Effect of a hold time on the fatigue crack growth rate da/dN as a function of ΔK . Hold times, t_h , and the values of D_c parameter are indicated (Pineau 1996a)

and forged Cr-Mo-V steels were analysed. The maximum hold time was ~ 200 h and values of D_c typically were found to range from 2×10^{-4} to 1×10^{-2} .

This model was also applied to test results obtained by Laiarinandrasana (1994) on 316L stainless steel tested at 600°C (Fig. 8.48). Moreover this author used the incremental equation derived from physical measurements of intergranular damage (8.56) with $\gamma = 2$. This equation was integrated numerically using the results of FE calculations. Laiarinandrasana (1994) defined the non-dimensional damage per cycle D_c as:

$$D_c = \int_0^{t_h} dD_c^i \bigg/ \int_0^{t_i} dD_c^i \quad (8.101)$$

where t_i represents the time to initiation under steady load, that is the creep initiation time defined earlier (8.76b) and D_c^i the intergranular damage per cycle at crack initiation. It was shown that the values of D_c inferred from Skelton's model, which is purely phenomenological, were in good agreement with those calculated from (8.101) which are based on metallographical observations.

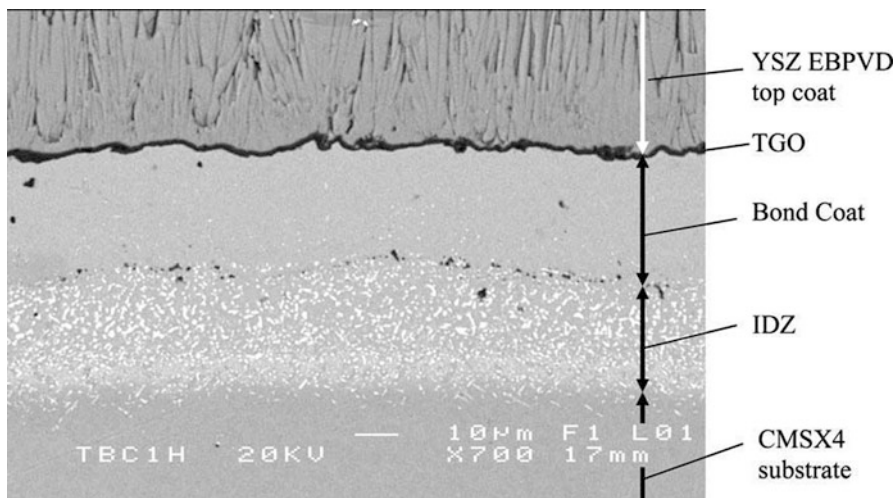


Fig. 8.49 Scanning electron micrograph of a section through a thermal barrier coating (TBC) system on a CMSX-4 substrate after exposure to air at 1,100°C for 1 h (Taylor et al. 2006)

It is therefore possible to model the fatigue crack growth rate behaviour in the LEFM regime for materials in which intergranular damage is accumulated during the hold times, progressively but not sufficiently to initiate CCG during each cycle.

8.6.6 High Temperature Materials

We finish this chapter by giving a few indications on the development of materials for high temperature applications. The presentation is limited to two applications: thermal barrier coatings and ceramic composite materials. A number of these indications have already been given in this volume (see Chap. 4, Sect. 4.5).

8.6.6.1 Thermal Barrier Coatings

One example of great technological importance is that of the blades used in modern gas turbines (see Fig. 1.2 and Sect. 4.5 in this volume). The metallurgical solutions adopted to increase the creep strength of the nickel-base alloys used to fabricate these blades have been briefly presented. It has been shown that the most recent materials are monocrystalline. However the increase of the temperature of combustion chambers necessitated the introduction of thermal barriers made of ceramics such as zirconia to decrease the working temperature of the nickel-base

material. Modern typical thermal barrier coating (TBC) systems are now widely used in aerospace and land-based gas turbines to improve the performance and efficiency of the engine. A typical TBC system consists of an Yttria-partially-stabilised zirconia (YSZ) top coat and aluminium-rich intermediate metallic layer or bond coat (BC) (see Fig. 8.49) (Busso and Qian 2006). One function of the BC is to provide a protective layer of thermally-grown oxide (TGO) – designed to be α -alumina – to the TBC system. Many current researches relate TBC failure to stress generation at or near the TGO which initiates cracks and spallation of the ceramic layer.

Methods for predicting spallation and failure in TBCs generally followed two main approaches. The first approach used empirical fatigue life models that link the life of the TBC to the total amount of damage due to oxidation and mechanical straining. The second approach used delamination models to consider edge cracking and buckling. This approach neglects the fact that there is generally a long period of subcritical crack nucleation and growth prior to the delamination stage. More recently finite element calculations have been introduced to predict the stress distribution accounting for oxide growth mechanical behaviour of TC and BC materials, and viscoplasticity of the matrix (see *e.g.* Busso et al. 2008). Moreover new top coat and bond coat systems, including in particular Pt-aluminised diffusion bondcoat, are being investigated (see *e.g.* Tawancy and Al-Hadhrani 2011, 2012; Taylor et al. 2006).

8.6.6.2 Ceramic Matrix Composites (CMCs)

The demand for increasing temperatures in gas turbines is such that the temperature reached by nickel-based superalloys used for the fabrication of blades is approaching the melting temperature, T_m , of metallic alloys. This explains why ceramic materials could also be good candidates for these applications. The use of ceramic materials in gas turbines is limited by the intrinsic brittleness of these materials. However non-oxide ceramic matrix composites (CMC), *i.e.* mainly those consisting of a SiC-based matrix reinforced with either carbon or SiC-fibres composites, have been extensively studied during the last two decades since their discovery in the mid-1970s (see Naslain et al. 1981; Naslain 2004). These tough ceramics matrix composites have the potential for being used up to 1,500°C, as structural materials, in different fields including advanced engines (see Vinci rocket engine nozzle exhaust, Fig. 8.50), gas turbines for power/stream cogeneration, heat-exchangers, etc.

The main advantage of CMCs with respect to their monolithic counterparts lies in the fact that they are relatively tough (tensile elongation of a few percent) although their constituents are intrinsically brittle (tensile elongation less than 1%). This property is obtained through a proper design of the fibre/matrix (F/M) interface. The chemistry and the fabrication of this interface allow crack arrest and crack deflection in the brittle matrix and prevent the early failure of the fibrous reinforcement (Boitier et al. 1999) (see also Chap. 10, Sect. 10.7). It was shown that the best F/M interface materials are those with a layered crystal structure, such as pyrocarbon (P_yC-SiC)_n or (BN-SiC)_n, the layers being deposited parallel to the fibre surface (Naslain 2004).

Fig. 8.50 Vinci engine used in Ariane V rocket. The nozzle extension is fabricated with ceramic composite SiC-SiC materials (Courtesy of E. Forterre, Snecma)



The properties of SiC-matrix composites can thus be optimised through material design at the level of each of their constituents (see Chermant et al. 2002). Pyrocarbon and boron nitride are prone to oxidation. One way to improve their resistance in oxidising environment is to replace part of them by $(\text{PyC-SiC})_n$ and (BN-SiC) multilayered interface. The oxidation resistance of the matrix can be improved by introducing a small amount of phases forming oxide glasses which fill the matrix cracks and slow down the diffusion of oxygen. Self-healing composite materials are thus obtained.

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Chapter 9

Contact Mechanics; Friction and Wear

Abstract Contact mechanics describes the stresses and the deformations in the neighbourhood of the contact between two pressed bodies. The following examples are given: concentrated and distributed normal and tangential loads, point load, pressure on a circular surface, hertzian contact for normal and tangential loadings. The analysis of sliding fatigue allows understanding fretting fatigue. The onset of plastic deformation is treated in the case of sliding cylinders. This forms the basis to analyse contact damage. The real area of contact of two solids is much smaller than the nominal area. It varies as the applied pressure is increased. Friction is understood as due to junctions between asperities. Typical values of the friction coefficients are given. The stick-slip mechanism is explained. Several different types of wear are distinguished: seizure, melt wear, oxidation wear, plasticity dominated wear. Wear is measured with the pin-on-disk apparatus. The temperatures of the friction surfaces are calculated. The mechanisms of the various types of wear are described. A convenient way to represent the results is to use wear mechanisms maps. Materials for friction and wear resistance are indicated.

9.1 Introduction

Contact mechanics is a subject of great theoretical and practical importance. The mechanical interaction between two contacting solids plays a key role in many situations: a car or a train could not move without friction, skiing or skating would be impossible without the formation of a thin film of water between the snow (or the ice) and the skis or the skates, machining such as drilling holes involves also large interactions between the tools and the materials, etc.

This subject may be said to have started to be studied seriously in 1882 with the publication of Heinrich Hertz¹ (1882a, b) of his classic paper *On the Contact of Elastic Solids*, as stated by K. L. Johnson (1985) in the preface of his classic book entitled *Contact Mechanics*. However the friction law was known since the early work by Leonardo de Vinci,² Coulomb³ and Amontons.⁴

Contact mechanics plays also a key role in *tribology*. The tribological interactions of a solid surface with interfacing materials and environment can result in loss of material from the surface. The process leading to loss of materials is known as “*wear*”. Estimated direct and consequential annual loss to industries due to wear is approximately 1–2% of gross national product (GNP) (Zum Gahr 1987). In recent years micro- and nano-tribology have been largely investigated because the interactions in microscopically small components are becoming important for the development of new products in electronics, life science, chemistry, sensors and by the extension of all modern technology.

This field is thus very wide and can be studied at several scales:

- the *atomic* or *molecular scale* where the interest is in the interaction between the atoms or the molecules,
- the *mesoscopic scale*, where the inevitable roughness (1–10 μm) of the contacting surfaces is a key factor and,
- the *macroscopic scale* where the interest lies in the stress-strain fields in the two solids, resulting from the contact.

In this chapter we start with the third scale where it is assumed that the contacting surfaces are perfectly smooth. This first part is largely based on the presentation of contact mechanics adopted by Johnson (1985). Then we will deal with the mesoscopic scale when the mechanisms linked to friction will be analysed. The chapter will end with an overview of the wear properties of materials when sliding occurs over long distances. This part devoted to wear relies heavily on the overview published by Lim and Ashby (1987). This chapter finishes with a brief description of the materials used for improving the wear properties, including metals and others materials, such as ceramics and polymers. In this presentation only dry solid-solid interactions are studied, that is lubrication is not considered. Further we shall be concerned only with *non-conforming contact*, that is, contact in which the two surfaces have different profiles and therefore do not interlock perfectly.

¹Heinrich Rudolph Hertz (1857–1894) was a German physicist. He taught at Technische Hochschule of Karlsruhe. He was working as a research assistant to Helmholtz in the University of Berlin when he discovered the Hertzian distribution of contact pressure.

²Leonardo da Vinci (1452–1519) was the first to enunciate the laws of friction.

³Charles Augustin de Coulomb (1736–1806) was a French physicist who left his name to two laws: that of the electrostatic interaction between electrically charged particles, and that of the friction between two solids.

⁴Guillaume Amontons (1663–1705) was a French scientific instrument inventor and physicist. In 1699, Amontons published his rediscovery of the laws of friction first put forward by Leonardo da Vinci. These laws were verified by Charles-Augustin de Coulomb in 1781.

9.2 Contact Mechanics

9.2.1 Contacting Surfaces: Relative Movement and Forces Transmitted

Figure 9.1 represents the contact between the two surfaces of the solids. Oxy is the common tangent plane and Oz is the common normal. The shapes of the solids before deformation are given by:

$$z_1 = f_1(x, z) \quad \text{and} \quad z_2 = f_2(x, z) \quad (9.1)$$

and the distance separating the surfaces by:

$$h = z_1 + z_2 = f(x, y) \quad (9.2)$$

We consider only two types of relative movement, *sliding* and *rolling*. In sliding the displacement vector has two components:

$$\Delta u_x = u_{x1} - u_{x2} \quad \text{and} \quad \Delta u_y = u_{y1} - u_{y2} \quad (9.3)$$

In rolling there are two angular components:

$$\Delta \omega_x = \omega_{x1} - \omega_{x2} \quad \text{and} \quad \Delta \omega_y = \omega_{y1} - \omega_{y2} \quad (9.4)$$

There also can be a relative “spin” in which either or both solids rotate about the axis Oz .

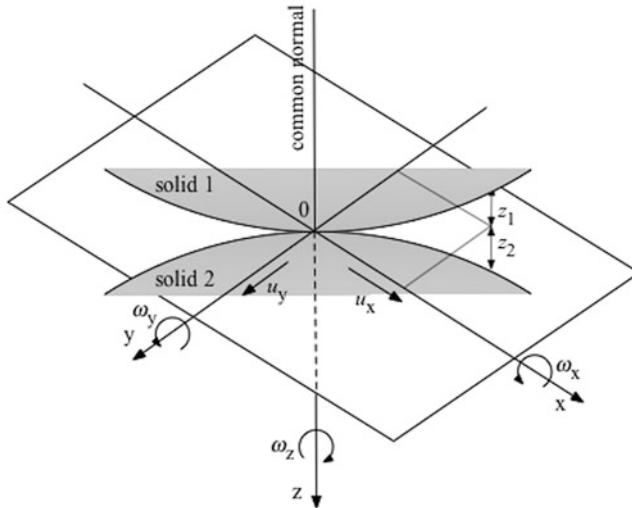
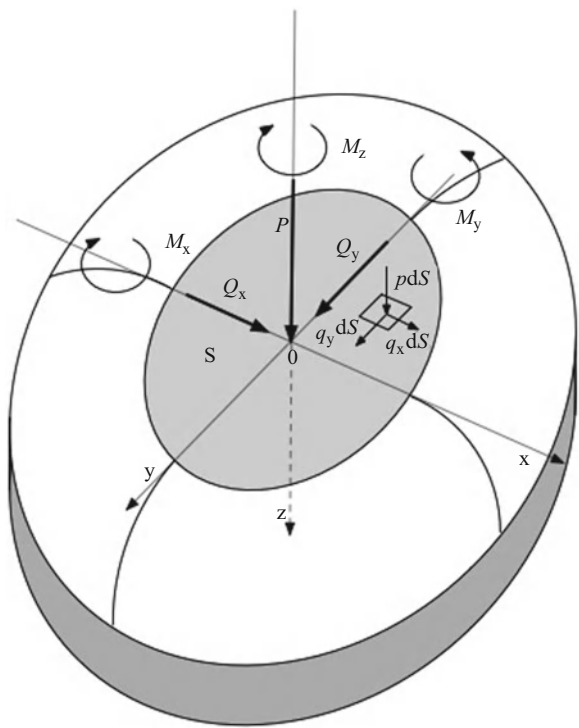


Fig. 9.1 Solids 1 and 2 in contact at O. The tangent plane xOy is defined

Fig. 9.2 Forces and moments acting on the contact area S



The resulting force transmitted from one surface to the other has two components (Fig. 9.2): normal, which is a compression P acting in the direction of the common normal, and tangential, which is a force Q acting in the common tangent plane. The force Q has two components Q_x and Q_y .

In the absence of sliding, Q must be less than μP where μ is the coefficient of friction (*Coulomb's law*). If the contact is one of pure sliding Q must reach this limiting value in the direction opposite to that of the relative velocity and we must have:

$$Q_x = -(\Delta u_x / |\Delta u|) \mu P \quad \text{and} \quad Q_y = -(\Delta u_y / |\Delta u|) \mu P \quad (9.5)$$

The contact force deforms the solids in such a way as to produce a *contact area*. Thus the contact will transmit not only a force but also moments, M_x and M_y of rolling and M_z of spin.

These forces and these moments are transmitted to the solids by surface distributions p and q of normal and tangential stresses respectively, such that:

$$P = \int_S p dS, \quad Q_x = \int_S q_x dS, \quad Q_y = \int_S q_y dS \quad (9.6)$$

$$M_x = \int_S p y dS, \quad M_y = - \int_S p x dS, \quad M_z = \int_S (q_y x - q_x y) dS \quad (9.7)$$

9.2.2 Elastic Analysis

9.2.2.1 The Problem

We have to study problems which differ in their two-dimensional or three-dimensional aspects; further we have to consider four different types of boundary conditions, according to the type of problem; with u_z^* , u_z^* representing prescribed values of the displacements, there are:

- Type I: $p(x)$ and $q(x)$ are given,
- Type II: $u_z^*(x)$ for $z=0$ and $q(x)$ are given, or $u_x^*(x)$ and $p(x)$ are given. Frictionless contact ($q(x)=0$) belongs to the first case,
- Type III: $u_z^*(x)$ and $u_x^*(x)$ for $z=0$ are given, when the distributions $p(x)$ and $q(x)$ are to be found,
- Type IV: $u_z^*(x)$ is given and there is a relation $q(x) = \pm \mu p(x)$ which corresponds to Coulomb's type friction.

Most of the solutions to these problems were obtained and can be found in, for example, Timoshenko and Goodier (1951). A classic relatively recent source is Johnson (1985), from which we have taken much material. We shall give only the solutions, without going into the details of proof: these can be found in Gladwell (1980).

9.2.2.2 Line Load on a Semi-infinite Solid

The elastic half-space is shown in cross-section in Fig. 9.3. The load is applied along Oy and is distributed over a band of width $(a+b)$ in the direction Ox. We assume plane strain conditions, $\varepsilon_{yy}=0$. It is required to find the stress components σ_{xx} , σ_{zz} and τ_{xz} at all points throughout the solid and the components u_x and u_z of the elastic displacement at any point from its undeformed position. In particular we are interested in the deformed shape of the surface $\bar{u}(z)$ (the bar is used throughout to denote values of the variable at the surface $z=0$).

This gives a problem in plane elasticity which can be solved in terms of Airy functions (see Sect. 3.3.7.2, Volume I).

Concentrated Normal Force

Let P be the force per unit length. It can be shown (see Volume III) that the solution is:

$$\sigma_{rr} = (2A/r) \cos \theta, \quad \sigma_{\theta\theta} = \sigma_{r\theta} = 0 \quad (9.8)$$

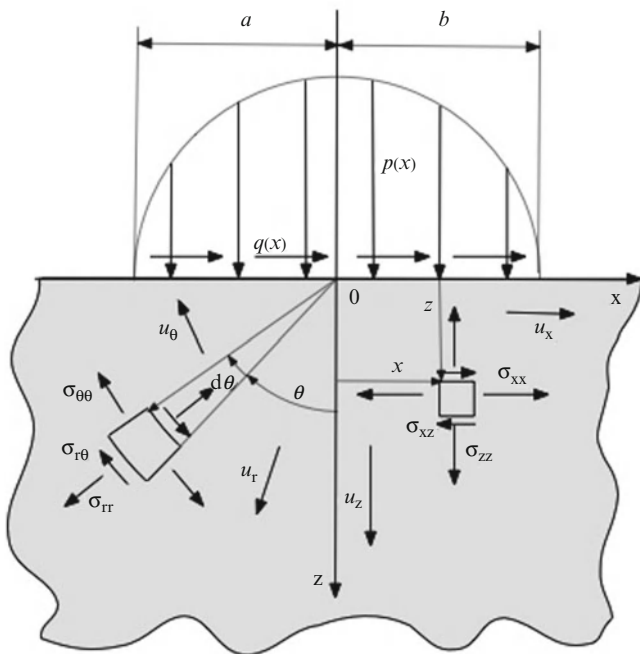


Fig. 9.3 Line loads $p(x)$ normal and $q(x)$ tangential. Displacement and stress fields below the contact area in rectangular cartesian coordinates and in cylindrical polars

The stress distribution is purely radial and has a singularity like $1/r$. The value of A in (9.8) can be found from the equilibrium equation:

$$-P = \int_{-\pi/2}^{\pi/2} \sigma_{rr} r \cos \theta \, d\theta = A\pi \quad (9.9)$$

Thus σ_{rr} can be written as:

$$\sigma_{rr} = -\frac{2P \cos \theta}{\pi r} \quad (9.10)$$

The radial stress has a constant value $-2P/\pi d$ on a circle of diameter d passing through the origin O (Fig. 9.4). The maximum shear stress, τ , at (r, θ) has the value $\sigma_{rr}/2$ and acts on planes inclined at 45° to the radial direction.

The deformation of the surface is shown schematically in Fig. 9.4. This also has a singularity of the order of $\log r$ and is given by:

$$[\bar{u}_\theta]_{\theta=\pi/2} = -[\bar{u}_\theta]_{\theta=-\pi/2} = \frac{(1-\nu^2)}{\pi E} 2P \log(r_0^*/r) \quad (9.11)$$

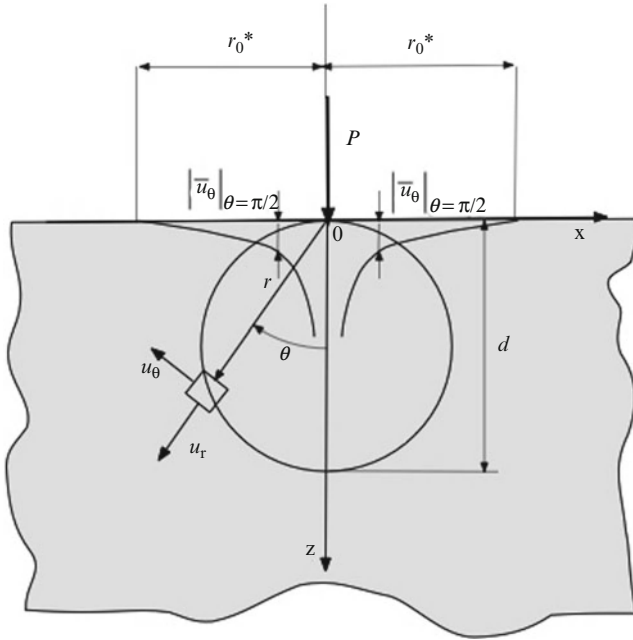


Fig. 9.4 Concentrated normal line load corresponding to an applied force per unit length, P . The deformed contact surface is shown

where E is Young modulus, ν , the Poisson coefficient and r_0^* is a constant. The infinite displacement at O is to be expected in view of the singularity in stress at that point.

In Cartesians, the stresses are:

$$\begin{aligned}\sigma_{xx} &= \sigma_{rr} \sin^2 \theta = -\frac{2P}{\pi} \frac{x^2 z}{(x^2 + z^2)^2} \\ \sigma_{zz} &= \sigma_{rr} \cos^2 \theta = -\frac{2P}{\pi} \frac{z^3}{(x^2 + z^2)^2} \\ \sigma_{xz} &= \sigma_{rr} \sin \theta \cos \theta = -\frac{2P}{\pi} \frac{x z^2}{(x^2 + z^2)^2}\end{aligned}\quad (9.12)$$

Concentrated Tangential Force

A tangential force Q per unit length generates a stress field that is similar to that for a normal force but is turned through 90° . If we measure Q clockwise from Ox (Fig. 9.5), we have:

Fig. 9.5 Concentrated tangential line load per unit length, Q

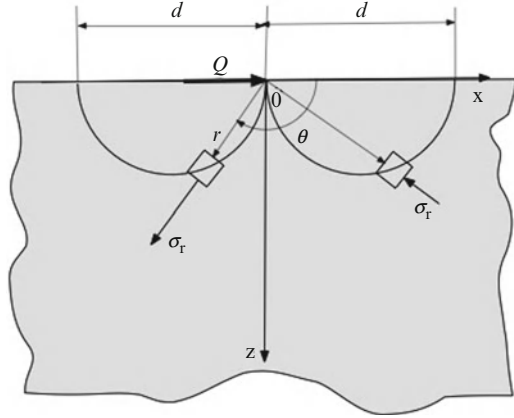
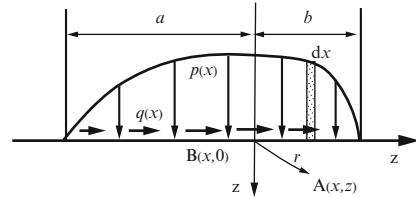


Fig. 9.6 Distributed line loads: $p(x)$ normal, $q(x)$ tangential



$$\sigma_{\pi\pi} = -\frac{2Q}{\pi} \frac{\cos \theta}{r} \quad \sigma_{\theta\theta} = \sigma_{r\theta} = 0 \quad (9.13)$$

The stress contours, shown in Fig. 9.5, are circles. In Cartesians:

$$\begin{aligned} \sigma_{xx} &= -\frac{2Q}{\pi} \frac{x^3}{(x^2 + z^2)^2} \\ \sigma_{zz} &= -\frac{2Q}{\pi} \frac{xz^2}{(x^2 + z^2)^2} \\ \sigma_{xz} &= -\frac{2Q}{\pi} \frac{x^2z}{(x^2 + z^2)^2} \end{aligned} \quad (9.14)$$

9.2.2.3 Distributed Normal and Tangential Line Forces

The loads are distributed, as shown in Fig. 9.6. The resulting stresses are obtained from the previous equations by integrating the elementary forces $p(x)dx$ and $q(x)dx$ along the x -axis. The results, however, are not always expressible in closed form. Here we give the results only for uniform distributions.

Fig. 9.7 Constant normal pressure over a strip O_1O_2 ($-a \leq x \leq a$)

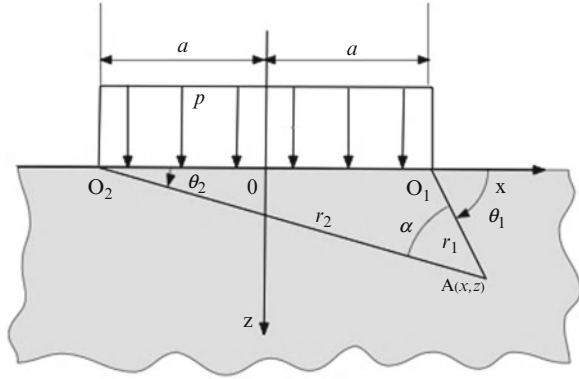
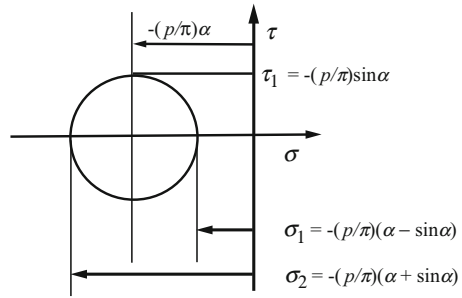


Fig. 9.8 Mohr circle for constant pressure. The angle α is defined in Fig. 9.7



Constant Normal Pressure

The simplest example of a distributed load arises when the pressure, p , is uniform over the strip ($-a \leq x \leq a$) with no shearing (Fig. 9.7).

$$\begin{aligned}\sigma_{xx} &= -\frac{p}{2\pi} \{2(\theta_1 - \theta_2) - (\sin 2\theta_1 - \sin 2\theta_2)\} \\ \sigma_{zz} &= -\frac{p}{2\pi} \{2(\theta_1 - \theta_2) + (\sin 2\theta_1 - \sin 2\theta_2)\} \\ \sigma_{xz} &= -\frac{p}{2\pi} (\cos 2\theta_1 - \cos 2\theta_2)\end{aligned}\quad (9.15)$$

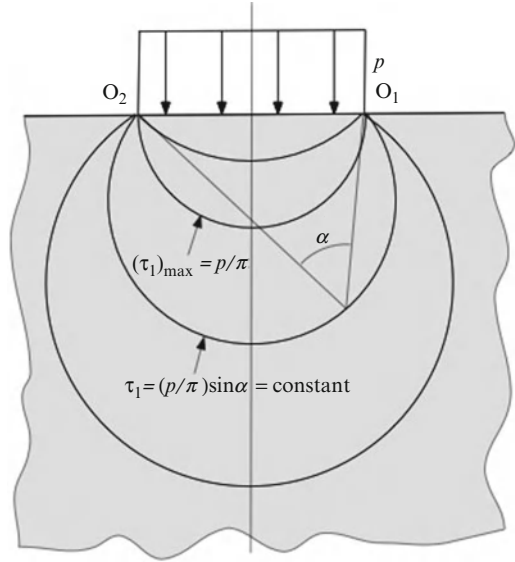
where $\tan \theta_{1,2} = z/(x \pm a)$.

The principal stresses on the Mohr circle (Fig. 9.8) can be expressed simply as:

$$\sigma_{1,2} = -(p/\pi)(\alpha \pm \sin \alpha), \quad \tau_1 = (p/\pi) \sin \alpha \quad (9.16)$$

Thus the contours for constant principal stress $\sigma_{1,2}$ and constant τ_1 are circles passing through O_1 and O_2 and that for maximum shear corresponding to $\alpha = \pi/2$ is a circle with O_1O_2 as diameter (Fig. 9.9). Along O_1O_2 , $\alpha = \pi$; the shear stress

Fig. 9.9 Contours of maximum shear stress for constant normal pressure, p , acting over a strip O_1O_2



vanishes and the stress state is one of compression, $\sigma_{xx} = \sigma_{zz} = -p$. This provides a qualitative explanation for the observation that, if there is *plastic deformation*, this is found not immediately under the line of application of the load but at some distance *below the surface*. The stresses are finite throughout the body of the solid except at O_1 and O_2 where there is a discontinuity in σ_{xx} .

The deformation of the surface is shown in Fig. 9.10; this is given by:

$$\bar{u}_z = -\frac{(1-\nu^2)}{\pi E} \left\{ (x+a) \log \left(\frac{x+a}{a} \right)^2 - (x-a) \log \left(\frac{x-a}{a} \right)^2 \right\} + C \quad (9.17)$$

where C is a constant of integration to be chosen. The curve of Fig. 9.10 corresponds to the condition $\bar{u}_z = 0$ for $x = \pm c$. The slope $d\bar{u}_z/dx$ is theoretically infinite at O_1 and O_2 .

Constant Tangential Tension

This corresponds to the situation shown in Fig. 9.11. The solution is:

$$\begin{aligned} \sigma_{xx} &= \frac{q}{2\pi} \left\{ 4 \log \left(\frac{r_1}{r_2} \right) - (\cos 2\theta_1 - \cos 2\theta_2) \right\} \\ \sigma_{zz} &= \frac{q}{2\pi} (\cos 2\theta_1 - \cos 2\theta_2) \\ \sigma_{xz} &= -\frac{q}{2\pi} \{ 2(\theta_1 - \theta_2) + (\sin 2\theta_1 - \sin 2\theta_2) \} \end{aligned} \quad (9.18)$$

where $r_{1,2} = [(x \pm a)^2 + z^2]^{1/2}$.

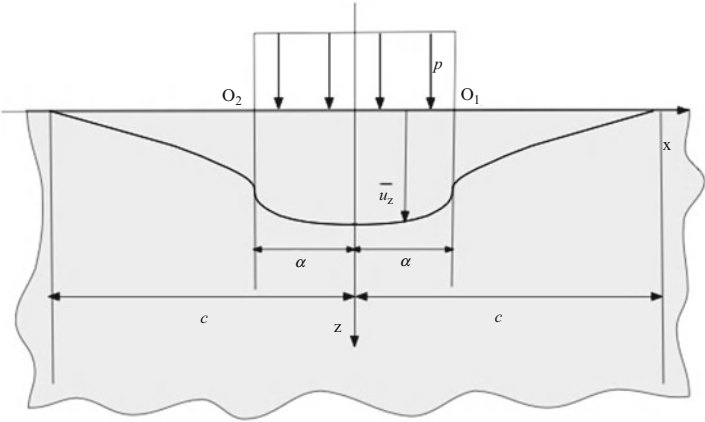


Fig. 9.10 Deformation of the surface produced by constant normal pressure, p , acting over a strip O_1O_2

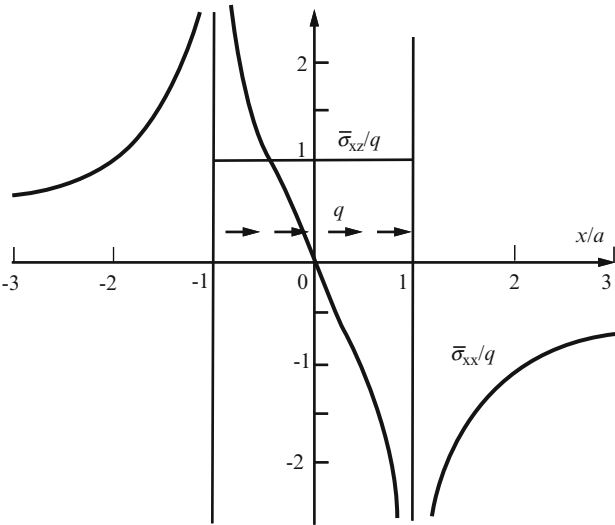


Fig. 9.11 Distribution of normal $\bar{\sigma}_{xx}$ and tangential $\bar{\sigma}_{xz}$ surface stresses produced by a constant tangential tension q acting over a band O_1O_2

The distribution of the associated surface stresses $\bar{\sigma}_{xx}$ and $\bar{\sigma}_{xz}$ is shown in Fig. 9.11. The shear stress is now distributed uniformly ($\bar{\sigma}_{xz} = -q/\pi$), but the tensile stress $\bar{\sigma}_{xx}$ has a singularity at O_1 and at O_2 . These stress concentrations certainly play an important role in *fretting-fatigue* damage that develops when a pair of surfaces in contact oscillates relative to each other (see Sect. 9.2.2.8).

9.2.2.4 Point Loading

General Methods of Solution

The load now is two-dimensional; the resulting stress distribution will be three-dimensional and, in general, all six components of the stress tensor will be non-zero. Problems of this type – the Boussinesq⁵ (1885) and Cerruti⁶ (1882) problem – can be solved using the potential theory (see Love⁷1952). There are special methods for treating problems in which the load is axisymmetric, meaning that the normal pressure p and the tangential stress q , are both functions of r only, independent of θ . The solutions are then simpler, since the shear components $\sigma_{r\theta}$ and $\sigma_{\theta z}$ vanish and the other stress components are independent of θ . We consider this case only.

Concentrated Normal Force

The normal point load P is applied at O as shown in Fig. 9.12.

The stresses are given by:

$$\begin{aligned}\sigma_{rr} &= \frac{P}{2\pi} \left\{ (1-2\nu) \left(\frac{1}{r^2} - \frac{z}{\rho r^2} \right) - \frac{3zr^2}{\rho^5} \right\} \\ \sigma_{\theta\theta} &= -\frac{P}{2\pi} (1-2\nu) \left(\frac{1}{r^2} - \frac{z}{\rho r^2} - \frac{z}{\rho^3} \right) \\ \sigma_{zz} &= -\frac{3P}{2\pi} \frac{z^3}{\rho^5} \\ \sigma_{rz} &= -\frac{3P}{2\pi} \frac{rz^2}{\rho^5}\end{aligned}\tag{9.19}$$

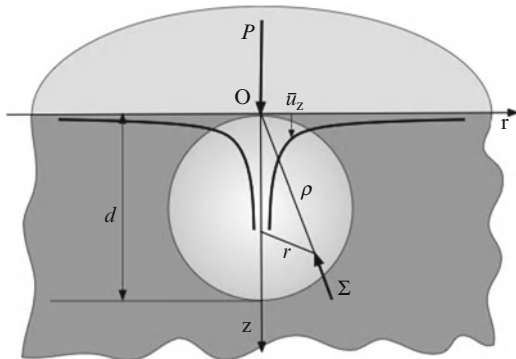
where $r^2 = x^2 + y^2$ and $\rho^2 = x^2 + y^2 + z^2$.

⁵Joseph Valentin Boussinesq (1843–1929) was a French mathematician and physicist who made significant contribution to the theory of hydrodynamics, vibration, light and heat.

⁶Valentino Cerruti (1850–1909) was an Italian mathematician from the University of Roma.

⁷Augustus Edward Hough Love (1863–1940) was an English mathematician famous for his work on the mathematical theory of elasticity.

Fig. 9.12 Normal point load P applied at O . The deformation of the surface is shown



In rectangular Cartesian coordinates, these stresses are written:

$$\begin{aligned}
 \sigma_{xx} &= \frac{P}{2\pi} \left[\frac{(1-2\nu)}{r^2} \left\{ \left(1 - \frac{z}{\rho}\right) \left(\frac{x^2 - y^2}{r^2}\right) + \frac{zy^2}{\rho^3} \right\} - \frac{3zx^2}{\rho^5} \right] \\
 \sigma_{yy} &= \frac{P}{2\pi} \left[\frac{(1-2\nu)}{r^2} \left\{ \left(1 - \frac{z}{\rho}\right) \left(\frac{y^2 - x^2}{r^2}\right) + \frac{zx^2}{\rho^3} \right\} - \frac{3zy^2}{\rho^5} \right] \\
 \sigma_{zz} &= -\frac{3P}{2\pi} \frac{z^3}{\rho^5} \\
 \sigma_{xy} &= \frac{P}{2\pi} \left[\frac{(1-2\nu)}{r^2} \left\{ \left(1 - \frac{z}{\rho}\right) \frac{xy}{r^2} - \frac{xyz}{\rho^3} \right\} - \frac{3xyz}{\rho^5} \right] \\
 \sigma_{xz} &= -\frac{3P}{2\pi} \frac{xz^2}{\rho^5} \\
 \sigma_{yz} &= -\frac{3P}{2\pi} \frac{yz^2}{\rho^5}
 \end{aligned} \tag{9.20}$$

The stresses σ_{zz} and σ_{rz} which act on planes parallel to the surface are independent of the Poisson ratio. The resultant stress Σ on these planes is:

$$\Sigma = (\sigma_{zz}^2 + \sigma_{rz}^2)^{1/2} = \frac{3Pz^2}{2\pi\rho^4} = \frac{3P}{2\pi d^2} = \text{constant} \tag{9.21}$$

and the direction of the resultant stress acts towards O . There is some analogy in this result with that for a concentrated line load (compare Figs. 9.4 and 9.12).

The displacements are:

$$\begin{aligned} u_r &= \frac{P}{4\pi\mu} \left\{ \frac{rz}{\rho^3} - (1-2\nu) \left(\frac{\rho-z}{\rho r} \right) \right\} \\ u_z &= \frac{P}{4\pi\mu} \left\{ \frac{z^2}{\rho^3} + \frac{2(1-\nu)}{\rho} \right\} \end{aligned} \quad (9.22)$$

The displacement at the free surface ($z=0$) is:

$$\bar{u}_r = -\frac{(1-2\nu)P}{4\pi\mu r}, \quad \bar{u}_z = \frac{(1-\nu)P}{2\pi\mu r} \quad (9.23)$$

There exists a strong singularity at O as for the case of concentrated normal line load (9.11).

9.2.2.5 Pressure Applied to a Circular Surface

The distribution of pressure p is axisymmetric and can be written in the general form:

$$p = p_0(1 - r^2/a^2)^n \quad (9.24)$$

Solutions were proposed for various values of n ; $n=0$ corresponds to a uniform pressure, $n=-1/2$ to a uniform displacement. In the following (Sect. 9.2.2.6) we discuss the *Hertz theory of contact* to which the value $n=1/2$ applies; consequently we consider only this value here. With this distribution the total load over a circle of radius a is

$$P = \frac{2\pi}{3} p_0 a^2 \quad (9.25)$$

Given the solution in Sect. 9.2.2.3.2 above, we can, by superposition, calculate the corresponding stress field and the associated displacement for the present case. The stresses along Oz can be calculated without difficulty by considering a ring of force concentrated on the circumference of a circle of radius r . They are given by:

$$\begin{aligned} \sigma_{rr} = \sigma_{\theta\theta} &= -p_0 \left[(1+\nu) \left(1 - \frac{z}{a} \arctan \frac{a}{z} \right) + \frac{1}{2} \left(1 + \frac{z^2}{a^2} \right)^{-1} \right] \\ \sigma_{zz} &= -p_0 \left(1 + \frac{z^2}{a^2} \right)^{-1} \end{aligned} \quad (9.26)$$

The principal stresses along Oz are σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} . It can be shown that the *maximum shear stress* $\tau = \frac{1}{2} (\sigma_{zz} - \sigma_{rr})$ is maximum at a distance $z = 0.48a$ below the surface and is equal to:

$$\tau_{1 \max} = \frac{1}{2} (\sigma_{zz} - \sigma_{rr}) = 0.31 p_0 = 0.47 P / \pi a^2 \quad (9.27)$$

The stresses on the surface of contact are:

– *in the interior of the circle*

$$\begin{aligned} \bar{\sigma}_{rr} &= p_0 \left\{ \left(\frac{1-2\nu}{3} \right) \left(\frac{a^2}{r^2} \right) \left[1 - \left(1 - \frac{r^2}{a^2} \right)^{3/2} \right] - \left(1 - \frac{r^2}{a^2} \right)^{1/2} \right\} \\ \bar{\sigma}_{\theta\theta} &= -p_0 \left\{ \left(\frac{1-2\nu}{3} \right) \left(\frac{a^2}{r^2} \right) \left[1 - \left(1 - \frac{r^2}{a^2} \right)^{3/2} \right] - 2\nu \left(1 - \frac{r^2}{a^2} \right)^{1/2} \right\} \\ \bar{\sigma}_{zz} &= -p_0 \left(1 - \frac{r^2}{a^2} \right)^{1/2} \end{aligned} \quad (9.28)$$

– *outside the circle*

$$\bar{\sigma}_{zz} = 0, \quad \bar{\sigma}_{rr} = -\bar{\sigma}_{\theta\theta} = \frac{1}{3} p_0 (1-2\nu) \frac{a^2}{r^2} \quad (9.29)$$

The radial stress here is thus tensile. It is maximum on the circumference ($r = a$) where it reaches the value:

$$(\bar{\sigma}_{rr})_{\max} = \frac{1-2\nu}{3} p_0 \simeq 0.13 p_0 \quad \text{with } \nu = 0.30 \quad (9.30)$$

This radial stress plays an important role in the fracture of brittle solids, such as ceramics or glasses crushed one against the other. In ductile materials, in contrast, the shear stress given by (9.27) produces a plastic sub-layer (see Sect. 9.2.3.2).

The deformation of the surface is given by:

– for $r < a$

$$\begin{aligned} \bar{u}_r &= -\frac{(1-2\nu)(1+\nu)}{3E} \frac{a^2}{r} p_0 \left[1 - \left(1 - \frac{r^2}{a^2} \right)^{3/2} \right] \\ \bar{u}_z &= \frac{1-\nu^2}{E} \frac{\pi}{4a} p_0 (2a^2 - r^2) \end{aligned} \quad (9.31)$$

– for $r > a$

$$\begin{aligned}\bar{u}_r &= -\frac{(1-2\nu)(1+\nu)}{3E}p_0\frac{a^2}{r} \\ \bar{u}_z &= -\frac{(1-\nu^2)}{E}\frac{p_0}{2a}\left[(2a^2-r^2)\arcsin(a/r) + ar\left(1-\frac{a^2}{r^2}\right)^{1/2}\right]\end{aligned}\quad (9.32)$$

9.2.2.6 Hertz Theory of Normal Contact

This theory was developed in 1882 (Hertz 1882a, b). The contacting surfaces are no longer perfectly flat. The Hertz theory predicts the dimensions of the contact area and the distribution of stresses and displacements. *It relates these distributions to the contact pressure and to the geometry and elastic moduli of the solids in contact.* We give here a simplified account. More details can be found in Johnson (1985).

Geometry of the Problem

We assume that the surfaces of the solids in contact are regular and that their profiles close to the origin can be adequately represented by a quadratic form. With an appropriate choice of axes the initial profiles can then be expressed in terms of the principal radii of curvature of the solids, R'_1 and R''_1 for solid 1 and R'_2 and R''_2 for solid 2 (Fig. 9.13).

Thus:

$$\begin{aligned}z_1 &= \frac{x_1^2}{2R'_1} + \frac{y_1^2}{2R''_1} \\ z_2 &= -\left(\frac{x_2^2}{2R'_2} + \frac{y_2^2}{2R''_2}\right)\end{aligned}\quad (9.33)$$

The separation of the surfaces is $h = z_1 - z_2$ and the coordinate system (x, y) can be chosen so that h can be written as:

$$h = Ax^2 + By^2 = \frac{x^2}{2R'} + \frac{y^2}{2R''}\quad (9.34)$$

where A and B are positive constants and R', R'' are the *relative principal radii of curvature*. When the axes (x_1, y_1) and (x_2, y_2) are inclined at an angle α , it can be shown that:

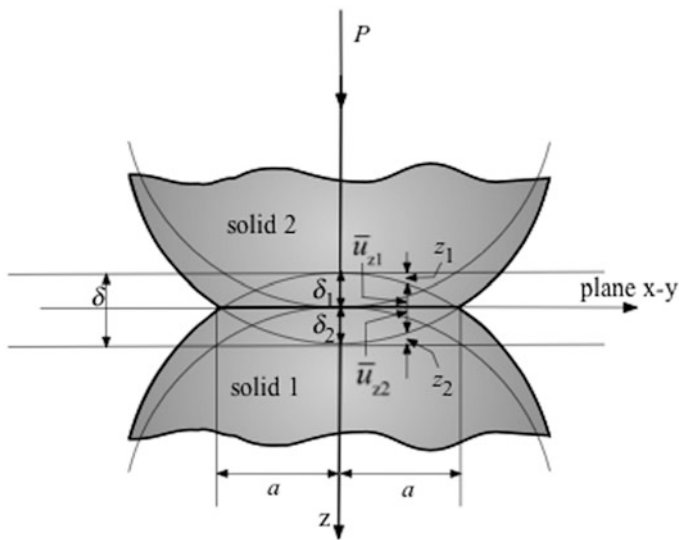


Fig. 9.13 Hertz contact of two solids 1 and 2. The undeformed surfaces are indicated with *thin lines*; the deformed surfaces are *shaded*

$$\begin{aligned}
 A + B &= \frac{1}{2} \left(\frac{1}{R'} + \frac{1}{R''} \right) = \frac{1}{2} \left(\frac{1}{R'_1} + \frac{1}{R''_1} + \frac{1}{R'_2} + \frac{1}{R''_2} \right) \\
 B - A &= \frac{1}{2} \left[\left(\frac{1}{R'_1} - \frac{1}{R''_1} \right)^2 + \left(\frac{1}{R'_2} - \frac{1}{R''_2} \right)^2 \right. \\
 &\quad \left. + 2 \left(\frac{1}{R'_1} - \frac{1}{R''_1} \right) \left(\frac{1}{R'_2} - \frac{1}{R''_2} \right) \cos 2\alpha \right]^{1/2} \quad (9.35)
 \end{aligned}$$

An equivalent radius R_e is often introduced, defined by:

$$R_e = (R' R'')^{1/2} = \frac{1}{2} (AB)^{-1/2} \quad (9.36)$$

These radii are taken as positive for convex surfaces and negative for concave.

For convenience, we consider two convex surfaces in contact under the effect of a normal load P (Fig. 9.13). Before contact, the distance between a pair of points $\delta_1 (x, y, z_1)$ and $\delta_2 (x, y, z_2)$ is given by (9.34). During the compression, two points T_1, T_2 approach one another by amounts δ_1, δ_2 , and if the solids were not deformed they would overlap each other, as shown by the heavy lines in the diagram. A contact pressure is created, under the influence of which each point on the surface of the solids is displaced parallel to Oz by a quantity $\bar{u}_{z1}$ or $\bar{u}_{z2}$. If after the deformation the points S_1, S_2 within the contact area coincide, then:

$$\bar{u}_{z1} + \bar{u}_{z2} + h = \delta_1 + \delta_2 = \delta \quad (9.37)$$

Thus the elastic displacement at the surface can be expressed as:

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - Ax^2 - By^2 \quad (9.38)$$

If S_1 and S_2 are located outside the contact area, they will not come into coincidence and for these:

$$\bar{u}_{z1} + \bar{u}_{z2} \leq \delta - Ax^2 - By^2 \quad (9.39)$$

To solve the problem we have to find the distribution of the pressure transmitted between the solids over the contact area, such that the displacements normal to this surface satisfy these boundary conditions.

Approximate Solution

To simplify the problem we consider only solids of revolution for which the contact area is a circle of radius a , or cylinders whose axes are parallel, for which this area is a band of width $2a$.

It can be noted that in Fig. 9.13, $\bar{u}_{z1}(0) = \delta_1$ and $\bar{u}_{z2}(0) = \delta_2$ so that (9.37) can be written in non-dimensional form:

$$\left(\frac{\bar{u}_{z1}(0) - \bar{u}_{z1}(x)}{a} \right) + \left(\frac{\bar{u}_{z2}(0) - \bar{u}_{z2}(x)}{a} \right) = \frac{1}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \frac{x^2}{a} \quad (9.40)$$

Putting $x = a$, the left side of (9.40) gives an approximation for the deformation of the pair of solids: if $\bar{u}_{zi}(0) - \bar{u}_{zi} = d_i$ ($i = 1, 2$), we have:

$$\frac{d_1 + d_2}{a} = \frac{a}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (9.41)$$

from which the mean contact pressure, p_m , is given by:

$$\frac{p_m}{E_1} + \frac{p_m}{E_2} \simeq a \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (9.42a)$$

that is:

$$p_m \simeq a \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \bigg/ \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \quad (9.42b)$$

For *contact between two cylinders*, $P = 2a p_m$

whence:
$$a \propto \left[P \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \bigg/ \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{1/2} \quad (9.43)$$

$$\text{and:} \quad p_m \propto \left[P \left(\frac{1}{R_1} + \frac{1}{R_2} \right) / \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \right]^{1/2} \quad (9.44)$$

showing that both the pressure, p_m , and the contact width, a , increase as the square root of the applied load.

For *contact between two spheres*, $P = \pi a^2 p_m$

$$a \propto \left[P \left(\frac{1}{E_1} + \frac{1}{E_2} \right) / \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{1/3} \quad (9.45)$$

$$\text{and:} \quad p_m \propto \left[P \left(\frac{1}{R_1} + \frac{1}{R_2} \right) / \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \right]^{1/3} \quad (9.46)$$

with now a and p_m varying as $p^{1/3}$.

Elastic Contact – Hertz Solution

The exact solution can be obtained with the following assumptions:

- the surfaces are continuous and non-conforming,
- the strains are small,
- each solid can be regarded as a semi-infinite body: $a \ll R_{1,2}$,
- there is no friction: $q_x = q_y = 0$.

We confine our treatment to solids of revolution ($R'_1 = R''_2 = R_1$ and $R'_2 = R''_2 = R_2$) and cylinders in contact along a common generator. Moreover only the results are given. Full details can be found elsewhere (*e.g.* Johnson [1985](#)).

Solids of Revolution

The contact area is a circle of radius a and $A + B = \frac{1}{2} (1/R_1 + 1/R_2)$ in (9.35). The boundary conditions are:

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - (1/2R) r^2 \quad (9.47a)$$

where R is the relative curvature defined by:

$$1/R = 1/R_1 + 1/R_2 \quad (9.47b)$$

and r is the distance to the origin in the contact circle. A parabolic distribution of pressure, $p = p_0 (1 - r^2/a^2)$, gives rise to displacements which satisfy (9.47a). This distribution gives normal displacements:

$$\bar{u}_z = \frac{1 - \nu^2}{E} \frac{\pi p_0}{4a} (2a^2 - r^2) \quad \text{for } r \leq a \quad (9.47c)$$

Substituting the expressions for $\bar{u}_{z1}$ and $\bar{u}_{z2}$ into (9.47a) we obtain:

$$a = \pi p_0 R / 2E^* \quad \text{and} \quad \delta = \pi a p_0 / 2E^* \quad (9.48a)$$

with:
$$\frac{1}{E^*} = \frac{1 - \nu^2}{E_1} + \frac{1 - \nu^2}{E_2} \quad (9.48b)$$

The total load is:
$$P = \int_0^a p(r) 2\pi r dr = \frac{2}{3} p_0 \pi a^2 \quad (9.49)$$

This leads to the following expressions for the contact area, a , the distance of approach of the two spherical solids, and the pressure p_0 , in terms of the load P :

$$\begin{aligned} a &= \left(\frac{3PR}{4E^*} \right)^{1/3} \\ \delta &= \frac{a^2}{R} \left(\frac{9P^2}{16RE^{*2}} \right)^{1/3} \\ p_0 &= \frac{3P}{2\pi a^2} \quad \text{with } p = p_0 \left(1 - \frac{r^2}{a^2} \right) \end{aligned} \quad (9.50)$$

The stress distribution within the solids has already been given (9.26); it is shown in Fig. 9.14, with the distribution due to a uniform pressure for comparison, the stresses being given in relation to the mean pressure, $p_m = (2/3) p_0$. The following important comments should be made.

1. The stresses over the contact area are all compressive, except on the boundary circle where σ_{rr} is tensile. The maximum value of this stress is $(1 - 2\nu)p_0/3$ at $r = a$, $z = 0$ (see (9.29)). This explains the formation of a cone shaped crack by indentation of brittle solids, such as glass; the extent of this crack allows estimating the fracture toughness.
2. The stress state at the centre ($r = 0$, $z = 0$) is:

$$\sigma_{rr} = \sigma_{\theta\theta} = (1 + 2\nu) p_0 / 2 \quad \text{and} \quad \sigma_{zz} = p_0 \quad (9.51)$$

Thus for an incompressible solid ($\nu = 0.5$) the stress state is purely hydrostatic.

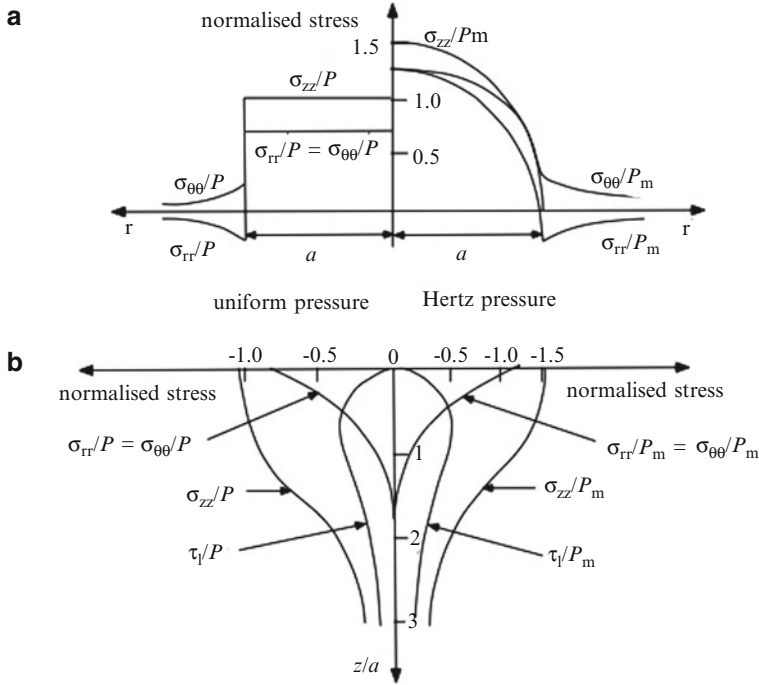


Fig. 9.14 Stress distribution at the surface (a) and along the axis of symmetry (b) due to uniform pressure (on the left) and Hertz parabolic distribution acting on a circular area radius, a

3. For $\nu = 0.30$ the shear stress $\tau_1 = (1/2)(\sigma_{rr} - \sigma_{\theta\theta})$ on the axis of symmetry ($r = 0$) is maximum at $z = 0.48a$ where the value is approximately $0.31p_0$. At the origin ($z = 0$) it is only $0.10p_0$ and on the boundary of the contact area ($r = a$, $z = 0$) it is $0.13p_0$.

We shall see later (Sect. 9.2.3.2) that this accounts for the development of plastic deformation in a sub-layer below the contact surface.

Contact of a Pair of Cylinders

We can use the solution given previously in Sect. 9.2.2.2. Thus we have:

$$h = z_1 + z_2 = Ax^2 = \frac{1}{2} (1/R_1 + 1/R_2) x^2 = \frac{1}{2} (1/R) x^2 \quad (9.52)$$

For points located on the contact area:

$$\bar{u}_{z1} + \bar{u}_{z2} = \delta - Ax^2 = \delta - \frac{1}{2} (1/R) x^2 \quad (9.53a)$$

For those outside this region:

$$\bar{u}_{z1} + \bar{u}_{z2} \geq \delta - \frac{1}{2} (1/R) x^2 \quad (9.53b)$$

We now assume, with Hertz, that the displacements $\bar{u}_{z1}$ and $\bar{u}_{z2}$ can be obtained by considering each cylinder as a semi-infinite body. However, the logarithmic singularity (9.17) gives rise to a difficulty in the calculation of the displacements. The local contact stresses alone no longer determine uniquely the distance δ , and the complete stress distribution in the bulk of the solids must be considered. Solutions were given for a number of specific problems, for example, a small cylinder compressed between two larger cylinders along diametrically opposite generators.

It can be shown that the pressure distribution remains parabolic, with:

$$p(x) = \frac{2P}{\pi a} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{1/2} \quad (9.54)$$

As in the case of solids of revolution, we can expect that the stress distribution does not differ greatly from that produced by a uniform pressure distribution (see Sect. 9.2.2.3.1). With this assumption the solution is:

$$\begin{aligned} a &= 2 \left(\frac{PR}{\pi E^*} \right)^{1/2} \\ p_0 &= \frac{2P}{\pi a} = \frac{4}{\pi} p_m = \left(\frac{PE^*}{\pi R} \right)^{1/2} \end{aligned} \quad (9.55)$$

Along Oz the stresses are:

$$\begin{aligned} \sigma_{xx} &= -\frac{p_0}{a} \left[(a^2 + 2z^2) (a^2 + z^2)^{-1/2} - 2z \right] \\ \sigma_{zz} &= -p_0 a (a^2 + z^2)^{-1/2} \\ \tau_1 &= p_0 a \left[z - z^2 (a^2 + z^2)^{-1/2} \right] \end{aligned} \quad (9.56)$$

The shear stress τ_1 reaches its maximum value of $0.3 p_0$ at $z = 0.78a$; the stress profile is qualitatively close to that obtained previously (Fig. 9.15). This figure shows how the principal stresses σ_{xx} , σ_{zz} vary along Oz and gives contours of equal values of the shear stress τ_1 .

Note: we have not taken into account any friction that may be generated by the normal contact; this will occur only when the solids have different elastic constants. This situation was also treated to some extent in the literature.

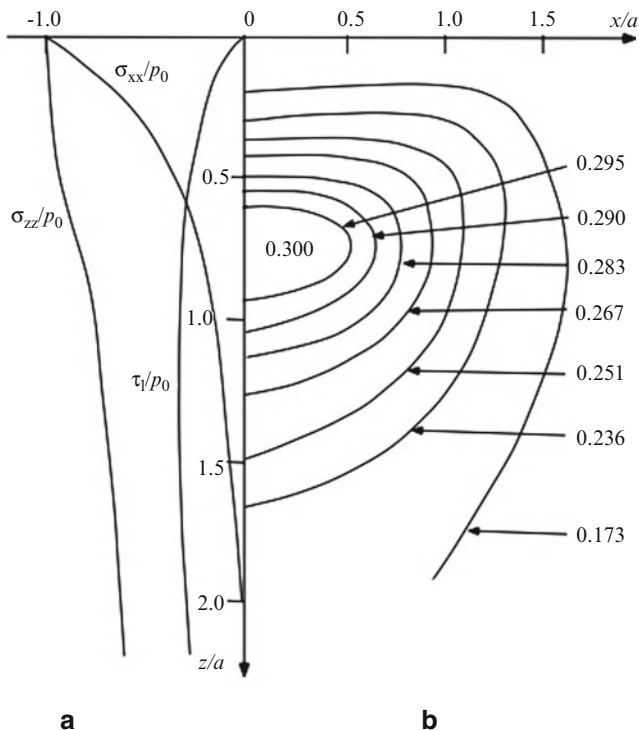


Fig. 9.15 Contact of two cylinders: (a) subsurface stresses along the axis of symmetry, (b) contours of shear stress τ_1

9.2.2.7 Hertz Theory of Tangential Loading and Sliding

Relative Sliding of Two Solids: Case of Two Cylinders

We consider only linear sliding (Fig. 9.16). The initial point of contact is taken as the origin and remains fixed; solid 1, below the contact surface, moves with linear velocity v from left to right with respect to solid 2 above, which remains fixed. In the absence of friction the normal force, P , would produce the contact area given by Hertz's theory (9.54).

The friction force Q , acting tangentially, is opposed to this relative movement and will modify the contact stresses. We assume that the two solids are in steady linear relative motion (*kinetic friction*). Later we consider the onset of sliding (*static friction*) corresponding to $Q < \mu P$ where μ is the coefficient of friction.

It is usual to take Coulomb's law for the friction, that is, that p and q are in a constant ratio at all points of the contact area:

$$|q(x, y)|/p(x, y) = |Q|/P = \mu \quad (9.57)$$

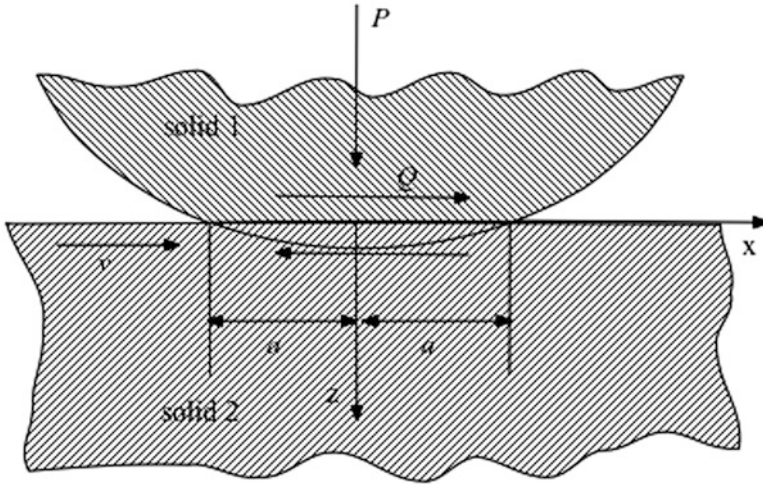


Fig. 9.16 Sliding contact

The only relative movement considered is that of one cylinder in a direction perpendicular to its axis, which corresponds to a two-dimensional problem. For the two cylinders in contact the Hertz solution for the distribution of normal pressure (Sect. 9.2.2.6.3) is given by (9.54) from which it follows that the tangential stress distribution is:

$$q(x) = \pm \frac{2\mu P}{\pi a} \left[1 - \left(\frac{x}{a} \right)^2 \right]^{1/2} \quad (9.58)$$

The conditions over the contact area, a band of width $2a$, are given by the superposition of these two distributions. As in the case of normal or tangential line loadings (Sect. 9.2.2.3), there is an analogy between the stress distributions. This is expressed by two relations:

$$\begin{aligned} \frac{(\sigma_{zz})_q}{q_0} &= \frac{(\sigma_{xz})_p}{p_0} \\ \frac{(\sigma_{xz})_q}{q_0} &= \frac{(\sigma_{xx})_p}{p_0} \quad \text{with } q_0 = \mu p_0 \end{aligned} \quad (9.59)$$

Thus from the solution given by Hertz one can obtain the components $(\sigma_{zz})_q$ and $(\sigma_{xz})_q$. The stress σ_{xx} , however, has to be determined separately. We have seen (Sect. 9.2.2.3.2) that for a uniform distribution $q(x) = q$ over the contact surface, $\bar{\sigma}_{xx}$ has a singularity, whilst for a parabolic distribution such that $q(x) = 0$ for $x = \pm a$ the situation is very different and this stress remains finite. It can be shown that the value is:

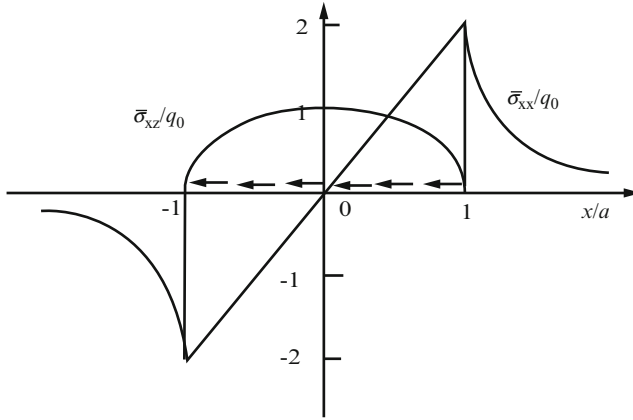


Fig. 9.17 Surface stresses generated by friction

$$\begin{aligned}
 (\bar{\sigma}_{xx})_q &= -2q_0x/a \quad \text{for } |x| \leq a \\
 &= -2q_0 \left[\frac{x}{a} \pm \left(\frac{x^2}{a^2} - 1 \right)^{-1/2} \right] \quad \text{for } x \geq a
 \end{aligned} \tag{9.60}$$

This stress distribution within the moving solid is shown in Fig. 9.17.

Over the contact surface the stresses due to the joint action of the load P and the friction force Q are obtained by superposing those due to the separate loads. The result is:

$$\begin{aligned}
 \bar{\sigma}_{xx} &= -p_0 \left[\left(1 - \frac{x^2}{a^2} \right)^{1/2} + 2\mu \frac{x}{a} \right] \\
 \bar{\sigma}_{zz} &= -p_0 \left(1 - \frac{x^2}{a^2} \right)^{1/2} \\
 \bar{\sigma}_{yy} &= \nu (\bar{\sigma}_{xx} + \bar{\sigma}_{zz}) \\
 \bar{\sigma}_{xz} &= -\mu p_0 \left(1 - \frac{x^2}{a^2} \right)^{1/2}
 \end{aligned} \tag{9.61}$$

The tensile stress $\bar{\sigma}_{xx}$ is maximum at the right-hand edge of the sliding surface, where it has the value:

$$(\bar{\sigma}_{xx})_{\max} = -2\mu p_0 \tag{9.62}$$

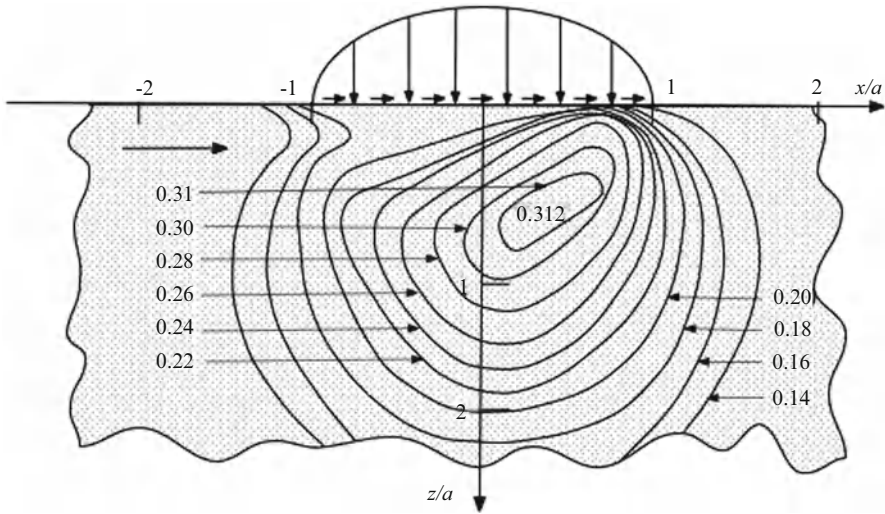


Fig. 9.18 Contours for the shear τ_1 in the sub-layer for sliding contact with $Q = 0.20P$

In the plane of contact the maximum shear stress τ_1 is:

$$\tau_1 = \frac{1}{2} \left[(\bar{\sigma}_{xx} - \bar{\sigma}_{zz})^2 + 4\bar{\sigma}_{xz}^2 \right]^{1/2} = \mu p_0 \quad (9.63)$$

As expected, the effect of sliding is to generate shear stresses on the contact surface. We shall see the influence of these stresses on the onset of plastic yielding (Sect. 9.2.3.2).

The effect of sliding is to move the position of the maximum value for τ_1 closer to the surface and in a direction parallel to that of slip. Figure 9.18 shows this effect for the case of $\mu = 0.20$.

In this figure the stress contours are given for normalised values τ_1/p_0 . It can be noted that the maximum is only very slightly above the value of 0.30 given by the Hertz theory for normal contact (see Fig. 9.15).

Stick and Incipient Slipping

It is a matter of common experience that the sliding over long distances of one body over another under a steady pulling force proceeds sometimes at constant or nearly constant velocity, on other occasions at velocities that fluctuate widely. In most practical sliding systems these fluctuations of the sliding velocity are considered a serious nuisance. All *stick-slip* processes are caused by the fact that the friction force does not remain constant as a function of some other variable, in particular the normal load (see Sects. 9.3.3 and 9.3.4).

Here we consider only *incipient slipping of elastic bodies*. A tangential force whose magnitude is less than the force of limiting friction, when applied to two bodies pressed into contact, will not give rise to a gross sliding motion but, nevertheless, will induce frictional tractions at the contact surface. In this part we shall examine the tangential surface tractions which arise from a combination of normal and tangential forces which does not cause the bodies to slide relative to each other. We shall consider two cylinders in contact with their axes parallel to the y axis, compressed by a normal force P per unit length, to which a tangential force Q per unit length ($< \mu P$) is subsequently applied. The contact width and the pressure distribution due to P are given by the Hertz theory. The quantities are assumed to be unaffected by the subsequent application of Q . We assume also that the coefficient of friction is sufficiently high to prevent slip throughout the whole contact area. Thus the complete strip $-a < x < a$ is a “stick” area in which the condition of no-slip applies, *i.e.*

$$\bar{u}_{x1} - \bar{u}_{x2} = \delta = \text{constant} \quad \text{for } -a \leq x \leq a \quad (9.64)$$

It can be shown that the required tangential stress distribution, $q(x)$, which gives rise to a constant normal displacement is given by:

$$q(x) = \frac{Q}{\pi(a^2 - x^2)^{1/2}} = q' \quad (9.65)$$

The traction given by (9.65) is plotted in Fig. 9.19. It rises to a theoretically infinite value at the edges of the contact. This result is not surprising when it is remembered that the original assumption that there should be no slip at the interface requires that the two bodies should behave as one. The points $x = \pm a$ then appear as the tips of two sharp deep cracks in mode II (see Sect. 2.2.4.1 for the definition).

This would result in a distribution of infinite stress at the edge of the contact band, which is physically impossible. *Microsliding* over two bands bordering the edges, of width $(a - c)$, as in Fig. 9.19, will take place. In these bands the distribution will be such that $q = q' = \mu p$, given by Eq. 9.65, while, in the central zone, the contact remains stick and $q(x) = q' - q''$. The extent of *microsliding* zone is found from the equilibrium conditions. Thus:

$$\begin{aligned} Q &= \int_{-a}^{+a} q(x) dx = \int_{-a}^{+a} q'(x) dx + \int_{-c}^{+c} q''(x) dx \\ &= \mu P - \frac{c^2}{a^2} \mu P \end{aligned} \quad (9.66)$$

whence

$$c/a = \left(1 - \frac{Q}{\mu P}\right)^{1/2} \quad (9.67)$$

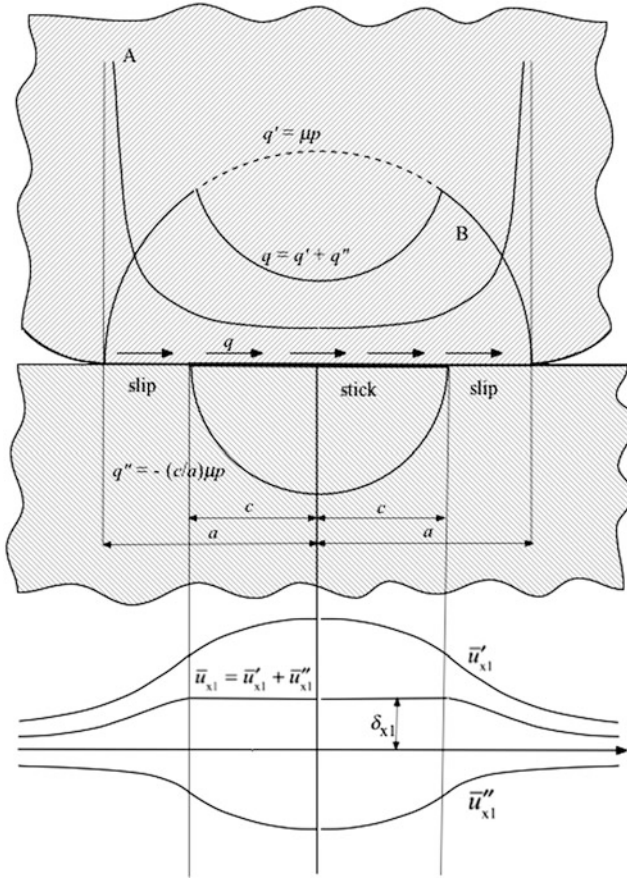


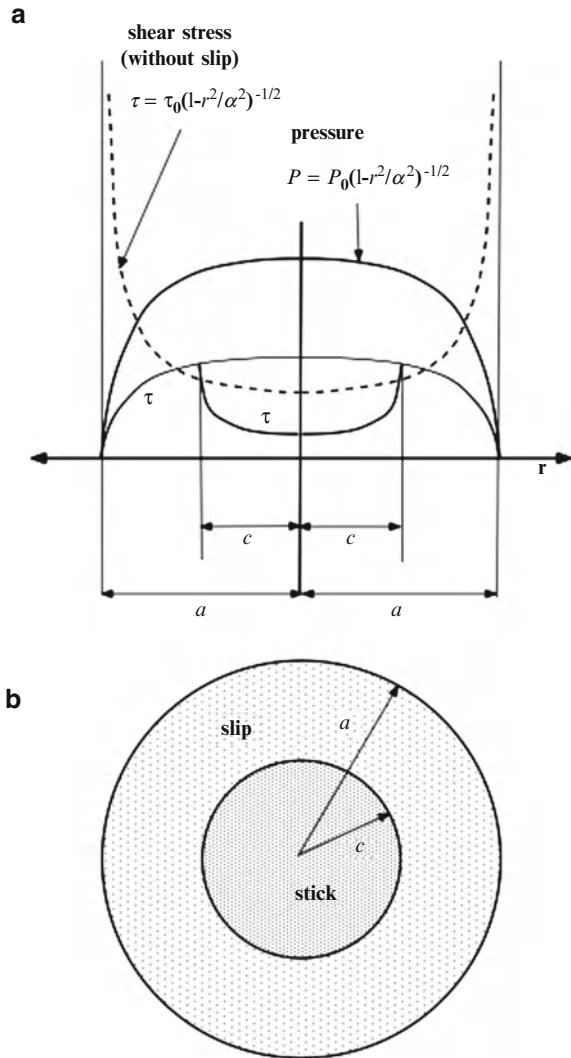
Fig. 9.19 Contact of two cylinders along a common generatrix. Surface tractions and displacements due to a tangential force $Q < \mu P$. Curve A: no slip; curve B: partial slip. $\left(q(x) = q'(x) + q''(x) = \mu p_0 \left[(a^2 - x^2)^{1/2} - (c^2 - x^2)^{1/2} \right] / a \right)$

Note: for the contact between a sphere and a plane, the zone in which stick occurs is a circle of radius c and *microsliding* occurs in a ring of radii c and a , (Fig. 9.20) such that:

$$c/a = \left(1 - \frac{Q}{\mu P} \right)^{1/3} \quad (9.68)$$

The corresponding stress distribution is shown in Fig. 9.21 for the values of $\mu = 0.25$ and 0.50 . In this figure the fixed solid is uppermost. In the *microslip* zone

Fig. 9.20 Contact between a sphere and a plane: (a) section; (b) plan view



the stress $\bar{\sigma}_{xx}$ ($z = 0$) is tensile and acts on the relative edge of the zone in relative motion with the maximum value $2\mu p_0$ obtained previously in (9.62). In the central zone, in particular at $x = 0$, this stress is lower than p_0 .

Thus the surface tensile stress $\bar{\sigma}_{xx}$ is much greater in the presence of *microsliding* than in the case of simple normal contact as shown in Fig. 9.21. This is one of the sources of *fretting-fatigue*, a form of damage which results from two surfaces rubbing against one another and which is often encountered in certain mechanical systems. This specific damage mechanism is briefly discussed in the next section.

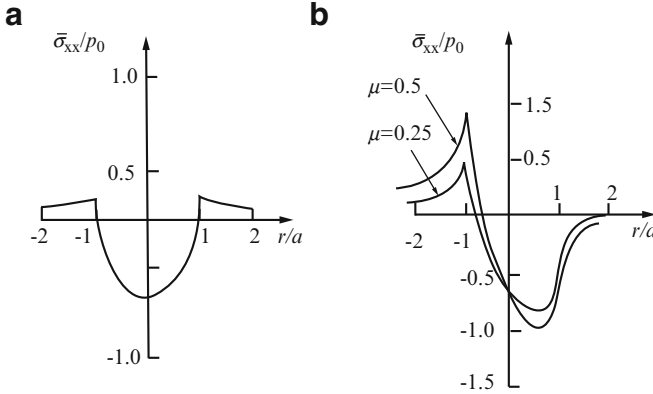
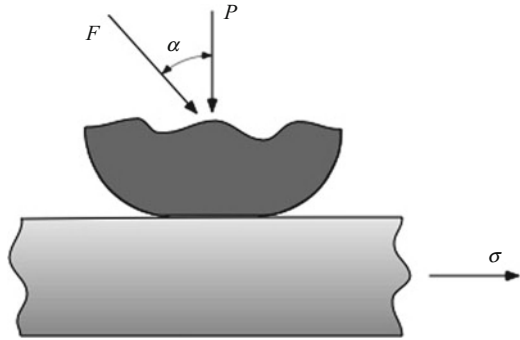


Fig. 9.21 Contact between a sphere and a plane: distribution of the surface stress $\bar{\sigma}_{xx}$ in the contact plane under conditions of (a) no sliding, (b) microslip ($\mu = 0.25$ and 0.50)

Fig. 9.22 Specimen subjected to fretting-fatigue



9.2.2.8 Fretting-Fatigue

Fretting is the oscillatory motion of small “slip” amplitude between two contacting surfaces. It is promoted by high frequency, low amplitude vibratory motion and commonly occurs in clamped joints, in key-ways, splines and dovetail joints (see *e.g.* Lindley 1997). The surface damage produced by fretting can take the form of fretting wear (with or without environmental assistance) or *fretting-fatigue* when the material is submitted to a cyclic load. Early research on fretting-fatigue employed a contact pad clamped to a fatigue specimen in order to determine the fatigue S-N curves, with or without fretting. The principle of the experiments is shown in Fig. 9.22. The specimen is loaded by a cyclic stress σ and the pad is clamped by a constant normal force P ; an inclined cyclic force F is applied.

The extent of the reduction in fatigue strength shown schematically in Fig. 9.23 is found to be affected by the following variables: (i) slip amplitude, (ii) magnitude and distribution of contact pressure, (iii) the particular material combination and condition of the mating surfaces, (iv) frictional forces and near surface stresses,

Fig. 9.23 Reduction in fatigue strength caused by fretting-fatigue

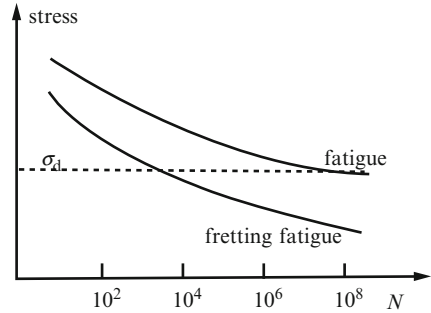
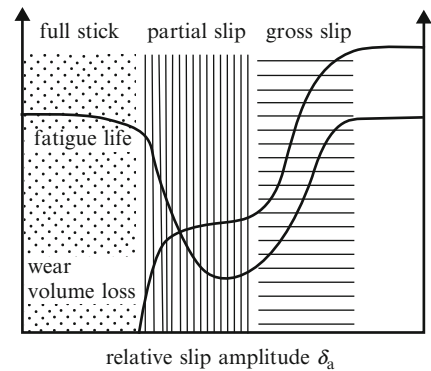


Fig. 9.24 Conditions for fretting-wear and fretting-fatigue (Baietto-Dubourg and Lindley 2011)



(v) cyclic frequency, (vi) temperature, and (vii) environment (see Waterhouse 1981a, b). Depending upon the value of relative slip amplitude between the two bodies in contact, fretting wear can involve full stick, partial slip or gross slip (Fig. 9.24). The full influence of fretting-fatigue has been associated with the partial slip regime which corresponds roughly to the minimum of fatigue life for given contact conditions (Baietto-Dubourg and Lindley 2011).

Fatigue S-N curves with and without fretting for 3.5%NiCrMoV steel specimen and 1%CrMo steel pads are given in Fig. 9.25. These results show a significant reduction of the fatigue strength even for small nominal contact pressure.

Several investigators attempted to quantify fretting-fatigue in terms of these parameters (see *e.g.* Waterhouse 1981a, b; Attia and Waterhouse 1992; Kuno and Waterhouse 1989; Hills and Nowell 1994; Lindley 1997; Dubourg et al. 2003).

This particular mode of damage was the subject of a large number of studies. More recently, interest concentrated on the formation of the cracks which develop preferentially in the microslip zone, as indicated schematically in Fig. 9.22 and as shown in Fig. 9.26 (King and Lindley 1980).

It is clear that fretting-fatigue cannot be analysed using only the elastic theory of contact mechanics. Other forms of damage occur in the contact between the specimen and the rubbing pad, in particular plasticity controlled mechanisms which will be discussed in Sect. 9.2.3. However the elastic analysis which shows the

Fig. 9.25 Fatigue S-N curves with and without fretting for 3.5%NiCrMoV steel specimen and 1%Cr steel specimen and 1%CrMo steel bridge pads at zero mean stress (Adapted from Lindley 1997)

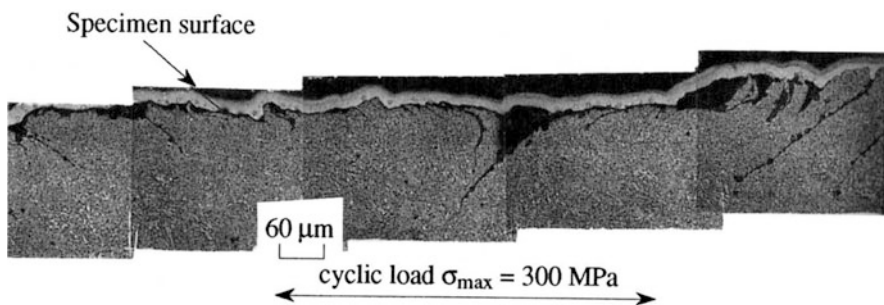
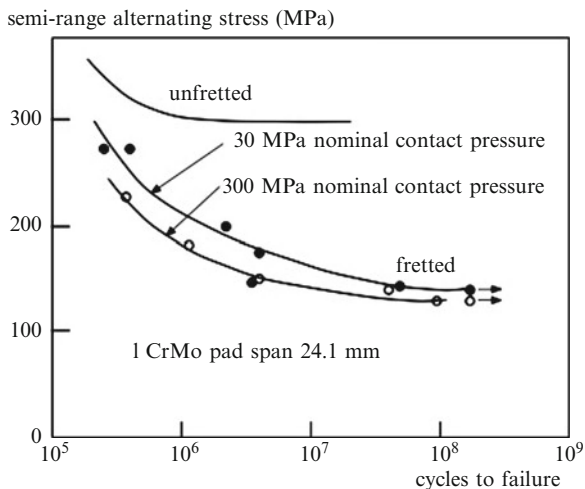


Fig. 9.26 3.5%NiCrMoV steel with cracks initiated by fretting-fatigue. A protective layer of nickel was deposited on the rubbing surface to make observation of the microcracks easier. Notice that the cracks make an angle of about 45 degrees with the direction of the applied cyclic load (King and Lindley 1980)

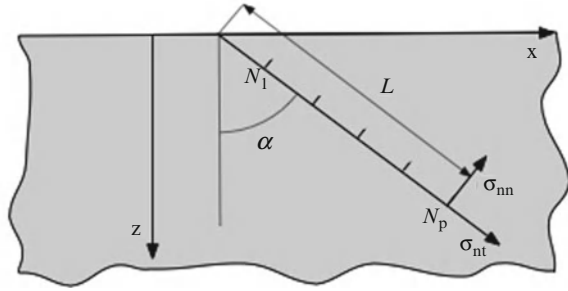
existence of high longitudinal tensile stresses acting on the leading edge of the zone in relative motion (see Fig. 9.21) and which are superimposed to the fluctuating applied stresses may be useful to explain, at least qualitatively, the phenomenon.

A brief description of the models proposed for the prediction of the initiation of contact fatigue microcracks as well as the stage I/stage II transition is given below. For the definition of these stages, see Chap. 6, Sect. 6.3.1.3.

Two stages in the fretting-fatigue damage are considered: crack initiation and crack growth.

Several studies showed that fretting-fatigue cracks could be initiated at a very early stage ($<5\%$) in the fretting-fatigue life (Endo and Goto 1976; Endo 1981; Lindley 1997). These cracks propagate first obliquely to the specimen surface (Fig. 9.25) under the combined action of tangential fretting forces and cyclic bulk stresses (see *e.g.* Hills and Nowell 1994). When the crack reaches a certain depth,

Fig. 9.27 Definition of the angle and the averaged parameters for an oblique crack initiated from a fretting-fatigue contact



$$\tau_m = \frac{1}{n_p} \sum_1^{n_p} \sigma_{nt} \quad \text{average shear stress}$$

$$\sigma_m = \frac{1}{n_p} \sum_1^{n_p} \sigma_{nn} \quad \text{average tensile stress}$$

which depends upon the applied cyclic stress, as well as the prevailing contact conditions, further crack propagation occurs perpendicularly to the applied stress. This deviation takes place essentially because the friction forces will then become insignificant.

Crack initiation and the early stages of crack propagation at an angle α with the surface are largely controlled by the friction stresses. Lamacq et al. (1997) and Lamacq and Dubourg (1999) proposed to average the macroscopic stress-strain field on small segments of length L (Fig. 9.27) along virtual crack planes inclined at various directions with respect to the contact plane. Two types of cracks were considered: tensile and shear cracks. For each type of cracks, relevant mechanical parameters were proposed by these authors to predict the position of the initiation sites and the direction of the propagation of microcracks. This approach, based on the critical plane concept is extended up to a given depth corresponding to the transition in the direction of the propagating macrocrack. (See also Dang Van 2011 for critical plane concept). The direction of the bifurcation ($\alpha = 90^\circ$) corresponds to that where the influence of mode I (for the definition see Sect. 2.2.4.1 in this volume) becomes predominant.

Crack growth is analysed using the fracture mechanics approach (Baietto-Dubourg and Lindley 2011). The effective stress-intensity factor has two components, one associated with the applied cyclic load, the other one with normal pressure load. Special attention must be given to the *short crack effect* (see Sect. 6.4.3.4) and to the level of *crack closure* (see Sect. 6.3.2.1a) which can reach high values because of the normal load.

This brief overview of fretting fatigue has shown that we are still very far from a perfect understanding of this complex phenomenon. In particular thermal effects which are known to have a strong influence on wear mechanisms for gross sliding (see Sect. 9.4.2) are not yet considered, although they play an important role on surface oxidation and thus sliding conditions.

9.2.3 Introduction to Plasticity in Contact Mechanics

9.2.3.1 Introductory Comments

The analysis of contact mechanics in the presence of plasticity is more complex than in the case of elasticity. There is non-linearity from two sources: the geometry of the Hertz contact, and the constitutive equations. Further, as in all problems involving plasticity, the choice of the constitutive equations is a matter of great importance (see Chap. 3 in Volume I); consequently we shall just make a few simple comments and limit the discussion to the case of *perfectly plastic solids*. We shall study only the onset of plastic yielding. It is well to remember that the problem of the limit load for punching was treated in Chap. 3 (Sect. 3.5.3.4) of Volume I.

9.2.3.2 Onset of Plastic Yield

A simple approach to the problem starts from the solutions already obtained for elasticity. We assume Tresca's conditions for plasticity, *i.e.* $\tau_1 = k = R_p/2$ where k and R_p are the yield strengths for shear and tension, respectively.

For *normal contact of two cylinders* we have shown (Fig. 9.15) that the shear stress τ_1 reaches a maximum value of $0.30p_0$ at a distance of $0.78a$ on the z -axis below the contact surface. Substituting in the Tresca criterion gives the Hertz pressure p_{0Y} corresponding to the onset of plastic yield:

$$p_{0Y} = 3.33k = 1.67R_p \quad (9.69a)$$

The von Mises criterion would lead to:

$$p_{0Y} = 3.10k = 1.79R_p \quad (9.69b)$$

The load for initial yield is then given by substituting the critical value of p_0 in (9.55) to give:

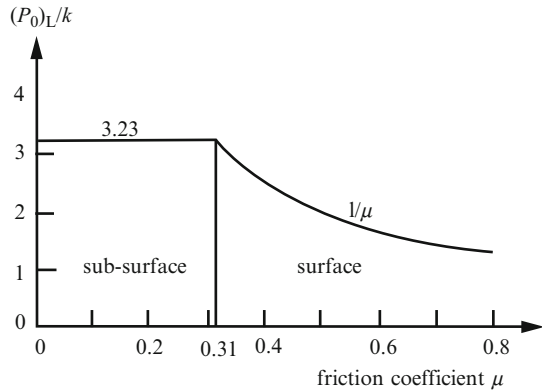
$$P_Y = \frac{\pi R}{E^*} (p_{0Y})^2 = 2.79 \frac{\pi R}{E^*} R_p^2 = 8.76 \frac{R}{E^*} R_p^2 \quad (9.70)$$

This equation shows that, for a given geometry, it is preferable to use materials with high yield strength and a low elastic modulus to avoid plastic deformation below the contact surface.

This effect is still more pronounced in the case of two *contacting spherical solids*. We know that τ_1 has its maximum value $0.31p_0$ at $z = 0.48a$ (Sect. 9.2.2.6), from which we find:

$$p_{0Y} = 3.23k = 1.61R_p \quad (9.71)$$

Fig. 9.28 Simplified analysis of the onset of plastic yielding



and
$$P_Y = 4.17 \frac{\pi^3 R^2}{6E^{*2}} R_p^3 = 21.54 \frac{R^2}{E^{*2}} R_p^3 \quad (9.72)$$

For *sliding contact* of two cylinders we have seen (Fig. 9.18) that not only is the distribution of $\tau_{1\max}$ changed below the contact surface but that on the surface this stress can reach very high values if the coefficient of friction is high. In the contact plane we have simply $\tau_1 = \mu p_0$, so that plastic yielding will occur at the surface as soon as:

$$p_0 = p_{0Y} = k / \mu = R_p / 2\mu \quad (9.73)$$

Below the surface the condition for plastic yielding will again be a function of μ (Fig. 9.18). To a first approximation, keeping the value $\tau_1 = 0.30p_0$ obtained for normal contact, the condition for sub-surface yielding is given by (9.56).

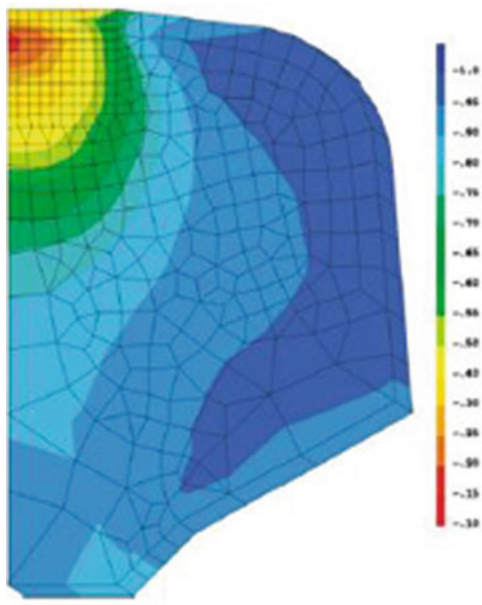
Thus taking together these two extreme cases, this simplified analysis is represented by the curve drawn in Fig. 9.28 which gives order of magnitude. A fuller treatment was given by Johnson (1985).

9.2.3.3 Implications for Damage Due to Contact

In certain mechanical assemblies it is possible that parts will be brought into contact repeatedly: for example, rolling mechanisms which have to support radial loads, for which the term *rolling contact fatigue* (RCF) is sometimes used.

RCF is a severe problem for railway companies. Rails can be damaged by the formation of cracks of different nature, depending on the considered period of time (Dang Van and Maitournam 2002). Nowadays, *squats* and *head checks* are the major defects which affect the rails. Squats are encountered on straight tracks or shallow curves. These defects initiate in deformed zones of the running surface. When these cracks reach a certain size (~ 5 mm) below the surface, they can propagate

Fig. 9.29 Contours of von Mises equivalent stress near the contact area in a rail (Dang Van and Maitournam 2002)



downwards to form progressive transverse cracks which propagate fast. That is the reason why the prevention of these *squat* defects is necessary.

Head checks are angle cracks observed in curves and on crossing rails. They initiate near the rail gauge corner and can join up to induce a gauge corner fracture. The reasons for the difference in the type of observed damages are: (i) the nature of the rail steel and (ii) the increasing load and power per axle.

The analysis of the formation of these defects is a difficult problem which necessitates overcoming two main difficulties.

1. The first one is the determination of the stress-strain field; because of the small size of the contact area, the stresses are beyond the yield strength. It is therefore necessary to take into account the effect of plastic deformation and residual stresses resulting from the sequence of repeated rolling. The presence of initial residual stresses and their redistribution must also be taken into account. Moreover rolling contact is a three-dimensional problem with friction and complex partial slip.
2. The second difficulty is the prediction of crack initiation and crack growth. This necessitates the use of multi-axial fatigue criteria able to deal with complex multi-axial and non-proportional loading situations.

The analysis of *squat and head cracking* defects was undertaken by a number of authors, in particular Dang Van and Maitournam (2002). These authors developed specific numerical procedures allowing the determination of stresses and strains in the rail subjected to repeated moving contacts. An example of calculated von Mises equivalent stresses near the contact is reproduced in Fig. 9.29. These authors

used the Dang Van criterion to predict the fatigue life (see Sect. 6.4.1.2 and Fig. 6.26). See also Dang Van (2011). By coupling this criterion with finite element calculations, these authors showed that it is possible to reproduce the main features associated with the apparition of *squat defects*.

Other studies (Bailetto et al. 2010; Gravouil et al. 2011a, b) based on the *extended finite element method* (X-FEM) showed that it was possible to predict the main features associated with fretting fatigue cracks, *i.e.* their orientation, their trajectory and their growth rate. The situation is complex because it is a three-dimensional problem with non-proportional loading, as stated earlier. A local approach to crack growth is required. It was assumed by these authors that at each increment of crack extension, the crack propagates in the direction where the *calculated local crack growth rate is maximum*. These studies showed how the crack length and the tangential loading due to friction modify the crack path during the propagation process.

9.3 Friction

9.3.1 Introduction

The practicable importance of friction is obvious; it is the origin of the wear mechanisms which will be discussed in Sect. 9.4. So far we have considered friction as a macroscopic phenomenon in terms of the coefficient μ in Coulomb's law. We now give some indication on surface interactions on a smaller scale-nanometre to micrometre – so as to take into account the large variations in the coefficient of friction that are encountered. We only deal with dry friction without lubrication. The presence of a lubricant between two solids in relative sliding or rolling motion changes the coefficient of friction, and especially the wear rate. The study of these effects (in particular hydrodynamic lubrication) makes use of the theory of fluid mechanics and is beyond the scope of this chapter.

Friction and wear are still only incompletely understood. Many aspects have to be taken into account, such as the topography and the modes of deformation and degradation of the solids on the scale of the surface roughness (micrometre) and the adhesion of the surfaces in contact.

9.3.2 The Real Area of Contact

Real surfaces are never perfectly smooth; all have asperities as shown schematically in Fig. 9.30. Because of these asperities, when a normal load P is applied, the surfaces are in contact over a set of discrete zones, as indicated in Fig. 9.31; the real area of contact A_r is some fraction of the nominal area A_n .

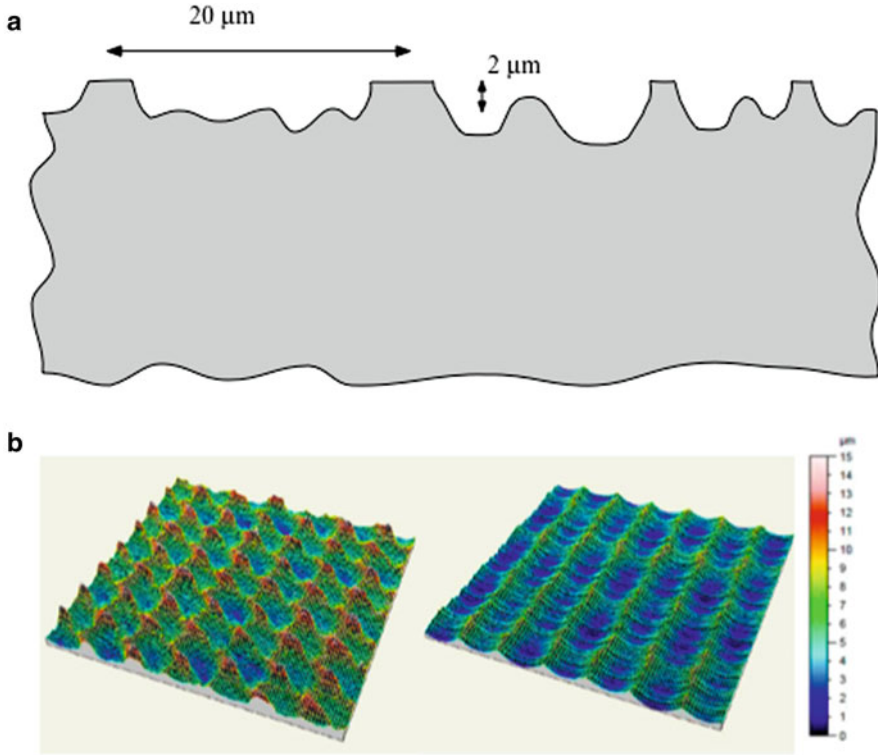


Fig. 9.30 (a) Diagram illustrating roughness of a machined surface; (b) topography of a machined surface

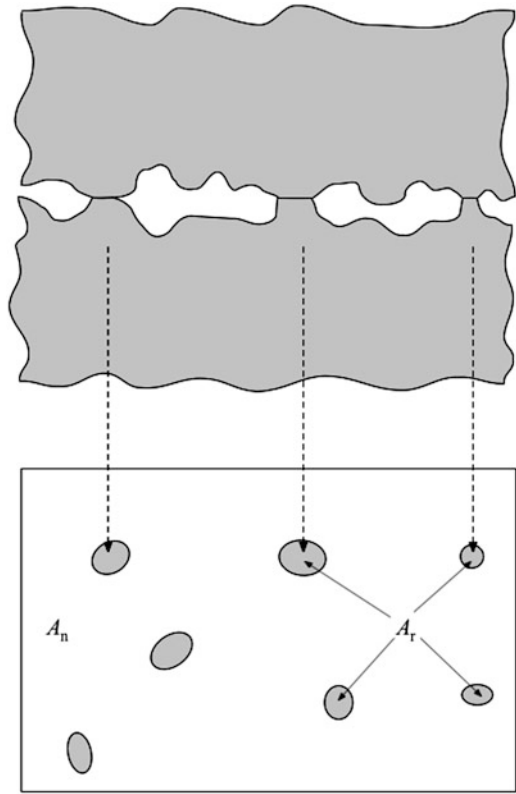
In the elastic regime, A_r is related to the microgeometry of the asperities. If we assume that the Hertz theory can be applied to this micrometric scale, A_r will be an increasing function of the applied load P ; for contact between spheres it will be proportional to $P^{2/3}$ and between cylinders to P (see (9.50) and (9.55)). These results apply to a single asperity, but the number of asperities in contact will increase with the applied load. Moreover purely elastic contact cannot be maintained for long on this scale and the asperities will be deformed plastically. As shown previously (Sect. 9.2.3.2) for a pair of spheres of radius R , if R_p is the yield strength, the limiting normal load for plastic yield is (9.72):

$$P_{Y0} = \text{const.} (R_p^3 / E^2) R^2 \quad (9.74)$$

Another criterion for plastic yield that is sometimes used involves the microgeometry of the surface irregularities. If R_i is the mean radius of the asperities and h_i the standard deviation of their heights the dimensionless parameter ψ is defined by:

$$\psi = (E / R_p) (h_i / R_i)^{1/2} \quad (9.75)$$

Fig. 9.31 Real (A_r) and nominal apparent (A_n) areas of contact of two bodies



If $\psi < 0.6$ the asperities remain purely elastic and the real area of contact is proportional to the value given by Hertz theory (9.50):

$$A_r \propto \pi a^2 = \pi \left(\frac{3 PR_i}{4 E} \right)^{2/3} \quad (9.76)$$

If $\psi > 1$, which is the case in most situations, the asperities are deformed plastically and the real area of contact is:

$$A_r = P / R_p \quad (9.77)$$

The strength of the material plastically deformed can be higher, and because of the punching effect (see Sect. 3.5.3 in Volume I), the value of the resistance to plastic yield can be larger than R_p . Usually the hardness H given by $H \approx 3R_p$ is taken.

For $0.6 < \psi < 1$ the situation corresponds to the transition between purely elastic and purely plastic microcontacts.

In the presence of gross sliding which is the situation discussed in Sect. 9.4 the number of contacts increases. Lim and Ashby (1987) proposed a simple expression giving this number N as:

$$N = \left(\frac{r_0}{r_a} \right) \left(\frac{P}{A_n H_0} \right) \left(1 - \frac{P}{A_n H_0} \right) + 1 \quad (9.78)$$

where r_a is the radius of an asperity, typically 10^{-5} m, r_0 is the radius of a pin sliding on a disk (situation of most wear tests, see Sect. 9.4) and H_0 is the hardness of the material at room temperature. This equation has proper limits. N increases linearly with P for small loads and approaches the value 1 for vanishingly small loads. At the other extreme, when the load approaches the *seizure load*, the “unit” contacts overlap and the true number of contacts again becomes small. In the limit of complete seizure, it again goes to 1. The ratio A_r/A_n can then be approximated as:

$$\frac{A_r}{A_n} \simeq \left(\frac{P}{A_n H_0} \right) \left(1 - \frac{P}{A_n H_0} \right) \quad (9.79)$$

The occurrence of plastic deformation implies dissipation of energy and heating. The repeated relative movements of two solids rubbing against each other will result in wear, as discussed later in Sect. 9.4. The materials are strongly work-hardened over a thin layer, and debris will be detached if the movement is great enough. Contact is then maintained through the intermediary of a bed of debris, which forms the “*third body*” often introduced in tribological analysis (Godet 1984, 1990).

9.3.3 Friction and Adhesion

In 1699 Amontons published the results of his experimental investigations of the friction of unlubricated solids. He found that the frictional force was independent of the area of the surface and was directly proportional to the normal load. To explain this result he assumed that irregularities on the surfaces of the two bodies interlocked, and the relative motion necessitated lifting the load from one position of interlocking to another one. Amontons conclusions were tested by many investigators (for a comprehensive history of friction, see *e.g.* Bowden⁸ and Tabor⁹1958). The most systematic work was done by Coulomb (1773) who examined the influence of a large number of variables on the coefficient of friction.

⁸Frank Philip Bowden (1903–1968) was a physicist and physical chemist, born in Australia, and who worked in Cambridge at the Cavendish Laboratory.

⁹David Tabor (1913–2005) was an English physicist. He worked in Cambridge with Philip Bowden in the Department of Chemistry. Tabor’s book “The Hardness of Metals” published in 1951, has had a major influence on the subject of indentation hardness.

In particular he showed that this coefficient was independent of the velocity of sliding. We will show later that this property is only valid at relatively slow sliding rate.

A much better understanding of friction was obtained in the middle of the twentieth-century thanks to the advances in plasticity. In particular Tabor (1959) presented an attractive theory of friction.

This theory attributes the major part of friction to the *formation of junctions* between the surfaces at the region of real contact. When the surfaces are first placed together under a normal load, P , plastic flow occurs, and the true area of contact is proportional to the load and independent of the size of the bodies. When a tangential stress is applied, combined stresses (compression + shear) at the regions of contact produce *junction growth*.

If the metals are thoroughly clean and ductile this process may continue until the whole of the geometric area of the bodies is in contact and *gross seizure* takes place. The coefficient of friction may then reach values of the order of 50 or more. If, however, some contaminant (for instance, oxide) is present on the surfaces the behaviour is different. Initially the interface is strong enough to transmit the requisite shear stress through it to the underlying metal and *junction growth* occurs as for clean surfaces. At later stage, however, the shear stress required to maintain the junction in a plastic state may exceed the shear strength of the interface; junction growth cannot continue and, if the tangential stress is further increased, growth sliding takes place.

Tabor (1959) indicated that generally the area of contact when sliding occurs is simply a numerical factor times the static area of contact. Since this static area is proportional to the load and independent of the geometric area of the surfaces, this may explain the two basic laws of friction, indicating that Q is proportional to P ($Q = \mu P$) and independent of the area of the surfaces.

The situation is more complex since it was shown that the *kinetic coefficient of friction* (measured under steady sliding) is lower than the *static coefficient of friction*. Moreover ageing time has also an influence: if, after sliding, the motion is stopped, in a number of cases it was observed that the time spent before sliding again had an influence on the transient static coefficient of friction. A detailed review of solid friction can be found in Baumberger and Caroli (2006).

In addition to the mechanical interactions which have been discussed, there are other physical forces acting across the interface. The origin of these forces is the sudden interruption of the periodic structure at a free surface, releasing bonds. This results in interactions of the types that contribute to the cohesion of solids – *metallic* (mobility of free electrons), *ionic* (electrostatic attraction) and *covalent* (sharing of valency electrons) (see Appendix A in Volume I). These are all *short range forces* (<1 nm) and depend on the nature of the material. There are also the *van der Waals forces*, which, although *long-range* and independent of the type of bonds, are relatively very weak, except in the case of polymers, where they are the only forces acting. The short-range forces, which can be expressed in terms of surface energy, are sufficient to constitute permanent bonds which are strong enough to create *adhesive junctions*.

Table 9.1 Coefficients of friction

Material	μ
Pure metals in vacuum	>5
In dry air	0.8–2
In moist air	0.1–0.5
Steel on other metals (lead, bronze)	0.1–0.5
Steel on ceramics (diamond, glass)	0.1–0.5
Ceramics on ceramics (<i>e.g.</i> carbides)	0.05–0.5
Polymers on polymers	0.05–1
Metals and ceramics on polymers (PE, PTFE, PVC)	0.04–0.5
Lubricants (MoS ₂ , graphite)	0.05–0.2
Hydrodynamic lubrication	0.001–0.005

In polymer-polymer friction (such as polyethylene [PE] on PMMA and PA66) it was shown that the coefficient of friction is directly related to the adhesion of PE on a rigid substrate, as determined from a peel test¹⁰ (see *e.g.* Lavielle 1991).

The only way to investigate experimentally these adhesive interactions is to break the bonds and measure the force P_a needed to separate the two surfaces, having first removed the load P which brought them into contact. The result is expressed in terms of a *coefficient of adhesion*, μ_a , defined by:

$$\mu_a = P_a / P \quad (9.80)$$

For two perfectly smooth surfaces of the same material, P_a is theoretically equal to the force necessary to fracture the bulk material. However, the adhesion is strongly affected by the reactions of the surfaces with the environment, both physical (adsorption) and chemical (chemisorption, oxidation, etc.). A detailed account of the mechanisms of metallic friction can be found in Bowden and Tabor (1958).

9.3.4 Typical Values of the Coefficient of Friction

Typical values of μ are reported in Table 9.1. Most of these values correspond to the *kinetic coefficient of friction*. These results show that, for very clean surfaces, the coefficient of friction can reach values of over four, whilst a lubricant can reduce it to values below 0.04. The presence of humidity in air leads to an increase of the coefficient of friction. This is likely related to the formation of an oxide film. The best method to reduce the friction is the use of hydrodynamic lubrication.

¹⁰The peel test measures the strength required to pull apart a bonded surface. It is useful in evaluating adhesives, adhesive tapes, or other attachment methods. See ASTM standards D 903-98 (2010).

Lim and Ashby (1987) collected the results of many authors who measured the value of the coefficient of friction for steel as a function of velocity. Their results can be expressed as:

$$\mu = 0.78 - 0.13 \log_{10} \tilde{v} \quad (9.81)$$

where $\tilde{v}$ is the normalised velocity defined as $\tilde{v} = vr_0/a$, v = sliding velocity (m/s), r_0 = radius of the pin in pin-to disk wear test (see Sect. 9.4) and a = thermal diffusivity of steel (m^2/s). The coefficient of friction varies from about 1 at very low velocity to about 0.1 at very high. These values will be used in the part devoted to wear (see Sect. 9.4).

These variations of the coefficient of friction with sliding velocity represented by (9.81) are due to several effects. The most important are the heating effect due to the dissipation of frictional energy (which reduces the local hardness of the material and generates oxidation of the surfaces) and the increase of the local hardness with increasing strain rate.

9.3.5 Stability of Steady Frictional Slipping – Stick-Slip

For many mechanical systems in sliding contact with an adjoining body, loading by the imposition of a constant relative displacement rate, directed parallel to the contact surface, is observed to lead to no constant slip motion at that surface. This unsteady motion is often referred to as “stick-slip”. It is exemplified by squeaking machinery and oscillating violin strings. On the other hand, motion with constant slip rate is often observed in situations that appear very similar.

The simplest, although not complete, approach to this problem is to ask if steady sliding is a possible stable motion. Classically this is analysed by assuming that the friction stress τ (at a fixed normal stress, σ) is dependent on the slip rate v , only, *i.e.* $\tau = \tau(v)$. If $d\tau(v)/dv > 0$, steady sliding is stable; if $d\tau(v)/dv < 0$, steady sliding is unstable. As the kinetic coefficient of friction is lower than the static one, one should always observe unsteady motion, which is not the situation observed in most cases.

A simple model of stick-slip behaviour can be developed in which it is assumed that the static coefficient of friction is an increasing function of time of contact (Rabinowicz 1966), but that the kinetic coefficient remains a constant. This time dependence may be related to various types of phenomena, such as creep in metals or viscoelasticity in polymers. The modelling of stick-slip behaviour was made by Rice and Ruina (1983) and, more recently, by Baumberger and Caroli (2006). In these models, the coefficient of friction is not a mere number but it depends also on the state of the surface, where the latter evolves with ongoing slip. The difficulty in the analysis lies in the fact that the mechanical response of sliding bodies is very much dependent on many variables, such as the weight, the damping characteristics, etc.

The approach developed by Maugis (1985) should also be considered to explain the stick-slip phenomenon. This author used the energy balance derived from

fracture mechanics concepts (see Chap. 2 in this volume, Sect. 2.2.1) to deal with this stick-slip effect observed in a number of materials including polymers (the phenomenon can be easily observed with scotch-tape), glasses and ceramics. He showed that the diagram giving the variation of the strain energy rate G (see Eq. 2.1 for the definition) as a function of the sliding velocity can reach a maximum due to viscoelastic effects. This behaviour can thus generate a stick-slip phenomenon.

A number of methods for preventing stick-slip are in use (Rabinowicz 1966). First, there are the methods which decrease the amplitude of the slip, or the sliding velocity during the slip, either by increased damping, by increased inertia, or by increased stiffness of the bodies. The second main method of preventing stick-slip is to arrange to have a positive friction-velocity characteristic at the sliding velocity employed. This can be achieved by using lubricating materials of several types, including rubbers, soap and plastics. These materials show the same behaviour, namely a positive characteristic at low speeds and a negative characteristic at high speeds.

9.4 Wear

9.4.1 Introduction

Wear is a very detrimental phenomenon generated when two rough surfaces slide on each other. It is a very complex process due to the many variables. In general the wear rate W (defined here as the volume loss per unit distance slid) depends on the bearing pressure P/A_n , on the sliding velocity, v , and on the material properties and geometry of the surface (Fig. 9.32) (Bowden and Tabor 1985; Ashby and Lim 1990):

$$W = f(P/A_n, v, \text{material properties, geometry}) \quad (9.82)$$

There are many mechanisms of wear, each dependent in a different way on the variables. The dominant mechanism, at any given P and v , is the one leading to the fastest rate of wear. Table 9.2 lists some of the mechanisms encountered in wear studies of metals and ceramics; it includes wear by melting, by chemical change induced by frictional heating, by low-temperature plasticity and by brittle fracture. These mechanisms will be studied individually relying on the comprehensive review by Lim and Ashby (1987).

Several experimental procedures are used to measure the wear rate. The most widely used is the pin-on-disc technique (Fig. 9.33). This technique which is capable of testing materials under a wide range of loads and sliding speeds was accepted by ASTM (American Society for Testing and Materials) as Standard G-99 (see *e.g.* Kato et al. 1994). The wear length of pin (with flat wear face of the order of 2 mm in diameter) is measured automatically as a function of sliding distance and

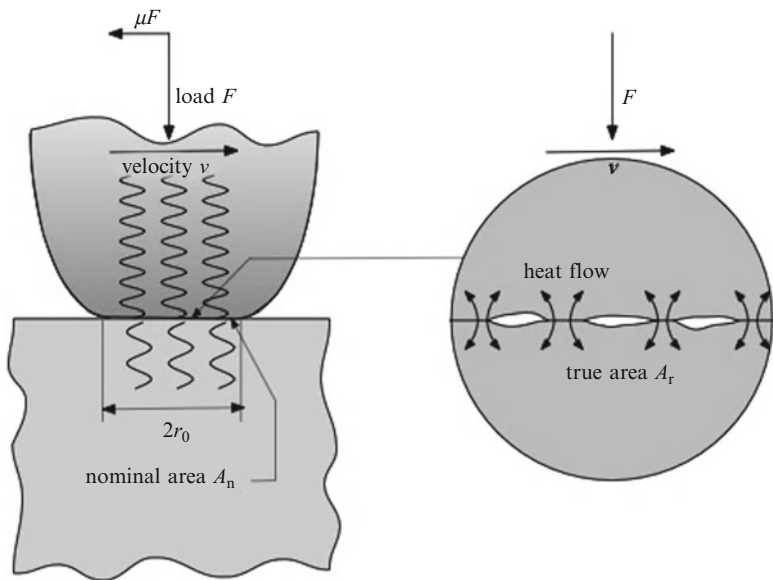


Fig. 9.32 A typical sliding couple. The nominal contact area is A_n but the true area of contact, A_r , is usually much smaller (Adapted from Ashby and Lim 1990)

Table 9.2 Mechanisms of wear (Ashby and Lim 1990)

Metals	Ceramics
Seizure	Seizure?
Melt wear	Melt wear
Severe oxidation wear	Thermally-induced structure change
Mild oxidation wear	Thermal cracking and spalling
Plasticity-dominated wear	Brittle spalling; indentation cracking
Ultra mild wear	Tribochemical wear

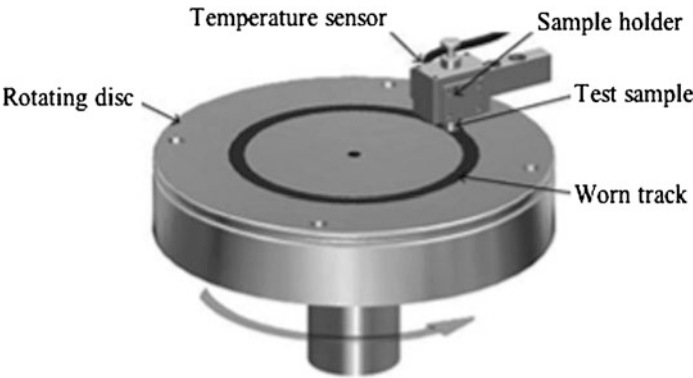


Fig. 9.33 Principle of the pin-on-disk test

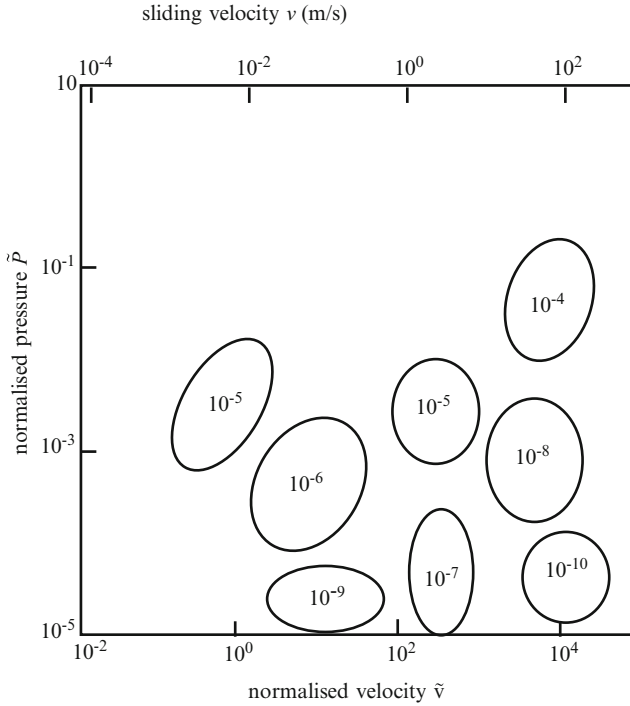


Fig. 9.34 The wear-rate data-map for low carbon steel. Data for pure iron are also included. The numbers given are $\log_{10} \tilde{W}$. The references are given in Figure 1 of Lim and Ashby (1987)

applied load P . This measure can be converted in wear rate as defined in (9.82). These pin-on disc tests can be performed under various relative humidity and at high temperature.

The results of wear tests performed on low carbon steel and pure iron were collected by Lim and Ashby (1987). These results are shown in Fig. 9.34. In this figure values for the normal load $\tilde{P}$, the sliding rate $\tilde{v}$ and the wear rate $\tilde{W}$, defined by Lim and Ashby, are reported. These normalised quantities are defined by:

$$\tilde{P} = \frac{P}{A_n H_0} \quad (9.83a)$$

$$\tilde{W} = \frac{W}{A_n} \quad (9.83b)$$

$$\tilde{v} = \frac{v r_0}{a} \quad (9.83c)$$

Here A_n is the nominal apparent contact area of the wearing surface, H_0 is its room-temperature hardness, a is the thermal diffusivity ($a \simeq 2.3 \cdot 10^{-5} \text{ m}^2/\text{s}$ for pure iron). Figure 9.34 shows a lot of scatter in the measurements. This may be due to different experimental techniques used to measure the wear rate. However an attempt is made to define domains of iso-values for $\bar{W}$. It is observed that, for a given velocity, the wear rate increases with $\bar{P}$, and that, for a given load $\bar{P}$, there is a critical velocity of 10^2 – 10^3 m/s corresponding to a minimum wear rate of about 10^{-7} . It can be calculated that this minimum in $\bar{W}$, taken as equal to 10^{-7} , observed at $\bar{v} = 10^{-3}$ corresponds to roughly 3 h to “wear” a contact of 1 mm^2 over a height of 1 mm.

Results such as those of Fig. 9.34 may be useful for practical purposes. However they do not give information on the operating mechanisms. Even if these mechanisms were identified, these results would remain essentially empirical. As first suggested by Tabor (1985) it might be useful to construct *wear-mechanism maps* based on physical modelling. This is what was proposed by Ashby and his colleagues (Lim and Ashby 1987; Lim et al. 1987; Ashby and Lim 1990; Lim 1998). These authors constructed maps using model-based equations describing the wear rate caused by each mechanism.

These maps show the total wear rate and the field of dominance of each mechanism. These models contain constants which are very often poorly known and which have to be calibrated using blocks of data lying in a given field. The construction of these maps is presented below. As in sliding wear heating plays a very important role, we present first the evaluation of the temperature in the contact (bulk and asperities) and then the wear mechanisms.

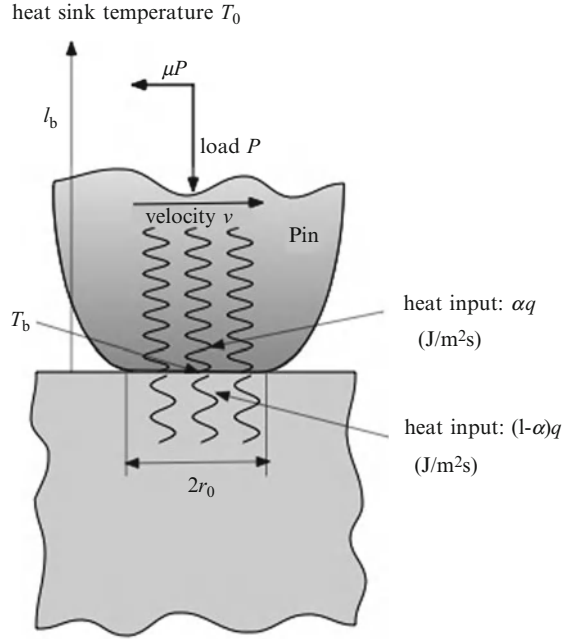
9.4.2 Temperature of Sliding Surfaces

9.4.2.1 Introductory Comments

When two surfaces slide together, most of the work done against friction is converted into heat. The resulting rise in temperature may modify the mechanical properties of the sliding surface. In the case of metals it may provoke oxidation or even melting. All these phenomena influence the rate of wear. The frictional heat is mainly generated at the asperities. The instantaneous temperature of these contact points (the “local” or “flash” temperature, T_f) is obviously higher than the average (or “bulk” temperature) of the surface.

Lim and Ashby (1987) and Ashby et al. (1991) estimated the bulk and flash temperatures to give a diagram showing the surface temperature as a function of the normalised pressure $\bar{P}$ and velocity $\bar{v}$ as defined by (9.83a) and (9.83c). The pin-on-disk geometry was chosen for the reasons given earlier.

Fig. 9.35 The bulk temperature generation at the pin-to-disk interface (Adapted from Lim and Ashby 1987)



9.4.2.2 Bulk Heating

When a pin slides on a disk, frictional heat is generated at the interface. The heat per unit area per second (units: $\text{J/m}^2 \text{ s}$) is:

$$q_b = \frac{\mu P v}{A_n} \quad (9.84)$$

A fraction α of the heat diffuses into the pin; the rest goes into the disk as shown in Fig. 9.35.

Let the mean heat diffusion distance be l_b . This distance can be evaluated as the mean diffusion distance (see Appendix B in Volume I) by:

$$l_b \simeq \left(\frac{\kappa_{\text{eff}}^{\text{th}}}{\rho C} \frac{2r_0}{v} \right)^{1/2} = \left(a \frac{2r_0}{v} \right)^{1/2} \quad (9.85)$$

where r_0 is the contact size, ρC the specific heat per unit volume ($\text{J/m}^3 \text{ K}$), $\kappa_{\text{eff}}^{\text{th}}$ the effective thermal conductivity (J/msK) of the pin-disk system, a the thermal diffusivity (m^2/s).

The heat flux (units: $\text{J/m}^2 \text{ s}$) is:

$$J = \kappa_m^{\text{th}} \left(\frac{T_b - T_0}{l_b} \right) \quad (9.86)$$

where κ_m^{th} is the thermal conductivity of the metal (J/msK), T_b is the bulk surface temperature and T_0 is the sink temperature which can be taken to be in most cases 20°C.

Equating (9.84) and (9.86) and using (9.85), one obtains:

$$T_b = T_0 + \frac{\alpha \mu P v l_b}{A_n \kappa_m^{\text{th}}} = T_0 + \frac{\alpha \mu P v}{A_n \kappa_m^{\text{th}}} \left(\frac{\kappa_{\text{eff}}^{\text{th}}}{\rho C} \frac{2r_0}{v} \right)^{1/2} \quad (9.87)$$

The α coefficient is calculated approximately from the time-dependent equations for heat flow (Carslaw and Jaeger 1959) (see Lim and Ashby 1987). This coefficient α was found as:

$$\alpha = \frac{1}{2 + l_b(\pi v / 8 a r_0)^{1/2}} \quad (9.88)$$

giving an expression for the *heat distribution coefficient*. This coefficient reduces to $\alpha = 1/2$ when $v = 0$.

At this stage the authors note that the quantity $(a H_0 / \kappa_m^{\text{th}})$ has the dimension of temperature, and that l_b and r_0 are both lengths and roughly of the same magnitude.

This temperature is defined as:

$$T^* = \frac{a H_0}{\kappa_m^{\text{th}}} \quad (9.89a)$$

For steels, $T^* \simeq 222$ K. They define a dimensionless number β :

$$\beta = \frac{l_b}{r_0} \quad (9.89b)$$

which is in general of the order of 1.

Rearranging the result in terms of the dimensionless variables $\tilde{P}$ and $\tilde{v}$ gives the equation for the bulk surface temperature:

$$T_b = T_0 + \frac{\mu T^* \beta}{2 + \beta(\pi \tilde{v} / 8)^{1/2}} \tilde{P} \tilde{v} \quad (9.90)$$

The variation of μ with velocity given by (9.81) can be included in (9.90). The calculation of the bulk temperature requires also taking a value for β . The authors adopted $\beta = 6$. Equation (9.90) can then be calculated with $T_b = 1,800$ K (the melting point for pure iron). It gives the conditions ($\tilde{P}$ and $\tilde{v}$) for which melting was observed by a number of authors. These observations were used to calibrate the equation giving the bulk temperature, in particular the coefficient β .

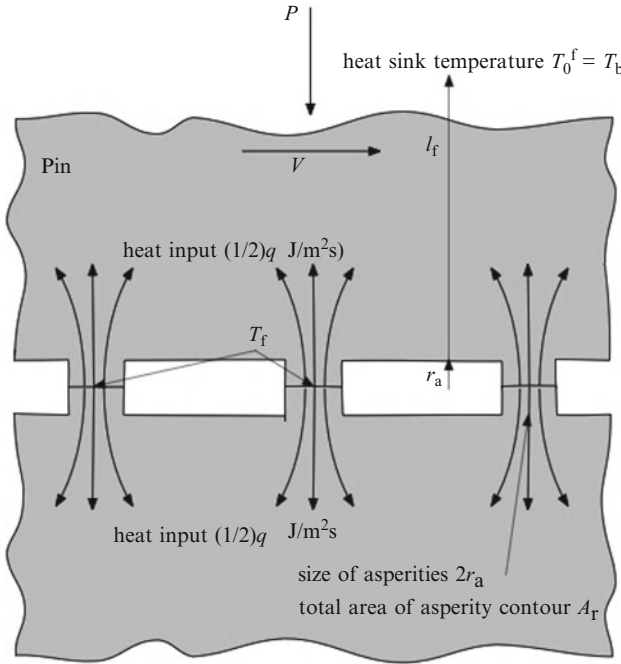


Fig. 9.36 Flash temperature generation at an asperity contact (Adapted from Lim and Ashby 1987)

9.4.2.3 Flash Heating

The real area of contact between the sliding surfaces, A_r , is much less than the nominal contact area, A_n , as discussed in Sect. 9.3.2 and as shown in Fig. 9.36. The heat input per unit area at an asperity, q_f , is larger than that given by (9.84) by the factor A_n/A_r , *i.e.*:

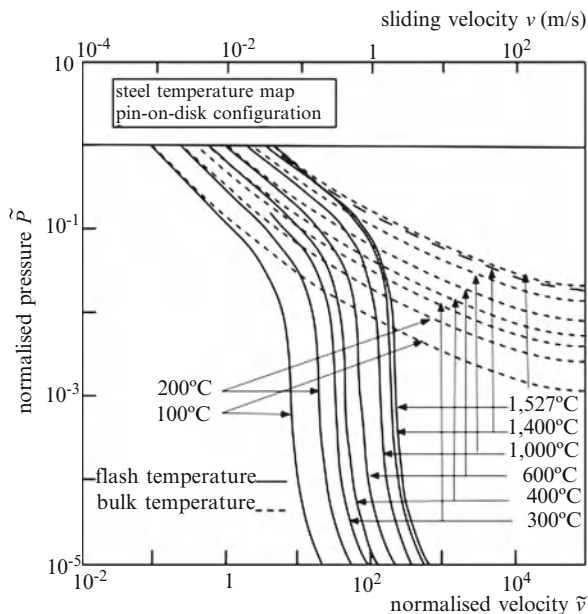
$$q_f = \frac{\mu P v}{A_n} \left(\frac{A_n}{A_r} \right) = \frac{\mu P v}{A_r} \quad (9.91)$$

To calculate q_f , we need this value for A_r , *i.e.* the number of contacting asperities, N . As indicated earlier, Lim and Ashby (1987) showed that N can be approximated by (9.78).

The solution to this problem depends on the relative value of contact time t_d ($=2r_0/v$) and the value of the characteristic time constant for heat flow for flash heating which is $t_h \simeq r_0^2/a$, as for bulk heating. The choice of α which depends on the ratio t_d/t_h has been discussed by Lim and Ashby (1987). These authors proposed a simplified expression for the *steady-state flash temperature* as:

$$T_f = T_b + \frac{\mu T_c^* \beta}{2N^{1/2}} \tilde{P}^{1/2} \tilde{v} \quad (9.92)$$

Fig. 9.37 Bulk (dotted line) and flash (thick lines) temperatures as a function of normalised load $\tilde{P}$ and normalised velocity, $\tilde{v}$, in pin-to-disk wear test (Adapted from Lim and Ashby 1987)



where T_b is given by (9.90), μ by (9.81) and T_e^* is given by:

$$T_e^* = \frac{aH_0}{\kappa_{\text{eff}}^{\text{th}}} \simeq 650 \text{ K} \quad \text{for steel} \quad (9.93)$$

which is similar to (9.89a) but where $\kappa_{\text{eff}}^{\text{th}}$ is the “effective” thermal conductivity which was calibrated from experiments.

Equation (9.92) predicts that, at low loads, T_f is almost independent of the applied load $\tilde{P}$, that when $\tilde{P} \rightarrow 0$, $(T_f - T_b) \sim \tilde{v} \tilde{P}^{1/2}$ and that when $\tilde{P} \rightarrow 1$, $(T_f - T_b) \sim \tilde{v}(1 - \tilde{P})^{-1}$. This explains the existence of a critical velocity above which the normalised pressure $\tilde{P}$ varies rapidly for a given value of the flash temperature. The results of these calculations are summarised in Fig. 9.37. This figure which was drawn assuming $r_a = 10^{-5} \text{ m}$, $r_o = 1.5 \times 10^{-3} \text{ m}$ and $\beta = 6$, gives an estimate of the temperatures that appear in a steel pin sliding on a steel disk during wear experiments. This map which is constructed with normalised axes contains some of the geometric (A_n , r_o , r_a), thermal (κ_m^{th} , μ) and mechanical (H_o , μ) variables of the problem.

9.4.3 Wear Mechanisms

Four broad classes of wear mechanisms are considered: (i) *seizure*, (ii) *melt wear*, (iii) *oxidation-dominated wear* and (iv) *plasticity-dominated wear*. Each class is discussed in turn.

9.4.3.1 Seizure

When metal surfaces are put in contact, the real area over which they touch, A_r , is usually very small, as indicated earlier (see Sect. 9.3.2). The large local pressures at the point of real contact can favour the formation of *metallic junctions* between the surfaces even under static conditions, as discussed earlier (Sect. 9.3.3). When the junctions are sheared during sliding the welded areas grow. Under large enough loads, the real area of contact grows until it reaches the nominal area, A_n . This corresponds to *seizure*. Effects such as plastic indentation and metallic transfer have been introduced in models for seizure (see *e.g.* Johnson (1985) who provided a physical basis for Tabor's equation discussed below).

The model for seizure introduced by Lim and Ashby (1987) is relatively simple (Fig. 9.38). The mean pressure on an asperity, P/A_r is simply equal to the local hardness, H , of the material, which may be written:

$$\frac{P}{A_r} = H \quad (9.94)$$

If the surfaces slide, each asperity is subjected to a shear stress $\tau = \mu P/A_r$ due to friction in addition to the normal pressure P/A_r , and the junction grows until:

$$\left(\frac{P}{A_r}\right)^2 + \alpha_t \tau^2 = H^2 \quad (9.95)$$

This is Tabor's equation (Tabor 1959). This author found that the constant $\alpha_t = 12$ fits experimental data well. Replacing τ in (9.95) by $\mu P/A_r$ gives a modified version of Tabor's equation:

$$\frac{P}{A_r} = \frac{H}{(1 + \alpha_t \mu^2)^{1/2}} \quad (9.96a)$$

The *seizure condition* is obtained approximately when $A_r = A_n$. This leads to the following expression:

$$\tilde{P} = \frac{1}{(1 + \alpha_t \mu^2)^{1/2}} \frac{H}{H_0} \quad (9.96b)$$

where H_0 is the hardness of the metal at room temperature. For simplicity sake it is adequate to set $H = H_0$ in (9.96b) because it should be kept in mind that the local hardness depends not only on temperature but also on strain rate. Temperature and strain rate have compensating effects. Lim and Ashby (1987) proposed an

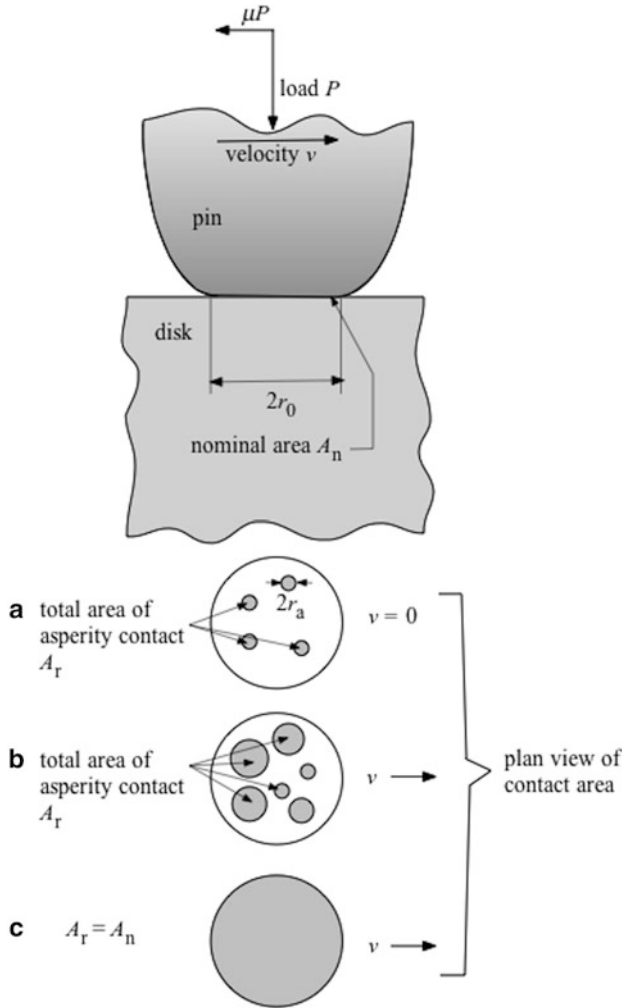


Fig. 9.38 An idealised model for the seizure mechanism proposed by Lim and Ashby (1987): (a) plastic deformation of the asperity contacts under the normal load, P , until the real area of contact, A_r , is given by P/H_0 ; (b) further increase of the normal load P and the presence of a tangential motion ($v \neq 0$) will increase the number of asperity contacts; (c) seizure takes place when the real area of contact, A_r , equals the nominal area of contact, A_n

expression of the seizure load which takes into account the temperature T_b , and the sliding velocity v . Here it is enough to assume that the load for seizure is independent of these two variables and is given by (9.96b) with $H/H_0 \simeq 1$.

9.4.3.2 Melt Wear

Localised melting can occur, in particular at high sliding velocities (10–1,000 m/s). Under these conditions the coefficient of friction decreases to reach very low values because of the presence of a film of liquid metal giving “melt lubrication”. The liquid layer supports the normal load by the usual hydrodynamic mechanism, but the heat generated in the layer continues to melt more solid. Under these conditions the wear rate is high even though the coefficient of friction is low.

Lim and Ashby (1987) developed a simple model for melt wear, adapting the analysis of the problem due to Evans et al. (1976). Their expression is written as:

$$\tilde{W} = \frac{T_m - T_0}{T^*} \frac{H_0}{L} \frac{1}{\beta \tilde{v}} \left[\alpha \mu \tilde{P} \tilde{v} \frac{T^* \beta}{(T_m - T_0)} - 1 \right] \quad (9.97)$$

where T_m is the melting temperature and L is the latent heat of fusion per unit volume (J/m^3). The other terms in (9.97) have been defined previously.

9.4.3.3 Oxidation-Dominated Wear

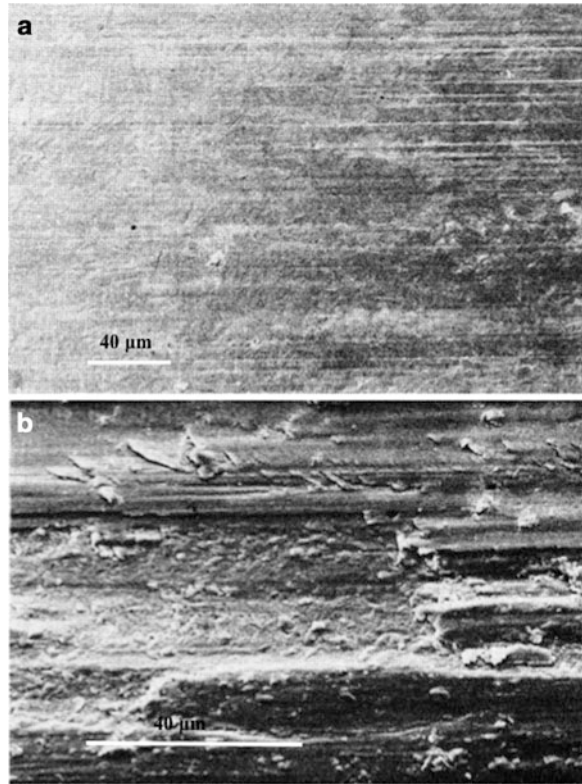
Introductory Comments

Many studies have shown that when steel surfaces slide, at velocities below about 1 m/s, the wear debris is largely metallic. Above this velocity it consists mainly of iron oxides (Fe_2O_3 and Fe_3O_4 , see *e.g.* Quinn et al. 1984; Kato et al. 1994). A close examination to the equation giving the flash temperature (9.92) and to Fig. 9.37 shows that the flash temperature is larger than 600°C, which is sufficient to develop a rapid oxidation of the sliding surfaces.

A change in wear rate is associated with the change to oxide-debris (see *e.g.* Kato et al. 1994). Part of this drop in wear rate may be due to local martensite formation in steels. However oxidation can also play a major role. Around 1 m/s and at low loads, the oxide film is thin, patchy and brittle as seen in Fig. 9.39a (Kato et al. 1994). At higher velocities (>10 m/s) and higher loads the film becomes thicker and continuous, covering the entire surface, as illustrated in Fig. 9.39b.

Under these conditions the friction heating is important; the metal is partly insulated by the oxide film, but the oxide itself is hot enough and can flow plastically or even melt. This is the reason why two regimes have been distinguished: (i) the regime of low velocities, $v \simeq 1\text{--}10^2$ m/s, named *mild-oxidation* wear and (ii) the regime of higher velocities named *severe-oxidation* wear. The denomination “mild” and “severe” refer to the extent of oxidation, not the wear rate, which is often lower during severe oxidation wear.

Fig. 9.39 Wear mechanism of nitrided steel. SEM of worn surfaces in pin-on-disk tests; (a) mild wear regime (sliding speed = 1 m/s, test load = 10 N); (b) severe wear regime (sliding speed = 1 m/s, test load = 160 N) (Adapted from Kato et al. 1994)



Mild-Oxidation Wear

This wear mechanism has been largely investigated, in particular by Quinn (see *e.g.* Quinn et al. 1984). An idealised model for this mode of wear is shown in Fig. 9.40. An oxide film is formed at the asperity contacts with a total area of A_r and grows according to a parabolic rate (Fig. 9.40a):

$$\Delta m^2 = k_p t \quad \text{where } k_p = A_0 \exp\left(-\frac{Q_0}{RT}\right) \quad (9.98)$$

Here A_0 is the Arrhenius constant, Q_0 is the activation energy for oxidation and R is the gas constant. Δm is the increase of mass of oxide per unit area. The oxide film reaches a critical thickness, t_c , at which the oxide layer spalls (Fig. 9.40b).

The broken off oxide layer becomes a wear fragment and an oxide layer is again formed on the newly exposed metallic surface while the oxide film on another asperity contact has reached the critical thickness, t_c , and has broken off. It is

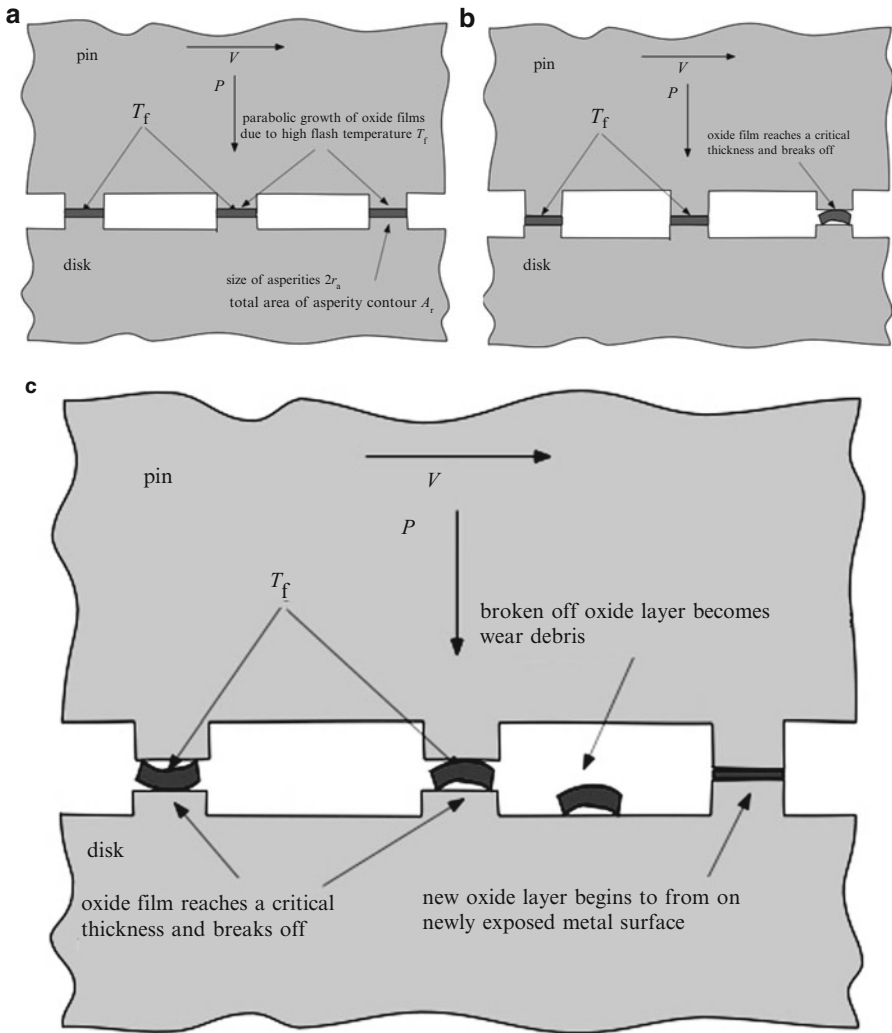


Fig. 9.40 An idealised mild-oxidational wear model: (a) The oxide film formed at the asperity contacts grows according to a parabolic rate; (b) the critical oxide thickness, t_c , is reached at one of the asperity contacts and the oxide layer breaks there; (c) the broken oxide layer becomes a wear fragment and formation of oxide film on another asperity contact reaching the critical thickness, t_c (Adapted from Lim and Ashby 1987)

assumed that, in the regime of mild-oxidation wear, the bulk heating is negligible and oxidation is only due to flash heating of the asperities. A simple equation giving the wear rate was derived by Lim and Ashby (1987).

$$\tilde{W} = \left(\frac{B^2 A_0 r_0}{t_c a} \right) \exp \left(-\frac{Q_0}{RT_f} \right) \frac{\tilde{P}}{\tilde{v}} \quad (9.99)$$

where B is a constant ($B = 3.4 \times 10^{-4} \text{ m}^3/\text{kg}$). A_0 and Q_0 are considered as adjustable parameters chosen to fit the measurements of wear rate. Q_0 is taken as equal to the activation energy for static oxidation of iron (138 KJ/mol). The Arrhenius constant A_0 , on the other hand, shows a wide scatter. A single value for A_0 was taken by Lim and Ashby: $10^6 \text{ Kg}^2/\text{m}^4\text{s}$. Moreover it was assumed that $t_c = r_a$.

It can easily be shown that in this regime of mild-oxidation wear, for a given wear rate, $\tilde{P}$ tends to increase with increasing values of $\tilde{v}$.

Severe-Oxidation Wear

Increased velocities generate higher temperatures (Fig. 9.37). This has two consequences: (i) the extent of oxidation is larger; (ii) the points of contact between the oxide-coated surfaces are so hot that the oxide is assumed to melt and flow (Fig. 9.41).

The most important contribution to heating comes from the latent heat of melting. The wear rate is calculated using a model close to the equation for melt wear (9.97). It is expressed as:

$$\tilde{W} = f_m \frac{(T_m^{\text{ox}} - T_b)}{L_{\text{ox}} a} \frac{(\tilde{P}N)^{1/2}}{\beta \tilde{v}} \left[\alpha \mu \frac{a H_0 \beta}{\kappa_{\text{ox}}^{\text{th}} (T_m^{\text{ox}} - T_b)} \left(\frac{\tilde{P}}{N} \right)^{1/2} \tilde{v} - 1 \right] \quad (9.100)$$

Here α has been given previously (9.88); T_b is given by (9.90); N by (9.78). T_m^{ox} is the melting temperature of oxide; $\kappa_{\text{ox}}^{\text{th}}$ is the thermal conductivity of oxide (J/msK); L_{ox} is the latent heat of fusion per unit volume for oxide (J/m³) and f_m is the volume fraction of molten material removed during sliding which is also an adjustable parameter. It was found that $f_m = 0.01$ gave a good fit to the data.

It can easily be checked that in this regime of severe oxidation wear, for a given wear rate, $\tilde{P}$ tends to decrease with increasing $\tilde{v}$. This indicates that there is a transition between the two oxidation wear mechanisms which can be calculated when equating the two corresponding wear rates, *i.e.* (9.99) and (9.100). This transition was determined by Lim et al. (1987) and is shown in Fig. 9.42 where the transition between mild and severe wear in the plasticity-dominated regime discussed below is also shown.

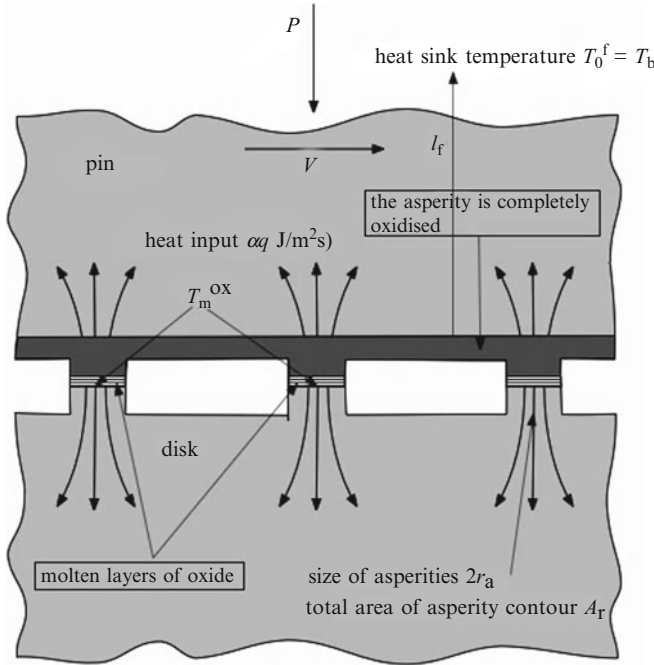


Fig. 9.41 An idealised severe-oxidation wear mechanism. The asperities are assumed to have completely oxidised and thin layers of molten oxide are formed at the contact asperities (Lim and Ashby 1987)

9.4.3.4 Plasticity-Dominated Wear: Adhesion and Delamination

At low sliding velocities ($\leq 0.1 \text{ m/s}$) surface heating (Fig. 9.37) is negligible. Thus the effect of friction is to deform the metal surface, as shown by a number of authors (e.g. Archard and Hirst 1956; Suh 1973). In this regime the wear rate follows the Archard's law (1953) which is simply:

$$\tilde{W} = k_A \tilde{P} \quad (9.101)$$

where k_A is the *Archard wear coefficient*. Typical values for k_A are 10^{-5} – 10^{-3} .

Any model for plasticity-dominated wear depends on the analysis of the plastic flow field at and near the contacting surfaces. This is provided by the slip-line field analysis (e.g. Johnson 1985). It was shown previously (see Sect. 9.2.3.2) that if the coefficient of friction μ is sufficiently small (~ 0.3) the plastic flow field beneath a sliding pin is contained below the surface; then the plastic strain accumulated per

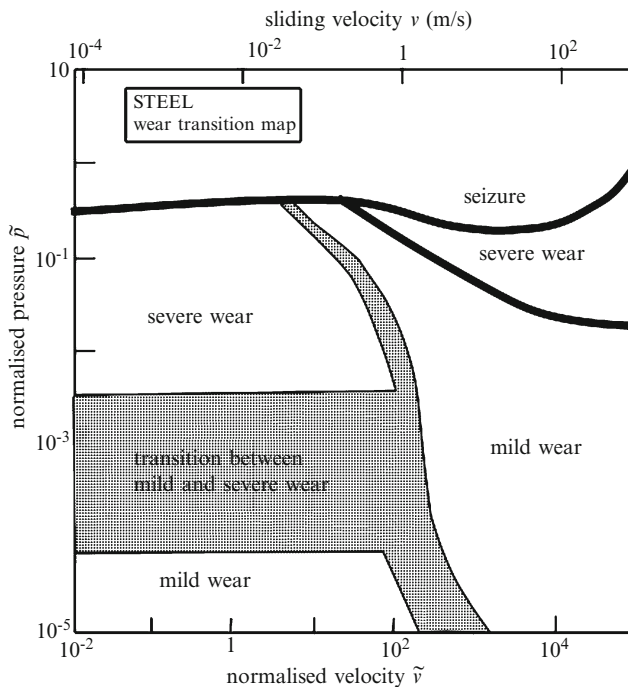


Fig. 9.42 The wear-transition map for the unlubricated sliding of steel (Adapted from Lim et al. 1987 which gives more details)

pass is small. But when μ is greater than 0.3 the flow field breaks through to the surface and the plastic strain per pass is larger. This suggests two regimes of wear depending on the value of the coefficient of friction.

Much evidence indicates that cracks are nucleated just below the surface which propagate, at first parallel to the surface, but later breaking out to give a metallic wear fragment. The modelling of this delamination was made in particular by Suh (1973).

The frictional traction on the sliding surface causes accumulating plastic shear, as shown in Fig. 9.43. Voids nucleate at inclusions below the surface at a critical depth, X_0 (Fig. 9.43a). This characteristic depth is of the same order as the size of an asperity ($\simeq 10 \mu\text{m}$). Figure 9.43b shows that the surface fraction of voids, f_A , on a plane parallel to the sliding surface and at a depth X_0 is:

$$f_A = N_v 2r_i \times 2r_i \times 2r_i \gamma \simeq 2f_v \gamma \quad (9.102)$$

where f_v is the volume fraction of inclusions and γ the cumulative plastic strain.

Lim and Ashby (1987) assumed that when this area fraction f_A reached a critical value f_A^* ($\simeq 0.50$), fast fracture occurred and caused a wear flake. Using this

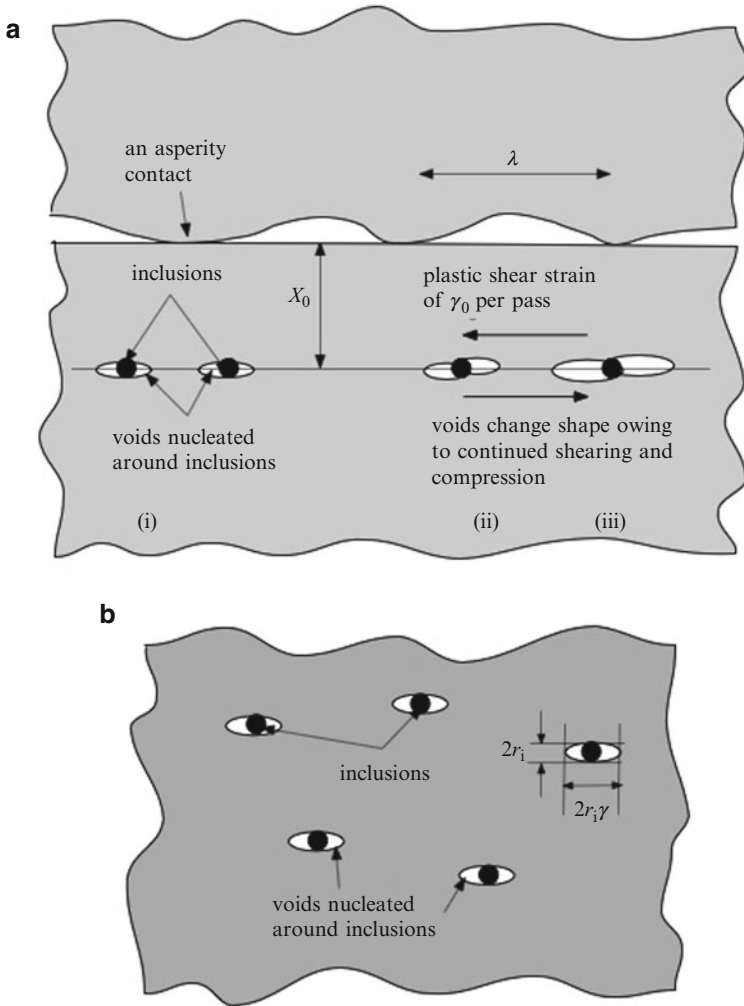


Fig. 9.43 An idealised plasticity-dominated (delamination) wear model: (a) voids nucleate around inclusions at a critical depth X_0 under the influence of plastic shear stress; (b) the voids around the inclusions on a plane parallel to and at a depth X_0 below the sliding surface; γ is the cumulative plastic shear strain in that plane (Adapted from Lim and Ashby 1987)

assumption simple calculations show that the wear rate can be calculated:

$$\tilde{W} = \frac{2\gamma_0 f_v}{f_A^*} \tilde{P} = k_A \tilde{P} \quad (9.103)$$

which is the Archard's law with $k_A = 2\gamma_0 f_v / f_A^*$. In (9.103) γ_0 is the plastic strain accumulated per pass.

The calibration of this simple model for plasticity-dominated wear leads to $k_A = 5 \times 10^{-5}$ for mild wear and $k_A = 5 \times 10^{-3}$ for severe wear. In this regime the wear rate is independent of the sliding velocity.

Five simple analytical models have thus been derived and calibrated using experimental wear-rate data obtained primarily from pin-on-disk wear tests. These models can be useful to build *wear-mechanism maps*, although, as stated by Lim and Ashby, it is obvious that they need further refinement. Two of these five models, those for seizure and for melt wear, contain no adjustable parameters. The other three, mild and severe-oxidation wear, and delaminating wear, do contain adjustable parameters which have been calibrated from experimental results.

9.4.4 Wear-Mechanisms Maps

Each mechanism can be described by a normalised wear-rate equation of the form:

$$\tilde{W} = f(\tilde{P}, \tilde{v}, \text{materials properties}) \quad (9.104)$$

In a graph $\log_{10} \tilde{P} - \log_{10} \tilde{v}$, the *field dominance* of each mechanism is drawn by using each equation in turn, selecting the expression which leads to the largest value of $\tilde{W}$. *Field boundaries* are the lines along which two mechanisms give equal rate. *Contours of constant normalised wear rate*, $\tilde{W}$, are plotted onto these fields. These contours are obtained by evaluating the wear-rate equation of the dominant mechanism acting in each field.

The result for steel is shown in Fig. 9.44. In this map it is observed that for a given load, $\tilde{P}$, there is an optimum velocity ($\simeq 10$ m/s) at which the wear rate is minimum. This is in qualitative agreement with the experimental results shown in Fig. 9.34.

Similar results, although less detailed and essentially experimental, were published for aluminium alloys (Antoniou and Subramanian 1988; Liu et al. 1991), for nitrided steel (Kato et al. 1994) and for thermal spray steel coatings deposited on aluminium alloys (Edrisy et al. 2005). Wear mechanism maps of tool steels used for drilling aluminium or magnesium alloys were also drawn (Zhang et al. 2001; Wang et al. 2008). However, it should be recognised that the comparison of these empirical maps with theoretical maps is still limited.

9.4.5 Materials for Use in Conditions of Friction and Wear

We are concerned here essentially with materials for bearings and machining. Most of the bearings are being lubricated with oil or grease, although some

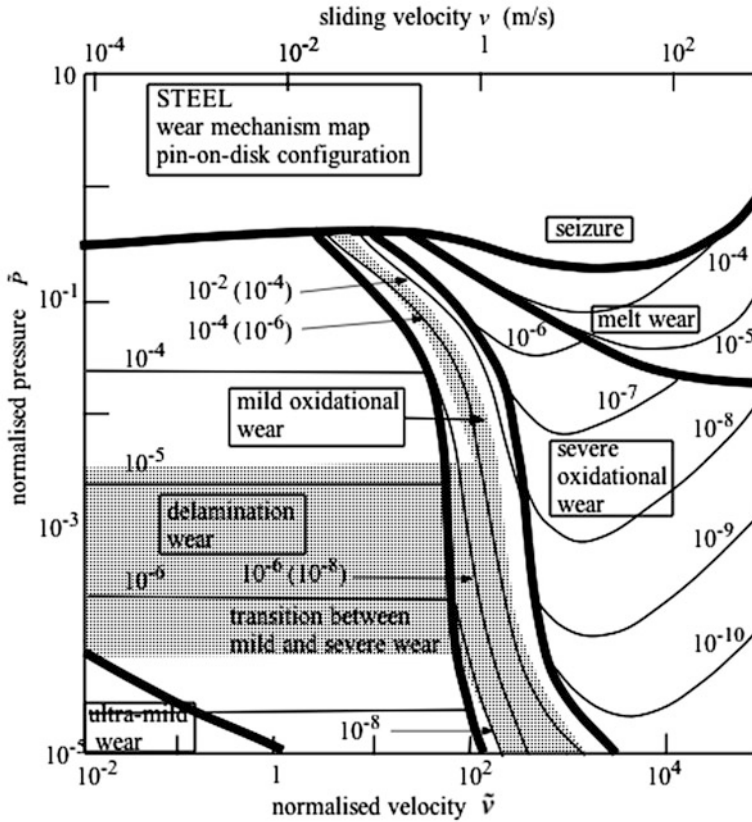


Fig. 9.44 The wear-mechanism map for a steel sliding pair using the pin-on-disk test. Contours of constant normalised wear rates are superimposed on field showing the regimes of dominance of different wear mechanisms. The *shaded* regions indicate a transition between mild and severe wear (Adapted from Lim and Ashby 1987)

polymer-based materials or composites incorporating a solid lubricant are used dry. Ceramic bearings are used for some specific applications, in particular at elevated temperature.

The choice of a given bearing material for any application will involve a compromise between:

- “adaptability”: if the alignment is poor the materials should not be too hard and brittle; play in bearings is usually of the order of 0.025 mm,
- resistance to fretting fatigue,
- resistance to *erosion* in the case of hydrodynamically-lubricated bearings.

Machining involves the use of very hard materials which can be metals or ceramics.

9.4.5.1 Metallic Materials for Bearings

Very often the best results are obtained by depositing a 0.2–0.5 mm layer of one of the following materials on a low-carbon steel substrate; this is the solution often adopted for engine bearings.

(a) *Alloys lead, antimony or tin*

Alloys of Pb, Sb and Sn are often used, for example 10% Sb, 6%Sn; or of Sn, Sb and Cu, for example 8% Sb, 3% Cu. In the first of these the microstructure consists of Sb-Sn compounds and a pseudo-eutectic Pb-Sb-Sn; in the second, a hard compound Cu_6Sn_5 and a relatively soft Sn-Sb solid solution.

(b) *Cuprous alloys*

These are either Cu-Pb alloys or bronzes (Cu-Sn) containing lead. In the first alloys two practically distinct phases, one of copper and the other of lead, the solubility in the other being very low. As with lead or tin alloys, the same principle applies, that of mixing a soft and a hard phase.

(c) *Aluminium alloys*

These much-used alloys have the advantage of good resistance to corrosion. Several are available and two large families, Al-Sn and Al-Pb are particularly popular.

(d) *Porous bronzes*

This much-used material is fabricated by powder metallurgy. The process is such as to leave a residual porosity of 10–25%, the pores being interconnected so that lubricating oil can circulate through the channels.

(e) *Cemented tungsten carbides and coatings*

The main use of tungsten, in the form of tungsten carbide, WC, is now in the manufacture of cemented carbide. This is a material made by “cementing” very hard tungsten monocarbide (WC) grains in a binder matrix of tough cobalt metal by liquid phase sintering. The high solubility of WC in cobalt at high temperature and a very good wetting of WC by the liquid phase result in an excellent densification during liquid phase sintering. The resulting material combines high strength, toughness and high hardness.

Cemented tungsten carbides are largely used for metal cutting tools (about 30,000 tons/year) and in mining and stone cutting tools, in particular in the oil industry.

This material is also used as a coating. The first coating was with a thin layer ($\sim 5 \mu\text{m}$ thick) of titanium carbide made by a chemical vapour deposition (CVD) process. It improved the lifetime of tools by a factor of 2–5. This technique has since been improved by using multilayer coatings, where layers of alumina, titanium nitride and other materials have been added which have further improved the lifetime by 5–10 times.

Thermal spray coatings are now used in a very broad range of wear resisting surfaces and for the repair of wear resisting surfaces. The main advantage of this technique is that it can develop the surface properties which are required while the

component substrate material can be chosen for appropriate purposes. Many thermal spray processes have been developed including combustion powder thermal spray, plasma thermal spray process and cold spray coating process.

9.4.5.2 Effect of Microstructure on the Wear Resistance of Steels

The effect of microstructure on wear resistance is not easy to summarise because of the variety of wear conditions. However many experimental data (see *e.g.* Zum Gahr 1987) showed that the wear resistance (measured as the reciprocal of the wear rate) of bulk iron based alloys increases with increasing carbon content. This may be caused by effects such as the increasing work-hardening rate capability of austenite, the increasing hardness of martensite, the increasing carbide volume in high carbon steels or cast irons or the increasing volume fraction of pearlite or martensite in hypoeutectoid steels due to increasing carbon content. For instance rail steels contain about 0.80 wt % C (*i.e.* 100% pearlite).

Figure 9.45 shows typical wear resistance of iron alloys as a function of carbon content and microstructure (Zum Gahr 1987). There is a general trend indicating that decreasing wear rates are obtained when increasing carbon content in steels and cast irons. However at a given carbon content, wear loss can differ substantially due to different microstructures, as schematically shown in this figure.

A simplified overview of the wear resistance of different microstructures in the pin abrasion test is presented in Fig. 9.46. The data of wear resistance of all microstructures are related to those of pure Armco iron. In general, wear resistance increases in the sequence ferrite, to ferrite plus spheroidal cementite, to ferrite plus lamellar pearlite, to pearlite, to martensite plus cementite, to martensite. The wear resistance of martensite increases with hardness and/or metastable retained austenite, and can be further enhanced by alloying and/or by carbides such as M_7C_3 or M_6C or MC . This has led to the development of tool steels, commonly used in tool bits and cutting tools. These steels are often used in power saw blades and drill bits. These materials are superior to the older high carbon steel tools in that they can withstand higher temperatures without losing their hardness. This property allows these steels to cut or to machine faster than high carbon steels, hence the name *high speed steel* (HSS or HS).

9.4.5.3 Polymers

These materials lead generally to a low coefficient of friction (see Table 9.2). They are often used in the form of composites consisting of various kinds of reinforcing fibres impregnated with a thermoset polymer such as polyester resin. A composite often used for dry bearings is porous bronze impregnated with a mixture of polytetrafluorethylene (PTFE) (see *e.g.* Mortensen 2007).

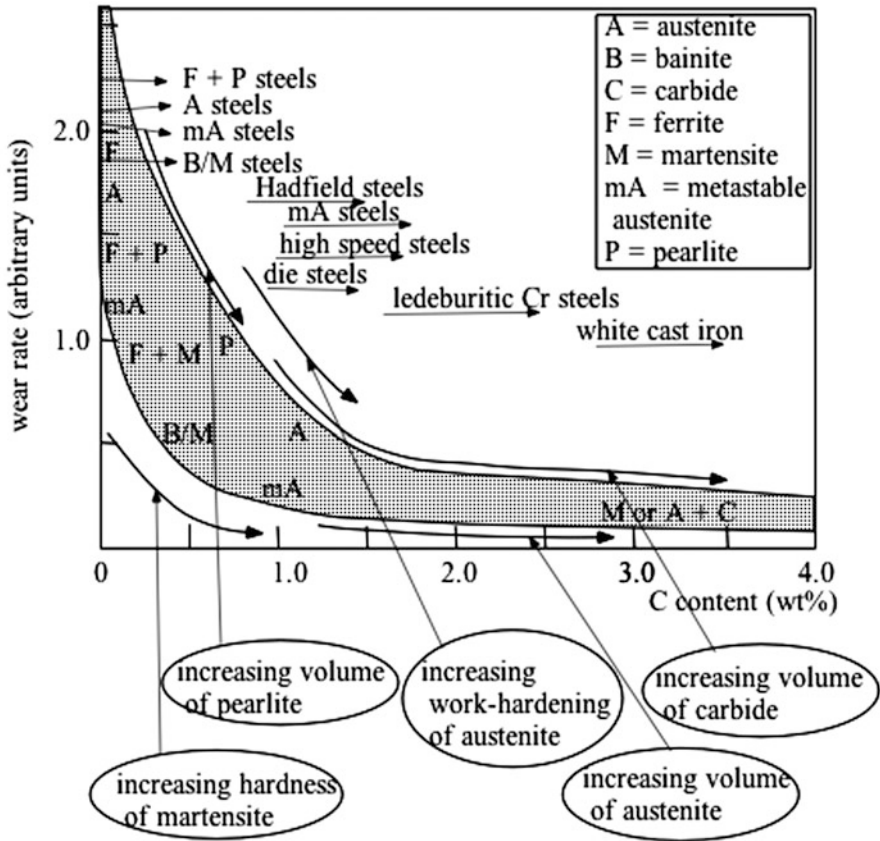


Fig. 9.45 Wear loss in an arbitrary scale measured by using pin abrasion tests versus carbon content of different Fe-alloys (Adapted from Zum Gahr 1987)

9.4.5.4 Ceramics

Excellent surface properties can be obtained by using ceramics such as alumina, zirconia, silicon carbide and silicon nitride. These ceramics are used as coatings, as discussed earlier. A number of recent studies have shown that decreasing the scale of coating structure below the micrometric scale was leading to an improvement of its tribological properties in terms of coefficient of friction and wear rate. *Suspension Plasma Spraying* (SPS) is a process which is able to manufacture relatively thick (10–50 μm) coatings with sub-micrometre sized or even nanometre-sized architecture (see e.g. Darut et al. 2010). These nanometric layered materials may exhibit excellent wear resistance. It should be added that ceramic materials can also be used as bulk materials as, for instance, in hip surgery where alumina is now largely used.

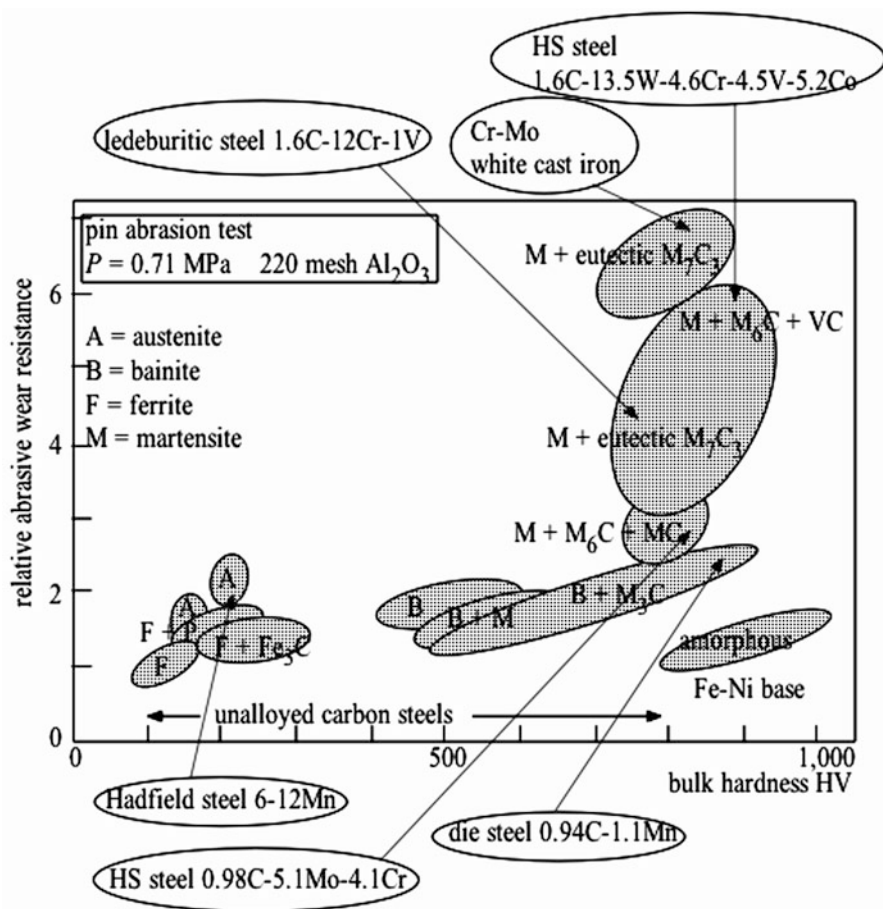


Fig. 9.46 Relative wear resistance of different materials in the pin abrasion test. The microstructures are indicated (Adapted from Zum Gahr 1987)

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Chapter 10

Damage and Fracture of Non-metallic Materials

Abstract The methods of fabrication of *ceramic* materials produce weak grain boundaries in general. The cavities and inclusions in these are critical for fracture. The damaged zone at the tip of cracks can be described by the fictitious crack model. Crack deflections and crack tip shielding mechanisms increase the fracture toughness. Size effects yield two different fracture regimes for short and long cracks. The fracture strength is statistically distributed. Environment effects produce delayed fracture of ceramics. In *glass* fracture is most often due to surface cracks. Fracture toughness can be enhanced by incorporating metallic particles. Slow crack growth of cracks in glass is the result of interaction with humidity. The complex structure of *concrete* at various scales results in cracks arrested on obstacles. Continuum damage mechanics is a useful tool in this context. Size effects are important in fracture toughness measurements. Evolution of chain configurations is linked to the homogeneous plastic yielding of glassy *polymers*. Heterogeneous plastic yielding goes along with crazing. In semicrystalline polymers plastic yielding produces amorphisation. Delayed fracture of polymers due to their viscoplastic behaviour takes place under static loading and more or less in cyclic loading according to the frequency. The microstructure of *wood* explains its very anisotropic behaviour. Fracture depends on the moisture content. Creep and fatigue of wood resemble that of polymers. Residual stresses due to differences in thermal expansion coefficients must not be neglected in *composite materials*. The direction of propagation of cracks in brittle matrix reinforced composites depends on the density of the packing of long fibres. The calculation of stresses in inclusions can explain the fracture behaviour of composites reinforced with small fibres. Various anisotropic macroscopic criteria of fracture exist.

10.1 Introduction

The various damage processes so far described have been studied mostly in metallic materials. But non-metallic materials should not be neglected. Engineers, who do

not design only with metals, need to know how damage and fracture take place for non-metallic materials so as to use them safely.

The general considerations of Chap. 1 in Volume I refer to many examples of non-metallic materials. Table 1.1 in the present volume lists the main damage mechanisms. It concerns any kind of materials. However, each kind of non-metallic materials displays particularities and fracture prevention methods may differ somewhat according to the material used. This deserves descriptions and discussions in addition to the content of the preceding chapters. Nevertheless, this book is not an encyclopaedia. The variety of materials is such that we will treat typical non-metallic materials and their most essential characteristics only. In spite of their interest, we shall not deal with materials like ice, papers, fabrics, bones or multi-materials such as tyres.

Attaching great importance to the relations between the overall behaviour at the macroscopic level and microstructures, the reader ought to refer to Appendix A of Volume I. It contains information about microstructure of materials, which will not be entirely repeated here.

We shall first address damage and fracture of ceramic materials, which, like metallic alloys, are crystalline, but, owing to strong valence bonding, allow almost no deformation by dislocation glide. Brittle sudden fracture is the essential feature for those materials. They are also affected by environment. They are heat resistant and their important uses at high temperature require understanding of their creep resistance. This has been already dealt in Chap. 8. Glasses will be treated next. They have many characteristics in common with ceramic materials. Then, we shall discuss concrete, a material for which progressive damage justifies the use of damage mechanics to a certain extent. We shall then give some information about the fracture of woods, very anisotropic materials, very much dependent on atmospheric conditions. The study of damage and fracture of polymers will follow. The end of this chapter will be devoted to composite materials, the variety of which will allow but general considerations.

10.2 Damage and Fracture of Ceramic Materials

10.2.1 *Microstructure of Ceramic Materials*

Except for some electronic applications (single crystals Si for instance), most ceramic materials are polycrystalline. There are two main routes for their fabrication:

- *Slip fabrication.* The ceramic is milled and screened to achieve the desired grading. The powder mixed with water produces a slip. This can be shaped as a green component by various techniques: extrusion to produce bars or tubes, casting in a porous mold which can absorb water, or which is emptied to keep only a layer of slip on its inner surface. Finally, after drying, the green component

is fired at the desired temperature. During this operation from 900 to 1,400°C a vitrous phase is formed, which binds the ceramic particles together.

- *Sintering*. The ceramic powder is pressed either by uniaxial compression or by isostatic pressing. It is then sintered at high temperature.

Leaving aside the functional ceramics, the most important structural ceramic materials are the following:

- *Alumina* Al_2O_3 – Fabricated by sintering it is used for instance in crucibles, tubes and rods for furnaces, sealing disks, thread guides, lamp tubes, prosthetic devices.
- *Zirconia* ZrO_2 – Different forms are distinguished: *Cubic stabilised zirconia* (CSZ) is obtained by sintering with stabilising oxides Y_2O_3 , MgO , CaO , which then are in solid solution. *Partially stabilised zirconia* (PSZ) is obtained with 5–15% stabiliser. *Tetragonal zirconia polycrystals* (TPZ) is metastable tetragonal zirconia obtained with 2–3% Y_2O_3 . Strain-induced transformation yields the stable monoclinic phase with volume expansion. This provides high toughness to the material. *Zirconia toughened ceramics* (ZTC) take advantage of this transformation.

These materials are used in particular in extrusion dies, machinery wear parts, piston heads, furnace elements.

- *Ytria* Y_2O_3 . It is largely used as sintering additive for silicon nitrides and as a stabilising agent for zirconia. It is also applied in electronics and optics.
- *Silicon nitride* Si_3N_4 – is obtained by sintering with additives like MgO , Al_2O_3 , Y_2O_3 . Porous, it is used as refractories; dense, as cutting tools to machine cast iron and for gas turbine applications for instance.
- *Silicon carbide* SiC – is obtained as recrystallised SiC (RSiC) or as liquid phase sintered (LPS-SiC) with Y_2O_3 , Al_2O_3 , AlN additives. It is used for instance for sealing disks, heat exchangers, burner nozzles.

In any case, these fabrication processes give polycrystalline materials the grain boundaries of which have a different composition from the matrix most often forming a *vitrous phase*. Moreover it is difficult to get rid of remaining *porosities*. Remaining *inclusions* are found preferentially in grain boundaries (Fig. 10.1).

Different microstructures are obtained: equiaxed grains, acicular grains, anisotropically grown grains, bimodal grain size distribution, dual phase materials. The ASM handbook (Täffner et al. 2004) displays a variety of such microstructures.

10.2.2 Cracking Mechanisms

10.2.2.1 Room Temperature Cracking Mechanisms

Inclusions and porosities constitute stress raisers. Cracks initiate there before propagating in the ceramic phase as cleavages. The stress concentration at the interface of inclusions or at the surface of porosities can be calculated using Eshelby's

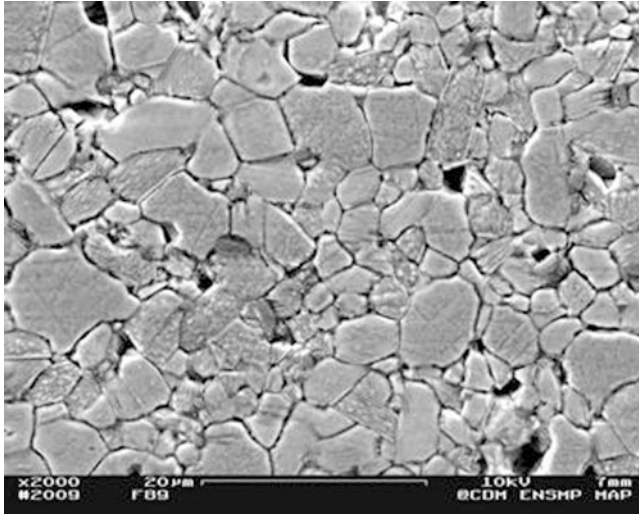


Fig. 10.1 Microstructure of sintered natural SiC with B and C additives (Courtesy of M Boussuge)

formulation (Sect. 2.7.3 in Volume I). Assuming uniaxial applied stressing, the stress σ at the interface of a spherical porosity can be expressed as

$$\sigma = K_T \Sigma \quad (10.1)$$

Σ being the remote applied stress. The stress concentration factor, K_T , depends only on the Poisson ratio; it is of the order of 3. The stress near the porosity decreases as $1/\rho^2$, ρ being the distance from the surface of the porosity.

When a penny-shaped crack of radius a propagates away from the porosity, the stress intensity factor (SIF) is the sum of the one due to the stress concentration and of the one due to the uniform stress Σ . Both SIFs can be obtained by integration of Eq. 2.70:

$$K_I = \frac{2}{\sqrt{\pi a}} \int_R^a \sigma(r) \frac{r}{a} \left[1 - \left(\frac{r}{a} \right)^2 \right]^{-1/2} dr \quad (10.2)$$

R being the radius of the spherical cavity.

The stress concentration being expressed as $K_T \Sigma (R/r)^2$ yields a SIF given by:

$$\begin{aligned} K_I &= \frac{2}{\pi} \left(\frac{R}{a} \right)^2 K_T \Sigma \sqrt{\pi a} \int_R^a \frac{a}{r} \left[1 - \left(\frac{r}{a} \right)^2 \right]^{-1/2} d\frac{r}{a} \\ &= \frac{2}{\pi} \left(\frac{R}{a} \right)^2 K_T \Sigma \sqrt{\pi a} \log \left| \cotan \left(\frac{\theta}{2} \right) \right| \quad \text{with } \theta = \text{Arcsin} \left(\frac{R}{a} \right) \end{aligned} \quad (10.3)$$

Starting from 0, this SIF increases to a maximum with the distance ρ and tends to 0 at long distances.

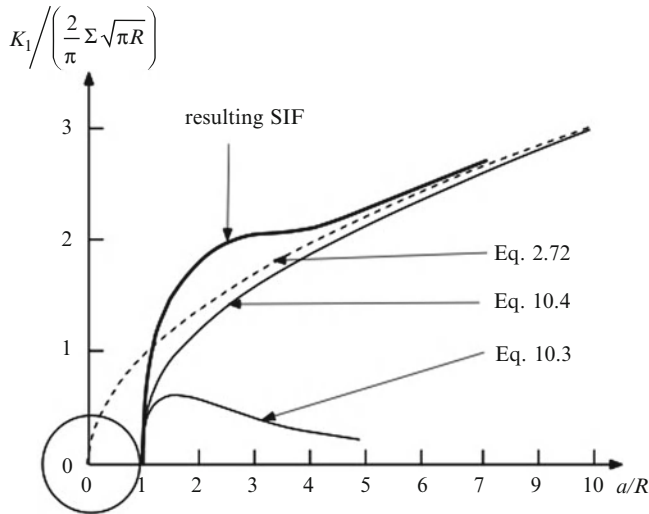


Fig. 10.2 Reduced stress intensity factor for a penny-shaped crack of radius a emanating from a spherical cavity of radius R as a function of the reduced size a/R

The SIF due to the constant stress Σ , obtained in the same way, is given by:

$$K_I = \frac{2}{\pi} \Sigma \sqrt{\pi a} \left[1 - \left(\frac{R}{a} \right)^2 \right]^{1/2} \quad (10.4)$$

At long distances the SIF tends to the expression given by Eq. 2.72 (Fig. 10.2). The resulting SIF, obtained by the addition of the two preceding solutions, agrees well with the solution given by Murakami (1987).

The same kind of evolution of the SIF would be found for a penny-shaped crack emanating from an inclusion. The preceding equations show that the SIF is an increasing function of the radius R of the cavity (or of the inclusion), in spite of the fact that the stress is independent of this size.

The cracks initiated on cavities and inclusions can stop at the obstacles offered by grain boundaries. The fracture stress, that is the critical value of Σ , thus depends on statistically distributed microscopic parameters: cavities, inclusions and grain sizes. It is itself statistically distributed; Weibull statistics is a useful tool in fracture of ceramics (Sect. 10.2.3).

10.2.2.2 Elevated Temperature Cracking Mechanisms

Clean ceramics behave at high temperature in much the same way as metallic materials. Creep initiates voids and makes them grow; the mechanisms are the same as the ones described in Chap. 8.

However, in most industrial ceramics, the grain boundaries, which contain glassy phases, are weak and viscous; microcracks are initiated there at high temperatures.

Stress concentrations at triple points constitute preferential initiation sites. Once a microcrack has propagated across a grain boundary, its propagation to the next grain boundary requires a higher stress, and it is likely to stop there. The fracture at high temperatures is intergranular. Filaments bridging the edges of microcracks in the glassy phases are observed; they resemble the filaments in the crazes of thermoplastic polymers (Clarke and Faber 1987) (refer to Sect. 10.5).

Above a certain stress level, rather than the development of damaged zones, fracture appears to take place by the growth of pre-existing flaws.

10.2.2.3 Damage Zones at Crack Tips

Stress singularity at the crack tip favours crack nucleation at sites of stress concentration. These microcracks are stopped at obstacles, mostly grain boundaries. Thus a damaged zone is created. Crack propagation across the whole polycrystal requires linking of the microcracks and a somewhat larger stress. It could be considered that damage zones are equivalent in these brittle materials to plastic zones in ductile ones.

A reasoning similar to the one used in the Irwin's model of the plastic zone (Sect. 2.3.3.1) provides a damaged zone size l_D given by:

$$l_D = \frac{1}{\pi} \left(\frac{K_I}{\sigma_f} \right)^2 \quad (10.5)$$

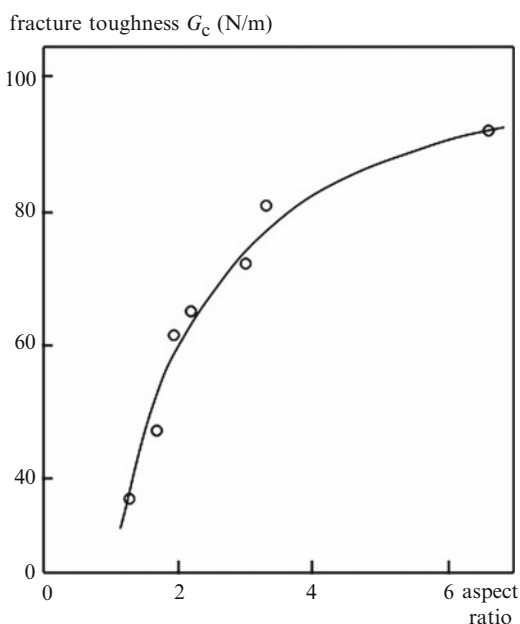
σ_f being the fracture strength.

A closed form solution was given by Bui and Ehrlacher (1981) for a running crack. In their model the material is elastic and breaks when the stress reaches the critical fracture strength σ_f . The crack is surrounded by a damaged zone of width $2h$, with

$$h = \left(\frac{K_I}{\sigma_f} \right)^2 \quad (10.6)$$

The *fictitious crack model* of Hillerborg et al. (1976) for concrete, but applicable as well for ceramics notwithstanding the scale, is quite similar to the Dugdale-Barenblatt model (Sect. 2.3.3.2). The difference is that the closure stress distribution at the crack tip is not constant but increases from 0 at the crack tip up to the damage zone boundary. The size of the damaged zone then differs slightly according to the distribution chosen for the closure stress.

Fig. 10.3 Fracture toughness of hot pressed silicon nitride ceramics as a function of the aspect ratio of grains (Adapted from Clarke and Faber 1987)



10.2.2.4 Crack Deflection

Crack deflection can occur as the result of the presence of less tough grain boundaries, of varying residual elastic stresses, of inclusions. The fracture surface area is then the larger the more important the deflections. The fracture energy is thus increased. However it is not increased in exact proportion to the surface area because deflected cracks are loaded in mixed mode. The problem was analysed by Faber and Evans (1983a). They showed that the deflection processes are more efficient for elongated grains (Fig. 10.3). Experiments (Faber and Evans 1983b) supported the theory. The analysis also demonstrates that the toughening by crack deflection depends on the stress state and is less pronounced when the stress state is more complex as in biaxial loading.

10.2.2.5 Crack Tip Shielding Mechanisms

Various crack tip shielding mechanisms operate (Clarke and Faber 1987).

- *Crack bridging.* Bridging grains left behind the crack front impede crack propagation. A small crack has a low probability of intersecting grains oriented in such a way that they do not break and then participate in the bridging phenomenon. The measured fracture toughness is the intrinsic one. As the crack

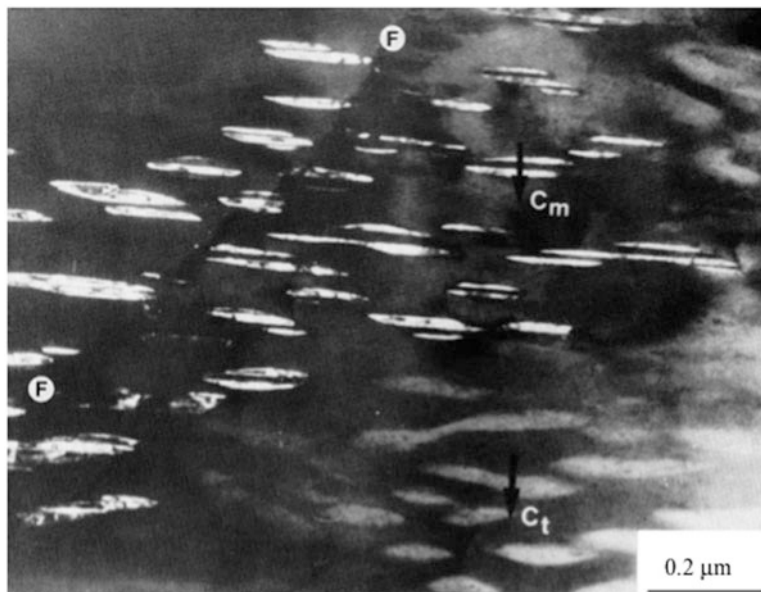


Fig. 10.4 Transformed tetragonal to monoclinic zirconia particles appearing *bright* on the *dark field* TEM micrograph taken across the crack marked FF (C_m refers to monoclinic, C_t to tetragonal). The material is partially transformed zirconia (PSZ) (Clarke and Faber 1987)

grows, it intersects more and more bridging grains and the apparent toughness increases. This produces an R-curve effect. Experiments demonstrating that it is indeed due to grain bridging consisted in determining the R-curve before and after machining out the crack behind the tip. Machining, eliminating the grain bridging, produced a large decrease of the apparent toughness when crack propagation was resumed (Steinbrech et al. 1983).

An evaluation of the magnitude of the bridging effect can be provided by Eq. 2.68, which gives the value of the SIF for a long crack loaded by distributed forces q over a distance d from the tip. In the case of bridging this SIF must be considered as negative and it reduces the applied one. The variable q is equal to the number of bridging grains per unit area multiplied by their resistance. Strictly speaking, this resistance, as well as the distance d over which the bridging effect remains, should be calculated as a function of the crack opening. This leads to involved calculations.

- *Transformation toughening.* In Volume I (Sect. 3.4.5.4) we discussed TRIP steels, the high strain-hardening of which was explained by a martensitic transformation occurring during deformation. A similar phenomenon exists in ceramic materials: the martensitic transformation of tetragonal metastable zirconia into the monoclinic form (Fig. 10.4). This phenomenon was discovered by Garvie et al. (1975). This transformation produces a 4–5% increase of the volume. In the stress field of a crack, the transformation thus creates negative stresses, decreasing the SIF and increasing crack closure. At the crack tip, the

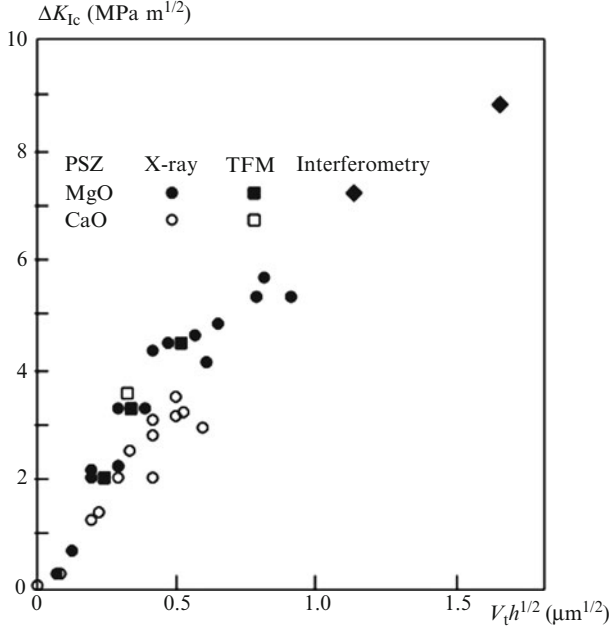


Fig. 10.5 Toughening enhancement for partially stabilised zirconia (PSZ) versus the measured transformation zone size and concentration (Adapted from Clarke and Faber 1987)

transformation occurs within a zone where the stress is larger than a critical value σ_t . In the wake of the crack, this zone is squeezed by the outside untransformed region. This hinders crack opening.

The size h of the zone where transformation occurs varies in proportion to $(K_I/\sigma_t)^2$. Hence, the fracture toughness should be proportional to the square root of h . It was found (McMeeking and Evans 1982) that

$$h = \frac{\sqrt{3}(1+\nu)^2}{12\pi} \left(\frac{\Delta K_{Ic}}{\sigma_t} \right)^2 \quad (10.7)$$

ΔK_{Ic} being the part of the SIF due to the transformation. It is given by:

$$\Delta K_{Ic} = 0.22 V_t \varepsilon^t \frac{E}{1-\nu} \sqrt{h} \quad (10.8)$$

V_t being the volume fraction of transformed phase, ε^t the strain associated with the transformation, E the Young modulus and ν the Poisson ratio.

The relation between the size of the transformed zone and the SIF was checked experimentally by measuring this dimension by various techniques as shown in Fig. 10.5. The figure shows that the effect is large, the increase in fracture toughness reaching $6 \text{ MPa m}^{1/2}$.

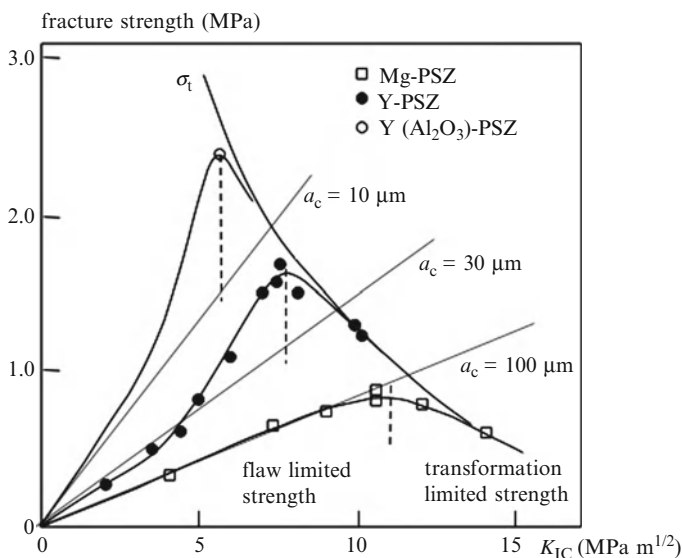


Fig. 10.6 Fracture strength versus fracture toughness for three partially stabilised zirconia (PSZ). The curve σ_T is the variation of the transformation stress. The lines through the origin represent the fracture strength for different critical crack sizes a_c (Adapted from Clarke and Faber 1987)

In the case of transformation toughening again an R-curve behaviour is observed, as for crack bridging and for the same reason.

For the transformation toughening effect to be observed, the temperature must be higher than the M_S temperature of the transformation. Stabilising by various additives: CeO_2 , Y_2O_3 , MgO , modifies the M_S temperature in the desired fashion.

Transformation toughening is accompanied by unusual relations between fracture toughness and fracture strength (Fig. 10.6). It can be observed that, above a certain value of the fracture toughness, the fracture strength decreases when the fracture toughness increases in contrast to the usual behaviour of ceramic materials. Below this critical value the fracture toughness is proportional to the fracture strength as given by the calculation of the critical crack size. Above this critical value, the strength is determined by the stress needed to trigger the transformation.

- *Microcrack softening.* In ceramics containing particles with a thermal expansion mismatch, residual thermal stresses are created upon cooling. They can be relieved by opening microcracks. This can take place in the stress field of the crack tip. The volume change plays then a similar shielding effect as in transformation toughening. However another effect is added, which contributes to the increase of the fracture toughness: the lowering of the apparent elastic modulus due to the presence of the microcracks.

Table 10.1 Experimental toughness values of some ceramic materials

Material	Various methods (MPa m ^{1/2})	SEPB (MPa m ^{1/2})	IF (MPa m ^{1/2})	IS (MPa m ^{1/2})
Alumina 998	3–5	–	–	–
999	3.57 ± 0.22 ^a 3.74 ± 0.40 ^a	–	–	–
Silicon carbide (SiC)	2–5 2.61 ± 0.18 ^a	–	–	–
Silicon nitride	4–6 5.5 ± 0.1 at 20°C ^c 4.2 ± 0.3 at 1,300°C ^c	–	–	–
HPSSN	5.36 ± 0.34 ^a	5.66 ^b	5.75 ^b	6.37 ^b
Zirconia (PSZ or TTZ)	5–10	–	–	–
Y-PSZ	5.34 ± 0.65 ^a	–	–	–
Mg-PSZ	9.44 ^c	–	–	–
Zirconia-Alumina composite (ZAC)	–	6.14 ^b	7.36 ^b	7.41 ^b

^aKübler (1999)^bAwaji et al. (1990)^cGogotsi (2003)

10.2.2.6 Experimental Values of the Fracture Toughness

Various methods are used to measure the fracture toughness of ceramics. The *fracture indentation* method (IF) is the most convenient (Anstis et al. 1981; Marshall and Evans 1981). It consists in measuring the length of cracks emanating from a Vickers hardness indentation. In the *fracture strength* method (IS) a bar with a Vickers indentation is tested in three-point bending (Chantikul et al. 1981). It is simpler than the fracture indentation because no crack length measurement is needed. *Three-point bend testing of single edge pre-cracked beam* (SEPB) is also of common use (Nose and Fujii 1988; Sadahiro 1981). These methods provide slightly different toughness values. Table 10.1 gives examples of experimental values of fracture toughness for a few ceramics.

10.2.3 Size Effects

Consider a crack of length a_0 in a plate of thickness B and width W . Applying a stress results in a stress singularity at the tip, which is shielded by the creation of a damage zone of length l_D at the tip (Bazant and Kazemi 1990).

The nominal applied stress at fracture is written:

$$\Sigma_N = Y_N \frac{F_m}{BW} \quad (10.9)$$

where Y_N is a coefficient pertaining to the loading geometry, tensile or bending for instance and F_m is the maximum load.

The strain energy release rate G is given by:

$$G = \frac{F^2 g(\alpha)}{E' B^2 W} \quad (10.10)$$

where F is the applied load, $g(\alpha)$ is a function of the ratio $\alpha = a/W$ depending on the geometry and E' is equal to the Young modulus E in plane stress and to $E/(1 - \nu^2)$ in plane strain. Note that the effective crack length a , the one which corresponds to the unloading compliance, is larger than a_0 : $a = a_0 + l_D$ accounting for the presence of the damaged zone, denoted also the process zone of length l_D . Let l_{Dc} be the limit of l_D at fracture for very large structures ($W \rightarrow \infty$). The corresponding value of G is then denoted G_c ($\alpha \rightarrow a_0/W$). The values of l_{Dc} and G_c are independent of the geometry. The ratio $G_f/g(\alpha)$ at maximum load should be equal to the ratio $G_c/g(\alpha_c)$, because G is a function of the ratio α . We can make the approximation $g(\alpha) = g(\alpha_0) + g'(\alpha_0)(\alpha_f - \alpha_0)$. From Eqs. 10.9 to 10.10, we now obtain:

$$\Sigma_N = Y_N \left(\frac{E' G_c}{W g(\alpha_0) + l_{Dc} g'(\alpha_0)} \right)^{1/2} \quad (10.11)$$

This equation shows that there is a size effect through the dimension of the structure W . It is also written (Bazant 1984):

$$\Sigma_N = \frac{C f_m}{\sqrt{1 + \beta}} \quad (10.12)$$

where $\beta = W/W_0$ is called the *brittleness number*. The parameter f_m is related to material properties and C is an empirical constant. Dimensional analysis allows justifying these equations (Bazant 2002)

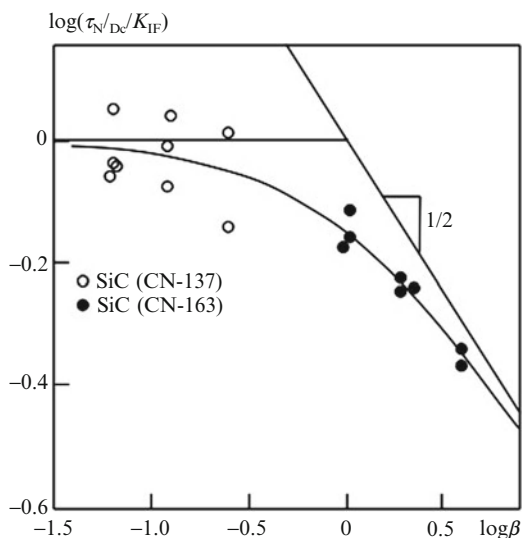
According to these equations the fracture stress becomes constant for low brittleness numbers, corresponding to small ratios W/l_{Dc} , and varies as $\beta^{-1/2}$ for large values. Equation 10.11 shows the size effect for constant ratio a_0/W and could be expressed in term of the crack size for constant dimension. This emphasises the fact that a large brittleness number corresponds to the applicability of LEFM, with the nominal stress varying as the crack size to the power $-1/2$. This reminds of Fig. 2.42 which can be interpreted as the evolution of the failure load as a function of the dimension for a given crack length. For small dimensions, the failure load is the limit load in plasticity; for smaller dimensions, it is the one given by LEFM.

Figure 10.7 shows as an example results obtained on SiC. It displays a dispersion of the results, which relates to the inherent dispersion of the fracture events at the microscopic scale. But the trend given by Eq. 10.12 is well observed.

The critical damaged zone size is related to the characteristic dimension of the microstructure, the grain size. Meaningful comparisons of the effect of this require testing at the same value of the brittleness number.

Fig. 10.7 Reduced fracture strength versus the brittleness number for SiC.

$\tau_N = \Sigma_N g^{1/2} / Y_N$. (Adapted from Bazant and Kazemi 1990)



It should be added that the statistical distribution of the cracks involves also a size effect, since the probability of the presence of a large defect, associated with a low strength, is greater in a big volume. This is discussed in the next section.

10.2.4 Statistical Distribution of the Fracture Strength

The fracture mechanisms at the microscopic scale yield an inherent statistical distribution of the fracture strength related to the distribution of inclusions, pores and grain sizes and shapes. The Weibull analysis described in Sect. 3.4.1 is well suited for the brittle ceramic materials. Figure 10.8 shows results obtained on various ceramics or glass ceramics materials. Plotted on Weibull scales, the data points would align with a slope not very different.¹

Figure 10.9 shows Weibull plots for two different kinds of alumina. It can be observed that the width of the distribution is large (the Weibull m exponents are low), much larger than for brittle metals.

However, in Sect. 3.4.3, a limitation to the applicability of the Weibull law, based on the weakest link assumption, was pointed out for the case of a collection of arrested cracks. Indeed Fig. 10.10 displays a deviation of the linearity on a Weibull plot at low probabilities. This represents a drawback in design, inasmuch as to insure a given probability of fracture, say 1%, the admissible stress is smaller when

¹It can be shown that at the inflexion point of the Weibull representation the slope dP_R/dR_m is equal to $(1 - 1/m)^{1/m}$.

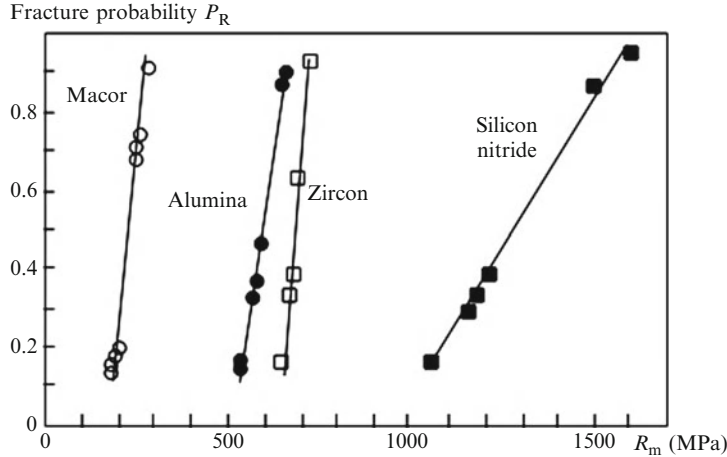


Fig. 10.8 Fracture probability as a function of the fracture strength for four different materials. Macor is a glass ceramic the crystallised phase of which is K fluorphlogopite, a mica of composition $6\text{SiO}_2 \cdot \text{Al}_2\text{O}_3 \cdot \text{MgO} \cdot 2\text{MgF}_2 \cdot \text{K}_2\text{O}$; Zircon is the Zr silicate ZrSiO_4

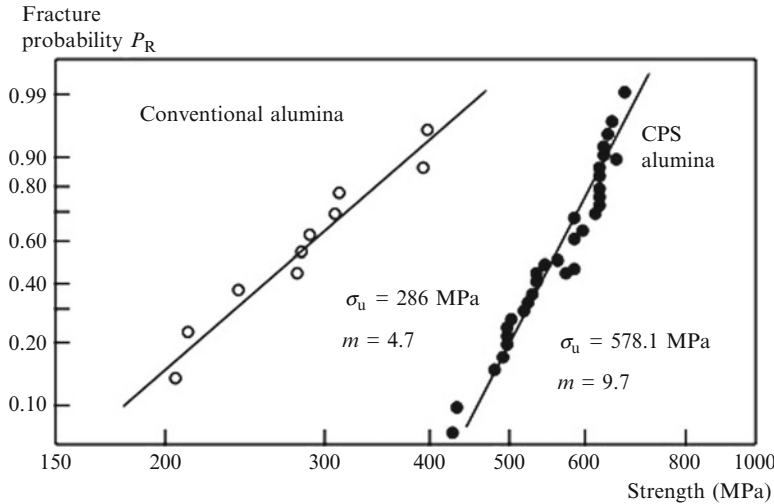
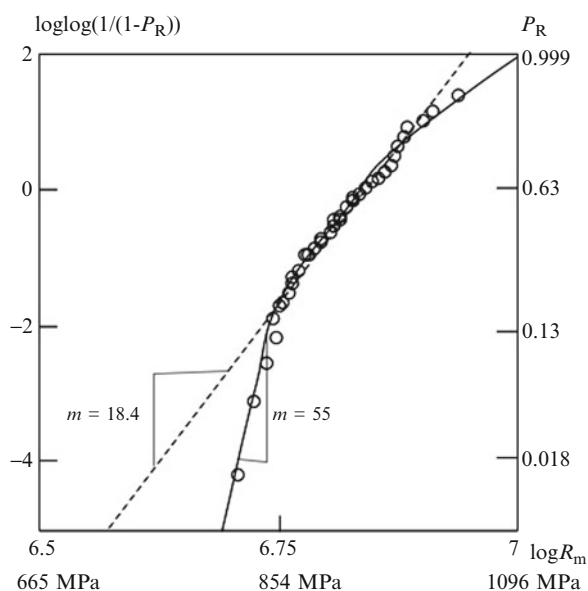


Fig. 10.9 Weibull plots for two different kinds of alumina, one prepared in the usual way, the other through controlled processing techniques leading to improved properties (Adapted from Meyers and Chawla 1999)

extrapolating the Weibull law than what it should really be. The deviation from the Weibull law is linked with the size effects discussed in the preceding section (Le and Bazant 2009). A small brittleness number, that is a structural part of small size, corresponds to a process zone size, which is a large fraction of the dimension of the structural part. The weakest link model does not apply in that case.

Fig. 10.10 Weibull plot for zirconia TZP (Adapted from Le and Bazant 2009)



10.2.5 Delayed Fracture of Ceramics

Similar to stress corrosion cracking of metallic materials, cracks in ceramics can propagate for stress intensity factors well below the fracture toughness K_{Ic} in certain environments, especially water. This is usually due to dissolution at the crack tip of the amorphous phases. We discuss it in the next section devoted to the fracture of glass.

We mention here only that the phenomenon can be observed as well in single crystals and in polycrystalline ceramics in which there is no intergranular glassy phase. It is due to molecular reactions and depends on the more or less ionic character of the oxide. A schematic representation of the chemical reaction is shown in Fig. 10.11. At the crack tip the strain extends the Si–O–Si bonds and a hydrogen bond can be created with water molecules. The final breaking of the hydrogen bond produces the rupture. This dissociative reaction cannot take place in the case of strong ionic bonding, like for instance in MgF_2 .

10.3 Fracture of Glass

10.3.1 Fracture Mechanisms

Notions about the structure of glass can be found in Appendix A of Volume I. As for other materials, the much lower fracture strength of glass compared with the

Fig. 10.11 Schematic representation of molecular reaction at the crack tip of water molecules with SiO_2 . (1) Hydrogen bond between H and O; (2) dissociative reaction; (3) rupture of hydrogen bond

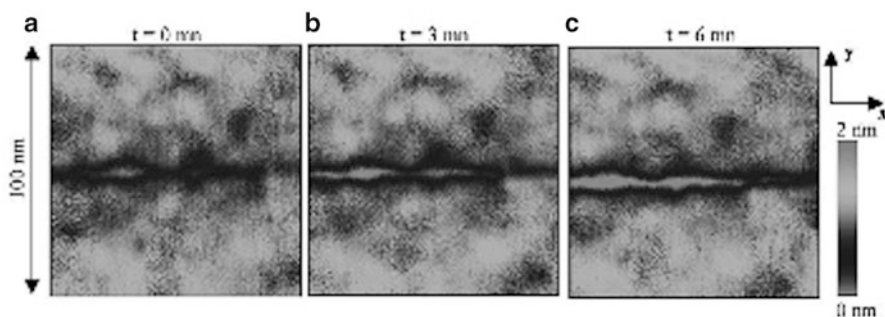
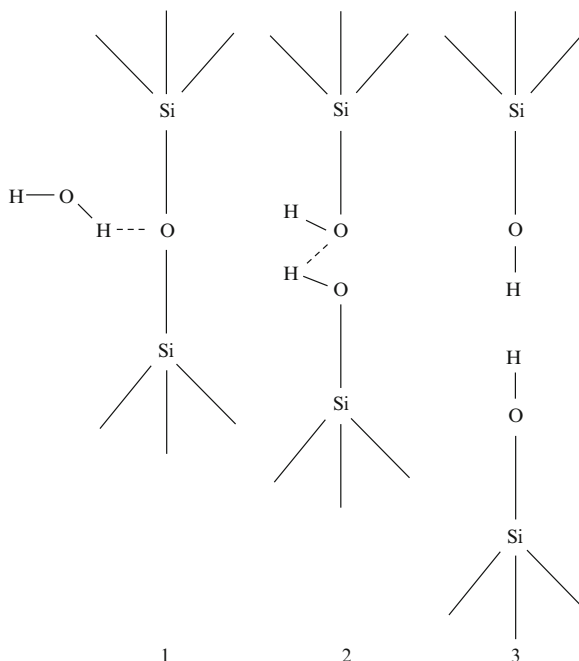


Fig. 10.12 Sequence of successive topographic AFM frames showing the crack propagation at the surface of glass. The crack propagates from *left* to *right* with a mean velocity of 3.10^{-11} m/s . (a) evidence of nanometric damage cavities ahead of the crack; (b) growth of the cavities; (c) crack advance by coalescence of cavities ($K_I = 0.43 \text{ MPa m}^{1/2}$) (Célarié et al. 2003)

theoretical fracture strength implies the presence of defects to create local stress concentrations. Initial microcracks are needed. We discuss their presence on the surface in the next section. Process damaged zones exist at the tip of cracks in glass (Fig. 10.12). They are of very small size (of the order of 100 nm); they are difficult to observe. Atomic force microscopy has allowed measuring the depression of the surface where a crack emerges. *Ab initio* calculations showed that small voids are created ahead of the crack tip. They are about 20 nm long.

10.3.2 Importance of Surface Cracks

In glass initial cracks are essentially surface microcracks. Five microns very acute cracks are sufficient to account for the difference with the theoretical fracture strength. Such cracks cannot be seen, but they can be detected by chemical etching.

The origin of surface cracks is in handling glass objects. As manufactured, they are much more resistant than after some use. Contact with abrasives, with sharp particles, impingement by dust create tiny surface cracks. Can occur also chemical reactions.

The resistance can then be improved by applying a protective layer, like a polymer coating, on the surface. Otherwise, an efficient way consists in creating a compressive superficial stress. There are two techniques to achieve this:

- *Quenching*. Fast cooling of a newly-cast sheet contracts surface layers more than the interior. But, the centre of the sheet remains viscous and it can follow the surface deformation and relax. Its contraction which follows when it cools down puts the surface layers in compression. The compressive stresses prevent surface cracks from opening. The residual stress distribution across the thickness is parabolic. Tensile stresses in the interior equilibrate the ones near the surface. If the loading is sufficient to trigger fracture, for instance when a wind screen is hit by a stone, the glass is shattered in many small pieces by these remaining tensile stresses.
- *“Chemical quenching”*. This consists in introducing chemical elements in the surface layers, which dilate the glass by ion exchange. This is achieved most often by replacing small sodium ions by larger potassium ones by diffusion. The resulting stress distribution of stresses through the glass thickness is different from that obtained by fast cooling: the zone in compression is much narrower after chemical quenching. The tensile stress in the interior, which balances the compressive one at the surface, is reduced compared with thermal quenching. The glass does not shatter the way it does in fracture after thermal quenching.

10.3.3 Reinforcement by Metallic Particles

Increasing the size of the process zone would be a mean to improve the fracture toughness of glasses. The idea is to introduce obstacles to stop the cracks. One way to achieve that is to incorporate in the glass metallic particles. Furthermore these particles dissipate energy by plastic deformation. Their dilatation coefficient must be carefully chosen. If the particles contract less than the glass when it is cooled after manufacture, tensile stresses are generated and they have an embrittling effect. In the converse case, the interfaces break and the inclusions behave as cavities. The best result is achieved if the coefficient of thermal expansion of the particles matches the one of the glass. In that case the cracks can be stopped at the particles and when they propagate they cross the particles. In glass M (% composition: 50 SiO₂, 37

Na_2O , 8 Al_2O_3 , 5 Li_2O) of Young modulus 68.9 GPa and of thermal expansion coefficient $16.10^{-6}/^\circ\text{C}$, aluminium particles meet the requirement. Incorporation of 20% of these in the glass raises the fracture toughness from 12 to 600 J/m^2 .

10.3.4 Delayed Fracture of Glass

10.3.4.1 Practical Consequences of Delayed Fracture of Glass

Slow crack propagation is observed for glass in certain environments, especially water or humid air. This phenomenon is designated also as *stress corrosion cracking* or *static fatigue*. It has practical consequences (Gy 2003). For instance, empirical design rules for architectural glazing distinguish between vertical windowpanes loaded occasionally by wind bursts (20 MPa design stress) and more or less horizontal glazings more permanently loaded (10 MPa design stress). In an aquarium the limit would be 6 MPa. Cracks produced on windscreens, hit by a gravel stone, can be observed to propagate slowly. Glaziers used to know that pitting on a diamond scratch could ease the cutting of a windowpane. Glass wool products for insulation are compressed during transportation, after which they do not recover their initial thickness, owing to the delayed fracture of highly bent fibres. Other applications such as optical fibres, reinforcement fibres, TV screens, require protection against delayed fracture of glass.

10.3.4.2 Mechanisms of Delayed Fracture of Glass

Figure 10.13 shows classical results obtained by Wiederhorn (1967). They display the three stages already described in the section devoted to stress corrosion cracking (Sect. 7.3.3.4): below a threshold the cracks do not propagate; in stage I, above the threshold, the crack velocity is strongly dependent on the stress intensity factor; in stage II, there is a plateau of the crack velocity; stage III precedes brittle fracture for $K_I = K_{Ic}$. Stage I is controlled by the chemical reaction kinetics at the crack tip; stage II by the hydrodynamic flow through the crack to the tip.

The chemical reaction taking place at the crack tip is shown schematically in Fig. 10.11: $\equiv\text{Si}-\text{O}-\text{Si}\equiv + \text{H}_2\text{O} \rightarrow \equiv\text{Si}-\text{OH} + \text{HO}-\text{Si}\equiv$. This reaction is thermo-activated with an energy of activation which is reduced by the local stress at the crack tip and which depends on the radius of curvature of the tip. An exponential dependency on the SIF results. This is close to a power law with a high exponent, which is easier to use for life predictions (Gy 2003).

Blunting of the crack tip would explain the *threshold*. This hypothesis derived from experiments of Michalske (1977) who observed markings on the fracture surface of specimens in which crack growth was stopped by decreasing the SIF below the threshold and then resumed by increasing the SIF again. However, AFM observations showed that the markings observed were not due to crack tip

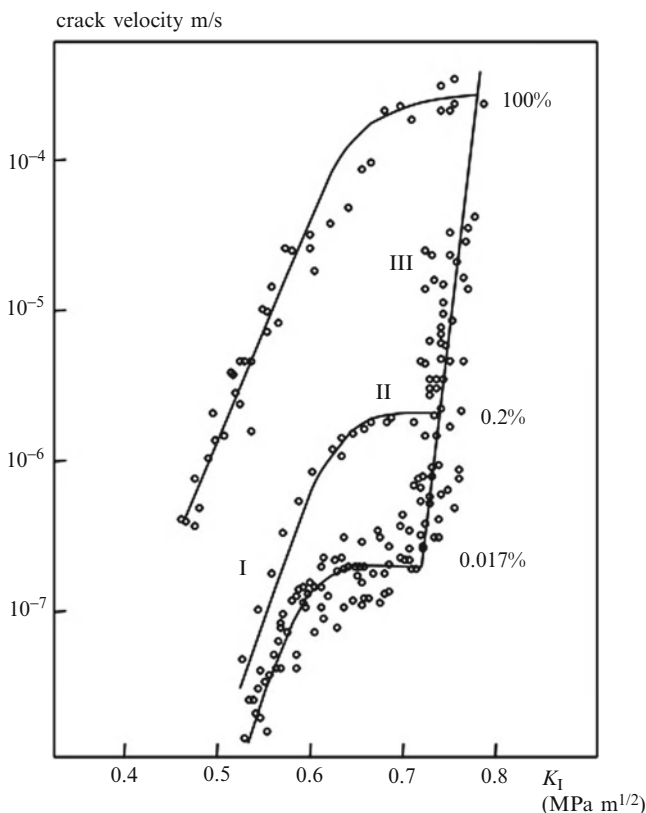


Fig. 10.13 Crack velocity as a function of the stress intensity factor showing the effect of relative humidity (Wiederhorn 1967)

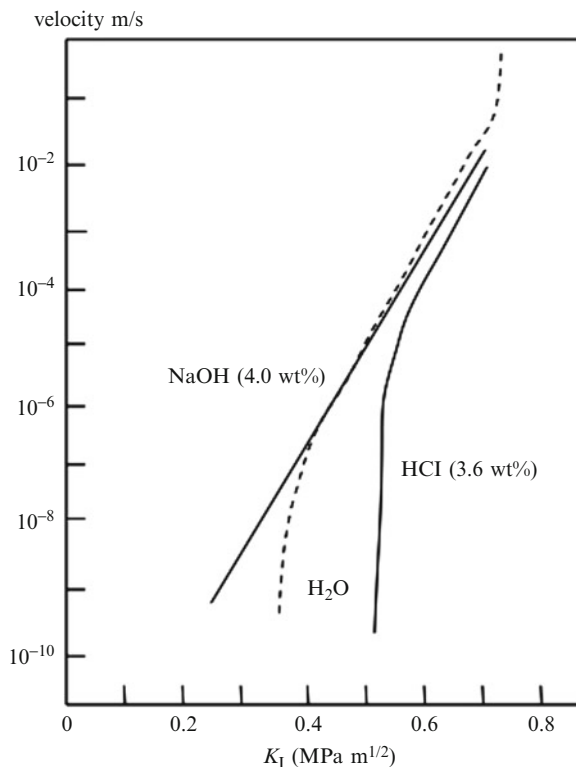
blunting but to crack deviation (Wiederhorn et al. 2002; Guin and Wiederhorn 2003). Presumably this is to avoid the chemically disturbed zone at the tip. Indeed, there are evidences that alkali ions are leached at the crack tip and exchanged with H ions (Gy 2003). This is supported by the strong effect of the pH of the active solution as shown in Fig. 10.14.

Cracks in glass can be healed. The SIF for crack propagation was found to be larger for cracks that had been aged in wet air after unloading than it was before (Gy 2003). The effect increases with ageing time and temperature. Healing does not take place in dry atmospheres.

10.3.5 Size Effects

We have seen that there exists a damaged process zone at the crack tip. In the same way as in ceramics (Sect. 10.2.3) this results in size effects. For glass the transition

Fig. 10.14 Effect of the pH on crack growth rate for $17\text{Na}_2\text{O}-8.2\text{SiO}_2-1\text{Al}_2\text{O}_3$ glass (Adapted from Gehrke et al. 1990)



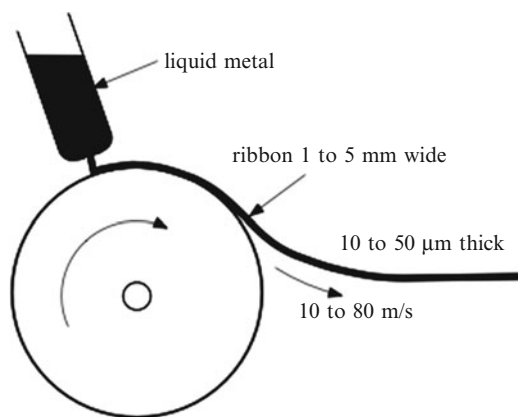
to low brittleness number behaviour occurs for very small sizes and the general behaviour is brittle fracture obeying linear elastic fracture mechanics. Nevertheless, low brittleness number might be the explanation of the behaviour of glass fibres, which chatter in small pieces at very high stresses (of the order of 3,000 MPa) in tensile loading, which do not heal and which fall out of the extrapolation of live calculations derived from long cracks growth rate (Gy 2003).

10.3.6 Metallic Glasses

10.3.6.1 Introduction

Solid-state materials of various bonding types, including ionic, covalent, van der Waals, hydrogen and metallic (see Appendix A in Volume I) can be made by various ways into amorphous solid forms. Metallic amorphous alloys (*i.e.* metallic glasses) are recent materials. The first metallic glass of $\text{Au}_{75}\text{Si}_{25}$ (the subscript designates the composition in atomic % otherwise stated) was discovered by

Fig. 10.15 Melt-spinning technique used for producing metallic glasses. The molten alloy is ejected onto the flat rim of a rapidly rotating wheel to produce a *thin ribbon* up to a few millimeters wide and 10–50 μm thick



Duwez² in 1960 (Klement et al. 1960). These researchers developed a rapid cooling technique for quenching metallic liquids at very high rates of 10^5 – 10^6 K/s (Fig. 10.15). They showed that in some alloy systems the process of nucleation and growth of crystalline phases could be kinetically by-passed to yield a frozen liquid configuration corresponding to a metallic glass. The importance of Duwez's work and of his group lies in the fact that their method permits large quantities of a metallic glass to be made as compared to other methods, for instance, vapour condensation. The researches devoted to metallic glasses have largely increased due to the scientific challenge and engineering applications of these materials (for a review, see e.g. Wang et al. 2004; Johnson 1989; Greer 1995; Inoue 2000; Schuh et al. 2007). One of the attractions of amorphous alloys is their mechanical properties. Amorphous alloys generally exhibit elastic moduli of the same order as conventional engineering metals, but have room-temperature strengths significantly in excess of those of polycrystals with comparable composition. At the same time, the amorphous structure leads to high temperature rheological properties well known in other glassy systems, including oxides and amorphous polymers: stable Newtonian flow at low stresses enables superplastic-like deformation that can be useful in shape-forming operations. The technique of melt quenching used to produce these metallic amorphous materials has been extensively developed for producing a wide variety of metallic glasses, as shown below in Table 10.2. However, this technique allows the production of thin films only with a typical thickness of 10–50 μm .

The production of bulk metallic glasses (BMG) requires that the glassy state can be obtained with much slower cooling rates, typically 1–100 K/s (Fig. 10.16). This figure shows a schematic TTT diagram (see Appendix B in Volume I for the

²Pol Duwez (1907–1984) was a Belgian-born materials scientist. While working at Caltech in USA, he first introduced glasses made through rapid liquid cooling.

Table 10.2 The composition of representative BMG systems, their glass transition temperature, T_g , onset temperature of crystallisation, T_x , and onset melting point, T_m

BMG	T_g (K)	T_x (K)	T_m (K)
Mg ₈₀ Ni ₁₀ Nd ₁₀	454.2	477.7	725.8
Mg ₆₅ Ni ₂₀ Nd ₁₅	459.3	501.4	743.0
Mg ₇₅ Ni ₁₅ Nd ₁₀	450.0	482.8	717.0
Mg ₇₀ Ni ₁₅ Nd ₁₅	467.1	494.1	742.5
Mg ₆₅ Cu ₂₅ Y ₁₀	424.5	484.0	727.9
Zr _{41.2} Ti _{13.8} Cu _{12.5} Ni ₁₀ Be _{22.5}	623.0	705.0	932.0
Zr _{46.75} Ti _{8.25} Cu _{7.5} Ni ₁₀ Be _{27.5}	622.0	727.0	909.0
Zr _{45.38} Ti _{9.62} Cu _{8.75} Ni ₁₀ Be _{26.25}	623.0	740.0	911.0
Zr _{42.63} Ti _{12.37} Cu _{11.25} Ni ₁₀ Be _{23.75}	623.0	712.0	933.0
Zr ₄₄ Ti ₁₁ Cu ₁₀ Ni ₁₀ Be ₂₅	625.0	739.0	917.0
Zr _{38.5} Ti _{16.5} Ni _{9.75} Cu _{15.25} Be ₂₀	630.0	678.0	921.0
Zr ₄₈ Nb ₈ Cu ₁₂ Fe ₈ Be ₂₄	658	751	1,009
Zr ₄₈ Nb ₈ Cu ₁₄ Ni ₁₂ Be ₁₈	656	724	997
Zr ₅₇ Ti ₅ Al ₁₀ Cu ₂₀ Ni ₈	676.7	725.4	1095.3
Zr ₅₇ Nb ₅ Cu _{15.4} Ni _{12.6} Al ₁₀	687	751	1,092
Zr ₅₃ Ti ₅ Cu ₁₆ Ni ₁₀ Al ₁₆	697	793	1,118
Zr ₆₆ Al ₈ Cu ₇ Ni ₁₉	662.3	720.7	1117.3
Zr ₆₆ Al ₈ Cu ₁₂ N ₁₄	655.1	732.5	1109.1
Zr ₆₆ Al ₉ Cu ₁₆ Ni ₉	657.2	736.7	1110.9
Zr ₆₆ Al ₉ Ni ₂₆	672.0	707.6	1188.5
Zr ₆₅ Al _{7.5} Cu _{17.5} Ni ₁₀	656.5	735.6	1108.6
Pd ₄₀ Ni ₄₀ P ₂₀	590.0	671.0	877.3
Pd _{81.5} Cu ₂ Si _{16.5}	633.0	670.0	1008.8
Pd ₄₀ Cu ₃₀ Ni ₁₀ P ₂₀	586.0	678.0	744.8
Pd _{42.5} Cu ₃₀ Ni _{7.5} P ₂₀	574.0	660.0	808.0
Pd _{77.5} Cu ₆ Si _{16.5}	637.0	678.0	1019.4
Pd _{42.5} Cu _{27.5} Ni ₁₀ P ₂₀	572.0	666.0	752.0
Cu ₆₀ Zr ₃₀ Ti ₁₀	713.0	763.0	1110.0
Cu ₅₄ Zr ₂₇ Ti ₉ Be ₁₀	720.0	762.0	1090.0
Cu ₆₀ Zr ₂₀ Hf ₁₀ Ti ₁₀	754	797	1,189
La ₆₆ Al ₁₄ Cu ₂₀	395.0	449.0	681.9
La ₅₅ Al ₂₅ Ni ₂₀	490.8	555.1	711.6
La ₅₅ Al ₂₅ Ni ₁₀ Cu ₁₀	467.4	547.2	662.1
La ₅₅ Al ₂₅ Cu ₂₀	455.9	494.8	672.1
La ₅₅ Al ₂₅ Ni ₅ Cu ₁₀ Co ₅	465.2	541.8	660.9
Nd ₆₀ Al ₁₀ Cu ₁₀ Fe ₂₀	485.0	610.0	773.0
Nd ₆₀ Al ₁₅ Ni ₁₀ Cu ₁₀ Fe ₅	430.0	475.0	709.0
Nd ₆₁ Al ₁₁ Ni ₈ Co ₅ Cu ₁₅	445.0	469.0	729.0
Ti ₃₄ Zr ₁₁ Cu ₄₇ Ni ₈	698.4	727.2	1119.0
Ti ₅₀ Ni ₂₄ Cu ₂₀ B ₁ Si ₂ Sn ₃	726.0	800.0	1230.0
Au _{77.8} Si _{8.4} Ge _{13.8}	293.0	293.0	606.0
Pr ₆₀ Cu ₂₀ Ni ₁₀ Al ₁₀	409	452	705
Pr ₅₅ Al ₁₂ Fe ₃₀ Cu ₃	551	626	845

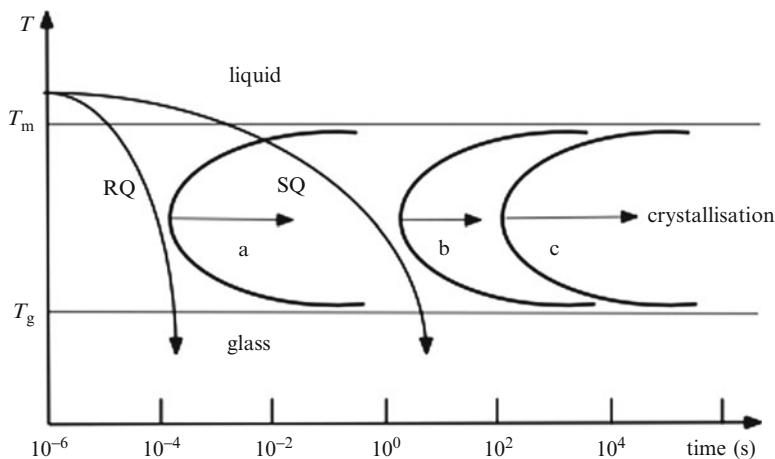


Fig. 10.16 Schematic time-temperature-transformation (TTT) diagrams comparing the crystallisation kinetics of (a) rapidly quenched, (b) slower quenched and (c) bulk-glass forming alloys. The two curves with arrows show the cooling history during rapid quenching (RQ) at 10⁶ K/s and relatively slow quenching (SQ) at 10² K/s

definition of TTT curves) for crystallisation of the under-cooled liquids. Duwez's glasses had nucleation kinetics in the under-cooled region between the melting point T_m and the glass-transition temperature T_g such that the time scale for crystallisation was in the hundreds of microseconds to milliseconds range at the “nose” of the nucleation curve. Other materials, many of them based on Zr or Pd, have been discovered by the Turnbull group (see *e.g.* Chen and Turnbull 1969). These new metallic glasses exhibited a TTT diagram with a “nose” in the range of 1–10s. Finally the alloys of the Vitreloy (*e.g.* Vitreloy 106a: $\text{Zr}_{58.5}\text{Cu}_{15.6}\text{Ni}_{12.8}\text{Al}_{10.3}\text{Nb}_{2.8}$) family have been developed. These glasses exhibit a TTT diagram at time scales of the order of 100s. Extensions of the earlier work on Pd-based glasses have also shown a TTT nose at time scales of 100s for quaternary Pd-Cu-Ni-P (Nishiyama and Inoue 1996). It has been shown that 1-cm fully glassy rods can be produced using copper mould casting of the $\text{Cu}_{44.25}\text{Ag}_{14.75}\text{Zr}_{36}\text{Ti}_5$ composition (Dai et al. 2006). All these metallic glasses can crystallise when they are heated above the temperature of crystallisation, T_x . Table 10.2 (Wang et al. 2004) recapitulates the composition of representative bulk metallic glasses, their glass transition temperature, T_g , the onset temperature of crystallisation, T_x and the onset of melting, T_m .

Some metallic glasses are used for their magnetic properties. Iron-based metallic glasses, not included in this table, are those which can be considered as one of the most attractive due to their excellent mechanical and magnetic properties (see *e.g.* Ashby and Greer 2006; Lewandowski et al. 2008; Greer 2009; Kim et al. 2005).

These glasses with a typical composition $\text{Fe}_{94.95}\text{Cu}_{4.45}\text{Si}_{0.4-0.8}\text{P}_{0.1-0.2}$ (Wt %) can be produced from cast iron in iron-making blast furnaces and steel-making converters. Fe-based metallic glasses have been commercialised as reinforcing

fibres in concrete, shot peening, golf balls, soft magnetic powdered cores in electronic devices, etc. (Inoue et al. 2003; Redon and Chermant 1999). The excellent soft-magnetic properties (low coercive force and high permeability) of iron-based compositions, have led to commercial applications such as transformer cores, magnetic read-heads and magnetic shielding (Ashby and Greer 2006).

10.3.6.2 Mechanical Properties of Metallic Glasses

The deformation mechanisms of metallic glasses were reviewed by Schuh et al. (2007), following the earlier work by Argon (1979) (see also Bletry et al. 2007). The *homogeneous plastic flow* is modelled using the formation of activated processes (see Volume I, Chap. 4, Sect. 4.3.1.2). The strain rate is written as:

$$\dot{\epsilon} = \dot{\epsilon}_0 \exp\left(-\frac{Q}{kT}\right) \sinh\left(\frac{\sigma V}{kT}\right) \quad (10.13)$$

where $\dot{\epsilon}_0$ is a constant, Q is the characteristic activation energy for the process and V is the activation volume. This equation can be rearranged to define the viscosity, η :

$$\eta = \frac{\sigma}{B \sinh(D\sigma/kT)} \quad (10.14)$$

where $B = \dot{\epsilon}_0 \exp(-Q/kT)$ and $D = V/kT$.

This expression predicts that at low stresses (*i.e.* low strain rates) the viscosity tends towards a constant equal to kT/BD and decreases with increasing σ at high stresses (*i.e.* high strain rates). The results reported in Fig. 10.17a and b show that the viscosity measured in a $\text{Zr}_{41.2}\text{Ti}_{13.8}\text{Cu}_{12.5}\text{Ni}_{10}\text{Be}_{22.5}$ BMG is strongly temperature dependent as predicted from (10.14) and is strain rate sensitive in particular when the temperature is decreased (Fig. 10.17a; Lu et al. 2003). The effect of temperature on the viscosity observed in various BMG is shown in Fig. 10.17b (Busch et al. 1998).

Constitutive laws for homogeneous deformation of metallic glasses become non-Newtonian at high stresses; this transition is also promoted by a decrease in temperature. The divergence from Newtonian behaviour corresponds to a decreasing rate sensitivity and a corresponding decline in flow stability (see Eq. 1.8 in Chap. 1, Volume I). Indeed, at very high stresses and lower temperatures, metallic glasses deform essentially through localisation processes. In these materials, there are a number of potential causes for strain softening and localisation, including the local production of free volume due to flow dilatation, local evolution of structural order, the redistribution of internal stresses, and local heat generation. Although all of these

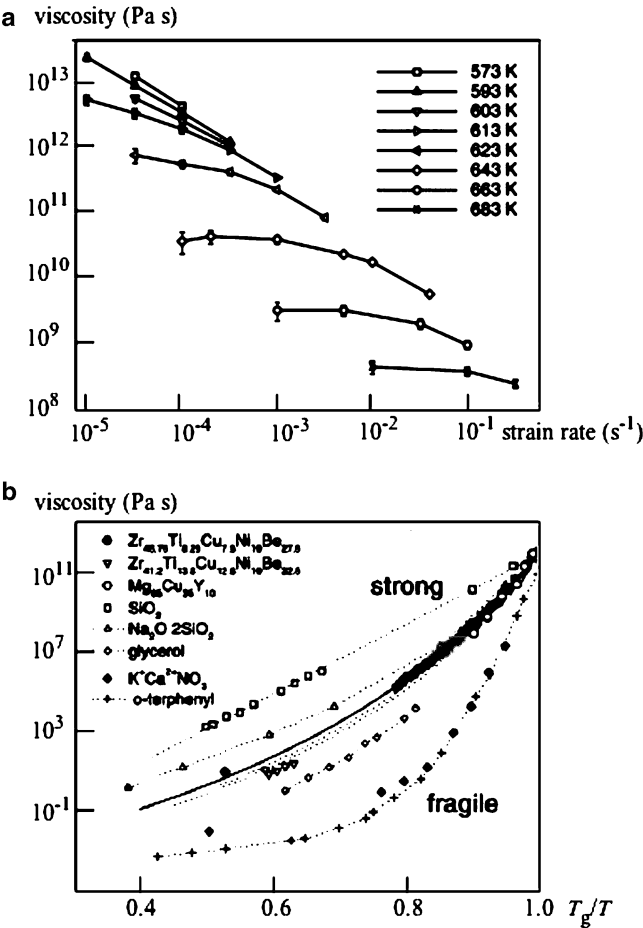


Fig. 10.17 (a) Viscosity in $Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$ bulk metallic glass as a function of strain rate at various temperatures (Lu et al. 2003); (b) Viscosity of the $Zr_{46.5}Ti_{8.25}Cu_{7.5}Ni_{10}Be_{27.5}$ alloy as a function of temperature. Two other bulk metallic glasses and several non-metallic “strong” and “fragile” glasses are included (Adapted from Busch et al. 1998)

causes were discussed in the literature, the dominant contributor to localisation is generally believed to be a local change in the state of the glass (*i.e.* a local increase in free volume or evolution of structural order). For a detailed discussion of all these causes, see Schuh et al. (2007).

The availability of BMG samples allowed better characterisation of the mechanical properties of metallic glasses, in particular the determination of their fracture toughness on fatigue pre-cracked specimens (Lewandowski et al. 2005). These studies showed that when metallic glasses exhibited plastic flow (*e.g.* those based on zirconium or palladium), it was localised into *shear bands*. The *local* plasticity within the shear band was very high (>100%) in these cases and formed part of

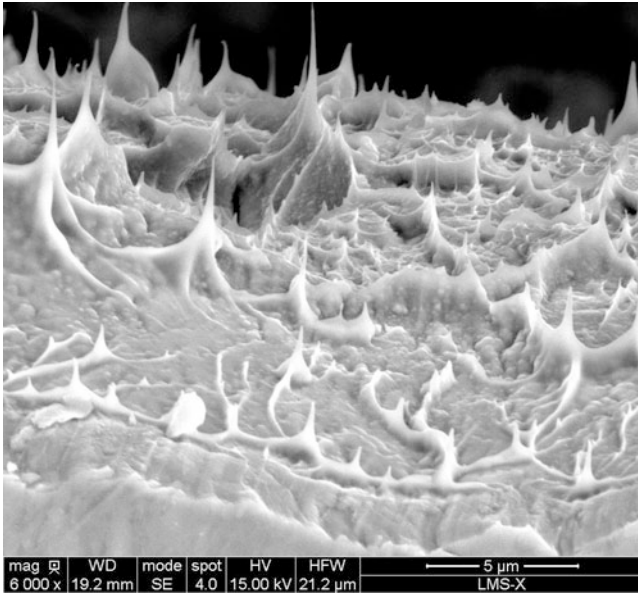


Fig. 10.18 Scanning electron micrograph of fracture surface of the bulk metallic glass. *In-situ* tensile test $\text{Zr}_{57}\text{Cu}_{20}\text{Al}_{10}\text{Ni}_8\text{Ti}_5$ (at %, Vitreloy 1TM). Vein pattern associated with localised plastic flow in shear bands in materials with significant toughness (Courtesy of E. Héripré)

the micro-mechanism of fracture (Spaepen 1997; Blety et al. 2004). The associated fracture surface illustrated in Fig. 10.18 shows a very characteristic vein pattern (resembling to some extent to the dimples observed in ductile fracture of metals) and evidence of high local plasticity arising from instabilities in the band of lowered “viscosity”. This shear-softening effect is in marked contrast to the work-hardening effect exhibited by crystalline metals. It prevents stable global plastic elongation in tension. Thus metallic glasses showing plasticity cannot properly be termed globally *ductile*, although they are *malleable* and can be deformed in compression or in bending. Plastic flow occurs at unusually large fraction of the theoretical strength and this combined with the capacity for *local* plastic flow can lead to high fracture toughness ($>20 \text{ MPa m}^{1/2}$) (Lowhaphandu and Lewandowski 1998; Wesseling et al. 2004).

The importance of the ratio μ/k (where μ is shear modulus and k the bulk modulus) for the mechanical properties of isotropic amorphous metallic structures was underlined (Lewandowski et al. 2005; He et al. 2011; Hennan and Anand 2009). It should be reminded that in isotropic materials (see Table 2.2 in Volume I, Chap. 2, Sect. 2.3.2.8):

$$\frac{\mu}{k} = \frac{3}{2} \frac{(1 - 2\nu)}{(1 - \nu)} \quad (10.15)$$

Fig. 10.19 SEM images showing multiple shear bands near the notch root and crack deflecting after fracture – $\text{Zr}_{48}\text{Cu}_{45}\text{Al}_7$ alloy (He et al. 2011)

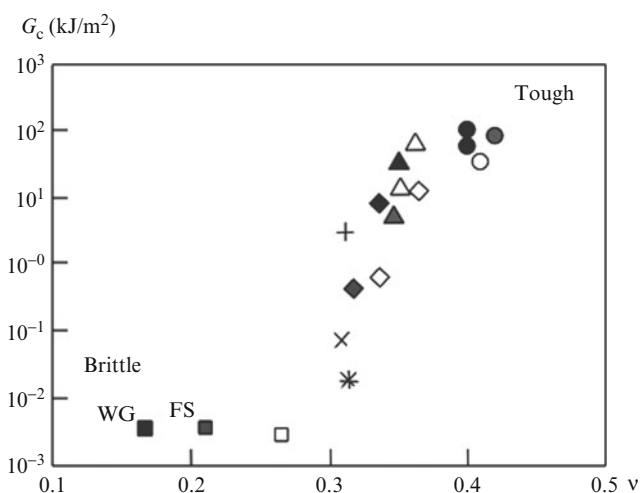
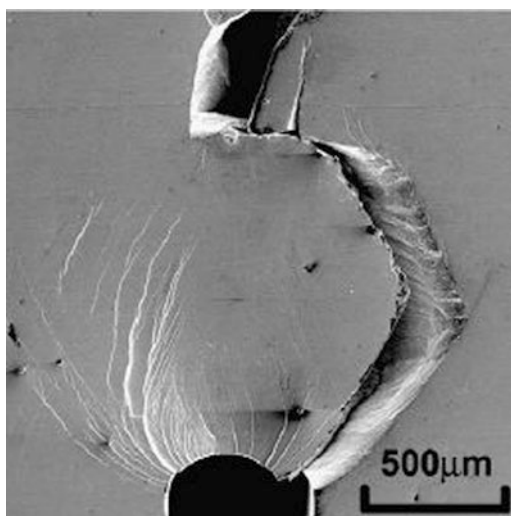


Fig. 10.20 The correlation of fracture energy G with Poisson's ratio, ν , for metallic glasses; Window glass (WG) and Fused silica (FS)

The importance of the ratio μ/k on the plasticity and cleavage fracture of crystalline metallic materials has already been underlined (see Volume II, Chap. 3, Sect. 3.2.2). With low values of μ/k , the glasses based on palladium, zirconium, copper or platinum, all have fracture energies well in excess of 1 KJ m^{-2} , exhibit extensive shear banding (see Fig. 10.19), and have vein-pattern fracture surfaces similar to those observed in Fig. 10.18. The correlation of fracture energy G with Poisson's ratio ν for the data collected by Lewandowski et al. (2005) is shown in Fig. 10.20. Higher values of ν give higher fracture energy; the transition

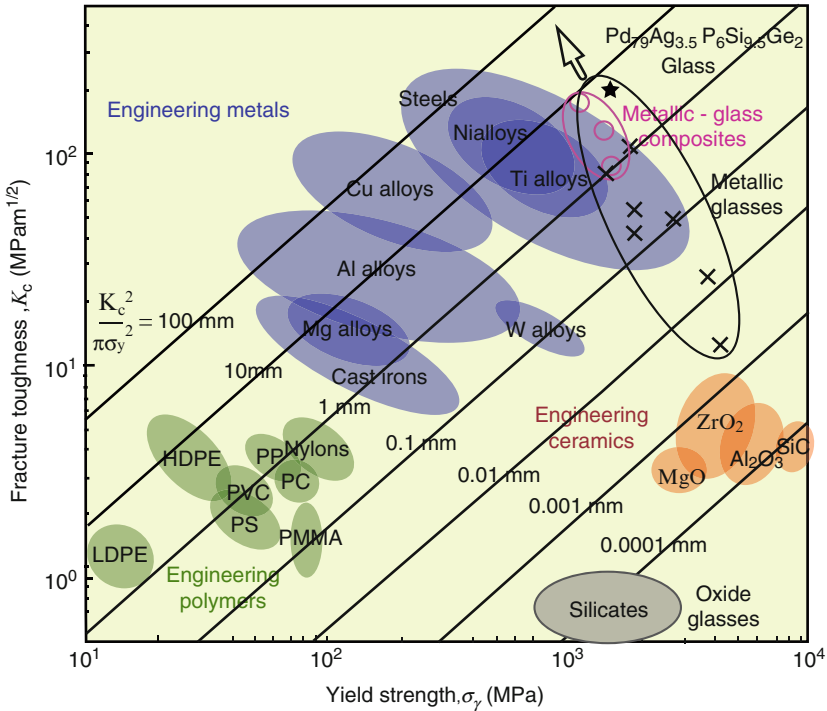


Fig. 10.21 Ashby map of the damage tolerance (toughness versus strength) of materials. Ranges of fracture toughness versus yield strength are indicated for various materials, including oxide glasses, engineering ceramics, engineering polymers and engineering metals, along with data for the $\text{Pd}_{79}\text{Ag}_{3.5}\text{P}_6\text{Si}_{9.5}\text{Ge}_2$ metallic glass and other BMG. Contours correspond to values for the plastic zone size, $K_c^2/\pi R_p^2$

between “brittle” and “tough” regimes (where there is a large increase in G beyond the oxide glass values) occurs when $\nu_{\text{crit}} = 0.31\text{--}0.32$. Thus metallic glasses with $\mu/k > 0.41\text{--}0.43$ or equivalently with $\nu < 0.31\text{--}0.32$, are brittle. The correlation between mechanical properties and elastic moduli strongly suggests that brittleness in metallic glasses can be alleviated by alloying with elements with low μ/k (or, equivalently, high ν) as constituents.

Results on a Palladium-based BMG showed that in this material it was possible to reach values of fracture toughness comparable to those of the toughest materials known (Demetriou et al. 2011). These results and those obtained on other BMG are reported on an Ashby map in Fig. 10.21 where it is seen that the combination of strength and toughness (*i.e.* damage tolerance) potentially accessible to metallic glasses extends beyond traditional benchmarks towards levels previously inaccessible to any material. However these BMG based on noble metals or rare earths will be expensive and this will limit their application. Fe-based BMG which are much cheaper should be more extensively developed.

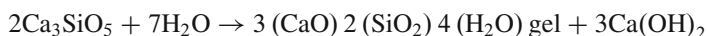
10.4 Damage and Fracture of Concrete

10.4.1 Concrete: A Complex Material

Concrete has been in use for centuries. There are really no major collapses due to concrete failure itself outside of earthquakes. Accidents involving concrete are due mostly to corrosion of reinforcements. Severe problems can be encountered because of alkali reactions between cement paste and aggregates producing expansion; this does not lead to complete destruction but to damages requiring repairs. We shall not here study these purely chemical problems. However, on May 23, 2004, the E2 terminal of the Roissy airport in France, which was made of concrete, collapsed. It seems that this was due to a daring design with little safety margin and carelessness in the construction, but not to faulty concrete. Nevertheless, the fact that the building had been in operation for some months might incriminate an evolution of the mechanical properties of the concrete itself, which is not abnormal. We will see that indeed it is a material in which water migrates and various reactions involving diffusion can intervene.

Concrete is a complex multi-scaled material. The basic component is cement. It is mixed with aggregates of various sorts. Water is added to this mix. Chemical admixtures are also often included. There are thus many different concretes according to the proportions of the constituents, and to their characteristics (composition, shape, size).

- *Cement.* The common type of cement is called *Portland cement*. This name comes from the similarity of its colour with the limestone quarried in the Island of Portland and used in many London buildings. Cement is made by heating limestone mixed with clay. The grinded product is *clinker*. Cement is a mixture of oxides of calcium, silicon and aluminium.³
- *Aggregates.* Sand, gravel and crushed stones are the common aggregates added to the cement. Their respective sizes allow filling the space. Fly ash is also used as it fills the empty space and produces high strength concretes. The aggregate is of very varied size, shape and composition, according to the sites from which it is quarried.
- *Water.* Hydration by water of the cementitious material forms the *cement paste*. The chemical reaction is:



It is otherwise written by cement chemists $\text{C}_3\text{S} + \text{H} \rightarrow \text{C-S-H} + \text{CH}$

³Cement industry is among the largest producers of green house gas emission.

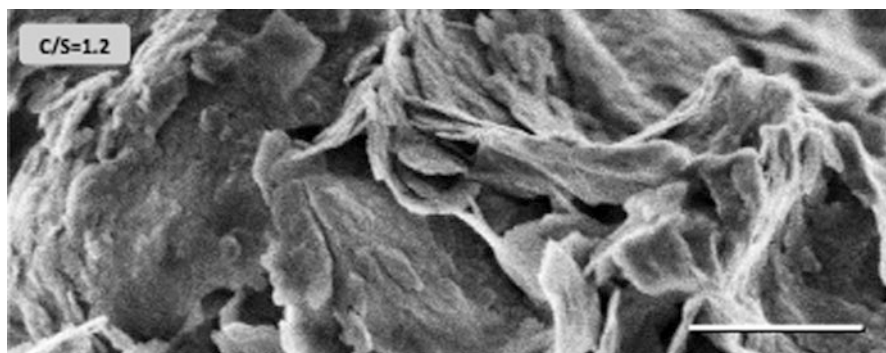


Fig. 10.22 Scanning electron images of C-S-H platelets. The *white bars* represent 500 nm (Alizadeh 2009)

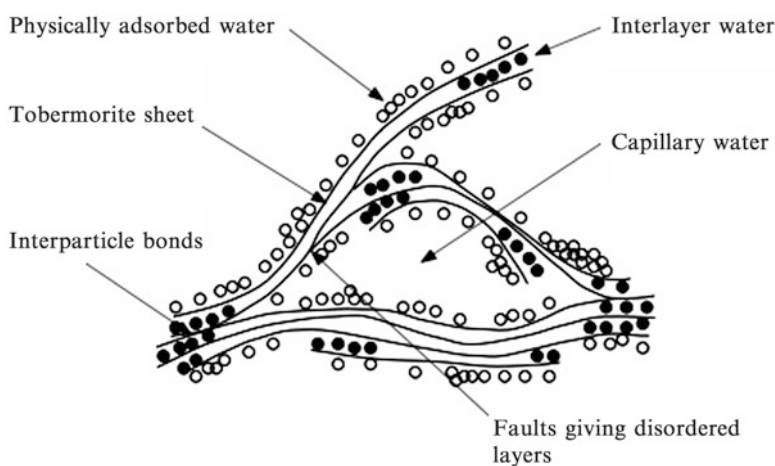


Fig. 10.23 Schematic drawing of hydrated Portland cement at a scale of some 10 nm

This hydration reaction develops over a long time. It is accompanied by the hardening of the cement paste with a decreasing rate. It is usual to record the mechanical properties of concrete at 28 days. The C-S-H is found as platelets of tobermorite (Fig. 10.22). Empty space remains between them. Figure 10.23 is a schematic drawing of hydrated Portland cement. Water remains adsorbed on the tobermorite platelets as well as capillary water and as water vapour. The water can migrate in the cement paste and exchange with the exterior. The storage conditions are important to be specified for a proper appreciation of the properties.

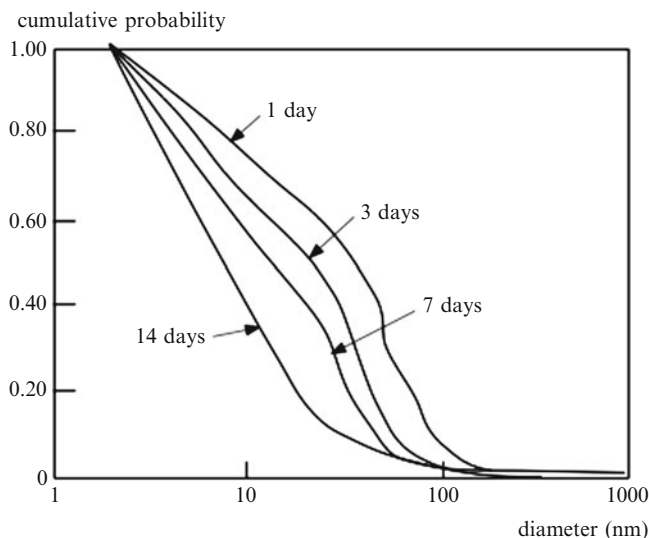


Fig. 10.24 Evolution of the size of pores during curing of mortar (Adapted from Shi et al. 1993)

The change of volume that takes place during setting gives rise to stresses and even cracks. The size of the pores decreases during curing of mortar (Fig. 10.24). The zone surrounding the aggregates has a slightly different composition. It is a weak transition ring of larger porosity. Cracks initiate more easily at the bottom part of the aggregates owing to the low resistance resulting from the water meniscus formed during set up. Thus concrete could be regarded as a multi-fissured material with cracks ranging in scale from the finest C–S–H platelets (a few nanometres) to the largest pebbles (a few centimetres).

- *Mineral admixtures.* Mineral admixtures having pozzolanic properties (similar to that of the Portland cement) are added so as to use less cement. They are *fly ashes*, a by-product of coal fired electric generating plants; *ground granulated blast furnace slag*; *silica fumes*, a by-product of the production of silicon or ferro-silicon, which are extremely fine; *metakaolin*. These last two additives provide concretes of very high strength. Metakaolin has the advantage of being white, which makes it interesting for architectural applications.
- *Fibres reinforcement.* Various kinds of short fibres are incorporated in concrete (Yin et al. 1990). The main objective is to reduce early age cracking and to improve the strength and the ductility. Asbestos fibres were used before being forbidden because of health hazards and costly replacements must be undertaken in buildings. Polypropylene fibres improve the impact resistance of concrete. Other artificial or natural polymeric fibres are being tested. Steel fibres improve structural strength and ductility. This is one of the main applications of amorphous steel products.

10.4.2 Concrete Damage

10.4.2.1 Damage Mechanisms

For each of the cracks present in concrete there is a corresponding critical propagation stress. It is not an easy matter to construct criteria for fracture on this scale, because of the different orientations of the fissures and the interactions between these and the aggregates. As a load is applied it is the cracks that are closest to the critical condition which will begin to propagate under normal stress. They would become unstable if they did not encounter aggregates, which in fact force them to divert and follow the interfaces, and eventually stop. Whilst cement paste is very weak, concrete is much less so because of the stabilising role of the aggregate, which allows multi-fissuring to develop. However, this is not so in high strength concrete containing silica fumes; the strength of the cement paste matches then that of the aggregates; the cracks can cross them and these concretes are more brittle.

In compression multi-fissuring is much more pronounced than in tension. Under compressive stresses relative displacements of the edges of the cracks are restrained by friction. Furthermore, the critical cracks are loaded in modes II and III; they divert and take an orientation corresponding to mode I, that is parallel to the loading compressive direction; in that position, the cracks are not loaded anymore and they are stabilised (Fig. 10.25). However, in concrete the stress field is not uniform on the scale of the aggregates and there can be local tensile stresses, averaging to zero over the test piece as a whole but sufficient to cause cracks to propagate, running parallel to the compression axis until they meet. Quantitative estimations of the local stress distribution were computed by Escoda et al. (2011) using fast Fourier transform method after segmentation of 3D images of concrete. Fracturing, resulting from a very diffuse fissuring, is progressive and absorbs much more energy than it would under tension, when rapid localisation of stress would generate serious instability. This is why concrete is much stronger in compression than in tension. The final fracturing of a test piece under compression is due to fragmentation into small columns parallel to the compression axis.

Considering the microstructure of concrete and the damage mechanisms, it is expected to find a statistical distribution in the measurements of the mechanical properties. This is indeed the case. Although using a Weibull statistical analysis is tempting, care is needed as the damage mechanisms cannot justify a weakest link approach.

10.4.2.2 Early-Age Cracking

Many phenomena take place during the first days of the life of a concrete structure: chemical reactions, hydration, drying, heat transfers. They can produce shrinkage

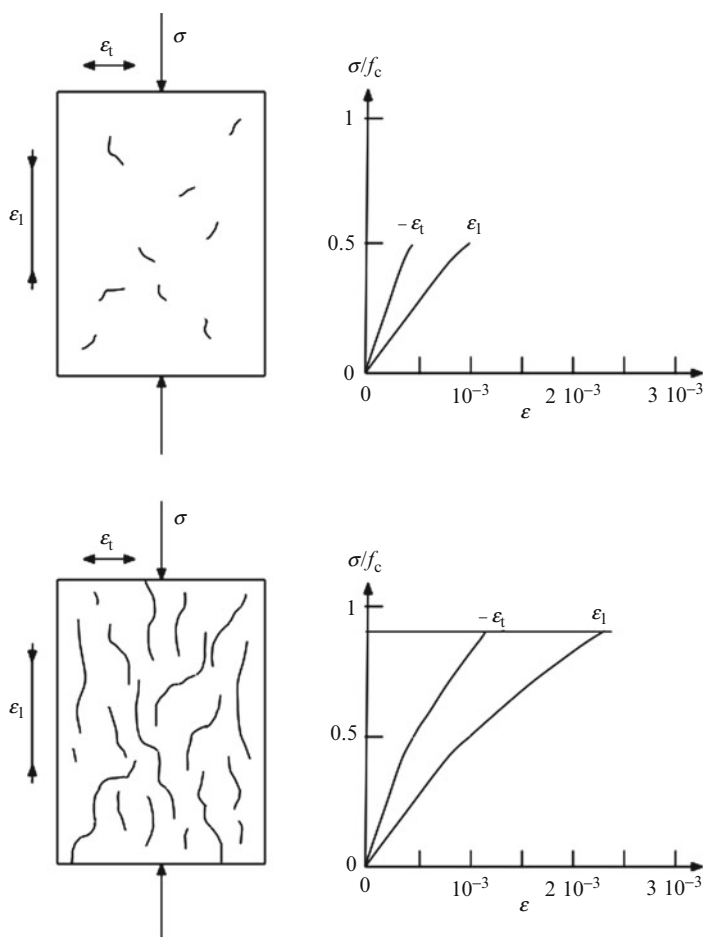


Fig. 10.25 Schematic representation of the progressive cracking in compression for concrete

deformations sufficient to create cracks. This damage is influenced also by the applied stresses. These phenomena are coupled and this complicates modelling. They are an intrinsic part of early-age creep of concrete.

Early-Age Shrinkage

Shrinkage occurs in the absence of applied stress. The chemical reactions during setting produce a volume decrease. This shrinkage was discovered by Le Chatelier⁴

⁴Henri Le Chatelier (1850–1936) was a French engineer and scientist who made important contributions in metallurgy and in materials science.

(1904). It is then called *Le Chatelier shrinkage*. The self-drying due to hydration produces the *autogenous shrinkage*. There are also exchanges of water with the environment and thus water migration within the structure: this produces *drying shrinkage*. For very thick structure, there can be a high temperature raise in the centre owing to the heat of reaction. Differential deformations result from thermal gradients and add a *thermal shrinkage*. These various forms of shrinkage can create internal stresses sufficient to initiate cracks within the structure even in the absence of any applied stress.

Early-Age Creep

Early-age creep was first observed by Eugène Freyssinet on the Le Veudre bridge on the river Allier in 1910. Two phenomena contribute: *autogenous creep*, only due to mechanical stressing, and *drying creep* linked with water migration. Although decreasing with time, this creep is not limited to early age. We will not give more details about these referring the readers to various publications (Acker 2001; Ghosh 1973; Bazant and Wittmann 1982; Bazant 1988; Ulm and Coussy 1998; Ulm et al. 1999)

10.4.2.3 Macroscopic Behaviour

Figure 10.26 shows a schematic stress-strain curve for concrete. Tensile loading develops cracks at low values of the stress and the fracture strength, denoted f'_t in the field of concrete, is low. In displacement-controlled tests it is possible to record unloading compliance. It is the larger the higher the strain. Irreversible deformation after unloading is also observed. It could be due to debris preventing complete crack closure. In compression, the cracks progressively close down, with a decrease of the compliance, until new cracks develop.

Fracture criteria for concrete cannot be expressed in the form of a critical stress since, in a compression test, cracks parallel to the compression axis open. However, a criterion for maximum deformation can be useful since in such a test the Poisson effect gives positive lateral deformations. Multiaxial tests show that the *criterion for fracture* can be taken to be a maximum positive strain of the order of 10^{-4} (van Geel 1998). Figure 10.27 gives fracture limits in biaxial compression loading.

Statistical distribution. Consider a representative volume element (RVE) of concrete, the dimension of which is about three times the maximum inhomogeneity size. This RVE is considered as the smallest element whose failure will cause the whole structure to fail (*e.g.* Bazant and Pang 2006). It is considered itself as elements working in series, like the links of a chain, and of elements working in parallel, as the fibres in a bundle. The cumulative probability function of the elements in series follows a Weibull law, while it can be demonstrated that the cumulative probability function of a large number of elements working in parallel follows a Gaussian distribution. This is reflected by a kink on the Weibull plots of the probability of

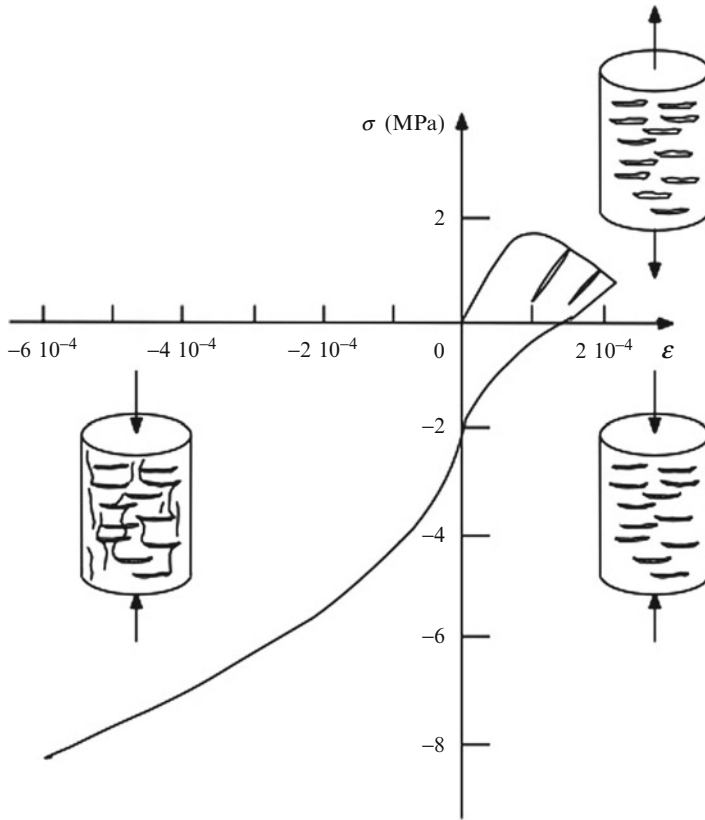


Fig. 10.26 Stress-strain curve for concrete

fracture versus the stress; the low probabilities obey a Weibull law, while the large ones obey a Gaussian distribution (Fig. 10.28). The probability corresponding to the kink increases with the size of the piece; large structures obey the Weibull statistics. Another statistical theory is that of non-local Weibull statistics (Bazant 1999). For a material such as concrete the fracture probability at a given point of a piece must be influenced by the strain in a surrounding region; that is a non-local behaviour. The non-local Weibull theory then yields the following equation for the mean fracture stress in a bending test of a plain bar:

$$\bar{\Sigma}_n = \sigma_0 \left[W_b / W + (W_b / W)^{2/m} \right] \quad (10.16)$$

where m is the Weibull exponent and W_b is the damaged zone size: $W_b = (3L/2W) l_{Dc}$. Large structures obey the Weibull law; for small ones the fracture stress is inversely proportional to the size (Fig. 10.29).

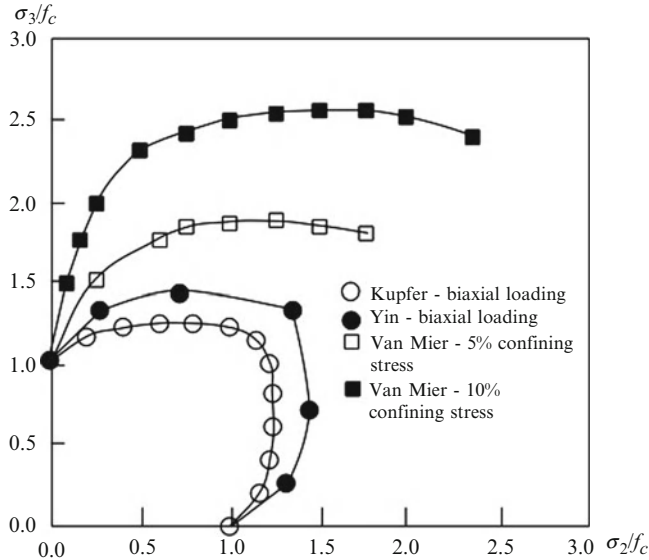


Fig. 10.27 Fracture limits in biaxial compression

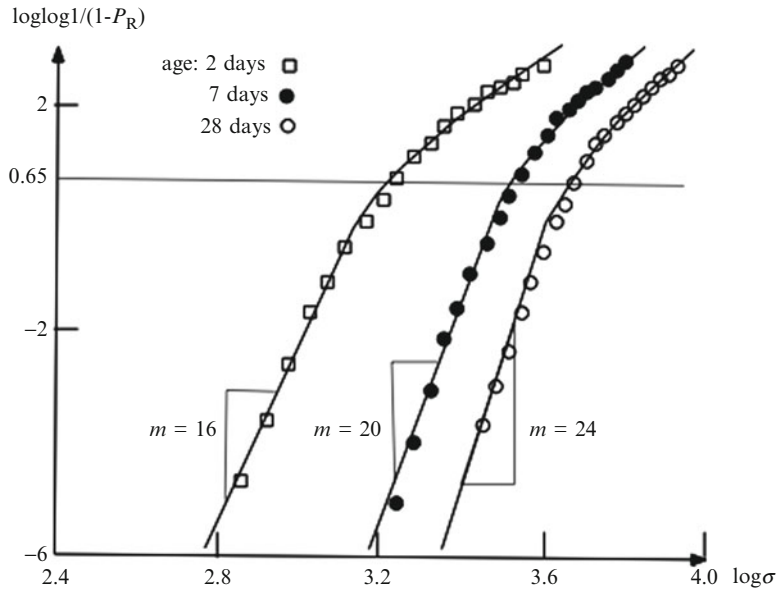
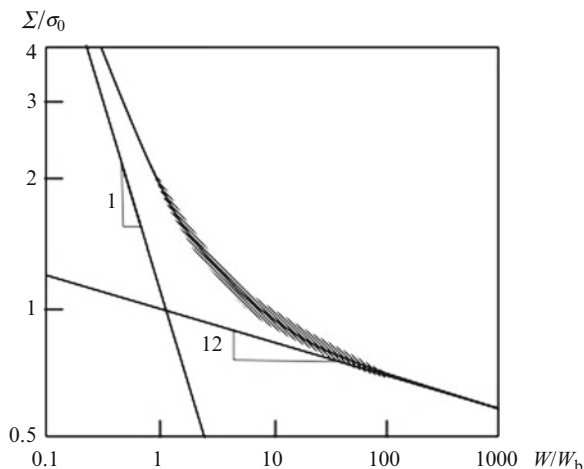


Fig. 10.28 Weibull's data points (1939) on mortar plotted on a Weibull probability paper displaying the kink separating the Weibull law from the Gaussian distribution (Adapted from Bazant and Pang 2006)

Fig. 10.29 Fracture modulus versus non-dimensional size of bend plain bars. The shaded area represents the data points dispersion. There are two asymptotic behaviours (Adapted from Bazant and Yavari 2005)



Note that, for concrete safety factors are not sufficient, and statistical dispersion of the fracture strength of concrete should be incorporated in recommendations for design (Bazant and Pang 2006).

10.4.2.4 Damage Mechanics

In Sect. 8.4.4 we introduced the continuous damage model of Kachanov (1958) and Rabotnov (1969). *A priori* this model should be well suited to describe the damage of concrete due to diffuse cracking. The damage parameter D of a test piece can be defined as the ratio of the voided and cracked area A^* to the net section area: A^*/A . It would be close to 0 before loading and failure would occur when D takes the value 1.

The presence of voids or cracks increases the compliance C of the test piece. This is a quantity easy to measure, whereas the fractional area of voids or cracks is not. Another definition of the damage parameter could then be:

$$D = 1 - \frac{C}{C^*} = 1 - \frac{E^*}{E} \quad (10.17)$$

C^* and E^* being respectively the compliance and the Young modulus of the damaged material. Unhappily these two definitions of the damage parameters do not coincide. For instance the elastic moduli of a porous material, from Sect. 2.7.3.3 of Volume I based on Eshelby's theory of inclusions, can be calculated. For spherical inclusions the bulk and the shear moduli can be estimated as:

Table 10.3 Damage model parameters for moderate strength concrete (Pijaudier-Cabot and Mazars 2001)

Young modulus	$E_0 \cong 30,000\text{--}40,000$ MPa
Poisson ratio	$\nu_0 \cong 0.2$
Equivalent strain	$\kappa_0 \cong 1 \times 10^{-4}$
Parameter A_t for uniaxial tension	$0.7 < A_t < 1.2$
Parameter B_t for uniaxial tension	$10^4 < B_t < 5 \times 10^4$
Parameter A_c for uniaxial compression	$1 < A_c < 1.5$
Parameter B_c for uniaxial compression	$10^3 < B_c < 2 \times 10^3$

$$\begin{aligned}
 k^* &= k \left[1 - \frac{3(1-\nu)}{2(1-2\nu)} f_v \right] \\
 \mu^* &= \mu \left[1 - \frac{15(1-\nu)}{7(1-2\nu)} f_v \right]
 \end{aligned} \tag{10.18}$$

f_v being the volume fraction of voids. These two equations yield the Young modulus E^* as $E^* = 9k^*/(1 + 3k^*/\mu^*)$. The fractional area being the same as the volume fraction, we see that the two definitions of the damage are very different. Calculating the compliance of a test piece containing penny-shaped cracks would lead to the same conclusion.

Nevertheless, damage mechanics models are much used for concrete (*e.g.* Pijaudier-Cabot and Mazars 2001). In keeping with the definition based on the effective moduli (Eq. 10.17), a scalar damage parameter is introduced in the stress-strain relation:

$$\varepsilon_{ij} = \frac{1}{E_0(1-D)} [(1+\nu_0)\sigma_{ij} - \nu_0\sigma_{kk}\delta_{ij}] \tag{10.19}$$

index 0 pertaining to the undamaged material.

The evolution of the parameter D as a function of the strain for uniaxial tension is given by:

$$D_t = 1 - \frac{\kappa_0(1-A_t)}{\kappa} - \frac{A_t}{\exp[B_t(\kappa - \kappa_0)]} \tag{10.20}$$

with the same expression for uniaxial compression, the indices t being replaced by c. Parameter κ stands for the equivalent strain $\kappa = \sqrt{\sum_1^3 (\langle \varepsilon_i \rangle_+)^2}$, ε_i being the principal strains. The bracket indicates the positive value.

Pijaudier-Cabot and Mazars (2001) give the values of the parameters as listed in Table 10.3 for moderate strength concrete.

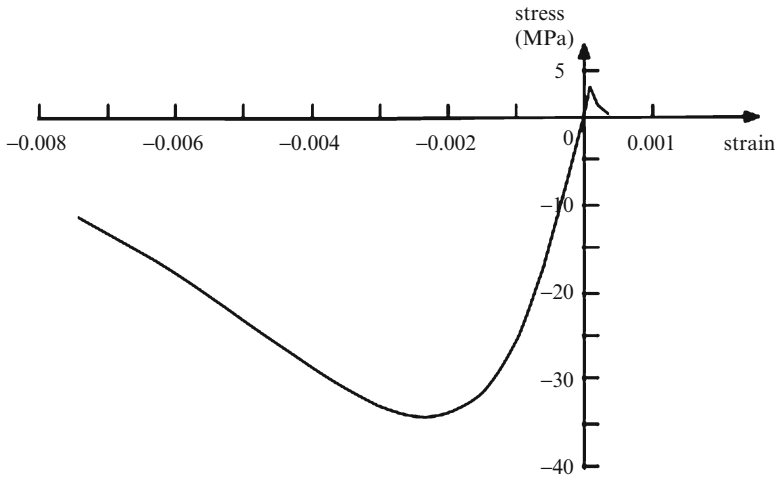


Fig. 10.30 Uniaxial response of the model (Eq. 10.19) with the parameters given in the text (Adapted from Pijaudier-Cabot and Mazars 2001)

Figure 10.30 shows the uniaxial response of the model for $E_0 = 30,000$ MPa, $\nu_0 = 0.2$, $\kappa_0 = 0.0001$, $A_t = 1$, $B_t = 15,000$, $A_c = 1.2$, $B_c = 1,500$. The fracture strength in tension is much lower than in compression. The post peak behaviour in compression is well represented.

The damage model is extended to take account for anisotropy. In that case the damage parameters are three scalars for three orthogonal directions. In other words, instead of one scalar damage parameter, a second-order tensorial damage parameter must be used. Actually, in the general case, a fourth-order tensor is needed. But this is scarcely used due to difficulties arising for the identification of the evolution laws of such a tensor.

When one aims at predicting the ultimate fracture of the studied structure by using a continuous damage model, the constitutive equations express necessarily at some stage a softening behaviour. This implies simultaneously a loss of ellipticity of the equations of the considered boundary value problem, which is responsible for fake localisations and, from a numerical point of view, for a physically insane dependency of the solution on the mesh size and orientation. This major deficiency can be overcome by use of regularisation techniques which could somehow force any singularity to spread about and diffuse in space or time. The most common way to do so is to use a non-local formulation of the continuous damage model (Bazant and Pijaudier-Cabot 1989): for example, in the preceding model defined by (10.20), the local equivalent strain $\kappa = \sqrt{\sum_1^3 (\langle \varepsilon_i \rangle_+)^2}$ can be replaced by the integral (and then non-local) expression

$$\bar{\varepsilon}_{\text{eq}}(\underline{x}) = \frac{1}{V_r(\underline{x})} \int_{\Omega} \psi(\underline{x} - \underline{y}) \bar{\varepsilon}_{\text{eq}}(\underline{y}) d\Omega \quad (10.21)$$

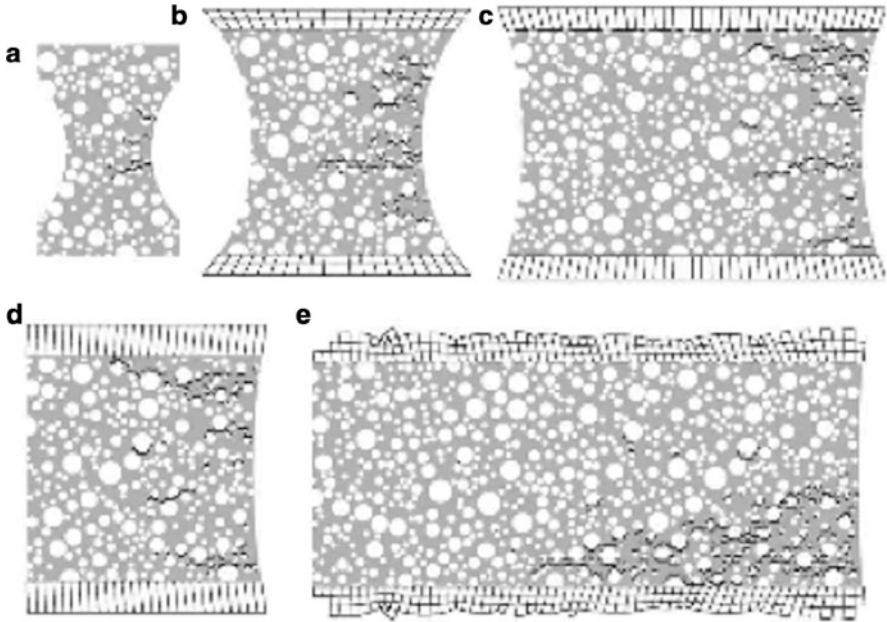


Fig. 10.31 Results of numerical simulation by lattice modelling of different sizes, increasing from (a) to (e), of test pieces of concrete. In figures (d) and (e) only half of the specimen is shown (van Mier and van Vliet 2003)

where $\psi(\underline{x} - \underline{y})$ is a non-local weight function (usually taken as a Gaussian function), $V_r(\underline{x})$ is the “representative” considered volume element at point $\underline{x}$ and Ω is the volume of the structure. The weight function $\psi(\underline{x} - \underline{y})$ introduces an internal length which can be given some physical content (*e.g.* the size of the diffuse cracked zone according to Bazant and Pijaudier-Cabot 1988).

Other techniques have been proposed for the same purpose (de Borst 2002), such as second-gradient (Peerlings et al. 1996) or higher order (de Borst 1991) formulations or viscous techniques (Suffis et al. 2003). Despite their practical interest from a numerical point of view, these approaches suffer from some lack of physical content. That is why a number of micromechanical analyses have been developed in order to meet this need, both for concrete and composites. Let us quote in particular for concrete the application of the so-called “peridynamic model” (Silling 2000; Gerstle et al. 2007) which is based on a reformulation of elasticity theory from a discrete analysis of interactions between particles.

10.4.2.5 Numerical Concrete

The constitutive equations just described incorporating damage or non-local behaviour are used in numerical modelling of concrete structures by finite elements. Otherwise, van Mier (2001, 2003) has made much use of *lattice modelling*.

He has represented concrete as a three-phase material: the homogeneous matrix, the aggregates represented by superimposed circles or digital images from the real material and the interfacial transition zone. The model consists of regular 2D or 3D triangular lattices of beams. Figure 10.31 is an example of the kind of results obtained in 2D simulation. It displays the size effect on the fracture behaviour. As any numerical model, it must be calibrated on experimental results. Its advantage is to allow studying the effect of a single parameter, as the size (Fig. 10.30). The randomness associated with variations of local material properties can be introduced by assigning lattice element properties according to a statistical distribution.

Other computation techniques for concrete have been used (Emery et al. 2007). We have mentioned the work of Escoda et al. (2011).

10.4.3 Fracture Toughness of Concrete

In concrete, the damaged zone ahead of a crack is large, of the order of a few centimetres. For instance, Gettu et al. (1990) chose the following values for a normal strength concrete: $K_{Ic} \approx 0.76 \text{ MPa m}^{1/2}$ and $f'_t \approx 3.5 \text{ MPa}$ with a damaged zone size of the order of 50 mm. Care is then needed in measuring the fracture toughness, as size effects can be important. In Fig. 10.32 the results show, for normal strength concrete, rather large deviations from LEFM. Things are better for high strength concrete for which, with $K_{Ic} \approx 0.95 \text{ MPa m}^{1/2}$ and $f'_t \approx 11 \text{ MPa}$, the damaged zone would be of the order of 7 mm. These uncertainties are such that there is no fracture toughness test standard for concrete.

To take account of the damaged zone ahead of a crack in concrete Hillerborg et al. (1976) proposed to consider a longer *fictitious crack* the tip of which was closed by cohesion stresses following a bilinear law. This is a Dugdale-Barenblatt model (Sect. 2.3.3.2) with a particular distribution of the closure stresses already mentioned in Sect. 10.2.2.3.

To introduce the statistical distribution of fracture toughness values, in the same way as for crack initiation in the bend tests of plain bars (Sect. 10.4.2.2), non-local Weibull statistics can be assumed. It is then found (Bazant 1999) a size effect described by a modification of Eq. 10.12 corresponding to deterministic behaviour:

$$\bar{\Sigma}_n = B\sigma_0 \frac{1}{\left[W/W_0 + (W/W_0)^{2/m} \right]^{1/2}} \quad (10.22)$$

where m is the Weibull exponent.

The behaviour is not very different from the one described in Fig. 10.7 for ceramic materials. Linear elastic fracture mechanics works well for large structures with no statistical effect (for structures with the same crack front length). This is due to the concentration of these effects in the damage zone of negligible size.

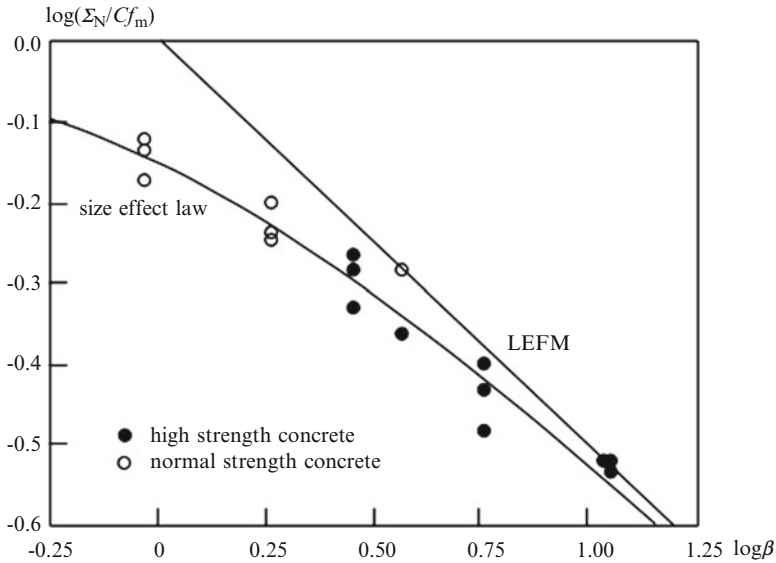


Fig. 10.32 Results of fracture toughness of concrete plotted in reference to Eq. 10.12 (Adapted from Gettu et al. 1990)

The fracture strength varies as $W^{-1/2}$. In contrast to the deterministic treatment, statistical distribution does not yield a constant fracture strength for small structures; it follows a Weibull law and it is the higher the smaller piece of materials.

Note that, as for other materials, Weibull statistics for long cracks introduces a thickness effect, which has not been considered above.

10.4.4 Fatigue of Concrete

As any other material, concrete fails after a certain number of cyclic loads depending on their amplitude. Plotted on a Wöhler diagram, stress amplitude versus fatigue life, results are much scattered. They are scattered in the same way as the fracture strength. It was found that a better way to represent fatigue tests results on concrete is to measure the deformation of the specimen during the test and to plot this as a function of the number of cycles (Alliche and François 1992; Thun et al. 2011). It is then found that the fatigue life is a function of the rate of deformation per cycle. This rate is found to be constant during the major part of a test, except at the beginning where it keeps decreasing and at the end when it accelerates until final fracture. The behaviour is such that an irreversible deformation accumulates at each cycle until the critical fracture deformation is reached. This critical fracture deformation is a

characteristic for a given concrete. The prediction of the fatigue life is then possible by extrapolation of the stabilised deformation rate after a few cycles.

Fatigue of concrete is a strong function of the water content. It is the faster the higher the humidity, in the same way as creep.

10.5 Plastic Yielding and Fracture of Polymers

For polymers plastic deformation and fracture are closely linked. In Volume I plastic deformation of metals was essentially treated. In this section it seems appropriate to cover both plastic deformation and fracture of polymers.

10.5.1 *Structure of Polymers, a Reminder*

Structure of polymers was studied in Appendix I of Volume I. We remind here essential notions. Polymers are made of long molecules, which are repetitive sequences of elementary monomers. They are mingled somewhat like noodles. In glassy disordered microstructures, the molecules are connected by physical entanglements and irreversible chemical bonds. The number of elementary chain units between entanglements or cross-links is denoted N_e . In semicrystalline structures the molecules are arranged in rigid crystalline lamellae interconnected by amorphous material.

Below the glass transition temperature T_g the amorphous polymers are in the glassy state. Above this temperature they are in the rubbery state. In the glassy state, under sufficient stress, small rotations and displacements of the molecular chains can produce irreversible plastic deformation. Above the glass transition temperature the only links remaining between molecular chains are the entanglements. Plastic deformation becomes much easier. In rubbers, vulcanisation creates some strong bonds at more or less distant locations along molecular chains. This results in rubber elasticity, which was studied in Sect 2.2.2 of Volume I. Rubbery state of polymers can be treated by the same models as the ones used for rubber elasticity.

10.5.2 *Plastic Yielding of Polymers*

10.5.2.1 Homogeneous Plastic Yielding of Glassy Amorphous Polymers

Let us consider an experiment where a glassy amorphous polymer is loaded in tension at constant strain rate $\dot{\epsilon}$ and the stress σ is recorded. The birefringence,

Fig. 10.33 Shape of the strain dependence of stress and birefringence for a polymer displaying homogeneous plastic yielding. Chain conformations are schematised at three distinct stages

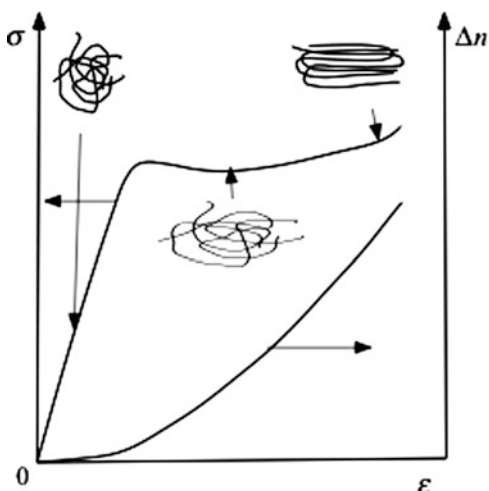
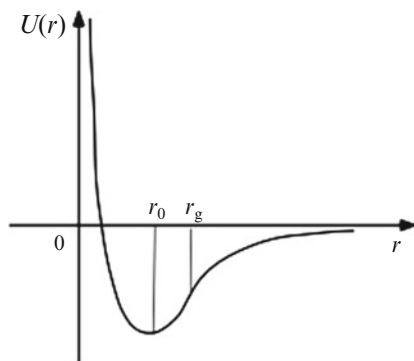


Fig. 10.34 Shape of the intermolecular potential versus average intermolecular distance



i.e. the difference of refractive indices $\Delta n = n_{\parallel} - n_{\perp}$ along the draw ($n_{\parallel}$) and the perpendicular ($n_{\perp}$) directions, is also recorded. Δn is a quantitative measurement of the chain orientation.

In a certain temperature interval $T_c < T < T_g$, where T_c depends on the polymer chemical structure, the behaviour schematised in Fig. 10.33 can be observed.

Yielding can be characterised as follows: sample elongation becomes partly irreversible. Chains become oriented in the tensile direction. Deformation becomes suddenly easier: $d\sigma/d\varepsilon$ drops by several orders of magnitude, the polymer behaving as if it was rubbery. A first approach of plastic yielding can be made from an expression of the intermolecular potential using for instance the Lennard-Jones expression (Fig. 10.34).

Mechanical tensile or shear loading induces a volume increase, *i.e.* an increase of the intermolecular distance. When the representative point reaches the inflection point of the curve ($r = r_g$), intermolecular forces collapse and the deformation becomes suddenly easier. According to Buchdahl (1958) the theoretical yield

stress σ_y would be equal to $E/30$. The average yield stress is slightly lower: $R_p \approx E/40$. That could be explained by the presence of “defects” responsible for stress concentrations.

A more fruitful point of view consists to assume that loading creates mobile defects: unstable conformations and/or free volume. When the defects reach a critical concentration, the polymer becomes suddenly rubbery and plasticity begins.

Taking for instance the theory of the *free volume* and considering a tensile test at $T < T_g - C_2$, *i.e.* in a region where the thermal free volume is null, the unique source of free volume is tension:

$$f = \frac{\Delta V}{V} = (1 - 2\nu) \varepsilon \quad (10.23)$$

where ν is the Poisson ratio.

The polymer becomes rubbery when the free volume fraction f approaches f_g , *i.e.* when the strain approaches ε_g such that

$$\varepsilon_g = \frac{f_g}{1 - 2\nu} \quad (10.24)$$

Taking the universal value of f_g (0.025) and $\nu = 0.4$, we obtain $\varepsilon_g \approx 6.25 \times 10^{-2}$. The value is higher than the experimental one but of the good order of magnitude.

The same approach can be made using the conformational state rather than the free volume (Robertson 1966). In certain cases, for instance polystyrene, the populations of trans and gauche conformations can be measured by IR spectrophotometry during the tensile test. Monnerie and his coworkers (Xu et al. 1989) demonstrated that yielding occurred effectively when the unstable conformation reaches a critical concentration.

It would be expected that the density of the polymer would decrease when strained in tensile loading. However G'sell et al. (2002) measured in PMMA density increase at the beginning of plastic deformation, up to an elongation of 0.5. They attributed this behaviour to the orientation of the molecular chains.

Figure 10.35 shows stress-strain curves for PMMA at different strain rates. The yield strength R_p is a decreasing function of temperature and an increasing function of strain rate $\dot{\varepsilon}$. According to Eyring (1936):

$$\dot{\varepsilon} = \dot{\varepsilon}_0 \exp \left(-\frac{H_p - V_p R_p}{RT} \right) \quad (10.25)$$

where H_p is the activation energy, V_p the activation volume. (This is the same equation as (3.80) in Volume I, with slightly different notations). At fixed strain rate this equation yields:

$$R_p = C_p (T_g - T) \quad (10.26)$$

Fig. 10.35 Stress-strain tensile tests recordings for PMMA at 298 K at different strain rates (Adapted from Nasraoui et al. 2009)

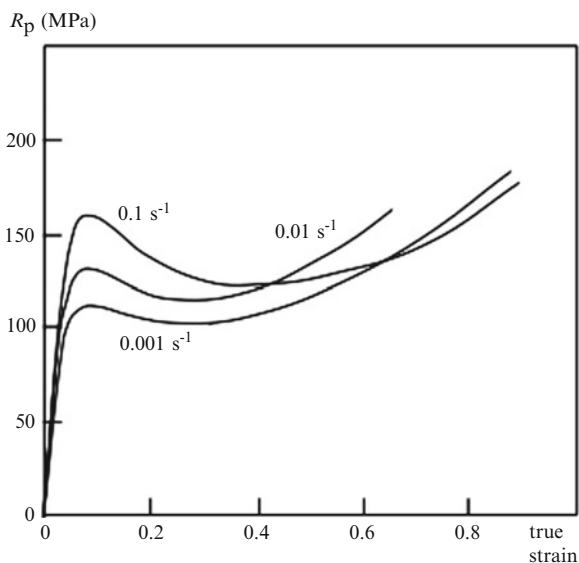
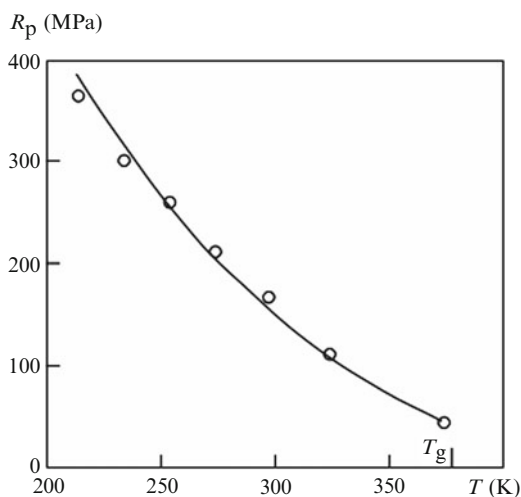


Fig. 10.36 Yield strength of PMMA versus temperature for a 0.1 s^{-1} strain rate (Adapted from Nasraoui et al. 2009)



with $C_p = (R/V_p) \log(\dot{\epsilon}_0/\dot{\epsilon})$ and $T_g = H_p/R \log(\dot{\epsilon}_0/\dot{\epsilon})$. It is effectively observed that the yield strength decreases linearly with temperature and vanishes at T_g (Fig. 10.36). The activation volume is often of the order of $0.001 \text{ m}^3/\text{mol}$, H_p is of the order of a few hundreds of kJ/mol, C_p is of the order of 1 MPa and increases with cohesive energy density.

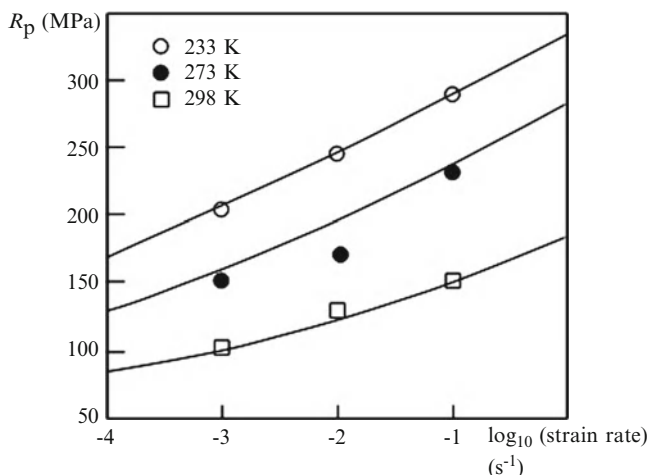


Fig. 10.37 Yield strength of PMMA versus strain rate for different temperatures (Adapted from Nasraoui et al. 2009)

According to Eyring's law (Eq. 10.20), the reduced strength R_p/T is expected to increase linearly with $\log \dot{\epsilon}$ (Fig. 10.37). Physical ageing does not modify the slope of the straight line, but induces its translation towards higher R_p/T values.

10.5.2.2 Heterogeneous Plastic Yielding in Glassy Amorphous Polymers

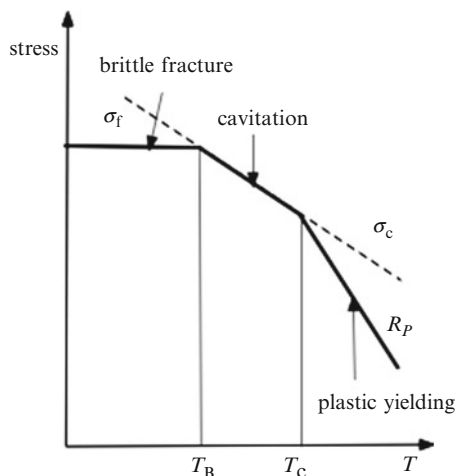
Homogeneous yielding always competes with *cavitation* processes. Cavitation is presumably initiated at defects (holes). It occurs at a macroscopic stress σ_c , which is a function of surface energy. In a first approach, the competition can be schematised by Fig. 10.38.

If the temperature is low enough and the strain rate high enough, cavitation initiates the sudden sample fragmentation. This brittle fracture, very similar to that of mineral glass, is characterised by a stress σ_f almost independent of temperature and strain rate. Certain authors suggested that it could be directly linked with the cohesive energy density; but, it depends also on molar mass, the key factor being the number of chains crossing the fracture plane (Vincent 1972).

At temperatures high enough, *crazing* competes with fragmentation. Crazing involves plastic deformation (with chain drawing); but, the latter remains localised into fibrils linking the crack edges (Figs 10.39 and 10.40) (Schirrer 2001). Observations of crazing in neck of tensile test pieces were achieved by tomography (Laiarinandrasana et al. 2010) (Fig. 10.41). The crazes were oriented along the tensile direction.

The nucleation of crazes involves three stages (Argon 2011). In the first stage, micro-shear bands are nucleated; they resemble dislocation loops. They get arrested by molecular heterogeneities. The stress concentration at the head of

Fig. 10.38 Schematic representation of the various behaviours of glassy polymers as a function of temperature. Maximum stress versus temperature



these micro-shear bands nucleate cavities. When locally the porosity is sufficient nucleation of crazes can occur. A sufficient hydrostatic stress is needed for crazing. Figure 10.42 shows the crazing criterion together with the yield criterion.

Crazes can be treated as cracks with local closing forces applied on the edges. This subtracts from the stress intensity factor due to the remote applied stress a stress intensity factor calculated with the help of Eq. 2.63 (or Eqs. 2.70 or 2.71 for a penny-shaped crack).

Crazes propagate by a mechanism of meniscus instability (Kausch et al. 2001). The crack front is shaped like fingertips (Fig. 10.43).

The material behaviour being viscoplastic, the material around the craze creeps. Experiments show that the size of the craze increases as $t^{1/2}$ (Schirrer 2001). In the case of linear polymers, it is well known that polymers having high chain characteristic ratios C_∞ (refer to Eq. 5.62 of Volume I) have long elastically active chains and low rubbery modulus values. Typical examples of this family are polystyrene and poly(methylmethacrylate). These polymers undergo crazing in a wide temperature interval. Polymers having low C_∞ values, for instance biphenol A polycarbonate or aromatic polysulfone, undergo homogeneous plastic yielding in a wide temperature interval below T_g .

Since cross-linking reduces the number of elementary chain units between entanglements N_e , it inhibits crazing; in the maximum stress-temperature graph (Fig. 10.38) the domain between T_c and T_B disappears. Here the interval $(T_g - T_B)$ is an increasing function of the activity of the beta transition and of the separation between T_β and T_g . It is relatively wide in amine cross-linked aromatic epoxies and relatively sharp in styrene cross-linked polyesters for instance.

Once a craze has grown for some time, fracture of some fibrils can occur (Schirrer 2001). As the fibrils extend by extracting molecular chains from the bulk, their oldest part is the midrib. A *propagating crack* is formed behind the craze, which has a length denoted S . Life-time $\tau(\sigma)$ can then be estimated by the time molecules entering the front of the craze uphold the stress σ until they break. We have

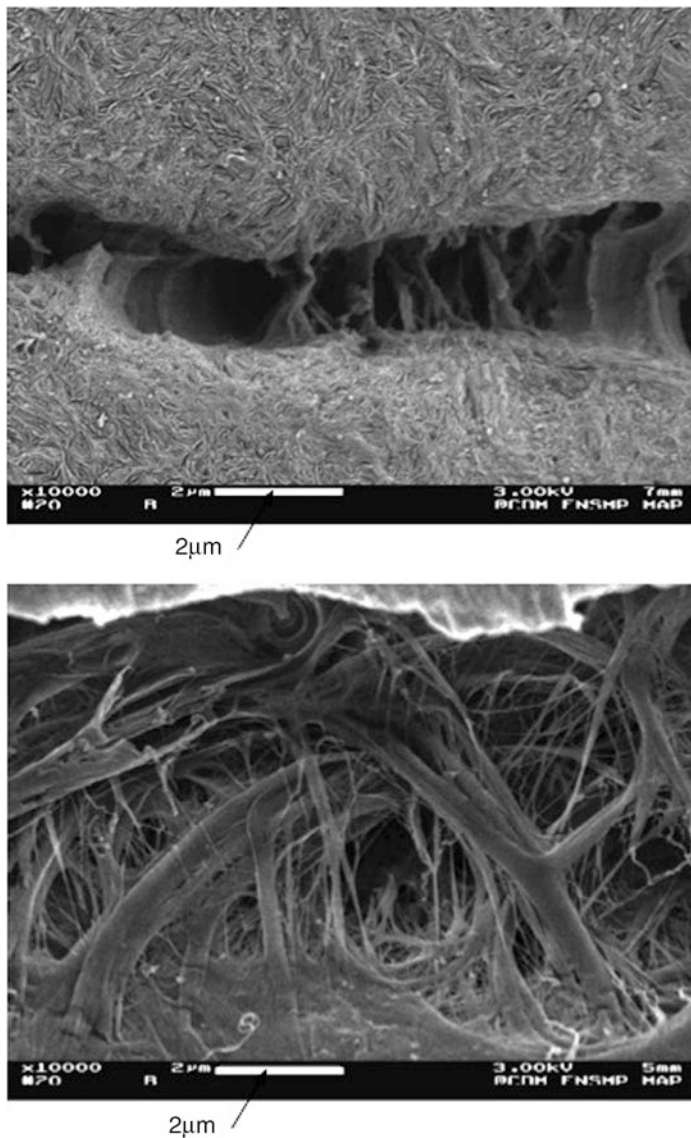


Fig. 10.39 Two aspects of crazes in polypropylene. Fibrils bridging the gap between the sides of a craze (*top image*); less stretched fibrils in a craze (*bottom image*) (Courtesy of L. Laiarinandrasana)

$\tau(\sigma) = S/\dot{a}$, if $\dot{a}$ is the crack growth rate. Now, the craze length can be calculated using the Dugdale-Barenblatt model (Eq. 2.102 in Sect. 2.3.3.2):

$$S = \frac{\pi K_I^2}{8\sigma_c^2} \quad (10.27)$$

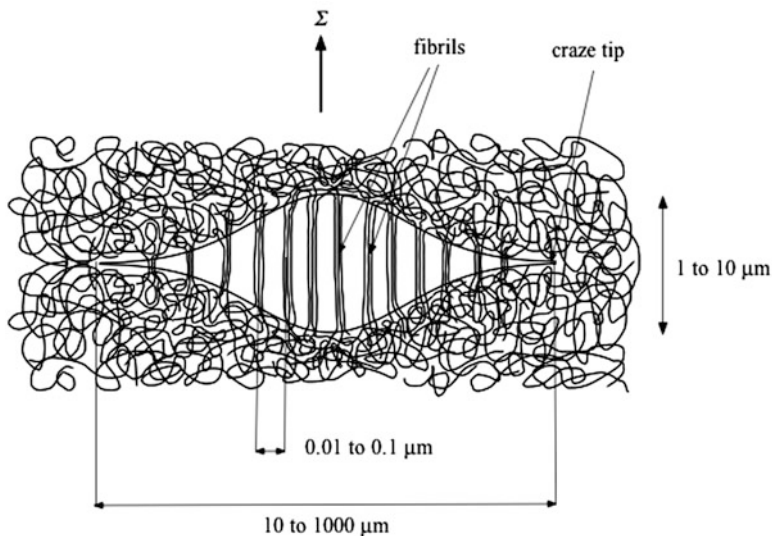


Fig. 10.40 Schematic drawing of a craze showing molecules being drawn out of the matrix forming fibrils

with σ_c the average stress applied on the craze edges by the restraining fibrils. Hence:

$$K_I = \sigma_c \left[\frac{8\tau(\sigma) \dot{a}}{\pi} \right]^{1/2} \quad (10.28)$$

This expression allows deducing σ_c and $\tau(\sigma_c)$ from measurement of the craze length S and of the fracture toughness K_{Ic} of a running crack.

This time-dependent growth of cracks in polymers is a kind of *static fatigue* (Knauss 1989). It is but a phenomenon of creep crack growth (refer to Chap. 8). Equation 10.28 shows that it results from viscoplastic stress redistribution around the crack tip. Figure 10.44 shows results of crack propagation rates obtained on PMMA.

10.5.2.3 Yielding in Semi-crystalline Polymers

Two cases need to be distinguished depending on the state of the amorphous phase. When $T < T_g$, the yielding behaviour is very similar to the one of the amorphous glassy polymers. When $T > T_g$, two yielding mechanisms can exist:

- According to the first one, the tie molecules linking two crystalline lamellae are first fully extended. Then chains will be extracted from crystals and the polymer will undergo an *amorphisation*, which can be observed for instance by wide angle

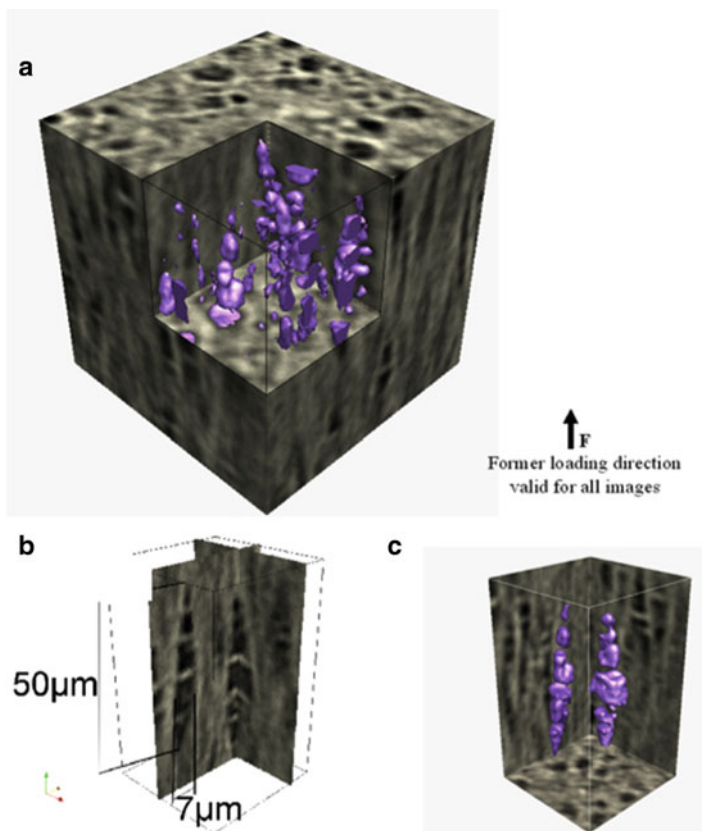


Fig. 10.41 X-ray tomography images within a necked tensile specimen. (a) 3D rendering of a $100 \times 100 \times 100 \mu\text{m}$ region. The voids were contoured by an isosurface; (b) orthogonal sections of two typical void columns; (c) 3D rendering of the two void columns characterised by elongated shape of the voids with small non broken segment of matter (*walls*) (Laiarinandrasana et al. 2010)

X-ray scattering (WAXS). In the sketch of Fig. 10.45, the interlamellar distance is progressively increased during loading. In stage (b) the macromolecule t_1 is fully extended. In stage (c) the macromolecule t_1 has extracted chain segments out of the crystal and the tie molecule t_2 is now fully extended. There the yield strength is expected to increase with the lamellar thickness as experimentally observed.

- The second mode of yielding involves *cavitation* in the amorphous phase. According to Pawlak and Galeski (2005), if the critical step is the nucleation of cavities of radius r , the critical negative pressure p_c for cavitation is:

$$p_c = \frac{2\gamma_s}{r} \quad (10.29)$$

Fig. 10.42 Crazing and yield criteria

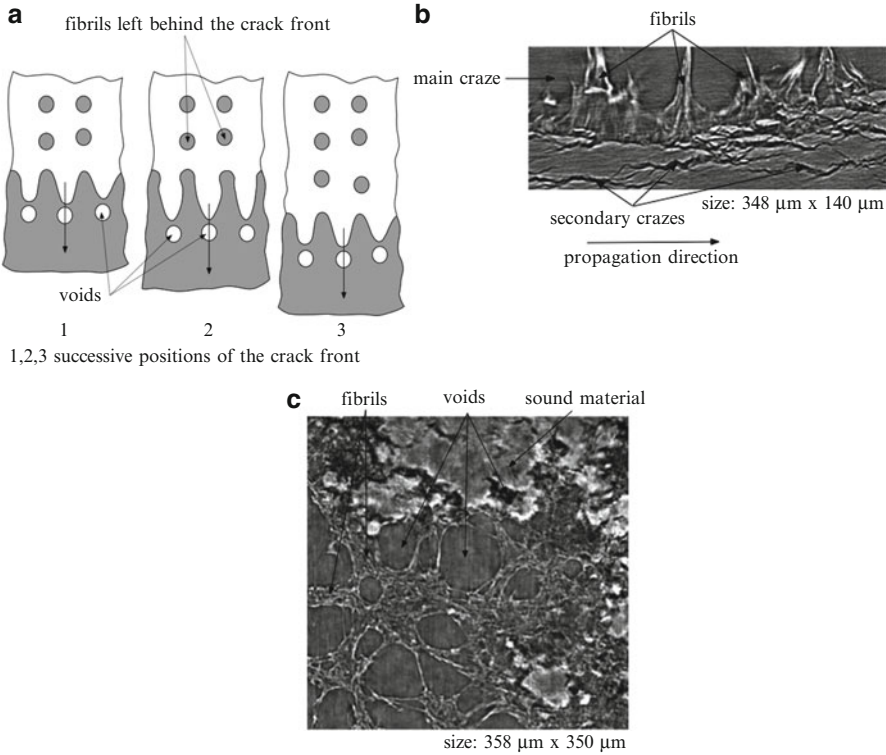
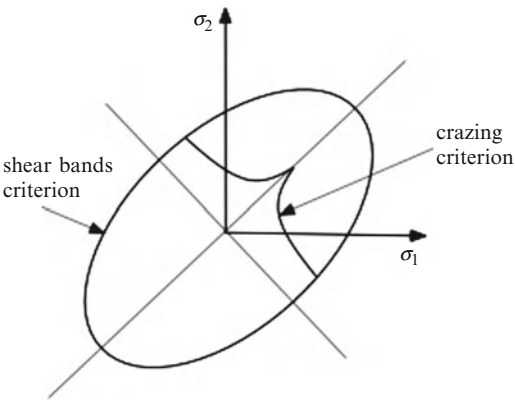


Fig. 10.43 (a) schematic *top view* of a growing craze with a crack front like finger-tips; (b) growing craze in PP showing broken fibrils, which bridged the craze; (c) *top view* of the craze (Courtesy of L. Laiarandrasana)

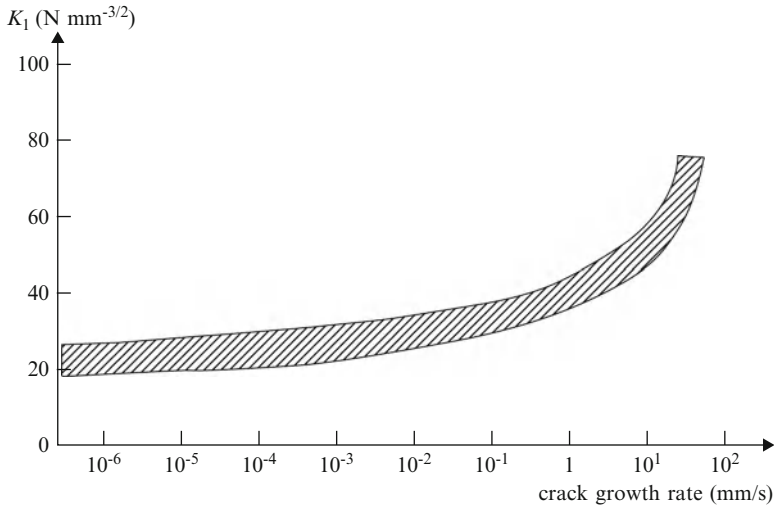


Fig. 10.44 Crack propagation rates in PMMA for different values of the stress intensity factor (Döll 1983)

where γ_s is the surface energy, for instance $35 \cdot 10^{-3} \text{ J/m}^2$ for polyethylene. Considering that the cavity size is of the order of half the interlamellar spacing, 10 nm for polyethylene, the yield strength, three times p_c , is found to be 21 MPa, close to the experimental value (Pawlak and Galeski 2005).

Note: environment can modify greatly cavitation and crack propagation.

10.5.3 Fracture of Polymers

10.5.3.1 Fracture of Glassy Polymers

At low temperature the fracture is brittle. The fracture stress σ_f is practically temperature and strain rate independent. It is linked to the cohesive energy density U_c and to the molar mass M :

$$\sigma_f = 0.25 U_c \left(1 - \frac{A_f}{M} \right) \quad (10.30)$$

where A_f depends on the material.

When the temperature is high enough, localised or generalised plastic yielding becomes activated, chain drawing absorbs a great part of the deformation energy, and the polymer molar mass becomes an important factor. The variation of the

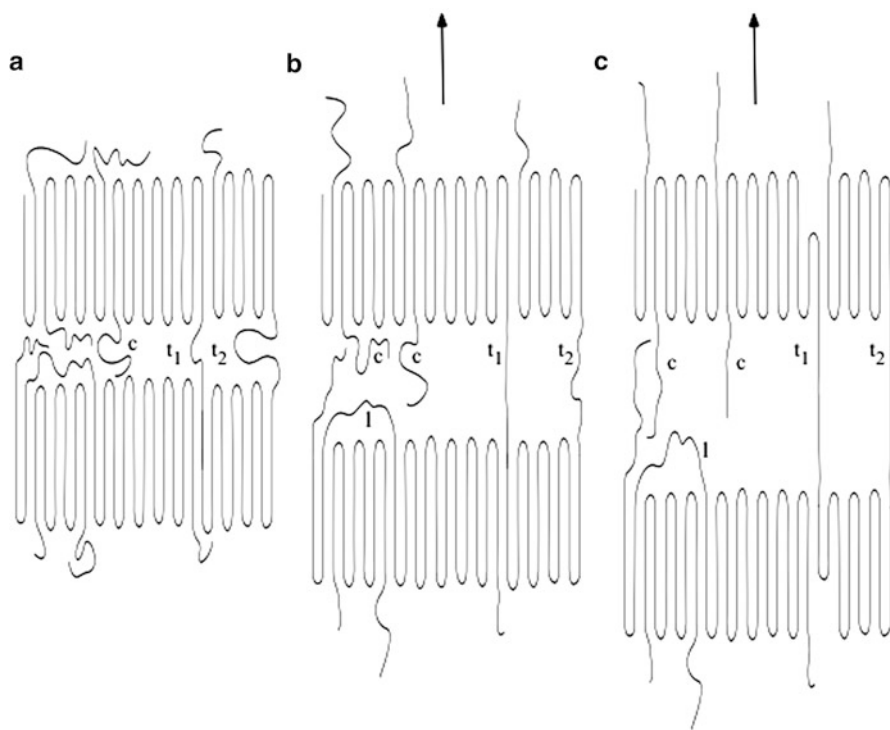


Fig. 10.45 Schematic drawing showing the role of tie molecules in plastic yielding of semi-crystalline polymers. c dangling chain (cilia), l loop, t tie molecule

fracture energy as a function of temperature shows a sharp transition from brittle to ductile behaviour, with a toughness gap of two or three decades (Fig. 10.46). It must be stressed that the viscoplastic behaviour of polymers invalidates many reported values of fracture toughness measured as if the material were elastic. This is only correct for brittle polymers. The right parameter to characterise the fracture toughness is C^* . In addition, the results for different temperatures should be reported as a function of the difference between the temperature and T_g (Laiarinandrasana et al. 2012). The fracture energy represents the contribution of chain drawing to the fracture energy. The critical molar mass M'_c separating both regimes is always of the order of 10–100 kg/mol. For $M < M'_c$, the flexible chains polymers, for instance polyethylene, behave as waxes. The stiff chain polymers, for instance polycarbonate, behave as highly brittle solids. It can be observed that, for glassy polymers, the critical molar mass is higher than the entanglement molar mass M_c , but of the same order: typically $2 < M'_c < 10$. The explanation is that plastic deformation involves chain drawing, but this is only possible if the chains participate in the network structure, *i.e.* if they are entangled and long enough to maintain this structure when they disentangle partially by sliding under load.

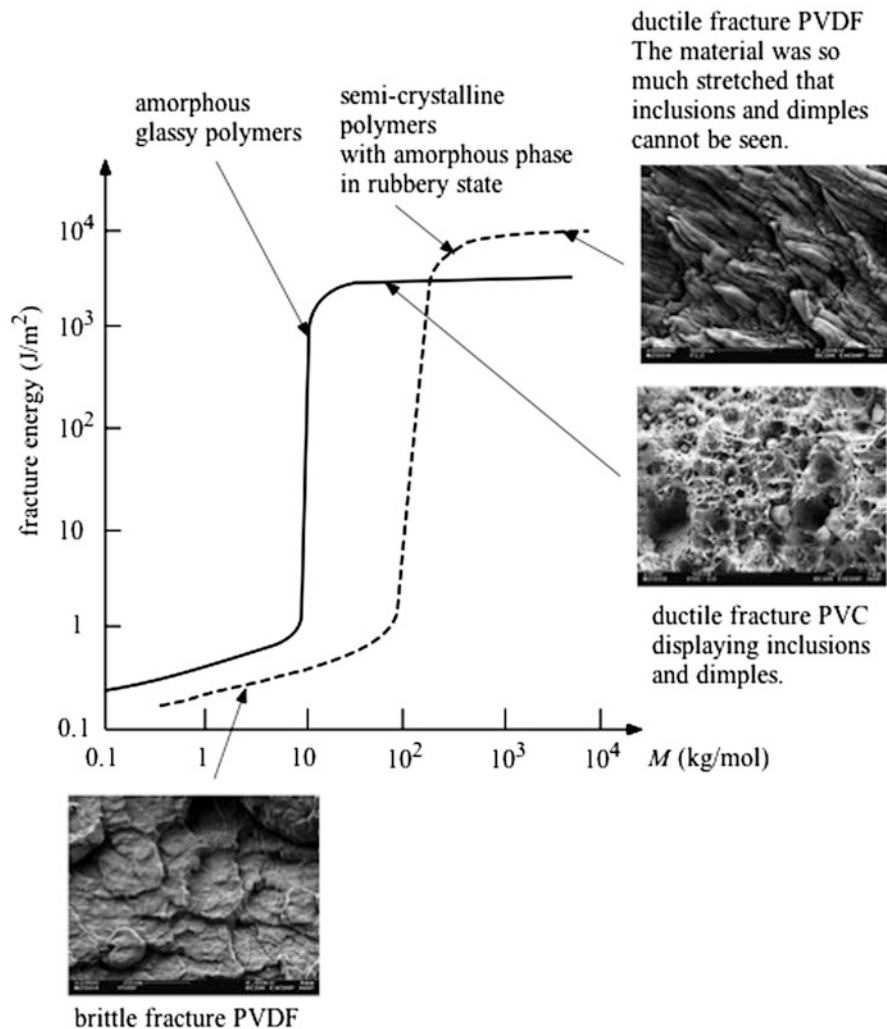
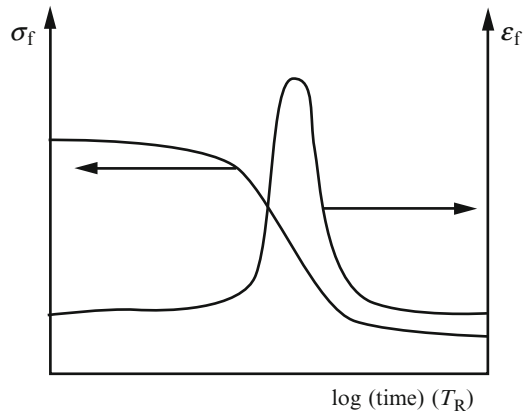


Fig. 10.46 Approximate shape of the molar mass dependence of fracture energy. For an amorphous polymer (*full line*) and for a semi-crystalline polymer having its amorphous phase in rubbery state (PE, PP, POM, PTFE) (*dashed line*) (Images provided by courtesy of L. Laiarinandrasana)

All the glassy amorphous polymers display *this brittle-ductile transition*. For $M > M'_c$ polymers such as for instance PS and PMMA remain brittle in tension, whereas polymers such as for instance PVC or polycarbonate are ductile in tension. Here brittleness in tension for $M > M_c$ is linked to crazing.

If the loading time and/or temperature are high enough, disentanglements can result only from molecular motions (chain *reptations*: De Gennes 1979). This is the

Fig. 10.47 Universal shape of master curves (at reference temperature T_k) of fracture coordinates for static tests



reason why polymers are brittle beyond a certain boundary in the time-temperature space. This is put in evidence in stress and strain master curves obtained from static tests (Fig. 10.47). Such curves can be built from results obtained at various temperatures, using the time superposition principle.

10.5.3.2 Fracture of Semi-crystalline Polymers

In the case of semicrystalline polymers, two cases must be distinguished. For polymers having their amorphous phase in the glassy state at the test temperature, there is no difference with glassy amorphous polymers. For poly(ethyleneterephthalate) or poly(etheretherketone), the critical molar mass M'_c is of the order of a few times the entanglement molar mass.

For polymers having their amorphous phase in rubbery state at the test temperature, for instance polyethylene or polypropylene, the critical molar mass is considerably higher: $M'_c/M_c \approx 50$ (Fayolle et al. 2008).

Here disentanglements cannot be the cause of the brittleness, at least in the case of short term resting.

In the case of polyethylene, Kennedy et al. (1994) clearly showed that the pertinent structural parameter is the *interlamellar spacing* l_a . There is a critical value l_{ac} , such as for $l_a < l_{ac}$, the polymer is always brittle (in tension) whereas it is always ductile for $l_a > l_{ac}$. For polyethylene and for polyoxymethylene $l_{ac} \approx 6$ nm.

It was shown that l_a depends mainly on the molar mass. It can be roughly written:

$$l_a \approx l_{a0} + a_c M_w^{1/2} \quad (10.31)$$

Thus the critical value of the interlamellar spacing corresponds more or less sharply to a critical value of the molar mass. The high value of the latter, for instance ≈ 70 kg/mol for polyethylene and ≈ 200 kg/mol for polypropylene, explains why the

Table 10.4 Mechanical properties of some polymers

Polymer	Young modulus E GPa	Yield strength R_p MPa	Fracture energy kJ/m ²
<i>Amorphous polymers</i>			
ABS (multiphase)	2.8	55	
PMMA	3.3	77	0.3–0.4
PS	3.1	45	0.3–2
PC	2.3	65	3.5–8
PSU	2.6	80	
PVC	3.0	60	
<i>Semicrystalline polymers</i>			
PA6 (50%RH)	1.0	50	
PA6 (dry)	3.2	90	
PA12	1.45	65	
PE	0.2–0.8		
PEhd	0.2–0.8	25	6–7
PEEK	3.6	100	
PET (orientated)	9.0	250	8
POM (homopolymer)	3.4	70	
POM (copolymer)	2.9	65	
PP	1.5	40	
UP (thermoset)	3.4	60	

PSU bisphenol A polysulfone, *PE* the modulus depends on the degree of crystallinity, *PET* fibres, *POM* the role of copolymerisation is to limit crystallisation, *UP* styrene cross-linked, unplasticised, unsaturated polyester

industrial grades of these polymers have always a molar mass considerably higher than for all other kinds of polymers.

Semi-crystalline polymers also undergo crazing. The highly stretched chains in the fibrils tend to crystallise in their oriented conformation.

To give orders of magnitude Table 10.4 lists mechanical properties of some polymers.

Table 10.5 gives glass transition temperatures of various polymers.

10.5.3.3 Fracture of Vulcanised Rubbers

Static testing on rubbers leads to master-curves having the same shape as for linear polymers (Fig. 10.47). At low temperature, near the glass transition temperature, the behaviour tends to be brittle because molecular relaxations tend to be too slow with regard to the loading time. At high temperatures and/or long loading times, the behaviour tends also to be brittle, as for linear polymers, despite the fact that, here, disentanglement by reptation is impossible. It can be supposed that, in this case, chemical ageing (thermal degradation for instance) plays the same role as disentanglement.

Table 10.5 Type and glass transition temperatures of common linear polymers

Polymer	Code	Type	T_g (K)
Polydimethylsiloxane	PDMS	E	152
Polybutadiene	PB	E	171
Polyoxytetramethylene	PTMG	E	190
Polyisobutylene	PIB	E	200
Plasticised polyvinyl chloride	PVC	E	230–270
Polyethylene	PE	SC	195 or 230 ^a
Polyoxymethylene	POM	SC	230
Poly(vinylidene fluoride)	PVDF	SC	240
Polypropylene	PP	SC	270
Polyamide 11	PA11	SC	300
Polyamide 6	PA6	SC	333
Polyamide 4–6	PA4-6	SC	358
Polyvinyl chloride	PVC	A	355
Polystyrene	PS	A	378
Poly(methyl methacrylate)	PMMA	A	403
Poly(butylenes terephthalate)	PBT	SC	333
Poly(phenylene sulfide)	PPS	SC	358
Poly(etheretherketone)	PEEK	SC	415
Poly(bisphenol A carbonate)	PC	A	425
Poly(bisphenol A sulfone)	PSF	A	463
Polyetherimide	PEI	A	483
Polyethersulfone	PES	A	493
Polypyromellitimide (Kapton)	PI	A	620

E uncrosslinked elastomer, *SC* semi-crystalline, *A* amorphous

^aTwo theories coexist in the literature

Smith (1964) proposed to use master-curves $\sigma_r^* = f(\log t)$ and $\Lambda_r = f'(\log t)$, to build a “rupture envelope” $\sigma_r^* = f(\Lambda_r)$; σ_r^* is a reduced stress $\sigma_r^* = \sigma_r T_{\text{ref}} / \rho T v_e$, σ_r being the fracture strength, T_{ref} a reference temperature arbitrary chosen, ρ the density, T the temperature and v_e the number of active chains (linking two reticulation bonds). Λ_r is the elongation at fracture. This curve resumes all the properties available for a given type of rubber, characterised by a given cross-link density.

10.5.4 Delayed Fracture of Polymers

10.5.4.1 Static Fatigue

Static fatigue has been already addressed in Sect. 10.5.3.1. We look here at the prediction of life-time. A considerable amount of research work concerned the control of polymer structures in view of improving the durability of *polyethylene pipes* used for gas or drinking water transportation at relatively low pressures, corresponding to stresses typically less than 10 MPa. The applications are economically interesting

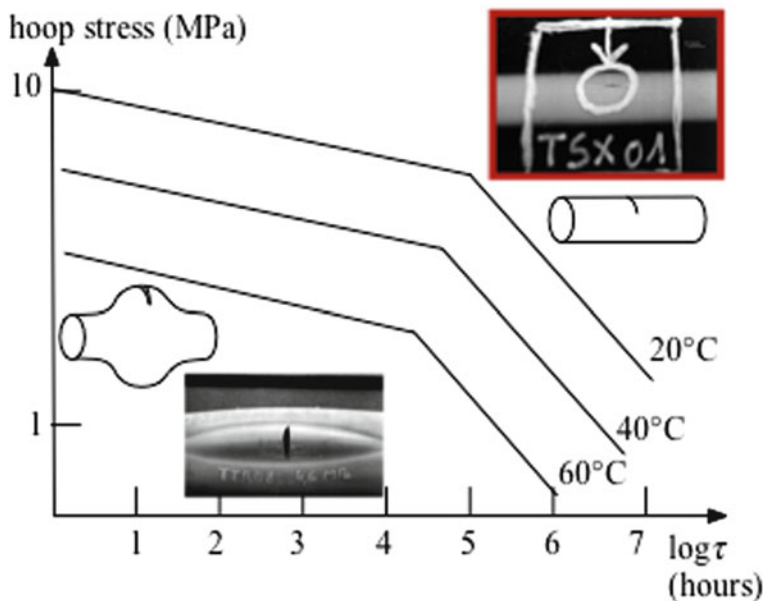


Fig. 10.48 Shape of the hoop stress-time to failure plots at three distinct temperatures. In the ductile domain, failure occurs after a large plastic deformation. In the brittle domain, a crack appears without any macroscopic deformation (Images provided by courtesy of L. Laiarinandrasana)

if the lifetime exceeds about 50 years. If the crack propagation rate has been determined as a function of the stress intensity factor as in Fig. 10.48, integration allows calculating the time to fracture. The main difficulty, in these studies, comes from the fact that, beyond a certain critical time, the fracture mechanisms change and become brittle (Fig. 10.48). The sketch of Fig. 10.48 is further illustrated by results obtained on PE at various temperatures (Fig. 10.49).

Fracture in the brittle region is presumably initiated by cavitation in the amorphous phase. Cavitation is favoured by disentanglement and hampered by an increase of the interlamellar spacing. Disentanglement is the slower the longer the chains. Theoretically, the disentanglement time must increase as $\approx M^3$. However, the melt viscosity increases in the same way and cannot be too high otherwise processing becomes impossible. Interlamellar spacing can be increased using comonomer units (butene, hexene, octene), which act as chain defects unable to crystallise and then responsible for a limitation of crystal thickness. The concentration of comonomer units must be however limited because the whole polymer crystallinity must be kept high enough to have a modulus and a yield strength able to resist ductile fracture at reasonable pressures. The spectacular improvement of pipe durability is a good example of rational use of structure-properties relations for the optimisation of industrial applications.

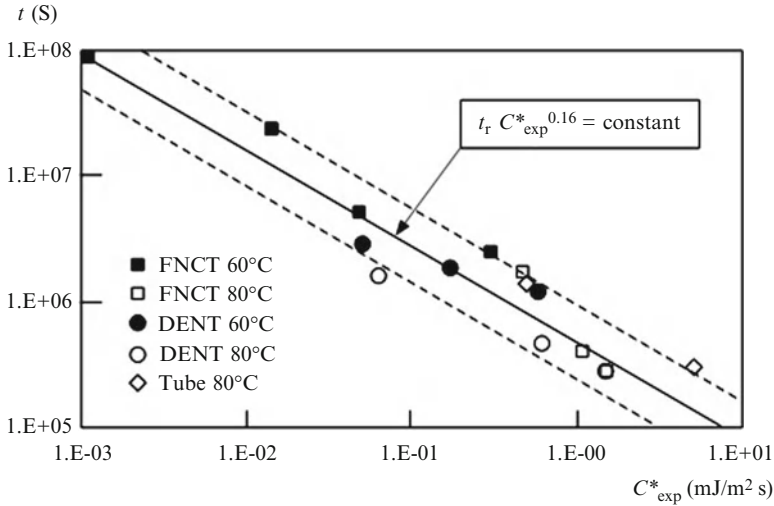


Fig. 10.49 Time to failure for PE at different temperatures versus the C^*_{exp} parameter (Adapted from Ben Hadj Hamouda et al. 2007)

10.5.4.2 Fatigue of Polymers

In a first approach, fatigue of polymers does not differ from fatigue of metals. Wöhler (or S-N) curves can be determined experimentally, crack propagation follows Paris law. However, in polymers, which are poor heat conductors, the energy dissipated by viscous deformation can produce large increase of the temperature. Two extreme situations can be encountered. At high stress amplitude and high frequencies, in the low-cycle fatigue regime, the temperature increase is so large that the glass transition temperature is exceeded and the polymer becomes rubbery. In that case the behaviour is viscoplastic and the crack singularity cannot be characterised by the stress intensity factor (SIF) K . Instead it is the C^* parameter which ought to be used (Ben Hadj Hamouda et al. 2007). Nevertheless, fracture studies on polymers most often keep using the SIF as the characterising parameter, which is legitimate in the case of brittle behaviour observed at low stress amplitude and low frequencies; in the high cycle regime, there is hardly an increase of the temperature, except in crack propagation at the crack tip. Figure 10.50 shows fatigue curves (analogous to Wöhler curves) of annealed or quenched polycarbonate (PC) (Janssen et al. 2008). The low-cycle fatigue (full symbols), thermally dominated domains, are separated from the high-cycle fatigue (open symbols), mechanically dominated domains, by two time-decades.

The crack propagation rate is a function of the stress intensity factor range, as for metals (Sect. 6.4.3) and it follows the Paris law (Fig. 10.51). In Fig. 10.51 the crack tip opening displacement δ (CTOD) was measured instead of the linked stress intensity factor. The crack growth per cycle is for most of the decades much less

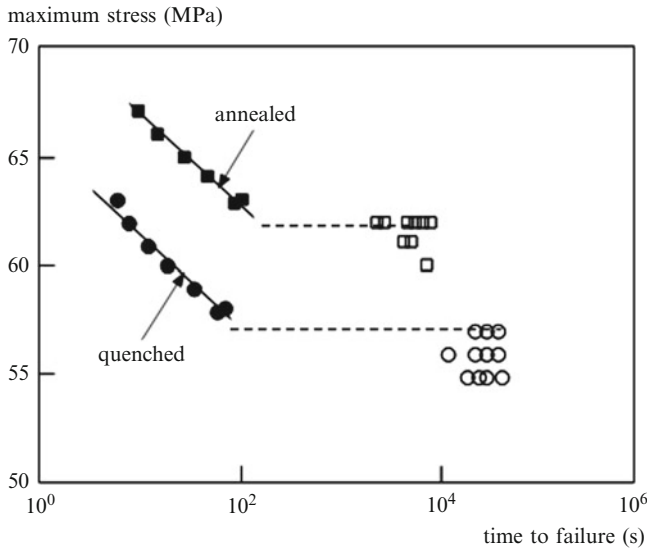


Fig. 10.50 Fatigue curves for annealed or quenched PC in which can be distinguished thermally dominated domains (*full symbols*) from mechanically dominated domains (*open symbols*) (Janssen et al. 2008)

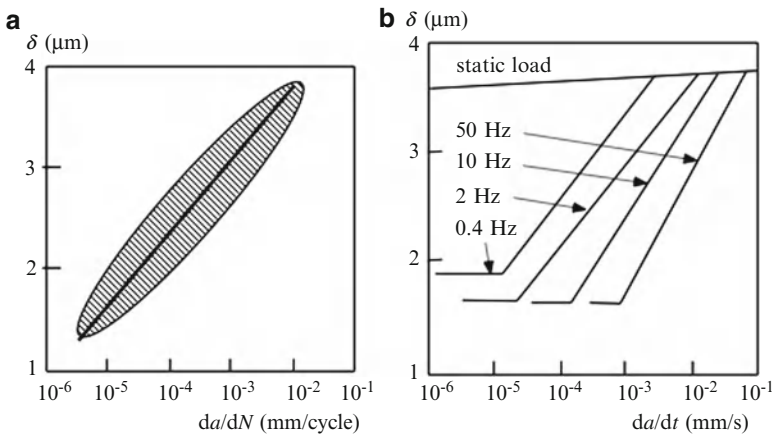
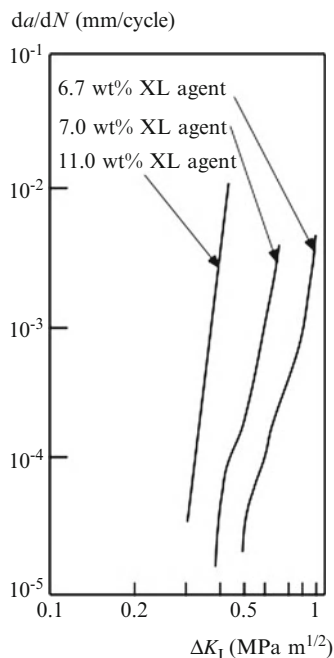


Fig. 10.51 Plot of the crack propagation rate as a function of the crack tip opening displacement δ (CTOD) at different frequencies for PMMA at 23°C. (a) crack propagation rate in mm/cycle, all the data point falling in the hatched area; (b) crack propagation rate in mm/s. The crack growth rate in static loading is also shown (Adapted from Knauss 1989)

than $\delta/2$. This is in keeping with the observation that in polymers the crack growth is discontinuous, the crack remaining idle for many cycles. In these materials, counting the lines left on the fracture surface by each crack advance does not correspond with the number of cycles.

Fig. 10.52 Paris curves for PMMA cross-linked to different degrees (Adapted from Herzberg and Manson 1980)



However, in view of the behaviour of polymers, it seems appropriate to consider crack propagation in cyclic loading as a succession of crack increments by viscoplastic mechanism. This is exemplified in Fig. 10.51b concerning PMMA, in which, instead of plotting the crack growth rate per cycle as in Fig. 10.51a, it is the crack growth rate per second which is plotted. The figure shows that the propagation of the crack is not thermally controlled, which would give a single curve when plotted in mm/s, but rather mechanically controlled. The fatigue crack propagation behaviour is related to the number of cross-links. This is shown in Fig. 10.52, in which different Paris curves are obtained for PMMA with various amounts of an additive modifying the degree of cross-linking.

The situation is quite different for elastomeric polymers. Figure 10.53, concerning polyurethane, shows that in that case the curves obtained for different frequencies merge together when the plot is in mm/s (Fig. 10.53b). It shows that the time is here the essential parameter.

The *local approach to fracture mechanics* has not yet been much applied in the field of polymers in spite of its interest not only for crack propagation predictions but also for characterising the fracture mechanisms. Ben Hadj Hamouda and his co-authors (2009) studied in this way the slow crack growth in a medium density ethylene-butene copolymer (MDPE). The failure criterion was expressed as a critical creep damage over a critical distance. The model allowed good representation of cracking experiments at 60°C of axisymmetric full notched creep tensile specimens and of double-edge notched tensile specimens.

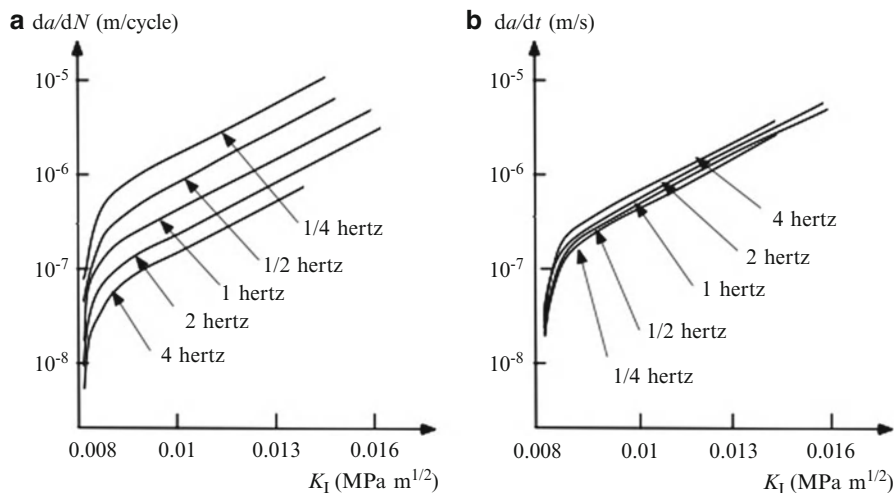


Fig. 10.53 Crack growth rate as a function of the stress intensity factor for polyurethane at 20°C for different frequencies. (a) crack growth rate expressed in m/cycle; (b) crack growth rate expressed in m/s (Adapted from Knauss 1989)

10.6 Fracture of Wood

10.6.1 Introduction

Wood is the first material to have been used in various constructions. It still is, and possibly more and more in view of climatic issues. Up to recent time, building with wood and fabricating furniture were only based on past experience and traditions of craftsmen. Nowadays, architects dare to design unconventional wood structures, especially with the introduction of glued-laminated timbers.⁵ For safe design, it is therefore needed to gain a better understanding of the mechanical properties of wood and to dispose of quantitative results. Keep in mind that the specific mechanical properties of wood (value of a given property divided by the density) are about the same, or even better, than those of metals.

There is a wide variety of wood species growing in different climates. Wood is a living material containing variable contents of water. In a tree old and new layers of wood coexist. Wood products can contain more or less knots. Under those conditions, it is hard to give a synthetic outline of mechanical properties, which depend on many parameters.

⁵Wood is not only used in buildings and furniture; de Havilland Mosquito bomber was made of wood.

The sought-after mechanical properties for wood are not different from those for other structural materials: stiffness, yield strength, fracture strength, fatigue resistance, creep resistance. We will give the main characteristics of these properties for wood. We will insist on two distinctive aspects found in wood: pronounced anisotropy and moisture content. It will be important to remember that the variability of the mechanical properties of wood is large. Deviations from the trends that we will describe are not uncommon. As we will shortly see, wood can be considered as a natural polymeric material. In particular, this confers it viscoplastic behaviour.

At first a description of the microstructure of wood will help to better understand its relation with the mechanical properties. We will then study fracture and we will give some indications about fatigue and creep.

10.6.2 Microstructure of Wood

A tree grows every year by increasing its diameter. This produces successive *growth rings*. The outer ones, which conduct water, minerals, oxygen, nitrogen, constitute the *sapwood*. The inner ones have ceased their conductive function and form the inactive *heartwood*. It is darker than the sapwood.

Tree species belong to two categories: *soft wood* (cedar, douglas fir, fir, hemlock, pine, redwood, spruce . . .) and *hard wood* (ash, balsa, birch, cherry, elm, maple, oak, poplar, walnut, willow . . .).

The growth rate is faster in the spring than it is in summer. Thus, each growth ring includes an inner layer of *early wood* and an outer one of *late wood*, with different microstructures. Three *structural directions* are distinguished: the longitudinal direction L parallel to the stem; the radial direction R perpendicular to the growth rings; the tangential direction T tangent to the growth rings (Fig. 10.54).

The microstructure consists of elongated cells called *tracheids*. Figure 10.55 shows such cells in soft wood. They are 3–5 mm long and 20–80 μm in diameter. They are less wide in late wood than in early wood. In hard wood the structure is slightly different: there are wood fibres similar to the tracheids of soft wood, but smaller (0.7–3 mm long and 20 μm in diameter) and not transporting fluids, and larger vessel elements of various sizes.

Figure 10.56a is a scanning electron micrograph of cells in soft wood (spruce) and Fig. 10.56b in hard wood (oak) in which it is possible to observe water. *Water* is present either adsorbed on the cell walls (bound water) or as free water. The free water is situated in the cell cavities called lumens. The *fibre saturation point* corresponds to the moisture limit when there is no more free water. It varies from species to species with an average of 28% (the moisture content is the percentage of water relative to the completely dry wood).

Figure 10.57 demonstrates the cleverness of nature in designing materials. It is a sketch of the successive layers in cell walls. They are made of nine layers: S_3 ,

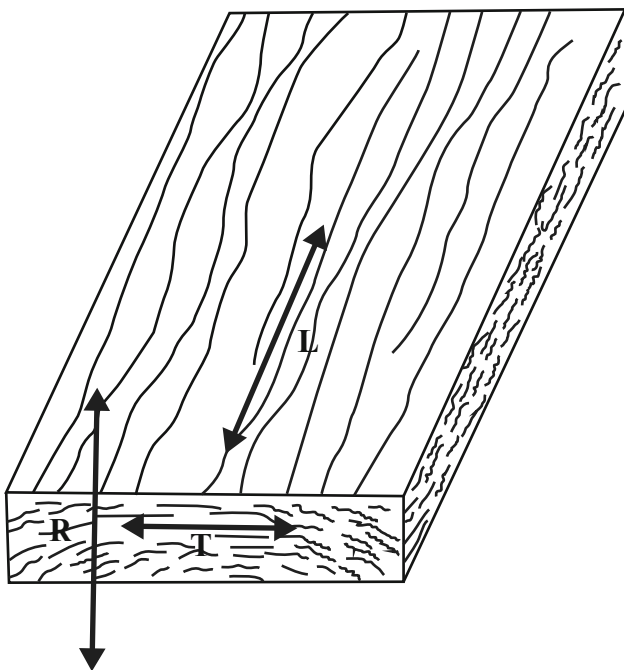


Fig. 10.54 The three structural directions in wood: *L* longitudinal, *R* radial, *T* tangential

S_2 , S_1 , P, M, S_1 , S_2 , S_3 . In layers S microfibrils form helices with orientations as sketched in Fig. 10.57. In layers P and M the orientation is random (Qing and Mishnaevski 2009). We are in the presence of a composite material optimising mechanical properties through the orientations of the fibrils.

The fibrils are bundles of about 1,500 molecules of cellulose, hemi-cellulose and lignin in various proportions. Cellulose is the major constituent. Figure 10.58a shows that this molecule, the most abundant on earth, is a linear polymer. Hemi-cellulose is a branched polymer with a variety of monosaccharides (sugars). Lignin is a three-dimensional network of three aromatic monolignols (Fig. 10.58b). The number of different types of lignin is extremely great according to wood species. The molecules are hydrophobic; they provide wood with resistance to decay.

The proportion of the constituents differs in the layers of the cell walls (Table 10.6).

10.6.3 Anisotropic Behaviour

The very microstructure of woods explains that their mechanical properties are very anisotropic. Table 10.7 gives various mechanical properties of different wood

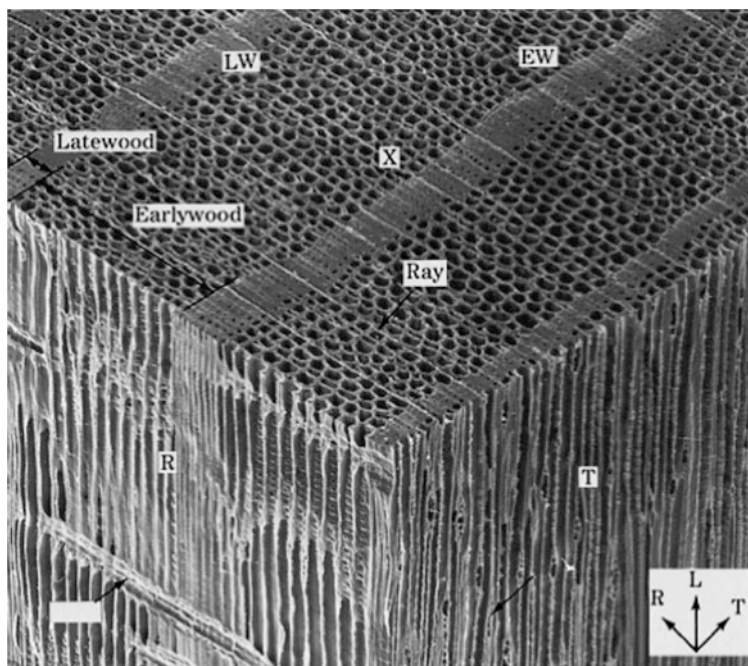


Fig. 10.55 SEM image of cells in soft wood

species. It should be remembered that there is a great variability of the results: for E it amounts to 22%, for the modulus of rupture to 16%, for tensile and compressive strengths to 25%.

The *Young modulus* given in Table 10.7 is about 10% lower than the one measured in a longitudinal direction. The moduli measured in the radial and tangential directions are a few hundredths of the modulus in the longitudinal direction. This is what is expected from the microstructure schematically consisting of stacks of parallel tubes. The deformation of a circular tube of radius r , of wall thickness e and of length l and Young modulus E , loaded by a load P along a generatrix, is $0.149P/(2\pi erE)$. The corresponding stress is $P/(2rl)$. Hence, the apparent Young modulus of the tube in this transverse direction is $(\pi e/0.149l)E$. With the cell dimensions given above we find that this is about $0.003E$. This rough calculation yields too low a value compared with experimental results, which is not so surprising, but at least, it gives a good explanation for this anisotropy of elastic moduli.

Table 10.7 shows walnut as the best wood for construction, combining high Young modulus and modulus of rupture. One is not surprised to find willow as a poor structural material.

We have not given the *Poisson ratios* in Table 10.7. The anisotropy requires the determination of 6 Poisson ratios. They are indeed different from one another. For instance Table 10.8 gives a few examples.

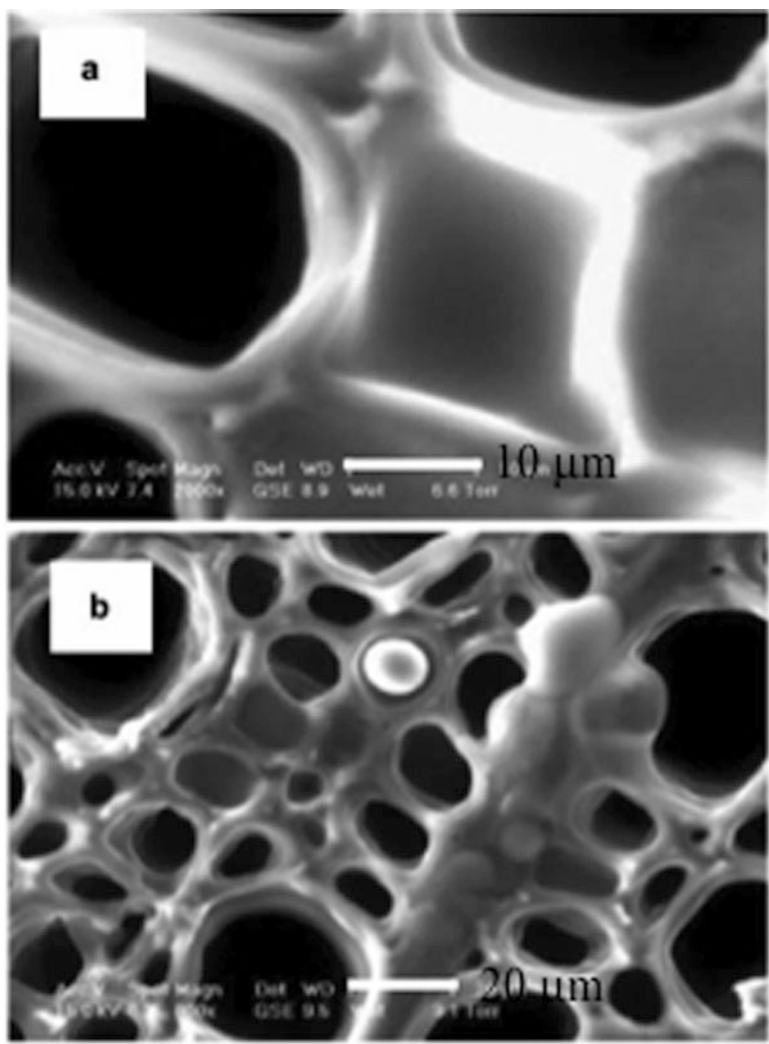


Fig. 10.56 Microstructure in the TR direction in the green state at 98% moisture content for (a) spruce; (b) oak (Vasic and Stanzl-Tscheegg 2007)

Table 10.8 shows that for some species, loading in the longitudinal direction produces almost no change of volume, in hemlock for instance. In walnut tensile longitudinal loading on the contrary produces a volume decrease ($\nu_{LR} + \nu_{LT} = 1.127$), which is unusual. Loading in the radial or in the tangential directions produces almost no deformation in the longitudinal direction whatever the wood. This anisotropic behaviour is connected with the microstructure of the cell walls. If, within a layer, fibrils are inclined of an angle θ with the tensile axis, the Poisson ratio is equal to $1/\tan^2\theta$ in the plane of the layer. It increases with the tensile deformation.

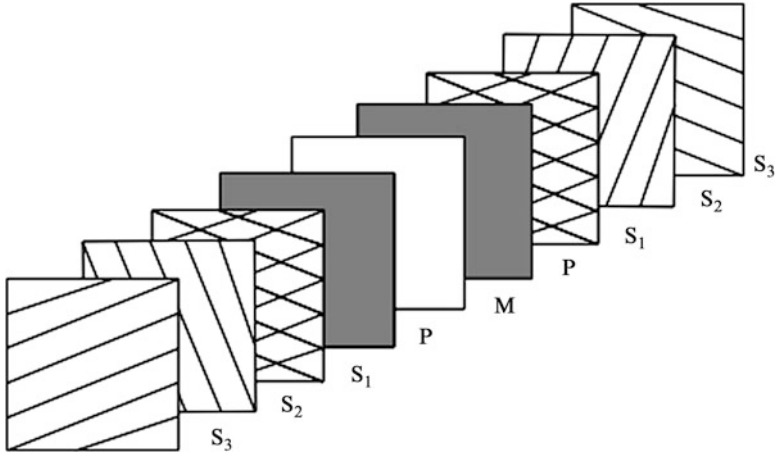


Fig. 10.57 Sketch showing the arrangement of fibrils in the successive layers of wood cells (Qing and Mishnaevski 2009)

Fig. 10.58 Monomers of the molecules found in wood.
(a) cellulose monomer;
(b) constituents of lignin:
H hydroxyphenyl, *G* gaiacyl,
S syringyl

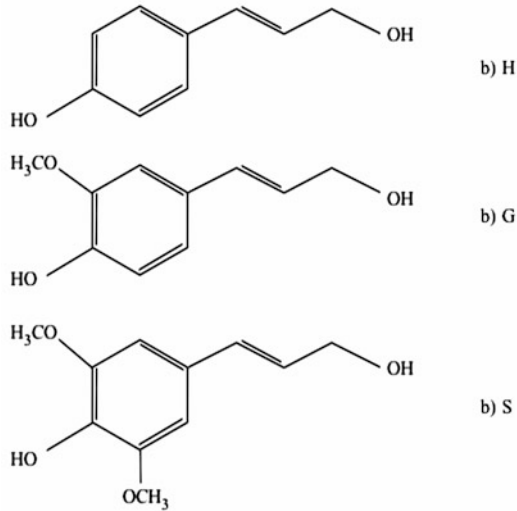
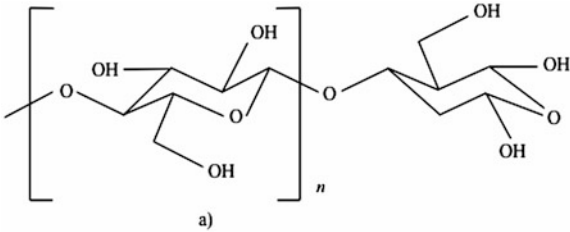


Table 10.6 Proportion of constituents in the layers of the cell walls (Qing and Mishnaevski 2009)

Layer	Thickness ^a (μm)	Cellulose (%)	Hemicellulose (%)	Lignin (%)
M	0.25	–	38	62
P	0.1	12	26	62
S ₁	0.2–0.3	35	30	35
S ₂	1.4–4.0	50	27	23
S ₃	0.03–0.04	45	35	20

^aThe first figure in the thickness column concerns early wood, the second one late wood

Table 10.7 Mechanical properties of various wood species for 12% moisture content

Species	<i>E</i> (GPa)	Modulus of rupture (MPa)	<i>R</i> _{mT} (MPa)	<i>R</i> _{mL} (MPa)	<i>R'</i> _{mL} (MPa)	<i>R'</i> _{mT} (MPa)
Ash	11.0	87	41.2	–	5.2	4.8
Beech	11.9	59	50.3	86	7.0	7.0
Chestnut	8.5	59	36.7	–	4.3	3.2
Elm	9.2	81	38.1	121	4.8	4.6
Maple	10.0	74	41.0	108	5.2	3.7
Oak	11.3	96	45	78	6.4	–
Walnut	11.6	101	52.3	–	7	4.8
Willow	7.0	54	28.3	73	3	–
Fir	10.0	63	36.4	78	2.8	1.2
Hemlock	8.3	61	37.3	90	4.5	–
Pine	8.5	59	33.1	73	3	2.1
Redwood	9.2	69	42.4	65	4.8	1.7
Spruce	11.1	74	41.1	85	3.8	–

Values selected from Green et al. (1999)

E the Young modulus, and the modulus of rupture are deduced from bend tests, *R*_{mT} is the tensile strength in a direction perpendicular to the grains, *R*_{mL} the tensile strength in a direction parallel to the grains, *R'*_{mL} the compressive strength in a direction parallel to the grains, *R'*_{mT} the compressive strength in a direction perpendicular to the grains

The Poisson ratio of the cell wall results from some complicated average of the Poisson ratios of the various layers, the angles θ of which differ. It is conceivable that this average could take a rather large value.

Looking at Table 10.7, the anisotropic nature of the other mechanical properties is obvious. As a general rule the properties in a direction parallel to the grains are superior to those in a direction perpendicular. Loading in a direction perpendicular to the elongated cells applies a stress on the boundaries between them, whereas in a direction parallel to them the boundaries are free of stress. Boundaries constitute weak links.

The published values of *fracture toughness* of wood are scarce and not always consistent. Who has split wood logs has no doubt about the much lower fracture toughness in the RL or TL directions than in other directions. The fracture toughness values are in the range 0.25–0.5 MPa m^{1/2}. Table 10.9 gives some results. The variability of results amounts to 34%.

Table 10.8 Poisson ratios of some wood species for 12% moisture content

Species	ν_{LR}	ν_{LT}	ν_{RT}	ν_{TR}	ν_{RL}	ν_{TL}
Ash	0.371	0.440	0.684	0.360	0.059	0.061
Birch	0.426	0.451	0.697	0.426	0.043	0.024
Maple	0.424	0.476	0.774	0.349	0.065	0.037
Oak	0.350	0.448	0.560	0.292	0.064	0.033
Walnut	0.495	0.632	0.718	0.378	0.062	0.035
Hemlock	0.485	0.423	0.442	0.382	–	–
Redwood	0.360	0.346	0.373	0.400	–	–
Spruce	0.372	0.467	0.436	0.245	0.040	0.025

Selected from Green et al. (1999)
 ν_{LR} refers to the deformation along the radial axis when the wood is stressed in the longitudinal direction. Other indices follow the same rule: the first one refers to the direction of loading, the second to the deformation direction

Table 10.9 Fracture toughness values for some wood species

Species	K_{Ic} RL direction (MPa m ^{1/2})	K_{Ic} TL direction (MPa m ^{1/2})
Douglas fir	0.32	0.36
Hemlock	0.375	–
Pine	0.25	0.26
Spruce	0.42	–
Oak	0.41	–
Maple	0.48	–

Selected from Green et al. (1999)

There is not a great difference between the fracture toughness values in the RL and TL directions.

The anisotropic nature of wood is expected to give different behaviour in *mixed-mode loading* than for isotropic materials. Whereas in the latter cracks deviate, in wood they keep following the longitudinal direction along tracheids (Jernkvist 2001). The ratio of the fracture toughness in pure mode II to that in pure mode I is large, from 2 to 4 for the RL direction, 5–7 in the TL one. This reflects differences of the fracture at the microscopic level. In mode I cracks split the interface between tracheids, whereas in mode II, while keeping a path along the longitudinal direction, they cut tracheids from place to place. The results in mixed-mode loading for pine (Hunt and Croager 1982) and for spruce (Mall et al. 1983) are found to coincide with the maximum stress criterion and not at all with a critical value of the strain energy release rate.

10.6.4 Effect of Moisture Content

The strength of wood is constant above the fibre saturation point. Below this point, it increases when the moisture content decreases. In other words, the free water has no influence on the properties of wood; what counts is the adsorbed water on the

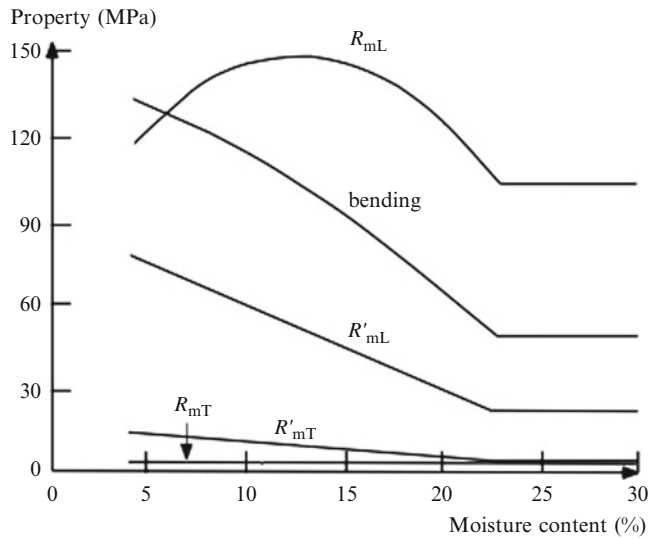


Fig. 10.59 Sketch showing the effect of moisture below the fibre saturation point on strength of wood (Adapted from Green et al. 1999)

Table 10.10 Values of M_p for various species of wood

Species	M_p (%)
Ash	24
Birch	27
Chestnut	24
Hemlock	28
Pine	21
Redwood	21
Spruce	27

Selected from Green et al. (1999)
 M_p is the moisture content below which the properties as a function of the moisture content change

molecules of cellulose. This trend is sketched in Fig. 10.59. Green et al. proposed a formula to calculate the evolution of a given property P as a function of the moisture content M (%) according to the fibre saturation point, or, more exactly to the moisture content M_p corresponding to the beginning of the effect on the properties. This value, which is not very different from the fibre saturation point, depends on the wood species (Table 10.10). The formula is:

$$P = P_{12} \left(\frac{P_{12}}{P_g} \right)^{\frac{12-M}{M_p-12}} \tag{10.32}$$

where P_{12} is the value for the property at 12% moisture content and P_g the value in the green state.

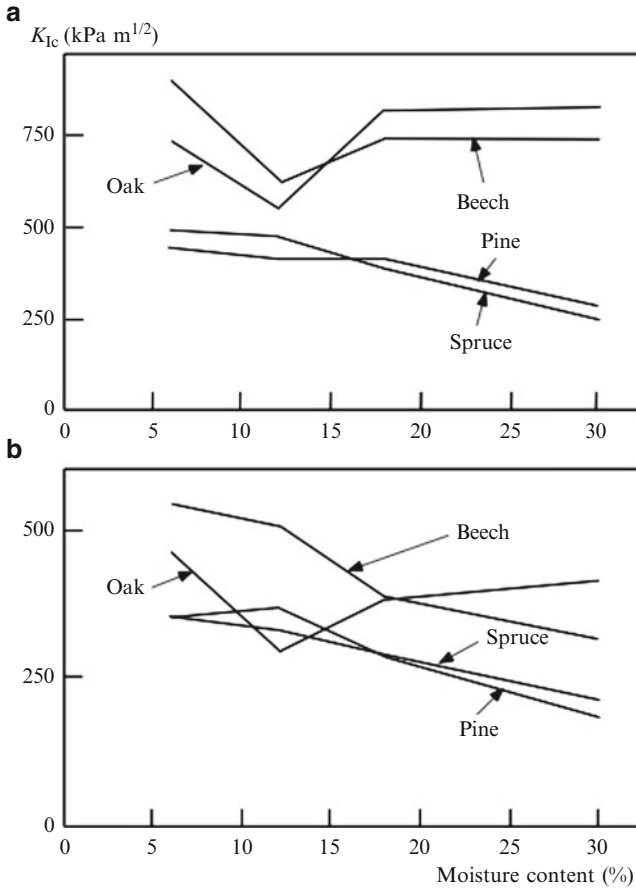


Fig. 10.60 Fracture toughness values as a function of moisture content. (a) RL direction; (b) TL direction (Adapted from Vasic and Stanzl-Tschegg 2007)

Concerning fracture toughness, practical knowledge tells that it is easier to split a dry log than a green one. It is expected that decreasing the moisture content would lower the fracture toughness. Figure 10.60 shows results that do not quite agree.

10.6.5 Delayed Fracture of Wood

10.6.5.1 Creep of Wood

Under constant load wood creeps. Figure 10.61 shows the time to fracture for different loads. For safe design, the load must be kept at a rather low percentage of the fracture strength, the more so as changes in climatic conditions can accelerate creep.

Fig. 10.61 Time to rupture of wood for different stresses expressed as the % of the fracture strength. (Based on a time to rupture of 28 s at 100%) (Adapted from Green et al. 1999)

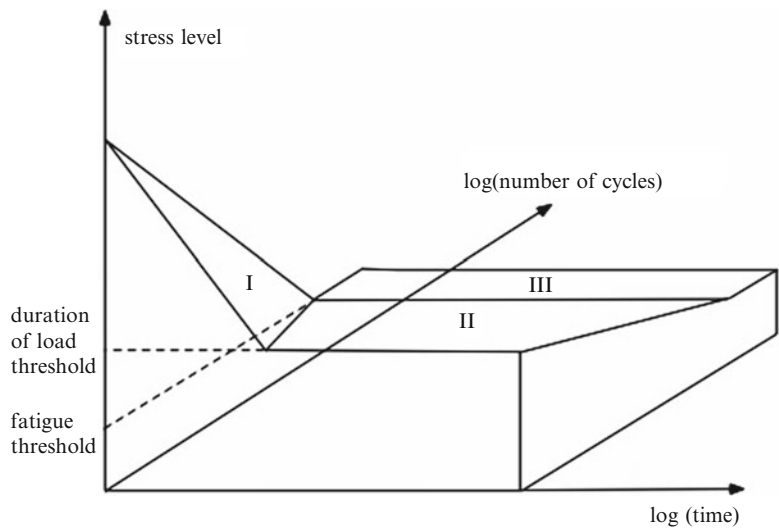
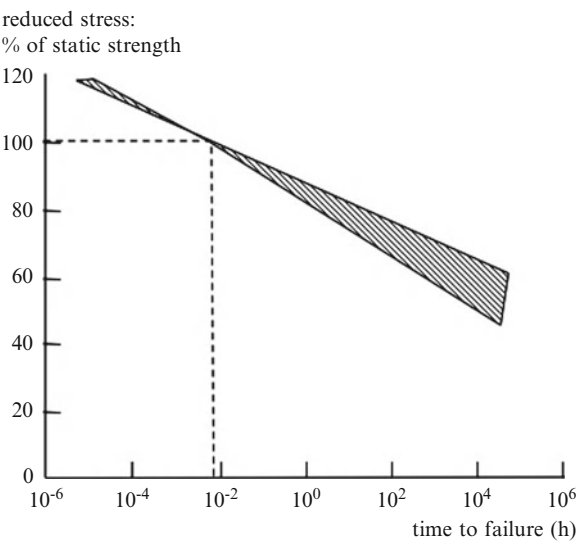
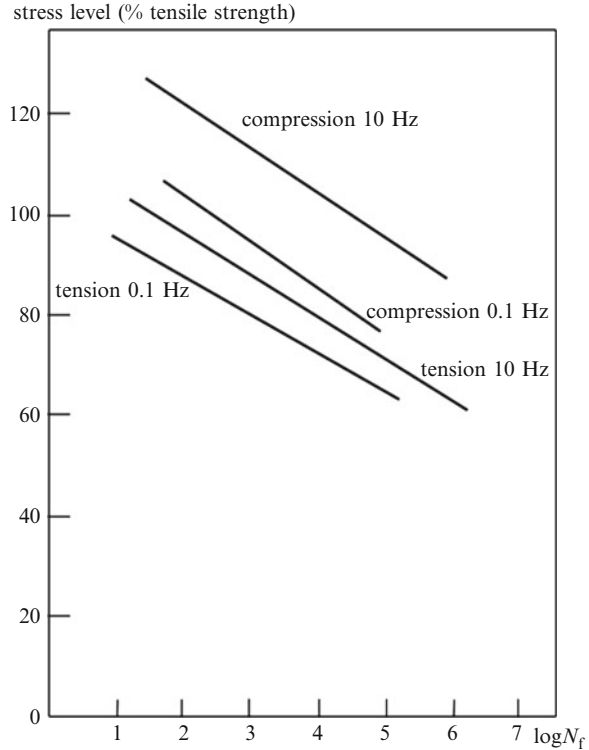


Fig. 10.62 Sketch of the fatigue life as a function of duration of load and of the number of cycles

10.6.5.2 Fatigue of Wood

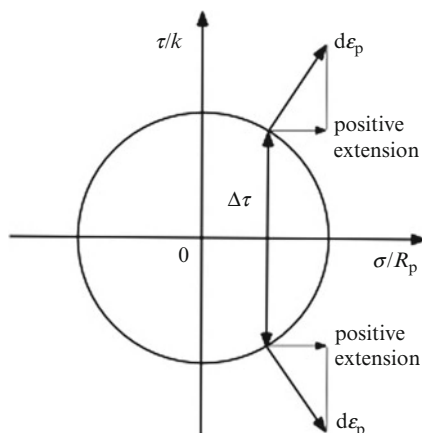
Fatigue of wood, as it is the case with concrete, can be due to purely mechanical mechanisms, or to repeated creep damage. In the first case fatigue is cycle dependent, in the second it is time dependent. Figure 10.62 is a sketch of the fatigue life enhancing three regimes: in regime I fatigue is a combination of cycle-dependent

Fig. 10.63 Fatigue life curves for spruce tested at different frequencies
(Adapted from Okuyama et al. 1984)



and time-dependent effects, life is a function of the frequency; in regime II the stress is below the duration of load threshold, and fatigue is cycle-dependent only with no effect of frequency; regime III corresponds to the endurance limit. The endurance limit of wood at 10^6 cycles is about 30% of the static strength. Figure 10.63 comes from results obtained on spruce loaded parallel to the cells (Okuyama et al. 1984). It clearly shows the large effect of frequency, especially in the compression tests. Clorius (2002) performed fatigue tests on pine. He discovered that frequency had no effect for a relative humidity of 65% whereas it was not the case for a relative humidity of 85%. He reported that the fatigue life at 65% RH was 1×10^6 cycles at a relative stress of 50% (of the short time static strength) and 2×10^4 at 65%. For 85% RH the results were the followings: for a relative stress of 50% the fatigue life was 3×10^5 at 1 Hz, 5×10^4 at 0.1 Hz and 2×10^4 at 0.01 Hz; for a relative stress of 65% the fatigue life was 1×10^4 at 1 Hz, 5×10^3 at 0.1 Hz and 2×10^3 at 0.01 Hz.

Fig. 10.64 Ratcheting. The circle represents the yield criterion. Normality rule shows that, the normal stress being positive, there is an accumulation of positive extensions at each reversal of the shear stress



10.7 Fracture of Composite Materials

The very large variety of composite materials makes it impossible to give an exhaustive account of damage and fracture in these. We shall give only a few indications here.

10.7.1 Effect of Residual Stresses

It is essential to consider first the residual stresses generated by differences among the coefficients of thermal expansion of the various constituents. A fibre that contracts on cooling more than the matrix generates compressive stresses in the latter. In the opposite case, it is the fibres or the particles, and in particular the boundaries, which are in compression, a state of affairs that opposes their cracking. Thus with *thermal cycling* there is risk of fatigue, and of *ratcheting*: every cycle extends the material (Fig. 10.64). Application of the normality rule (Volume I, Sect. 3.5.2.3a) shows that at each reversal of the shear a material under constant traction accumulates a positive extension.

10.7.2 Composite with Long Fibres, More Brittle Than the Matrix

A typical example is that of glass fibres in an epoxy resin. The shear lag model (Sect. 3.4.6.3 of Volume I) shows that, in the composite under a tensile stress, plastic deformation of the matrix results in a tensile stress in the fibres, increasing from the ends. These will break at the point where this stress reaches the fracture strength

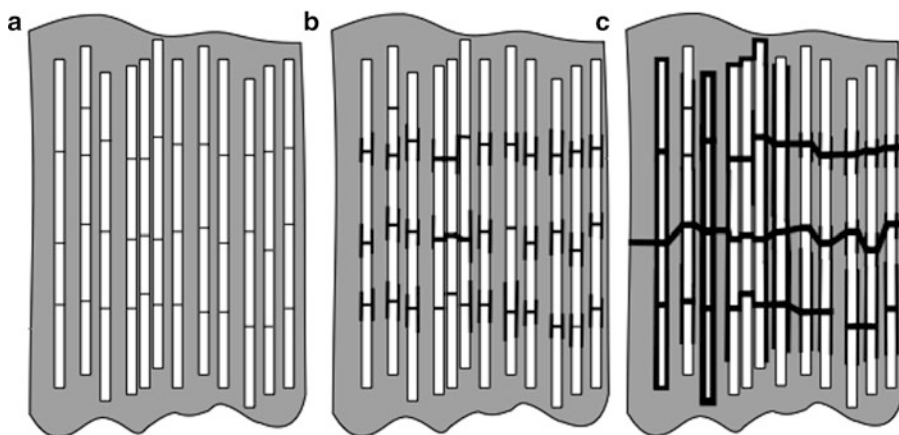


Fig. 10.65 Schematic drawing of successive steps in fracture of composite with brittle fibres. (a) fracture of the fibres; (b) debonding of interfaces and development of cracks joining adjacent cracks in fibres close together; (c) further development of cracks and debonding

R_m . In a composite in which brittle fibres of various lengths and of diameter d_f are embedded in a ductile medium, the first ruptures occur in the fibres of length equal to the critical value:

$$l_c = d_f \frac{R_m^f}{R_p^m} \quad (10.33)$$

where R_p^m is the yield strength of the matrix material.

The further fibre fractures are distant of this critical length. This is in fact a mean distance, since the fracture is of a statistical nature. Figure 10.65 shows schematically how cracking develops. Cracks in fibres, which are close together, link, whereas in more isolated fibres the interface with the matrix breaks by shear.

Figure 10.66 is an illustration of this mechanism. It can be observed on this SEM image an isolated glass fibre the extremity of which was pulled out of the epoxy matrix and a bundle of four fibres close together which broke while remaining bonded to the matrix.

The distribution of fibres in the composite is never uniform. Numerical calculations show that when a crack, in which several fibres and the surrounding matrix are involved, propagates, the shear stress that it generates on the fibre boundaries increases faster than the normal stress on the plane of the crack. Beyond a certain size the cracks are stopped and branch, shearing the boundaries. The final rupture consists of a splitting of the composite.

Where the fibres are less dense they are loosened from the matrix. It is clear therefore that departures from homogeneity are of fundamental importance in the damaging of composites. Misalignment of the fibres also has important effects: those that are most closely aligned in the direction of the traction bear the highest

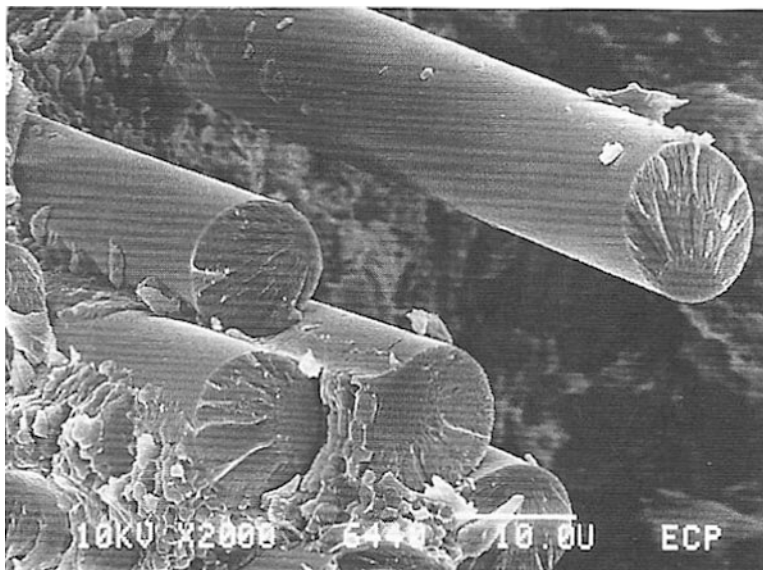


Fig. 10.66 SEM image of the fracture of an epoxy-glass fibre composite. In the regions where the fibre density is high they break without pull-out; where this is low they are pulled-out of the matrix

stress and fail first. Finally, it is very important to take *edge effects* into account, as it is here that splitting occurs most easily.

10.7.3 Composites Reinforced with Short Fibres

A typical example is a metallic matrix reinforced with short carbon fibres, whiskers or particles of silicon carbide.

Damage results either from the growth of cavities in the matrix at the ends of the fibres or particles under traction, or from fracturing at the boundaries; it is followed by coalescence of these cavities, usually by shearing in the regions where the volume fraction of the reinforcement is high (Fig. 10.67).

The threshold at which an applied stress will initiate damage can be determined from the stress on the inclusions and especially on the boundaries, calculated using Eshelby's theory, given in Volume I, Sect. 2.7.3. It is usual to add these to the stresses resulting from differences in thermal expansion. It is often possible to assume that there is no chemical binding at the boundary, so that damage will start as soon as the applied stresses cancel the residual compressive stresses there; a profile of the damage surface can then be constructed, of which Fig. 10.68 is an example.

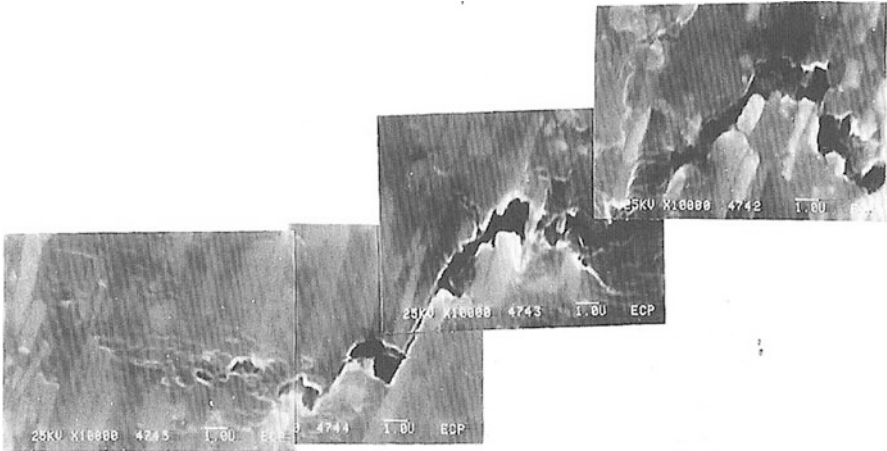
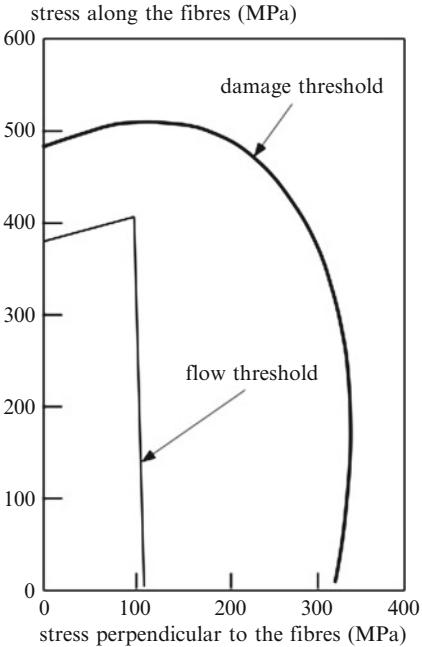


Fig. 10.67 Propagation of a crack in aluminium reinforced with silicon carbide whiskers

Fig. 10.68 Yield strength and damage threshold for aluminium reinforced with carbon fibres under biaxial stress



10.7.4 Criteria for Macroscopic Fracture

It is seldom possible to carry through calculations on the micromechanics scale so as to determine, by homogenisation, a macroscopic criterion for the rupture we have been discussing. A more arbitrary macroscopic criterion might be envisaged,

incorporating the effects of anisotropy and of hydrostatic stress, which latter would oppose the opening of cracks.

Taking first unidirectional composites under simple loading along the axes of anisotropy, the behaviour, and in particular the fracture load, can be approximated very simply by treating these as consisting of elements that work in parallel. It can then be assumed, for example, that if the fibres break then the matrix alone bears a tensile load in the direction of the length.

A simple criterion in this case would be to state that the stresses in the directions of the axes of the material must not exceed the fracture stresses for those directions, determined by simple tensile tests; this would give, for example, the following limits:

$$\begin{aligned} \sigma_{1R}^C < \sigma_1 < \sigma_{1R}^T & \text{ along the fibre axis} \\ \sigma_{2R}^C < \sigma_2 < \sigma_{2R}^T & \text{ perpendicular to the fibre axis} \\ \tau < \tau_c & \text{ in a shear test} \end{aligned}$$

where T, C refer to tension and compression respectively.

This gives a surface in stress space that has several sheets, with angles between these, which can pose computational problems.

Tsaï has proposed a criterion derived from that of Hill for anisotropic elastoplastic materials (see Volume I, Sect. 3.5.2.1b); the latter reads:

$$\begin{aligned} F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 \\ + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = 1 \end{aligned} \quad (10.34)$$

The von Mises criterion is a particular case of Hill criterion, for isotropic materials.

For loads restricted to being in the plane of the composite (10.34) becomes

$$\left(\frac{\sigma_1}{\sigma_{1R}}\right)^2 + \left(\frac{\sigma_2}{\sigma_{2R}}\right)^2 - \left(\frac{\sigma_1}{\sigma_{1R}}\right)\left(\frac{\sigma_2}{\sigma_{2R}}\right) + \left(\frac{\tau}{\tau_R}\right)^2 = 1 \quad (10.35)$$

in which there is no distinction between tension and compression. This restriction is removed by the following criterion due to Tsaï and Wu:

$$\begin{aligned} \frac{\sigma_1^2}{\sigma_{1R}^T \sigma_{1R}^C} + \frac{\sigma_2^2}{\sigma_{2R}^T \sigma_{2R}^C} + \frac{\tau^2}{\tau_R^2} + 2F_{12}\sigma_1\sigma_2 + \left(\frac{1}{\sigma_{1R}^T} - \frac{1}{\sigma_{1R}^C}\right)\sigma_1 \\ + \left(\frac{1}{\sigma_{2R}^T} - \frac{1}{\sigma_{2R}^C}\right)\sigma_2 = 1 \end{aligned} \quad (10.36)$$

Here F_{12} is a coupling term which can be determined by conducting a biaxial test with $\sigma_1 = \sigma_2 = \sigma$. If this gives a fracture stress σ_R , we have, from (10.36):

$$F_{12} = \frac{1}{2\sigma_R^2} \left[1 - \left(\frac{1}{\sigma_{1R}^T} - \frac{1}{\sigma_{1R}^C} + \frac{1}{\sigma_{2R}^T} - \frac{1}{\sigma_{2R}^C} \right) \sigma_R - \left(\frac{1}{\sigma_{1R}^T} \frac{1}{\sigma_{1R}^C} + \frac{1}{\sigma_{2R}^T} \frac{1}{\sigma_{2R}^C} \right) \sigma_R^2 \right] \quad (10.37)$$

Experience has shown that no serious error results from taking $F_{12} = 1/\sigma_{1R}^T \sigma_{1R}^C$, for the sake of simplicity.

10.8 Final Survey

A look at Fig. 10.21 allows us to judge the mechanical efficiency of non-metallic materials and to make comparisons with metallic alloys. Concrete, which has not been included, would be at the same level as LDPE. Considering this only map, non-metallic materials would appear less interesting than metals. Woods, taken in the TL or RL directions, are even outside the limits of the figure. Nevertheless, materials like HDPE or metallic glasses approach the performances of metallic alloys in view of fracture toughness versus fracture resistance. However, other properties are essential according to applications. For instance, specific resistance, that is the mechanical resistance divided by the density, is very important in energy saving. This would put materials like polymers, woods, even ceramics, in a much more favourable position. Composite reinforced materials, which are not included in Fig. 10.21, offer also very good solutions. Ceramic materials are of course much used when high temperature properties are needed. Aesthetic considerations can make preferable sometimes materials like glass which might not be the best choice from a purely mechanical point of view.

This last chapter, incomplete as it is, tried to provide some ideas about the mechanical behaviour of various non-metallic materials so that they should not be forgotten in design, even if traditional solutions would tend to overlook other alternatives.

Lastly, this chapter attempted again to stress the importance of the microstructures and their relations with mechanical properties. This is of great importance for safe design.

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Appendix A: Diffusion Coefficients

The values of self-diffusion in a number of metals are given in Table A1 (Peterson 1968), those of the heterodiffusion in nickel are reported in Table A2 (Adda and Philibert 1966), and those of the self-diffusion coefficients in strongly misoriented grain boundaries are given in Table A3 (Gibbs and Harris 1969). In the second column of Table A2 the concentration of the diffusing element in homogeneous systems is given in At %. In the third column of this table the concentration of the diffusing element which is most often a tracer in homogeneous systems is small and is noted ε . In diffusion couples C_D designates the concentration of the diffusing element (At %) for which the values of the diffusion coefficient D (D_0 and Q) are indicated in the other columns. The references where these data have been published can be found in these three books which give more details. Further details can be found in chapter 13 of *Smithells Metals Reference Book* (Gale and Totemeier 2004).

Table A1 Self-diffusion in metals (Peterson 1968)

Element	Q (kcal/mole)	D_0 (cm ² s ⁻¹)
Aluminium	28.750	0.035
Antimony c	48.000	56.0
⊥ c	35.800	0.1
Beryllium c	39.400	0.62
⊥ c	37.600	0.52
Cadmium c	18.200	0.05
⊥ c	19.100	0.10
Chromium	73.700	0.20
Cobalt (paramagnetic)	67.700	0.83
Copper	49.560	0.62
Germanium	68.500	7.8
Gold	41.700	0.091
Hafnium	38.700	0.0012
Indium c	18.700	2.7
⊥ c	18.700	3.7
Iron α (paramagnetic)	57.300	2.0
Iron γ	67.860	0.49
	64.500	0.18
Iron δ	57.000	1.9
Lead	26.060	1.37
Lithium	13.490	0.39
Magnesium c	32.200	1.0
⊥ c	32.500	1.5
Molybdenum	92.200	0.1
Nickel	68.000	1.9
Niobium	96.000	1.1
Palladium	63.600	0.21
Platinum	68.200	0.33
Potassium	9.750	0.31
Silicon	110.000	1,800
Silver	44.270	0.44
Sodium	10.090	0.145
Tantalum	98.700	0.124
Thallium α c	22.900	0.4
⊥ c	22.600	0.4
Thallium β	20.000	0.7
Thorium α	76.600	1.2
Tin β c	25.600	7.7
⊥ c	25.100	10.7
Titanium β	See below	
Tungsten	153.000	42.8
Uranium γ	26.500	0.00112
Vanadium 880–1,360 °C	73.650	0.36
1,360–1,830 °C	94.140	214.0
Zinc c	21.900	0.13
⊥ c	23.000	0.18
Zirconium	See below	
Titanium β: $D = 3.58.10^{-4} \exp (-31.200/RT) + 1.09 \exp (-60.000/RT)$		
Zirconium β: $D = 8.5.10^{-5} \exp (-27.700/RT) + 1.34 \exp (-65.200/RT)$		

Table A2 Heterodiffusion in nickel (Adda and Philibert 1966)

Addition element	Atomic composition %	C_D atomic %	Tracer	Temperature °C	D cm ² s ⁻¹	D_0 cm ² s ⁻¹	Q kcal/mole
Al		0.7		1,099–1,280		1.87	64
		0.08		800–920		1.1	59.5
Au	0	ϵ	Au ¹⁹⁸	900–1,100		2	65
		2		850–975		1.8×10^4	85.2
C	0	ϵ	C ¹⁴	600–900		0.08	33
	0	ϵ	C ¹⁴			0.13	34.5
	0.48	ϵ	C ¹⁴	600–1,400		0.12	32.9
				700–1,300		2.49	40
		0.17–0.12		700–1,300		2.48	40
Co	0	ϵ	Co ⁶⁰	1,193	1.71×10^{-10}		66.3
				1,095	3.42×10^{-11}		66.3
	0	ϵ	Co ⁶⁰	748–1,192		0.75	64.7
	0	ϵ	Co ⁶⁰	1,154–1,373		1.11	64.9
	0	ϵ	Co ⁶⁰	900–1,250		1.46	68.3
	5.2	ϵ	C ¹⁴	600–900		0.4	37
Cr	0	ϵ	Cr ⁵¹	900	1×10^{-12}		
				1,000	9×10^{-12}		66
				1,100	7×10^{-11}		
	0	ϵ	Cr ⁵¹	1,100–1,268		1.1	65.1
	0	ϵ	Cr ⁵¹	600–900		0.03	40.8
	0	ϵ	Cr ⁵¹	350–600		5.45×10^{-9}	13.7
	0.84	ϵ	C ¹⁴	600–900		0.15	34
	5.2	ϵ	C ¹⁴	600–900		0.3	37
Cu	0	ϵ	Cu ⁶⁴	1,054–1,359		0.57	61.7
	0	ϵ	Cu ⁶⁴	800–1,050		0.724	61
Fe	0	ϵ	Fe ⁵⁹	950–1,250		9×10^{-3}	50.7
	0	ϵ	Fe ⁵⁹	950–1,130		8.4×10^{-3}	51
	0	ϵ	Fe ^{55–59}	900–1,150		0.8	61
		0.1		1,100–1,300		0.34	67.5
Mo	0	ϵ	Mo ⁹⁰	900–1,200		1.6×10^{-2}	51
	3.14	ϵ	Co ⁶⁰	1,050–1,250		0.078	66
	1.8	ϵ	C ¹⁴	600–900		1	38
	1.8	1		1,150–1,400		3	68.9
W	0	ϵ	W ¹⁸⁵	1,100–1,275		1.13	71
	0	ϵ	W ¹⁸⁵	1,100–1,295		2	71.5
	1.7	ϵ	W ¹⁸⁵	1,100–1,295		2.2	73.1
	5.3	ϵ	W ¹⁸⁵	1,100–1,295		17	80.5
	9.2	ϵ	W ¹⁸⁵	1,100–1,295		1.4	74.5

Table A3 Self-diffusion coefficients in strongly misoriented grain-boundaries (Gibbs and Harris 1969)

Element	Structure	D_{0j} cm ² s ⁻¹	Q_j kcal/mole
Ag	FCC	0.03–0.12	20.2–21.5
		0.024	20.3
		0.063	18.9
Ni	FCC	0.018	26
		0.01	26.6
		0.07	27.4
		1.75	28.2
Pb	FCC	1.6	15.7
		1.6	4.7
Fe γ	FCC	2×10^{-4}	30.6
		3–5	39
		1.5	38.0
Fe α	BCC		45
		13	40
		0.014	33.7
		22.4	41.5
Cr	BCC		46
W	BCC	6.7	92
U γ	BCC	0.02	42.7
Zn	CPH	0.22–0.38	14.3–14.6
			12.3
Cd	CPH	1	13
Co	CPH	4	39
Te	Hexagonal	0.22	20
Sn	Quadratic		9.4
		0.06	9.55
Sb	Rhombohedral	3–6	22–23
U α	Orthorhombic	320	44.3

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